L-1/T-1/ChE

Date : 04/11/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **EEE 155** (Electrical Engineering Fundamentals)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

The symbols have their usual meaning. Assume suitable values for any missing data.

1. (a) Discuss the significance of the statement: 'Superposition is based on linearity.' Use the principle of superposition to find the voltage V_x in the network shown in Fig. for Q. 1(a). **(18)**

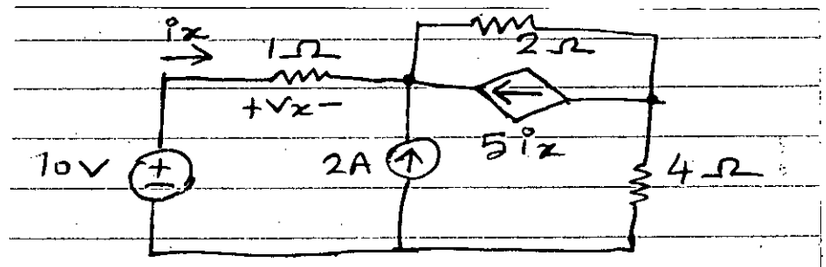

Fig. for Q. 1(a)

(b) Obtain the Norton equivalent circuit at terminals a-b of the network shown in Fig. for Q. 1(b). **(12)**

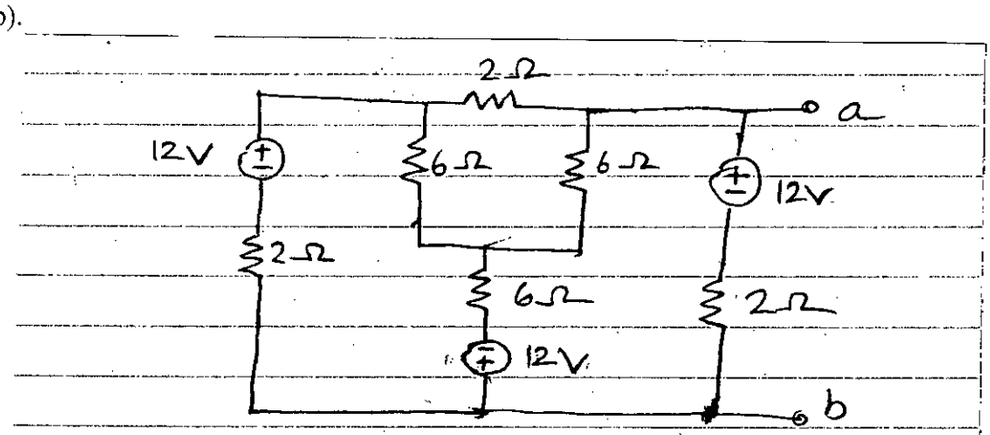

Fig. for Q. 1(b)

2. (a) For the circuit shown in Fig. for Q. 2(a), use the nodal analysis techniques to find the power dissipated in the 1 Ω resistors and the voltage across the 2Ω resistor. **(20)**

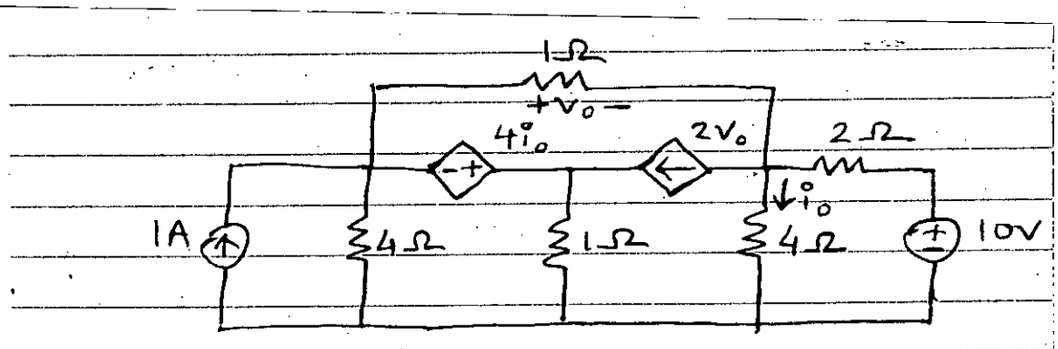

Fig. for Q. 2(a)

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Contd... Q. No. 2

(b) The network shown in Fig. for Q. 2(b) has a total of 14 resistors. Find the equivalent resistance Rab in the circuit. **(15)**

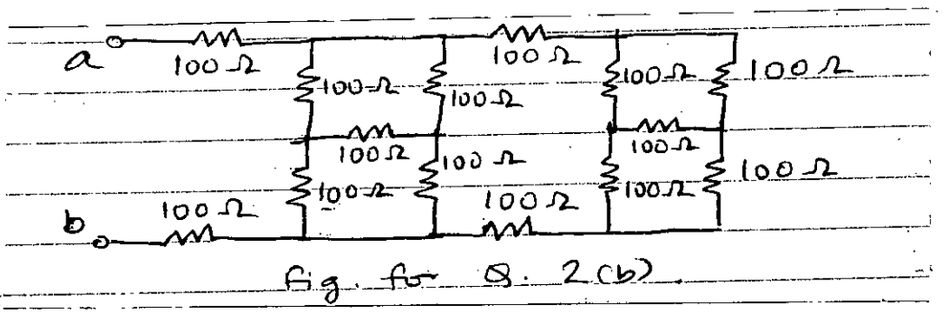

Fig. for Q. 2(b).

3. (a) For the circuit shown in Fig. for Q. 3(a), what resistor connected across terminals a-b will absorb maximum power? What is that power? **(17)**

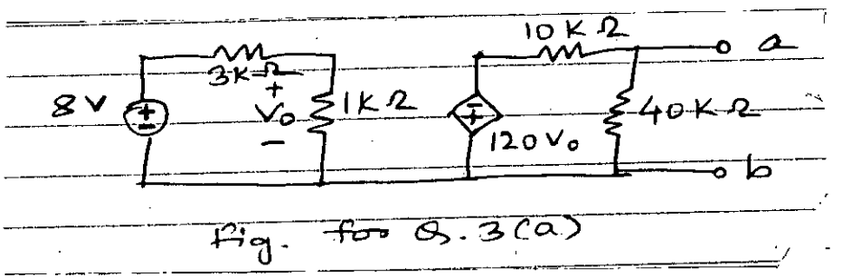

Fig. for Q. 3(a)

(b) For the circuit shown in Fig. for Q. 3(b), if a 5Ω resistor is connected between terminals a and b, find the current through it with the help of the Thevnin's theorem. **(18)**

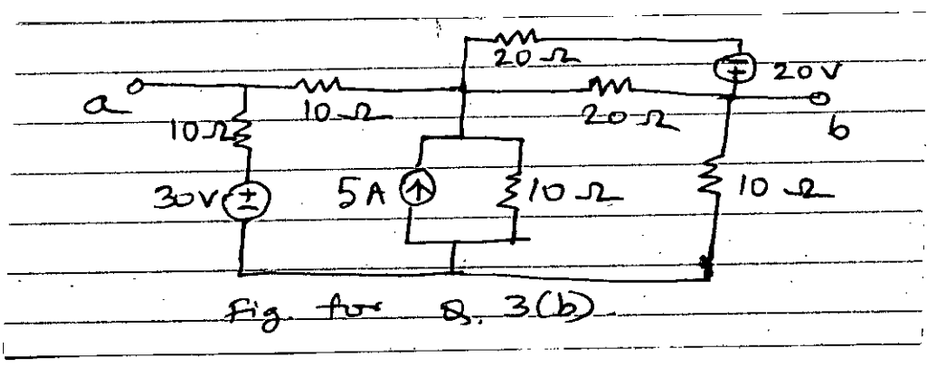

Fig. for Q. 3(b).

4. (a) For the circuit shown in Fig. for Q. 4(a), use an appropriate technique to determine the power absorbed (or delivered) by the source V_2. **(17)**

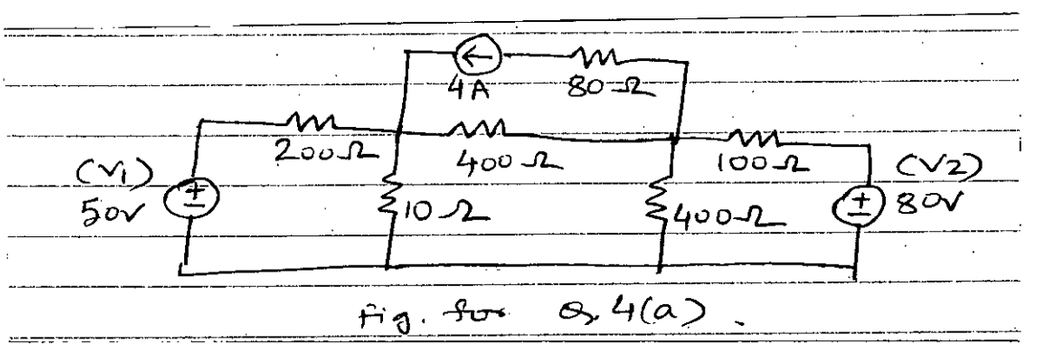

Fig. for Q. 4(a).

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Contd... Q. No. 4

(b) Find the current I in the circuit shown in Fig. for Q. 4(b). **(18)**

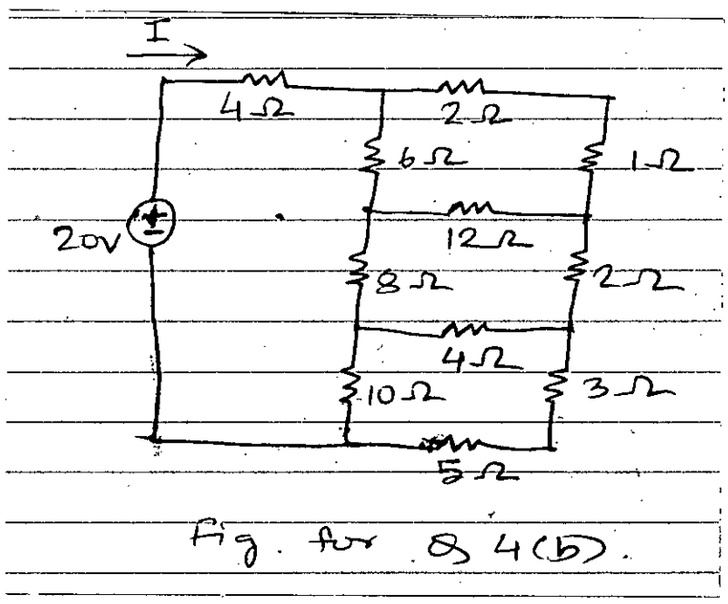

Fig. for Q 4(b).

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Derive the maximum average power transfer theorem for sinusoidal steady state of ac circuits. Explain why the power factor is 1 at the maximum power transfer condition. **(8+2)**

(b) If the current i(t) shown in Fig. for Q. 5(b) is passed through a 4Ω resistor, find the average power absorbed by the resistor. **(15)**

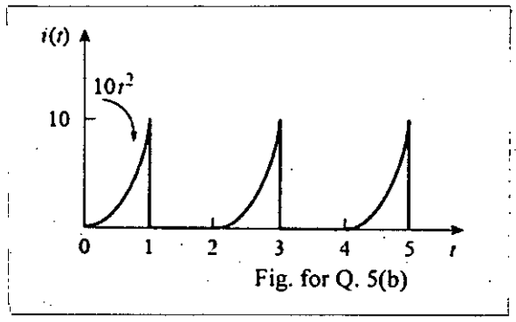

Fig. for Q. 5(b)

(c) From the circuit given shown in Fig. for Q. 5(c) calculate total (i) Real Power (ii) Reactive power (iii) Power factor and (iv) Apparent Power. **(10)**

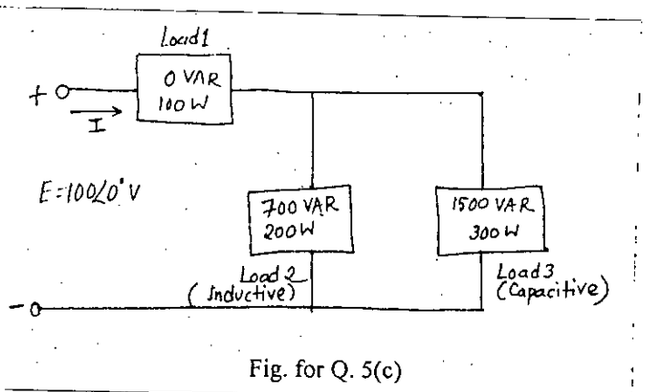

Fig. for Q. 5(c)

Contd P/4

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6. (a) Draw the Phasor Diagram of the circuit given shown in Fig. for Q. 6(a) with V_c as reference. **(10)**

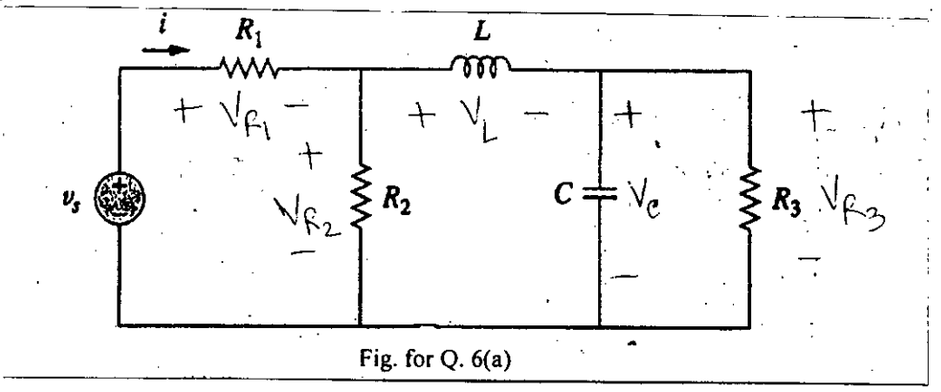

Fig. for Q. 6(a)

(b) Determine the Norton equivalent circuit for the circuit shown in Fig. for Q. 6(b) as seen right from terminals a and b. Use the equivalent circuit to find I_o. Also determine the value of load impedance for maximum average power transfer. What will be the value of power transferred to the load (10-5jΩ) at maximum power transfer condition? **(15+3+5+2)**

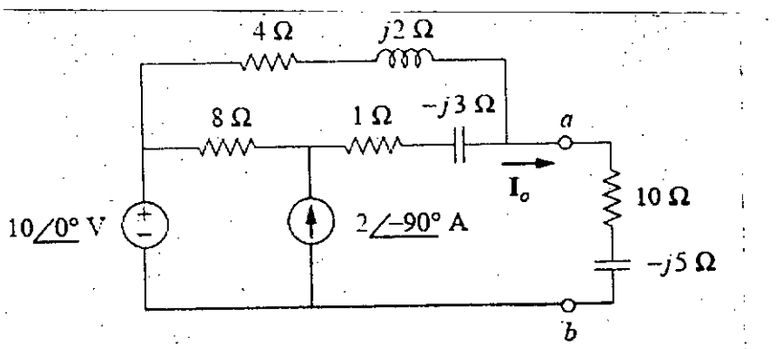

Fig. for Q. 6(b)

7. (a) Determine the equivalent impedance when looking to the right from terminals (a-b) for the circuit given in Fig. for Q.7(a). If a voltage source 100 < 0° volts is connected across terminals a and b, what will be the current leaving the source? Also find the amount of real and reactive supplied by the source. **(18)**

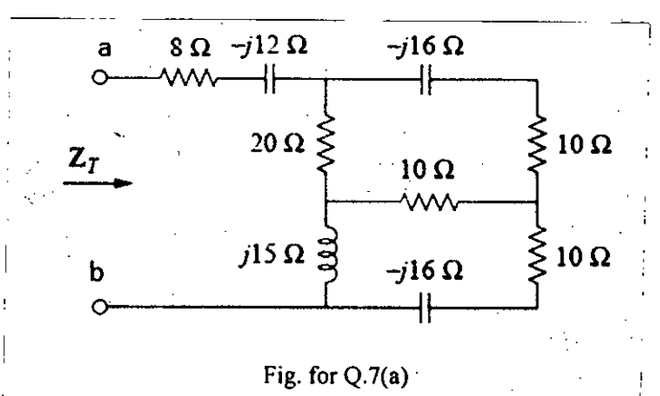

Fig. for Q.7(a)

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Contd... Q. No. 7

(b) Calculate $v_o(t)$ in the circuit given in Fig. for Q.7(b) using Superposition theorem. **(17)**

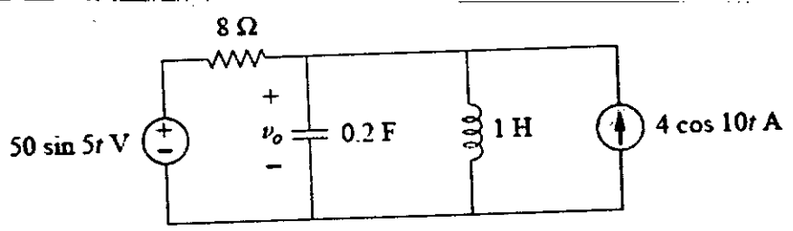


Fig. for Q. 7(b)

8. (a) Determine the current I_1 required to establish a flux of $\emptyset = 2 \times 10^{-4}$ Wb in the magnetic circuit of Fig. for Q.8(a). Neglect Fringing. (Magnetization curve supplied). **(15)**

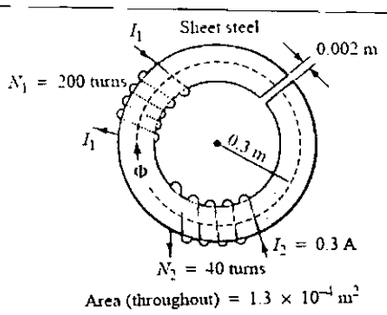
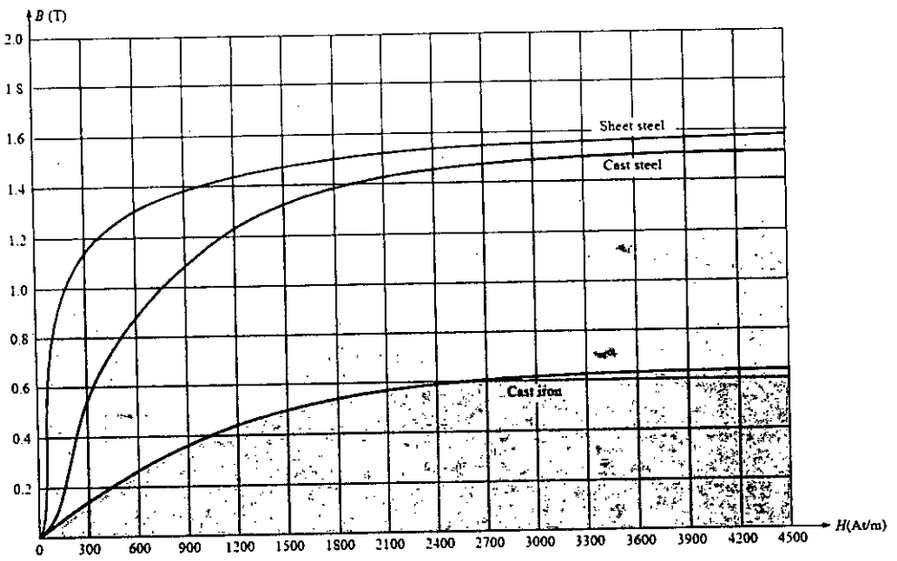


Fig. for Q. 8(a).



Magnetization Curve

(b) Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume that the load is supplied by a 110-V (rms), 60-Hz line. **(12)**

(c) Show that a pure capacitor does not consume any power in a full cycle of the sinusoidal input voltage. **(8)**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub: **ME 141** (Engineering Mechanics)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B as shown in Fig. 1 (a). Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D. **(17)**

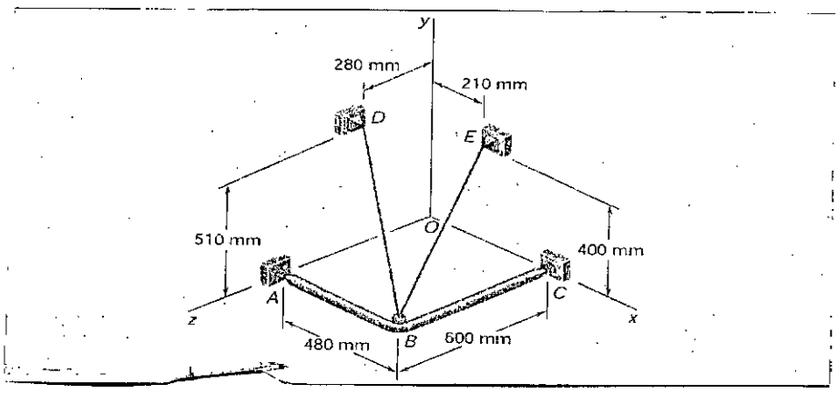


Fig. for Question. No. 1 (a)

(b) Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods as shown in Fig. 1 (b). If a force **P** = (341 N) **j** is applied to collar A, determine (i) the tension in the wire when y = 155 mm, (ii) the magnitude of the force **Q** required to maintain the equilibrium of the system. **(18)**

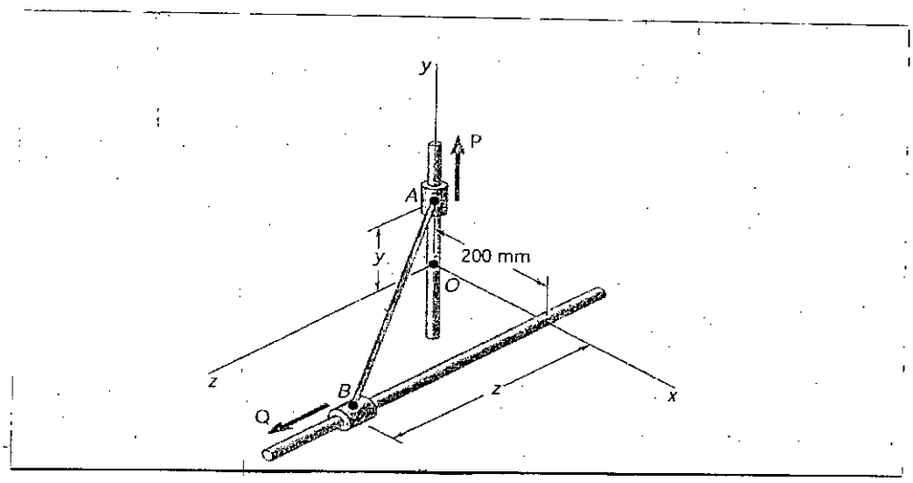


Fig. for Question. No. 1 (b)

Contd P/2

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2. (a) A single force P acts at C in direction perpendicular to the handle BC of the crank as shown in Fig. 2 (a). Determine the moment M_x of P about the x axis when $\theta = 65^0$, knowing that $M_y = -15$ Nm and $M_z = -36$Nm **(18)**

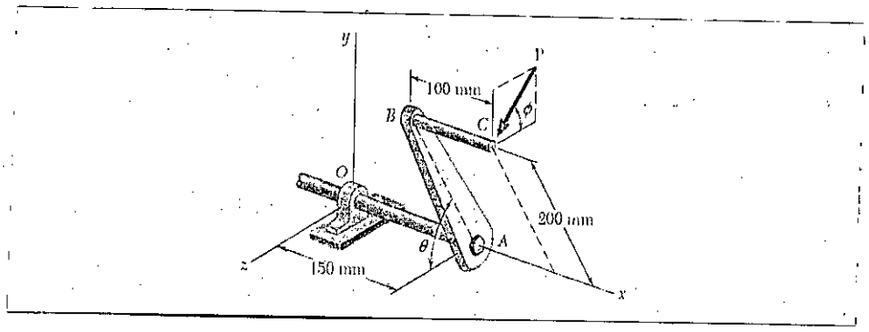

Fig. for Question. No. 2 (a)

(b) A 50-kg crate is attached to the trolley-beam system as shown in Fig. 2 (b). Knowing that a = 1.5 m, determine (i) the tension in cable CD, (ii) the reaction at B. **(17)**

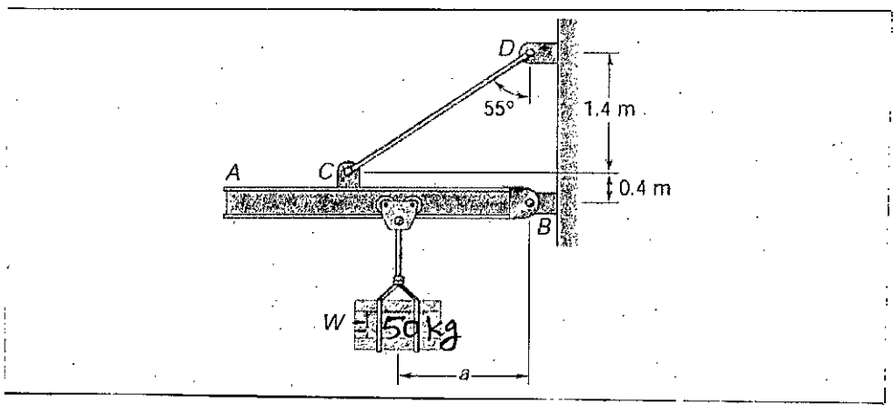

Fig. for Question. No. 2 (b)

3. (a) A wastebasket as shown in Fig. 3(a), designed to fit in the corner of a room, is 16 in. high and has a base in the shape of a quarter circle of radius 10 in. Locate the center of gravity of the wastebasket, knowing that it is made of sheet metal of uniform thickness. **(17)**

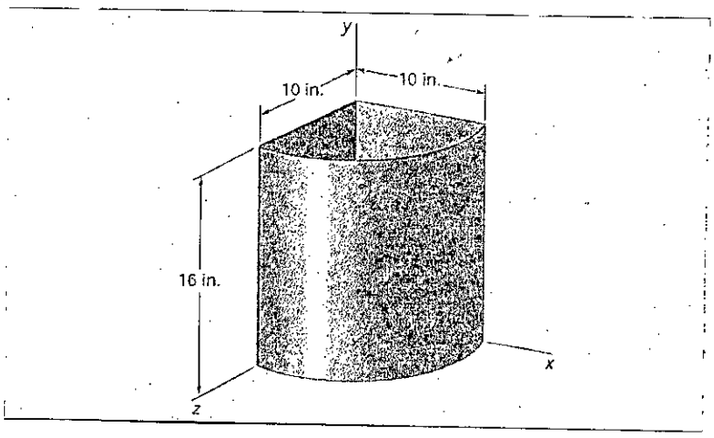

Fig. for Question. No. 3 (a)

Contd P/3

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Contd... Q. No. 3

(b) A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown in Fig. 3(b). Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (i) of the reaction at E, (ii) of the force exerted at C on member CDE. **(18)**

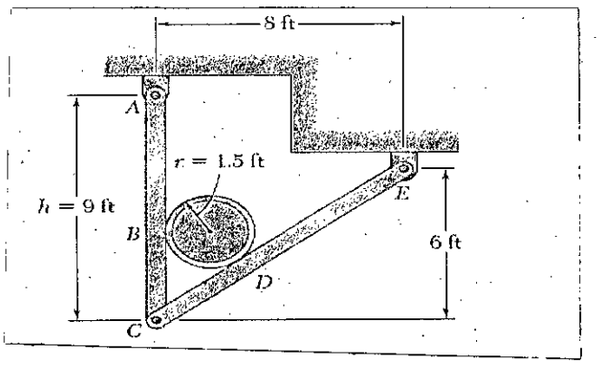

Fig. for Question. No. 3 (b)

4. (a) The position of the automobile jack as shown in Fig. 4(a) is controlled by a screw ABC that is single-threaded at each end (right-handed thread at A, left-handed tread at C). Each tread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple M that must be applied to raise the automobile. **(18)**

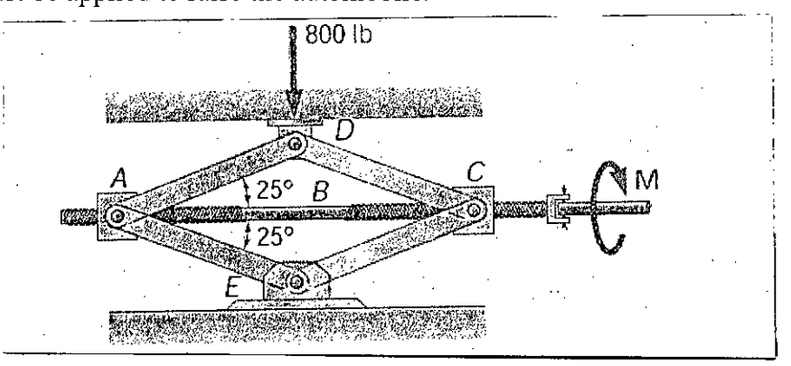

Fig. for Question. No. 4 (a)

(b) Determine the moment of inertia and the radius of gyration of the shaded area shown in Fig. 4(b) with respect to the x axis. **(17)**

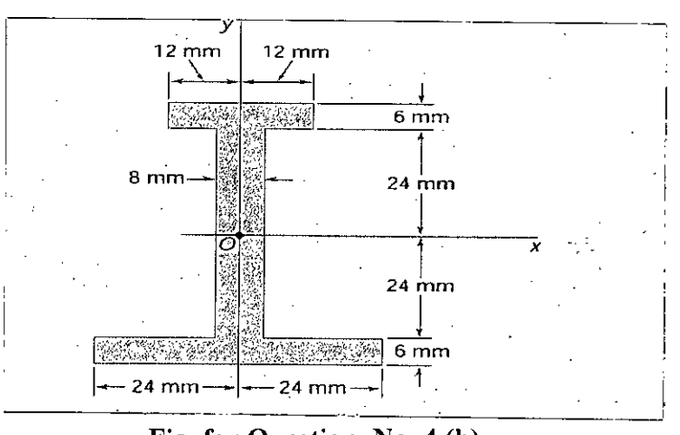

Fig. for Question. No. 4 (b)

Contd P/4

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SECTION – B

There are FOUR questions in this section. Answer any THREE.

5. (a) The rotation of the 0.9-m arm *OA* about *O* is defined by the relation $\theta = 0.15t^2$, where θ is expressed in radians and t in seconds. Collar B slides along the arm in such a way that its distance from *O* is $r = 0.9 - 0.12t^2$, where *r* is expressed in meters and t in seconds. After the arm *OA* has rotated through 30^0, determine (i) the total velocity of collar, (ii) the total acceleration of the collar, (iii) the relative acceleration of the collar with respect to the arm. See figure for Ques. No. 5 (a) **(18)**

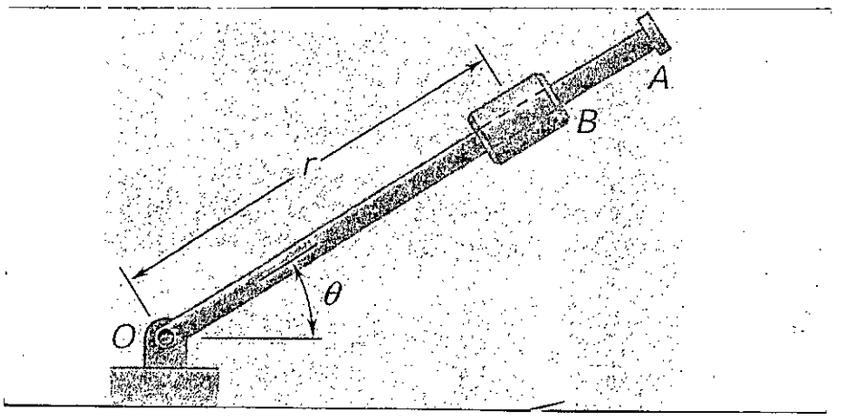

Fig. for Question. No. 5 (a)

(b) The masses of blocks *A*, *B*, *C*, and *D* are 9 kg, 9kg, and 7 kg, respectively. Knowing that a downward force of magnitude 120 N is applied to block D, determine (i) he acceleration of each block, (ii) the tension in cord ABC. Neglect the weights of the pulleys and the effect of friction. See figure for question No. 5 (b) **(17)**

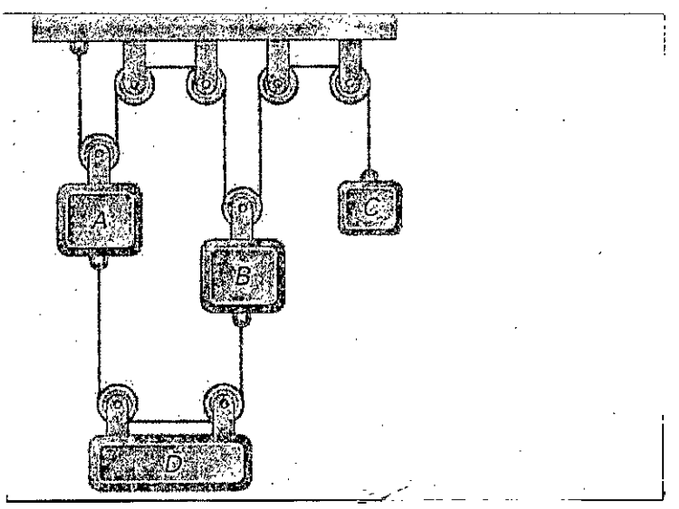

Fig. for Question. No. 5 (b)

Contd P/5

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6. (a) The system shown is at rest when a constant 30-1b force is applied to collar *B*. (i) If the force acts through the entire motion, determine the speed of collar *B* as it strikes the support at *C*. (ii) After what distance d should the 30-1b force be removed if the collar is to reach support *C* with zero velocity? See figure for question No. 6 (a) **(17)**

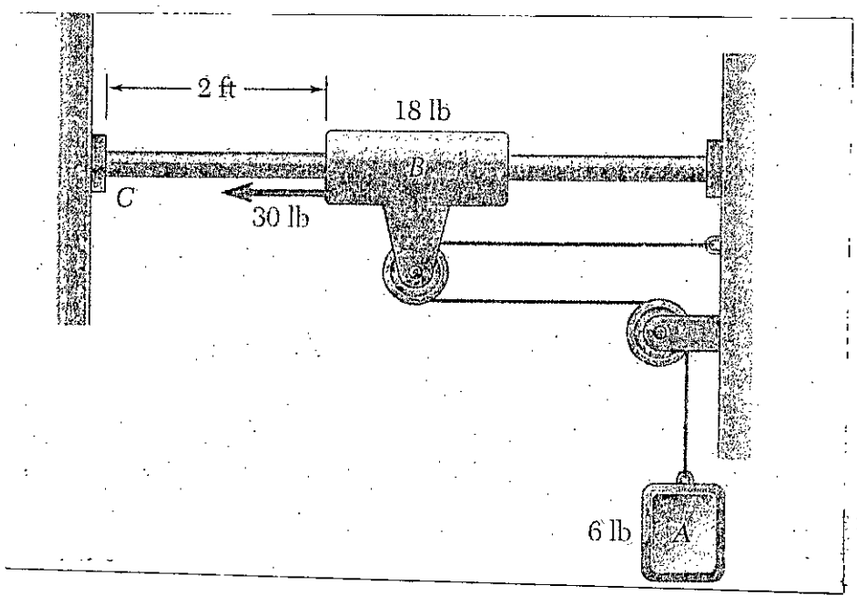

Fig. for Question. No. 6 (a)

(b) A 3-lb collar is attached to a spring and slides without friction along a circular rod in a *horizontal* plane. The spring has an undeformed length of 7 in. and a constant k = 1.5 1b/in. Knowing that the collar is in equilibrium at *A* and is given a slight push to get it moving, determine the velocity of the collar (i) as it passes through *B*, (ii) as it passes through *C*. See figure for question No. 6 (b) **(18)**

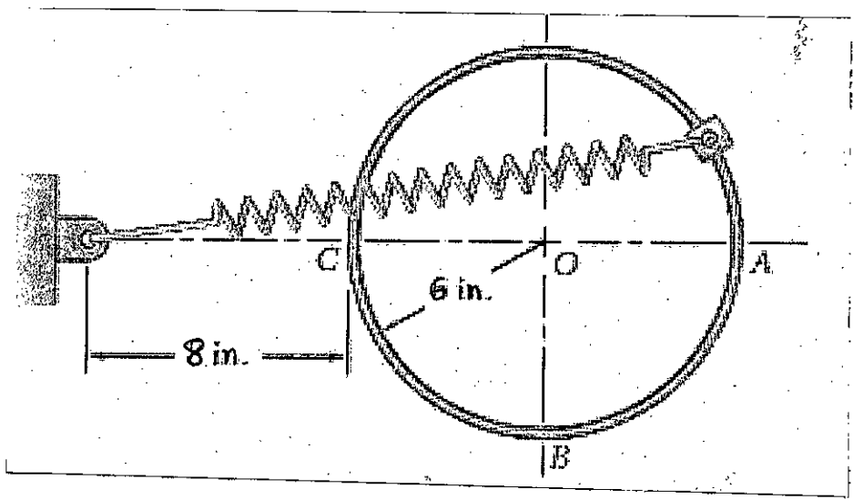

Fig. for Question. No. 6 (b)

Contd P/6

ME 141

7. (a) A circular plate of 120-mm radius is supported by two bearings *A* and *B* as shown. The plate rotates about the rod joining *A* and *B* with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of point *C* is directed to the right, determine the velocity and acceleration of point *E*. See figure for question No. 7 (a) **(17)**

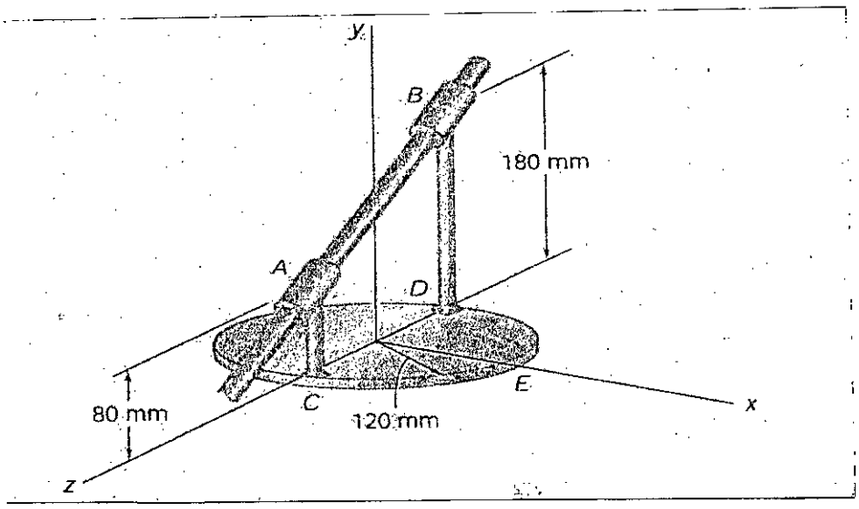

Fig. for Question. No. 7 (a)

(b) Two rods *AB* and *DE* are connected as shown. Knowing that point *D* moves to the left with a velocity of 40 in./s, determine (i) the angular velocity of each rod, (ii) the velocity of point *A*. See figure for question No. 7 (b) **(18)**

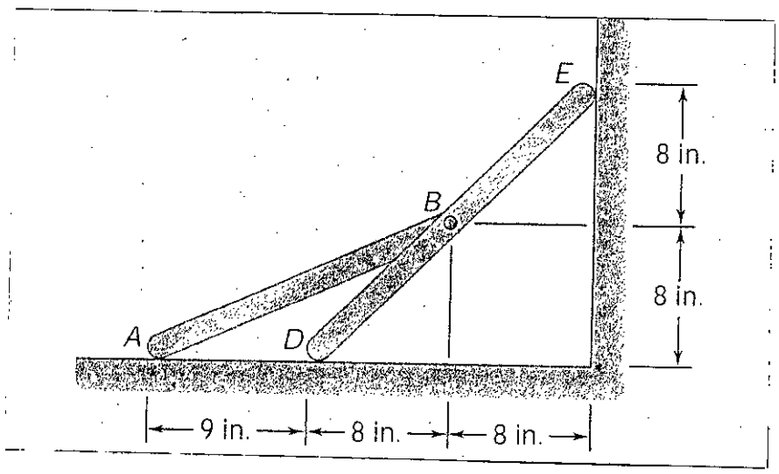

Fig. for Question. No. 7 (b)

Contd P/7

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8. (a) A completely filled barrel and its contents have a combined mass of 90 kg. A cylinder C is connected to the barrel at a height h = 550 mm as shown. Knowing $\mu_s = 0.40$ and $\mu_k = 0.35$, determine the maximum mass of C so the barrel will not tip. See figure for question No. 8 (a) **(17)**

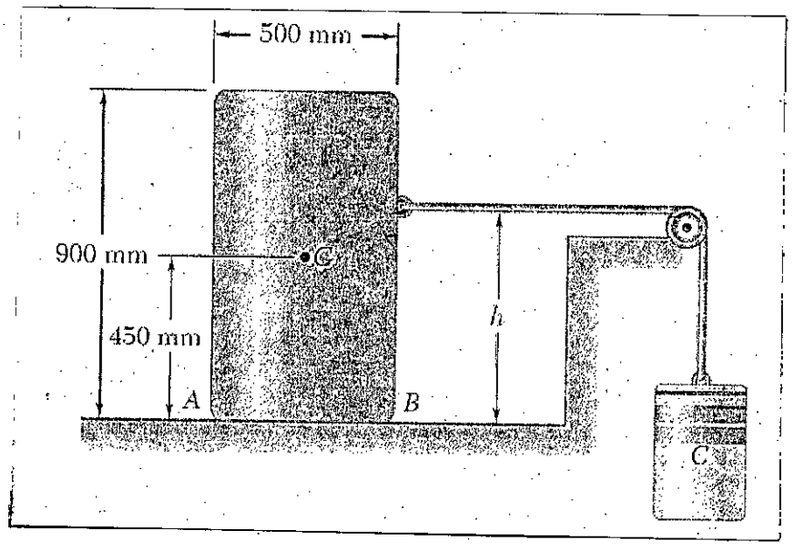

Fig. for Question. No. 8 (a)

(b) The uniform rods AB and BC weigh 2.4 kg and 4 kg, respectively, and the small wheel at C is of negligible weight. If the wheel is moved slightly to the right and then released, determine the velocity of pin B after rod AB has rotated through 90°. See figure for question No. 8 (b) **(18)**

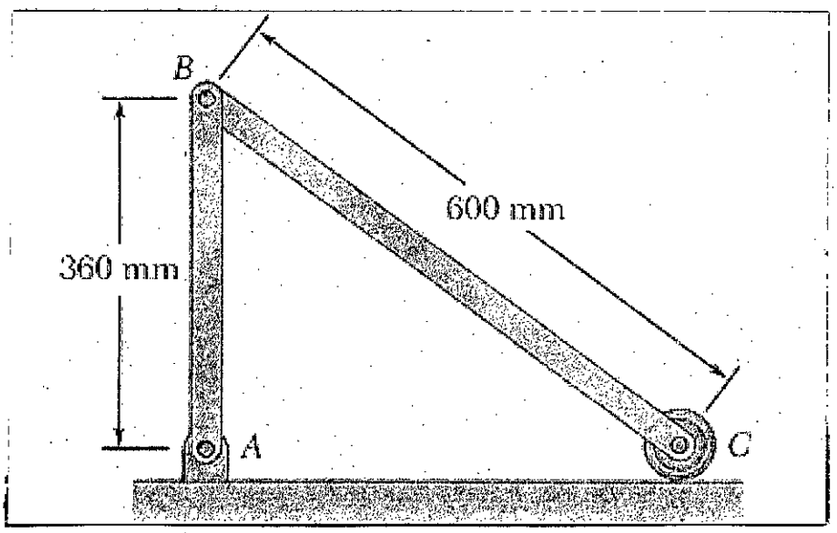

Fig. for Question. No. 8 (b)

L-1/T-1/CHE Date : 27/10/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **PHY 111** (Heat and Thermodynamics, Waves and Oscillation, Physical Optics)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define the terms **(9)**

(i) Root mean square velocity

(ii) Mean free path

(iii) Degrees of freedom

(b) Discuss the deficiencies of the perfect gas equation and obtain the Van der Waal's equation of state for real gas using the pressure correction and volume correction **(20)**

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

Where the symbols have their usual meaning.

(c) At room temperature 27°C, the rms speed of the molecules of a certain diatomic gas is found to be 1920 m/sec. What is the name of this gas? **(6)**

2. (a) Explain the following concepts as related to thermodynamics: **(9)**

(i) thermodynamic system

(ii) Surroundings and boundary

(iii) State of a system

(b) Distinguish between isothermal and adiabatic processes. Obtain an expression for the work done by a perfect gas in expanding (i) isothermally and (ii) adiabatically from a volume V_1 to volume V_2. **(18)**

(c) Derive the following relations for a quasistatic adiabatic process of an ideal gas. **(8)**

(i) pV^{γ} = constant; (ii) $TV^{\gamma-1}$ = constant

3. (a) Define entropy and state briefly its physical significance. Discuss the meaning of the term 'heat death' of the universe. **(10)**

(b) Show that the entropy remain constant in an isolated reversible process and increase in an irreversible process. **(19)**

(c) Calculate the change in entropy of a system containing 1 kg ice at 0°C which melts at the same temperature. Latent heat of ice is 79.6 kcal/kg. **(6)**

4. (a) What are reverberation and reverberation time? What are the requirements of a good auditorium? **(7)**

Contd P/2

PHY 111

Contd... Q. No. 4

(b) What are the assumptions of Sabines? Deduce an expression for the decay of intensity of sound in a room and hence found an expression for reverberation time. **(20)**

(c) Find reverberation time of a room 8 m wide, 12 m long and 3 m high. The ceiling and wall are concrete the floor is tiles and there are 50 people standing in the room. Absorption coefficient of concrete and titles is 0.03 and 0.01 respectively. Absorption power per person is 0.5 Sabine. **(8)**

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What are Lissajous' figure? On what factors do they depend? Explain how these figures are useful in the laboratory? **(7)**

(b) Derive a general expression for the resultant vibration of particle simultaneously acted upon by two initially perpendicular simple harmonic vibrations having same frequency but different phase and amplitude. What happens if the two vibrations are in (i) the same phase, and (ii) opposite phase? **(20)**

(c) A particle of mass 100 g is oscillating in a field of potential $U = 5x^2 + 10$ ergs/g. Find the frequency of oscillation. **(8)**

6. (a) What is phase velocity? Find the relation between group velocity and phase velocity. When does the group velocity become equal to the phase velocity? **(7)**

(b) Define energy density and energy current of a plane progressive wave. Obtain expression for them. **(20)**

(c) A progressive wave is represented by the equation **(8)**

$y = 0.25 \sin \frac{\pi}{65}(68000t - 2x)cm$. If the wave is travelling through a medium of density 0.00129 g/cm^3, find the flow of energy across a square centimeter per second.

7. (a) Explain the principle of superposition. What is meant by "Interference of light"? **(7)**

(b) Describe Young's double-slit experiment. Discuss interference of light analytically and obtain the conditions of maximum and minimum intensities. **(20)**

Contd P/3

PHY 111

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(c) What is fringe width? In Young's double-slit experiment, the separation of the slits is 1.9 mm and the fringe spacing is 0.31 mm at a distance of 1 m from the slits. Calculate the wavelength of light. **(8)**

8. (a) What is diffraction of light? **(5)**

(b) Derive an expression for the intensity distribution in the Fraunhofer diffraction pattern at N-slits. **(12)**

(c) State Brewster's law. Show that at the polarizing angle of incidence, the reflected and refracted rays are perpendicular to each other. **(10)**

(d) What is quarter wave plate? The critical angle for certain wavelength of light in the case of a piece of glass is 40°. Find the polarizing angle for glass. **(8)**

L-1/T-1/ChE Date : 11/11/2019

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **CHEM 111** (Inorganic Chemistry)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) State Werner theory of Coordination compounds with its limitations. How did Alfred Werner develop the theory? **(8+4)**

   (b) How does splitting of d-orbitals occur when octahedral legend field interacts with transition metal? **(5)**

   (c) What do you mean by stability constant of a complex compound? Establish a relationship between stepwise and overall stability constant. **(6)**

   (d) The complexes $[Fe(NH_3)_6]^{+2}$ and $[Fe(CN)_6]^{4-}$ in solution give maximum absorption bands at the frequencies of 12.25 and 32.90 cm^{-1} respectively. The pairing Energy (E_p) value for Fe^{2+} is 50 k.Cal./mol. **(12)**

   (i) Calculate the Crystal field splitting energy (CFSE) of the complexes in K.Cal./mol. for high and low spin complexes.

   (ii) Do the complexes have high and low spin configurations?

   (iii) Draw the 'd' orbital splitting diagram for each.

2. (a) What do you mean by binary and ternary complex compounds? Illustrate your answer with examples. **(7)**

   (b) What are the methods to express quantitatively the stability of a ternary complex? Discuss their merits and demerits. **(10)**

   (c) Discuss the mechanism of the substitution reaction in metal complex compounds. **(10)**

   (d) Discuss John Teller effect in the formation of complex compounds. **(8)**

3. (a) Discuss the classification of Lewis acids and ~~bafer~~ Ligands. Give two examples from each class. **(8)**

   (b) Explain Hard and Soft Acid and Base principle with examples. **(7)**

   (c) What are the characteristics of Hard and Soft Acids and Bases? Explain with example. **(10)**

Contd P/2

CHEM 111

Contd ... Q. No. 3

(d) You are supplied with H_2SO_4 of specific gravity 1.85. Calculate the amount of Distilled water required to convert its specific gravity 1.70. **(10)**

4. (a) Justify the following statements: **(18)**

(i) Coordination number of Co^{3+} ion is six, and Coordination number of Zn^{2+} ion is four.

(ii) $[Fe(CN)_6]^{3-}$ is a low spin complex but $[Fe(H_2O)_6]^{2+}$ is a high spin complex.

(iii) Complex compounds are generally colored.

(iv) Perchloric acid is stronger than sulphuric acid.

(v) BF_3 is an acid but NH_3 is a base.

(vi) k_1^H is greater than k_2^H for oxalic acid.

(b) Discuss the factors which causes huge impact for determining the stability constant of the ternary complexes. Illustrate your answer with example. **(17)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) How does de Broglie's hypothesis account for the fact that the energies of the electron in a hydrogen atom are quantized? **(5)**

(b) Apply your knowledge about atomic absorption spectra to explain the quantization of energy. What is the ratio of the energy of a ground-state He^+ ion to that of a Be^{3+} ion? **(5+5)**

(c) Why does the Bohr model of Hydrogen atom violate the uncertainty principle? "The concept of electron density is better suited to describe the position of an electron in the quantum mechanism model of an atom." – justify. **(6+4)**

(d) According to the Born interpretation, the probability of finding an electron in a volume element $d\tau$ is proportional to $\psi^2 d\tau$. (i) What is the most probable location of an electron in an H atom in its ground state? (ii) What is its most probable distance from the nucleus, and why is this different? **(4+6)**

6. (a) Calculate the energies needed to move an electron from the n = 5 state to the n = 1 state in the Li^{2+} ion. Determine is the region of electromagnetic radiation of the emitted photon using it's wavelength in nm. The Rydberg constant for hydrogen is $(2.18 \times 10^{-18}$ J). **(7)**

Contd P/3

CHEM 111

Contd ... Q. No. 6

(b) Using the Na (Z = 11) atom as an example, describe the effect of shielding on the 1st ionization energy. Sketch the outline of the periodic table and show group and period trends in the first acidity of oxides, metallic character and atomic size of the elements. **(4+6)**

(c) Magnesium metal is easily deformed by an applied force, whereas magnesium fluoride is shattered. Why do these two solids behave so differently? **(8)**

(d) Explain how the lattice energy of an ionic compound such as MgO can be determined using the Born-Haber cycle. On what law is this procedure based? **(9+1)**

7. (a) Describe the interactions that occur between individual chlorine atoms as they approach each other and form Cl_2. What combination of forces gives rise to the energy holding the atoms together and to the final inter-nuclear distance? **(8)**

(b) In AB_4E arrangement (A is the central atom and E represents lone pair), why does the lone pair occupy an equatorial position rather than axial position? Sketch the bond moments and resultant dipole moments for the following molecules : H_2O, PCl_5, NH_3 **(3+6)**

(c) Find out the Hybridization, geometry and shape of the following ions/molecules by drawing Lewis structure: IF_4^-, IF_2^+, O_3 and N_2O (N is the central atom). **(3×4=12)**

(d) What effect of 2s–2p mixing is observed in relative energy level of molecular orbitals (MOs) in homonuclear diatomic molecules of period 2 elements? **(6)**

8. (a) Use an MO diagram and the bond order you obtain from it to answer: (i) Is Be_2 stable? (ii) Is Be_2^+ diamagnetic? (iii) What is the outer (valence) electron configuration of Be_2^+? **(7)**

(b) Three p orbitals from one atom and Three p orbitals from another atom are combined to form molecular orbitals for the joined atoms. How many MOs will result from this combination? Draw the shapes of the resultant MOs. Arrange them according to energy. **(2+4+4)**

(c) How do nuclear reactions differ from ordinary chemical reactions? Apply Einetein's equation, $E = mc^2$ (m = mass, c = velocity of light in vacuum) to calculate nuclear binding energy for any atom. **(5+5)**

(d) Write a short note on application of radioactive isotopes in medicine. **(8)**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2018-2019

Sub : **MATH 121** (Differential Equations and Co-ordinate Geometry)

Full Marks: 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Discuss the continuity and differentiability at $x = \pi/2$ of the function **(20)**

$$f(x) = \begin{cases} 1 & ; \quad x < 0 \\ 1 + \sin x & ; \quad 0 \le x < \pi/2 \\ 2 + \left(x - \dfrac{\pi}{2}\right)^2 & ; \quad x \ge \dfrac{1}{2} \end{cases}$$

Also, sketch the curve.

(b) Evaluate: (i) $\lim\limits_{x \to 0} \dfrac{xe^x - \ln(1+x)}{x^2}$ **(8)**

(ii) $\lim\limits_{x \to \pi/2} (\sec x)^{\cot x}$ **(7)**

2. (a) If $y = \dfrac{x^2}{(x-1)^2(x+2)}$, find y_n. **(11)**

(b) If $y = \left[x + \sqrt{1+x^2}\right]^m$, then prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(12)**

(c) Expand lnx in power of (x -2). Also, find Lagrange's form of remainder. **(12)**

3. (a) If $u = \sin^{-1}\left(\dfrac{x^2 + y^2 + z^2}{x + y + z}\right)$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = \tan u$. **(11)**

(b) Verify Rolle's theorem for $f(x) = e^{-x}\sin x$ in the interval $(0, \pi)$. **(12)**

(c) Prove that the curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $\dfrac{x^2}{a_1^2} + \dfrac{y^2}{b_1^2} = 1$ will cut orthogonally if $a_1^2 - b_1^2 = a^2 - b^2$. **(12)**

4. (a) Find the pedal equation of the cardioid $r = a(1 + \cos\theta)$. **(11)**

(b) If $u = \dfrac{1}{r}$ and $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$, then prove that $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial y^2} + z\dfrac{\partial^2 u}{\partial z^2} = 0$, where *a, b, c* are constants. **(12)**

(c) Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle. **(12)**

Contd P/2

MATH 121

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used here bear usual meaning

5. (a) Transform the equation $32x^2+52xy-7y^2-64x-52y-148=0$ in rectangular coordinates using suitable translation and rotation of axes so as to remove the terms in x, y and xy. Then identify the conic. **(17)**

(b) Show that the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines if $\frac{a}{h}=\frac{h}{b}=\frac{g}{f}$ and also that the distance between the parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$. **(18)**

6. (a) Find the equation of the pair of lines joining the origin to the points of intersection of the line $\frac{x}{a}+\frac{y}{b}=1$ with the circle $x^2+y^2=c^2$ and deduce that if the line is tangent to the circle then $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$. **(18)**

(b) Prove that if polar of a point P with respect to the circle $x^2+y^2=37$ touches the circle $(x-3)^2+(y+2)^2=25$, the locus of P must be a conic. Find the equation of that conic. **(17)**

7. (a) Find the equation of the circle whose diameter is the common chord of the circles $x^2+y^2+6x+2y+6=0$ and $x^2+y^2+8x+y+10=0$. **(17)**

(b) Find the coordinates of the limiting points of the system of circles co-axial with the circles $x^2+y^2-2x+8y+11=0$ and $x^2+y^2+4x+2y+5=0$. **(18)**

8. (a) Show that the locus of the middle points of the chords of the parabola $y^2=4ax$ which subtends a right angle at the vertex is $y^2-2ax+8a^2=0$. **(17)**

(b) A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the ends of a chord which subtends a right angle at the centre intersect on the ellipse $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$. **(18)**
