L-2/T-2/EEE **Date : 25/09/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub : **EEE 205** (Energy Conversion II)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

All the symbols and notations used in this section bear their usual meanings.

1. (a) Why the emf generated in the armature of a DC motor is called the back emf? Describe a method that can prove the existence of back emf in the armature of a running DC motor. **(12)**

   (b) Draw and explain the torque-speed characteristics of series, shunt and compound DC motors. **(10)**

   (c) The armature resistance of a DC shunt motor is 0.5 Ω. It draws 20 A from a 220 V DC source and is running a speed of 80 radian per second. For this motor, determine (i) back emf (ii) electromagnetic torque and (iii) speed in rpm. **(13)**

2. (a) List various methods of controlling speed of DC shunt motors. Among those methods, describe one method that is used for controlling speed above base speed of the motor. **(12)**

   (b) What is commutation of a DC machine? Briefly describe the remedies for commutation problems. **(8)**

   (c) A 240 V series motor takes 40 A when giving its rated output at 1500 rpm. It's armature and series field resistances are 0.18 Ω and 0.12 Ω respectively. Find the external resistance which must be added to obtain rated torque (i) at starting and (ii) at 1000 rpm. **(15)**

3. (a) Show that a single-phase sinusoidal field can be replaced by two fields rotating around the air gap in opposite directions. **(10)**

   (b) Explain the construction (with sketch) and working principle of a capacitor-start capacitor-run single-phase induction motor. **(15)**

   (c) A single-phase induction motor draws a current of 0.5 A at 230 V and 0.6 lagging p.f. If it runs at a speed of 100 radian per second and develops an output torque of 0.3 N-m, find its output power and efficiency. **(10)**

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EEE 205

4. (a) In reference to photovoltaic technology, define the following components (i) solar cell (ii) cell module (iii) solar panel, and (iv) panel array. **(8)**

   (b) Draw the real I-V characteristic of a p-n junction solar cell and show effect of change in (i) series resistance, (ii) shunt resistance, and (iii) temperature on that characteristic. **(9)**

   (c) Design and draw the circuit/block diagrams for the following PV arrangements: **(18)**

   (i) A battery backed-up PV solar powered DC pump system.

   (ii) A grid-connected hybrid PV solar system with standby generator.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) For a synchronous generator, what are the factors that cause the difference between the internal generated voltage E_A and terminal voltage V_ϕ? With necessary diagrams, explain armature reaction. Ignoring the effect of salient-pole rotor shapes, develop the equivalent circuit of a synchronous generator. **(20)**

   (b) For a synchronous motor operating at a leading power factor with no R_A, the torque angle is given by **(15)**

$$\delta = \frac{X_S I_A \cos\theta}{V_\varphi + X_S I_A \sin\theta}$$

   With necessary phasor diagram, derive an equation for the torque angle of the synchronous motor if the armature resistance is included.

6. (a) A 480-V 375-kVA Y-connected synchronous generator has a synchronous reactance of 0.4 Ω and a negligible armature resistance. This generator is supplying power to a 480-V 80-kW Y-connected synchronous motor with a synchronous reactance of 1.1 Ω and a negligible armature resistance. The synchronous generator is adjusted to have a terminal voltage of 480 V when the motor is drawing the rated power at unity power factor. **(20)**

   (i) Calculate the magnitudes and angles of E_A for both machines.

   (ii) If the flux of the motor is increased by 10 percent, what happens to the terminal voltage of the power system? What is its new value?

   (b) Draw the power flow diagram for a synchronous generator. Assuming $X_S >> R_A$, prove that $P_{conv} = \frac{3V_\varphi E_A}{X_S}\sin\delta$, and draw the corresponding phasor diagram. **(15)**

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7. (a) With necessary diagrams, explain the problem associated with starting a synchronous motor and how it can be started using Amortisseur windings. **(20)**

(b) Assume that two generators (generator 1 and generator 2) are supplying a load. Generator 1 has a no-load frequency of 61.5 Hz and a slope s_{p1} of 1 MW/Hz. Generator 2 has a no-load frequency of 61.0 Hz and a slope s_{p2} of 1 MW/Hz. The two generators are supplying a real load of 2.5 MW at 0.8 PF lagging. **(15)**

(i) What is the operating frequency of this system? How much power is being supplied by each of the generators?

(ii) Suppose an additional 1 MW load is attached to this power system. What would the new system frequency be, and how much power would generator 1 and generator 2 supply now?

(iii) With the system in the configuration described in part (ii), what will the system frequency and generator powers be if the governor set points on G2 are increased by 0.5 Hz?

8. (a) A 480-V, 50-Hz, Y-connected, 6-pole synchronous generator has a per-phase synchronous reactance of 1.0 Ω. Its full-load armature current is 60 A at 0.8 PF lagging. This generator has friction and windage losses of 1.5 kW and core losses of 1.0 kW at full load. Assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480 V at no load. **(20)**

(i) What is the terminal voltage of the generator when it is loaded with rated current at 0.8 PF lagging?

(ii) What is the terminal voltage of the generator when it is loaded with rated current at 0.8 PF leading?

(iii) What is the efficiency of the generator when it is operating at the rated current at 0.8 PF lagging?

(iv) How much shaft torque must be applied by the prime mover at full load? How large is the induced counter-torque?

(v) What is the voltage regulation of the generator when it is operating at the rated current at 0.8 PF lagging?

(b) Assuming a synchronous generator is operating in parallel with an infinite bus, discuss the effects of changing (i) the governor se points; and (ii) the field current with necessary phasor diagrams. **(15)**

L-2/T-2/EEE Date : 01/10/2022

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub : **EEE 207** (Electronic Circuits II)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Derive an expression for the output voltage, v_0 in the operational amplifier circuit shown in Fig. for Q. 1(a). Sketch the time variation of v_0 for the given V_{dc} and V_{ac}. Assume a time constant (RC) of 1 ms and $V_{cc} = \pm 12$ V. **(20)**

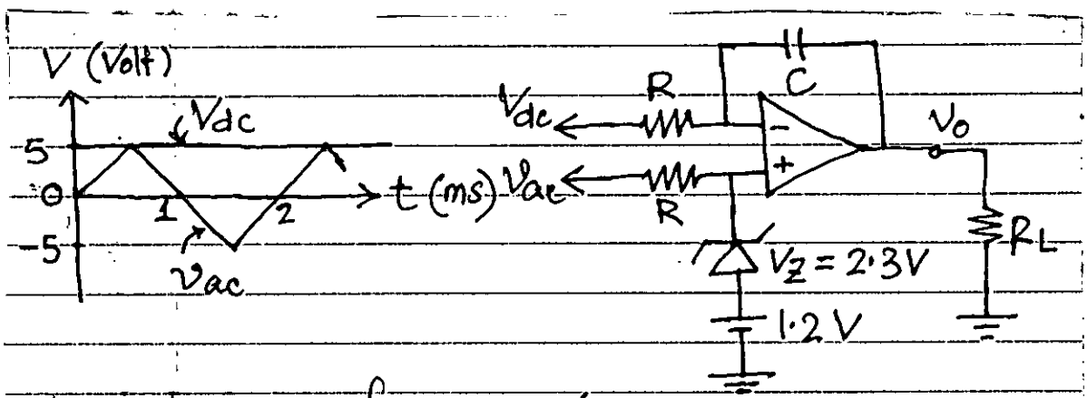

Fig. for Q. 1(a)

(b) Design a noninverting amplifier circuit using only ONE operational amplifier IC and other necessary elements so that three dc voltage sources (V_1, V_2, V_3) applied to the input appear at the output as $V_0 = V_2 - V_3 + V_1$. **(15)**

2. (a) Draw the circuit of an active narrowband filter and show that the resonant frequency can be controlled by a circuit element without changing the bandwidth or gain of the filter. **(20)**

(b) Illustrate the internal circuit of 555 Timer IC and briefly describe the function of each segment. **(15)**

3. (a) The power supply for biasing the operational amplifier ICs in the circuit shown in Fig. for Q. 3(a) is switched ON at time, t = 0. Explain operation of the circuit with necessary illustration and sketch time variation of the output voltage, V_F. **(20)**

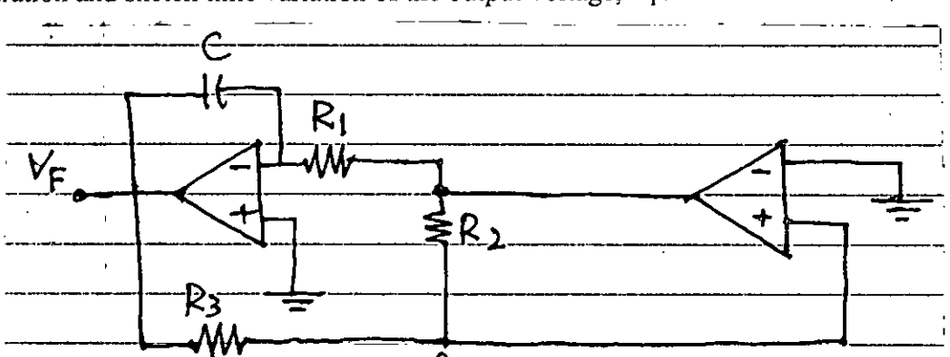

Fig. for Q. 3(a)

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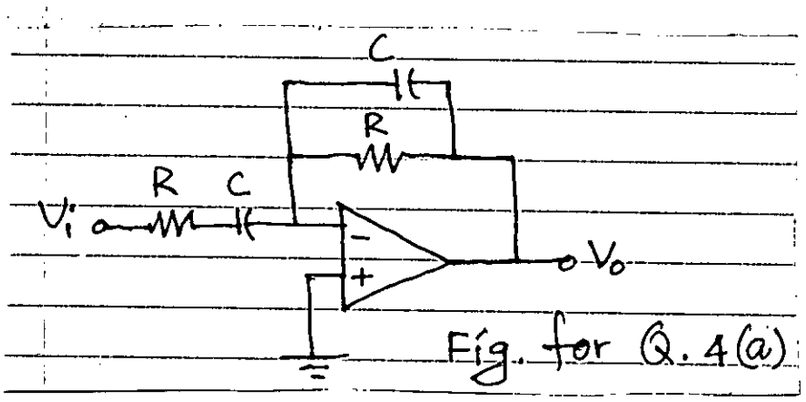
Contd ... Q. No. 3

(b) Design a circuit using operational amplifier ICs and other necessary elements that can give solution to the following equation and explain the operation of the designed circuit. **(15)**

$$2\frac{d^2V}{dt^2} - 3\frac{dV}{dt} + 4V + 5 = 0$$

4. (a) Derive an expression for the transfer function of the circuit shown in Fig. for Q. 4(a). Sketch Bode plot for the magnitude and phase of the transfer function. **(20)**



(b) Design a monostable multivibrator circuit using 555 Timer IC and explain the operation with necessary diagrams. **(15)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

All the symbols have their usual meanings.

Assume reasonable values for missing data.

5. (a) Establish and state Barkhausen criteria for sinusoidal oscillators. Draw the circuit diagram of Colpitts oscillator using op-amp and other necessary elements and explain how the Barkhausen criteria are satisfied in this circuit. Also, derive the expression for oscillating frequency and the conditions for sustainable oscillation. **(20)**

(b) Design the Colpitts oscillator using op-amp and other necessary elements to produce a 50 KHz output frequency. Use a 110 mH inductor and an OP-AMP with a ± 12 V supply. **(15)**

6. (a) For the common-emitter amplifier, as shown in Fig. for Q. 6(a), prove that **(15)**

$$I_{CQ} = \frac{V_{CC}}{R_{ac} + R_{dc}}$$

under maximum output swing condition.

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Contd ... Q. No. 6(a)

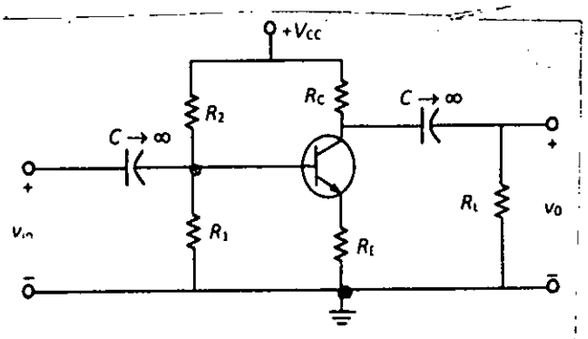

Fig. for Q.6(a)

(b) Design a class-A transformer-coupled power amplifier for a current gain of 100. Assume V_{CC} = 15 V, β = 100, turns ratio = 10 and R_L = 8 Ω. Find the power supplied to the load, power required from the supply, power rating of transistor and conversion efficiency. **(20)**

7. (a) Derive the expression of conversion efficiency for a diode-compensated class-B push-pull amplifier and hence prove that the maximum conversion efficiency is 78.5%. **(15)**

   (b) Design a diode-compensated Class-B push-pull amplifier to drive a 4-Ω load with 1 W of power for a low-frequency cutoff of 40 Hz. Use a 12 V power supply and silicon transistors with β = 100. The diodes have forward resistance of 8 Ω. Determine the current gain, power delivered to the amplifier, power rating of the transistors, and conversion efficiency. **(20)**

8. (a) Write down the advantages and drawback of negative feedback. Derive the expressions of R_{if} and R_{of} for the feedback amplifier topology shown in Fig. for Q. 8(a). **(15)**

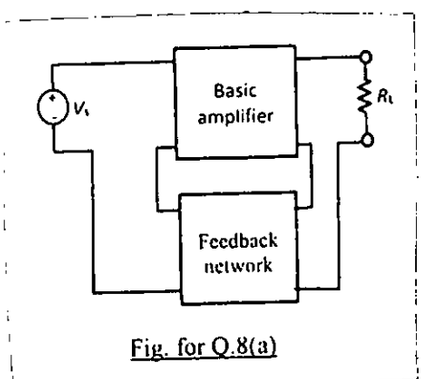

Fig. for Q.8(a)

Contd P/4

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Contd ... Q. No. 8

(b) Identify the topology of feedback amplifier and calculate the gain with feedback A_f, input resistance R_{if} and output resistance (including load) R'_{of} and voltage gain with feedback A_{vf} for the circuit shown in Fig. for Q. 8(b), given that $\mu = 10^4$ V/V, $R_{id} = \infty$, $r_o = 100\ \Omega$ $R_F = 10$ kΩ, $R_s = 1$ kΩ, $R_L = 1$ kΩ. **(20)**

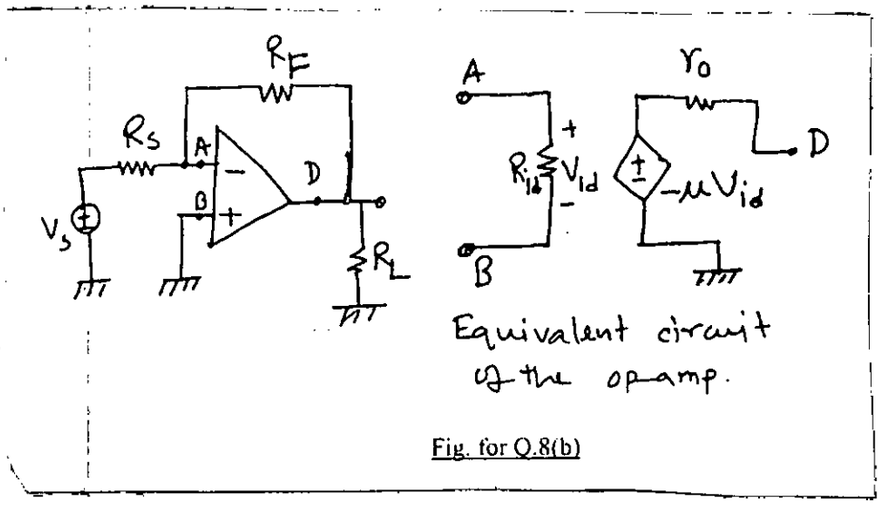

Fig. for Q.8(b)

L-2/T-2/EEE **Date: 16/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **EEE 209** (Engineering Electromagnetics)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Define Electric field intensity. Use Gauss's law to determine the electric field intensity $\vec{E}$ of an infinite surface charge of uniform surface charge density P_s (c/m^2) in Air. Assume the surface is in the x - z plane of the Cartesian co-ordinate system. **(10)**

(b) Three concentric spherical shells r = 1m, r = 2m, and r = 3 m, respectively, have charge distributions of 2 μ c/m^2, - 4 μ c/m^2 and 5 μc/m^2. **(10)**

(i) calculate the flux through r = 1.5 m and r = 2.5 m.

(ii) Find $\vec{D}$ at r = 0.5 m, r = 2.5 m and r = 3.5 m.

(c) Write down the equation of V_{21}, which represent the difference in electric potential energy of unit charge between point P_2 and point P_1, in an electric field $\vec{E}$. Simplify the expression when P_1 is considered at infinity. Define Electric dipole. For an electric dipole along the z-axis. Show that the potential V at a distant point P(R, θ, ϕ) is $\frac{\vec{p}.\hat{a}_R}{4\pi \in}.\frac{1}{R^2}$. Where $\vec{p}$ is the electric dipole moment. **(15)**

2. (a) State Image theory. Give the conditions to be satisfied for applying the image theory. An infinite line charge of 50 (nC/m) lies 4 m above the ground, which is at zero potential. Choosing the ground as the xy-plane and the line charge as parallel to the x-axis, find the following by the method of images **(12)**

(i) E at (0, 4, 4)

(ii) E and Ps at (0, 4, 0). All dimension are in meters

(b) Derive the Laplace's and Poission's equation for the cylindrical co-ordinate system. **(11)**

Contd P/2

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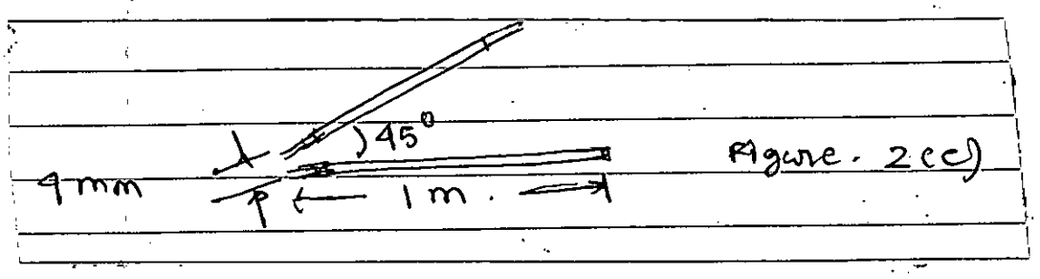
Contd... Q. No. 2

(c) Two conducting plates of 1 m and 5m length are inclined at 45° to each other with a gap of width 4 mm separating them as shown in the figure 2(c). Determine an approximate value of the charge per plate if the plates are maintained at a potential difference of 60 V. Assume that the medium between them has $\epsilon_r = 3.0$. **(12)**



3. (a) Define vector magnetic potential. Show that (i) $\nabla^2 \vec{A} = -\mu_0 J$ (for non magnetic media) (ii) $\nabla.\nabla\times \vec{A}=0$ and (iii) $\phi = \oint \vec{A}.\vec{dl}$. All symbols have their usual meanings. A current distribution gives rise to the vector magnetic potential $\vec{A} = x^2 y\hat{a}_x + y^2 x\hat{a}_y - 4xy\hat{a}_z$ Wb/m. Calculate the flux through the surface defined by z = 1, $0 \le x \le 1$, $-1 \le y \le 4$. **(15)**

(b) Starting from the magnetic force on a complete circuit of contour C, carrying steady current I in magnetic field $\vec{B}$, derive the expression of Ampere's law of force between two current carrying circuits. Also show that $\vec{T} = \vec{m}\times\vec{B}$. Where $\vec{m}$ is the magnetic Dipole moment (A - m^2). **(10)**

(c) A rectangular coil of area 10 cm^2 carrying current of 50 A lies on plane 2x + 6y - 3z = 7 such that the magnetic moment of the coil is directed away from the origin. Calculate its magnetic moment. **(10)**

4. (a) Derive the expression of magnetic stored energy in terms of field quantities. By using stored magnetic energy, determine the inductance per unit length of an air co-axial transmission line that has solid inner conductor of radius r_i and a very thin outer conductor of inner radius r_0. **(13)**

(b) Region 1, describe by $3x + 4y \ge 10$, is free space whereas region 2, describe by $3x + 4y \le 10$, is a magnetic material for which $\mu = 10\ \mu_0$. Assuming that the boundary between the material and free space is current free. Find $\vec{B}_2$ if $\vec{B}_1 = 0.1\hat{a}_x + 0.4\hat{a}_y + 0.2\hat{a}_z$ Wb/m^2. **(10)**

Contd P/3

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(c) What is polarization vector? The polarization vector in a dielectric sphere of radius, b is P = $\hat{a}_x 5$ (i) Determine the surface and volume charge densities, and (ii) Show that the total bound charge is zero. **(12)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

All the Symbols have their usual meaning. Assume a reasonable value for any missing data.

5. (a) Derive the charge-current continuity equation and explain it's physical significance. Also, obtain the boundary conditions of steady current at the interface of two media with different conductivities. **(10)**

(b) What do you mean by Retarded Potentials? How do they differ from the static potentials? **(10)**

(c) A current element of length l = 2.0cm is located in air. It is oriented along the z-axis of rectangular coordinates with its mid-point at the origin. It carries a current of I = 5cos ($2\pi \times 10^6$t) Amp. Find the vector magnetic potential at a point P(20, 25) and at time t = 1.0 μs. **(15)**

6. (a) Derive the wave equations governing the $\overline{E}$ and $\overline{H}$ fields in a source free conducting medium with constitutive parameters ε, μ, and σ. Also, obtain the corresponding Helmholtz equations for time harmonic fields. **(10)**

(b) For a uniform plane wave propagating in + z direction in a lossless dielectric medium, write expressions of $\overline{E}$ field in phasor form, determine the associated $\overline{H}$ field and find the intrinsic impedance of the medium. **(10)**

(c) A parallel-plate capacitor is filled with a lossy dielectric material of relative permittivity $\varepsilon_r = 4$ and conductivity $\sigma = 2.5$ S/m. The separation between the plates is d = 0.5 cm, and each plate is of area A = 4 cm^2. The capacitor is connected to a time-varying voltage source V(t) = 10cos ($3\pi \times 10^3$t) V. Find (i) the conduction current flowing between the plates inside the capacitor, (ii) the displacement current flowing inside the capacitor, and (iii) show the equivalent circuit representation of the capacitor. **(15)**

Contd P/4

EEE 209

7. (a) How does the Poynting vector, $\overline{P} = \overline{E} \times \overline{H}$ represent the instantaneous power density associated with electromagnetic wave? Explain. **(10)**

(b) A wave traveling in a lossless, nonmagnetic medium has an electric field amplitude of 24.56 V/m and an average power density of 2.4 W/m^2. Determine the phase velocity of the wave. **(10)**

(c) The electric-field phasor of a uniform plane wave traveling downward in water is given by **(15)**

$$\overline{E} = \hat{a}_x \, 5e^{-0.2z} \, e^{-j0.2z} \ (V/m)$$

where $\hat{a}_z$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4$ S/m, (i) obtain an expression for the average power density, (ii) determine the depth at which the power density has been reduced by 40 dB.

8. (a) What is standing wave? Explain how does the standing wave give rise from normal incidence of EM wave on a plane perfect conductor. Also, find an expression of standing wave ratio (SWR) in terms of reflection coefficient. **(15)**

(b) A 200-MHz left-handed circularly polarized plane wave with an electric field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 4$ and occupying the region defined by $z \geq 0$. **(20)**

(i) Write an expression for the electric field phasor of the incident wave, given that the field is positive maximum at $z = 0$ and $t = 0$.

(ii) Calculate the reflection and transmission coefficients.

(iii) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(iv) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

L-2/T-2/EEE **Date: 26/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **ME 267** (Mechanical Engineering Fundamentals)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

All symbols used have their usual meaning and interpretation.

Assume reasonable values for missing data.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) If you are asked to choose a refrigerant, what are the properties of refrigerant you will consider for the selection and why? **(10)**

(b) What are the functions of an air handling unit in central air conditioning system? Illustrate with a necessary schematic diagram. **(10)**

(c) Two airstreams are mixed steadily and adiabatically as shown in Figure Q. 1(C). The first stream enters at 35°C and 30 percent relative humidity at a rate of 15 m^3/min, while the second stream enters at 12°C and 90 percent relative humidity at a rate of 25 m^3/min. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, the dry-bulb temperature, and the volume flow rate of the mixture. **(15)**

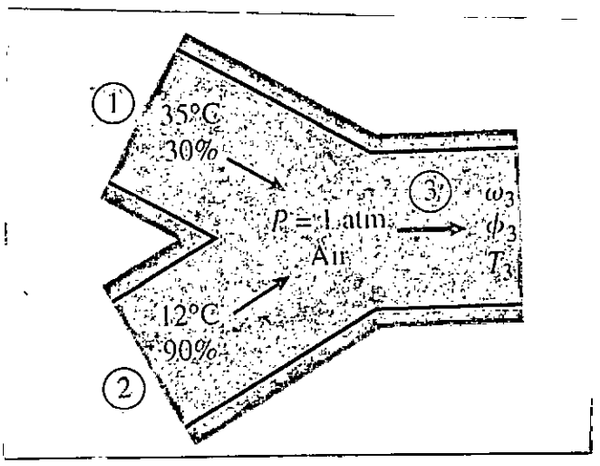

Fig. for Q. 1(c)

2. (a) What is the cutoff ratio? How does it affect the thermal efficiency of a Diesel cycle? **(5)**

(b) You have a 3000 cc, V8, square spark ignition engine car. Calculate Bore and stroke length of that engine. Will you be able to use Diesel in your car? Explain. **(10)**

(c) An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (i) the thermal efficiency and (ii) the mean effective pressure. Assume constant specific heats for air at room temperature. The properties of air at room temperature are c_p = 1.005 kJ/kg.K, c_v = 0.718 kJ/kg.K, R = 0.287 kJ/kg.K, and k = 1.4. **(20)**

Contd P/2

ME 267

3. (a) What is the physical significance of IC Engine Firing Order? **(5)**

(b) With the help of schematic diagram briefly explain how the engine ignition system works. **(10)**

(c) A multifluid container is connected to a U-tube, as shown in Figure Q. 3(c). For the given specific gravities (SG) and fluid column heights, determine the gage pressure at A. Also determine the height of a mercury column that would create the same pressure at A. Specific gravity of mercury is 13.6. **(20)**

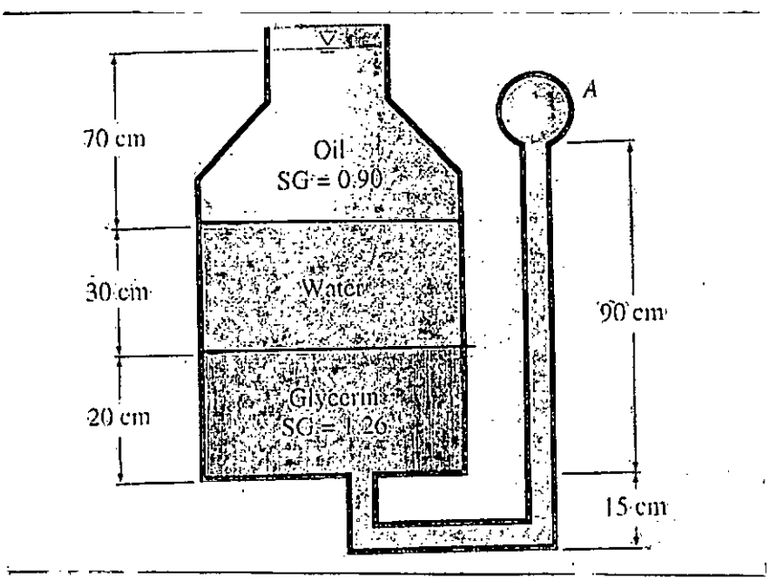

Fig. for Q. 3(c)

4. (a) Draw main characteristics curves of a typical centrifugal pump. A centrifugal pump delivers 0.055 m^3/s (Q) of water to a total height of 16 m. The diameter of the pipe is 150 mm (d) and it is 22 m (l) long. If the overall efficiency is 75%, calculate the power (**P**) required to drive the pump. Take $f = 0.05$ for the pipe. **(15)**

(b) A cylindrical water tower of diameter 3.0 m supplies water to a house. The level of water in the water tower is 35m above the point where the water enters the house through a pipe that has an inside diameter 5.1 cm. The intake pipe delivers water at a maximum rate of 2.0×10^{-3} m^3/s. The pipe is connected to a narrower pipe leading to the second floor that has an inside diameter 2.5 cm. What is the pressure and speed of the water in the narrower pipe at a point that is a height 5.0 m above the level where the pipe enters the house as shown in Figure Q. 4(b)? **(20)**

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ME 267

Contd... Q. No. 4(b)

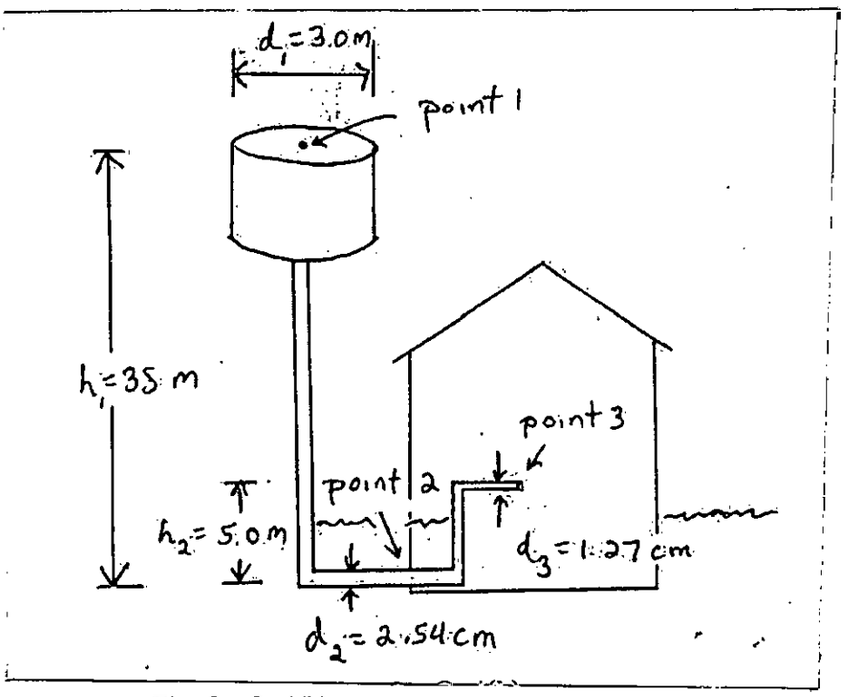

Fig. for Q. 4(b)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Discuss the availability, present scenario, and the future potential of solar energy in Bangladesh. **(10)**

(b) What are the different ways of increasing the efficiency of boilers? Briefly explain one such method. **(10)**

(c) What the help of simple schematic(s), explain the working principle and important features of a water tube boiler. **(15)**

6. (a) Why is Carnot cycle not practicable for an actual steam power plant? How does the actual vapor cycle deviate from the idealized one? **(10)**

(b) Why is the shape of the blades so important in the design and performance of steam turbines? **(10)**

(c) A steam engine utilizes 450°C steam which is later exhausted at 270°C. (i) What is the maximum possible efficiency that such a heat engine can have? (ii) Since 270°C steam is still quite hot, a second steam engine is operated using the exhaust of the first engine. What is the maximum efficiency of the second engine if its exhaust has a temperature of 150°C? (iii) What is the overall efficiency of these two engines? (iv) Would the overall efficiency obtained for (iii) be same as the efficiency of a single Carnot engine operating between 450°C and 150°C? Explain. **(15)**

Contd P/4

ME 267

7. (a) Consider steady heat transfer through the wall of a room in winter. The convection heat transfer coefficient at the outer surface of the wall is three times that of the inner surface as a result of the wind outside. On which surface of the wall do you think the temperature will be closer to the surrounding air temperature? Why ? **(7)**

(b) Solid *A* has a higher thermal conductivity than solid *B*. Which of these solids will have a higher value of the slope dT/dx? Why? **(8)**

(c) The interior of a refrigerator having inside dimension of 50 cm × 50 cm base area and 120 cm height needs to be maintained at 4°C. The walls of the refrigerator are constructed of two mild steel sheets (each 3 mm thick) and a glass wool insulation layer of 5 mm between them. The convective heat transfer coefficient inside the refrigerator is 10 $W/m^2°C$. The refrigerator is placed in a kitchen at 25°C with a convective heat transfer coefficient of 20 $W/m^2°C$. Assuming that the refrigerator gains heat only by the side walls, determine the rate of heat gain. **(20)**

Assume, *k* (mild steel) = 46.4 W/m °C, and *k* (glass wool) = 0.0464 W/m °C.

8. (a) Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why? **(7)**

(b) "Depending on the application, we might want to be on the higher or lower side of Critical Radius of Insulation" – Why? Explain. **(8)**

(c) A copper wire of radius 0.5 mm is insulated uniformly with plastic (k = 0.5 W/m K) sheathing 1 mm thick. The wire is exposed to an atmosphere at 30°C and the outside heat transfer coefficient is 8 W/m^2K. Find the maximum safe current (in ampere) carried by the wire so that no part of the insulated plastic is above 75°C. Also calculate the critical thickness of insulation. **(20)**

For copper: Thermal conductivity = 400 W/m.K, specific electrical resistance = 2×10^{-8} ohm-m

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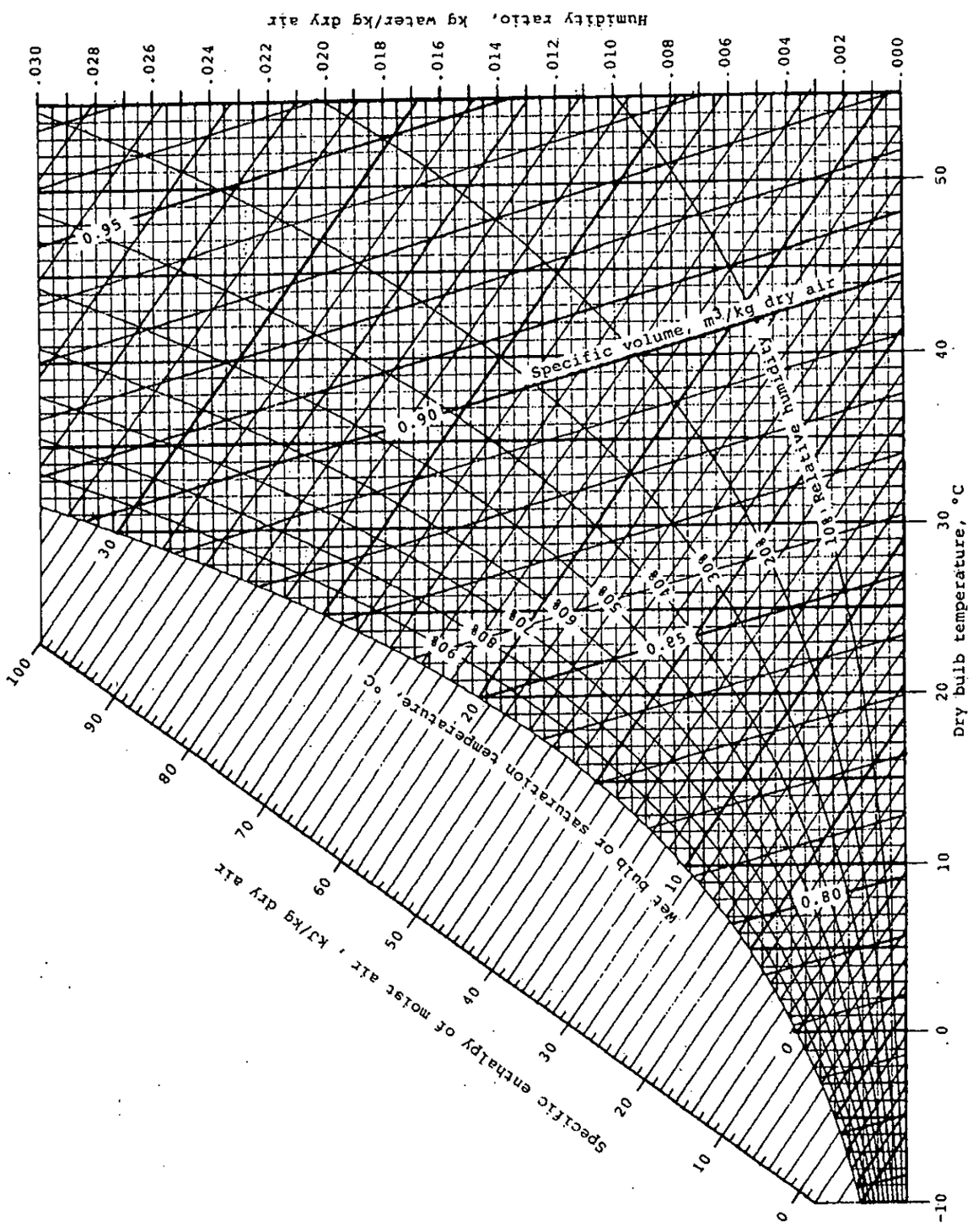
Psychrometric chart at 1 atm total pressure.



L-2/T-2/EEE **Date: 20/10/2022**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **MATH 357** (Probability and Statistics)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols used have their usual meanings.

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1. (a) A large industrial firm uses three local motels to provide overnight accommodations for its clients. From past experience it is known that 20% of the clients are assigned rooms at the Ramda Inn, 50% at the Sheraton, and 30% at the Lakeview Motor Lodge. If the plumbing is faulty in 5% of the rooms at the Ramda Inn, in 4% of the rooms at the Sheraton, and in 8% of the rooms at the Lakeview Motor Lodge. **(10)**

   i. What is the probability that a client will be assigned a room with faulty plumbing?

   ii. What is the probability that a person with a room having faulty plumbing was assigned accommodations at the Lakeview Motor Lodge?

   (b) Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The probability density function that characterizes the proportion Y that make a profit is given by: **(12)**

$$f(y)=\begin{cases} ky^4(1-y)^3, & 0\le y\le 1, \\ 0 & elsewhere. \end{cases}$$

   i. What is the value of k that renders the above a valid density function?

   ii. Determine the probability that at least 80% of the firms make a profit in the first year.

   (c) Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pounds per square inch (psi). Let X denote the actual air pressure for the right tire and Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density function **(13)**

$$f(x,y)=\begin{cases} k(x^2+y^2), & 30\le x<50,\ 30\le y<50, \\ 0, & elsewhere. \end{cases}$$

   i. Determine the value of k. Hence find $P(30 \le x \le 40$, and $40 \le y < 50)$.

   ii. Find the probability that both tires are underfilled.

Contd P/2

MATH 357

2. (a) Computer technology has produced an environment in which robots operate with the use of micro-processors. The probability that a robot fails during any 6-hour shift is 0.10. What is the probability that a robot will operate through at most 5 shifts before it fails? **(10)**

(b) According to USA Today (March 18, 1997), of 4 million workers in the general workforce, 5.8% tested positive for drugs. Of those testing positive, 22.5% were cocaine users and 54.4% marijuana users. **(12)**

i. What is the probability that of 10 workers testing positive, 2 are cocaine users, 5 are marijuana users, and 3 are users of other drugs?

ii. What is the probability that of 10 workers testing positive, all are marijuana users?

(c) A government task force suspects that some manufacturing companies are in violation of federal pollution regulations with regard to dumping a certain type of product. Twenty firms are under suspicion but not all can be inspected. Suppose that 3 of the firms are in violation. **(13)**

i. What is the probability that inspection of 5 firms will find no violations?

ii. What is the probability that the plan above will find two violations?

3. (a) The length of life, in hours, of a drill bit in a mechanical operation can be calculated using continuous distributions. A drill bit is a mechanical part that certainly have significant wear over time. **(10)**

i. Describe some continuous distributions with examples that have memoryless property?

ii. For the abovementioned scenario, what could be the most appropriate continuous distribution for calculating the lifetime problem.

(b) In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma distribution with conventional parameters $\alpha = 2$ and $\beta = 3$. If the daily capacity of that city is 9 million liters of water. **(12)**

i. What is the relationship between gamma and exponential distribution?

ii. What is the probability that on any given day the water supply is inadequate?

(c) A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live. **(13)**

i. more than 32 months;
ii. between 37 and 49 months.
(Necessary Table 1 and 2 are attached)

Contd P/3

MATH 357

4. (a) In a scientific paper, one researcher describe the reliability of DNA paternity testing as follows: "To get a completely accurate result, you would have to be tested, and so would (the man) and your mother. The test is 100% accurate if the manufacturer is not the father and 99.9% accurate if he is." Consider using the results of DNA paternity testing to decide between the following two hypotheses:

H_0: a particular manufacturer is the father

H_a: a particular manufacturer is not the father

In the context of this problem, describe Type I and Type II errors.

(b) The hospitalization period, in days, for patients following treatment for a certain type of kidney disorder is a random variable Y = X + 4, where X has the density function: **(12)**

$$f(x)=\begin{cases}\dfrac{32}{(x+4)^3}, & x>0\\ 0, & \textit{elsewhere,}\end{cases}$$

Find the average number of days that a person is hospitalized following treatment for this disorder.

(c) The mean score for freshmen on an aptitude test at a certain college is 540, with a standard deviation of 50. Assume the means to be measured to any degree of accuracy. What is the probability that two groups selected at random, consisting of 32 and 50 students, respectively, will differ in their mean scores by **(13)**

i. more than 20 points
ii. an amount between 5 and 10 points
(Necessary Table 1 and 2 are attached)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) For each of the following distributions, decide whether it is possible to find the mean and whether it is positions to find the median. Explain your answers and if possible, calculate the mean and the median. **(17)**

Grade	Frequency
40-50	5
50-60	18
60-70	27
70-80	15
80-90	6

Weight	Frequency
Less than 90	3
90-100	14
100-110	22
110-120	19
More than 120	7

IQ	Frequency
70 or less	21
70-75	42
75-80	14
80-85	11
85-90	17

Contd P/4

MATH 357
Contd... Q. No. 5

(b) An analysis of companies resulted in the following distribution: **(18)**

Profit (million)	15-25	25-35	35-45	45-55	55-65	65-75	75-85
No. of Companies	10	14	16	9	12	8	11

Calculate the first four moments about the assumed mean. Convert the result into moments about the mean. Compute the values of y_1 and y_2 and comment on the results.

6. (a) An examination of ten applicants for a post was taken by a firm. Formation the marks obtained by the applicants in the Mathematics and English papers, Compute the rank correlation coefficient and comment on its value. **(17)**

Student	1	2	3	4	5	6	7	8	9	10
Marks in Math	65	70	76	75	85	78	83	84	85	90
Marks in English	30	35	25	40	38	43	50	55	43	45

(b) An experiment was conducted to determine if the weight of an animal can be predicted after a given period of time on the basis of the initial weight of the animal and the amount of feed that was eaten. The following data, measured in kilograms, were recorded: **(18)**

Final Weight, y	95	77	80	100	97	70	50	80
Initial Weight, x_1	42	33	33	45	39	36	32	41
Feed Weight, x_2	272	226	259	292	311	183	173	236

(i) Fit a multiple regression equation of the formation

$$\mu_y |x_1, x_2 = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

(ii) Predict the final weight of an animal having an initial weight of 35 kilograms that is given 250 kilograms of feed.

7. (a) Write short notes on (i) Null and Alternative hypotheses, (ii) Type-I and Type-II errors, and (iii) One-tailed and two tailed test. **(8)**

(b) A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram, Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance. (Given that, for 1% level of significance $z = \pm 2.58$) **(12)**

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Contd... Q. No. 7

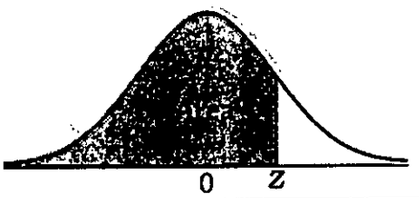
(c) A UCLA researcher claims that the average life span of mice can be extended by as much as 8 months when the calories in their diet are reduced by approximately 40% from the time they are weaned. The restricted diets are enriched to normal levels by vitamins and protein. Suppose that a random sample of 10 mice is fed a normal diet and has an average life span of 32.1 months with a standard deviation of 3.2 months, while a random sample of 15 mice is fed the restricted diet and has an average life span of 37.6 months with a standard deviation of 2.8 months. Test the hypothesis, at the 0.05 level of significance, that the average life span of mice on this restricted diet is increased by 8 months against the alternative that the increase is less than 8 months. Assume the distributions of life spans for the regular and restricted diets are approximate normal with equal variances. (Necessary Table 3 is attached) **(15)**

8. (a) A teacher wishes to test three different teaching methods, A, B, C. To do this, three groups of 5 students each are chosen at random, and each group is taught by a different method. The same examination is then given to all the students, and the grades in the following table are obtained. Determine at the 0.05 significance level whether there is a significant difference in the teaching methods. (Necessary Table 4 is attached) **(15)**

Method A	75	62	71	58	73
Method B	81	85	68	92	90
Method C	73	79	60	75	81

(b) A new fertilizer has been developed to increase the yield of crops, and the makers of the fertilizer want to better understand which of the three formulations (blends) of this fertilizer are most effective for wheat, corn, soybeans, and rice (crops). They test each of the three blends on one sample of each of the four types of crops. **Determine at the 0.05 significance level, whether the** crop yields for the 12 combinations are as shown in the following table, vary significantly with differences in (i) the formulations (blends) and (ii) the type of corn. (Necessary Table 4 is attached) **(20)**

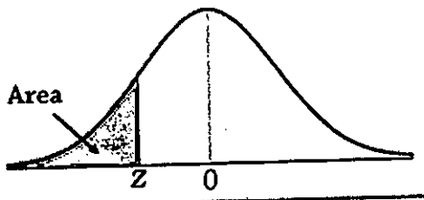
	Wheat	Cron	Soy	Rice
Blend X	123	138	110	151
Blend Y	145	165	140	167
Blend Z	156	176	185	175



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table 1: Necessary Table for Question No. 3(c) and 4(c)

Table-01



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table 2: Necessary Table for Question No. 3(c) and 4(c)

Table-02

For question No 7 (b) (c)

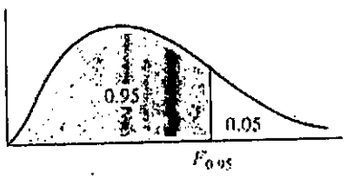
Table A.4 Critical Values of the t-Distribution

	α						
v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042

Table - 3

For question No 8 (a) and 8 (b)

95th Percentile Values (0.05 Levels), $F_{0.95}$, for the F Distribution



ν_1 degrees of freedom in numerator
ν_2 degrees of freedom in denominator

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Table-04