

The figures in the margin indicate full marks.

Assume reasonably any missing data. Symbols indicate their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Three elastic posts support a beam. Assume the horizontal beam to be rigid and supports the distributed load as shown in Fig. for Q. 1(a). Determine the angle of tilt of the beam after the load is applied. Each support consists of a wooden post having a diameter of 120 mm. Take $E_w = 12$ GPa. Neglect the self-weight of the beam and the column effect in the posts. (18)
- (b) A rod consisting of two cylindrical portions AB and BC is restrained at both ends as shown in Fig. for Q. 1(b). Portion AB is made of steel ($E_s = 200$ GPa, $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha = 20.9 \times 10^{-6}/^\circ\text{C}$). Knowing that the rod is initially unstressed, determine the compressive stresses induced in AB and BC when there is a temperature rise of 50°C . Diameter of portion AB is 30 mm and portion BC is 50 mm. (17)
2. (a) The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rpm as shown in Fig. for Q. 2(a). Determine the minimum diameter required of the shaft AB if the allowable shear stress is 200 MPa and the angle of twist of the shaft to 0.05 rad. The shaft is 0.6 m long and made from alloy steel ($G = 80$ GPa). (18)
- (b) Stresses are recorded at the center and corner points of an ultra-thin microprocessor component. The two in-plane principal stresses have been found at the center as -250 MPa and -200 MPa. At the corner point, the following stress state has been found: $\sigma_x = 100$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = 50$ Mpa. Determine which point is critical if (i) normal stress is considered, (ii) shear stress is considered. Ignore the out of plane stresses developed in this ultra-thin component. (17)
3. (a) A ship propeller shaft is to be subjected to a compressive load of 5 kN and torsional load of 10 kN-m. The shaft is made of steel material having the yield strength of 250 MPa. Determine the safe diameter of the shaft according to the maximum shear stress theory. Use a factor of safety 2 in your calculation. (18)

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- (b) A strain rosette is mounted on the surface of the wing of an airplane as shown in Fig. for Q. 3(b). The following readings are obtained from each gauge: $\epsilon_a = 650 \times 10^{-6}$, $\epsilon_b = 300 \times 10^{-6}$, $\epsilon_c = 480 \times 10^{-6}$. Determine, (i) the in-plane principal strains and (ii) the maximum in-plane shear strain. (17)
4. (a) A thick walled cylindrical pressure vessel having 200 mm ID and 240 mm OD is made of steel material with shear yield strength as 120 MPa. Determine the maximum internal pressure that can be applied to this cylinder so that the shear stress limit will not be exceeded. (18)
- (b) A solid aluminum shaft ($G = 55$ GPa) having 50 mm diameter and 500 mm length is subjected to a torsional load of 5 kN-m as shown in Fig. for Q. 4(b). At the center of the shaft, determine the deflection of a point on the surface following the strain energy technique.? (17)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) An axially loaded, fixed-hinged Al columns has to fulfill following design data: Cross-section is as shown in Fig. for 5(a), $E = 69$ GPa, unsupported length = 1000 mm. (20)
- (i) Calculate the maximum bending stiffness, buckling load and slenderness ratio.
With freehand sketches show followings:
- (ii) buckled shape of the column
 (iii) buckling direction wrt to cross-section
 (iv) the load-deflection curve of the column.
- (b) For an eccentrically loaded column, the mid-span transverse deflection can be given by,
- $$\delta = e \sec 0.5 L \sqrt{P/EI},$$
- where symbols carry their usual meanings. Using the above equation, (i) explain how the buckling load can be determine (ii) derive the expression of maximum compressive stress in the column. (15)
6. (a) Is the beam shown in Fig. 6(a) statically determinate? Justify your answer with necessary equations. (10)
- (b) Draw the shear force and bending moment diagrams for the beam shown in Fig. 6(b). Locate the points of the maximum bending moment and shear force. Ignore self-weight. (25)

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7. (a) Using double integration method, find the maximum deflection and slope of the elastic curve for the beam shown in Fig. for 7(a). Ignore self-weight. (18)
- (b) Using area moment method, find the deflection and slope of the tip (free end) of the elastic curve for the beam shown in Fig. for 7(b). Use separated moment diagrams to solve the problems. Ignore self-weight. (17)
8. (a) Calculate the maximum flexural stress, maximum shear stress in the beam shown in Fig. for Q. 8(a). Also calculate the radius of curvature at mid-span. Given, the beam has 40×40 mm solid square cross-section and $E = 207$ GPa. Ignore self-weight. (18)
- (b) The Figure for Q. 8(b) shows cross-section of an RC beam designed for a moment M . Also given, $n = 8$, $d = 500$ mm, $A_s = 1000$ mm², $f_s = 150$ Mpa, $f_c = 8$ MPa, where symbols carry their usual meanings. Calculate, for balanced reinforcement: b and M . If 13 mm dia steel rods are used, how many steel rods are required? (17)
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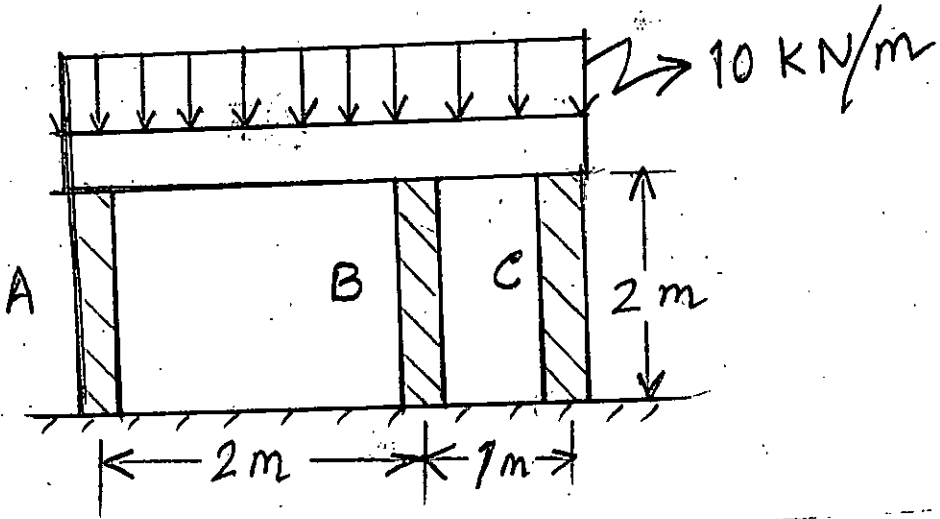


Fig. for Q. No. 1(a)

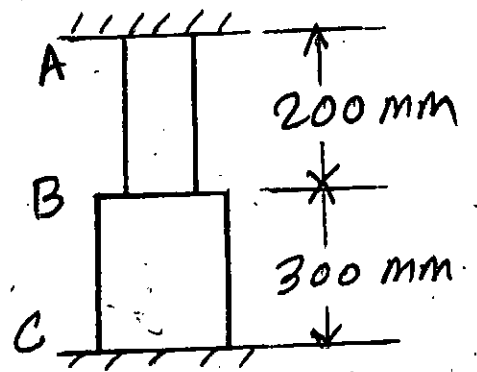


Fig. for Q. No. 1(b)

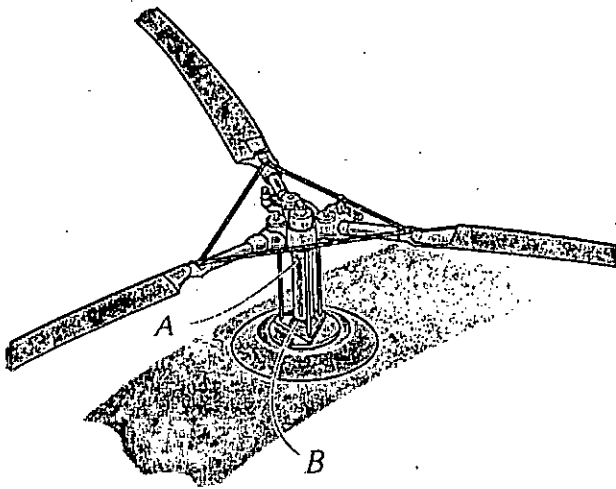


Fig. for Q. No. 2(a)

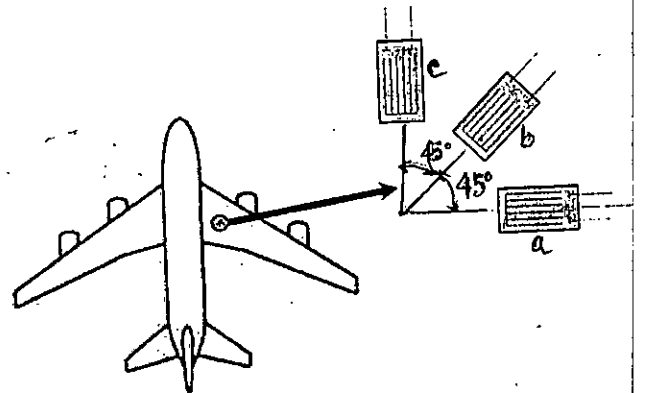


Fig. for Q. No. 3(b)

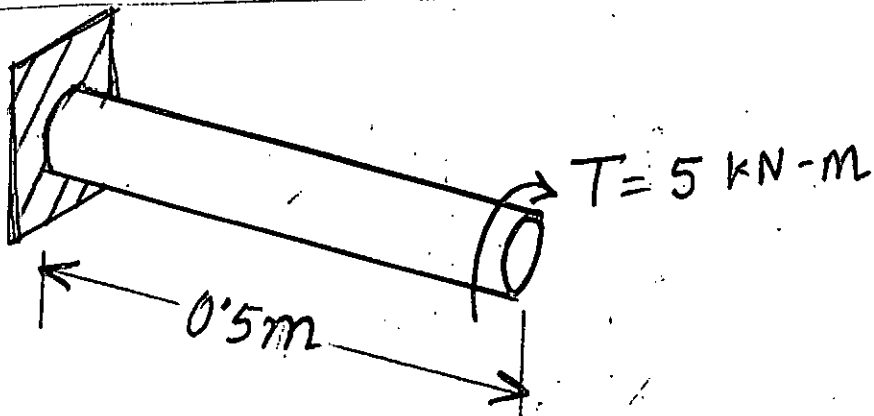
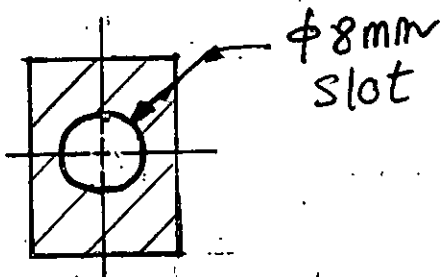


Fig. for Q. No. 4(b)



10x15mm
FIG. for Q.5(a)

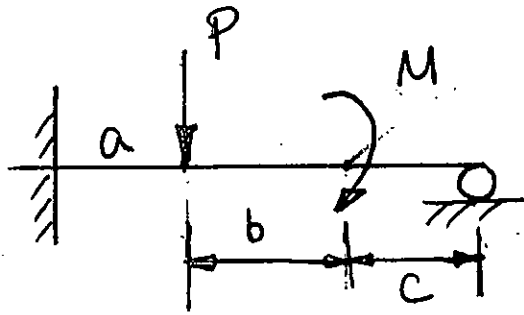


FIG. for Q.6(a)

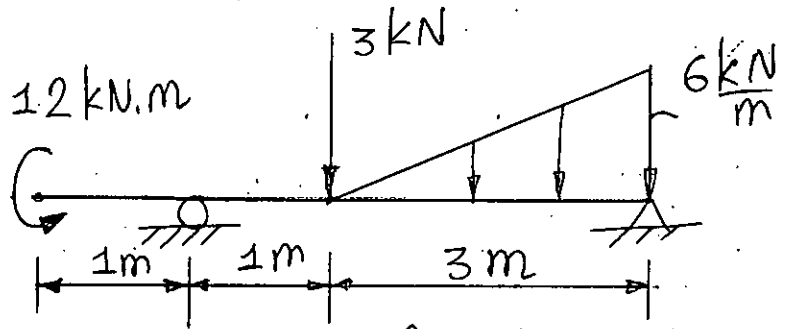


FIG. for Q.6(b)

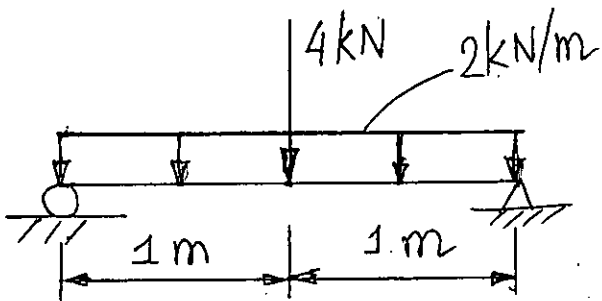


FIG. for Q.7(a)

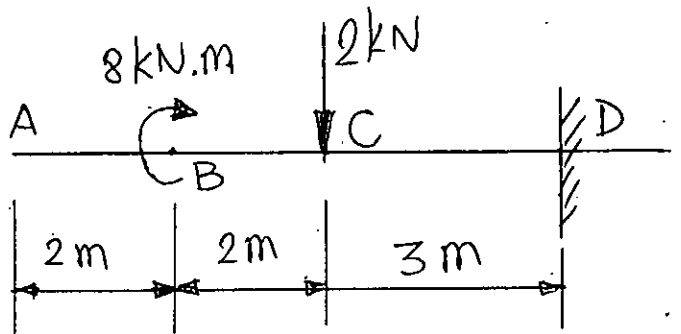


FIG. for Q.7(b)

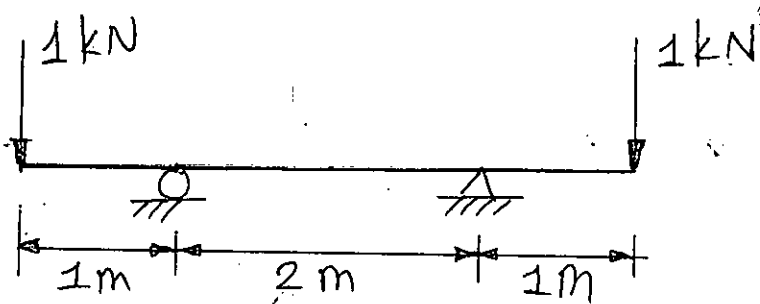


FIG. for Q.8(a)

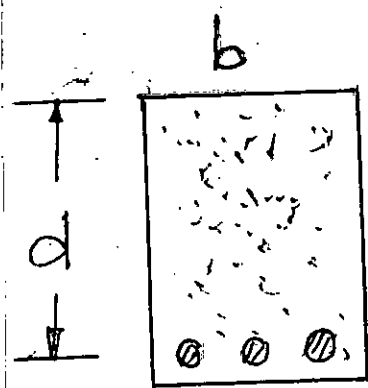


FIG. for Q.8(b)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2020-2021

Sub: **ME 249** (Engineering Mechanics – II)

Full Marks: 210

Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

Assume a reasonable value for any missing data.

The figures in the margin indicate full marks

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Packages are thrown down an incline at A with a velocity of 1 m/s as shown in Fig. 1(a). The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. The distance $d = 7.5$ m and coefficient of kinetic friction $\mu_k = 0.25$ between the packages and the surface AB , 0.30 between the packages and the surface BC , and 0.35 between the packages and the conveyor belt. Determine (i) the speed of the package at C , (ii) the distance a package will slide on the conveyor belt before it comes to rest relative to the belt. (17)
- (b) The 10-kg sphere C is released from rest when $\theta = 0^\circ$ and the tension in the spring is 100 N (when $\theta = 0^\circ$) as shown in Fig. 1(b). Determine the speed of the sphere at the instant $\theta = 90^\circ$. Neglect the mass of rod AB and the size of the sphere. Solve the problem by principle of conservation of energy. (18)
2. (a) A 60-g model rocket is fired vertically. The engine applies a thrust P which varies in magnitude as shown in Fig. 2(a). Neglecting air resistance and the change in mass of the rocket, determine (i) the maximum speed of the rocket as it goes up, (ii) the time for the rocket to reach its maximum elevation. Solve the problem by principle of impulse and momentum. (17)
- (b) A sphere A of mass $m_A = 2$ kg is released from rest in the position shown and strikes the frictionless, inclined surface of a wedge B of mass $m_B = 6$ kg with a velocity of magnitude $v_0 = 3$ m/s as shown in Fig. 2(b). The wedge, which is supported by rollers and may move freely in the horizontal direction, is initially at rest. Knowing that $\theta = 60^\circ$ and that coefficient of restitution $e = 0.8$, determine the velocities of the sphere and of the wedge immediately after impact. (18)
3. (a) A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown in Fig. 3(a). Knowing that $\theta = 30^\circ$, determine immediately after rope CF has been cut, (i) the acceleration of the plate, (ii) the tension in ropes AD and BE . (17)

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(b) A 1.5 kg slender rod is welded to a 5-kg uniform disk as shown in Fig. 3(b). The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10 rad/s clockwise, determine (i) the angular acceleration of the assembly, (ii) the components of the reaction at C . (18)

4. (a) The 10-kg rod AB is pin connected at A and subjected to a couple of moment $M = 15 \text{ N.m}$ as shown in Fig. 4(a). If the rod is released from rest when the spring is unstretched at $\theta = 30^\circ$, determine the rod's angular velocity at the instant $\theta = 60^\circ$. Note that as the rod rotates, the spring always remains horizontal, because of the frictionless roller support at C . Solve the problems by principle of work and energy. (17)

(b) A slender 4-kg rod can rotate in a vertical plane about a pivot at B . A spring of constant $k = 400 \text{ N/m}$ and of unstretched length 150 mm is attached to the rod as shown in Fig. 4(b). Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 45° . Solve the problems by principle of conservation of energy. (18)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) The elevator E shown in Fig. 5(a) moves downward with a constant velocity of 4 m/s. Determine (i) the velocity of the cable C , (ii) the relative velocity of the cable C with respect to the elevator (iii) the relative velocity of counterweight W with respect to the elevator. (17)

(b) Knowing that at the instant shown in Fig. 5(b) assembly A has a velocity of 0.2 m/s and an acceleration of 0.15 m/s^2 both directed downward, determine (i) the velocity of block B , (ii) the acceleration of block B . (18)

6. (a) As shown in Fig. 6(a) the coefficients of friction between blocks A and C and the horizontal surfaces are $\mu_s = 0.24$ and $\mu_k = 0.20$. Knowing that $m_A = 5 \text{ kg}$, $m_B = 10 \text{ kg}$, and $m_C = 10 \text{ kg}$, determine (i) the tension in the cord, (ii) the acceleration of each block. (17)

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- (b) Two spheres, each of mass m , can slide freely on a frictionless, horizontal surface as shown in Fig. 6(b). Sphere A is moving at a speed $v_0 = 5$ m/s when it strikes sphere B which is at rest and the impact causes sphere B to break into two pieces, each of mass $m/2$. Knowing that 0.7 s after the collision one piece reaches Point C and 0.9 s after the collision the other piece reaches Point D, determine (i) the velocity of sphere A after the collision, (ii) the angle θ and the speeds of the two pieces after the collision. **(18)**
7. (a) The nozzle discharges water at the rate of 1.3 m³/min as shown in Fig. 7(a). Knowing the velocity of the water at both A and B has a magnitude of 20 m/s and neglecting the weight of the vane, determine the components of the reactions at C and D. **(17)**
- (b) At the instant as shown in Fig. 7(b), the angular velocity of rod AB is 15 rad/s clockwise. Using the method of instantaneous center of rotation, determine (i) the angular velocity of rod BD, (ii) the velocity of the midpoint of rod BD. **(18)**
8. As shown in Fig. 8, a straight rod PQ, 180 mm long, forms part of a mechanism. The end P of the rod is constrained to move in a straight vertical path. The rod PQ slides in a small block pivoted at a fixed point O. At the instant shown P is situated 60 mm to the left, and 45 mm above the point O. The velocity and acceleration of P are respectively, 0.816 m/s and 14.8 m/s², both directed upward. Determine the velocity and acceleration of Q. Draw necessary vector diagrams. **(35)**
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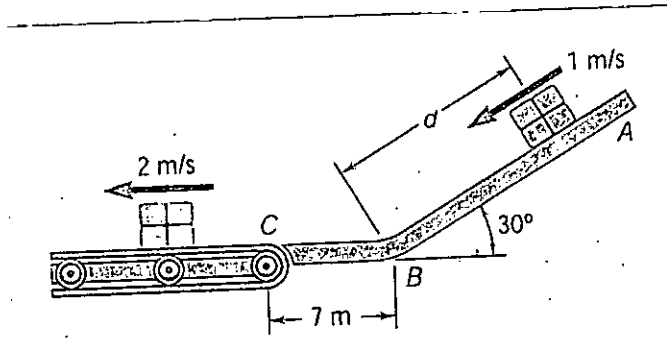


Fig. 1(a)

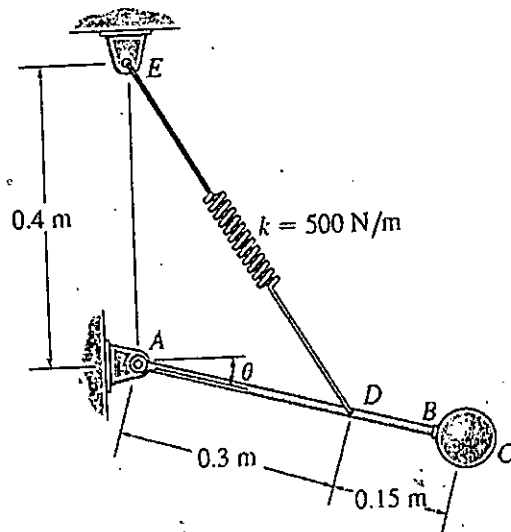


Fig. 1(b)

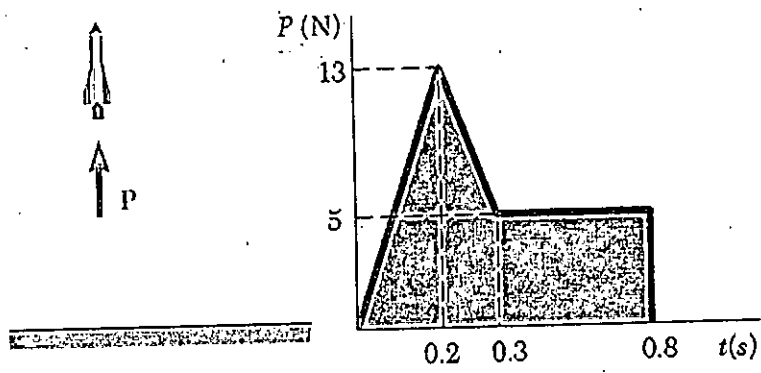


Fig. 2(a)

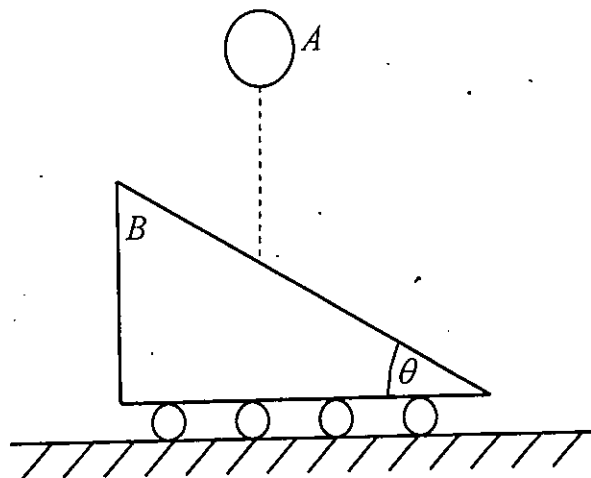


Fig. 2(b)

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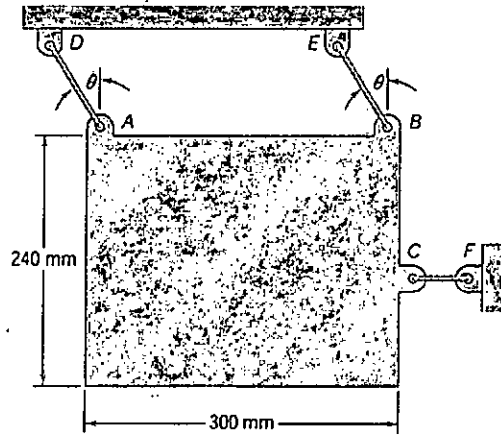


Fig. 3(a)

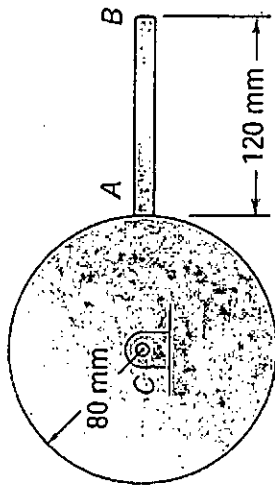


Fig. 3(b)

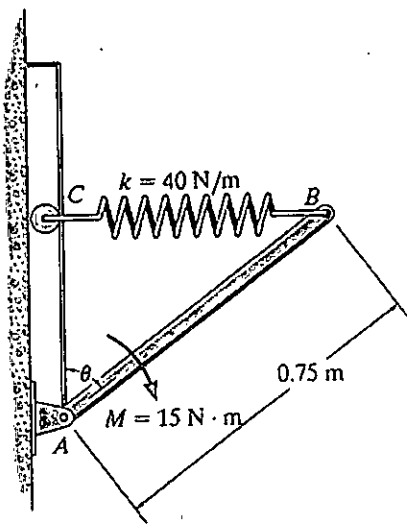


Fig. 4(a)

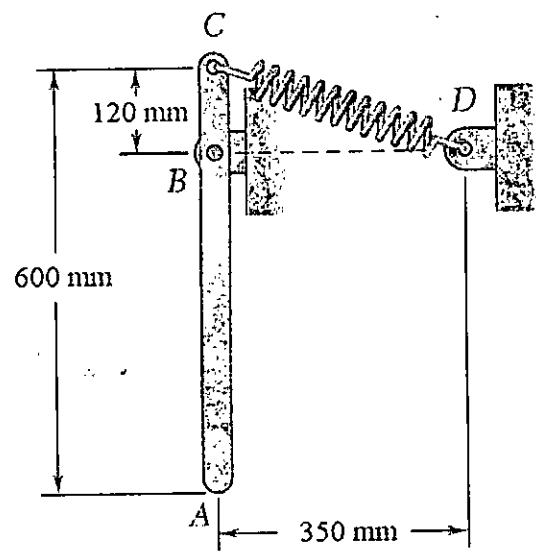


Fig. 4(b)

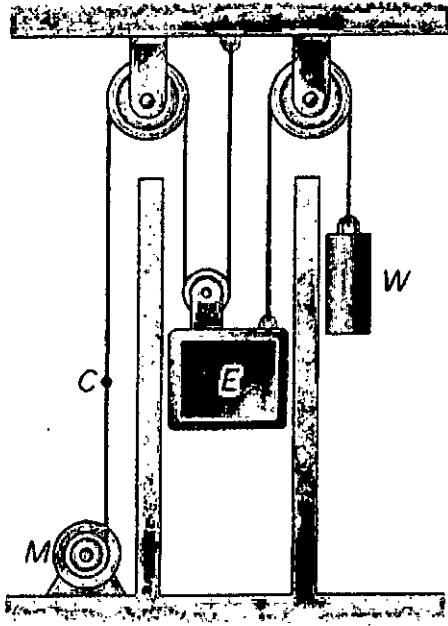


Fig. 5(a)

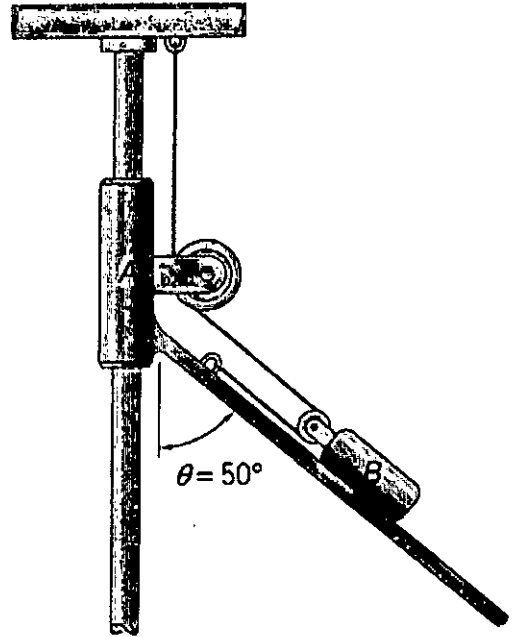


Fig. 5(b)

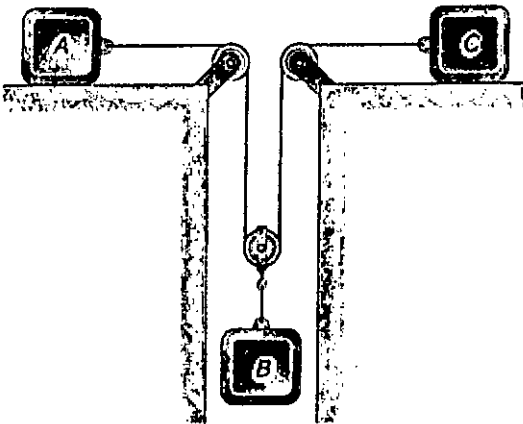


Fig. 6(a)

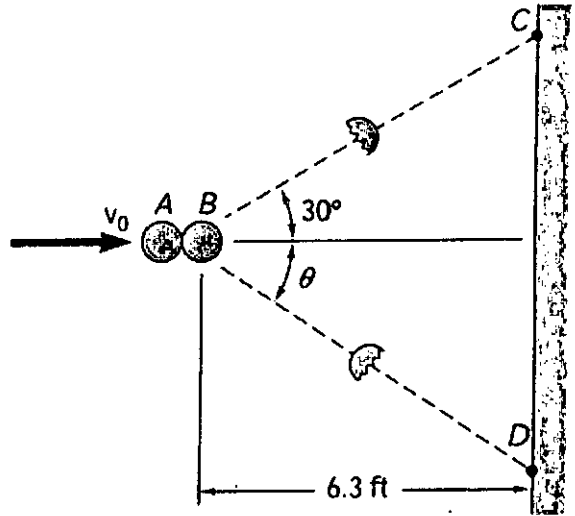


Fig. 6(b)

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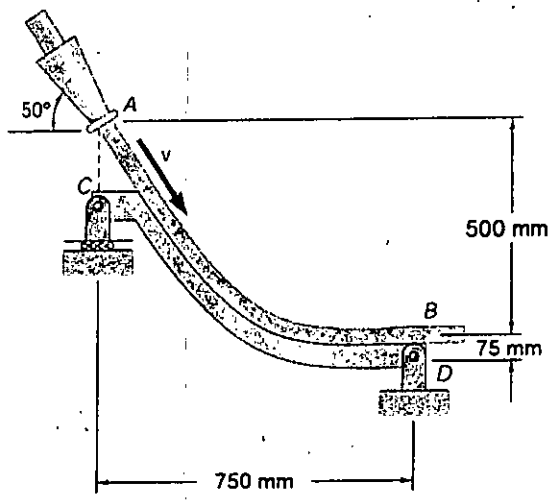


Fig. 7(a)

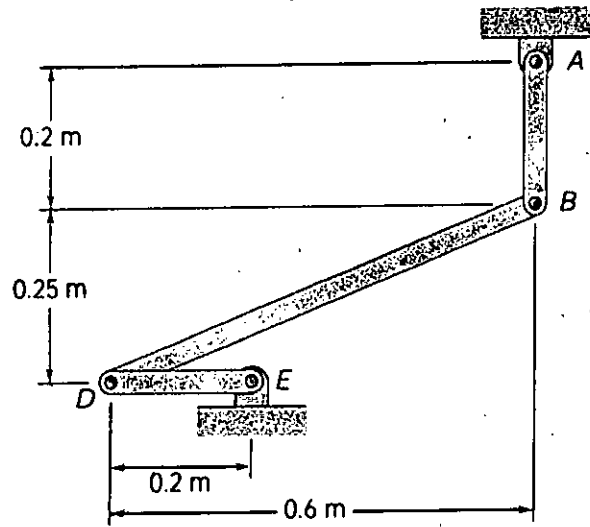


Fig. 7(b)

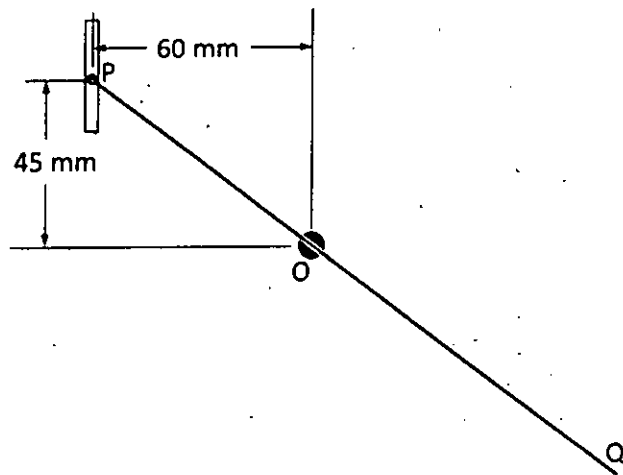


Fig. 8

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols have their usual meanings.

1. (a) Compare and contrast the following two methods of root finding: (18)

(i) Fixed-point iteration method

(ii) Newton-Raphson method

'Newton-Raphson method is superior to fixed-point iteration method' — Justify the statement with necessary mathematical proof.

- (b) Compare the performances of fixed-point and Newton-Raphson methods to compute the square root of 10. (17)

Show the results of first five iterations and comment on the results. Start iteration with $x_0 = 10$.

2. (a) The three-dimensional state of stress is given by the following matrix equation in terms of the principal stress, σ_p . (18)

$$[\sigma] \{x\} = \sigma_p \{x\}$$

$$\text{where, } [\sigma] = \begin{bmatrix} \sigma_x & \sigma_{xy} & \sigma_{zx} \\ \sigma_{xy} & \sigma_y & \sigma_{yz} \\ \sigma_{zx} & \sigma_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Find the corresponding characteristic equation of principal stress σ_p and the $[\sigma]^{-1}$ matrix using Faddeev Leverrier's method.

- (b) Find the smallest principal stress for the problem stated in Q2(a) and the corresponding eigen vector using a suitable iterative method. Show the results of at least three iterations. (17)

3. (a) With reference to least-squares curve fitting, write short notes on the following three cases with suitable examples: (15)

(i) Multiple linear regression

(ii) Weighted least-squares regression

(iii) Non-linear regression

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(b) Find the best linear least-squares fit to the following three sets of data points: (20)

- (i) points (0,0), (1,1) and (2,1)
- (ii) points (0,0), (1,1), (2,1) and (2,1)
- (iii) points (0,0), (1,1) and (2,1), giving the last point double weight.

Comment on the results.

4. (a) Give physical, graphical and mathematical interpretations of the following methods for solving initial-value problems of first-order ODE: (18)

- (i) Ralston's method
- (ii) Third-order R-K method
- (iii) classical fourth-order R-K method

(b)

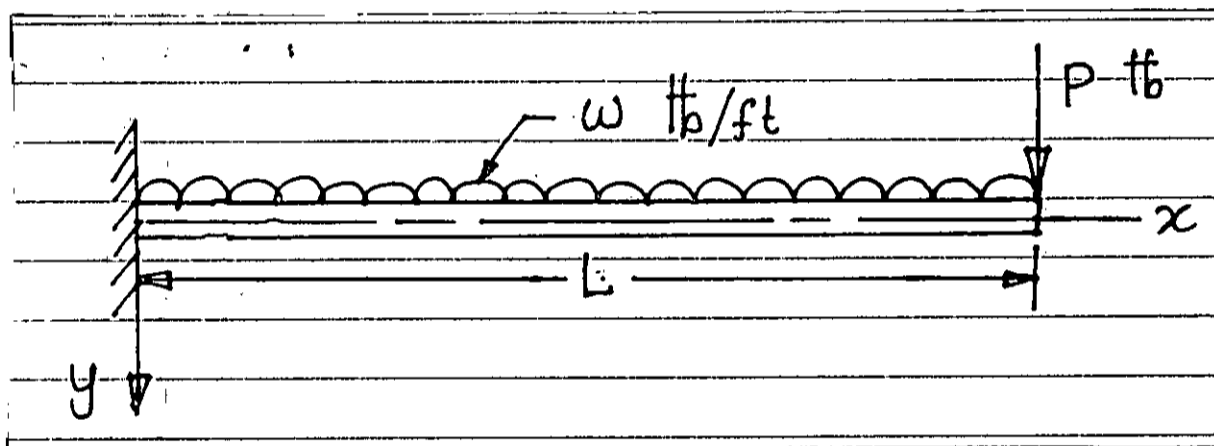


Fig. for Q 4(b)

The lateral deflection at the free end of the cantilever beam shown in Fig. for Q 4(b), is given by

$$y = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} \dots\dots(1)$$

where, w = uniformly distributed loading = 50 ± 2 lb/ft

L = Beam length = 30 ± 0.1 ft

E = Modulus of elasticity = $1.5 \times 10^8 \pm 0.01 \times 10^8$ lb/ft²

I = Moment of inertia = 0.06 ± 0.0006 ft⁴

P = a concentrated load = 1000 lb

The value of concentrated load may be taken as an exact one.

Use a first-order error analysis to determine the sensitivity of the deflection prediction in terms of both absolute and percentage error. (17)

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SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used have their usual meanings and interpretation.

Assume a reasonable value for any missing data.

5. (a) With necessary diagrams, describe. Forward Difference, Backward Difference, and Central Difference approaches of numerical differentiation. Explain their applicability and relative advantages/disadvantages in context to practical problems. (12)

(b) The rate of cooling of a body as shown in Figure for Q. No. 5(b) can be expressed as

$$\frac{dT}{dt} = -k(T - T_a),$$

where, T is the temperature of the body ($^{\circ}\text{C}$), T_a is the temperature of the surrounding medium ($^{\circ}\text{C}$), and k is a proportionality constant (per minute). Thus, this equation (called *Newton's law of cooling*) specifies that the rate of cooling is proportional to the difference in the temperature of the body and of the surrounding medium. If a metal ball heated to 80°C is dropped into water that is held constant at $T_a = 20^{\circ}\text{C}$, the temperature of the ball changes, as in (23)

Time, min	0	5	10	15	20	25
T, $^{\circ}\text{C}$	80	44.5	30.0	24.1	21.7	20.7

Use appropriate numerical differentiation schemes to determine dT/dt at each value of time. Plot dT/dt versus $T - T_a$ and explain the best possible way to determine the following:

- (i) the value of constant k within the linear fit of dT/dt versus $T - T_a$,
- (ii) the value of dT/dt at $t = 12.5$ min.
- (iii) the value of dT/dt at $t = 21.8$ min.

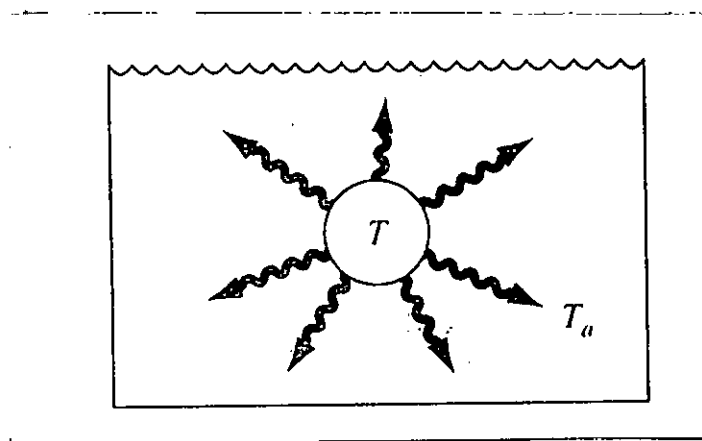


Figure for Q. No. 5(b)

6. (a) How can one improve the accuracy of numerical integration? Discuss. (9)

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(b) Show that the global truncation error associated with Simpson's 1/3 rule of integration is $-\frac{b-a}{90} h^4 f''''(\xi)$. Under what condition one can obtain the exact result using this rule - discuss. (11)

(c) Water exerts pressure on the upstream face of a dam as shown in Figure for Q. No. 6c. The pressure can be characterized by (15)

$$p(z) = \rho g (D - z),$$

where $p(z)$ is the pressure in Pa exerted at an elevation z meter above the reservoir bottom; ρ is the density of water, which for this problem is assumed to be a constant 10^3 kg/m^3 ; g is the gravitational acceleration, and D is the elevation (in m) of the water surface above the reservoir bottom. According to this equation, pressure increases linearly with depth, as depicted in Figure for Q. No. 6c(a). Omitting atmospheric pressure (because it works against both sides of the dam face and essentially cancels out.), the total force f_t can be determined by multiplying pressure times the area of the dam face (as shown in Figure for Q. No. 6c(b)). Because both pressure and area vary with elevation, the total force is obtained by evaluating

$$f_t = \int_0^D \rho g w(z) (D - z) dz,$$

where $w(z)$ is the width of the dam face (m) at elevation z (Figure for Q. No. 6c(b)). Use Simpson's rule to compute f_t .

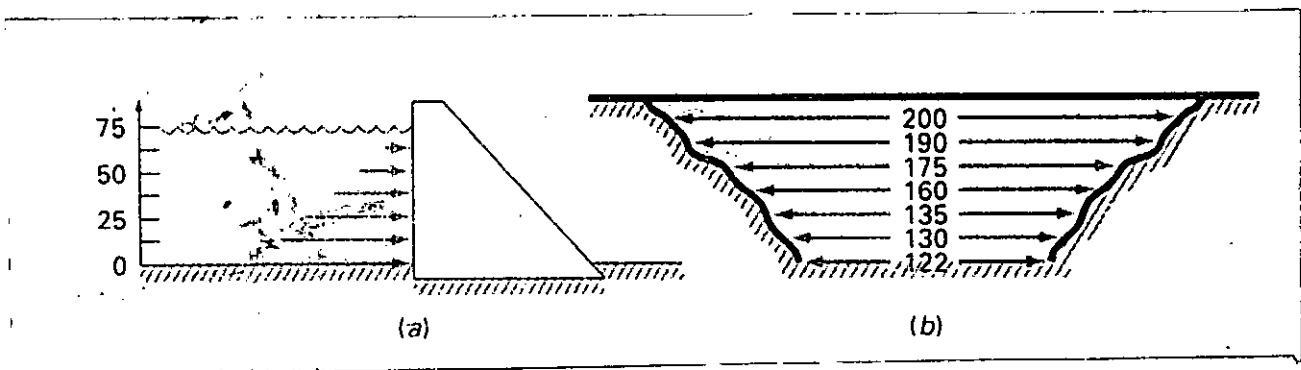


Figure for Q. No. 6c: Water exerting pressure on the upstream face of a dam (a) side view showing face increasing linearly with depth, (b) front view showing width of dam in meters.

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7. (a) The specific volume of superheated steam is listed in steam tables for various temperature. For example, at a pressure of 3000 lb/in², absolute. (15)

T, °C	370	382	394	406	418
v, Lt ³ /kg	5.9313	7.5838	8.8428	9.796	10.5311

Determine v at T = 750°F using any suitable numerical technique.

- (b) Classify and define Matrix Norms. How can you classify the condition of a system of linear algebraic equation? Discuss graphically. (10)

- (c) Mention the convergence criteria for Gauss-Seidel method used to solve liner systems of equations. How does the Gauss-Seidel iteration method differ from the Jacobi method? (10)

8. Idealized spring-mass systems have numerous applications throughout engineering. An arrangement of four springs in series being depressed with a force of 2000 kg. At equilibrium, force-balance equations can be developed defining the interrelationships between the springs,

$$\begin{aligned}
 k_2(x_2 - x_1) &= k_1x_1 \\
 k_3(x_3 - x_2) &= k_2(x_2 - x_1) \\
 k_4(x_4 - x_3) &= k_3(x_3 - x_2) \\
 F &= k_4(x_4 - x_3)
 \end{aligned}$$

where the k's are spring constants. k₁ through k₄ are 150, 50, 75, and 225 N/m, respectively.

- (a) Solve the above problem using Gauss Elimination method. (20)

- (b) Suppose now this problem needs to be solved using LU decomposition method. Find the [L] and [U] matrices. Also, show how the solution vector {x} can be found using [L] and [U] matrix (only show the matrix equations). (9)

- (c) Can you solve this problem using iterative methods? Explain the justification of using SOR (successive over-relaxation) to solve this problem. (6)

Sub : **MATH 263** (Complex Variables, Harmonic Analysis
and Partial Differential Equations)

Full Marks : 280

Time : 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Let C_R denote the upper half of the circle $|z| = R$ for some $R > 1$. Show that (11)

$$\left| \frac{e^{iz}}{z^2 + z + 1} \right| \leq \frac{1}{(R-1)^2} \text{ for all } z \text{ lying on } C_R.$$

- (b) Compute the principal value of the complex power $(2i)^{1-i}$. Also find all solutions to the equation $\sin z = 5$. (12)

- (c) Determine where the following functions are differentiable. (12)

(i) $f(z) = 2x^2 + y + i(y^2 - x)$ (ii) $f(z) = z \operatorname{Im}(z)$

Hence discuss the analyticity of these functions.

- (d) Given the transformation $w = f(z) = z^2$ which lies in the area in the first quadrant of the z plane bounded by the axes and circles $|z| = a$ and $|z| = b$, ($a > b > 0$). Discuss the transformation in the w plane and check whether if it is a conformal mapping. (11 $\frac{2}{3}$)

2. (a) Write down Cauchy Riemann equations in polar form. Test the differentiability of the function $f(z) = e^{-\theta} \cos(\ln r) + ie^{-\theta} \sin(\ln r)$, ($r > 0$, $0 < \theta < 2\pi$) in the indicated domain and hence show that $f'(z) = i \frac{f(z)}{z}$. (16 $\frac{2}{3}$)

- (b) Show that $v(x, y) = \frac{x}{x^2 + y^2} + \cosh x \cos y$ is a harmonic function. Find an analytic function $f(z) = u(x, y) + iv(x, y)$ and express $f(z)$ in terms of z . (20)

- (c) Evaluate $\int_C \frac{1}{z} dz$ along the curve $x = \cos t$, $y = \sin t$, $0 \leq t \leq 2\pi$. (10)

3. (a) State Cauchy's integral formulas and evaluate $\oint_C \frac{z+2}{z^2(z-1-i)} dz$, along the indicated closed contours: (i) $|z| = 1$ (ii) $|z-1-i| = 1$. (15)

- (b) Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$. (15)

- (c) Evaluate $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ by Cauchy's residue theorem, where the contour C is the circle $|z| = 4$. (16 $\frac{2}{3}$)

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Form a partial differential equation by eliminating arbitrary functions from (10)

$$z = f(x + iy) + g(x - iy)$$

- (b) Find the integral surface of the differential equation, $2y(z - 3)p + (2x - z)q = y(2x - 3)$

which passes through the circle $z = 0, x^2 + y^2 = 2x$. (12)

- (c) Find the complete integral by using Charpit's method: (13)

$$(p^2 + q^2)y = qz$$

2. Solve the following PDEs:

(a) $(D_x^3 - 4D_x^2D_y + 5D_xD_y^2 - 2D_y^3)z = (y + x)^{\frac{1}{2}}$. (10)

(b) $(D_x^2 - D_xD_y - 2D_y^2 + 2D_x + 2D_y)z = e^{3x+4y} + xy$. (12)

(c) $(x^2D_x^2 - xyD_xD_y - 2y^2D_y^2 + xD_x - 2yD_y)z = \log\left(\frac{y}{x}\right) - \frac{1}{2}$. (13)

3. (a) Prove that the product of two matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and

$\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$. (10)

- (b) For what values of λ and μ the following system of equations: (13)

$$x + y + z = 6$$

$$x + 2y + 5z = 10$$

$$2x + 3y + \lambda z = \mu$$

has a unique solution. Also find the solution for $\lambda = 8$ and $\mu = 8$.

- (c) Use elementary row operations to find the inverse of (12)

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

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4. (a) Find non-singular matrices P, Q so that PAQ is a normal form where (17)

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

- (b) Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$ to canonical form and write down the corresponding transformations. Also find the rank, index and signature. (18)

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Verify Cayley-Hamilton theorem for the matrix A where (15)

$$A = \begin{bmatrix} 2 & 3 & 0 \\ -5 & 7 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

- (b) Determine Eigen vector of the matrix (20)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

6. (a) An analysis of companies resulted in the following distribution (15)

Profit (lakhs)	10-20	20-30	30-40	40-50	50-60
No. of companies	18	20	30	22	10

Calculate first four moments about assumed mean. Convert the result into moments about mean. Also find β_1 and β_2 .

- (b) An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.95 and 0.98, respectively. Assume the components are independent. (10)

- (i) Draw a tree diagram to demonstrate all the outcomes along with probabilities.
- (ii) Determine the cumulative distribution function of the number of components in the assembly that meet specification. Also calculate the standard deviation of that random variable.

- (c) Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, find $P [[X, Y] \in A]$ where A is the region $\{(x, y) : x + y \leq 1\}$. (10)

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7. (a) The probability that your call to a service line is answered in less than 30 seconds is 0.75. Assume that your calls are independent. (18)

(i) If you call 10 times, what is the probability that exactly 9 of your calls are answered within 30 seconds?

(ii) If you call 20 times, what is the mean number of calls that are answered within 30 seconds?

(iii) If you call 20 times, what is the probability that at least 16 calls are answered in less than 30 seconds?

(b) A computer terminal can pick up an erroneous signal from the keyboard that does not show up on the screen. This creates a silent error that is difficult to detect. Assume that for a particular keyboard the chance that this will occur per entry is 0.001. In 12000 entries find the chance that 3 silent errors occur. What about at least 2 silent error occurs? (17)

8. (a) Find the Karl Pearsons correlation coefficient between the sales and expenses from the data given below and interpret its value. (17)

Firm	1	2	3	4	5	6	7	8	9	10
Sales (Lakhs)	50	50	55	60	65	65	65	60	60	50
Expenses (Lakhs)	11	13	14	16	16	15	15	14	13	13

(b) Prices of shares of a company on the different days in a month were found to be 66, 65, 69, 70, 69, 71, 70, 63, 64, 68. Discuss whether the mean price is 65. Test at 5% level of significance. Given $t = 2.26$ for 9 degrees of freedom. What type of error may occur in this problem? (18)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Suppose you are analyzing a failed component of red alpha brass that operated in an ammonia rich environment and cracked after few days in service. What is the possible Phenomenon that occurred in this case? What are its reasons and how can it be prevented? (18)
- (b) Which type of cast iron is obtained by heat treating white cast iron? Explain the mechanism of formation of its microstructure using appropriate diagram. (17)
2. (a) "Gray cast iron has low strength and almost zero ductility in tension but has better properties in compression" explain the assertion with figure. (10)
- (b) Suppose you have to design a heat treatment cycle for a high alloy tool steel component. During tempering of the component which factors can affect the hardness and how they contribute depending on the tempering temperature? (12)
- (c) "Despite having a little amount of alloying elements, high strength low alloy steels (HSLA) display great mechanical properties." – explain the reason behind this phenomenon. (13)
3. (a) "Iron making is reductive process, while steelmaking is an oxidative process" – Justify the assertion with proper reasoning. (10)
- (b) Describe steel making using the LD process. If LD process is so fast and efficient, why is it not used in our country? (15)
- (c) Using necessary diagram, explain how you can remove the dust particles from blast furnace gas. (10)
4. (a) In case of annealing of hypereutectoid steels, why heating above upper critical is not done like hypo eutectic steels, even though both types of steels are heated to austenite region in case of normalizing? (10)
- (b) In case of normalizing of steels, what are the effects of faster cooling on:
(i) proeutectoid constituents. (15)

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Contd... Q. No. 4(b)

- (ii) austenite transformation temperature.
(iii) fineness of pearlite.
- (c) In carburizing process, after carburization heat treatment is necessary to get desired hardness whereas in nitriding process, no such post treatment is needed - why? (10)

SECTION - B

There are **EIGHT** questions in this section. Answer any **SIX**.

5. Find out if the packing density of zinc is higher than that of chromium or not. Show the detail calculation and list the closely packed crystallographic planes and directions for both crystal structures. (17 $\frac{1}{2}$)
6. Describe how a binary isomorphous phase diagram can be constructed using a series of cooling curves. For a binary eutectic alloy, explain how coring structure is developed and how this can be eliminated completely. (17 $\frac{1}{2}$)
7. With the necessary diagrams explain the micro-structural change that take place during cooling of steel once it crosses the upper critical temperatures and when this phase transformations cease. Show how these temperatures vary as a function of the chemical composition. (17 $\frac{1}{2}$)
8. Evaluate the effect of crystallographic structure and bonding type on the ease of slip for mechanical deformation along with the effect of temperature and stress. It is easier for slip to take place in a single crystal than in polycrystal – argue this statement. (17 $\frac{1}{2}$)
9. Two metallic bars of chromium were broken at 20°C and -200°C. Fracture surfaces for both are shown in Fig. for question no. 9. Failure occurred at much lower strength at the latter temperature. Compositional purity was the same for both the pieces. Predict the most probable reason for the change in fracture behavior and describe how the predicted mechanism can be proved. (17 $\frac{1}{2}$)

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10. Two alloys of a binary system, A-B, are cooled down to room temperature from high temperature single phase alpha, which is a solid solution of B in A. During cooling, the alloys intersect the solvus line, and this results in a significant increase in yield strength of both the alloys. However, the strength of alloy 1 was much higher than alloy 2 where cooling rates were different for the alloys of the same composition. Explain the mechanism for the observed differential enhanced strength for alloys 1 and 2 with necessary sketches.

(17½)

11. Fig. for question no 11 shows a stress-strain curve for the tensile test of an engineering alloy – identify if the alloy is ferrous or non-ferrous and how yield strength along with the modulus of elasticity can be determine from this curve. Relate the elastic and plastic deformation behaviour with the aspects of bond stretching and plane shearing for this alloy.

(17½)

12. Fig. for question no. 12 shows the fracture surface of a broken metallic shaft, find the reason of this brittle mode of fracture. List the features of the fracture surface that support your opinion and describe how such a failure can be prevented.

(17½)

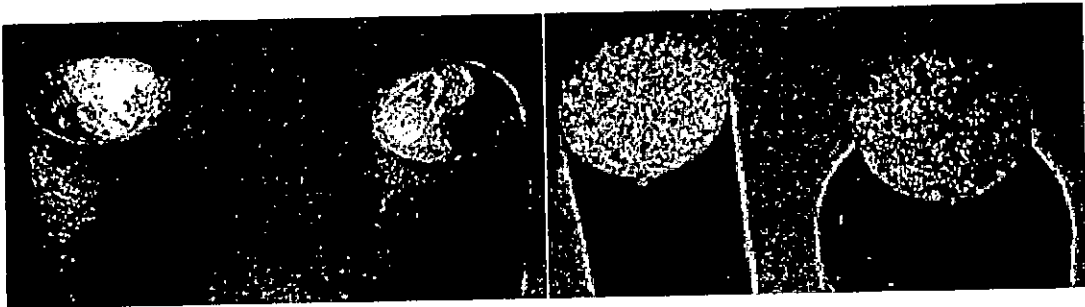


Fig. for question no. 9 showing fracture surfaces at (left) 20 C and (right) - 200 C

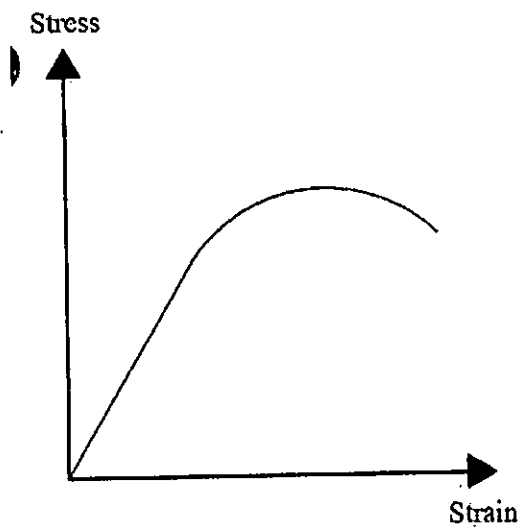


Fig. for question no. 11

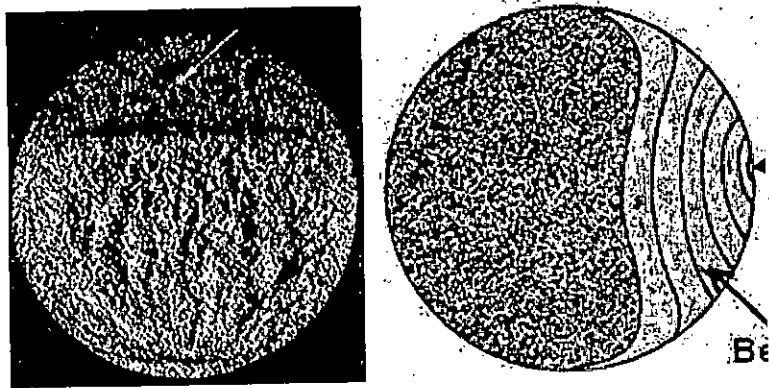


Fig. for question no. 12