L-2/T-2/CE **Date: 27/03/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub: **CE 205** (Numerical Methods)

Full Marks: 140 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Explain the graphical method to find roots with relevant theorems. **(10)**

(b) Find the solution for the following equation using Extended Newton Raphson Method: $f(x) = \cos(x) - xe^{0.9x}$. **(13⅓)**

2. (a) Define Gauss quadrature. Also derive associated points and weighing coefficients for n = 3. **(10)**

(b) For the following data find **(13⅓)**

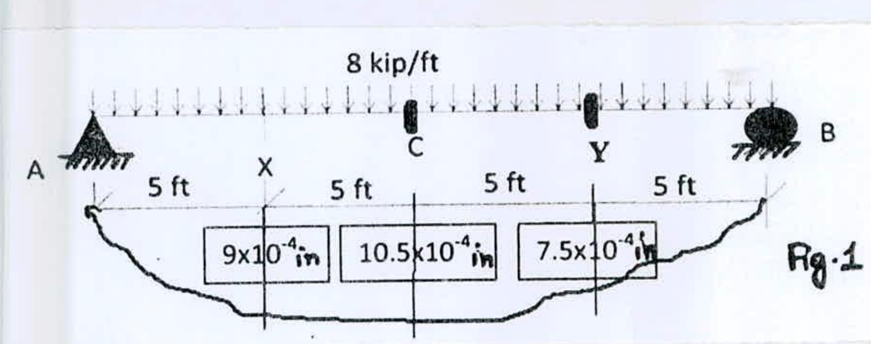
(i) General Polynomial equation

(ii) Function value for x = 0.295

x	0.21	0.25	0.34	0.41	0.49
$f(x)$	2.43	2.58	2.77	2.98	3.36

3. (a) Prove for Newton Raphson method that at each iteration absolute error is proportional to the square of the previous error and the convergence is quadratic. **(8)**

(b) For the loaded beam and deflected shape (Fig. 1), estimate slope, moment and shear forces at points X, C and Y. Consider E and I values for A to C to be 15×10^6 psi and 1200 in^4 and from C to B to be 10×10^6 psi and 1500 in^4, respectively. **(15⅓)**



4. (a) Derive the general expression for area calculation in Simpson's rule. **(10)**

(b) Estimate the value of the following integral using Romberg's quadrature **(13⅓)**

$$I = \int_0^5 \frac{1}{3+x^3}\,dx$$

Contd P/2

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) It has been found that the mass-transfer coefficient of dissolved oxygen K_L (m/d) is related to a river's mean water velocity U (m/s) and depth H (m) by the following equation: **(15)**

$$K_L = a_0 U^{a1} H^{a2}$$

The following data was collected in a laboratory flume:

U	0.5	2	10	0.5	2	10	0.5	2	10
H	0.15	0.15	0.15	0.3	0.3	0.3	0.5	0.5	0.5
K_L	0.48	3.9	57	0.85	5	77	0.8	9	92

Use these data and apply least squares regression to evaluate the constants a_0, a_1 and a_2

(b) Explain why 'appropriate error' is used instead of 'true error' in most numerical computations. Also, explain the difference between initial value problems and boundary value problems with examples. **(8 ⅓)**

6. (a) Solve the following system of linear equations using LU decomposition: **(12)**

$$-8x + y - 2z = -20$$
$$2x - 6y - z = -38$$
$$-3x - y + 7z = -34$$

(b) Determine the matrix inverse of the system of linear equations from the previous problem using LU decomposition. **(11 ⅓)**

7. (a) Use zero to 4th order Taylor series expansion to predict $f(2.5)$ for **(11)**

$$f(x) = \ln(x)$$

using a base point $x = 1$. Compute the true percent relative error (ε_t) for each approximation.

(b) Solve the following equation for $y(0.4)$ **(12 ⅓)**

$$10\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 6x = 0$$

using a step size of 0.2 with $y(0) = 1$ and $y'(0) = 0$. Use Mid-point method.

Contd P/3

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8. (a) Suppose the position of a falling object is governed by the following differential equation, **(11)**

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} - g = 0$$

where, c = drag coefficient = 12.5 kg/s, m = mass = 70 kg, g = 9.81 m/s^2. For the boundary conditions: $x(0) = 0$ and $x(12) = 500$ and step-size, $\Delta x = 2$, write down the finite difference equations (in matrix form) to solve the equation for position of the falling object. No need to solve the problem.

(b) Solve the equation $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 2x^2y^2$ over the square domain $0 \le x \le 3$ and $0 \le y \le 3$. Given the boundary conditions as shown in Figure 2 below, find the temperatures T_1, T_2, T_3, T_4. **(12⅓)**

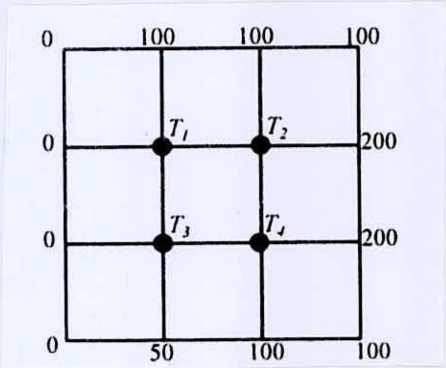

Figure – 2

L-2/T-2/CE **Date : 07/04/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub : **CE 207** (Applied Mathematics for Engineers)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols used have their usual meaning and interpretation. Assume reasonable value for any missing data.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) What is the Gibb's Phenomenon? What is its significance in engineering applications? **(5)**

(b) Is the function shown in Fig. 1 a periodic function? Is it an even or old function? Can we express the function as a Fourier Series? Why and how? If yes, please express the function as a Fourier Series and justify the values of the coefficients of the series. **(15)**

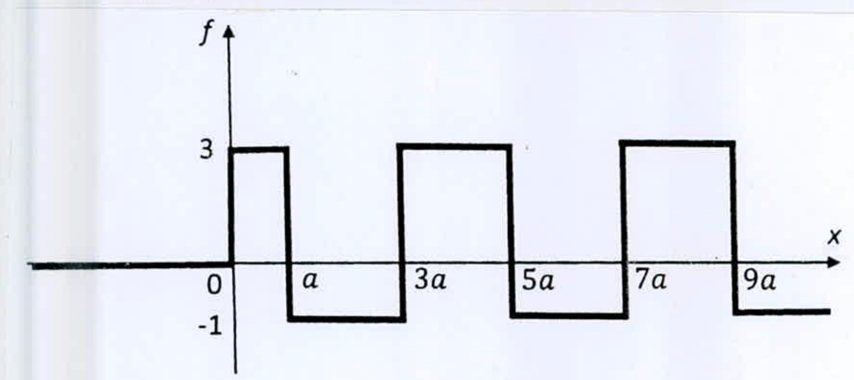

Fig. 1

(c) A rigid body A (Fig. 2) weighing 20 lb. is resting on a 45° inclined plane or which $f = 0.1$ (kinetic friction). A horizontal dynamic force $Q = 10$ lb. acts on the body. Time history of Q is shown in Fig. 3. If the body starts from rest, what is its displacement after 5 sec.? **(15)**

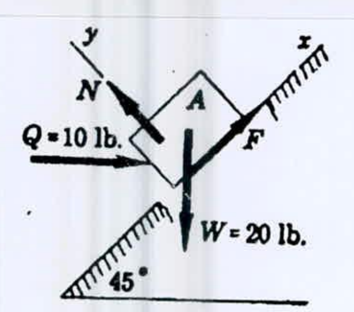

Fig. 2

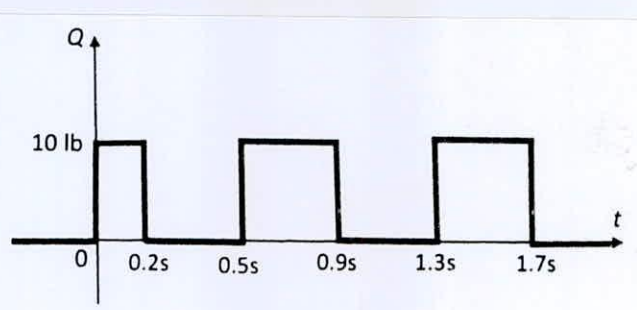

Fig. 3

2. (a) Determine the inverse Fourier transform of the following functions: **(15)**

(i) $\dfrac{1}{i\omega-\omega^2}$

(ii) $-\sqrt{\pi}(\omega-2)^2 e^{\frac{(\omega-2)^2}{4}}$

(iii) $-\dfrac{d^2}{d\omega^2}\left(e^{-3i\omega}\sin 2\omega\right)$

Contd P/2

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Cont... Q. No. 2

(b) The governing equation of a vibrating string subjected to a concentrated force is given below. Solve the equation using Fourier transform. **(20)**

$$\frac{d^2\psi}{dx^2} + \beta^2\psi = \delta(x-\xi)$$

3. (a) What is Dirichlet's discontinuous factor? Plot the function, $\int_0^{\infty} e^{-|x|} \frac{\cos wx \sin w}{w} dw$ **(20)**

and determine the Fourier integral function.

(b) The general solution of a Legendre's ODE $[(1-x^2)y'' - 2xy' + n(n+1)y = 0]$ can be written as $y(x) = a_0 y_{even} + a_1 y_{odd}$

Where,
$y_{even} = 1 - \frac{(n)(n+1)}{2!}x^2 + \frac{(n-2)n(n+1)(n+3)}{4!}x^4 - \frac{(n-4)(n-2)n(n+1)(n+3)(n+5)}{6!}x^6 + \cdots$ and
$y_{odd} = x - \frac{(n-1)(n+2)}{3!}x^3 + \frac{(n-3)(n-1)(n+2)(n+4)}{5!}x^5 - \frac{(n-5)(n-3)(n-1)(n+2)(n+4)(n+6)}{7!}x^7 + \ldots$

Show that, for n = 1 $P_1(x) = x$ and $y_{even} = 1 - \frac{1}{2}x \ln\frac{1+x}{1-x}$ **(15)**

4. (a) Under what circumstance(s) $y = \sum_{m=0}^{\infty} a_m (x-x_0)^m$ could NOT be a valid power series solution of the 2nd order ODE $y'' + p(x)y' + q(x)y = r(x)$? Under such circumstance(s) what condition(s) need to be satisfied for $y = \sum_{m=0}^{\infty} a_m (x-x_0)^{m+r}$ to be the valid power series solution of the same ODE? **(11)**

(b) Apply appropriate power series method to the following Bessel's equation and answer the following questions. Show all the relevant steps. **(24)**

$$x^2 y'' + xy' + \left(x^2 - \frac{4}{49}\right)y = 0$$

(i) Derive the indicial equation and calculate the two roots of the indicial equation.

(ii) Derive a formulae that relates the coefficients of the power series expansion (often referred to a recursive formulae) and write down the recursive formulae for the two roots of the indicial equation.

(iii) Comment on the odd values of the coefficients (i.e., a_1, a_3, a_5, ...) from the formulae derived in part (ii).

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) If X_1, X_2, …, X_n are independent Normal random variables with each having the same parameters μ and σ, determine the maximum likelihood estimators of μ and σ (Derive expressions). **(7)**

(b) An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at sometime within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a non-accident-prone person. **(14)**

(i) If we assume that 30 percent of the population is accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

(c) Sample of 10 grains of metallic sand taken from a large sand heap has following lengths (mm): 2.2, 3.4, 1.6, 0.8, 2.7, 3.3, 1.6, 2.8, 2.5 and 1.9. It is known that the size of an individual particle will have an approximate lognormal distribution. Estimate the percentage of sand grains with length between 2 and 3 mm. **(14)**

6. (a) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email? **(15)**

(b) Civil Engineers believe that W, the amount of weight (in units of 1000 pounds) that a certain span of a bridge can withstand without structural damage resulting, is normally distributed with mean 400 and standard deviation 40. Suppose that the weight (again, in units of 1000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. How many cars would have to be on the bridge span for the probability of structural damage to exceed 0.1? **(10)**

(c) Daily probability that a major earthquake occurs $P[E] = 10^{-5}$. Probability that premonitory event A or B occurs given that major earthquake occurs is $P[A|E] = P[B|E] = 0.1$. Probability that premonitory event A or B occurs given that major earthquake does not occur is $P[A|E^c] = P[B|E^c] = 0.001$ **(10)**

(i) Determine the probability of a major earthquake, given that premonitory event A is observed.

(ii) Also, determine the probability of a major earthquake, given that both premonitory events A and B are observed at the same time.

Contd ………. P/4

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7. (a) A traffic surveyor records the number of cars that approach an intersection. He finds that an average of 1.6 cars approach the intersection in every minute. What is the probability that 3 or more cars will approach the intersection within a minute? **(6)**

(b) The managers at Hunter Chemical Company claim that their major product contains on average μ = 4.0 fluid ounces of caustic material per gallon. They further state that the distribution of caustic material is normal with a standard deviation of σ = 1.3 fluid ounces. If there is too much caustic material, the product will be dangerous. A government inspector is brought in to test the product. She randomly selects a sample of 100 gallon-size containers of the product and finds that their mean weight of caustic materials is $\bar{X}$ = -4.5 fluid ounces per gallon. **(14)**

(i) What are the null and alternative hypotheses that the inspector is exploring? Explain your reasoning?

(ii) Conduct the hypothesis test using test statistics and state your conclusion.

(c) If the occurrences of high winds and earthquake are unrelated, and if, at a location the probability of a "high" wind occurring throughout any single minute is 10^{-5} and the probability of a "moderate" earthquake during any single minute is 10^{-8}. **(15)**

(i) Find the probability of the joint occurrence of the two events during any minute. Building codes do not require the engineer to design the building for the combined effects of these loads. Is this reasonable?

(ii) Find the probability of the occurrence of one or the other or both during any minute. For rare events, i.e. events with small probabilities of occurrence, the engineer frequently assumes $P[A \cup B] \approx P[A] + P[B]$. Comment.

(iii) If the events in succeeding minutes are mutually independent, what is the probability that there will be no moderate earthquakes in a year near this location? What is the probability in 10 years?

8. (a) Check the exactness of the following differential equation and find the particular solution for the given initial condition. **(18)**

$$e^{2x}(2\cos y\,dx - \sin y\,dy) = 0$$
$$\text{Initial condition}: y(0) = 0$$

(b) What do you understand by the following operations as they apply to power series method for solving ordinary differential equation? Explain with one example for each operation. **(9)**

(i) Term wise differentiation

(ii) Term wise addition

(iii) Vanishing of all coefficients

(c) Determine the radius of convergence of the following series. **(8)**

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{k^m} x^{2m}$$

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Table of Fourier Transforms

	$f(x)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$
1.	$\dfrac{1}{x^2+a^2} \quad (a>0)$	$\dfrac{\pi}{a}e^{-a\lvert\omega\rvert}$
2.	$H(x)e^{-ax} \quad (\mathrm{Re}\,a>0)$	$\dfrac{1}{a+i\omega}$
3.	$H(-x)e^{ax} \quad (\mathrm{Re}\,a>0)$	$\dfrac{1}{a-i\omega}$
4.	$e^{-a\lvert x\rvert} \quad (a>0)$	$\dfrac{2a}{\omega^2+a^2}$
5.	e^{-x^2}	$\sqrt{\pi}e^{-\omega^2/4}$
6.	$\dfrac{1}{2a\sqrt{\pi}}e^{-x^2/(2a)^2} \quad (a>0)$	$e^{-a^2\omega^2}$
7.	$\dfrac{1}{\sqrt{\lvert x\rvert}}$	$\sqrt{\dfrac{2\pi}{\lvert\omega\rvert}}$
8.	$e^{-a\lvert x\rvert/\sqrt{2}}\sin\left(\dfrac{a}{\sqrt{2}}\lvert x\rvert+\dfrac{\pi}{4}\right) \quad (a>0)$	$\dfrac{2a^3}{\omega^4+a^4}$
9.	$H(x+a)-H(x-a)$	$\dfrac{2\sin\omega a}{\omega}$
10.	$\delta(x-a)$	$e^{-i\omega a}$
11.	$f(ax+b) \quad (a>0)$	$\dfrac{1}{a}e^{ib\omega/a}\hat{f}\left(\dfrac{\omega}{a}\right)$
12.	$\dfrac{1}{a}e^{-ibx/a}f\left(\dfrac{x}{a}\right) \quad (a>0,\ b\text{ real})$	$\hat{f}(a\omega+b)$
13.	$f(ax)\cos cx \quad (a>0,\ c\text{ real})$	$\dfrac{1}{2a}\left[\hat{f}\left(\dfrac{\omega-c}{a}\right)+\hat{f}\left(\dfrac{\omega+c}{a}\right)\right]$
14.	$f(ax)\sin cx \quad (a>0,\ c\text{ real})$	$\dfrac{1}{2ai}\left[\hat{f}\left(\dfrac{\omega-c}{a}\right)-\hat{f}\left(\dfrac{\omega+c}{a}\right)\right]$
15.	$f(x+c)+f(x-c) \quad (c\text{ real})$	$2\hat{f}(\omega)\cos\omega c$
16.	$f(x+c)-f(x-c) \quad (c\text{ real})$	$2i\hat{f}(\omega)\sin\omega c$
17.	$x^n f(x) \quad (n=1,2,\ldots)$	$i^n\dfrac{d^n}{d\omega^n}\hat{f}(\omega)$

Linearity of transform and inverse:

18.	$\alpha f(x)+\beta g(x)$	$\alpha\hat{f}(\omega)+\beta\hat{g}(\omega)$

Transform of derivative:

19.	$f^{(n)}(x)$	$(i\omega)^n\hat{f}(\omega)$

Transform of integral:

20.	$f(x)=\int_{-\infty}^{x} g(\xi)\,d\xi$, where $f(x)\to 0$ as $x\to\infty$	$\hat{f}(\omega)=\dfrac{1}{i\omega}\hat{g}(\omega)$

Fourier convolution theorem:

21.	$(f*g)(x)=\int_{-\infty}^{\infty} f(x-\xi)g(\xi)\,d\xi$	$\hat{f}(\omega)\hat{g}(\omega)$

TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A2 *Values of* $\chi^2_{\alpha,n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	13.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.484	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.772	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other chi-square probabilities:
$\chi^2_{.9,9} = 4.2$ $P(\chi^2_{16} < 14.3) = .425$ $P(\chi^2_{11} < 17.1875) = .8976.$

TABLE A3 *Values of* $t_{\alpha,n}$

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t probabilities:
$P(T_3 < 2.541) = .9825$ $P(T_8 < 2.7) = .9864$ $P(T_{11} < .7635) = .77$ $P(T$[illegible]$ < .941) = .81$ $P(T$[illegible]

L-2/T-2/CE **Date : 21/03/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub : **CE 213** (Mechanics of Solids II)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols and notations have their usual meanings.

SECTION – A

There are **FIVE** questions in this section. Answer **ALL** questions.

1. **Answer either (a) or (b)** **(21)**

(a) For the beam shown in Fig. 1. Determine the vertical deflection and slope at point A following direct integration method.

(b) For the beam shown in Fig. 2, determine the vertical deflection and slope at point B following direct integration method.

2. **Answer either (a) or (b)** **(21)**

(a) For the beam shown in Fig. 3, determine the vertical deflection at point A following moment-area method. In Fig. 3, consider a=12 ft. P= 25 kip, and EI=1000 k-ft^2.

(b) For the beam shown in Fig. 4, determine the vertical deflection at point A following moment-area method. In Fig. 4, consider a=12 ft. M_1 = 25 kip-ft, and EI=1000 k-ft^2.

3. **Answer either (a) or (b)** **(21)**

(a) If each cable segment can support a maximum tension of 75 lb, determine the largest load P that can be applied for the cable shown in Fig. 5.

(b) For the parabolic cable shown in Fig. 6, Young's modulus, E=30,000 ksi and allowable tensile stress = 40 ksi. Determine the following: (i) Horizontal component of the cable tension, H, (ii) mid-span sag, h, (iii) equation of the shape of the cable w.r.t left support, (iv) maximum cable-tension, T_{max}, (v) stretch, ΔS and (vi) original cable length, S_0.

4. **Answer either (a) or (b)** **(21)**

(a) An air chamber for a pump, the sectional side view of which is shown in Fig. 7 (dimensions are in mm), consists of two pieces. Compute the number of 19-mm dia. bolts (net area 195 mm^2) required to attach the chamber to the cylinder at planes A-A and B-B. The allowable tensile stress in the bolts is 50 MPa, and the water and air pressure is 1.5 MPa.

(b) Consider a cylindrical pressure vessel made of steel plate (Youngs modulus = 200 GPa, Poisson's ratio = 0.03) as shown in Fig. 8 where measurements are shown in millimeters. The internal gas pressure is 1.20 MPa. Calculate (i) the longitudinal and hoop stresses on the cylindrical body, (ii) the change in diameter of the cylinder and (iii) change in the length of the cylindrical part.

Contd P/2

CE 213

5. Based on elastic strain energy principle, determine the vertical deflection of the truss at point D as shown in Fig. 9. Cross sectional area of all the members is 2.5 in^2 and Young's modulus is 29000 ksi. **(21)**

SECTION – B

There are **FIVE** questions in this section. Answer **ALL** questions.

6. **Answer either (a) or (b)** **(21)**

(a) A tilted simple supported beam is to span 15 ft and is to carry a uniformly distributed load of 1000 lb/ft including own weight. The cross section of the beam is shown in Fig. 10(a). Determine required dimension of the beam so that the maximum stress due to bending does not exceed 1500 psi. Also, locate neutral axis and show its position on sketch.

(b) A full sized 2 in by 4 in cantilever projects 5-ft from a wall in a tiled position as shown in Fig. 10(b). At the free end a vertical force of 100 lb is applied which acts through centroid of the section. Determine maximum flexural stress at built-in end and locate neutral axis. Neglect self wt of beam.

7. **Answer either (a) or (b)** **(21)**

(a) Determine the normal stress developed at corners A, B, C and D of the column loaded as shown in Fig. 11(a).

(b) A cantilever beam has a rectangular cross section and subjected to the loading as shown in Fig. 11(b). Determine the normal stress at A and B.

8. **Answer either (a) or (b)** **(21)**

(a) The state of stress at a point is shown in Fig. 12(a). Using Mohr's Circle, determine (i) principal stress (ii) the maximum in plane shear stress and corresponding normal stress. Specify the orientation of the element in each case. Show the result on each element.

(b) The state of stress at a point is shown in Figure 12(b). Using Mohr's Circle, determine (i) principal stress, (ii) the maximum in plane shear stress and corresponding normal stress. Specify the orientation of the element in each case. Show the result on each element.

9. **Answer either (a) or (b)** **(21)**

(a) The A-36 steel column can be considered pinned at its top and bottom and braced against its weak axis at mid height, Fig. 13(a). Determine the maximum allowable force P that the column can support without buckling. Apply a F.S. = 1.5 against buckling.

Given: $A = 7.4 \times 10^{-3}\ m^2$, $I_x = 87.3 \times 10^{-6}\ m^4$, $I_y = 18.8 \times 10^{-6}\ m^4$.

(b) The pin-connected aluminum-alloy frame section carries a concentrated force F. Assuming buckling can only occur in the plane of frame. The orientation and support condition of the frame is shown in Fig. 13(b). Determine the value of F that this frame can support without buckling of its members. Given: $E = 10 \times 10^6$ psi and both members have 4 in by 4 in cross section.

10. A cantilever rectangular bar is subjected to the force of 10 kip as shown in Figure 14. Using the stress transformation equation, determine the principal stress at point 'A'. **(21)**

Contd P/3

Parabolic Cable Formulae (symbols and notations have their usual meanings)

$$y = \frac{4hx}{L^2}(x - L) + x\tan\gamma$$

$$T_{max./min.} = H(1 + 16\theta^2 + \tan^2\gamma \pm 8\theta\tan\gamma)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{8\theta x}{L} - 4\theta + tan\,\gamma$$

$$S = \frac{L\sec\gamma}{2}\left(1 + \frac{16\theta^2}{\sec^4\gamma}\right)^{1/2} + \frac{L\sec^3\gamma}{8\theta}\ln\left[\frac{4\theta}{\sec^2\gamma} + \left(1 + \frac{16\theta^2}{\sec^4\gamma}\right)^{1/2}\right]$$

$$T_{av} = \frac{HL}{S}\left[1 + \frac{16}{3}\theta^2 + tan^2\gamma\right]$$

Fig.1

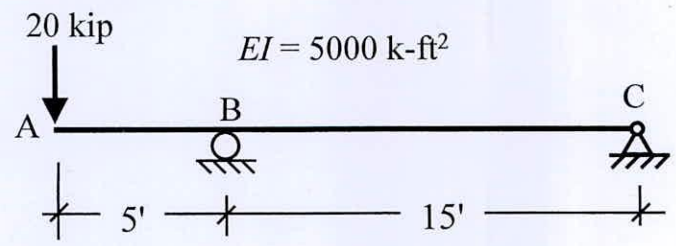

Fig.2

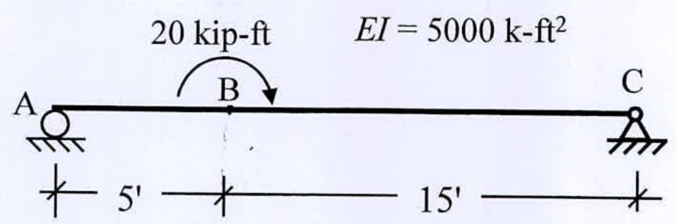

Fig.3

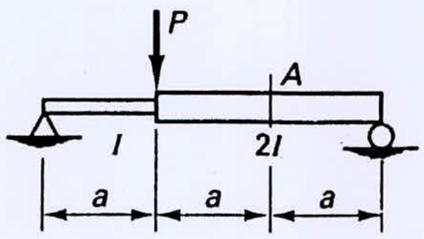

Fig.4

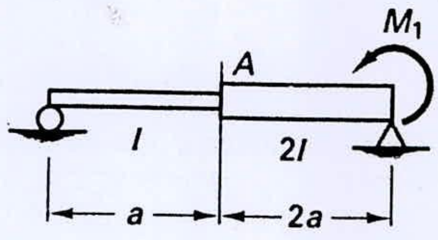

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Fig.5

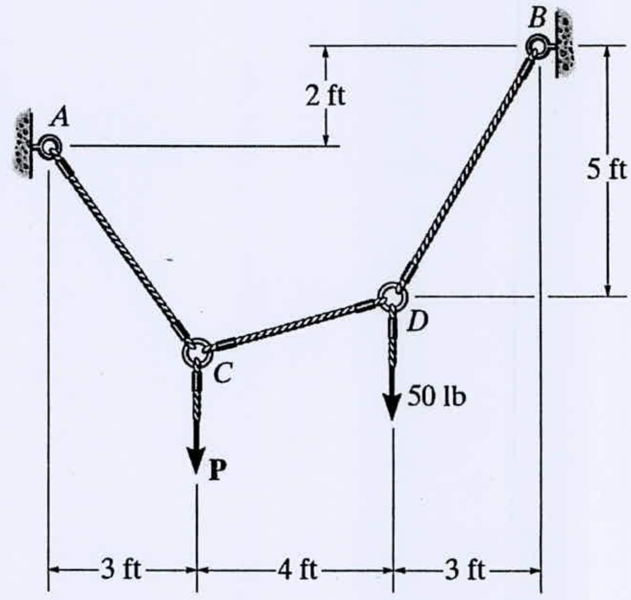


Fig.6

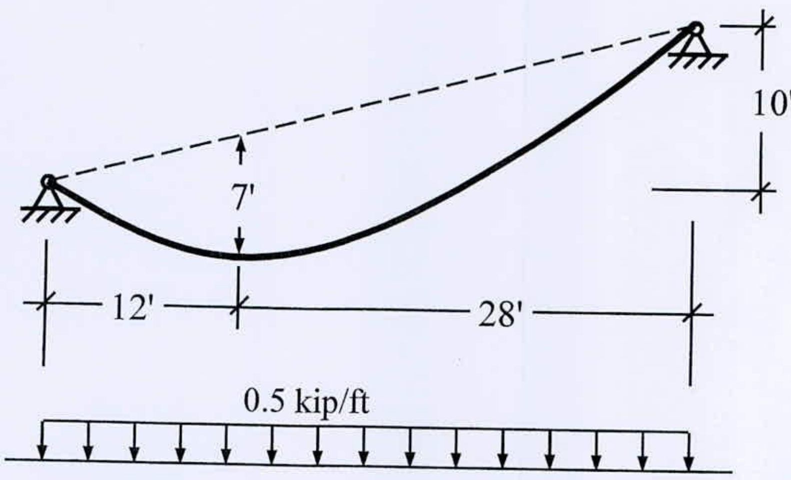


Fig.7

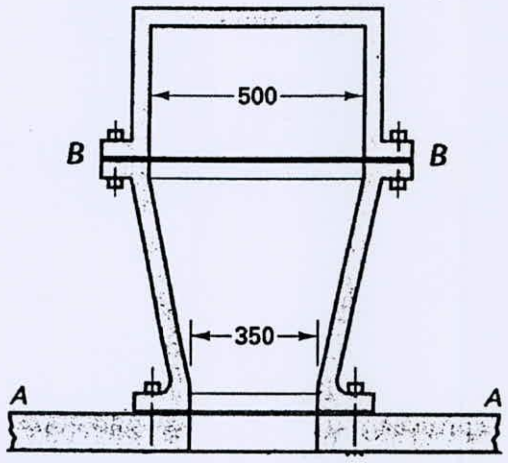


Fig.8

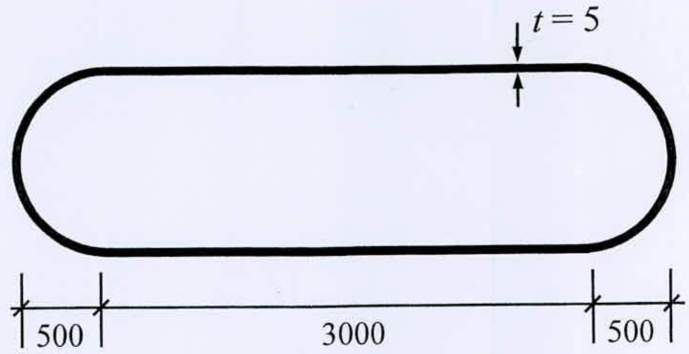
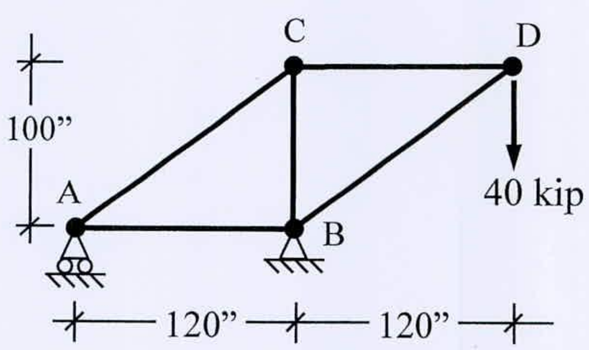


Fig.9



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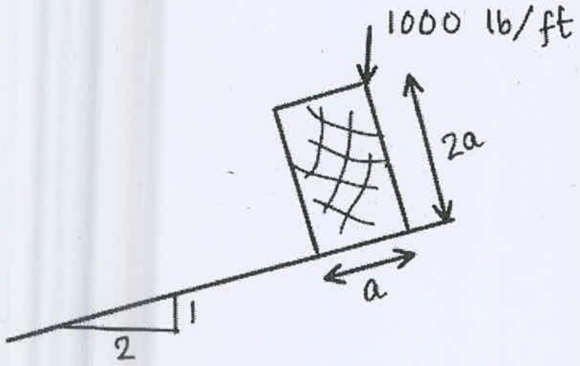

Fig. 10(a)

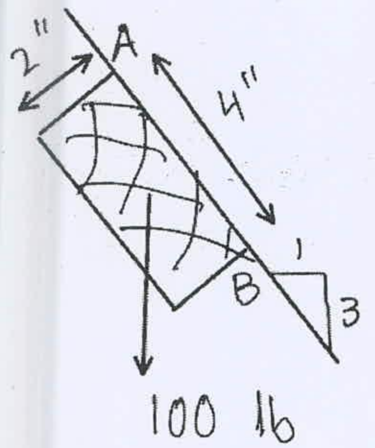

Fig. 10(b)

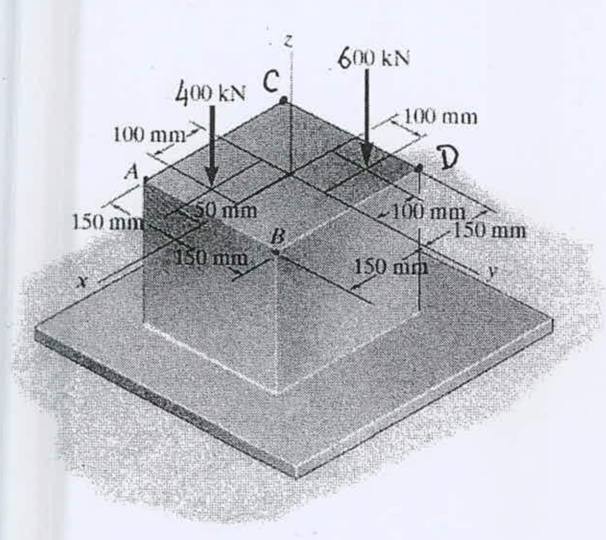

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Fig. 11(a)

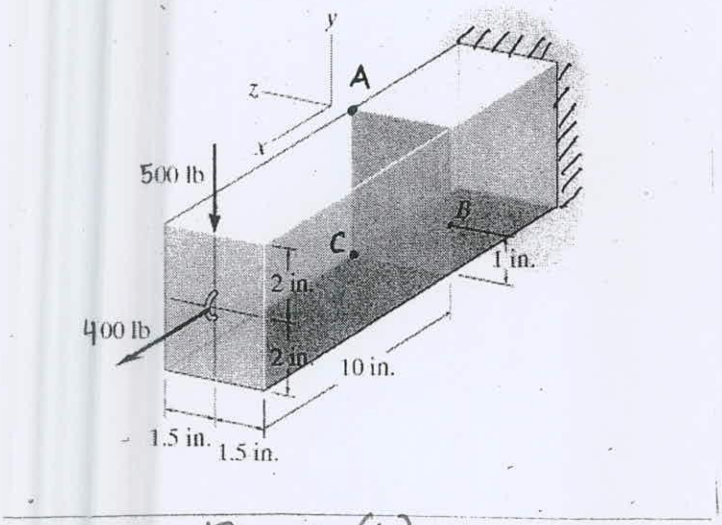


Fig. 11(b)

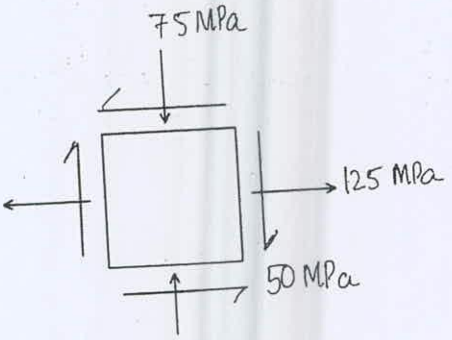


Fig. 12(a)

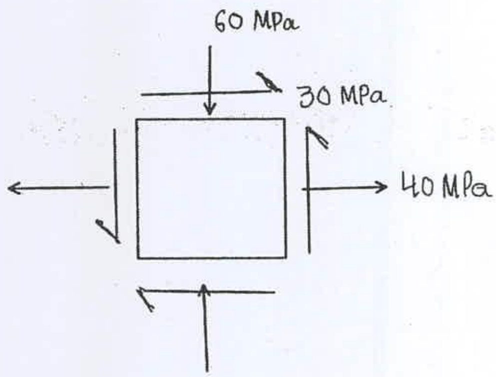


Fig. 12(b)

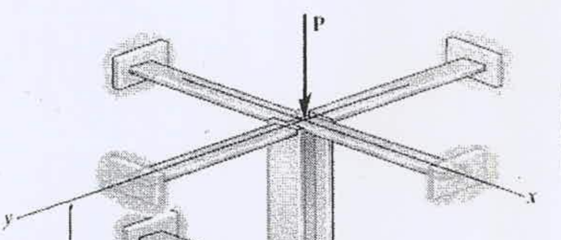


Fig. 13(a)

Contd... P/7

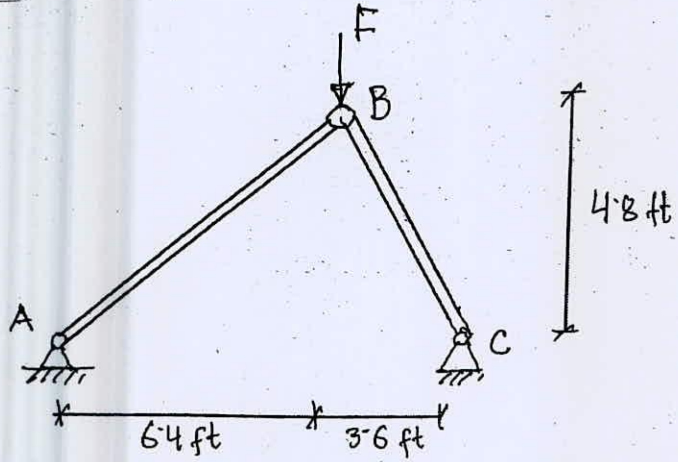


Fig. 13(b)

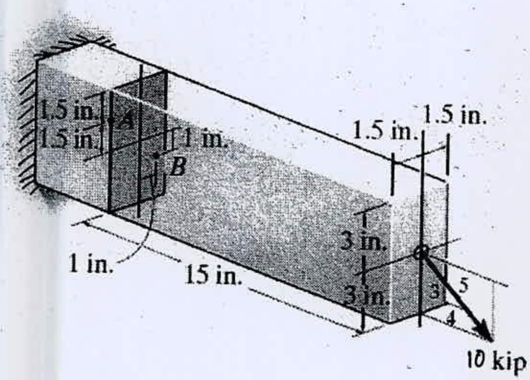


Fig. 14

—— X ——

L-2/T-2/CE **Date: 16/04/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub: **WRE 211** (Fluid Mechanics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Distinguish between the following items: **(15)**

(i) Dynamic viscosity and kinematic viscosity

(ii) Hydraulically rough and smooth boundary

(iii) Simple manometer and differential manometer

(b) Water is moving through a pipe. The velocity profile at a section is shown in figure 1 and is given mathematically as $v = 0.25\frac{\beta}{\mu}(\frac{d^2}{4} - r^2)$, where v = velocity of water at any position r, β = a constant, μ = viscosity of water, d = pipe diameter, r = radial distance from the centerline. **(12)**

(i) What is the shear stress at the wall of pipe due to the flowing water?

(ii) What is the shear stress at a position r = d/4?

(iii) If the given profile persists a distance L along the pipe, what drag is induced on the pipe by water in the direction of flow over the distance?

(c) A cylindrical tank (diameter = 10.0 m and depth = 5.0) contains water at 25°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Here, specific weight of water at 25°C and 60°C are 9.78 kN/m^3 and 9.64 kN/m^3 respectively. **(8)**

2. (a) Define absolute pressure, vacuum pressure and gage pressure. How are they interrelated? **(6)**

(b) Define buoyance. Write down the criteria for stability of a floating body with neat sketch. **(9)**

(c) The canal section shown in figure 2 runs 40.0 m perpendicular to the paper. Determine the location of center of pressure and the magnitude of the resultant hydrostatic force. **(12)**

(d) For the configuration shown in figure 3, calculate the weight of the piston if the gauge pressure reading is 70.0 kPa. **(8)**

3. (a) Write short notes on **(9)**

(i) Reynolds number

(ii) Surface tension

(iii) Minor losses in pipe flow

Contd P/2

WRE 211/CE

Contd... Q. No. 2

(b) The tank in Figure 4 is filled with oil of sp. gr. 0.8 and accelerated as shown. There is a small opening in the tank A. Determine the acceleration a_x required to make the pressure at B zero. **(11)**

(c) The flow into and out of a two loop pipe system are as shown in figure 5, determine the flow in each pipe by Hardy-Cross method. Use two trials. **(15)**

4. (a) Show that in circular pipe shear stress varies linearly with radius. **(7)**

(b) Calculate the discharge through the syphon in Figure 6 without the conical diffuser. Use friction factor, f = 0.016 for the pipe and minor loss co-efficients of 1.0 for both the entrance and exit and 0.9 for each elbow. **(10)**

(c) Three parallel pipes carry water at standard temperature and pressure. The pipe data are: **(18)**

Pipe	Length (m)	Diameter (m)	Roughness height (mm)
1	100	8	0.003
2	150	6	0.002
3	80	4	0.005

Calculate the flow rate for a total head loss of 20.3 m. Neglect all minor losses. Use Moody diagram (Figure 7) for finding pipe's friction factor.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

Students will attach the figure 12 with their answer scripts.

5. (a) Derive the mathematical relationship between stream function and equipotential line for solving practical engineering problems e.g.., flow around a body, spillway, seepage analysis, flow in conduits, etc. **(9)**

(b) A fluid has velocity components of $u = (y^2 - x^2)$ m/s and $v = (2xy)$ m/s, where x and y are in meters. If the pressure at A(3 m, 2m) is 600.0 kPa, determine the pressure at point B(1 m, 3m). Also, what is the potential function (ϕ) for the flow? Consider $\gamma = 9.81$ kN/m^3 and elevation of both points are on same level. **(20)**

(c) Suppose, you have been visited the Pohela Boishakh Fair and started riding on a Ferris Wheel (nagordola), how can you correlate your circular movement on the Ferris wheel with fluid particle movements? Is it rotational or irrotational, why? **(6)**

6. (a) What is Navier-Stokes Equation? Why it is considered the backbone of computational fluid dynamics (CFD)? **(6)**

(b) Water is to be pumped from one large, open tank to a second large, open tank as shown in figure 8. The pipe diameter throughout is 8.0 inch, and the total length of the pipe between the pipe entrance and exit is 350.0 ft. Minor loss coefficients for the entrance, exit, and the elbow are shown, and the friction factor for the pipe can be assumed constant and equal to 0.02. What would be the flowrate between the tanks? Do you think this pump would be a good choice? The performance characteristics curve of the centrifugal pump is attached at the end. **Please attach the performance characteristics curve with your answer script. (Figure 12).** **(20**

(c) Write short notes on: Kinetic-energy correction factor, (ii) Momentum correction factor, and (iii) Streaklines. **(9)**

7. (a) Water enters at A with a velocity of 10.0 m/s and pressure of 80.0 kPa (Figure 9). If the velocity at C is 12.0 m/s, determine the horizontal and vertical components of the resultant force that must activity on the transition to hold it in place. Neglect the size of the transition. State, what would happen if we have to consider the transition size and friction in the pipe system for solving the problem? **(20)**

(b) Mathematically prove that in a real fluid flow, the Rotation at a point is half of the Vorticity considering the necessary assumption. **(10)**

(c) Why the velocity through a propeller disk is the numerical average of the velocities at some distance ahead and behind the propeller? **(5)**

8. (a) The pump draws water from the large reservoir A and discharge it at 0.4 m^3/s at C (Figure 10). If the diameter of the pipe is 200.0 mm, determine the power the pump delivers to the water. Consider frictional head loss inside the pump is 0.7 m and a friction loss of 1.1 m for every 5.0 m length of pipe. The pipe extends 3.0 m from the reservoir to B, then 12.0 from B to C. Construct the EGL and HGL for the pipe considering a datum at B. **(20)**

(b) The nozzle in Figure 11 has a diameter of 50.0 mm. If it discharges water with a velocity of 25.0 m/s against the fixed blade, determine the horizontal force exerted by the water on the blade. Assume the blade divides the water evenly at angles of $\theta_1 = 35^\circ$ and $\theta_2 = 55^\circ$. **(15)**

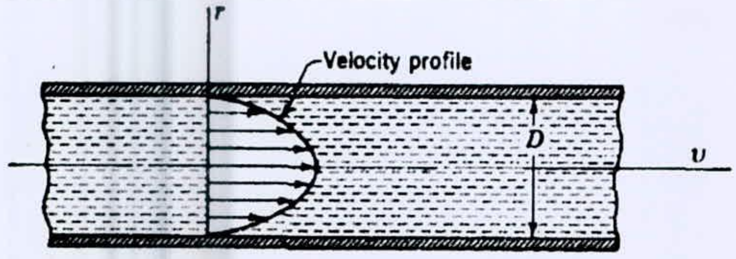


Figure 1 for Q 1 (b)

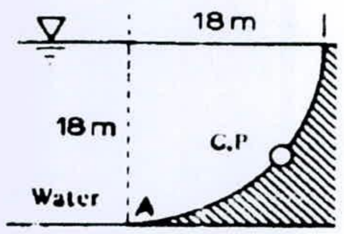


Figure 2 for Q 2 (c)

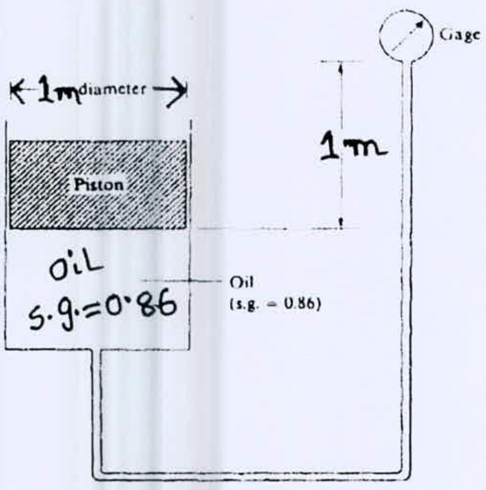


Figure 3 for Q 2 (d)

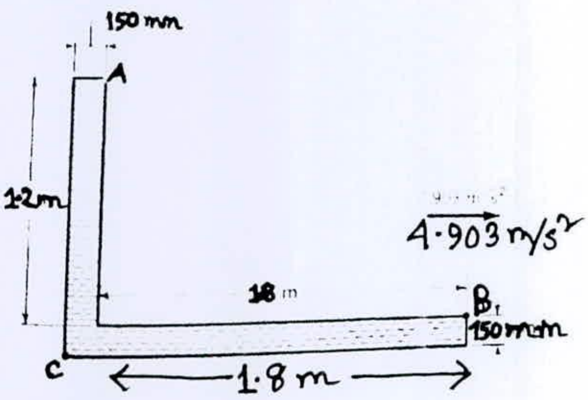


Figure 4 for Q 3 (b)

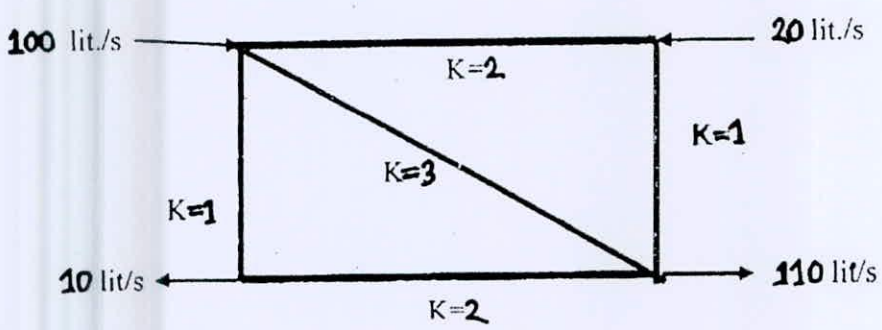


Figure 5 for Q 3 (c)

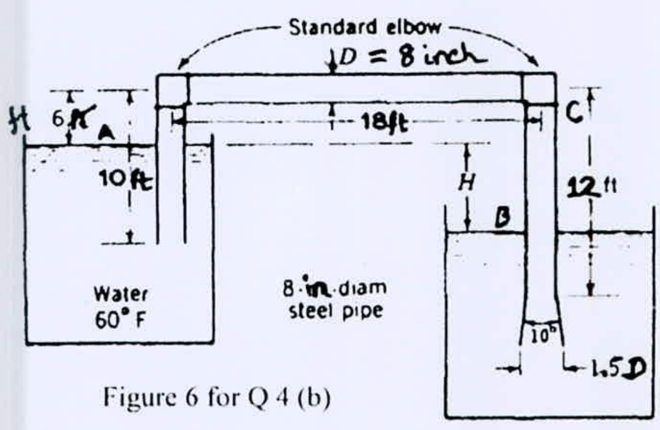


Figure 6 for Q 4 (b)

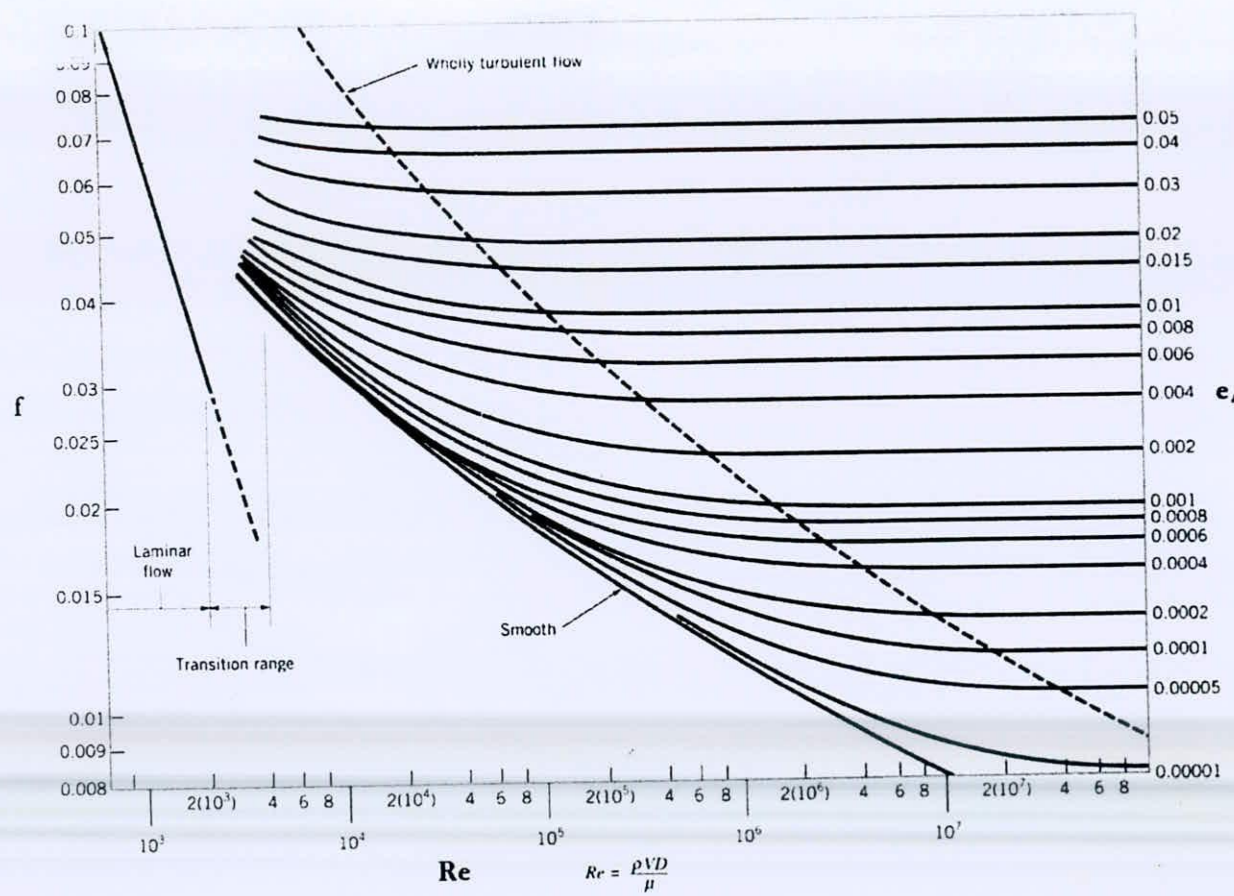


Figure 7 for Q4(c)

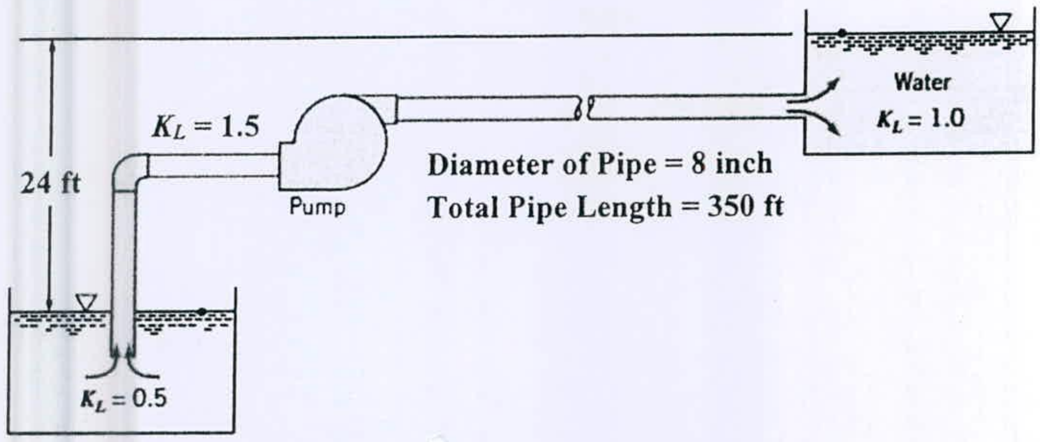


Figure 8 for Question 6 (b)

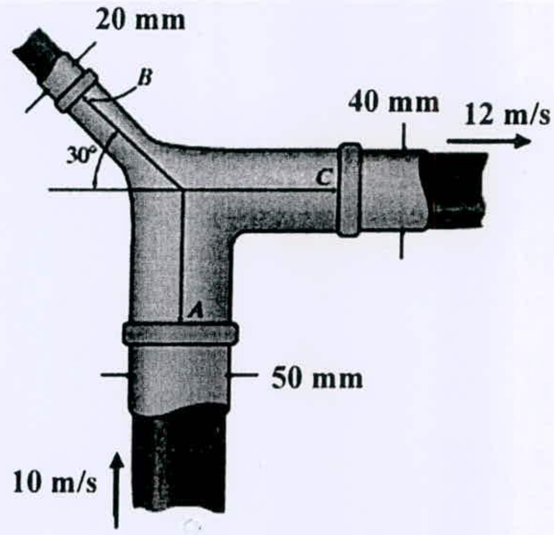


Figure 9 for Question 7(a)

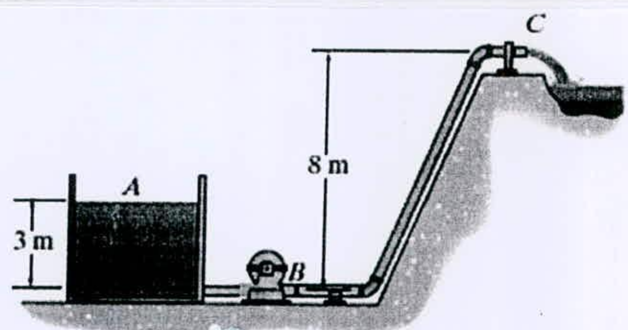


Figure 10 for Question 8 (a)

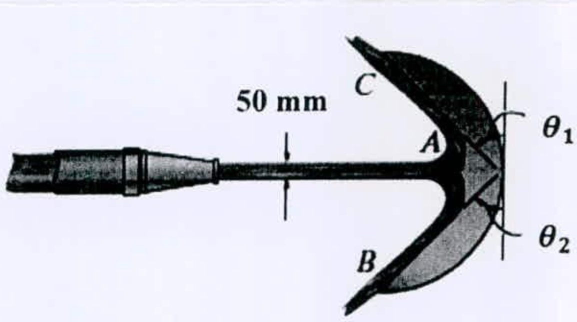


Figure 11 for Question 8 (b)

Supplement Equations if Required

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$dv = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$dw = \frac{\partial w}{\partial t}dt + \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_2}{\gamma} - \frac{P_1}{\gamma} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + z_2 - z_1 = -\frac{2\tau L}{\gamma r}$$

$$\frac{P_1}{\gamma_1} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\gamma_2} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

$$dp = \rho\frac{V^2}{r}\,dr$$

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{\omega^2}{2g}(r_2^2 - r_1^2) + \frac{v_1^2 - v_2^2}{2g}$$

$$\sum \vec{F} = \rho_2\, Q_2 \vec{V}_2 - \rho_1\, Q_1 \vec{V}_1$$

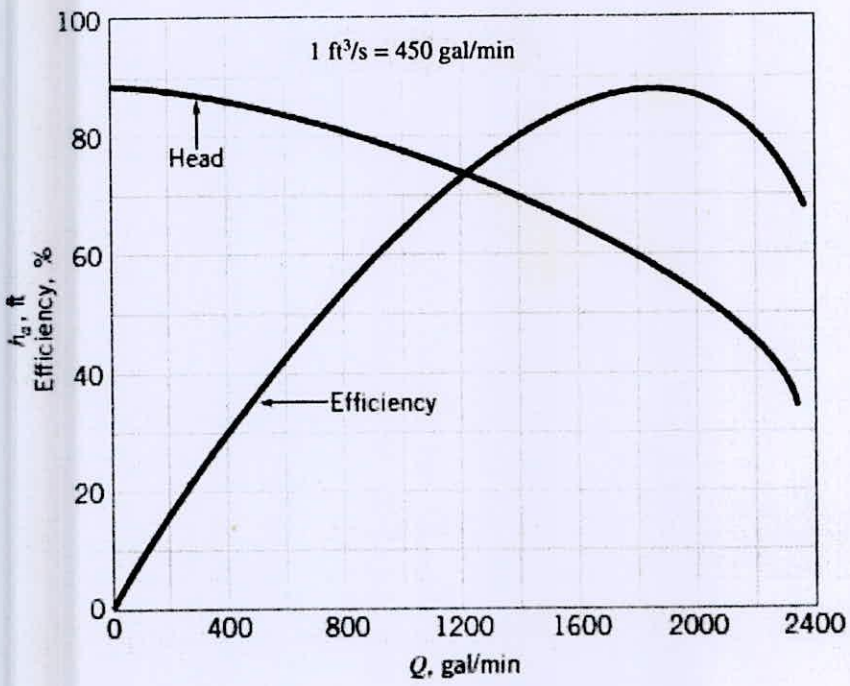
$$\sum F_x = P_1 A_1 - P_2 A_2 - \left(F_{R/F}\right)_x = \rho Q(V_2 - V_1)$$

$$-F_x = \dot{m}_r[(v-u)\cos\theta - (v-u)]$$

$$F_y = \dot{m}_r\,(v-u)\sin\theta$$

$$h_a = z_2 - z_1 + \sum h_L$$

$$h_a = z_2 - z_1 + KQ^2$$



12:
Figure: Pump Characteristics Curve for Problem 6 (b)

L-2/T-2/CE **Date: 31/03/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub: **HUM 217** (Engineering Economics)

Full Marks: 140 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** from the rest.

Symbols indicate their usual meaning

1. (a) Explain the law of diminishing marginal utility with numerical as well as graphical presentations. Using the axiom of diminishing marginal utility show that the positive segment of marginal utility curve represents the demand curve of a commodity. **(10)**

   (b) How would you prove that the equilibrium conditions are identical in the cardinalist approach and in the indifference-curves (ordinalist) approach to utility analysis? **(13 ⅓)**

2. (a) Following are the demand and supply functions of Maxell super ball pen respectively, **(10)**

   $Q_{Dm} = 1950 - 48P_m$

   $Q_{Sm} = 990 + 63P_m$

   Find the equilibrium price and quantity of Maxell super ball pen. If a 18% supplementary tax is imposed on unit price, what would be the new equilibrium price and quantity? What is the proportion of this tax that the suppliers are likely to bear?

   (b) Construct a market supply curve using a hypothetical supply schedule. Explain how the interactions between market demand and market supply curves determine equilibrium price and quantity of a commodity. **(13 ⅓)**

3. (a) Given the demand function of a commodity X **(10)**

   $Qdx = 2050 - 25Px + 0.005\ M + 3.8\ Py - 7Pz$

   Where, price of X (Px) is tk. 90. Price of Y (Py) is tk. 140, price of Z (Pz) is tk. 110 and the level of income (M) is tk. 60,000. Find the income elasticity and cross-price elasticities of dema

   (b) How would you derive the formula for measuring cross price elasticity of demand? Explain three important implications of elasticity of demand in business. **(13 ⅓)**

Contd …… P/2

HUM 217

4. Write short notes on any THREE of the following **(23⅓)**

i) Split-up of price effect into income effect and substitution effect

ii) Determinants of elasticity of demand

iii) Indifference map, budget line, and price consumption curve

iv) Main economic challenges.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols indicate their usual meaning

5. (a) A manufacturer has a fixed cost of \$ 40,000 and a variable cost of \$1.60 per unit made and sold. Selling price is \$ 2 per unit. **(10)**

i) Find the revenue, cost and profit functions using q for the number of units.

ii) Compute profit if 150000 units are made and sold.

iii) Compute profit if 1500 units are made and sold.

iv) Find the break-even quantity.

v) Construct the break-even chart. Label the cost and revenue lines, the fixed cost line, and the break-even point.

(b) Complete the following table and sketch the graph explaining the relations among the various short run cost curves. **(13⅓)**

Quantity of output	Total fixed cost	Total variable cost	Total cost	Average fixed cost	Average variable cost	Average Total cost	Marginal cost
1	70	30					
2	70	40					
3	70	45					
4	70	55					
5	70	75					
6	70	120					

6. (a) From the following revenue and cost functions calculate the profit maximizing level of output and maximum profit. **(10)**

$$R = 111\,Q - 2Q^2$$

$$C = \frac{1}{3}Q^3 - 8Q^2 + 122Q + 50$$

Contd …… P/3

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Contd... Q. No. 6

(b) Graphically explain the short run equilibrium of a firm under perfect competition. **(13⅓)**

7. (a) Explain producer's equilibrium with the help of iso-cost and isoquant curves. **(13⅓)**

(b) From the following functions calculate the amount of labour and capital that maximizes output. What is the maximum amount of output? **(10)**

$$Q = 200L^{0.5}K^{.05}$$

$$3000 = 35L + 45K$$

8. (a) What do you understand by localization of industries? What are the causes of localization of industries? **(10)**

(b) Explain the advantages and disadvantages of localization of industries. **(13⅓)**
