L-2/T-2/CE Date : 01/10/2022

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub : **CE 205** (Numerical Methods)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define Gauss quadrature. Also derive associated points and weighing coefficients for n = 3. **(8)**

   (b) A circle of radius R = 3.2 unit and shaded area A = 14.75 unit is shown in Figure 1. Using Newton-Raphson method find θ in degree for corresponding value of A. **(15⅓)**

   $|\theta_{n+1} - \theta_n| \leq 0.001$.

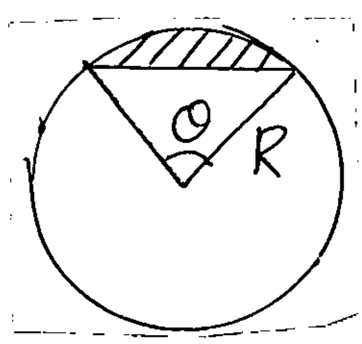

Figure 1

2. (a) Explain the Graphical method for solving mathematical equations with relevant theorems. **(8)**

   (b) Find the deflection at point **a** for the beam shown in Figure 2. **(15⅓)**

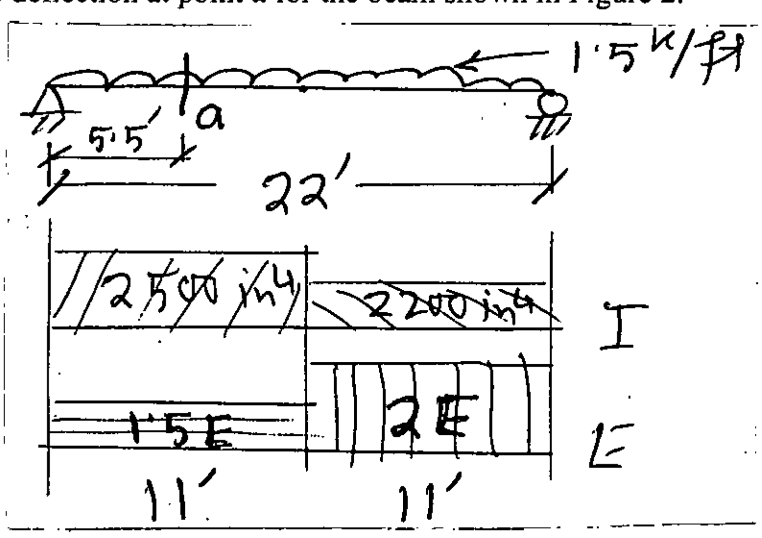

Figure 2

Contd P/2

3. (a) Derive the final expression for the Gregory-Newton Interpolation formula. **(10)**

(b) In the Geotechnical Engineering laboratory, the result of an unconfined compression strength test shows the following stress-strain data. Set a polynomial equation passing through all the points and find the stress corresponding to 12% strain without using the difference table. **($13\frac{1}{3}$)**

Strain %	1	2	3	5	7	10	15	20
Stress in psi	16	30	63	99	131	158	187	195

4. (a) Derive the mathematical molecule for D^3y for forward difference. **(10)**

(b) Estimate the value of the following integral using Gauss Quadrature. **($13\frac{1}{3}$)**

$$I = \int_0^{\pi} \frac{\sin x}{x^2}\, dx$$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Determine the roots of the simultaneous nonlinear equations using the Newton-Raphson method. **(14)**

$$f_1(x, y) = x^2 + 1 - y$$
$$f_2(x, y) = 2\cos(x) - y$$

Use $x_0 = 0.7$ and $y_0 = 1.5$ as your initial guess. Perform two iterations and determine the approximate error at each iteration.

(b) Write down the system of linear equations (in matrix form) to solve the 2-D Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $u(x, y)$ in the region bounded by $0 \le x \le 4$ and $0 \le y \le 4$. **($9\frac{1}{3}$)**

The boundary conditions are:

$u(0, y) = 0$

$u(4, y) = 8 + 2y$

$u(x, 0) = x^2/2$

$u(x, 4) = x^2$

Assume $\Delta x = \Delta y = 1$.

Contd P/3

CE 205

6. (a) Given the table of points **(15)**

x	−1	0	1	2	3
y	6.62	3.94	2.17	1.35	0.89

use least square regression to fit the curve $y = \dfrac{1}{Ax + B}$. Also find the R-square value of the fit.

(b) Derive the expression for the system of linear equations (in matrix form) that can be used to find the best-fit coefficients a_0, a_1 and a_2 for the following relationship: **(8 ⅓)**

$$y = a_0 + a_1x_1 + a_2x_2,$$

where y is s linear function of x_1 and x_2.

7. (a) Estimate y(4) for the following differential equation: **(11)**

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = 2t^2 + 10t + 8$$

Given, $y(0) = 1, y'(0) = -1, y''(0) = 3$.

Use Heun's Method with a step-size, $h = 2$.

(b) Consider the boundary value problem **(12 ⅓)**

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 1$$

with boundary conditions $y(0) = 0$, $y(1) = 0$.

Use finite difference to solve for $y(0.25)$, $y(0.5)$ and $y(0.75)$.

8. (a) The temperature distribution of a long, thin rod of length 0.05 m is described by the equation: **(11 ⅓)**

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where, $k = 1.413 \times 10^{-5}$ m²/sec. Write down the simultaneous equations (in matrix form) to solve the temperature distribution of the rod at $t = 3$ sec using Crank-Nicolson method given the following:

$$\Delta x = 0.01 \text{ m}, \Delta t = 3 \text{ sec},$$

initial condition: the temperature of the rod is 20°C at $t = 0$

boundary condition: $T(0) = 100$°C and $T(0.05) = 25$°C at all times.

(b) Solve the simultaneous equations from part (a) using Gauss-Seidel method. Use any suitable initial guess and perform 3 iterations. **(8)**

(c) What is the difference between explicit and implicit methods of solving partial differential equation? What is the advantage of the implicit method? **(4)**

L-2/T-2/CE **Date: 26/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **CE 207** (Applied Mathematics for Engineers)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Determine the value of $\Gamma\left(\frac{1}{2}\right)$. Use this value to prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x.$ **(15)**

(b) Find the basis of solutions by the Frobenius method for the following ordinary differential equation. **(15)**

$$xy'' + 2y' + xy = 0$$

(c) Write down Legendre Equation and Legendre Polynomial. Use general notations. **(5)**

2. (a) What do you understand by periodic extension of a non-periodic function? What is the limitation in its use in solving differential equations? **(5)**

(b) Is the function shown in Fig. 1 an even or odd function? Express the function as a Fourier Series. **(15)**

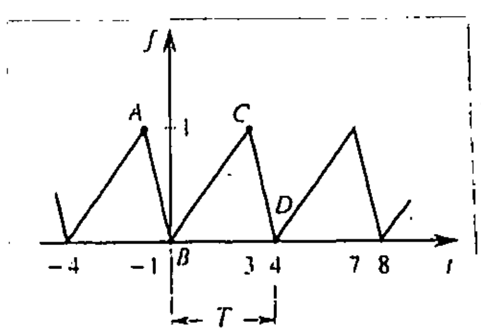

Fig. 1

(c) Solve the linearized differential equation, given below, of a simple pendulum (Fig. 2) for the forcing function shown in Fig. 3. **(15)**

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\theta=F(t)$$

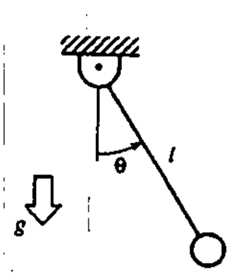

Fig. 2

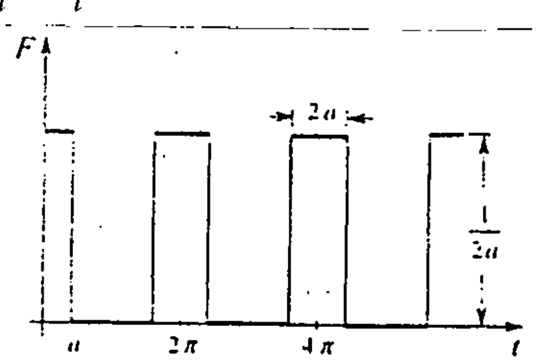

Fig. 3

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CE 207

3. (a) Determine the Fourier integral of the function $H(x)e^{-ax}$. **(15)**

(b) What is the difference in applicability of Fourier Transform and Laplace Transform? **(5)**

(c) Determine the motion of a simple oscillator (Fig. 4) for any arbitrary forcing function F(t) governed by the following differential equation. **(15)**

$$mx'' + kx = F(t)$$

Then derive the solution for the impact $\delta(t)$.

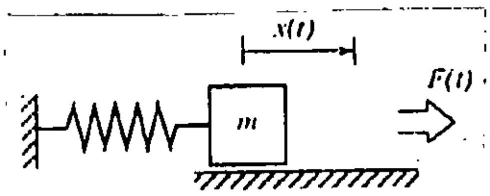

Fig. 4

4. (a) Show the wave equation, given below, is a hyperbolic equation. **(5)**

$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

(b) Solve the following differential equation for the given boundary conditions. **(15)**

$$u'' - 9u = 50e^{-2x}$$

Given : $u'(0) = v_0$ and $u(\infty)$ is bounded.

(c) Determine Fourier Transform of the following vector using FFT. **(15)**

$$f = [0\ 1\ 4\ 9]^T$$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A statistics professor plans classes so carefully that the lengths of her classes are uniformly distributed between 47.0 and 57.0 minutes. Find the probability that a given class period runs between 50.25 and 51.5 minutes. (Round to three decimal places as needed.) **(7)**

(b) A manufacturer of car batteries claims that his batteries will last, on average, 3 years with a variance of 1 year. If 5 of this batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, construct a 95% confidence interval for σ^2 and decide if the manufacturer's claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be approximately normally distributed. **(14)**

Contd P/3

CE 207
Contd... Q. No. 5

(c) A thermometer, with reading at 5°C, is brought into a room whose temperature is 25°C. One minute later the thermometer reading is 12°C. How long does it take until the reading is practically 25°C? Consider that the time rate of change of temperature of a body is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. Also, show the formation of the governing differential equation and solution procedure in details. **(14)**

6. (a) If X_1, X_2, ... X_n are independent Normal random variables with each having the same parameters μ and σ, determine the maximum likelihood estimators of μ and σ (Derive expressions). **(7)**

(b) There are five machines in a factory. Of these machines, three are working properly and two are defective. Machines which are working properly produce articles each of which has independently a probability of 0.1 of being imperfect. For the defective machines this probability is 0.2. A machine is chosen at random and five articles produced by the machine are examined. What is the probability that the machine chosen is defective given that, of the five articles examined, two are imperfect and three are perfect? **(10)**

(c) Solve the following ODE using power series method. Show details. **(18)**

$$y'' - y' + x^2 y = 0$$

7. (a) Define n-th partial sum and remainder with respect to power series solution of differential equation. Show an example. **(5)**

(b) The National Institute of Standards and Technology provides exact data on conductivity properties of materials. Following are conductivity measurements for 11 randomly selected pieces of a particular type of glass: 1.11; 1.07; 1.11; 1.07; 1.12; 1.08; .98; .98; 1.02; .95; .95. Is there convincing evidence that the average conductivity of this type of glass is greater than one? Use a significance level of 0.05. **(15)**

(c) If the occurrences of high winds and earthquakes are unrelated, and if, at a location, the probability of a "high" wind occurring throughout any single minute is 10^{-5} and the probability of a "moderate" earthquake during any signal minute is 10^{-8}. **(15)**

(i) Find the probability of the joint occurrence of the two events during any minute. Building codes do not require the engineer to design the building for the combined effects of these loads. Is this reasonable?

(ii) Find the probability of the occurrence of one or the other or both during any minute. For rate events, i.e. events with small probabilities of occurrence, the engineer frequently assumes $P[A \cup B] \approx P[A] + P[B]$. Comment.

Contd P/4

CE 207

Contd... Q. No. 7(c)

(iii) If the events in succeeding minutes are mutually independent, what is the probability that there will be no moderate earthquakes in a year near this location? In 10 years?

8. (a) X and Y are independent random variables with common density function: **(5)**

$f(x) = e^{-x}$ for $x > 0$

$= 0$ for $x \leq 0$

Determine density function of X/Y.

(b) The tensile strength of a glued joint is related to the glue thickness. A sample of six values gave the following results: **(12)**

Glue Thickness (inches)	0.12	0.12	0.13	0.13	0.14	0.14
Tensile Strength (lbs.)	49.8	46.1	46.5	45.8	44.3	45.9

(i) Calculate the sample correlation coefficient r for these data.

(ii) Use the fitted least squares regression line to predict the tensile strength of a joint for a glue thickness of 0.14 inches.

(c) Check the exactness of the following differential equation and find the general solution.

$$3(x^2 + y^2)dx + x(x^2 + 3y^2 + 6y)dy = 0$$ **(18)**

Table of Fourier Transform

	$f(x)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
1.	$\dfrac{1}{x^2 + a^2} \quad (a > 0)$	$\dfrac{\pi}{a} e^{-a\lvert\omega\rvert}$
2.	$H(x) e^{-ax} \quad (\operatorname{Re} a > 0)$	$\dfrac{1}{a + i\omega}$
3.	$H(-x) e^{ax} \quad (\operatorname{Re} a > 0)$	$\dfrac{1}{a - i\omega}$
4.	$e^{-a\lvert x\rvert} \quad (a > 0)$	$\dfrac{2a}{\omega^2 + a^2}$
5.	e^{-x^2}	$\sqrt{\pi} e^{-\omega^2/4}$
6.	$\dfrac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a > 0)$	$e^{-a^2\omega^2}$
7.	$\dfrac{1}{\sqrt{\lvert x\rvert}}$	$\sqrt{\dfrac{2\pi}{\lvert\omega\rvert}}$
8.	$e^{-a\lvert x\rvert/\sqrt{2}} \sin\left(\dfrac{a}{\sqrt{2}}\lvert x\rvert + \dfrac{\pi}{4}\right) \quad (a > 0)$	$\dfrac{2a^3}{\omega^4 + a^4}$
9.	$H(x + a) - H(x - a)$	$\dfrac{2 \sin \omega a}{\omega}$
10.	$\delta(x - a)$	$e^{-i\omega a}$
11.	$f(ax + b) \quad (a > 0)$	$\dfrac{1}{a} e^{ib\omega/a} \hat{f}\left(\dfrac{\omega}{a}\right)$
12.	$\dfrac{1}{a} e^{-ibx/a} f\left(\dfrac{x}{a}\right) \quad (a > 0,\ b \text{ real})$	$\hat{f}(a\omega + b)$
13.	$f(ax) \cos cx \quad (a > 0,\ c \text{ real})$	$\dfrac{1}{2a}\left[\hat{f}\left(\dfrac{\omega - c}{a}\right) + \hat{f}\left(\dfrac{\omega + c}{a}\right)\right]$
14.	$f(ax) \sin cx \quad (a > 0,\ c \text{ real})$	$\dfrac{1}{2ai}\left[\hat{f}\left(\dfrac{\omega - c}{a}\right) - \hat{f}\left(\dfrac{\omega + c}{a}\right)\right]$
15.	$f(x + c) + f(x - c) \quad (c \text{ real})$	$2\hat{f}(\omega) \cos \omega c$
16.	$f(x + c) - f(x - c) \quad (c \text{ real})$	$2i\hat{f}(\omega) \sin \omega c$
17.	$x^n f(x) \quad (n = 1, 2, \ldots)$	$i^n \dfrac{d^n}{d\omega^n} \hat{f}(\omega)$
	Linearity of transform and inverse:	
18.	$\alpha f(x) + \beta g(x)$	$\alpha \hat{f}(\omega) + \beta \hat{g}(\omega)$
	Transform of derivative:	
19.	$f^{(n)}(x)$	$(i\omega)^n \hat{f}(\omega)$
	Transform of integral:	
20.	$f(x) = \int_{-\infty}^{x} g(\xi)\, d\xi$, where $f(x) \to 0$ as $x \to \infty$	$\hat{f}(\omega) = \dfrac{1}{i\omega} \hat{g}(\omega)$
	Fourier convolution theorem:	
21.	$(f * g)(x) = \int_{-\infty}^{\infty} f(x - \xi) g(\xi)\, d\xi$	$\hat{f}(\omega) \hat{g}(\omega)$

Tables of Fourier Cosine and Sine Transforms

$f(x)$	$\hat{f}_C(\omega) = \int_0^\infty f(x)\cos\omega x\,dx$
1C. e^{-ax} $(a > 0)$	$\dfrac{a}{\omega^2 + a^2}$
2C. $x^n e^{-ax}$ $(a > 0)$	$\dfrac{n!\,\mathrm{Re}\,(a + i\omega)^{n+1}}{(\omega^2 + a^2)^{n+1}}$ (Re = real part)
3C. $\dfrac{1}{x^2 + a^2}$ $(a > 0)$	$\dfrac{\pi}{2a} e^{-a\omega}$
Linearity of transform and inverse:	
4C. $\alpha f(x) + \beta g(x)$	$\alpha \hat{f}_C(\omega) + \beta \hat{g}_C(\omega)$
Transform of derivative:	
5C. $f'(x)$	$\omega \hat{f}_S(\omega) - f(0)$
6C. $f''(x)$	$-\omega^2 \hat{f}_C(\omega) - f'(0)$
Convolution theorem:	
7C. $\dfrac{1}{2}\int_0^\infty [f(\lvert x-\xi\rvert) + f(x+\xi)]g(\xi)\,d\xi$	$\hat{f}_C(\omega)\hat{g}_C(\omega)$

$f(x)$	$\hat{f}_S(\omega) = \int_0^\infty f(x)\sin\omega x\,dx$
1S. e^{-ax} $(a > 0)$	$\dfrac{\omega}{\omega^2 + a^2}$
2S. $x^n e^{-ax}$ $(a > 0)$	$\dfrac{n!\,\mathrm{Im}\,(a + i\omega)^{n+1}}{(\omega^2 + a^2)^{n+1}}$ (Im = imaginary part)
3S. $\dfrac{x}{x^2 + a^2}$ $(a > 0)$	$\dfrac{\pi}{2} e^{-a\omega}$
Linearity of transform and inverse:	
4S. $\alpha f(x) + \beta g(x)$	$\alpha \hat{f}_S(\omega) + \beta \hat{g}_S(\omega)$
Transform of derivative:	
5S. $f'(x)$	$-\omega \hat{f}_C(\omega)$
6S. $f''(x)$	$-\omega^2 \hat{f}_S(\omega) + \omega f(0)$
Convolution theorem:	
7S. $\dfrac{1}{2}\int_0^\infty [f(\lvert x-\xi\rvert) - f(x+\xi)]g(\xi)\,d\xi$	$\hat{f}_C(\omega)\hat{g}_S(\omega)$

TABLE A1 Standard Normal Distribution Function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\,dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A2 Values of $\chi^2_{\alpha,n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other chi-square probabilities:
$\chi^2_{[illegible]}$ = 4.2 $P(\chi^2_{[illegible]} <$ [illegible]$)$ = [illegible] $P(\chi^2_{[illegible]} <$ [illegible]$)$ = [illegible]

TABLE A3 Values of $t_{\alpha,n}$

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t probabilities:
$P(T_8 < 2.541) = .9825$ $P(T_8 < 2.7) = .9864$ $P(T_{11} < .7635) = .77$ $P(T_{11} < .934) = .81$ $P(T_{11} < 1.66) = .94$ $P(T_{12} < 2.8) = .984$.

L-2/T-2/CE **Date : 25/09/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub : **CE 213** (Mechanics of Solids II)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FIVE** questions in this section. Answer **All** questions.

1. Derive the Euler load equation for a column fixed at one end and pinned at the other. Using this equation determine the largest axial compressive load for the column (as shown in Fig. 1) with rectangular structural tubular cross-section. **(10+11)**

   Given: Length = 5 m; σ_y = 250 MPa and E = 200 GPa.

2. **Answer either (a) or (b).** **(21)**

   (a) A simply supported beam is loaded as shown in Fig. 2(a). Determine the stresses at the four corners of the rectangular beam section where maximum bending moment occurs. Draw the stress distribution diagram along the X-section and locate the neutral axis.

   (b) A cantilever beam is loaded as shown in Fig. 2(b). Determine the stresses at the four corners of the rectangular beam section where maximum bending moment occurs. Draw the stress distribution diagram along the X-section and locate the neutral axis.

3. **Answer either (a) or (b).** **(21)**

   (a) A concrete dam has the shape as shown in Fig. 3(a). Determine the maximum compressive stress (bearing stress) on section p-q if the depth (h) of water behind the dam is 10 meter.

   Given: $\gamma_{concrete}$ = 24 kN/m^3; γ_{water} = 10 kN/m^3

   (b) A cantilever beam has the profile as shown in Fig. 3(b). A load P = 35 kN is applied at point 'D'. Determine the resultant stresses at A and B at the support.

4. **Answer either (a) or (b).** **(21)**

   (a) For the structure shown in Fig. 4(a) determine the normal and shear stresses at point 'A'. Sketch an element at point 'A' and show the magnitude and sense of the stresses on each face. Now using Mohr's Circle of stresses determine the principal stress at 'A' and show their sense on a properly oriented element.

   (b) For the structure shown in Fig. 4(b) determine the normal and shear stresses at point A. Sketch an element at A and show the magnitude and sense of the stresses on each face. Now using Mohr's Circle of stresses determine the principal stress at 'A' and show their sense on a properly oriented element.

Contd P/2

5. **Answer either (a) or (b).** **(21)**

(a) The state of stress at a point is shown in Fig. 5(a). Using the stress transformation equation, determine the normal and shear stress acting on the indicated inclined planes. Show the results on properly oriented element.

(b) The state of stress at a point is shown in Fig. 5(b). Using the stress transformation equation, determine the normal and shear stresses acting on the indicated inclined planes. Show the results on properly oriented element.

SECTION – B

There are **FIVE** questions in this section. Answer **All** questions.

6. **Answer either (a) or (b).** **(21)**

(a) For the beam shown in Fig. 6 determine the vertical deflection and slope at point C following direct integration method. Consider EI constant throughout the length.

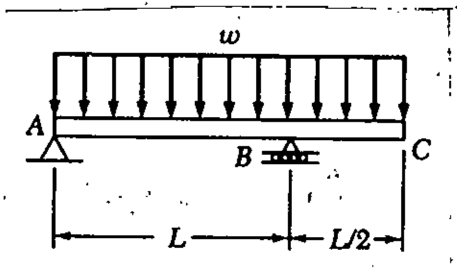

Fig. 6

(b) For the beam shown in Fig. 7 determine the vertical deflection at C and slope at point B following direct integration method. Consider EI constant throughout the length.

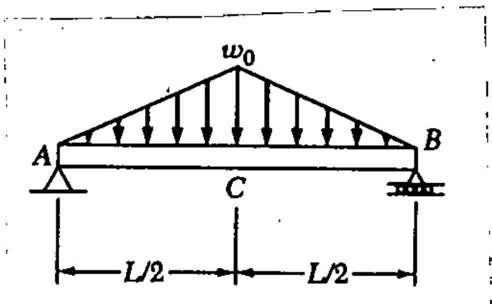

Fig. 7

7. **Answer either (a) or (b).** **(21)**

(a) In Fig. 8, the rod is constructed from two shafts for which the moment of inertia of AB is I and for BC is $2I$. Determine the maximum slope and deflection of the rod due to the loading. Consider $L = 8$ ft, $P = 25$ kip and $EI = 1000$ k-ft^2.

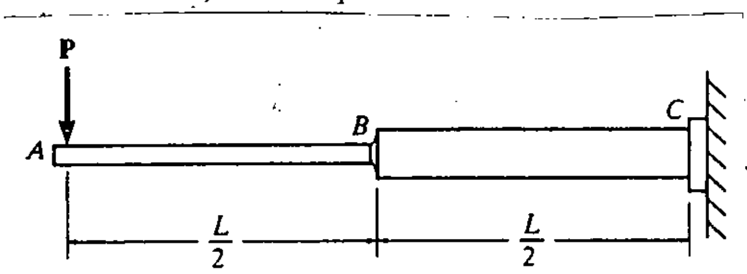

Fig. 8

(b) For the beam shown in Fig. 9, determine the moment at A and vertical reactions at A and B following moment-area method.

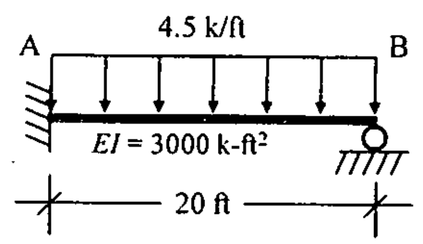

Fig. 9

8. **Answer either (a) or (b).** **(21)**

(a) Determine the unstretched length of the cable shown in Fig. 10. Given, cross sectional area of the cable is 0.10 in^2, Young's modulus = 29000 ksi.

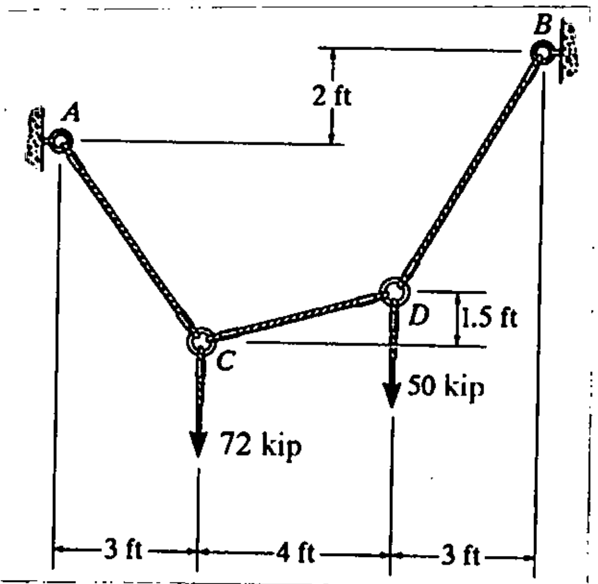

Fig. 10

(b) For the parabolic cable shown in Fig. 11, the maximum allowable cable tension is 40 kip. Determine the location (coordinates x_c and y_c) of the lowest point C from the left support A.

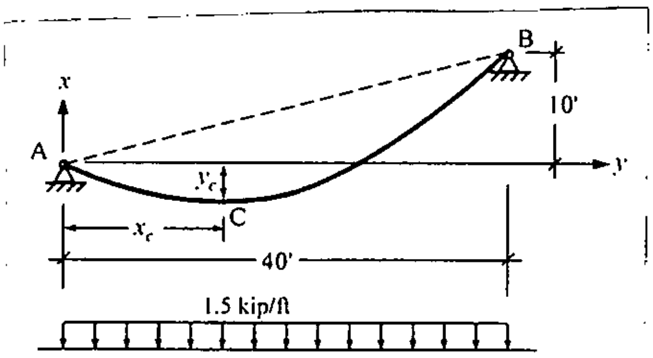

Fig. 11

CE 213

9. **Answer either (a) or (b).** **(21)**

(a) A riveted cylindrical pressure vessel as shown in Fig. 12 is constructed by riveted lap joints with 8 mm dia. rivets. Rivet A are spaced 45 mm apart while Rivet B are spaced 20 mm apart. If the allowable shear capacity of each rivet is 5.0 kN, what maximum internal pressure can be allowed in the vessel?

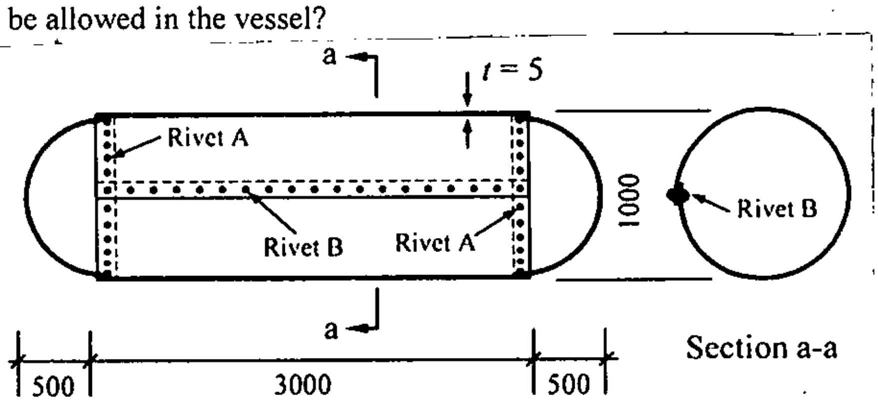

Fig. 12

(b) Determine the maximum magnitude of force P that can be safely applied on the pistons shown in Fig. 13, if the allowable tensile stress in the cylinder material is 3 MPa. The wall thickness of the cylinder is 2.0 mm.

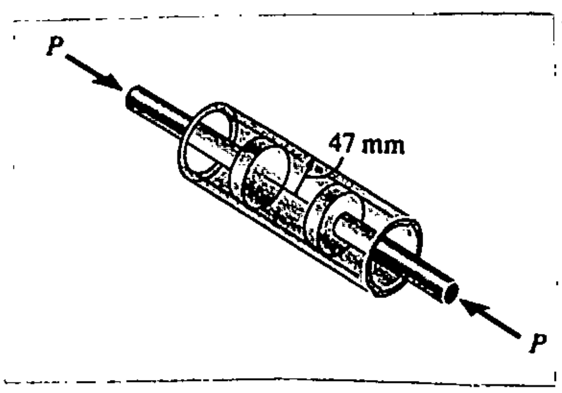

Fig. 13

10. **Answer either (a) or (b).** **(21)**

(a) Based on elastic strain energy principle, determine the vertical deflection of the beam at point C as shown in Fig. 14. For this beam, consider $EI = 29 \times 10^4$ k-in^2.

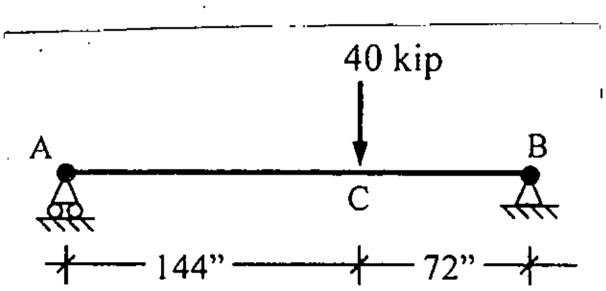

Fig. 14

CE 213

(b) In Fig. 15, members of the truss shown consist of sections of aluminum pipe with the cross-sectional areas indicated. Using Young's modulus = 73 GPa, determine the vertical deflection of point E caused by the load P based on energy principle.

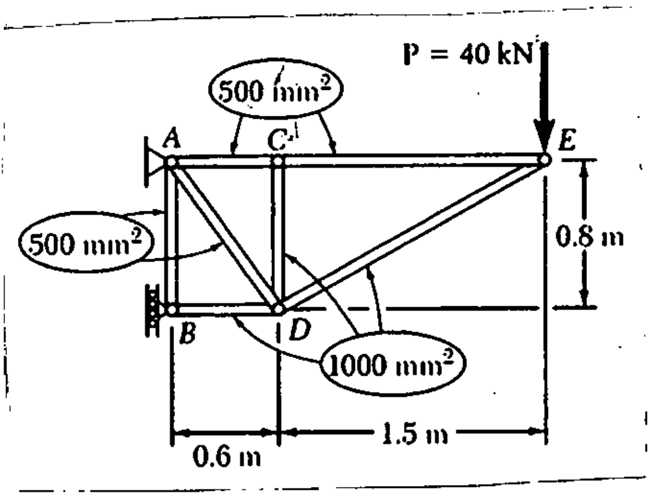

Fig. 15

<u>Parabolic Cable Formulae (symbols and notations have their usual meanings)</u>

$$y = \frac{4hx}{L^2}(x - L) + x\tan\gamma$$

$$T_{max./min.} = H(1 + 16\theta^2 + \tan^2\gamma \pm 8\theta\tan\gamma)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{8\theta x}{L} - 4\theta + tan\,\gamma$$

$$S = \frac{L\sec\gamma}{2}\left(1 + \frac{16\theta^2}{\sec^4\gamma}\right)^{1/2} + \frac{L\sec^3\gamma}{8\theta}\ln\left[\frac{4\theta}{\sec^2\gamma} + \left(1 + \frac{16\theta^2}{\sec^4\gamma}\right)^{1/2}\right]$$

$$\Delta S = \frac{HL}{AE}\left[1 + \frac{16}{3}\theta^2 + \tan^2\gamma\right]$$

$$T_{av} = \frac{HL}{S}\left[1 + \frac{16}{3}\theta^2 + tan^2\gamma\right]$$

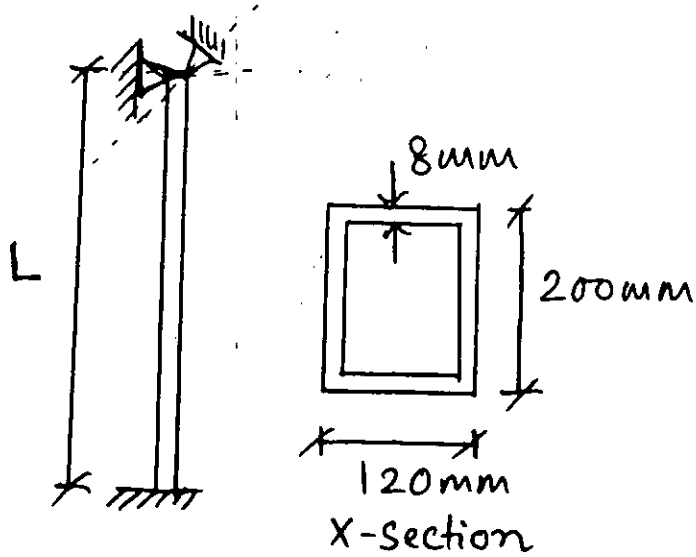

Fig. 1

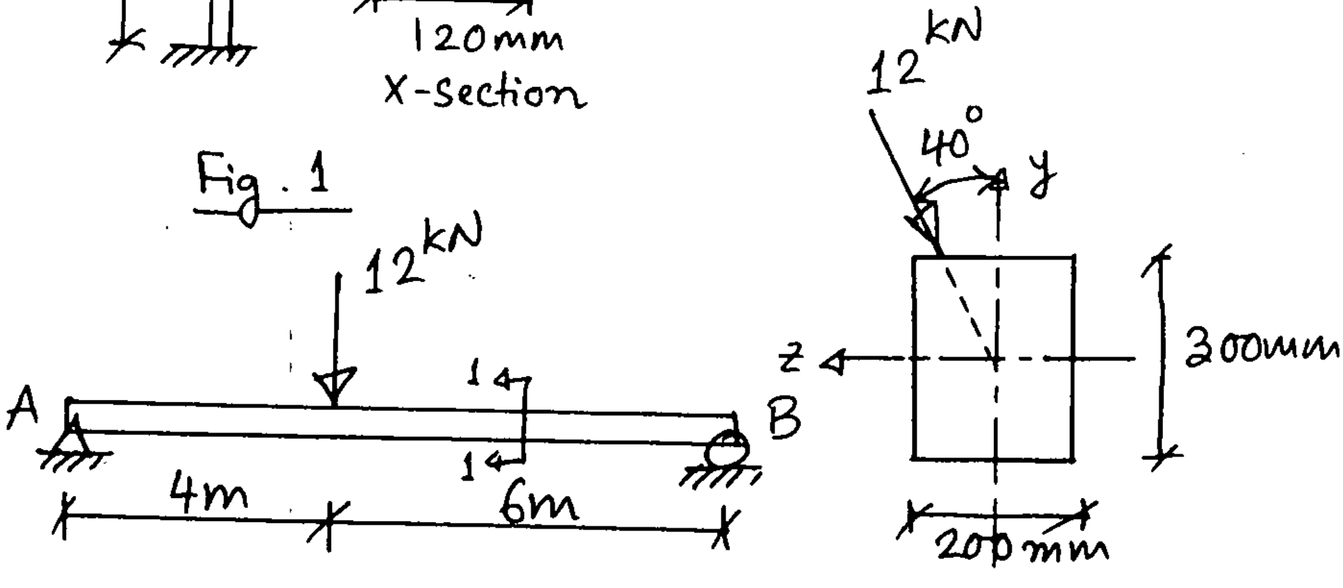

Fig. 2(a)

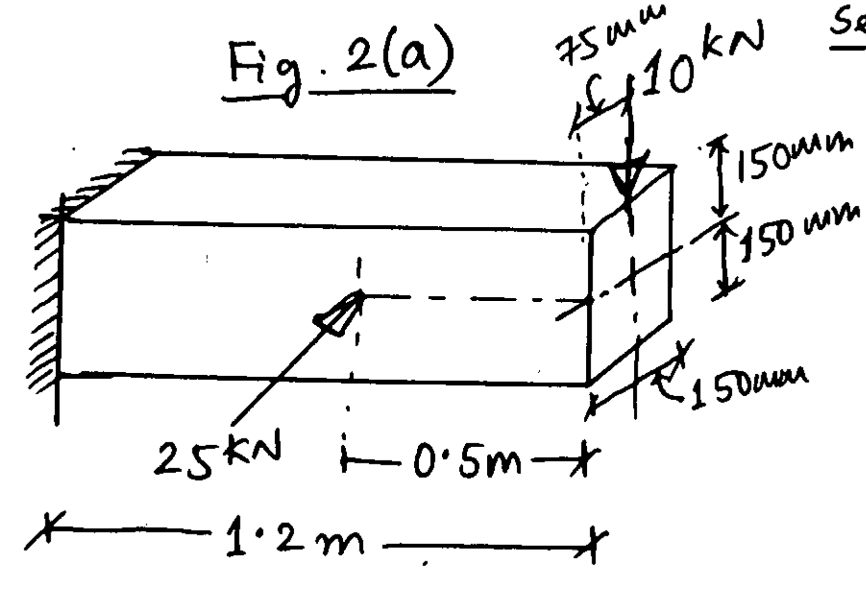

Fig. 2 (b)

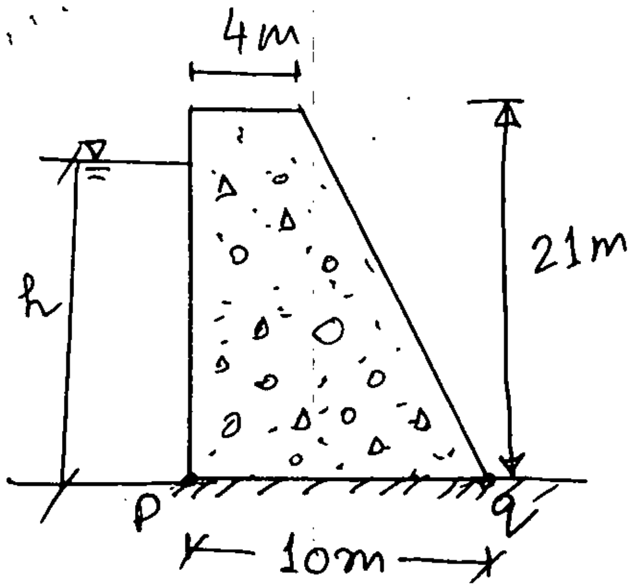

Fig. 3(a)

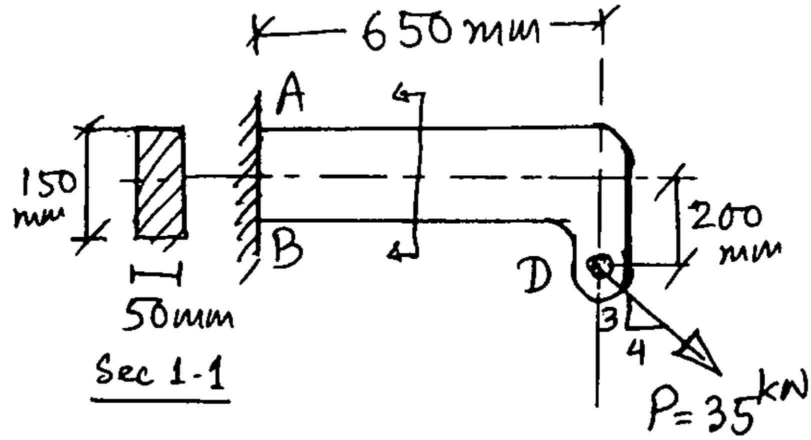

Fig. 3(b)

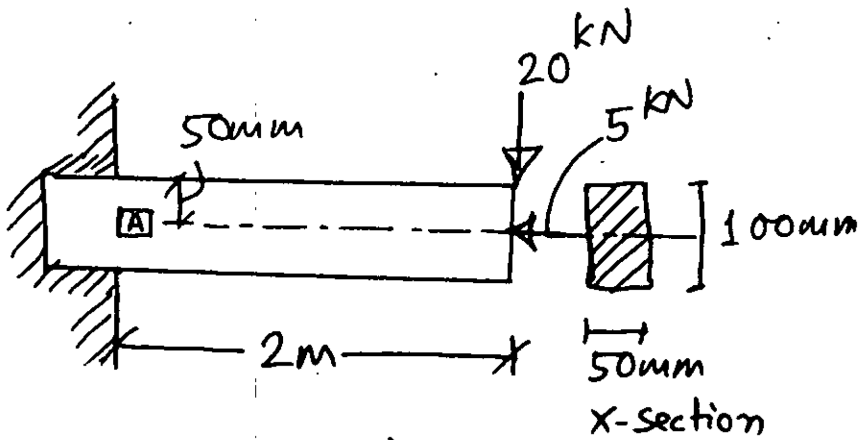

Fig. 4(a)

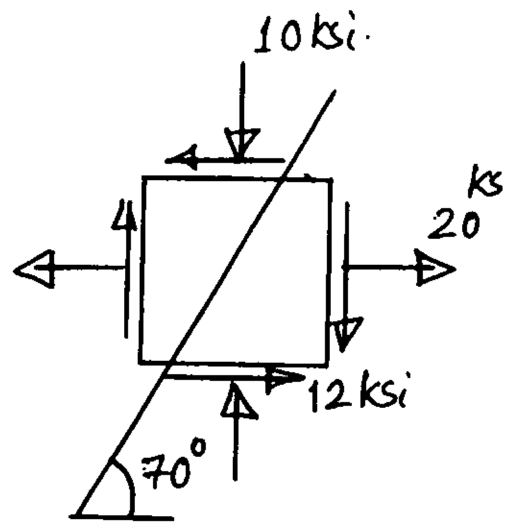

Fig. 5(a)

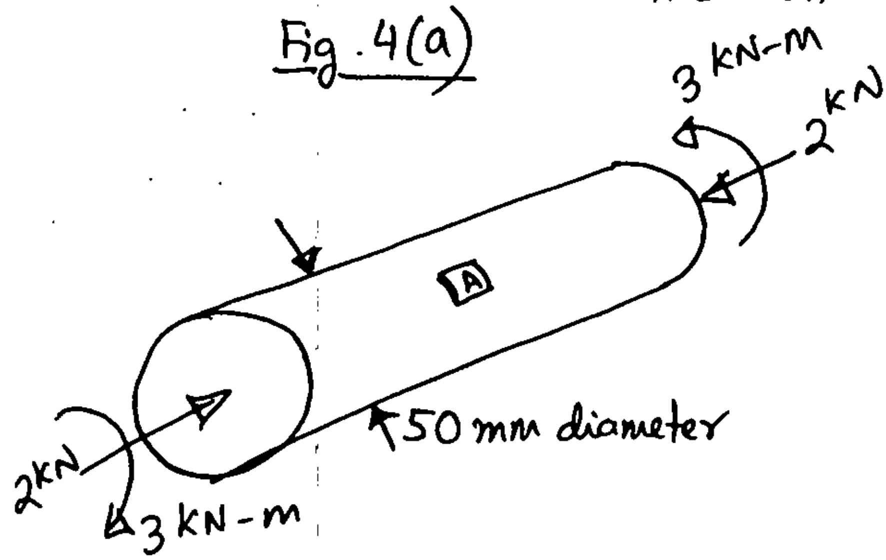

Fig. 4(b)

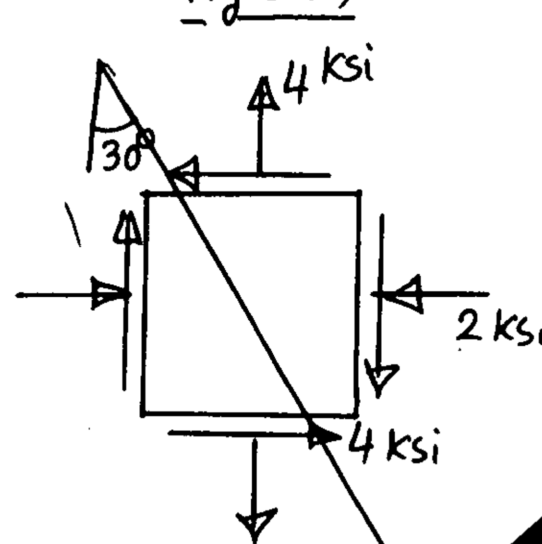

Fig. 5(b)

L-2/T-2/CE **Date: 16/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **WRE 211** (Fluid Mechanics)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Write short notes on viscosity and center of pressure. **(6)**

(b) A small circular jet with a diameter of 200.0 μm comes out from an opening. Given that, the liquid type is mercury. What is the pressure difference between the inside and outside of the jet at 20°C? Given that, surface tension of mercury at 20 °C is 0.514 N/m. **(7)**

(c) A large plate moves with speed V_0 over a stationary plate on a column of oil. If the velocity profile is parabolic, what would be the shear stress on the moving plate from the oil? If a linear velocity profile is assumed, what will be the shear stress on the upper plate? **(12)**

(d) Water in a tank is pressurized to 85.0 cm Hg (Figure 1). Determine the hydrostatic force per meter width on panel AB. **(10)**

2. (a) Prove that for an inclined plane surface submerged in a static fluid, center of pressure is always below center of gravity. **(6)**

(b) Pressurized water fills the tank as shown in Figure 2. Compute the net hydrostatic force on conical surface ABC. **(11)**

(c) Water flows through a vertical 150.0 mm diameter pipe with a velocity of 3.0 m/s. The pipe is standing in a body of water, where its lower end 1.0 m below the surface. Considering all the losses, and friction factor, f = 0.022, find the pressure at a point 3.0 m above the surface of the water for the following conditions. **(18)**

(a) when the flow occurs upward

(b) when the flow occurs downward

3. (a) Derive generalized equation for frictional losses in pipe flow. **(15)**

(b) The flows into and out of a two loop pipe system and the k-values for each pipe are shown in Figure 3. Using Hardy Cross method, determine the flow in each pipe. The head loss is given by $h_L = KQ^2$. Show calculation for two trials. **(20)**

Contd P/2

WRE 211/CE

4. (a) Differentiate between **(12)**

   (i) Absolute pressure and gauge pressure

   (ii) Piezometer and manometer

   (iii) Dynamic viscosity and kinematic viscosity

   (b) A small cylindrical drum 32.0 cm in diameter and 52.0 cm height, weighing 27.0 N, contains perfume (specific gravity = 0.83) to a depth of 22.0 cm. **(12)**

   (a) When placed in water, what will be the water depth at the bottom of the drum?

   (b) How much perfume can the drum hold and still float?

   (c) Two long pipes are used to convey water between two reservoirs whose water surfaces are at different elevations. One pipe has a diameter twice that of the other. If both pipes have the same value of a friction factor, f, and if minor losses are neglected, what will be the ratio of the flow rates through two pipes? **(11)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.
Assume any reasonable value where necessary

5. (a) Why is the kinetic energy correction factor used? Derive the expression for kinetic energy correction factor and find α for a two dimensional laminar flow as between two flat plates, for which the velocity profile is parabolic. **(15)**

   (b) Show different types of deformation that a fluid may undergo in sketch. **(8)**

   (c) What is the difference between cylindrical forced vortex and spiral forced vortex? Derive the equation for steady motion along a streamline for real fluid. **(12)**

6. (a) Differentiate between (i) Steady and unsteady flow; (ii) Compressible and incompressible flow. For the velocity field $V = 2txi - t^2yj + 3xzk$, is the flow steady or unsteady? Is it two or three dimensional? Also compute the total acceleration at the point (x, y, z,) = (2, - 2, 0). **(12)**

   (b) A two dimensional flow field is given by $u = 2y$, $v = x$. Sketch the flow field. Derive a general expression for the velocity and acceleration. **(12)**

   (c) What are the uses and limitations of flow net? An incompressible ideal fluid flows at a rate of 15L/s through a circular pipe into a conically converging nozzle as shown in Figure 4. Determine the average velocity of flow at sections A and B. **(11)**

Contd P/3

7. (a) Differentiate between (i) streamline and streaklines, (ii) static pressure and stagnation pressure, (iii) source and sink, (iv) velocity potential and stream function **(12)**

(b) A 5cm circular orifice at the end of a 7.5 cm diameter pipe discharges into the atmosphere where measured flow is 17L/s and the pressure in the pipe is 70 kPa. The jet velocity is found to be 12m/s. Find the values of the coefficient C_V, C_C and Cd. Also find the head loss from inlet to throat. **(11)**

(c) Assuming ideal flow in a horizontal plane, calculate the magnitude and direction of the resultant force on the stationary blade as shown in **Figure 5**. The entering jet has 150 mm diameter and velocity of 12 m/s, which is divided by the splitter so that one third of the total water is diverted towards A. **(12)**

8. (a) A Styrofoam cylinder filled with water, sits on a table. You poke and create a small hole through the side of the cylinder, 20 cm below the top of the water surface. What is the speed of the fluid emerging from the hole? Assume the area of the hole is very small compared to the opening of the cylinder. **(10)**

(b) A nozzle that discharges a 60 mm diameter water jet into the air is on the right side of a horizontal 120 mm diameter pipe. In the pipe the water has a velocity of 4 m/s and a gauge pressure of 400 kPa. Find the magnitude and the resultant axial force the water exerts on the nozzle and the head loss in the nozzle. **(12)**

(c) A radial flow turbine has the following dimension: $r_1 = 0.5$ m, $r_2 = 0.3$ m and $\beta_1 = 80°$.The width of the flow passage between the two sides of the turbine is 0.25 m. At 300 rpm, the flow rate through the turbine is 4 m^3/s. Find **(13)**

(i) the blade angle β_2 such that the water exits from the turbine in the radial direction.

(ii) the torque exerted by the water on the runner

(iii) the head utilized by the runner and the resulting power

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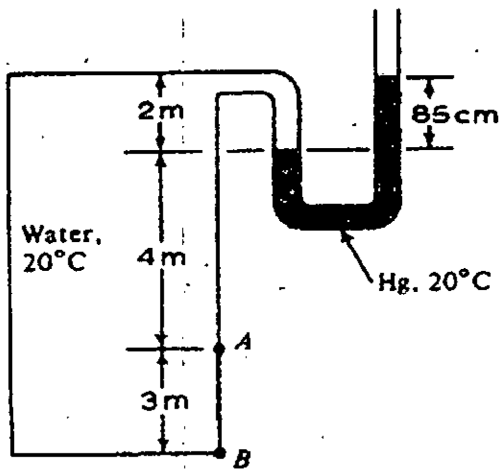

Figure 1 for question 1(d)

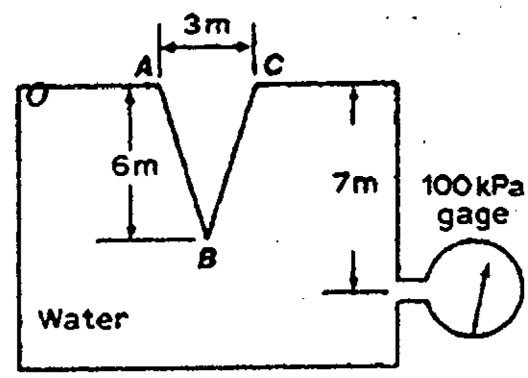

Figure 2 for question 2(b)

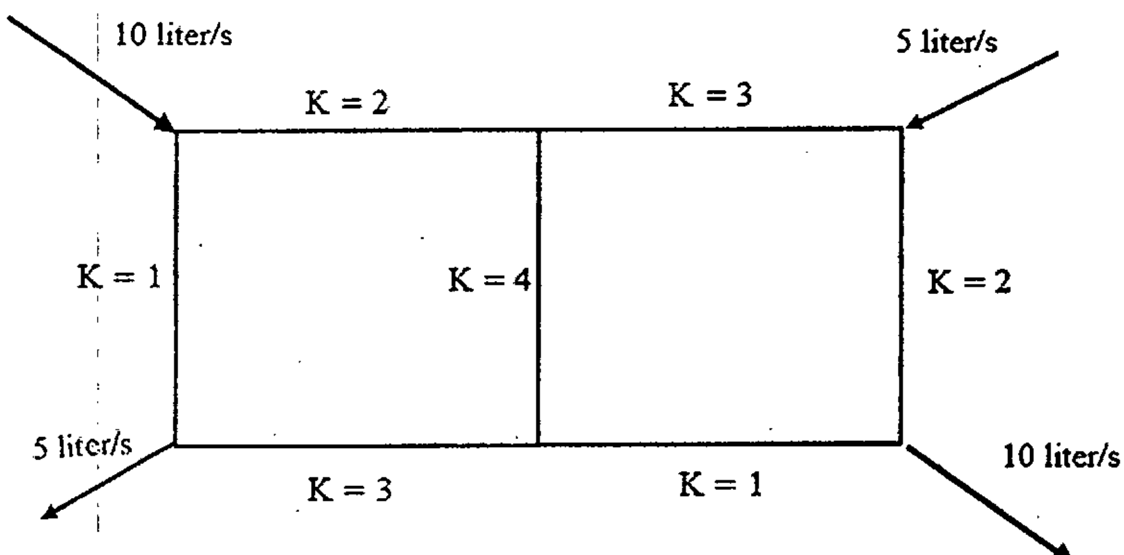

Figure 3 for question 3(b)

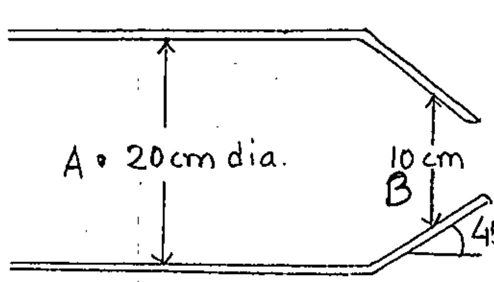

figure 4 for Q 6(c)

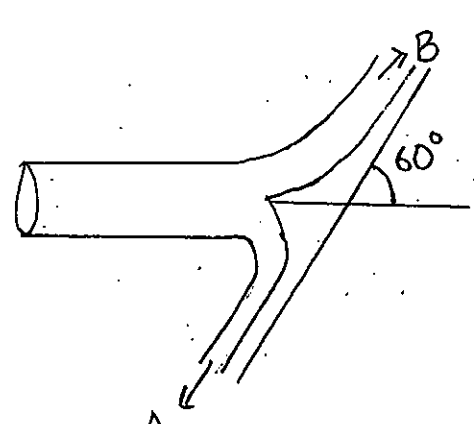

figure 5 for Q 7(c)

L-2/T-2/CE **Date: 20/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **HUM 217** (Engineering Economics)

Full Marks: 140 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Define demand function. **(5)**

(b) What are the main determinants of demand? Explain. **(10)**

(c) Why do demand curves generally slope downward? **($8\frac{1}{3}$)**

2. (a) How would you measure price elasticity of demand at any point of a straight-line demand curve? Explain graphically. **($13\frac{1}{3}$)**

(b) From the following table calculate elasticity of demand if you move from point A to C and explain what you understand from result. **(10)**

POINT	Y	Q
A	5000	500
B	6000	600
C	7000	700

3. (a) Explain the properties of an indifference curve. **(10)**

(b) Explain consumer's equilibrium with the help of budget line and indifference curve. **($13\frac{1}{3}$)**

4. (a) How is price determined in an open economy? What will happen to the price and quantity due to simultaneous change in supply and demand? **(10)**

(b) From the following demand and supply functions, calculate equilibrium price and quantity and show the result in a graph. **($13\frac{1}{3}$)**

$$P = 0.20\,Q + 150$$
$$P = -\,0.60Q + 230$$

Contd P/2

HUM 217/CE
Contd... Q. No. 4(b)

(i) What will happen to the equilibrium price and quantity if government imposes a unit tax of TK 10 per unit?

(ii) Describe the change in equilibrium. Show the equilibrium coordinates on the same graph.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Define optimization. How can optimization be achieved? Why is optimization necessary with reference to the production of a firm? **($6\frac{1}{3}$)**

(b) What is the basis of distribution of production among the factors of production? If all the factors of production are paid as rewards equal to their marginal products, would the total product be just exactly exhausted? Explain. **(10)**

(c) What are the various returns to scale of production? Briefly explain them. **(7)**

6. (a) Make a comparison between perfectly competitive market and monopolistic market **(5)**

(b) What are the conditions of monopoly market? Discuss them. Graphically explain the short run equilibrium of a firm under monopoly market. **($5\frac{1}{3}$)**

(c) What is meant by the shut-down point of production of a firm? Explain graphically the shut-down point of production of a firm under perfect competition. **(5)**

(d) Given the following total revenue (TR) and total cost (TC) functions for a firm **(8)**

$$TR = 4000Q - 33Q^2$$
$$TC = 2Q^3 - 3Q^2 + 400Q + 5000$$

where Q = Quantity of output

(i) Set up the profit function

(ii) Find the critical value(s) and

(iii) Calculate the maximum profit.

7. (a) What is market failure and what are the different causes of market failure? **(5)**

(b) Suggest some solutions to market failure. **(5)**

Contd P/3

Contd... Q. No. 7

(c) What is meant by the concept of long run in Economics? How would you derive a long run average cost (LAC) curve of a firm from its short run cost curves? Why is LAC curve often called the planning curve? **($5\frac{1}{3}$)**

(d) Complete the following table and sketch a graph explaining the relations among the various short run cost curves. **(8)**

Quantity of output	Total Fixed cost	Total Variable cost	Total cost	Average Fixed cost	Average variable cost	Average Total cost	Marginal cost
1	80	35					
2	80	45					
3	80	50					
4	80	60					
5	80	80					
6	80	125					

8. (a) What are the flows in the market economy that go from firms to households and from households to firms? Explain with circular flow diagram. **(5)**

(b) What is inflation? What are the causes of inflation? **(5)**

(c) Briefly discuss the various policies for controlling inflation with reference to the context of Bangladesh. **($5\frac{1}{3}$)**

(d) Calculate national income from the following information: **(8)**

GNP = Tk. 1,20,000 crore

Depreciation = TK. 11,000 crore

Indirect tax = TK. 12,500 crore

Subsidy is 20% of indirect tax.
