

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub : **ME 243** (Mechanics of Solids)

Full Marks : 210

Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE** questions.

Assume any missing data. Symbols have their usual meanings.

1. (a) The joint in Fig. for Q. No. 1(a) is fastened together using two bolts having a diameter of 10 mm. Determine the maximum force  $F$  that can be applied if the allowable normal and shear stresses in bolt are 450 MPa and 350 MPa, respectively. Also, what would be the maximum value of this force  $F$  if the stresses in plate material are considered? Consider the allowable normal and shear stresses in plate material as 300 MPa and 200 MPa, respectively. **(18)**
- (b) The composite rod in Fig. for Q. No. 1(b) is stress-free before the axial loads  $P_1$  and  $P_2$  are applied. Assuming that the walls are rigid, calculate the stress in each material if  $P_1 = 150$  kN and  $P_2 = 90$  kN. The cross sectional areas of the aluminum, steel and bronze sections of the rod are 900, 2000, and 1200 mm<sup>2</sup>, respectively. Also, the elastic modulus of aluminum, steel, and bronze materials are 70, 200, and 83 GPa, respectively.
2. (a) The center rod CD of the assembly as shown in Fig. for Q. No. 2(a) is heated from  $T_1 = 30^\circ\text{C}$  to  $T_2 = 180^\circ\text{C}$ . Also, the two end rods AB and EF are heated from  $T_1 = 30^\circ\text{C}$  to  $T_2 = 50^\circ\text{C}$ . At tgeh lower temperature, the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm<sup>2</sup>. CD is made of aluminum and has a cross-sectional area of 375 mm<sup>2</sup>. Conisder,  $E_{st} = 200$  GPa,  $E_{al} = 70$  GPa  $\alpha_{st} = 12 \times 10^{-6}/^\circ\text{C}$ , and  $\alpha_{al} = 23 \times 10^{-6}/^\circ\text{C}$ . **(18)**
- (b) Draw the shear force and bending moment diagrams of the beam loaded as shown in Fig. for Q. No. 2(b). Also, write down the necessary equations to draw the diagrams. **(17)**
3. (a) A beam carries a concentrated load  $W$  and a uniformly distributed load of  $4 W$  as shown in Fig. for Q. No. 3(a). If the allowable compressive stress in the beam is 120 MPa and the allowable shear stress is 80 MPa, determine the maximum value of  $W$  that can be applied. **(18)**
- (b) The steel shaft ( $E = 200$  GPa) is used to support a rotor that exerts a uniform load of 5 kN/m within the region CD of the shaft as shown in Fig. for Q. No. 3(b). Determine the slope of the shaft at the bearing A, and deflection at the midpoint. The bearings exert vertical reactions only on the shaft. **(17)**

**ME 243( ME)**

4. (a) A steel tube having an outer diameter of 75 mm is used to transmit 3.5 hp when turning at 30 rpm. Determine the inner diameter  $d$  of the tube if the allowable shear stress in the tube is 70 MPa. If the length of the tube is 1 m, find the angular displacement of one end of the tube relative to its other end. Consider, the modulus of rigidity of the tube material is 90 MPa. **(18)**
- (b) A helical compression spring having 150 mm spring diameter is subjected to an axial load of 100 N. If the allowable shearing stress in the material is 200 MPa, determine the wire diameter required to support the load. If the spring consists of 20 turns, and the shear modulus of the material is 80 GPa, find the maximum deflection of spring. **(17)**

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What is (i) column (ii) ideal column (iii) critical load and (iv) slenderness ratio? **(8)**
- (b) Determine the critical stress for an ideal column fixed at the base and free at the top. **(15)**
- (c) The A-36 steel rod BC has a diameter of 50 mm and is used as a strut to support the beam as shown in Figure Q. 5(c). Determine the maximum intensity  $w$  of the uniformly distributed load that can be applied to the beam without causing the strut to buckle. Take F.S. = 3 against buckling,  $E_{st} = 200$  GPa,  $\sigma_Y = 250$  MPa. **(12)**

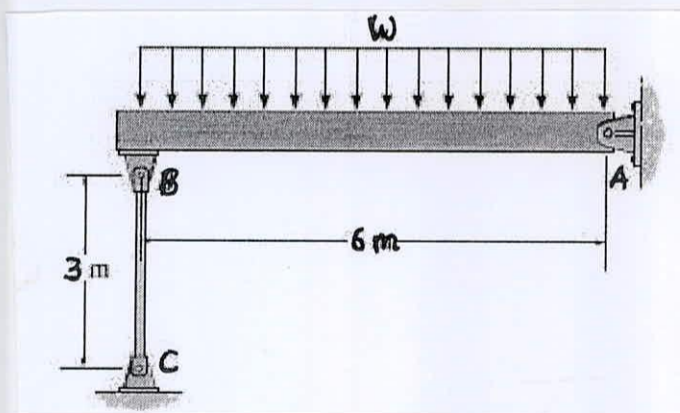


Figure Q. 5(c)

6. (a) Derive the Secant formula of an eccentrically loaded column. Hence explain how columns with large slenderness ratios tend to fail at or near the Euler critical load regardless of the eccentricity ratio. **(18)**
- (b) The W200 × 22 A-36-steel column is fixed at its base as shown in Figure Q. 6(b). Its top is constrained to rotate about the  $y$ - $y$  axis and free to move along the  $y$ - $y$  axis. Also, the column is braced along the  $x$ - $x$  axis at its mid-height. If  $P = 25$  kN, determine the maximum normal stress developed in the column. Take,  $E_{st} = 200$  GPa,  $\sigma_Y = 250$  MPa. **(17)**

**ME 243 (ME)**  
 Contd...Q.No. 6(b)

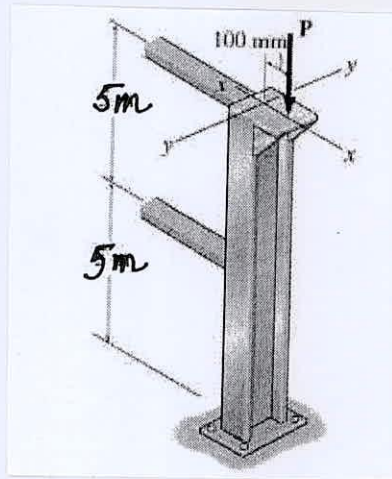


Figure Q. 6(b)

7. (a) What does the term “Thin Walled” mean in the analysis of Pressure Vessels or Cylinders? **(5)**
- (b) The spherical gas tank as shown in Figure Q. 7(b) is fabricated by bolting together two hemispherical thin shells. If the 8-m inner diameter tank is to be designed to withstand a gauge pressure of 2 MPa, determine the minimum wall thickness of the tank and the minimum number of 25-mm diameter bolts that must be used to seal it. The tank and the bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. **(12)**

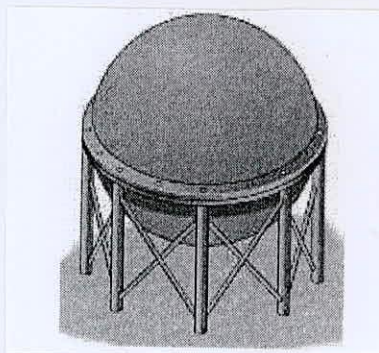


Figure Q. 7(b)

- (c) Derive the expressions for radial and tangential normal stresses in a long thick-walled cylinder with axially restrained end whose inside radius is  $r_i$ , the outside radius  $r_o$ , the internal pressure  $p_i$  and the outside or external pressure  $p_o$ . **(18)**
8. (a) The state of plane stress at a point is shown on the element in Figure Q. 8(a). Construct the Mohr’s circle and determine the maximum in-plane shear stress at this point. **(18)**

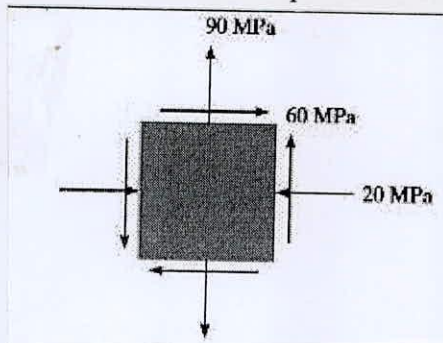


Figure Q. 8(a)

**ME 243 (ME)**

Contd...Q.No. 8

(b) What is a strain rosette? Using the strain transformation equation show that for a  $60^\circ$  strain rosette. (10)

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_b - \epsilon_c)$$

Notations have usual meaning.

(c) Determine the elastic strain energy for axial loading of a cylindrical bar of constant cross section. (7)

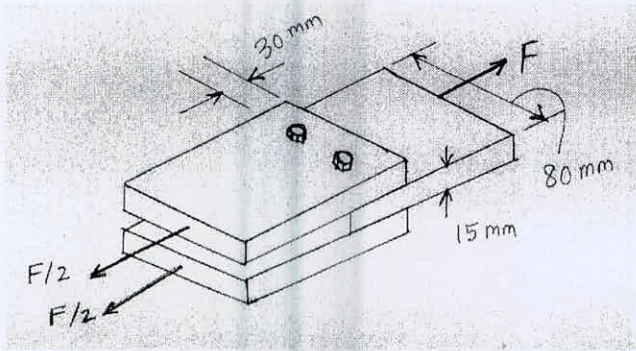


Fig. for Q. No. 1(a)

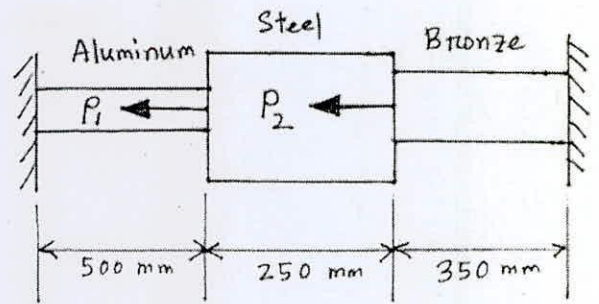


Fig. for Q. No. 1(b)

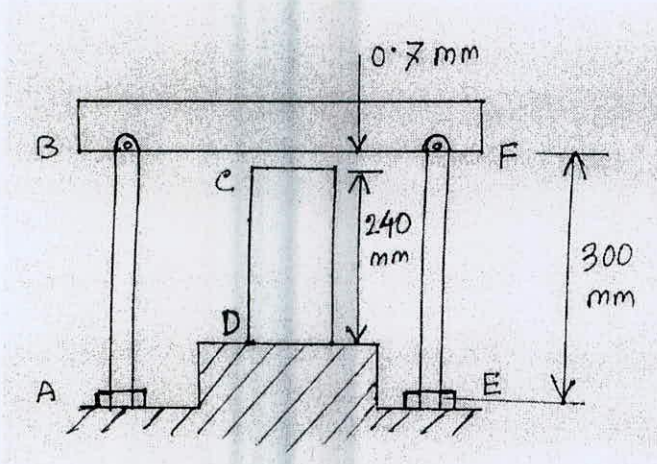


Fig. for Q. No. 2(a)

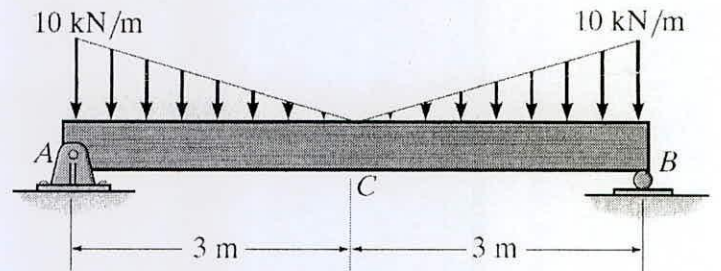


Fig. for Q. No. 2(b)

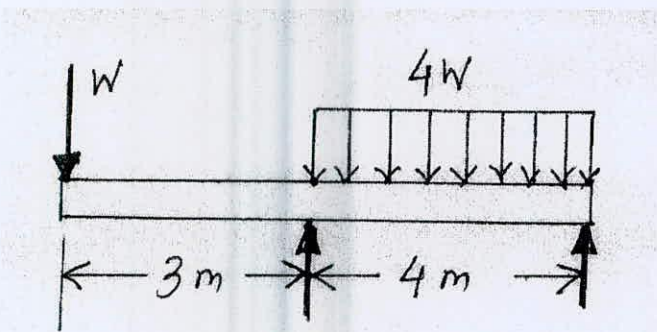


Fig. for Q. No. 3(a)

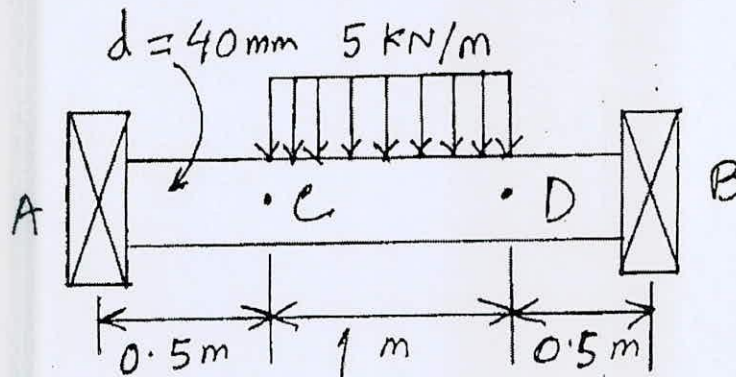
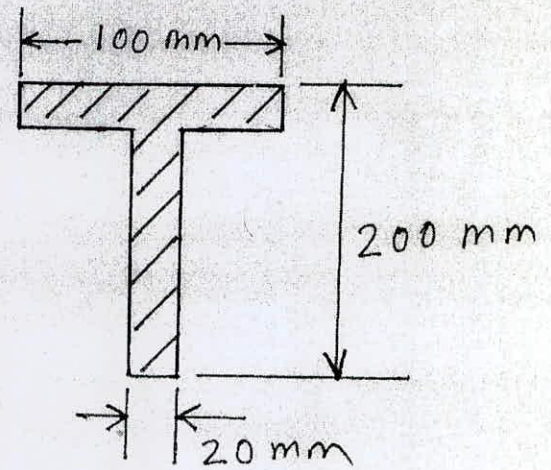
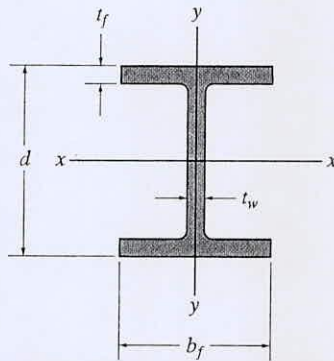


Fig. for Q. No. 3(b)

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| Wide-Flange Sections or W Shapes SI Units |                 |            |                                    |                         |                             |                                 |                                 |      |                                 |                                 |      |
|---|-----------------|------------|------------------------------------|-------------------------|-----------------------------|---------------------------------|---------------------------------|------|---------------------------------|---------------------------------|------|
| Designation                               | Area<br>A       | Depth<br>d | Web<br>thickness<br>t <sub>w</sub> | Flange                  |                             | x-x axis                        |                                 |      | y-y axis                        |                                 |      |
|   |                 |            |                                    | width<br>b <sub>f</sub> | thickness<br>t <sub>f</sub> | I                               | S                               | r    | I                               | S                               | r    |
| mm × kg/m                                 | mm <sup>2</sup> | mm         | mm                                 | mm                      | mm                          | 10 <sup>6</sup> mm <sup>4</sup> | 10 <sup>3</sup> mm <sup>3</sup> | mm   | 10 <sup>6</sup> mm <sup>4</sup> | 10 <sup>3</sup> mm <sup>3</sup> | mm   |
| W310 × 129                                | 16 500          | 318        | 13.10                              | 308.0                   | 20.6                        | 308                             | 1940                            | 137  | 100                             | 649                             | 77.8 |
| W310 × 74                                 | 9 480           | 310        | 9.40                               | 205.0                   | 16.3                        | 165                             | 1060                            | 132  | 23.4                            | 228                             | 49.7 |
| W310 × 67                                 | 8 530           | 306        | 8.51                               | 204.0                   | 14.6                        | 145                             | 948                             | 130  | 20.7                            | 203                             | 49.3 |
| W310 × 39                                 | 4 930           | 310        | 5.84                               | 165.0                   | 9.7                         | 84.8                            | 547                             | 131  | 7.23                            | 87.6                            | 38.3 |
| W310 × 33                                 | 4 180           | 313        | 6.60                               | 102.0                   | 10.8                        | 65.0                            | 415                             | 125  | 1.92                            | 37.6                            | 21.4 |
| W310 × 24                                 | 3 040           | 305        | 5.59                               | 101.0                   | 6.7                         | 42.8                            | 281                             | 119  | 1.16                            | 23.0                            | 19.5 |
| W310 × 21                                 | 2 680           | 303        | 5.08                               | 101.0                   | 5.7                         | 37.0                            | 244                             | 117  | 0.986                           | 19.5                            | 19.2 |
| W250 × 149                                | 19 000          | 282        | 17.30                              | 263.0                   | 28.4                        | 259                             | 1840                            | 117  | 86.2                            | 656                             | 67.4 |
| W250 × 80                                 | 10 200          | 256        | 9.40                               | 255.0                   | 15.6                        | 126                             | 984                             | 111  | 43.1                            | 338                             | 65.0 |
| W250 × 67                                 | 8 560           | 257        | 8.89                               | 204.0                   | 15.7                        | 104                             | 809                             | 110  | 22.2                            | 218                             | 50.9 |
| W250 × 58                                 | 7 400           | 252        | 8.00                               | 203.0                   | 13.5                        | 87.3                            | 693                             | 109  | 18.8                            | 185                             | 50.4 |
| W250 × 45                                 | 5 700           | 266        | 7.62                               | 148.0                   | 13.0                        | 71.1                            | 535                             | 112  | 7.03                            | 95                              | 35.1 |
| W250 × 28                                 | 3 620           | 260        | 6.35                               | 102.0                   | 10.0                        | 39.9                            | 307                             | 105  | 1.78                            | 34.9                            | 22.2 |
| W250 × 22                                 | 2 850           | 254        | 5.84                               | 102.0                   | 6.9                         | 28.8                            | 227                             | 101  | 1.22                            | 23.9                            | 20.7 |
| W250 × 18                                 | 2 280           | 251        | 4.83                               | 101.0                   | 5.3                         | 22.5                            | 179                             | 99.3 | 0.919                           | 18.2                            | 20.1 |
| W200 × 100                                | 12 700          | 229        | 14.50                              | 210.0                   | 23.7                        | 113                             | 987                             | 94.3 | 36.6                            | 349                             | 53.7 |
| W200 × 86                                 | 11 000          | 222        | 13.00                              | 209.0                   | 20.6                        | 94.7                            | 853                             | 92.8 | 31.4                            | 300                             | 53.4 |
| W200 × 71                                 | 9 100           | 216        | 10.20                              | 206.0                   | 17.4                        | 76.6                            | 709                             | 91.7 | 25.4                            | 247                             | 52.8 |
| W200 × 59                                 | 7 580           | 210        | 9.14                               | 205.0                   | 14.2                        | 61.2                            | 583                             | 89.9 | 20.4                            | 199                             | 51.9 |
| W200 × 46                                 | 5 890           | 203        | 7.24                               | 203.0                   | 11.0                        | 45.5                            | 448                             | 87.9 | 15.3                            | 151                             | 51.0 |
| W200 × 36                                 | 4 570           | 201        | 6.22                               | 165.0                   | 10.2                        | 34.4                            | 342                             | 86.8 | 7.64                            | 92.6                            | 40.9 |
| W200 × 22                                 | 2 860           | 206        | 6.22                               | 102.0                   | 8.0                         | 20.0                            | 194                             | 83.6 | 1.42                            | 27.8                            | 22.3 |
| W150 × 37                                 | 4 730           | 162        | 8.13                               | 154.0                   | 11.6                        | 22.2                            | 274                             | 68.5 | 7.07                            | 91.8                            | 38.7 |
| W150 × 30                                 | 3 790           | 157        | 6.60                               | 153.0                   | 9.3                         | 17.1                            | 218                             | 67.2 | 5.54                            | 72.4                            | 38.2 |
| W150 × 22                                 | 2 860           | 152        | 5.84                               | 152.0                   | 6.6                         | 12.1                            | 159                             | 65.0 | 3.87                            | 50.9                            | 36.8 |
| W150 × 24                                 | 3 060           | 160        | 6.60                               | 102.0                   | 10.3                        | 13.4                            | 168                             | 66.2 | 1.83                            | 35.9                            | 24.5 |
| W150 × 18                                 | 2 290           | 153        | 5.84                               | 102.0                   | 7.1                         | 9.19                            | 120                             | 63.3 | 1.26                            | 24.7                            | 23.5 |
| W150 × 14                                 | 1 730           | 150        | 4.32                               | 100.0                   | 5.5                         | 6.84                            | 91.2                            | 62.9 | 0.912                           | 18.2                            | 23.0 |

B

**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) The stream of water as shown in Fig. 1(a) flows at a rate of 9.1667 kg/s and moves with a velocity of magnitude 18 m/s at both  $A$  and  $B$ . The vane is supported by a pin and bracket at  $C$  and by a load cell at  $D$  which can exert only a horizontal force. Neglecting the weight of the vane, determine the components of the reactions at  $C$  and  $D$ . (17)
- (b) Arm  $BD$  is connected by pins to a collar at  $B$  and to crank  $DE$  as shown in Fig. 1(b). Knowing that the velocity of collar  $B$  is 400 mm/s upward, using the method of instantaneous center of rotation, determine (i) the angular velocity of arm  $BD$ , (ii) the velocity of point  $D$ . (18)
2. Figure 2 shows a quick-return motion in which the driving crank  $OA$  rotates at 120 rev/min in a clockwise direction. For the position shown, draw the necessary velocity and acceleration vector diagrams with suitable scale and graphically determine the magnitude and direction of (a) the acceleration of block  $D$ ; (b) the angular acceleration of the slotted bar  $QB$ . (35)
3. (a) The flywheel as shown in Fig. 3(a) has a radius of 500 mm, a weight of 120 kg, and a radius of gyration of 375 mm. A 15-kg block  $A$  is attached to a wire that is wrapped around the flywheel, and the system is released from rest. Neglecting the effect of friction, determine the acceleration of block  $A$ . (17)
- (b) The ends of the 10-kg uniform rod  $AB$  are attached to collars of negligible mass that slide without friction along fixed rods as shown in Fig. 3(b). If the rod is released from rest when  $\theta = 25^\circ$ , determine immediately after release (i) the angular acceleration of the rod, and (ii) the reaction at  $A$ . (18)
4. (a) A rope is wrapped around a cylinder of radius  $r$  and mass  $m$  as shown in Fig. 4(a). The cylinder is released from rest. Using principle of work and energy, determine the velocity of the center of the cylinder after it has moved downward a distance  $s$ . (17)
- (b) A bullet weighing 10 g is fired with a horizontal velocity of 550 m/s into the lower end of a slender 7.5-kg bar of length  $L = 800$  mm as shown in Fig. 4(b). Knowing that  $h = 300$  mm and that the bar is initially at rest, determine the angular velocity of the bar immediately after the bullet becomes embedded. (18)

**ME 249**

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols used have their usual meaning and interpretation.

5. (a) The elevator shown in the Figure 5a moves downwards with a constant velocity of 6 m/s. Determine (i) the velocity of the cable C, (ii) the velocity of the counterweight W, (iii) the relative velocity of the cable C with respect to the elevator E, (iv) the relative velocity of the counterweight W with respect to the elevator. **(15)**
- (b) Racecar A is traveling on a straight portion of the track while racecar B is traveling on a circular portion of the track. At the instant shown in Figure 5b, the speed of A is increasing at the rate of  $10 \text{ m/s}^2$ , and the speed of B is decreasing at the rate of  $6 \text{ m/s}^2$ . For the position shown, determine (i) the velocity of B relative to A, (ii) the acceleration of B relative to A. **(20)**
6. (a) Block B of mass 10-kg rests on the upper surface of a 22-kg wedge A as shown in Figure 6a. Knowing that the system is released from rest and neglecting friction, determine (i) the acceleration of B (ii) the velocity of B relative to A at  $t = 0.5 \text{ s}$ . **(20)**
- (b) A 54-kg pilot flies a jet trainer in a half vertical loop of 1200-m radius so that the speed of the trainer decreases at a constant rate. Knowing that the pilot's apparent weights at Points A and C are 1680 N and 350 N, respectively, determine the force exerted on her by the seat of the trainer when the trainer is at Point B as shown in Figure 6b. **(15)**
7. (a) As shown in Figure 7a, a package is projected up a  $15^\circ$  incline at A with an initial velocity of 8 m/s. Knowing that the coefficient of kinetic friction between the package and the incline is 0.12, determine (i) the maximum distance  $d$  that the package will move up the incline, (ii) the velocity of the package as it returns to its original position. **(15)**
- (b) A 600-g collar C may slide along a horizontal, semicircular rod ABD shown in Figure 7b. The spring CE has an undeformed length of 250 mm and a spring constant of 135 N/m. Knowing that the collar is released from rest at A and neglecting friction, determine the speed of the collar (i) at B, (ii) at D. **(20)**
8. (a) The Mars Pathfinder spacecraft used large airbags to cushion its impact with the planet's surface when landing (shown in Figure 8a). Assuming the spacecraft had an impact velocity of 18 m/s at an angle of  $45^\circ$  with respect to the horizontal, the coefficient of restitution is 0.85 and neglecting friction, determine (i) the height of the first bounce, (ii) the length of the first bounce. (Acceleration of gravity on the Mars =  $3.73 \text{ m/s}^2$ ). **(20)**



**ME 249**  
**Contd... Q. No.8**

(b) A 18-kg cannonball and a 12-kg cannonball are chained together and fired horizontally with a velocity of 165 m/s from the top of a 15-m wall as shown in Figure 8b. The chain breaks during the flight of the cannonballs and the 12-kg cannonball strikes the ground at  $t = 1.5$  s, at a distance of 240 m from the foot of the wall, and 7 m to the right of the line of fire. Determine the position of the other cannonball at that instant. Neglect the resistance of the air.

**(15)**

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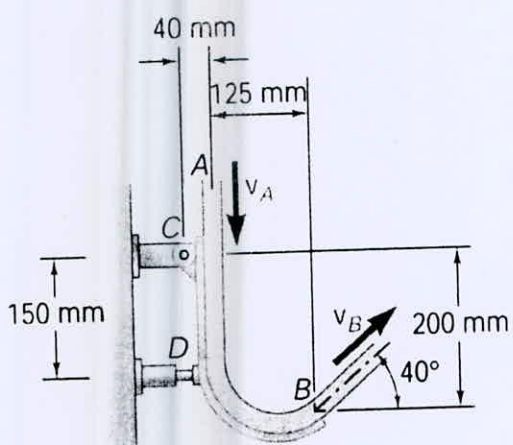


Fig. 1(a)

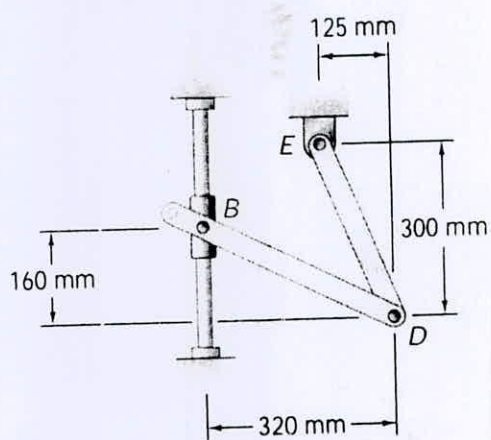


Fig.1(b)

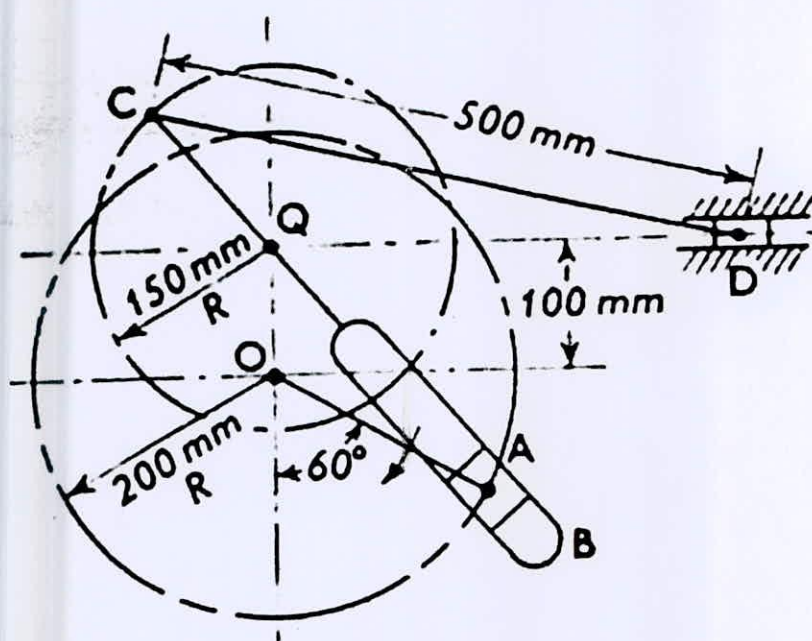


Figure 2

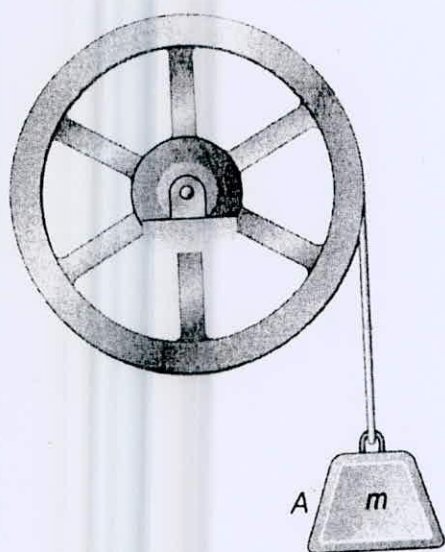


Fig. 3(a)

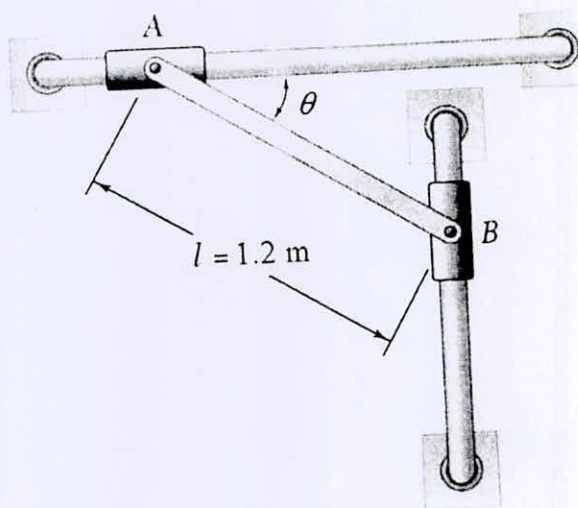


Fig. 3(b)

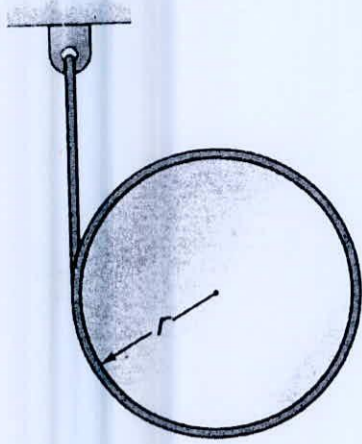


Fig. 4(a)

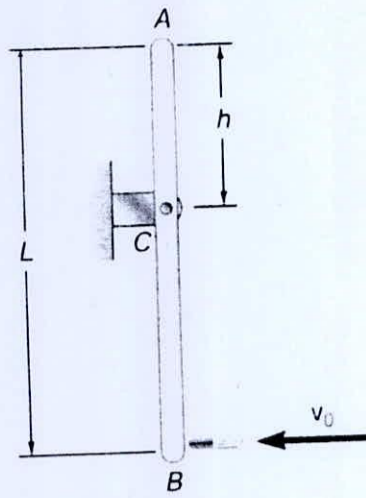


Fig. 4(b)

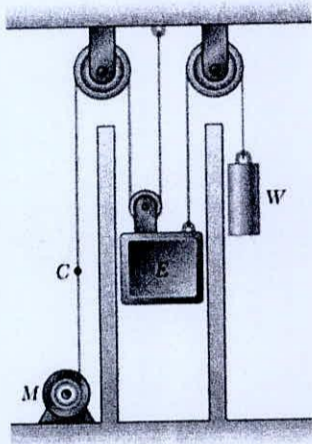


Figure 5a

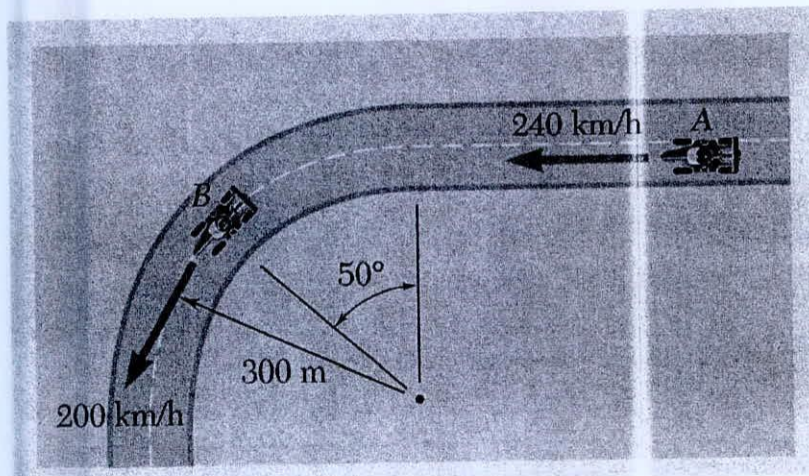


Figure 5b

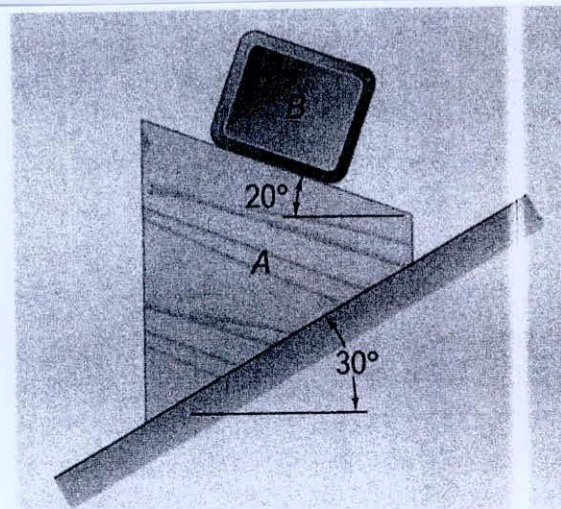


Figure: 6a

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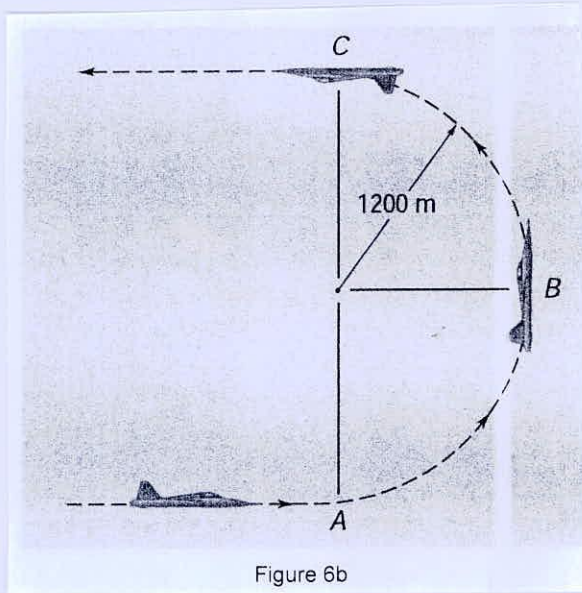


Figure 6b

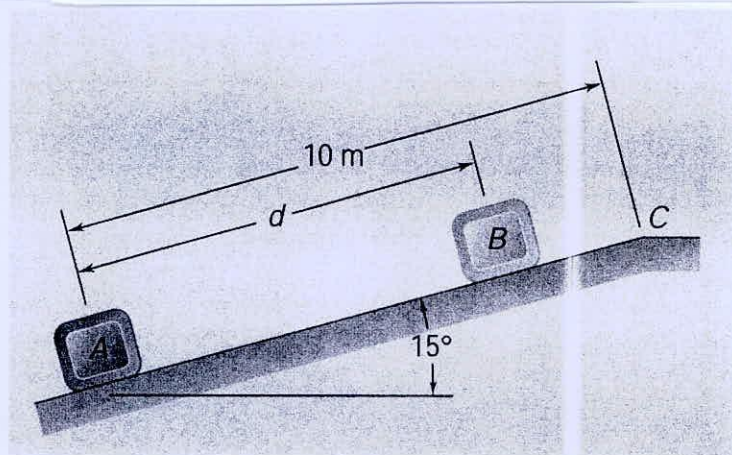


Figure 7a

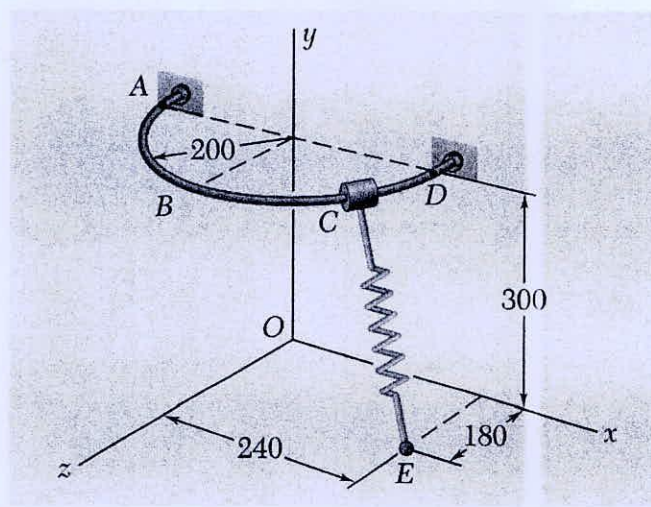


Figure 7b

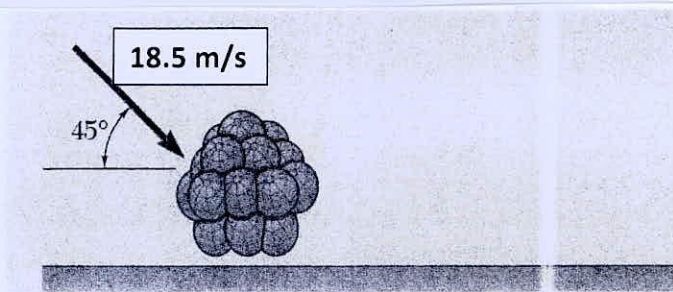


Figure 8a

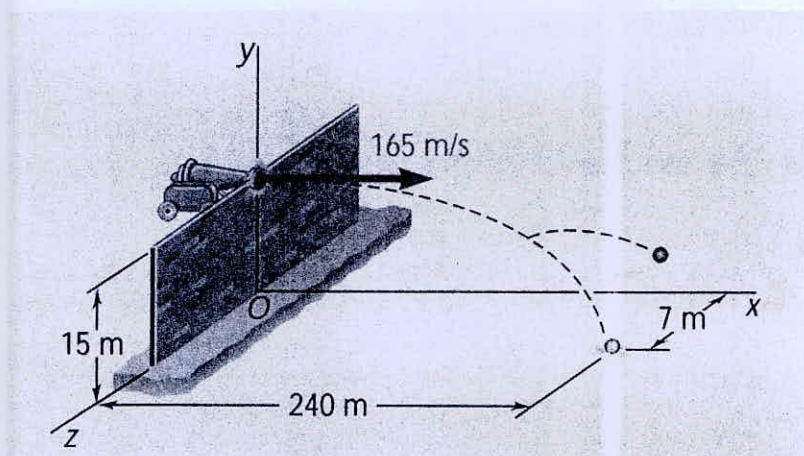


Figure 8b

**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE** from the rest.

1. (a) For symmetrical stress distribution in a Circular body, the stress function  $\phi$  is governed by the following DEQ, (20)

$$\frac{d^4 \phi}{dr^4} + \frac{2}{r} \frac{d^3 \phi}{dr^3} - \frac{1}{r^2} \frac{d^2 \phi}{dr^2} + \frac{1}{r^3} \frac{d\phi}{dr} = 0$$

Derive the corresponding central-difference algebraic equation with an error of  $O(h^2)$ .

- (b) The electrical voltage drop across an inductor, according to Faraday's law, is given by: (15)

$$V_L = L \frac{di}{dt}$$

where  $V_L$  = voltage drop (V),  $L$  = inductance (Henry),  $i$  = current (A),  $t$  = time (sec).

|   |   |      |     |      |     |     |
|---|---|------|-----|------|-----|-----|
| i | 0 | 0.15 | 0.3 | 0.55 | 0.8 | 1.9 |
| t | 0 | 0.1  | 0.2 | 0.3  | 0.5 | 0.7 |

From the above data set, determine the voltage at the following time steps for an inductance of 4 Henry: (i)  $t = 0.15$ , (ii)  $t = 0.4$  and (iii)  $t = 0.7$ . All the results should conform to the accuracy level of  $O(h^2)$ .

2. (a) Using the data of the following Table for Q. No. 2(a), construct a divided difference table. Write a fourth order Newton's divided difference interpolating polynomial ( $P_4$ ). Use the polynomial to interpolate for  $x = 1.5$ . (20)

Table for Q. No. 2(a)

|          |           |           |           |           |           |
|----------|-----------|-----------|-----------|-----------|-----------|
| $i$      | 0         | 1         | 2         | 3         | 4         |
| $x_i$    | 1.0       | 1.3       | 1.6       | 1.9       | 2.2       |
| $f(x_i)$ | 0.7651977 | 0.6200860 | 0.4554022 | 0.2818186 | 0.1103623 |

- (b) Derive Newton's first order interpolating polynomial. Also derive the Lagrange form from that interpolating polynomial. (15)

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3. (a) Suppose there are  $n$  number of data points to be fitted by (15)

$$y = a_0 + a_1x^2$$

Determine the generalized expressions of  $a_0$  and  $a_1$  using least-squares regression.

- (b) Determine the constants  $\alpha$  and  $\beta$  by the method of least-squares such that the following function fits the given data set shown below: (20)

$$y = \alpha e^{\beta x}$$

|     |       |        |        |        |        |
|-----|-------|--------|--------|--------|--------|
| $x$ | 2     | 4      | 6      | 8      | 10     |
| $y$ | 4.077 | 11.084 | 30.128 | 81.897 | 222.62 |

4. Solve the following DEQ, (35)

$$\frac{dy}{dx} = 1 + 2x - 5\sqrt{x}, y(0) = 1$$

over the interval from  $x = 0$  to 1 using a step size of 0.5 by

- (i) Analytical method  
(ii) Classical 4<sup>th</sup> order RK method

Find the accuracy of your numerical result.

### SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Use the False Positions Method to find a root of  $f(x) = \cos x - x$  in the interval  $[0, 1]$  correct to at least four significant figures. (14)

- (b) If the above problem (Q5(a)) is solved using the Bisection Method, predict how many iterations will be needed for the required accuracy? (6)

- (c) Find the root of the equation  $xe^{0.5x} + 1.2x = 5$  using the Newton's method. Use  $\epsilon_s = 0.1\%$ . (15)

6. (a) Define round off and truncation error. How these errors can be minimized? (9)

- (b) Evaluate and interpret the condition number for  $f(x) = \frac{10}{1-x^2}$  at  $x = 0.99$ . (6)

- (c) Define absolute error and relative error. Can you use absolute error in all cases? Justify your answer with proper example. (8)

- (d) Show that the local truncation error associated with the Trapezoidal rule of integration is  $\frac{-h^3}{12} f''(\xi)$ . How can one improve the results of integration? (12)

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7. (a) (i) Solve the following system using the Gauss elimination method. (18)

$$3x_2 + 2x_3 - 2x_4 = 4$$

$$x_1 + x_2 + x_3 + 5x_4 = -2$$

$$2x_1 + x_2 - 2x_3 = -1$$

$$3x_1 + x_2 - x_3 + 3x_4 = 4$$

- (ii) using the results obtained in (i), find the [L] matrix required for solving the system with LU decomposition method. (5)

- (b) Briefly discuss how Gauss-Seidel method can be improved using relaxation. (7)

- (c) Discuss the difference between Gauss-seidel and Jacobi method. (5)

8. (a) The total mass of variable density rod is given by: (18)

$$m = \int_0^L \rho(x) A_c(x) dx$$

where  $m$  = mass,  $\rho$  = density,  $A_c$  = cross-sectional area,  $x$  = distance along the rod, and  $L$  = the total length of the rod. The following data has been measured for a 10-m length rod. Determine the mass in kilograms using appropriate rule(s) of integration.

|                        |      |      |      |      |      |      |      |
|------------------------|------|------|------|------|------|------|------|
| $x, m$                 | 0    | 2    | 3    | 4    | 6    | 8    | 10   |
| $\rho, \text{gm/cm}^3$ | 4.00 | 3.95 | 3.89 | 3.80 | 3.60 | 3.41 | 3.30 |
| $A_c, \text{cm}^2$     | 100  | 103  | 106  | 110  | 120  | 133  | 150  |

- (b) Use Faddeev-Leverrier Method to find eigen values and eigen vectors of the following matrix. (17)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

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Sub : **MATH 263** (Complex Variable, Fourier Series, Harmonic Functions and Partial Differential Equations)

Full Marks : 280

Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

Symbols have their usual meanings.

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) An airplane travels 100 miles southeast, 50 miles due west, 200 miles  $30^\circ$  north of east and then 150 miles northeast. Determine (i) analytical and (ii) graphically how far and in what direction it is from its starting point. (12)

- (b) Compute the principal value of the complex logarithm  $\text{Log } z$  for  $z = 1 + i$ . Also show that  $|\sin z|^2 = (\sin x)^2 + (\sinh y)^2$  for all complex numbers  $z = x + iy$ . (11)

- (c) Guess at a possible value for  $\lim_{z \rightarrow 2+i} \frac{z^2 - 2iz}{z^2 + 4}$  and investigate the correctness of your guess. (10)

- (d) Find the image of the circle  $|z| = r_0, r_0 \neq 1$  under the bilinear transformation

$$T(z) = \frac{z-i}{z+i}. \text{ What happens if } r_0 = 1? \text{ Which point will map onto } 1+i. \quad (13 \frac{2}{3})$$

2. (a) Write down Cauchy-Riemann equations in polar form. Test the differentiability of the function  $f(z) = \sqrt{r} e^{i\frac{\theta}{2}}, (r > 0, 0 < \theta < 2\pi)$  in the indicated domain and hence show that  $f'(z) = \frac{1}{2f(z)}$ . (15  $\frac{2}{3}$ )

- (b) Show that  $u(x, y) = e^{-x}(x \sin y - y \cos y)$  is a harmonic function. Find an analytic function  $f(z) = u(x, y) + iv(x, y)$  and express  $f(z)$  in terms of  $z$ . (16  $\frac{2}{3}$ )

- (c) Evaluate  $\int_C (x^2 - iy^2) dz$  along the straight lines from  $z = 1 + i$  to  $z = 1 + 8i$  and then along a line parallel to  $y$ -axis from  $z = 1 + 8i$  to  $z = 2 + 8i$ . (15)

3. (a) Use Cauchy integral formula to evaluate  $\oint_C \frac{z+1}{z^4 + 2iz^3} dz$ , where the contour  $C$  is the circle  $|z| = 1$ . (15)

- (b) Expand  $f(z) = \frac{1}{(z+1)(z+3)}$  in a Laurent series valid for, (i)  $1 < |z| < 3$ , (ii)  $|z| > 3$ . (15)

- (c) Evaluate  $\oint_C \frac{1}{(z-1)^2(z-3)} dz$  by Cauchy residue theorem, where the contour  $C$  is the circle  $|z| = 2$ . (16  $\frac{2}{3}$ )



**MATH 263**

4. (a) Using the method of contour integration, show that  $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2} = \frac{5\pi}{32}$ . (23)

(b) Evaluate the following integral using the method of contour integration

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$$
(23 2/3)

**SECTION - B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Apply Lagrange method to solve,  $p \cos(x+y) + q \sin(x+y) = z$  (15)

(b) Find the integral surface of  $x^2 p + y^2 q + z^2 = 0$ , which passes through the hyperbola  $xy = x + y, z = 1$ . (16 2/3)

(c) Apply Charpit's method to find complete, singular, and general integrals of  $(p^2 + q^2)y = qz$ . (15)

6. (a) Solve the following partial differential equations:

(i)  $(D_x^3 + D_x^2 D_y - D_x D_y^2)z = e^x \cos(x + 2y)$ . (15)

(ii)  $(D_x^3 - 4D_x^2 D_y + 5D_x D_y^2 - 2D_y^3)z = (y + x)^{1/2}$ . (15)

(iii)  $(x^2 D_x^2 - 4y^2 D_y^2 - 4y D_y - 1)z = x^2 y^3 \ln y$ . (16 2/3)

7. (a) Find the Fourier series of  $f(x) = \begin{cases} 0 & -5 < x < 0 \\ 2 & 0 < x < 5 \end{cases}$ , period = 10. How should  $f(x)$  be defined at  $x = -5, x = 0$  and  $x = 5$  in order that the Fourier series will converge to  $f(x)$  for  $-5 \leq x \leq 5$ ? (20)

(b) Using Fourier cosine integral representation of an appropriate function, show that,

$$\int_0^{\infty} \frac{\cos wx}{k^2 + w^2} dw = \frac{\pi e^{-kx}}{2k}; x > 0, k > 0.$$
(10)

(c) Find the Fourier transform of  $f(x) = \begin{cases} a - |x| & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$ . Hence find

$$\int_0^{\infty} \left( \frac{\sin t}{t} \right)^2 dt.$$
(16 2/3)

8. (a) Solve and give physical interpretation of the boundary value problem,  $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$ ;  $y(0, t) = y(5, t) = 0, y(x, 0) = 0, y_t(x, 0) = 3 \sin 2\pi x - 2 \sin 5\pi x$ , where,  $(0 < x < 5, t > 0)$ . (26 2/3)

(b) Deduce Laplace's equation in polar coordinates and hence find the circular harmonics of degree 0 and of degree  $n$ . (20)

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**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Mention the operating temperature and major functions of different sections of blast furnace proper. (15)
- (b) How can you remove the ultrafine particles of blast furnace gas? (7)
- (c) Explain the steps involved in steelmaking by LD process. (13)
  
2. (a) 'Despite having a little alloying element, high strength low alloy steels (HSLA) show great mechanical properties.' — explain the reasons behind this phenomenon. (12)
- (b) Suppose you have ordinary low carbon steel ( $\%C < 0.2$ ) and the option of adding C, Ni, Cr as alloying elements. Your target is to make steel blades suitable for surgical instruments. State with clear reasoning(s) which alloying elements you will add and in what amount. (15)
- (c) What happens to the mechanical properties of tool steels after tempering? (8)
  
3. (a) A ferrous alloy is required that has good castability, good wear resistance and a high degree of ductility. (6+6)
  - (i) Suggest an alloy and explain how each desired property is present in your alloy.
  - (ii) If a little more strength is required instead of ductility, can you think of a way to fulfill the requirement without changing the composition of your chosen alloy?
- (b) Suppose, two samples of 60-40 brass are produced by casting and hot rolled method. What will be the difference in microstructure and mechanical properties of these two samples? (8)
- (c) How can you induce hardness in heat treatable aluminium alloys? (8)
- (d) What is the reason behind the unusual strength of Ni-based superalloys at high temperatures? (7)
  
4. (a) With necessary diagram briefly describe the operating steps of any method, which you think is suitable in detecting internal cracks of a non-ferrous component. (13)
- (b) Which material will be suitable for bone plates which help in pulling bones together to promote healing? Give the reasons for choosing this material. (12)
- (c) Using schematic diagrams of fracture surfaces, briefly state the typical features observed in a material after fatigue failure. (10)

**MME 291/ME**

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) What information may be obtained from an equilibrium diagram? **(5)**  
(b) Bismuth (melting point 520°F) and cadmium (melting point 610°F) are assumed to be completely soluble in the liquid state and completely insoluble in the solid state. They form a eutectic at 290°F containing 40 percent cadmium. Draw the equilibrium diagram to scale on a piece of graph paper labeling all points, lines and areas. For an alloy containing 70 percent cadmium (i) give the temperature of initial solidification (ii) give the temperature of final solidification (iii) give the chemical composition and relative amounts of the phases present at a temperature of 100°F below (i); (iv) Sketch the microstructure at room temperature. **(30)**
6. (a) What is corning in metallurgy? With schematic view show the development of microstructures of an isomorphous alloys during fast (non-equilibrium) cooling. **(2+20)**  
(b) Draw a phase diagram for two metals completely soluble in the liquid state but only partly soluble in the solid state. Label completely the phase diagram and discuss the significance of each line on the diagram. **(5+8)**
7. (a) What is allotropic transformation of iron? **(5)**  
(b) Explain the microstructural changes for hypoeutectoid and hypereutectoid steels cooled slowly from fully austenitic region down to room temperature. **(30)**
8. (a) Distinguish between the following properties of materials: (i) strength and hardness, (ii) modulus of elasticity and modulus of resilience, (iii) stiffness and toughness, (iv) ductility and malleability. **(20)**  
(b) The addition of impurity atoms to a metal will result in the formation of a solid solution and/or a new second phase, depending on the kinds of impurity and their concentrations— explain. **(15)**
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