A CYCLIC BEHAVIORAL MODELING ASPECT TO UNDERSTAND THE EFFECTS OF VACCINATION AND TREATMENT ON EPIDEMIC TRANSMISSION DYNAMICS

by

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This work is dedicated

to

My Family

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Abstract

Evolutionary epidemiological models have played an active part in analyzing various contagious diseases and intervention policies in the biological sciences. The design in this effort is the addition of compartments for treatment and vaccination, so the system is designated as susceptible, vaccinated, infected, treated, and recovered (SVITR) epidemic dynamic. The contact of a susceptible individual with a vaccinated or an infected individual makes the individual either immunized or infected. Inventively, the assumption that infected individuals enter the treatment and recover state at different rates after a time interval is also deliberated through the presence of behavioral aspects. The rate of change from susceptible to vaccinated and infected to treatment is studied in a comprehensive evolutionary game theory with a cyclic epidemic model. To show stable conditions, we theoretically investigate the cyclic SVITR epidemic model framework for disease-free and endemic equilibrium. Then, the embedded vaccination and treatment strategies are present using extensive evolutionary game theory aspects among the individuals in society through a ridiculous phase diagram. Extensive numerical simulation suggests that effective vaccination and treatment may implicitly reduce the community risk of infection when reliable and cheap. The results exhibited the dilemma and benefitted situation, in which the interplay between vaccination and treatment evolution and coexistence are investigated by the indicators of social efficiency deficit and socially benefited individuals.

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Nomenclature

Symbol	Description
S	Susceptible
V	Vaccination
I	Infected
T	Treatment
R	Recovered
N	population
h_c	Herd immunity
L_f	Lyapunov function
R_0	Basic reproduction number
R_e	Effective reproduction number
R_{SN}	Strength number
$\bar{c}$	Average rate of contact
d	Duration of infectiousness
C_V	Vaccination cost
C_T	Treatment cost
C_i	Infection cost
ε_0	Disease-free equilibrium
β	Infection rate (per person per day)
γ	Natural Recovery rate(per day)
ω	Immunity loss rate
x	Vaccination rate
τ	Treatment rate
δ	Treatment to recovery rate

Chapter - 1

Introduction

1.1 Introduction

In recent years, the spread of the virus has created attention among people [1,2], which affects people's lives, such as the covid-19 pandemic [3], monkeypox [4], seasonal influenza [5], and others. Apart from country initiatives, its impact on individual or group initiatives is also noticeable [6]. Infectious diseases could be driven towards eradication if adequate and timely steps (e.g., vaccination, treatment, self-defense measure, and refinement campaign) are taken in the course of the epidemic [7]. However, many of these diseases eventually become endemic in our society due scarcity of adequate policies and timely interventions to mitigate the spread of the viruses [8]. Consequently, there is a need for proactive and retroactive steps toward controlling the spread of infectious diseases, particularly those for which both vaccines and treatment are available. Here, the theoretical studies of vaccination and treatment strategies have considered different effectiveness, associated costs, payoff structures, and time scales.

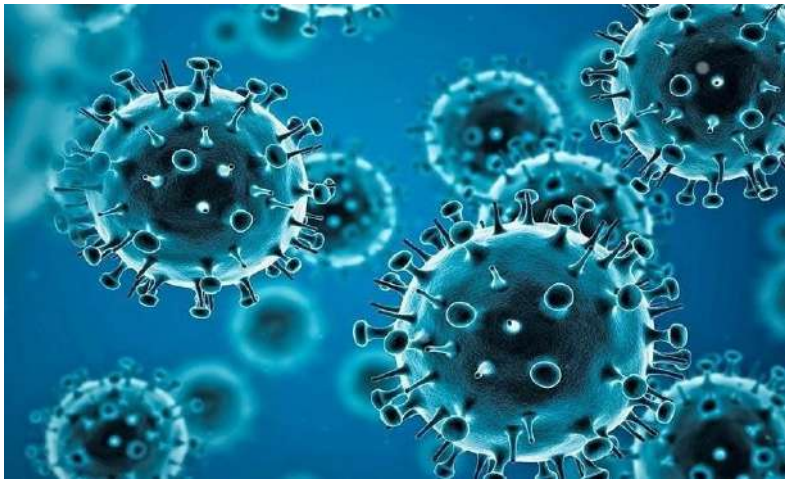

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Figure 1.1: The covid-19 pandemic, an infectious disease caused by severe Acute respiratory syndrome coronavirus 2 (SARS-coV-2) virus [3]

Based on the theory of Kermark and Mckendrick [9], the dynamics of infectious diseases can usually be described mathematically based on compartmental models such as SIR (susceptible–infected–recovered) or SIRS models, with each term referring to a "compartment" in which an individual can reside. Recently, Covid-19 inclined the attention of mathematicians, and they have tried to inflict an approximate solution by bearing different models [10]. To understand the mechanism of infectious disease transmission, several authors have studied various kinds of epidemic models by considering other compartment models such as SI [11], SIS [12], SIR [13-19], SIRS [20], SEIR [21, 22], SVEIR [23], and many more. Mathematical modeling has been successfully used in constructing control strategies with suitable interventions for various diseases, such as vaccination, treatment, and quarantine [24-30]. Variations of standard, SIR, SIRS, and SEIR epidemiological models are considered to determine the sensitivity of these models to different parameter values that may not be fully known when the models are used to investigate emerging diseases [31]. Previous works above explored that vaccination, quarantine, and treatment would retrench contagious disease in a simple dynamic aspect on local time scales. The current study aims to develop a theoretical epidemic model embedding both vaccination and treatment as a cyclic model.

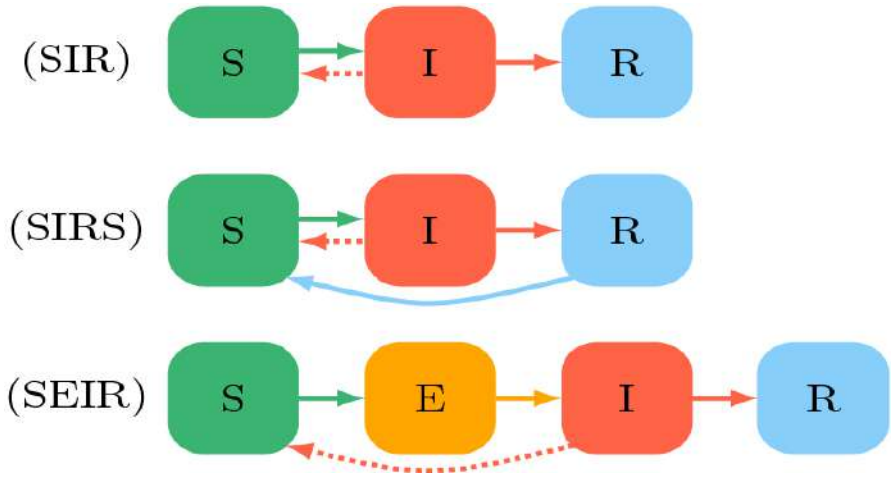

Figure 1.2: Classes of flowchart including Susceptible, Infections, Expose and Recovered compartment to SIR-type cyclic epidemic models [13 -22]

An extensive evolutionary game theory study studied the influence of people's vaccination decisions on imposed control policies, reducing the epidemic's spreading severity [32-35]. Prior research has shown that a game-based approach to epidemiological vaccination may

accurately forecast infection risk in vaccinated and unvaccinated people [36-37]. The vaccine properties, people's judgment, social networks, and neighbors' choices influence how people make apposite decisions to control the disease. As a result, it is crucial to examine how different elements affect both vaccine and treatment acceptance [38-42]. Here, use the SVITR-type epidemic model to thoroughly analyze the combined impact of these two types of protective measures: proactive and retroactive. In this situation, the vaccine's efficiency and treatment duration serves as a control parameter. Individuals' ability to update their methods by emulating those who appear to have adopted more effective techniques must be integrated into the model [43]. To characterize this process formally, it need to build a model that incorporates mathematical epidemiological with game-theoretic dynamics. For example, Bauch et al. [44] made and tested a model that blends epidemiological dynamics with replicator dynamics from evolutionary game theory to describe individuals' copying behavior during disease outbreaks. However, people's attitudes toward vaccination and treatment reflect their inherent recognition to choose between vaccine acceptance and risk of infection. This approach incorporates two types of game aspects (proactive and retroactive) on a local time scale [45] to illuminate the framework of cyclic disease dynamics embedded with vaccination and treatment provision.

(Collected from Google)

Figure 1.3: Coronavirus COVID-19 vaccine [7]

This work introduces a new indicator, “socially benefited individual,” termed SBI for vaccination and treatment provisions. This approach benefits individuals from society who

get advantages from participating in the vaccine program or treatment. Besides, explore the idea of a dual dilemma by considering the “social efficiency deficit” (SED) [46, 47] that presents the roles of vaccination (before infection) and treatment (after infection) games. Both evolutionary games occur on a local time scale (single season) that is affected by various factors concerning vaccine efficacy, vaccine cost, treatment cost, and treated time (facilities). Here, impose a pre-emptive intervention policy that controls disease before the infection spreads at an early stage that depends on the individual’s choice. On the other hand, the treatment strategy can be considered a “let-down” intervention in which people will recover faster. Utilizing our new idea assisted by the evolutionary game theory approach on epidemiology for vaccination and treatment game model, explore the impact of the dual-dilemma and social benefit situations by presenting line graphs and phase diagrams. Such a social benefit approach and dual-dilemma situation in the same framework, perhaps quite omnipresent in the real world, has not been studied in related earlier works.

1.2 Some definition of Model Compartments

Susceptible (S)

The term "susceptible" generally means being vulnerable or open to influence, harm, or disease. It can be used to describe individuals, organisms, or systems that are more likely to be affected by external factors. For example, a person may be described as susceptible to a particular disease if they have a weaker immune system or if they have not been vaccinated against it.

Vaccinated (V)

"Vaccinated" refers to the process of receiving a vaccine, which is a preparation of weakened or dead pathogens that are introduced into the body to stimulate the immune system and develop immunity against specific diseases. Vaccines have been developed for a wide range of diseases, including measles, polio, hepatitis B, and COVID-19. Vaccination is considered one of the most important public health interventions of the modern era and has contributed to the eradication of smallpox, the control of many other diseases, and the prevention of millions of deaths globally.

Infected (I)

Infection can occur through various routes, including inhalation, ingestion, contact with contaminated surfaces, or bites from infected animals. The severity and duration of the infection depend on various factors, such as the virulence of the pathogen, the susceptibility of the host, and the effectiveness of the host's immune response. The term "infected" refers to the invasion and multiplication of pathogenic microorganisms, such as bacteria, viruses, fungi, or parasites, within a host organism. This invasion and multiplication of pathogens can lead to disease or illness in the host.

Treatment (T)

Treatment can take various forms, depending on the type and severity of the condition, and may involve medications, surgery, lifestyle changes, physical therapy, or other interventions. The choice of treatment depends on various factors, such as the patient's age, medical history, and preferences, as well as the availability and effectiveness of different treatment options.

Recovered (R)

The duration of recovery can vary depending on the type of disease, the severity of the infection, and the overall health of the individual. Some illnesses may resolve quickly, while others may require a longer period of convalescence. In the context of COVID-19, an individual is typically considered "recovered" if they have tested positive for the virus, but have completed a period of isolation and are no longer showing symptoms. Some health organizations also require additional negative test results before considering someone fully recovered.

Final epidemic size (FES)

The final epidemic size is a key metric used in infectious disease modeling to estimate the total number of individuals who will become infected during an epidemic. In the context of the SVITR model, the final epidemic size can be calculated by integrating the differential equations until the epidemic has run its course and all individuals have either recovered or died.

Vaccination coverage (VC)

Vaccination coverage refers to the proportion of a population that has received a particular vaccine, typically expressed as a percentage. It is a measure of the extent to which a population has been vaccinated against a particular disease and is an important indicator of the level of protection against that disease within the population. The final epidemic size in the SVITR model is influenced by several factors, including the vaccination coverage. Vaccination coverage refers to the proportion of the population that has received a vaccine against the disease. In the SVITR model, vaccination coverage is represented by the V compartment, which includes individuals who are vaccinated and therefore protected from infection. Increasing the vaccination coverage can reduce the final epidemic size by reducing the number of susceptible individuals who can become infected. This is because the vaccinated individuals are less likely to become infected and therefore less likely to transmit the disease to others. The effect of vaccination on reducing the final epidemic size is dependent on the vaccine efficacy, the coverage rate, and the timing of the vaccine deployment. If the vaccination coverage is high enough, the final epidemic size may be small enough that the disease is effectively eliminated from the population. This is known as herd immunity, where the proportion of immune individuals in the population is high enough to provide indirect protection to susceptible individuals. On the other hand, if the vaccination coverage is low, the final epidemic size may be larger, leading to more infections and potentially more severe consequences such as hospitalization and death.

Average social payoff (ASP)

The average social payoff is a metric used in game theory to quantify the expected benefit or cost of a particular strategy or action. The Average Social Payoff (ASP) is a measure of the net benefit to society of a particular intervention or program. It takes into account both the costs and benefits of the intervention and is typically expressed as a monetary value. The ASP is calculated by subtracting the total social costs of the intervention from the total social benefits and dividing the result by the total number of individuals affected by the intervention. This provides an estimate of the average net benefit per individual. The ASP provides a useful tool for evaluating the efficiency of interventions and can help inform policy decisions by providing information on the net benefits of different programs and policies. In the context of the SVITR model, the average social payoff can be used to

evaluate the effectiveness of different disease control strategies and their impact on the overall population.

Fraction of treated individuals (FTR)

The fraction of treated individuals stands for Fraction of Treated Individuals. It is a measure of the proportion of individuals within a population who have received a particular treatment or intervention. The FTR is often used in the context of healthcare interventions, where it is used to evaluate the uptake and effectiveness of different treatment options. For example, if a particular treatment is recommended for a certain medical condition and the FTR is low, it may suggest that there is a need for better education and awareness about the benefits of the treatment. The FTR can also be used to evaluate the impact of policies in other areas, such as education or social welfare. It is an important tool for policymakers, as it provides information on the proportion of individuals who are benefiting from a particular intervention or policy. The FTR is an important indicator of the level of access to treatment within a population and is used to monitor progress in efforts to improve health outcomes. High FTRs are generally associated with better health outcomes and are an important goal of public health programs and policies.

Socially Benefitted Individuals (SBI)

Socially Benefitted Individuals refers to the number of individuals who receive a net benefit from a particular intervention or policy. The term "net benefit" refers to the overall positive impact that the intervention has on the individual's well-being, taking into account both the costs and benefits of the intervention. The SBI is an important measure of the effectiveness of interventions, as it indicates the number of individuals who have experienced a positive change as a result of the intervention. It is typically calculated by subtracting the number of individuals who experienced a negative impact from the number of individuals who experienced a positive impact. The SBI can often be used to evaluate the impact of policies in other areas, such as education or social welfare. It is an important tool for policymakers, as it provides information on the number of individuals who are likely to benefit from a particular intervention or policy.

The Social Efficient Deficit (SED)

The Social efficiency Deficit is a measure of the difference between the optimal level of an intervention or policy and the actual level that is implemented. It is calculated by comparing the social welfare that would be achieved if the intervention were implemented at the optimal level, to the social welfare that is achieved at the actual level of implementation. The SED is an important measure of the efficiency of interventions and policies. It provides information on the potential gains that could be achieved if the intervention were implemented more effectively. The SED can also help policymakers identify areas where improvements can be made to increase the overall effectiveness of interventions. The SED is often used in the context of healthcare interventions, where it is used to evaluate the effectiveness of different treatment options. For example, if a particular treatment is implemented at a lower level than optimal, the SED will be positive, indicating that there is potential to improve the social welfare that is achieved through the intervention.

1.3 Deterministic epidemic models

Start with simplest case of SIS [56] and SIR [57] disease model. The SIS model has twocompartments: susceptible (S) and infected (I), whereas the SIR model has additional recovered (R) compartment. The model is inscribed in the form of simple ODEs. The SIS and SIR model(figure 1.4) can be written as,

Susceptible-infected- susceptible (SIS) model:

$$\dot{S}(t) = -\beta S(t)I(t) + \gamma I(t),$$

$$\dot{I}(t) = \beta S(t)I(t) - \gamma I(t).$$

Susceptible-infected- recovered (SIR) model:

$$\dot{S}(t) = -\beta S(t)I(t),$$

$$\dot{I}(t) = \beta S(t)I(t) - \gamma I(t),$$

$$\dot{R}(t) = \gamma I(t).$$

(A) (B)

Figure 1.4: Schematic diagram of (A) the SIS epidemic model and (B) the SIR epidemic model.

Here, $S(t), I(t)$ and $R(t)$ denote the size of susceptible, infected and recovered at time t, compartments and the disease transmission rate is defined as β, while the rate of recovery is assumed as γ. As constraint, it presumed $S(t) + I(t) = 1$ for SIS model and $S(t) + I(t) + R(t) = 1$ for SIRmodel. Hence, the single differential equation for $I(t)$ (Both SIS and SIR) is $\dot{I}(t) = \beta S(t)I(t) - \gamma I(t)$, which can be easily solved by separation of variables method. This equation may have only two equilibria at steady stat, namely, the disease-free equilibrium (DFE), $I_{DFE} = 0$ and the endemic equilibrium, $I_E = (1 - \gamma/\beta)$. If the basic reproduction number (ratio) is defined as $R_0 = \frac{\beta}{\gamma}$ then the endemic steady state exists if $R_0 = \beta/\gamma > 1$. Thus, if $R_0 < 1$, the disease-free equilibrium is stable, while for $R_0 > 1$, the disease-free equilibrium is unstable, and the endemic equilibrium is stable.

To be more specific, the system tends to a DFE state as time goes to infinity; however, this DFE depends on the boundary conditions. Therefore, it can deduce the final epidemic size, $R(\infty)$, for the limit of $t \to \infty$. According to the SIR model with boundary conditions $S(0) \approx 1$, $R(0) = 0$, $I(0) \approx 0$ and $S(\infty) = 1 - R(\infty)$, then have, $R(\infty) = 1 - exp\,[-R_0 R(\infty)]$. $Here,\ R(\infty)$ is the portion of population who were once infected with the diseases called final epidemic size (FES). This implicit equation can be solved to arbitrary accuracy by iteration method, for instant, using Newton-Raphson method.

1.4 Game theory

Game theory is the study of the mathematical model of rational decision-making, where several players must make choices to maximize their own payoffs [61]. It has been applied in many disciplines such as social science, political science, economics, business, logic, system science, information science, computer science, and biology. John von Neumann first introduced the modern game theory in 1928 [62]. In 1944, John Nash [63] presented his dissertation about noncooperative games, to state equilibrium point (steady state) called Nash equilibrium. In the 1960s and 1970s, game theory was broadly applied to solve the problems caused by wars and economics [64, 65]. Game theory has also been used to biology, called evolutionary game theory, first introduced by John Maynard Smith and G.R. Price in 1973 [66].

1.5 Classical game theory

Classical game theory encompasses two different types of games: cooperative games with the central concept of Pareto optimum and non-cooperative games with the idea of Nash equilibrium [62]. Such a game, players always make consistent decisions in the face of certain and uncertain alternatives to maximize their payoffs. Under certain circumstances, players may face the dominant or dominated strategies (players choose either the dominant strategy or avoid the dominant strategy)

1.6 Vaccination Game

Pre-emptive voluntary vaccination benefits public health enormously. Vaccination, which is driven by people's attitude and decision, is one of the best public health provisions for preventing epidemics of infectious diseases [67-72]. The aspect of the human decision-making process may be the result of a trade-off between protection and risks. People in a society choose the best strategy to maximize their payoffs, based on others' (neighbors) strategies. When a large fraction of individuals is vaccinated and have immunity against the disease, some people who have not vaccinated benefit from indirect protection. Thus, it might seem that an individual's decision is affected by others' perceived vaccination behavior that can lead to the vaccination level, and this level is suboptimal for the whole society. The expected disease prevalence and vaccine coverage can be estimated and analyzed by coupling the epidemic compartment model and game theory.

1.7 Herd immunity

Herd immunity is the indirect protection from infectious disease provided by vaccinated individuals, which derives from mass vaccination or previous infection [73, 74]. When a large fraction of individuals in a population is vaccinated and has immunity against the disease, the remaining population who is not vaccinated benefits from the indirect protection. As the fraction of the vaccinated individuals reaching to the herd immunity threshold, there is less possibility to spread disease since there are not many suspected susceptible individuals, the chain of infection is congested, and the transmission of disease is prevented. The herd immunity threshold h_c is obtained from the simple SIR compartment

model for well mixed and infinite population [75]. The expression in terms of the basic reproduction number R_0 is, $h_c = 1 - 1/R_0$.

1.8 Free rider

The term "free-rider" is a type of market failure that occurs a problem because while not paying for the good, they may continue to take advantage of the good. For example, people using a bus without paying the fare are called a free rider. The free-rider problem arises when too many people take the free-riding advantage. The view of free riding in vaccination [76] means that, when mass people perceived vaccination to avoid infection, an alternative strategy can appear not vaccinating, thus avoiding any risk of vaccine side effects and spending cost for vaccination. The unvaccinated population tries to keep themselves under the coverage of free riding to provide herd immunity.

1.9 Social efficient deficit (SED) on vaccination game

The situation of social dilemma is of great interest in evolutionary game theory because of its importance in explaining the evolution of cooperation in biological systems. The dilemma strength parameters [77–79] have been used to numerically characterize the social dilemma existing in a game. However, these parameters can only be defined in pairwise games and some specific multiplayer games having a simple payoff structure but are not able to apprehend more complicated games; for example, vaccination game. To reveal the existence of social dilemma associated with vaccination game systems, "social efficiency deficit" has been introduced to quantify the payoff difference between social optimum (the desired state of affairs) and Nash equilibrium (The content of SED is based on the article [80]). Thus, the SED indicates that the payoff can be improved from that at the NE. Mathematically, the SED is given by

SED = (social optimum payoff) − (payoff at Nash equilibrium)

Here, SED = 0 implies no social dilemma, while any social dilemma causes a positive SED. According to the abovementioned conceptual definition, SED is given by,

$SED = ASP_{social}^{opt} - ASP^{NE}$.

1.10 Motivation

The key to developing the SVITR cyclic epidemic model lies in addressing the complexities and nuances associated with disease spread, vaccination, and treatment dynamics in a population. Conventional epidemic models, such as the classic SIR (Susceptible-Infectious-Recovered) model, provide valuable insights into disease dynamics but often overlook the role of vaccination and treatment strategies. The use of evolutionary game theory in the SVITR model provides a novel approach to studying the behavioral aspects of vaccination and treatment decisions among individuals in a society. Game theory allows researchers to analyze how individual choices influence the overall dynamics of disease control. It considers factors such as the effectiveness of vaccination and treatment, associated costs, and individual payoffs, making it a powerful tool for policy analysis and decision-making. The cyclic nature of the SVITR model further enhances its utility. Diseases often exhibit cyclical patterns, with outbreaks and periods of low activity. The model's ability to simulate these cyclical dynamics allows researchers to explore disease-free and endemic equilibria, providing insights into the stability and persistence of infections over time. Overall, the motivation behind the SVITR cyclic epidemic model is to provide a more comprehensive and realistic representation of disease spread, vaccination, and treatment dynamics. By incorporating evolutionary game theory and considering cyclical patterns, the model offers valuable insights into designing effective vaccination and treatment strategies, quantifying social benefits and dilemmas, and guiding public health interventions to control and prevent contagious diseases.

1.11 Objective

This research will enable us to develop an SVITRS (susceptible-vaccinated-infected-treatment-recovered susceptible) epidemic model for the disease spread and the embedded vaccine and treatment behavioral dynamics by using extensive evolutionary game theory among the individuals in societies. Evolutionary epidemiological models have played an active part in analyzing various contagious diseases and intervention policies in the biological sciences. The specific objectives of this research are as follows:

- To design the compartments for treatment and vaccination, so the system is modeled as susceptible, vaccinated, infected, treated, recovered and susceptible (SVITRS) epidemic dynamic.

- To study the rate of change from susceptible to vaccinated and infected to treatment in a comprehensive 2 evolutionary game theory with a cyclic epidemic model.
- To investigate the cyclic SVITRS epidemic model framework for disease-free and endemic equilibrium to show stable conditions. To present the embedded vaccination and treatment strategies using extensive evolutionary game theory aspects among the individuals in society through a ridiculous phase diagram.

Chapter - 2

Mathematical Preliminary

2.1 Introduction

The basic SIR epidemic model is a mathematical framework used to study the spread of infectious diseases. It consists of three compartments: susceptible (S), infected (I), and recovered (R). The dynamics of the model are described by a system of ordinary differential equations. The reproduction number, denoted as R_0, is a key parameter in the model that represents the average number of secondary infections caused by a single infected individual in a completely susceptible population. If $R_0 > 1$, the disease can spread in the population, while if it is less than 1, the disease will die out. The stability of equilibrium points in the SIR [13-19], model can be analyzed using various mathematical techniques, such as Lyapunov functions and LaSalle's invariance principle. A Lyapunov function is a scalar function that measures the distance of the system's state from the equilibrium point and can be used to determine stability properties. LaSalle's invariance principle states that the trajectories of the system will converge to the largest invariant set contained in the region where the Lyapunov function is decreasing. Local asymptotic stability for equilibria can be established using the linearization of the model around the equilibrium point. The next-generation method is a powerful approach to calculate the reproduction number in more complex epidemic models. It involves identifying the "next-generation matrix" that captures the interactions between different compartments in the model and calculating its dominant eigenvalue, which corresponds to the reproduction number. To apply the next-generation method, one needs to specify the model equations, identify the infectious compartments, and derive the expressions for the transmission rates. Global asymptotic stability for equilibria is a desirable property in epidemiological models, indicating that the disease will eventually be eliminated from the population. Establishing global stability requires additional mathematical techniques, such as Lyapunov functions or comparison theorems. In summary, the SIR model and its analysis techniques, including the

reproduction number and stability analysis, provide valuable insights into the dynamics and control of infectious diseases [31].

2.2 Mathematical Explanation

This chapter introduces some of the important mathematical theories and methodologies that are very relevant to the thesis.

2.2.1 Basic SIR epidemic model and its Reproduction number

The basic reproduction number $R_{0,}$ is defined as the expected number of secondary cases produced by a single (typical) infection in a completely susceptible population. It can use the fact that R_0 is a dimensionless number to help us in calculating it

$R_0 \propto \left(\frac{infection}{contact}\right).\left(\frac{contact}{time}\right).\left(\frac{time}{infection}\right)$

More specifically:

$R_0 = \tau \cdot \overline{c} \cdot d$

Where τ is the transmissibility (i.e., probability of infection given contact between a susceptible and infected individual), $\overline{c}$ is the average rate of contact between susceptible and infected individuals, and d is the duration of infectiousness.

The simple SIR model assumes that the population is closed, meaning there is no migration, births, or deaths during the course of the epidemic. It also assumes that individuals only transition between the three compartments and that recovered individuals acquire lifelong immunity. The dynamics of the SIR model are described by a set of ordinary differential equations (ODEs) that govern the flow of individuals between the compartments. The equations are as follows:

$\dot{S}(t) = -\beta S(t)I(t)$

$\dot{I}(t) = \beta S(t)I(t) - \gamma I(t)$

$\dot{R}(t) = \gamma I(t)$

Where $\beta = \tau \cdot \overline{c}$ and is known as the effective contact rate, γ is the recovery rate. By assumption all rates are constant. This means that the expected duration of infection is simply the inverse of the removal rate: $d = \gamma^{-1}$.An epidemic occurs if the number of infected individuals increases, i.e. $\dot{I} > 0$

$\beta S(t)I(t) - \gamma I(t) > 0$

$\frac{\beta S(t)}{\gamma} > 1$

At the outset of an epidemic, nearly everyone (except the index case) is susceptible. So we can say that $S \approx 1$. Substituting $S = 1$, we arrive at the following inequality

$\frac{\beta}{\gamma} = R_0 > 1$

Since $\beta = \tau c^-$ and $d = \gamma^{-1}$, it can be seen that the expression for R_0 given in first equation. This little bit of mathematical trickery explains why we have that cumbersome phrase "in a completely susceptible population" tacked onto our definition for R_0.

2.2.2 Stability of equilibrium point

The stability of an equilibrium point in a dynamic system can be analyzed mathematically using different methods. Consider a system described by differential equations:

$\dot{x} = f(x)$

Where x represents the vector of state variables and $f(x)$ represents the vector-valued function that describes the dynamics of the system.

To determine the stability of the equilibrium point, analyze the eigenvalues of the Jacobian matrix J. The Jacobian matrix represents the partial derivatives of a vector-valued function with respect to its variables. The eigenvalues λ of J provide information about the behavior of the system near the equilibrium point. The stability conditions are as follows:

- If all eigenvalues have negative real parts $(R_0(\lambda) < 0)$, the equilibrium point is stable. The system converges to the equilibrium point as time progresses.
- If at least one eigenvalue has a positive real part $(R_0(\lambda) > 0)$, the equilibrium point is unstable. The system diverges from the equilibrium point.
- If there are eigenvalues with zero real parts $(R_0(\lambda) = 0)$, further analysis is required. Higher-order terms of the Taylor expansion need to be considered to determine stability. Methods like center manifold analysis or normal form theory can be employed in such cases.

It is clear from this context that stability analysis can become more complex for nonlinear systems, and other techniques like Lyapunov stability analysis or bifurcation analysis may be necessary to assess stability in those cases. By mathematically analyzing the stability of equilibrium points, it can understand dynamic systems' long-term behavior and predictability, which are crucial in various fields such as physics, engineering, biology, and economics.

2.3 Lyapunov functions and LaSalle's invariance Principle

Lyapunov functions and LaSalle's invariance principle are mathematical tools used in the stability analysis of dynamical systems. They help determine the behavior and stability properties of equilibrium points or invariant sets.

2.3.1 Lyapunov Functions

A Lyapunov function is a scalar function that measures the stability properties of a dynamical system. It is typically used to determine the stability, or asymptotic stability, of an equilibrium point. Let's consider a system described by $\dot{x} = f(x)$, where x is the vector of state variables and $f(x)$ represents the system dynamics.

A Lyapunov function $V(x)$ is a scalar function defined in the system's state space. It has the following properties:

- $V(x)$ is positive definite: $V(x) > 0$ for all $x \neq 0$, and V(x) = 0 only at the equilibrium point(s).
- $V(x)$ is radially unbounded: $V(x) \to \infty$ as $||x|| \to \infty$, where $||x||$ represents the norm of the state vector.
- $V(x)$ has a negative definite derivative: $\dot{V}(x) < 0$ for all $x \neq 0$.

If a Lyapunov function satisfying these properties can be found, it implies that the equilibrium point is stable or asymptotically stable. Specifically:

The equilibrium point is globally asymptotically stable if $\dot{V}(x) < 0$ for all x $\neq$ 0.

If $\dot{V}(x) < 0 \leq 0$ for all x $\neq$ 0, the equilibrium point is globally stable but not necessarily asymptotically stable.

2.3.2 LaSalle's Invariance Principle

LaSalle's invariance principle is a result that provides information about the behavior of trajectories within a region of attraction around an equilibrium point. It states that if a Lyapunov function V(x) exists for the system such that its derivative $\dot{V}(x)$ is non-positive within a compact set D containing the equilibrium point, then the trajectories of the system starting in D will eventually converge to the largest invariant set contained in D where $\dot{V}(x) = 0$.

In simpler terms, LaSalle's invariance principle helps to identify the set of points where the trajectories of a system converge and remain, even if they do not converge to a specific equilibrium point. It provides a more general notion of stability and is particularly useful when dealing with systems that have multiple equilibrium points or limit cycles.

By utilizing Lyapunov functions and LaSalle's invariance principle, stability properties of dynamical systems can be analyzed, and conclusions can be drawn about the behavior of the system near equilibrium points or within certain invariant sets. These tools are fundamental in various fields of study, including control theory, robotics, and dynamical systems analysis.

2.4 Method for local asymptotic stability for equilibria

2.4.1 Next generation method

A next-generation method is a mathematical approach used in epidemiology to estimate the basic reproduction number (R_0) in infectious disease models. R_0 represents the average number of secondary infections caused by a single infectious individual in a completely susceptible population. The next-generation method allows for the calculation of R_0 by analyzing the transmission dynamics of the disease. It involves identifying the key factors that contribute to disease transmission and quantifying their impact on the spread of the infection. The next-generation matrix (also known as the next-generation operator) is a different concept used in mathematical models to describe the transmission dynamics of infectious diseases. It is commonly used in the context of compartmental models such as the SIR model.

Example of generation matrix

Consider a simplified example where are two compartments: Susceptible (S) and Infected (I). In this case, the next-generation matrix represents the average number of new infections caused by individuals in the Infected (I) compartment to individuals in the Susceptible (S) compartment.

Suppose the following next-generation matrix:

$$F = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$$

In this matrix, the element f_{ij} represents the average number of new infections in the Susceptible (S) compartment caused by each infected individual in the Infected (I) compartment. The next-generation matrix is used in combination with the population vector

and other parameters of the disease model to calculate the basic reproduction number (R_0). R_0 is the dominant eigenvalue of the next-generation matrix and provides insight into the potential for disease transmission.

2.4.2 Basic steps involved in applying the next generation method

Define the compartmental model

Construct a compartmental model that represents the different population groups involved in disease transmission. Common models include the SIR (Susceptible-Infected-Recovered) or SEIR (Susceptible-Exposed-Infected-Recovered) models.

Identify the transmission pathways

Determine the pathways through which the infection spreads in the population. This involves identifying the interactions between different compartments and the associated transmission rates.

Calculate the next-generation matrix

Construct a square matrix, often denoted as F, which represents the expected number of secondary infections caused by an infected individual in each compartment. The elements of matrix F quantify the transmission rates between compartments.

Compute the eigenvalue

Find the largest eigenvalue (spectral radius) of the next-generation matrix F. This can be done numerically or analytically. By estimating R_0, one can assess the potential for disease spread. If $R_0 > 1$, it suggests that the disease is likely to cause an epidemic, as each infected individual, on average, infects more than one susceptible individual. If $R_0 < 1$, the disease is expected to die out over time. The next-generation method provides a quantitative measure of disease transmissibility and is useful for understanding the impact of various interventions on controlling or mitigating the spread of infectious diseases. It's worth noting that the next-generation method assumes a well-mixed population and certain simplifications in disease dynamics. It may not capture all the complexities of real-world scenarios and may require adaptations or refinements for specific diseases or contexts.

2.5 Global asymptotic stability for equilibria

Global asymptotic stability of equilibria refers to a property of a dynamical system in which all trajectories starting from any initial condition in the system's state space converge to an equilibrium point as time goes to infinity. To determine the global asymptotic stability of equilibria, various mathematical methods, and criteria can be applied. Some common approaches include Lyapunov's direct method, LaSalle's invariance principle, and the use of Lyapunov functions.

2.6 Reproduction number

The reproduction number (R_0), also known as the basic reproduction number, is a fundamental concept in epidemiology that measures the average number of new infections generated by each infected individual in a population where everyone is susceptible. Mathematically, the reproduction number can be defined as the product of the contact rate (denoted by β) and the average duration of infectiousness (denoted by I), divided by the recovery rate (denoted by γ). This can be expressed as:

$R_0 = ((\beta * I))/(\gamma)$

Here's a breakdown of the components:

- Contact rate (β): This represents the average rate at which an infected individual comes into contact with susceptible individuals and can transmit the disease. It depends on factors such as population density, social interactions, and the nature of the disease.
- Average duration of infectiousness (I): It refers to the average period during which an infected individual remains infectious and capable of transmitting the disease to others. The duration may vary depending on the specific disease.
- Recovery rate (γ): It represents the rate at which infected individuals recover from the disease or are removed from the infectious population through other means (such as hospitalization or death).

If R_0 is less than 1 ($R_0 < 1$), it indicates that each infected individual, on average, infects fewer than one susceptible individual, leading to a decline in the number of cases over time. In this case, the disease is likely to die out in the long run. On the other hand, if R_0 is greater than 1 ($R_0 > 1$), it suggests that each infected individual, on average, infects more than one susceptible individual, leading to sustained transmission and the potential for an epidemic outbreak.

Chapter - 3

Susceptible- Vaccinated- infected- treated and recovered (SVITR) Model

3.1 Introduction

The SVITR epidemic model is an extension of the classical SIR model that incorporates behavioral dynamics and social interactions into the disease transmission process. The model structure includes five compartments: susceptible (S), vaccinated (V), infected (I), treated (T), and recovered (R). The formulation of the SVITR model involves a system of ordinary differential equations that describe the flow of individuals between these compartments, accounting for the effects of vaccination, treatment, and behavioral changes. The model ensures the positivity and boundedness of the solutions, meaning that the population sizes in each compartment remain non-negative and finite. The average social payoff in the SVITR model quantifies the net benefits or costs experienced by individuals due to their behaviors, vaccination, and treatment decisions. The social efficiency deficit (SED) measures the discrepancy between the optimal state of the system and the actual state resulting from individual decisions [46, 47]. Social benefitted individuals (SBI) represent the individuals who gain positive benefits from their actions in terms of reduced infection risk or improved health outcomes. The mathematical analysis of the SVITR model involves deriving the basic reproduction number **(** R_0**)** and the effective reproduction number (R_e). R_0 represent the average number of secondary infections caused by a single infected individual in a completely susceptible population, while R_e accounts for the effects of behavioral changes, vaccination, and treatment on disease transmission. The existence of an endemic equilibrium in the model signifies a stable persistent state where the disease remains prevalent in the population. The model also examines the existence of a uniformly stable solution, which ensures that the population eventually converges to a disease-free state. Additionally, a modified version of the model with the second derivative considers the strength number, which captures the effectiveness of treatment in reducing disease transmission.

3.2 Model Structure

This section presents the dynamic model for disease spreading and the embedded vaccine and treatment behavioral dynamics among the individuals in society. Assume the SVITRS cyclic epidemic model, in which the total population is divided into five epidemiological compartments: susceptible(S), vaccinated (V), infected(I), treatment(T), and recovered (R) individuals. Our model is governed by the following assumptions: All parameters are nonnegative, the total population size is constant, vaccination is introduced to the susceptible individuals, susceptible individuals are recruited by birth or immigration, the treated individuals cannot transmit disease to the susceptible population, and by losing temporary immunity the recovered individuals become susceptible again. The compartments and parameters of the model are described in (figure 1).

3.2.1 Formulation of the SVITR model

Mathematical models are important tools to gain a big understanding of the ongoing trends for COVID-19. They are also useful for obtaining a basic reproduction number, determining sensitivities to change in parameter values, estimating key parameters from the data that contribute to identifying trends, making general forecasts, and estimating uncertainties [60]. Epidemiological models play a fundamental role in the study of the dynamics of COVID-19. With regard to the studies carried out so far, a few mathematical modeling studies have been done about transmission of the pandemic.

Here, use two control measures, namely, vaccination and treatment, to control more optimally the spread of infection from a community because sometimes only one control variable may be challenging to eradicate the disease successfully. A susceptible individual becomes infected at a disease transmission β and alternatively can participate vaccine program at a dynamic rate x. A vaccinated may be infected at the rate $(1-\eta)\beta$, even after participating in the vaccine program, where η denotes vaccine effectiveness $(0 \leq \eta \leq 1)$. After infection, two different cases can happen to an infected person. Either individual will get treatment at rate, τ or they will recover naturally at rate γ. A treated individual recovers after treatment at a rate δ. Finally, an individual who has recovered after infection is immune; they can get susceptible again at a waning immunity rate ω.

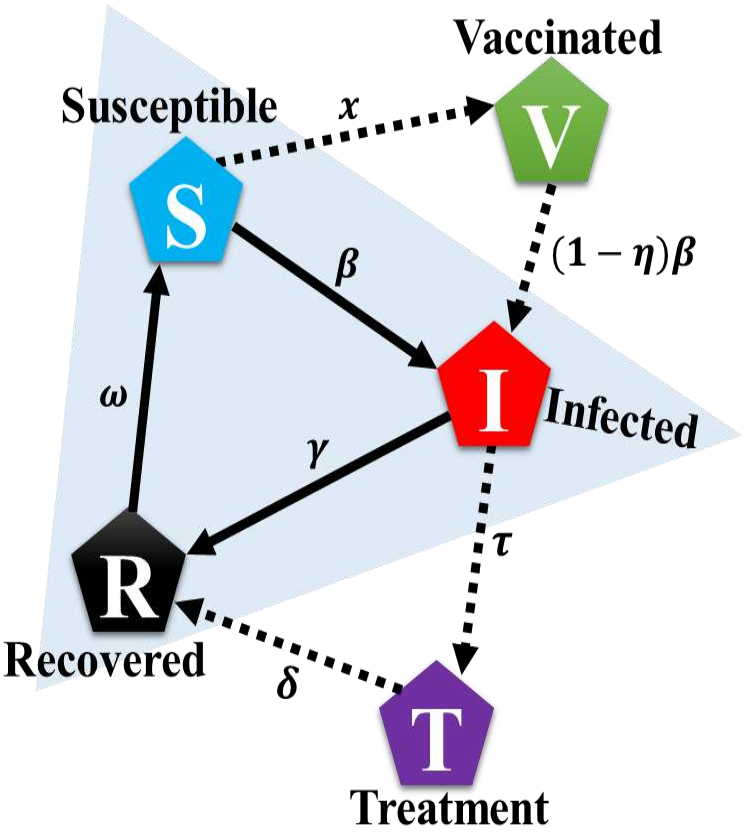

Figure 3.1: SVITR Model Structure

The dynamical equation of SVITRS is given by,

$$\dot{S} = -\beta SI - xS + \omega R, \quad (1)$$

$$\dot{V} = xS - (1-\eta)\beta VI, \quad (2)$$

$$\dot{I} = \beta SI + (1-\eta)\beta VI - \gamma I - \tau I, \quad (3)$$

$$\dot{T} = \tau I - \delta T, \quad (4)$$

$$\dot{R} = \delta T + \gamma I - \omega R. \quad (5)$$

Here, $S(t) + V(t) + I(t) + T(t) + R(t) = 1$. To solve the above sets of differential equations (1-5) against time numerically, we consider explicit finite difference method with the initial values as, $S(0) \approx 1.0$, $V(0) \approx 0.0$, $I(0) \approx 0.0$, $T(0) = 0.0$, and $R(0) = 0.0$.

3.2.2 Validation of model

An epidemic of influenza occurred in a boarding school in the north of England. The boarding school housed a total of 763 boys, who were at risk during the epidemic. On January 22, three boys were sick. The table below gives the number of boys ill on the nth day after January 22 ($n = 1$).

Table 3.1: Daily number influenza infected boys

Day	No. infected	Day	No. infected
3	25	9	192
4	75	10	126
5	227	11	71
6	296	12	28
7	258	13	11
8	236	14	7

[Data taken from "Influenza in a Boarding School," British Medical Journal, 4 March 1978]

Since these are outbreak data, using an epidemic model without demography as discussed in Chap. 2, the SIR model without demography is appropriate for this case.

The best-fitted solution with Mathematica and the data are plotted in Figure 3.2. Mathematica can also provide 95% confidence intervals. A 95% confidence interval (CI) is an interval calculated from many observations, in principle different from data set to data set, that 95% of the time will include the parameter of interest if the experiment is repeated. The CI for the above fitting are [0.4257, 0.5037] for α and [0.0022099, 0.00254] for β.

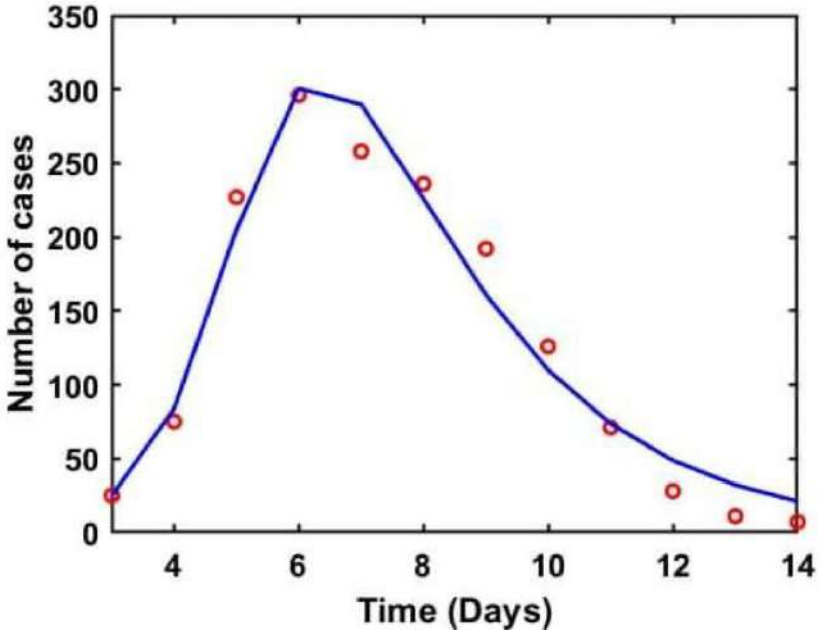

Figure 3.2: Validation of an SIR model with English boarding school data

3.2.3 Behavioral Dynamics

Individuals can take vaccination and treatment based on their interests and their strategy by observing how many people are infected at a given time in a single season. In the behavioral dynamics, each participant can choose proactive intervention whether to participate vaccine program or not, depending on the vaccine cost and associated factors. Further, if infected individuals want to take treatment facilities termed retroactive provision, they compare their strategies to recover time, treatment cost, and disease incidence. Thus, if the individual becomes vaccinated and treated at a vaccination rate (x) and a treatment rate (τ), the equation that describes the human behavioral dynamics are,

$$\dot{x} = mx(1-x)[-C_V V + C_i I], \quad (6)$$

$$\dot{\tau} = m\tau(1-\tau)\left[-C_T T + C_i I + \left(\frac{1}{\gamma} - \frac{1}{\delta}\right)\right]. \quad (7)$$

Here, C_v is the relative cost of vaccination and C_τ *is the* relative cost of treatment as compared with the disease cost, C_i $(= 1)$. The term in equation (6), $[-C_T V + C_i I]$, is the payoff difference (gain) between cooperation and defection and its sign (positive or negative) decides whether to vaccinate or not. Similarly, in equation (7), the term $\left[-C_T T + C_i I + \left(\frac{1}{\gamma} - \frac{1}{\delta}\right)\right]$ is the payoff gain for treatment strategies potentially indicates how much more beneficial instead taking treatment (treated) is than not taking treatment (untreated) is in terms of disease duration. With increase of δ as compared with γ, the treatment provides a patience an immediate recovery from illness. Thus, $\left(\frac{1}{\gamma} - \frac{1}{\delta}\right)$ implies 'willingness' amid infected people to take the treatment than doing nothing.

3.2.4 Model's positivity and boundedness of the solutions

Here, some of the actual results related to the theoretical analysis of the model will be present [48–53]. The primary goal is to get an asymptotic understanding of how the virus will propagate, ensuring that the model's explanations are accurate by requiring positivity and boundedness. The asymptotic local stability study by finding the model's disease-free equilibrium will be verified and also calculate the reproduction number and the existence of a uniformly stable situation for the exactness of the solution.

Mainly focus on the proposed model's positivity and boundedness, which certifies the exactness of the model's solutions in this part. Thus, for infected class $I(t)$, it may write, $I(t) \geq I^0 e^{-(\gamma+\tau)t}, \forall t \geq 0$.

Similarly, the treatment class $T(t)$ is expressed as follows,

$T(t) \geq R^0 e^{-\delta t}, \forall t \geq 0.$

Furthermore, the recovered class $R(t)$ is,

$R(t) \geq R^0 e^{-\omega t}, \forall t \geq 0.$

Hence, the norm of the domain D_φ where $\varphi \in D_\varphi$ [53], put out in the following way

$$\|\varphi\|_\infty = \sup_{t \epsilon D_\varphi} |\varphi(t)|$$

Analogously, utilizing the overhead norm, the vaccinated and susceptible classes are likewise represented as follows,

$$\begin{aligned}
\dot{V} &= xS - (1-\eta)\beta VI, \forall t \geq 0 \\
&\geq -\{(1-\eta)\beta I\}V, \forall t \geq 0 \\
&\geq -\{(1-\eta)\beta |I|\}V, \forall t \geq 0 \\
&\geq \{-(1-\eta)\beta \sup_{t \epsilon D_I} |I|\}V, \forall t \geq 0 \\
&\geq -\{(1-\eta)\beta \|I\|_\infty\}V, \forall t \geq 0
\end{aligned}$$

Therefore, $V(t) \geq V^0 e^{-\{(1-\eta)\beta \|I\|_\infty\}t}, \forall t \geq 0.$ $\dot{S} = -\beta SI - xS + \omega R$

In the same way, $S(t) \geq S^0 e^{-(\beta \|I\|_\infty + x)t}, \forall t \geq 0.$

Finally, it concluded that the suggested model and its solution are both positive and bounded.

3.3 Average Social Payoff

To establish the average social payoff (ASP) at the end of the epidemic season, consider the combined effect of vaccination and treatment in the same con-text. The ASP for the evolutionary game theory aspect at Nash equilibrium is given by,

$$ASP^{NE} = -C_V V(\infty) - C_T \int_0^\infty T(\theta)\, d\theta - R(\infty). \quad (8)$$

Where, ASP^{NE} indicates the payoff at Nash equilibrium (NE), estimated when both games (vaccination and treatment) have arrived at a steady state on the local time scales.

3.4 Social Efficiency Deficit (SED)

Social efficiency deficit (SED) defines as the difference between the result of the evolutionary train (which can be evaluated by the Nash equilibrium NE) and the optimum solution (without EGT). The payoff at the NE is obtained by taking an evolutionary game presence, whereas the optimal social gain is remark-able in a model of any complicacy. Therefore, one can evaluate the SED in any context and forecast the phenomenon of social

dilemma; if the SED is positive, the gap exists; if it is zero, the evolutionary train matches the optimum (SED=0 implies no social dilemma). The SED is given by,

$$SED = ASP^{SO} - ASP^{NE}. \quad (9)$$

where, ASP^{SO} and ASP^{NE} define the payoff at the optimal social situation and Nash equilibrium, respectively. Here, the explicitly reveals the underlying social dilemmas in the vaccination and treatment game by introduce SED. The social dilemma exists under certain combinations of the model parameter, such as vaccine efficacy, treatment duration and their associated cost. Now, consider the combined impact of vaccination and treatment in the same context.

In according to the abovementioned conceptual definition, SED in the current model for both vaccination and treatment games are given by,

$$SED_V = ASP_{x_k}^{X_{social}^{opt.}} - ASP_{x_k,\tau_k}^{NE}, \quad (10.1)$$

$$SED_T = ASP_{\tau_k}^{T_{social}^{opt.}} - ASP_{x_k,\tau_k}^{NE}. \quad (10.2)$$

The ASP is the quantity of payoff. The superscript 'Opt' and subscript 'social' together indicate the social optimal. In which, ASP_{x_k,τ_k}^{NE} indicates the average social payoff at the NE, estimated when both games, vaccination and treatment have occurred together on the local time scales. To understand the $ASP_{x_k}^{X_{social}^{opt.}}$ and $ASP_{\tau_k}^{T_{social}^{opt.}}$ The terms $X_{social}^{opt.}$ and $P_{social}^{opt.}$ reflect the statement that the maximum ASP is obtained for varying *x* ranging from 0 to 1 (for fixed τ_k) and varying τ from 0 to 1(for fixed x_k), respectively.

3.5 Social Benefitted Individuals (SBI)

To introduce the concept of socially benefitted individuals (SBI) on an epidemic model embedding with EGT, consider the fraction of individuals who benefit from either vaccination or treatment. For SBI, we first calculate the final epidemic size (FES) in the absence of vaccination and treatment, FESNV and FESNT, respectively, at equilibrium. Next, vaccination and treatment game strategy are calculated at NE in the presence of either vaccination or treatment strategies, defined by FESV and FEST. Finally, formulate the SBI as follows,

$$SBI_V(\infty) = \mathrm{FES}_{NV}(\infty) - \mathrm{FES}_V(\infty), \quad (11.1)$$

$$SBI_T(\infty) = \mathrm{FES}_{NT}(\infty) - \mathrm{FES}_T(\infty). \quad (11.2)$$

To solve the proposed epidemic model that belongs to the sets of differential equations by Consider the explicit finite difference method for a single season numerically. Here, presume the initial values as, $S(0) \approx 1.0, V(0) = 0.0,\ I(0) \approx 0.0,\ T(0) = 0.0,$ and $R(0) = 0.0$. Throughout the time step is to consider $\Delta t = 1$, meaning both strategy and epidemic dynamics update daily (per day).

3.6 Mathematical Analysis

3.6.1 Disease-free equilibrium (DFE) point and its stability

The disease-free equilibrium, symbolized by $\mathcal{E}_0$, is the point at which there is no infection in the population at equilibrium stage and all infected classes will have a zero value. To calculate the DFE of the proposed model, put $I = 0$ in the system (1-5). Then get the DFE of the current model is $\mathcal{E}_0 = (S^0, V^0, I^0, T^0, R^0) = (S^*, V^*, 0,0,0);\ S^* + V^* = N(= 1)$.

3.6.2 Derivation of the basic reproduction (R_0) and effective reproduction number (R_e)

Calculate the basic reproduction number, R_0 to show stable equilibrium conditions to analyze the preliminary theoretical investigation. Consider the next-generation matrix [45] technique to evaluate the basic reproduction number, as follows:

$$F = \begin{bmatrix} \beta s^* + \beta(1-\eta)V^* & 0 \\ 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} \gamma + \tau & 0 \\ -\tau & \delta \end{bmatrix},$$

$$FV^{-1} = \frac{1}{\delta(\gamma+\tau)} \begin{bmatrix} \delta[\beta s^* + \beta(1-\eta)v^*] & 0 \\ 0 & 0 \end{bmatrix}.$$

As basic reproduction number is the most considerable eigenvalue of FV^{-1} thus,

$$R_0 = \frac{\beta s^* + \beta(1-\eta)v^*}{\gamma + \tau}.$$

Furthermore, the time-dependent reproduction number is known as the effective reproduction number R_e is,

$$R_e(t) = \frac{\beta}{\gamma+\tau} S(t) + \frac{(1-\eta)\beta}{\gamma+\tau} V(t).$$

Theorem 1. If $R_0 < 1$, then the disease-free equilibrium E_0 is locally asymptotically stable. If $R_0 > 1$, the disease-free equilibrium is unstable.

Proof: Let us compute the proposed model's Jacobian matrix is as

$$J = \begin{bmatrix} -\beta I - x & 0 & -\beta S & 0 & \omega \\ x & -(1-\eta)\beta I & -(1-\eta)\beta V & 0 & 0 \\ \beta I & (1-\eta)\beta I & \beta S + (1-\eta)\beta V - \gamma - \tau & 0 & 0 \\ 0 & 0 & \tau & -\delta & 0 \\ 0 & 0 & \gamma & \delta & -\omega \end{bmatrix}$$

Substituting the value of the DFE point E_0, then obtain

$$J(E_0) = \begin{bmatrix} -x & 0 & -\beta s^* & 0 & \omega \\ x & 0 & -(1-\eta)\beta v^* & 0 & 0 \\ 0 & 0 & \beta s^* + \beta(1-\eta)v^* - \gamma - \tau & 0 & 0 \\ 0 & 0 & \tau & -\delta & 0 \\ 0 & 0 & \gamma & \delta & -\omega \end{bmatrix}$$

The characteristics equation $|J(E_0) - \lambda I| = 0$ has five roots, which are

$\lambda_1 = -\omega, \lambda_2 = -\delta, \lambda_3 = -x\,, \lambda_4 = -\lambda - \tau + \beta s^* + \beta(1-\eta)v^* \quad \lambda_5 = 0.$

As all eigenvalues are negative or equal to zero, therefore, conferring to Routh-Hurwitz criteria [56], it can easily accomplish that the model is locally asymptotically stable at the disease-free equilibrium point E_0 whenever $R_0 < 1$ and unstable whenever $R_0 > 1$.

3.6.3 Existence of endemic equilibrium

In this part, an investigation of endemic equilibrium points denoted by $\mathcal{E}_* = (S^*, V^*, I^*, T^*, R^*)$ is whether exists or not. For the endemic equilibrium point, we consider the equations as follows,

$0 = -\beta SI - xS + \omega R,$

$0 = xS - (1-\eta)\beta VI,$

$0 = \beta SI + (1-\eta)\beta VI - (\gamma + \tau)I,$

$0 = \tau I - \delta T,$

$0 = \delta T + \gamma I - \omega R.$

After some simplification,

$S^* = \frac{\gamma+\tau}{\beta I^* + x} I^*,$

$V^* = \frac{x(\gamma+\tau)}{\beta(\beta I^* + x)(1-\eta)},$

$T^* = \frac{\tau}{\delta} I^*$

$R^* = \frac{\gamma+\tau}{\omega} I^*.$

Theorem 2. Let the Lyapunov function L_f for the endemic equilibrium point $\mathcal{E}_*$ is $\{S, V, I, T, R\}, L_f < 0$. Have to prove $\mathcal{E}_*$ is globally asymptotically stable for $R_0 > 1$.

Proof: Let us suppose that the Lyapunov function is,

$$L_f(S,V,I,T,R) = \left(S - S^* - S^* log\frac{S^*}{S}\right) + \left(V - V^* - V^* log\frac{V^*}{V}\right) + \left(I - I^* - I^* log\frac{I^*}{I}\right)$$
$$+ \left(T - T^* - T^* log\frac{T^*}{T}\right) + \left(R - R^* - R^* log\frac{R^*}{R}\right) \quad (12)$$

After applying the first derivative, on both sides of the equation (12) for t, obtain

$$\dot{L_f} = \left(\frac{S - S^*}{S}\right)\dot{S} + \left(\frac{V - V^*}{V}\right)\dot{V} + \left(\frac{I - I^*}{I}\right)\dot{I} + \left(\frac{T - T^*}{T}\right)\dot{T} + \left(\frac{R - R^*}{R}\right)\dot{R} \quad (13)$$

Substituting the values of $\dot{S}, \dot{V}, \dot{I}, \dot{T}, \dot{R}$ from the equation (1-5) in equation (13), find that,

$$\dot{L_f} = \left(\frac{S - S^*}{S}\right)\{-\beta SI - xS + \omega R\} + \left(\frac{V - V^*}{V}\right)(xS - (1-\eta)\beta VI)$$
$$+ \left(\frac{I - I^*}{I}\right)(\beta SI + (1-\eta)\beta VI - (\gamma + \tau)I) + \left(\frac{T - T^*}{T}\right)(\tau I - \delta T)$$
$$+ \left(\frac{R - R^*}{R}\right)(\delta T + \gamma I - \omega R) \quad (14)$$

Substitute $S = S - S^*, V = V - V^*, I = I - I^*, T = T - T^*, R = R - R^*$ in equation (14),

$$\dot{L_f} = \dot{L_f} = \left(\frac{S - S^*}{S}\right)\{-\beta(S - S^*)(I - I^*) - x(S - S^*) + \omega(R - R^*)\}$$
$$+ \left(\frac{V - V^*}{V}\right)\{x(S - S^*) - (1-\eta)\beta(V - V^*)(I - I^*)\}$$
$$+ \left(\frac{I - I^*}{I}\right)\{\beta(S - S^*)(I - I^*) + (1-\eta)\beta(V - V^*)(I - I^*)$$
$$- (\gamma + \tau)(I - I^*)\} + \left(\frac{T - T^*}{T}\right)\{\tau(I - I^*) - \delta(T - T^*)\}$$
$$+ \left(\frac{R - R^*}{R}\right)\{\delta(T - T^*) + \gamma(I - I^*) - \omega(R - R^*)\}. \quad (15)$$

Equation (15) can be written as

$$\dot{L_f} = \psi_1 - \psi_2 \quad (16)$$

where,

$$\psi_1 = \omega\frac{S - S^*}{S}R + \beta\frac{(S - S^*)^2}{S}I^* + x\frac{V - V^*}{V}S + (1-\eta)\beta\frac{(V - V^*)^2}{V}I^*$$
$$+ \beta\frac{(I - I^*)^2}{I}S + (1-\eta)\beta\frac{(I - I^*)^2}{I}V + \tau\frac{T - T^*}{T}I + \delta\frac{R - R^*}{R}T$$
$$+ \gamma\frac{R - R^*}{R}I,$$

and

$$\psi_2 = \beta\frac{(S-S^*)^2}{S}I + x\frac{(S-S^*)^2}{S} + \omega\frac{S-S^*}{S}R^* + x\frac{V-V^*}{V}S^* + (1-\eta)\beta\frac{(V-V^*)^2}{V}I$$
$$+ \beta\frac{(I-I^*)^2}{I}S^* + (1-\eta)\beta\frac{(I-I^*)^2}{I}V^* + (\gamma+\tau)\frac{(I-I^*)^2}{I}$$
$$+ \tau\frac{T-T^*}{T}I^* + \delta\frac{(T-T^*)^2}{T} + \delta\frac{R-R^*}{R}T^* + \gamma\frac{R-R^*}{R}I^*$$
$$+ \omega\frac{(R-R^*)^2}{R}.$$

It is evident that $\dot{L_f} < 0$ when $\psi_1 < \psi_2$.

Therefore, for $S = S^*, V = V^*, I = I^*, T = T^*, R = R^*$

From equation (16),$\Rightarrow 0 = \psi_1 - \psi_2$, Implies$\Rightarrow \dot{L_f} = 0.$ (17)

In that case, according to Lasalle's invariance principle, the endemic equilibrium point $\mathcal{E}_*$ is globally asymptotically stable in Γ when $\psi_1 < \psi_2$ for compact invariant set,

$\{(S^*, V^*, I^*, R^*) \in \Gamma: \dot{L_f} = 0\}.$ (18)

3.6.4 Existence of a uniformly stable solution

To establish the exactness of a uniformly stable solution, assume that

$\dot{S} = -\beta SI - xS + \omega R = f_1(S,V,I,T,R),$

$\dot{V} = xS - (1-\eta)\beta VI = f_2(S,V,I,T,R),$

$\dot{I} = \beta SI + (1-\eta)\beta VI - (\gamma+\tau)I = f_3(S,V,I,T,R),$

$\dot{T} = \tau I - \delta T = f_4(S,V,I,T,R),$

$\dot{R} = \delta T + \gamma I - \omega R = f_5(S,V,I,T,R).$

For the total population $N(t)(=1)$, it may write $\Pi = \{(S(t) + V(t) + I(t) + T(t)\ R(t)) \in R^{15}: |\zeta(i)| \leq N(t)\ and\ t \in [0, T(time\ period)]$.

Thus, over Π, then

$\frac{\partial f_1}{\partial S} = -\beta I - xS \Rightarrow \left|\frac{\partial f_1}{\partial S}\right| \leq a_{11}; \frac{\partial f_1}{\partial V} = 0 = f_1(V) = a_{12};$

$\frac{\partial f_1}{\partial I} = -\beta S \Rightarrow \left|\frac{\partial f_1}{\partial I}\right| \leq a_{13}; \frac{\partial f_1}{\partial T} = 0 = f_1(T) = a_{14}; \frac{\partial f_1}{\partial R} = \omega \Rightarrow \left|\frac{\partial f_1}{\partial R}\right| \leq a_{15};$

$\frac{\partial f_2}{\partial S} = -x \Rightarrow \left|\frac{\partial f_2}{\partial S}\right| \leq a_{21}; \frac{\partial f_2}{\partial V} = -(1-\eta)\beta I \Rightarrow \left|\frac{\partial f_2}{\partial V}\right| \leq a_{22};$

$\frac{\partial f_2}{\partial I} = -(1-\eta)\beta V \Rightarrow \left|\frac{\partial f_2}{\partial I}\right| \leq a_{23}; \frac{\partial f_2}{\partial T} = 0 = f_2(T) = a_{24}; \frac{\partial f_2}{\partial R} = 0 = f_2(R) = a_{25};$

$\frac{\partial f_3}{\partial S} = \beta I \Rightarrow \left|\frac{\partial f_3}{\partial S}\right| \leq a_{31}; \frac{\partial f_3}{\partial V} = (1-\eta)\beta I \Rightarrow \left|\frac{\partial f_3}{\partial V}\right| \leq a_{32};$

$$\frac{\partial f_3}{\partial I} = \beta S + (1-\eta)\beta V - (\gamma+\tau) \Rightarrow \left|\frac{\partial f_3}{\partial I}\right| \leq a_{33}; \frac{\partial f_3}{\partial T} = 0 = f_3(T) = a_{34};$$

$$\frac{\partial f_3}{\partial R} = 0 = f_3(R) = a_{35};$$

$$\frac{\partial f_4}{\partial S} = 0 = f_4(S) = a_{41}; \frac{\partial f_4}{\partial V} = 0 = f_4(V) = a_{42}; \frac{\partial f_4}{\partial I} = \tau \Rightarrow \left|\frac{\partial f_4}{\partial I}\right| \leq a_{43};$$

$$\frac{\partial f_4}{\partial T} = -\delta \Rightarrow \left|\frac{\partial f_4}{\partial T}\right| \leq a_{44}; \frac{\partial f_4}{\partial R} = 0 = f_4(R) = a_{45};$$

$$\frac{\partial f_5}{\partial S} = 0 = f_5(S) = a_{51}; \frac{\partial f_5}{\partial V} = 0 = f_5(V) = a_{52}; \frac{\partial f_5}{\partial I} = \gamma \Rightarrow \left|\frac{\partial f_5}{\partial I}\right| \leq a_{53};$$

$$\frac{\partial f_5}{\partial T} = \delta \Rightarrow \left|\frac{\partial f_5}{\partial T}\right| \leq a_{54}; \frac{\partial f_5}{\partial R} = -\omega \Rightarrow \left|\frac{\partial f_5}{\partial R}\right| \leq a_{55};$$

Here, the constants $a_{ij} (i \geq 1 \text{ and } j \leq 5)$ all are positive. Therefore, the suggested model's five functions, namely $f_1, f_2, \cdots, f_5$ all are satisfied well-known Lipchitz condition [54-57].

3.6.5 Strength number

Use the suitable strength numbers approach to find out the waving tendency in the proposed epidemic dynamics [53].To determine the recommended model's strength number under the assumption of a limited population, N, analyze the partial first derivative of the infected class using next-generation matrix techniques as follows:

$$\beta SI = \frac{\beta SI}{N}, (1-\eta)\beta VI = (1-\eta)\beta \frac{VI}{N}.$$

Therefore,

$$\frac{\partial^2}{\partial I^2}\left[\beta \frac{SI}{N} + (1-\eta)\beta \frac{VI}{N} - (\gamma+\tau)\frac{I}{N}\right] = \beta S \frac{\partial}{\partial I}\left(\frac{N-\dot{N}I}{N^2}\right) + (1-\eta)\beta V \frac{\partial}{\partial I}\left(\frac{N-\dot{N}I}{N^2}\right)$$
$$= -\beta \frac{S}{N^2} - (1-\eta)\beta \frac{V}{N^2}.$$

Then,

$$F = \begin{bmatrix} -\frac{\beta}{N^2} - \frac{(1-\eta)\beta}{N^2} & 0 \\ 0 & 0 \end{bmatrix},$$

and

$$FV^{-1} = \frac{1}{\delta(\gamma+\tau)}\begin{bmatrix} \delta\left[-\frac{\beta}{N^2} - \frac{(1-\eta)\beta}{N^2}\right] & 0 \\ 0 & 0 \end{bmatrix}.$$

Therefore, as previous, from the spectral radius of $\rho(FV^{-1})$ for defining the epidemic wave, the desired strength number is denoted by R_{SN},

$$R_{SN} = -\frac{\beta + (1-\eta)\beta}{N^2(\gamma+\tau)} \tag{19}$$

When $R_{SN} \leq 0$, the disease can only produce one wave, and the infection class would quickly drop below or equal to the equilibrium of disease-free conditions. However, when $R_{SN} \geq 0$, multi-waving scenarios, are revealed. Here, all parameters of the suggested model are well-defined and positive. More precisely, $\beta, \gamma, \tau \geq 0$, and $0 \leq \eta \leq 1$, which shows that the proposed model's strength number $R_{SN} \leq 0$ represents only one wave.

3.6.6 Geometrical interpretation of Strength Number

The second-order derivative usually depicts the concavity or curvature of any graph. In epidemic models, a concept like this from fundamental calculus is routinely applied to observe the situation of several layers or waves of epidemic disease cases. To illustrate the second-order time derivative study of our suggested model, then exemplify it below as follows:

$$\ddot{S} = -\beta\dot{S}I - \beta S\dot{I} - x\dot{S} + \omega\dot{R},$$
$$\ddot{V} = x\dot{S} - (1-\eta)\beta\dot{V}I - (1-\eta)\beta V\dot{I},$$
$$\ddot{I} = \beta\dot{S}I + \beta S\dot{I} + (1-\eta)\beta\dot{V}I + (1-\eta)\beta V\dot{I} - (\gamma+\tau)\dot{I},$$
$$\ddot{T} = \tau\dot{I} - \delta\dot{T},$$
$$\ddot{R} = \delta\dot{T} + \gamma\dot{I} - \omega\dot{R}. \quad (20)$$

Putting the value of the first derivative $\dot{S}, \dot{V}, \dot{I}, \dot{T}, \dot{R}$ from equation (1-5) in equation (20), and getting,

$$\ddot{I} = \beta(-\beta SI - xS + \omega R)I + (1-\eta)\beta\{xS - (1-\eta)\beta VI\} + \{\beta S + (1-\eta)\beta V - (\gamma+\tau)\}\{\beta SI + (1-\eta)\beta VI - (\gamma+\tau)I\},$$
$$\ddot{T} = \tau\{(\beta SI + (1-\eta)\beta VI - (\gamma+\tau)I)\} - \delta(\tau I - \delta T). \quad (21)$$

By using the disease-free equilibrium point, we can demonstrate the concavity of the system of nonlinear ODEs (18). The inflection point occurs when the time derivative of the second order equals zero. Concave up arises if it is more significant than zero and concaves down if it is less meaningful than zero. Using the system (21) and the disease-free equilibrium point $\mathcal{E}_0$, it may conclude that $\mathcal{E}_0$ cannot be concave up or down,

$$\ddot{I} = 0,$$
$$\ddot{T} = 0. \quad (22)$$

Equation (22) shows that for all second-order time derivatives utilized in the computation of concavity, it only have the case for the inflection or stationary points. In conclusion, the model (22) only provides the infection or the fixed points for the second-order model (21) at the disease-free equilibrium points $\mathcal{E}_0$ instead of the concave up and concave down.

Chapter – 4

Results and Discussion

4.1 Introduction

The result and discussion of the SVITR epidemic model highlight the interplay between vaccination and treatment strategies in controlling the spread of infectious diseases. The model investigates the dynamics of a game where individuals make decisions regarding their vaccination and treatment choices based on the perceived costs and benefits. The analysis reveals that the effectiveness of vaccination and treatment depends on various factors, including the coverage rate, efficacy, and accessibility of interventions. The interplay between vaccination and treatment can have significant implications for disease control [45]. For instance, if the vaccination strategy is highly effective and accessible, it may reduce the number of infections and consequently lower the demand for treatment. On the other hand, if treatment is more effective or cost-effective compared to vaccination, individuals may opt for treatment rather than vaccination, potentially leading to a higher disease burden in the population [32-35]. The model also explores the interplay between vaccination and treatment costs. Higher vaccination costs can discourage individuals from getting vaccinated, resulting in lower coverage rates and compromised disease control. Similarly, higher treatment costs can deter individuals from seeking treatment, leading to delayed or inadequate care and potentially exacerbating the spread of the disease [8]. These findings emphasize the importance of considering the cost-effectiveness and accessibility of vaccination and treatment strategies in public health decision-making [38-42]. Overall, the study provides valuable insights into the complex interactions between vaccination, treatment, and cost factors, shedding light on the optimal allocation of resources and interventions for effective disease control and prevention.

4.2 Interplay between vaccination and treatment game

In this section, numerically explore the SVITR model; the results are presented for the line graphs and 2D phase diagrams in the aspect of the evolutionary game theory and cyclic epidemic model. The convene impact of proactive vaccination and the retroactive treatment policy based on human behavior depends on the vaccination cost, treatment cost, and corresponding factors. It extensively analyzed the vaccination and treatment cost, vaccination effectiveness (η), and recovery rate, considering the other sensible parameters. At first pursued the line graphs of infected individuals without the game and with game cases for varying control parameters. In the second case, present the two-dimensional phase diagram of final epidemic size (FES), vaccination coverage (VC), the fraction of treated people (FTR), and average social payoff (ASP) as a measure of policy burden to society while varying two parameters. Also introduce the SED (social efficiency deficit), the expresses the radical social dilemmas in the vaccination and treatment games.

First, briefly analyze the current model and its solution theoretically for the only epidemic model without using evolutionary game theory (EGT) (see Appendix). The model and its solution are positive and bounded for the finite population N(t) [58]. Here explored the existence of a uniformly stable solution for the model using the well-known Lipchitwz stability theorem. This model examined by reproduction number (basic and effective), local and global stability, and strength number to analyze stability conditions and wave properties. Finally, the Lyapunov function backed by second derivatives were also studied, providing information regarding the tendency toward curvature.

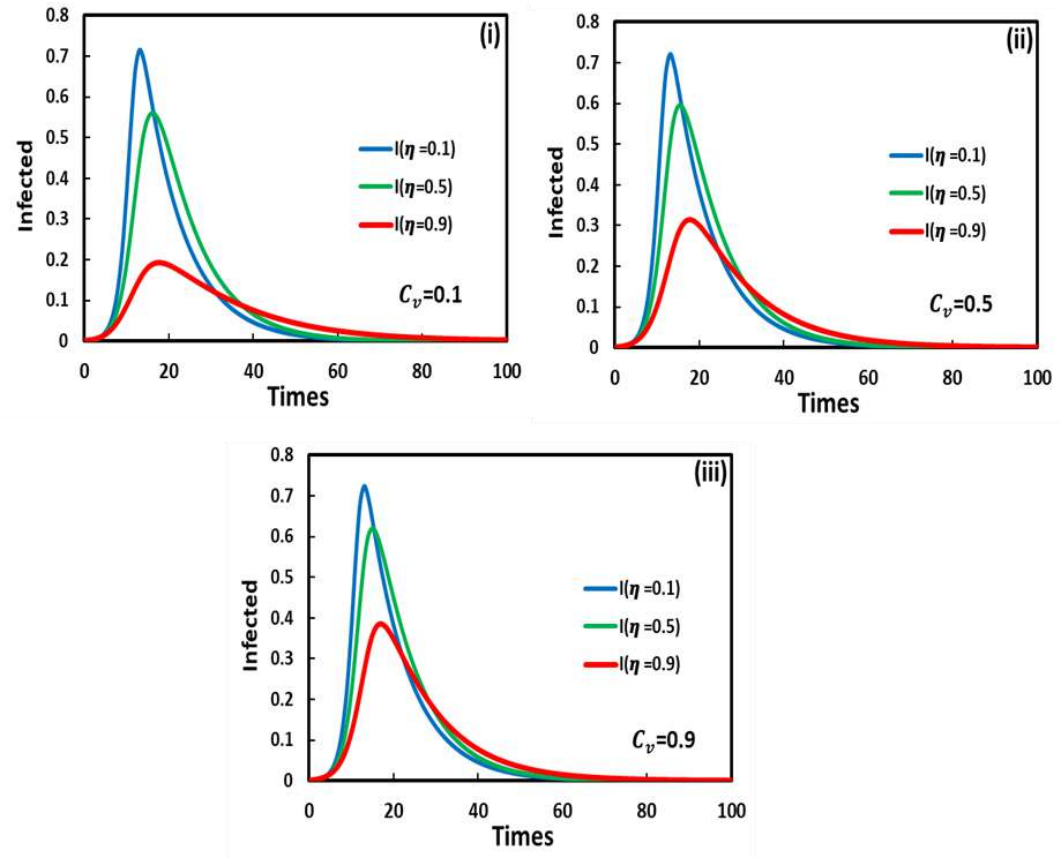

Figure 4.1: Vaccinated game and without treatment game.

Figure 4.1 shows the interplay between vaccination cost and vaccine efficiency when the treatment strategy is inactive. In each panel, the fraction of infected individuals will be minimal when vaccine effectiveness is higher indicating that higher reliability of vaccine attracts individuals to participate in vaccine programs in reducing disease. So, the primary benefit of vaccine efficiency is to reduce infection rates. However, vaccine efficiency depends on other factors, such as vaccination cost. While vaccination cost increases, at the same time, the ability to buy vaccines is decreased, and consequently, the infection rate increases. If a vaccine is cheap, the population responds in a couple of manners whereby everyone vaccinates or not. To compare panel (i) and panel (ii) for vaccine cost $C_V = 0.1$ and $C_V = 0.9$, if the vaccination cost is low, then the infection is reduced.

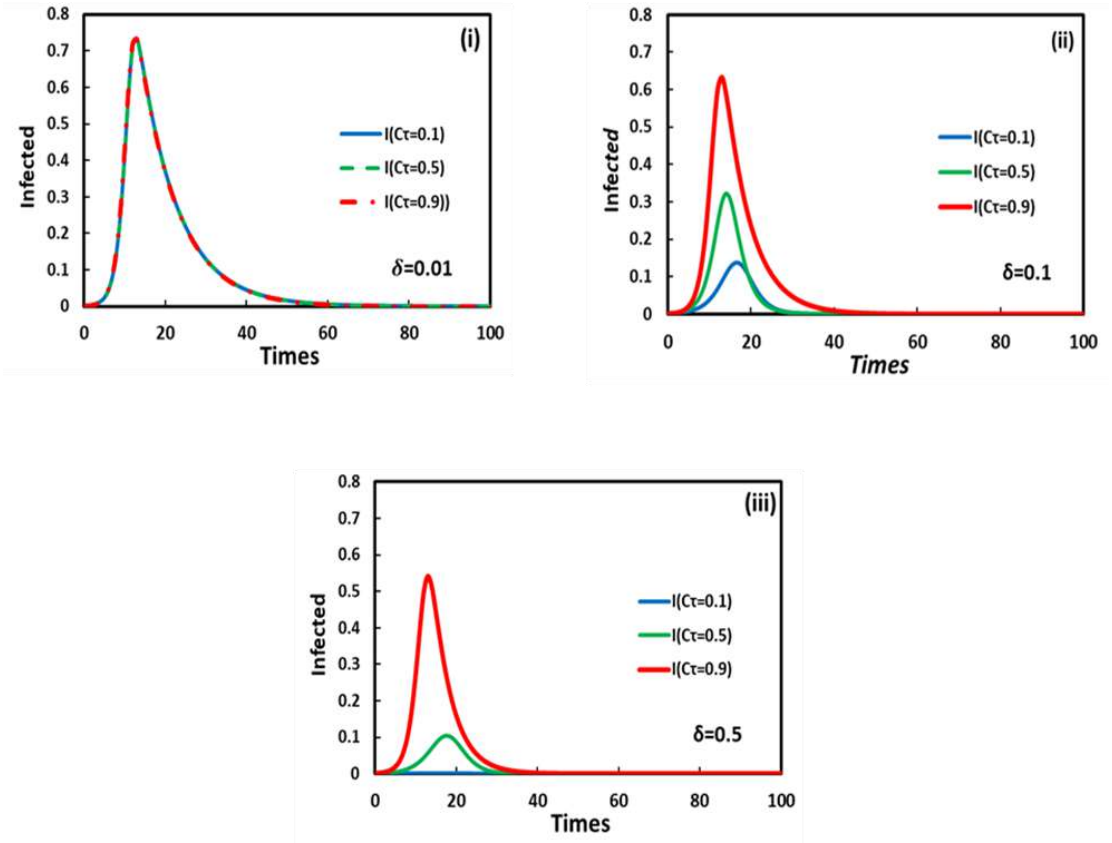

Figure 4.2: Treatment game and without vaccinated game.

To examine the treatment effect on the epidemic (without vaccination), represent figure 4.2. Here, the impact of the natural and treated recovery rates is considered to show the human behavior on treatment services or hospital facilities against a particular disease. When the treated recovery rate (δ) is less than the natural recovery rate (γ) (panel (ii)), the treatment strategy is not working. As a result, infections spread because of a lack of proper treatment. Besides, on treatment cost for $\delta > \gamma$, whenever treatment cost increases in such a periodic time, individuals lose their hope to take treatment (panel (i) and (ii)). The most expensive treatment costs are avoided irrespective of recovery rate, and eventually, the fraction of infections during the epidemic season is maximized. Thus, it's clear that if the recovery rate is higher and treatment costs are lower, the number of infected individuals will be lower.

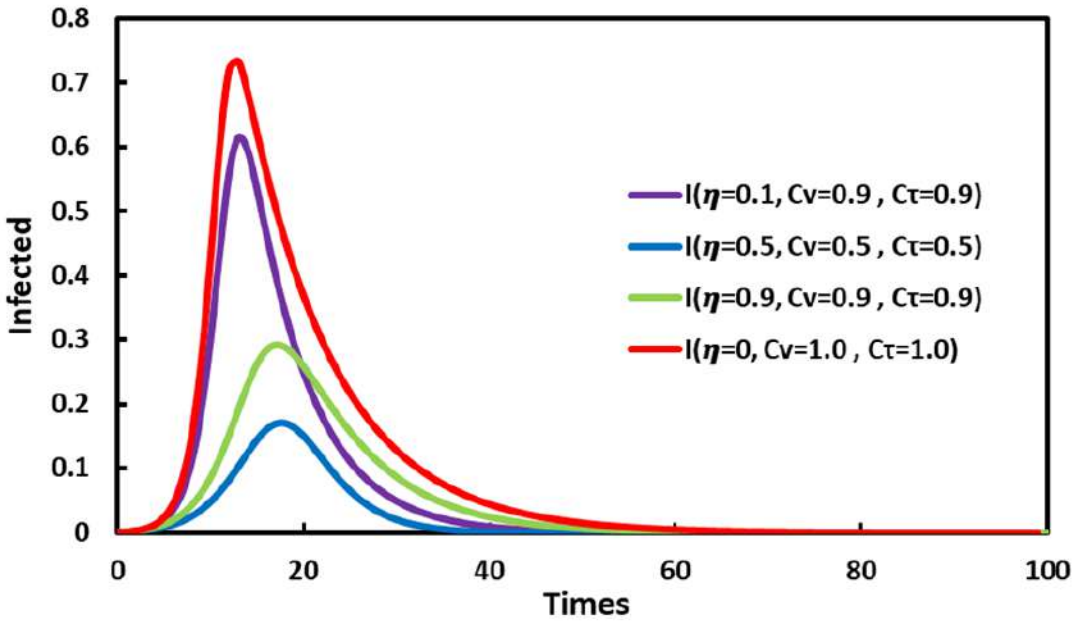

Figure 4.3: Vaccinated and Treatment game.

In figure 4.3, both vaccination and treatment strategies are considered to show the combined impact of intervention policies on the epidemic. Here, the infected individuals depend on vaccine efficiency, vaccination, and treatment costs. When the vaccination and treatment are cheap, individuals can take more vaccines and get treatment, but the infection rate among the individuals remains down. The reduced tendency of infected individuals controls remarkably when the vaccine reliability is increased (higher effectiveness). The most expensive rate of vaccines is rejected even though they are very productive. Curiously, cheaper vaccines may achieve less coverage with increased efficiency, but this is because they are better at controlling outbreaks. As opposed, treatment increases vaccination coverage when the vaccine is rightly priced and sufficiently efficacious.

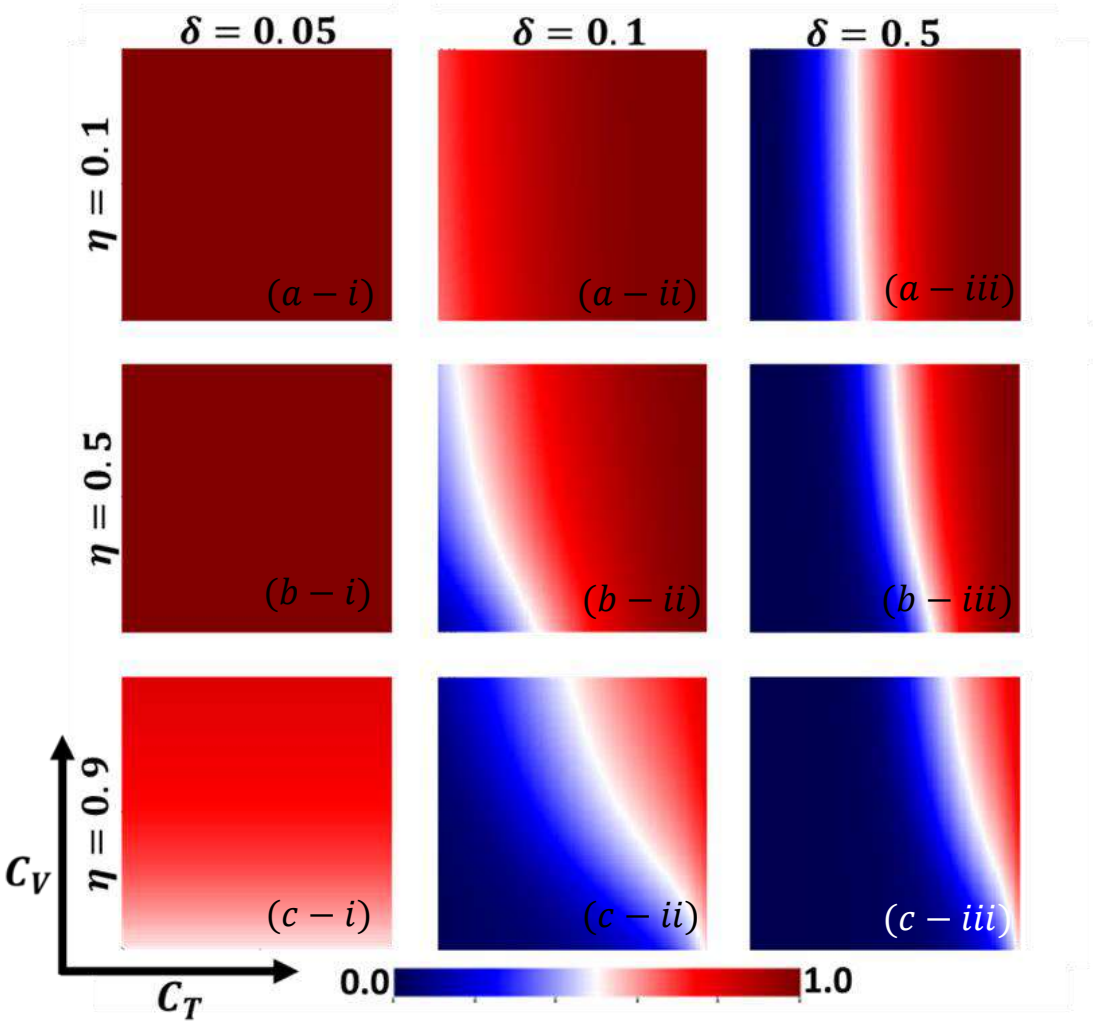

Figure 4.4: Phase diagram of the final epidemic size (FES)

The phase diagram of the final epidemic size (FES) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

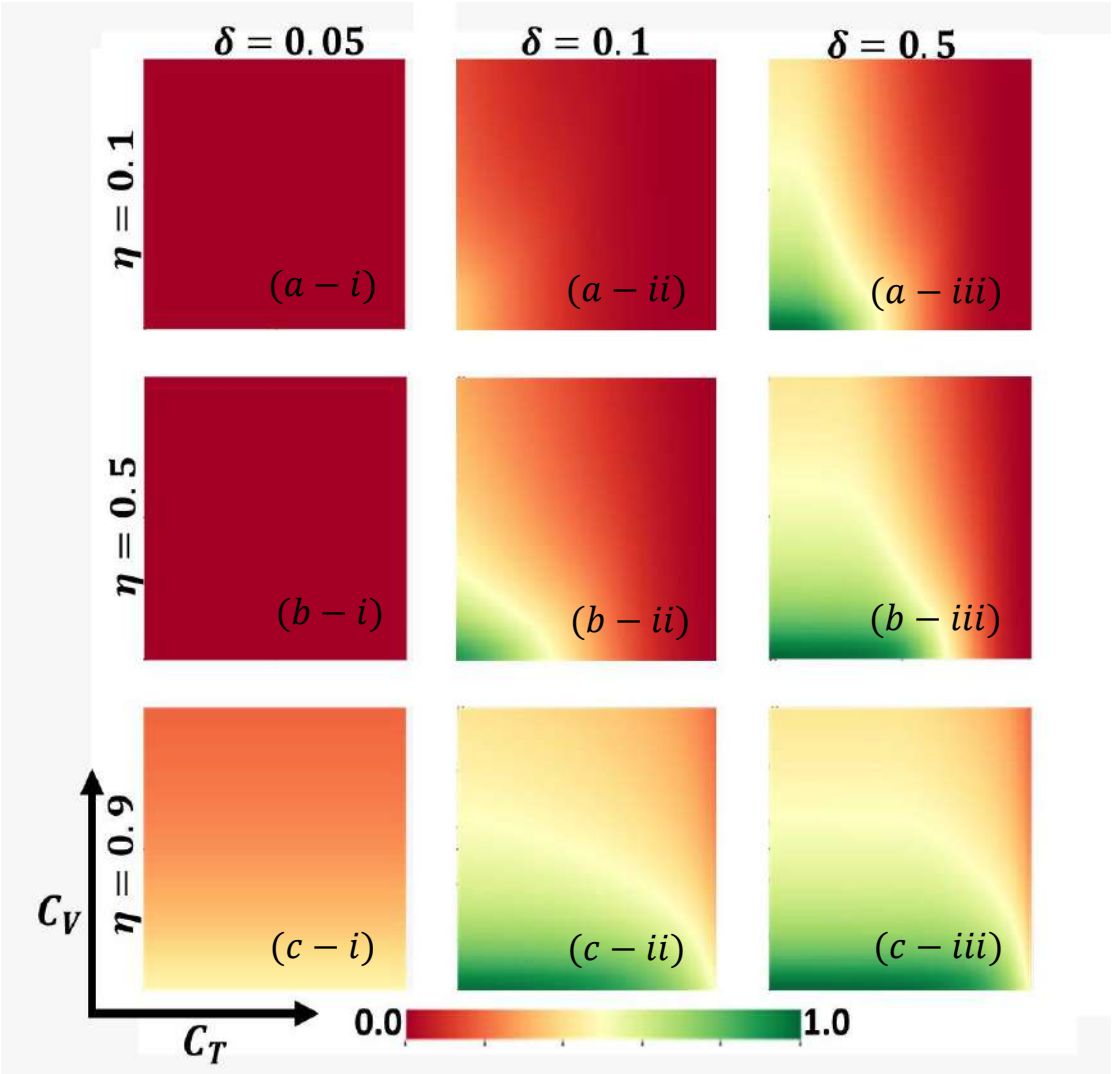

Figure 4.5: Phase diagram of the vaccination coverage (VC)

The phase diagram of the vaccination coverage (VC) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

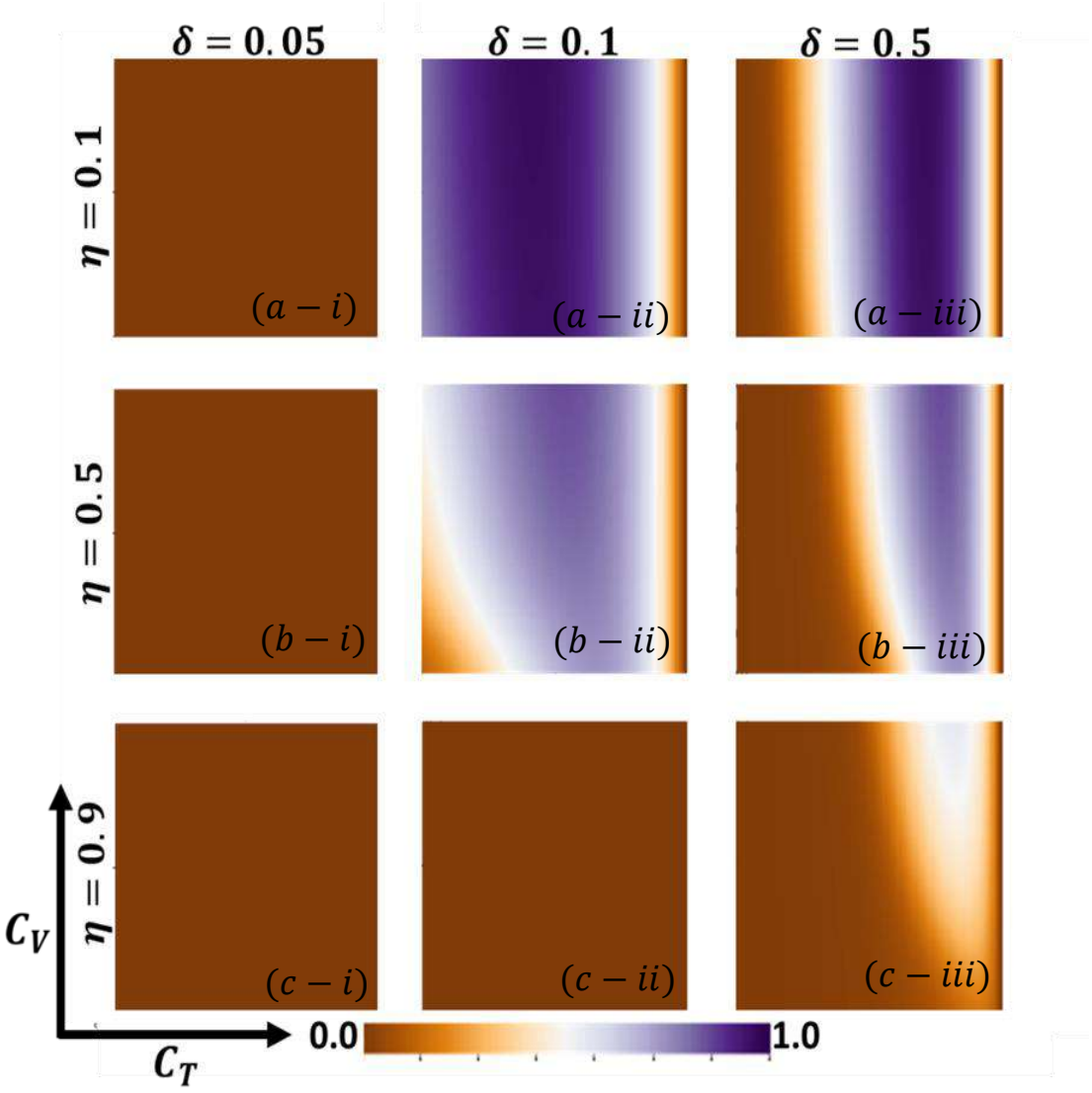

Figure 4.6: Phase diagram of the fraction of treated individuals (FTR)

The phase diagram of the fraction of treated individuals (FTR) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

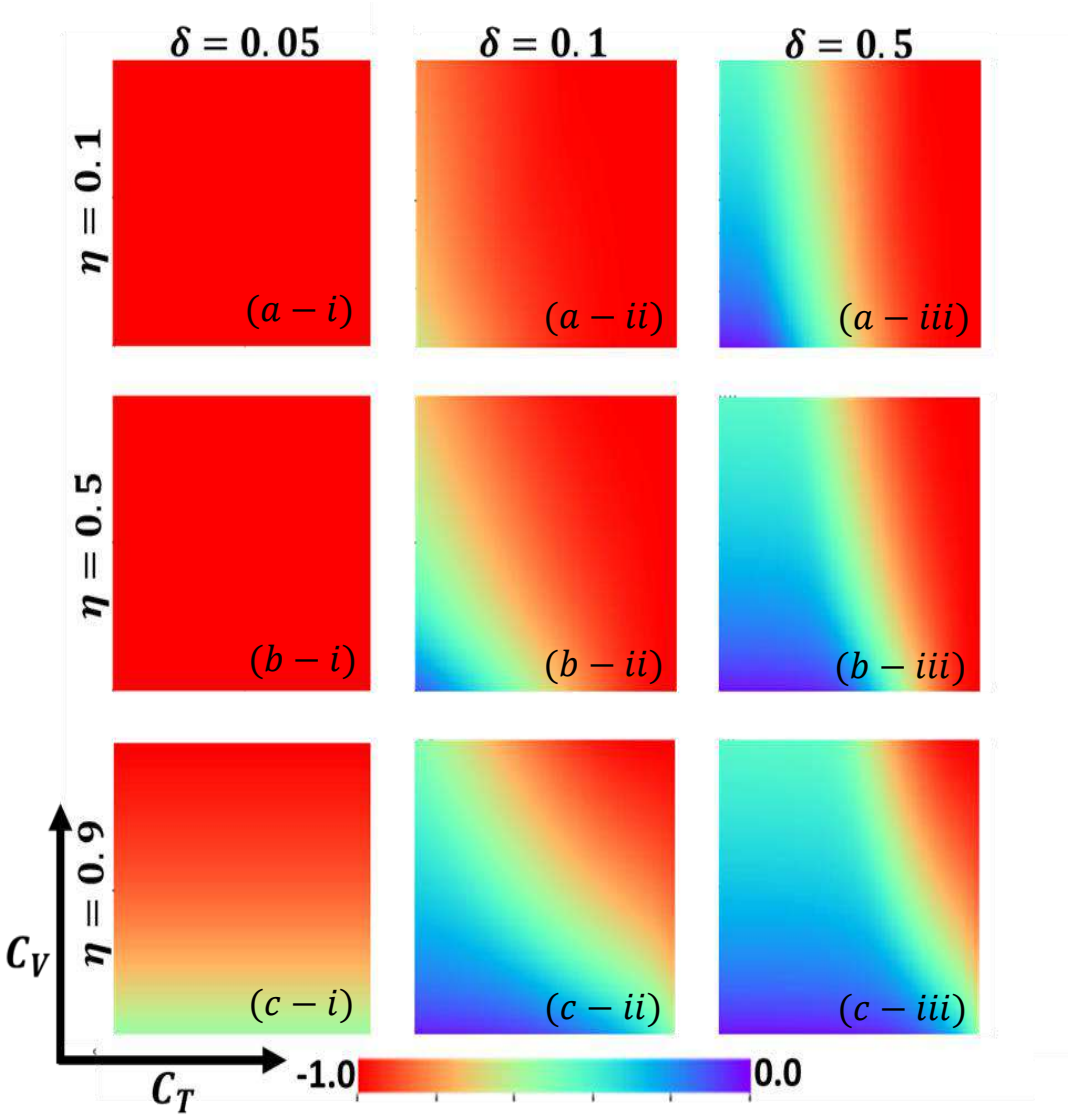

Figure 4.7: Phase diagram of the average social payoff (ASP)

The phase diagram of the average social payoff (ASP) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

4.3 Interplay between vaccination and treatment costs

Here, numerically explore the SVITR model; the results are presented for 2D phase diagrams in the aspect of the evolutionary game theory and cyclic epidemic model. The convene impact of proactive vaccination and the retroactive treatment policy based on human behavior depends on the vaccination cost, treatment cost, and corresponding factors. Also, extensively analyzed the vaccination and treatment cost, vaccination effectiveness (η), and recovery rate, considering the other sensible parameters. Aside from the line graph along the time step, now draw another set of results as a phase diagram in figures 4.4, 4.5, 4.6, and 4.7 at the equilibrium point, expressed by the parameters of treatment cost (C_{T}) and vaccination cost (C_{V}) that describes the underlying social dilemmas in the vaccination and treatment game. Figures 4.4, 4.5, 4.6, and 4.7 display the final epidemic size (FES), vaccination coverage (VC), treated individuals (TR), and average social payoff (ASP). Throughout, the first, second, and third columns show the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$ in panels (i), (ii), and (iii), respectively. On the other hand, the first, second, and third rows display the result of varying the treated recovery rate, $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$ depicted in panels (a), (b) and (c), respectively. The final epidemic size in the SVITR model is influenced by several factors, including the vaccination coverage. Vaccination coverage refers to the proportion of the population that has received a vaccine against the disease. In the SVITR model, vaccination coverage is represented by the V compartment, which includes individuals who are vaccinated and therefore protected from infection. Increasing the vaccination coverage can reduce the final epidemic size by reducing the number of susceptible individuals who can become infected. This is because the vaccinated individuals are less likely to become infected and therefore less likely to transmit the disease to others. The effect of vaccination on reducing the final epidemic size is dependent on the vaccine efficacy, the coverage rate, and the timing of the vaccine deployment. If the vaccination coverage is high enough, the final epidemic size may be small enough that the disease is effectively eliminated from the population. This is known as herd immunity, where the proportion of immune individuals in the population is high enough to provide indirect protection to susceptible individuals.

On the other hand, if the vaccination coverage is low, the final epidemic size may be larger, leading to more infections and potentially more severe consequences such as hospitalization and death. Therefore, increasing vaccination coverage is an important strategy for controlling and preventing the spread of infectious diseases. the higher final epidemic size (FES) was observed for the lower treated recovery rate ($\delta = 0.05$) and less vaccine efficacy ($\eta = 0.1$). Only reduced FES is obtained for high vaccine efficacy ($\eta = 0.9$); people are taking a vaccine for low vaccine cost and reduced infection. Decreasing the vaccination cost at higher vaccine efficacy with treated period remains constant; it occurs less infection, which means individuals encourage to take vaccination and infection level become less. Thus, if the treatment is not favorable for individuals ($\delta < \gamma$), people only participate vaccine program (figure 4.5(c-i)) and avoid taking treatment (figure 4.6(c-i)). Next, for the higher medical (treatment) facilities that accelerate recovery (delta greater than gamma) duration, lower treatment and vaccination cost reduce the infection level, remarkably (panel (ii) and (iii)). Lower treatment costs with higher vaccine costs, individuals take treatment and are less infected (Figures 4.4, 4.5, and 4.6). On the other hand, with higher treatment costs for lower vaccine costs and higher vaccine reliability, individuals take vaccines and avoid infection. In panels (c-ii) and (c-iii), for maximum vaccine efficiency, most individuals take the vaccine to avoid infection when the low vaccination cost, but higher treatment cost arises. Therefore, VC (vaccination coverage) depends on the increase or decrease of vaccine efficiency and treatment duration. However, increasing the vaccination cost and reducing the treatment cost attracted the individual to the treatment, which enhanced the FTR when the treatment duration was high. In addition, the higher vaccine efficiency and treatment recovery rate attract individuals to the treatment strategy. In the SVITR model, the fraction of treated individuals is represented by the infected compartment, which includes individuals who are currently infected and receiving treatment. The treatment can reduce the infectiousness and duration of the disease, which can help to reduce the spread of the disease and the final epidemic size.

The fraction of treated individuals can be increased by improving access to healthcare and treatments for the disease. This can include providing adequate medical supplies and facilities, training healthcare workers, and developing effective treatments such as antiviral drugs. Increasing the fraction of treated individuals can reduce the final epidemic size by reducing the infectiousness of infected individuals and shortening the duration of the infectious period. This can help to slow down the transmission of the disease, leading to

fewer new infections and ultimately a smaller epidemic. In contrast, a lower vaccine efficiency and recovery speed hamper the treatment-seeking behavior (lower FTR).

Now, by comparing Figures 4.5 and 4.6, the results show some interesting phenomena when the treatment duration is greater than the natural recovery rate and the vaccine efficacy of higher. Although it's expected that lower treatment cost attracts people to treatment in hospitals or clinics, our results show a distinct tendency when both vaccine and treatment are considered. If vaccine reliability is higher, irrespective of lower treatment cost or highly facilitated treatment, people are prone to take vaccines rather than be treated or infected. Finally, figure 4.7 represents the phase diagram of average social payoff (ASP) while varying the vaccination and the treatment cost with vaccine efficiency and treatment duration. In the SVITR model, the average social payoff can be defined as the net benefit or cost of the disease to society, taking into account the economic and social impacts of the epidemic. The average social payoff is influenced by several factors, including the final epidemic size, the duration and severity of the epidemic, and the costs of treatment and prevention measures. If the final epidemic size is small, the average social payoff is likely to be positive, as the costs of the epidemic, such as healthcare costs, lost productivity, and social disruption, are reduced. On the other hand, if the final epidemic size is large, the average social payoff is likely to be negative, as the costs of the epidemic are increased. The duration and severity of the epidemic also play a role in determining the average social payoff. A long and severe epidemic can lead to more significant economic and social impacts, increasing the negative social payoff. The costs of treatment and prevention measures, such as vaccines and public health interventions, can also impact the average social payoff. While these measures can be costly, they may ultimately result in a positive social payoff by reducing the final epidemic size and associated costs. It observed that lower vaccine and treatment cost brings higher ASP, meaning society reaches its optimal position when vaccine efficacy is higher, and $\delta > \gamma$.

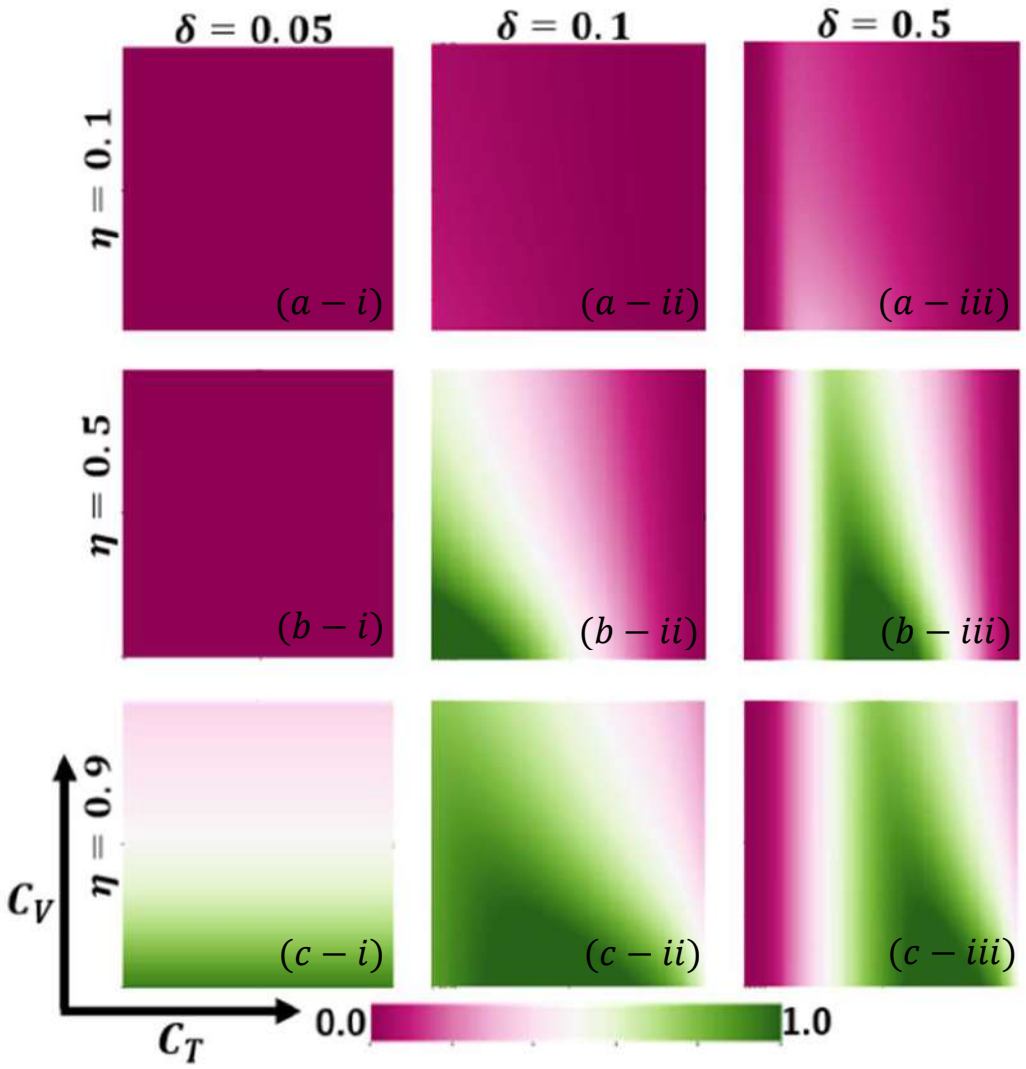

Figure 4.8: Phase diagram of the socially benefitted individuals from vaccination (SBI_V)

The phase diagram of the socially benefitted individuals from vaccination (SBI_V) is present by varying two parameters: the x-axis contains treatment cost (C_T) and the y-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

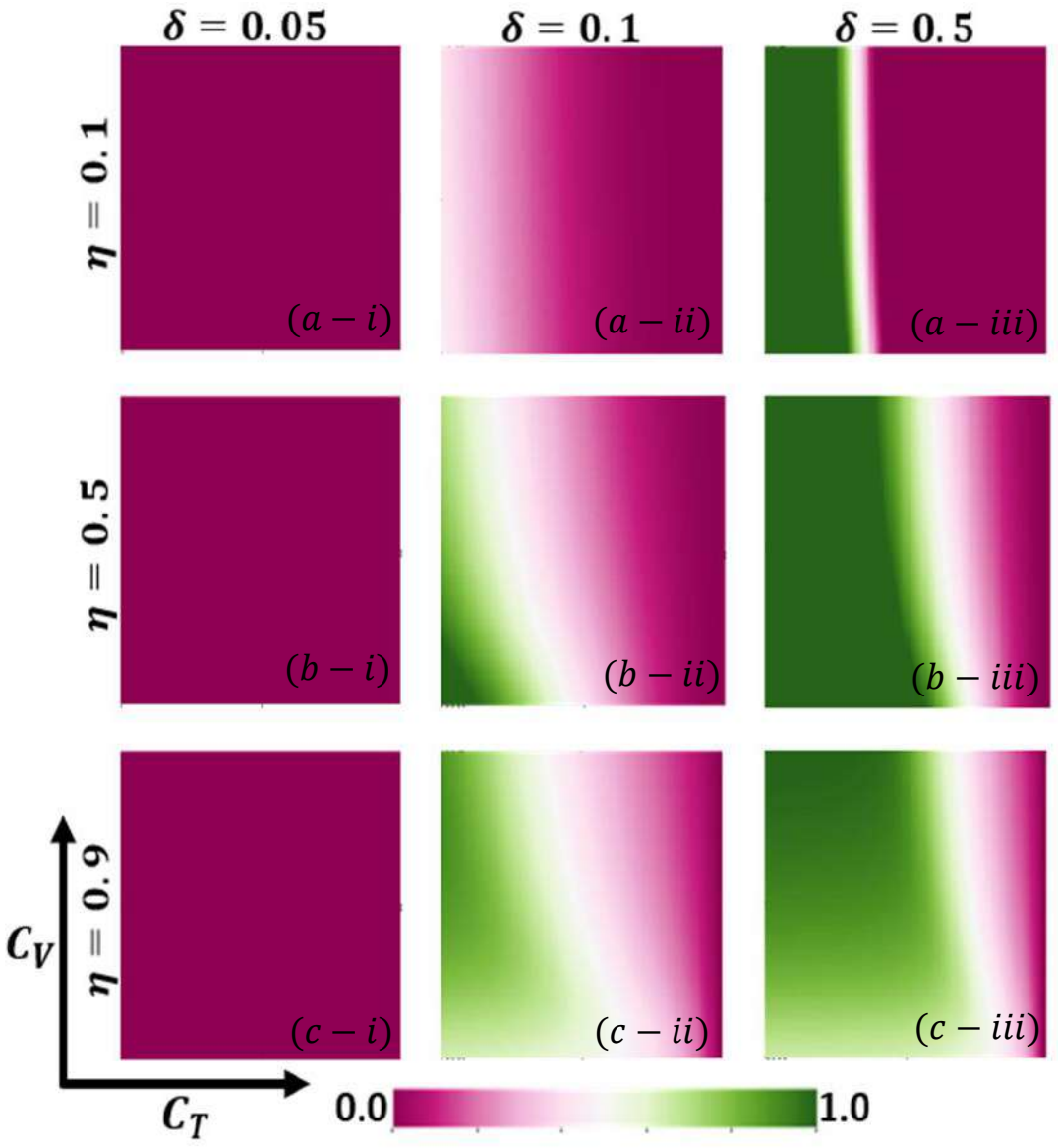

Figure 4.9: Phase diagram of the socially benefitted individuals from treatment (SBI_T)

The phase diagram of the socially benefitted individuals from treatment (SBI_T) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

To realize how different parameter settings affect the benefitted individuals from intervention games: vaccination, or treatment, interpreted socially benefitted individuals (vaccination) and (treatment) in figures 4.8 and 4.9, respectively. Here, adopt a new indicator termed socially benefitted individuals (SBI), referring to equations (11.1) and (11.2) as the fraction of the population gap between FES of control strategies and without controls at equilibrium. In the SVITR model, vaccination can lead to social benefits by reducing the number of individuals who become infected with the disease. Socially benefiting individuals from vaccination can be defined as those who are protected from infection as a result of vaccination. The number of socially benefiting individuals from vaccination depends on several factors, including vaccine efficacy, vaccination coverage, and the timing of vaccination deployment. If the vaccine efficacy is high and the vaccination coverage is sufficient, a large proportion of the population can be protected from infection, resulting in a significant number of socially benefiting individuals.

The socially benefiting individuals from vaccination may include not only those who receive the vaccine directly but also those who are indirectly protected through herd immunity. When a high proportion of the population is vaccinated, the disease has a reduced ability to spread, and even individuals who are not vaccinated may be protected from infection. The socially benefiting individuals from vaccination can include individuals who may be particularly vulnerable to the disease, such as the elderly, young children, and individuals with weakened immune systems. By protecting these vulnerable individuals, vaccination can help reduce the burden of the disease on society, including healthcare costs and lost productivity. Overall, vaccination can lead to significant social benefits by reducing the number of individuals who become infected with the disease and protecting vulnerable populations. Maximizing vaccination coverage and vaccine efficacy is crucial for maximizing the number of socially benefiting individuals from vaccination. In Figure 4.8, it can observe that individuals participating in vaccine programs can get more advantages from vaccination when efficacy is higher and vaccine cost is minimal. However, irrespective of the lower cost of vaccination, if the treatment cost is low, people benefit less (panels b-iii and c-iii).

In the SVITR model, treatment can lead to social benefits by reducing the severity and duration of the disease in infected individuals, thereby reducing the overall burden of the disease on society. Socially benefiting individuals from treatment can be defined as those who receive treatment and experience a reduction in the severity and duration of the disease. The number of socially benefiting individuals from treatment depends on several

factors, including the efficacy of the treatment, its availability, and the timing of its deployment. If the treatment is effective and widely available, a large proportion of infected individuals can benefit from it, resulting in a significant number of socially benefiting individuals. The socially benefiting individuals from treatment may include individuals who may be particularly vulnerable to the disease, such as the elderly, young children, and individuals with weakened immune systems. By reducing the severity and duration of the disease in these vulnerable individuals, treatment can help reduce the burden of the disease on society, including healthcare costs and lost productivity. Treatment can also lead to social benefits by reducing the risk of transmission of the disease to other individuals. By reducing the infectiousness of infected individuals, treatment can help to slow down the transmission of the disease and ultimately reduce the number of new infections, leading to a smaller epidemic size and associated social benefits. Overall, treatment can lead to significant social benefits by reducing the severity and duration of the disease in infected individuals, protecting vulnerable populations, and reducing the risk of transmission of the disease. Maximizing the availability and efficacy of treatments is crucial for maximizing the number of socially benefiting individuals from treatment.

In figure 4.9, it can see that there are no treated benefited people for treatment (SBI_T) when $\delta < \gamma$; treatment duration (treatment to recover) is higher than natural recovery (see panel (i)). These phenomena indicate that if the medical/clinic/drug facilities need to be better equipped and work effectively to recover from disease faster, people are not getting social benefits from treatment. However, for $\delta > \gamma$, individuals benefit from treatment when treatment costs are lower. Interestingly, increases in vaccine efficacy somehow increase the fraction of aided people. But for higher vaccine efficacy ($\eta = 0.9$), the treated benefitted people will decrease when vaccine cost is cheaper (lower vaccine prices).

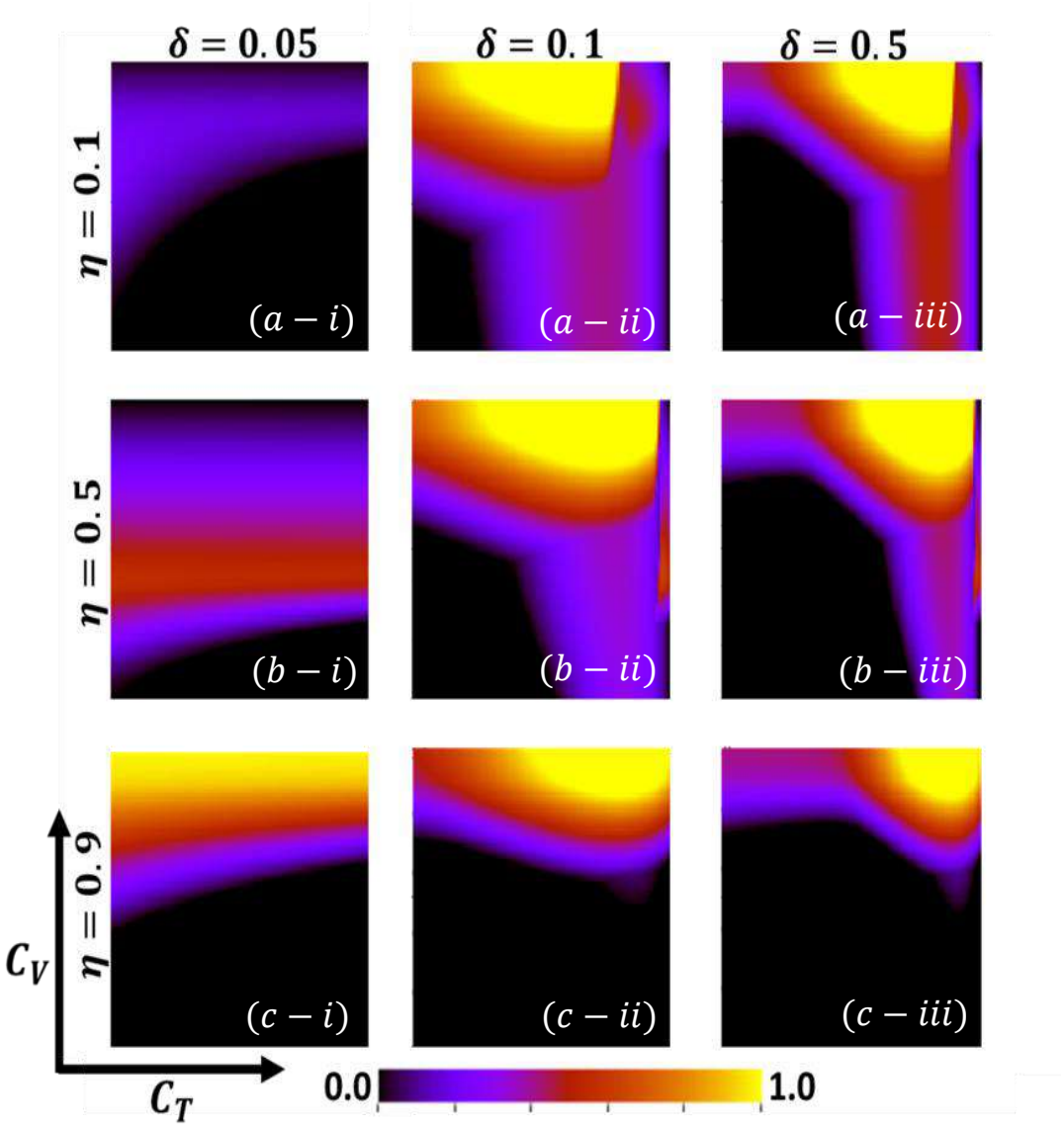

Figure 4.10: Phase diagram of the social efficient deficit (SED_V)

The phase diagram of the social efficient deficit (SED_V) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$.

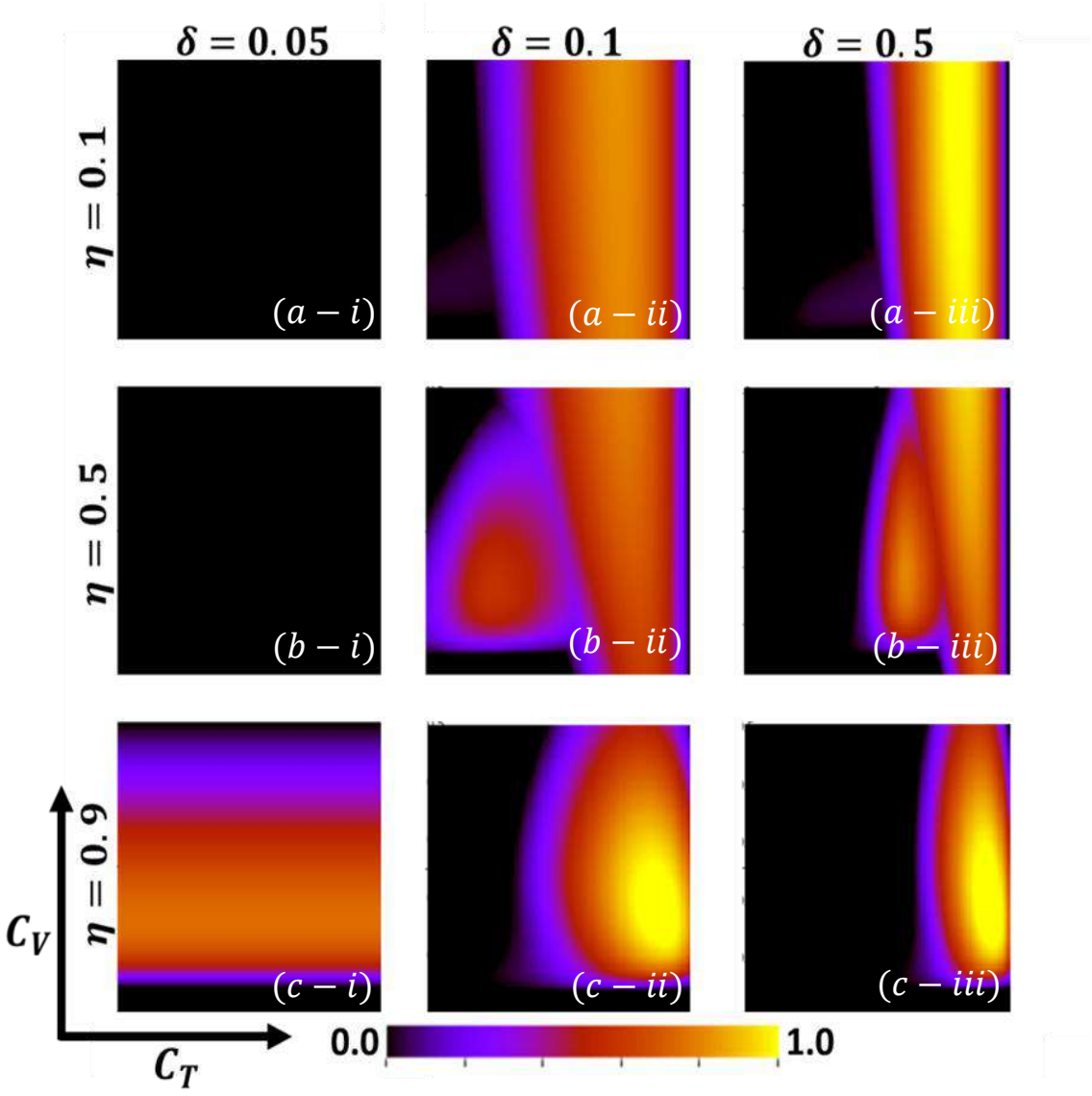

Figure 4.11: phase diagram of the social efficient deficit (SED_T)

The phase diagram of the social efficient deficit (SED_T) is present by varying two parameters: the *x*-axis contains treatment cost (C_T) and the *y*-axis is vaccination cost (C_V). In this figure, the first, second, and third rows display the results of varying the vaccine efficiency, $\eta = 0.1$, $\eta = 0.5$, and $\eta = 0.9$. Also, the first, second and third columns show the results of varying the treatment duration rate: $\delta = 0.05$, $\delta = 0.1$, and $\delta = 0.5$. Other parameters are, $\beta = 0.8333$, $\gamma = 0.333$, and $\omega = 0.01$

In figure 4.10 and figure 4.11, explore the idea of social efficiency deficit (SED) that explicitly reveals the underlying social dilemmas in the vaccination and treatment game. An individual's decision on taking provision (vaccine or treatment) that infers cooperation (C) or not accepting provision indicates defection (D) on the relative cost of vaccine and treatment. By exploring SED for vaccination and treatment, generate the 2D heat map for SED_V (figure 4.10) and SED_T (figure 4.11) to visualize how SED varies as a function of C_V and C_T. The region-colored black presented having no SED (no dilemma) in which society reached its stable situation, and the payoff at NE cannot be improved anymore.

The Social Efficient Deficit (SED_V) is a measure of the inefficiency of vaccination programs that takes into account both the costs of vaccination and the health benefits it provides to society. It is calculated by comparing the social costs of vaccination (including the cost of the vaccine, the cost of administering the vaccine, and any associated costs such as transportation and storage) to the social benefits of vaccination (including the reduction in disease transmission, the reduction in healthcare costs, and the improvement in overall health outcomes).The SED_V provides a measure of the net benefits of vaccination programs and can be used to evaluate the efficiency of different vaccination strategies. If the $SED_V$is positive, it indicates that the social benefits of vaccination exceed the social costs, and the vaccination program is considered socially efficient. On the other hand, if the $SED_V$is negative, it indicates that the social costs of vaccination exceed the social benefits, and the vaccination program is considered socially inefficient. The SED_V is an important tool for policymakers and public health officials when making decisions about the allocation of resources for vaccination programs. It can help identify the most efficient vaccination strategies, taking into account both the costs and benefits of vaccination. Additionally, it can help to ensure that public health resources are used in the most effective way possible to maximize the benefits to society. In (figure 4.10), with a lower level of vaccination cost, $SED_V = 0$, SED reaches its minimum point, meaning when vaccination cost is low, people will participate in the vaccine program (ALLC). Afterwards, it shows a monotonic increase when vaccine efficacy increases; the situation with a more effective vaccine might inspire some people to participate in vaccine program. Interestingly, we could also see the effect of treatment cost to arise dilemma situation. When $\eta \leq 0.5$, comparatively lower reliability of vaccines, people somehow think about treatment provision. But, when the treatment cost is higher SED_V is also arises, because people are trope in a dilemma situation. According to the findings, it can be concluded that we can minimize SED_V by either lower the vaccine

cost or improving the vaccine efficacy. Overall, the SED_V provides a useful framework for evaluating the efficiency of vaccination programs and can help to guide decision-making in public health policy.

The Social Efficient Deficit (SED_T) is a measure of the inefficiency of treatment interventions that takes into account both the costs of treatment and the health benefits it provides to society. It is calculated by comparing the social costs of treatment (including the cost of medication, hospitalization, and other associated costs such as transportation and lost productivity) to the social benefits of treatment (including the reduction in disease transmission, the reduction in healthcare costs, and the improvement in overall health outcomes). Similar to the SED_V for vaccination programs, the SED_T provides a measure of the net benefits of treatment interventions and can be used to evaluate the efficiency of different treatment strategies. If the $SED_T$is positive, it indicates that the social benefits of treatment exceed the social costs, and the treatment program is considered socially efficient. On the other hand, if the $SED_T$is negative, it indicates that the social costs of treatment exceed the social benefits, and the treatment program is considered socially inefficient. The SED_T is an important tool for policymakers and public health officials when making decisions about the allocation of resources for treatment interventions. It can help to identify the most efficient treatment strategies, taking into account both the costs and benefits of treatment. Additionally, it can help to ensure that public health resources are used in the most effective way possible to maximize the benefits to society. Overall, the SED_T provides a useful framework for evaluating the efficiency of treatment interventions and can help to guide decision-making in public health policy. In figure 4.11, panel (a-i) and (b-i) present completely no dilemma situation for $SED = 0$. In the aspect of evolutionary game theory, this no-dilemma situation is arises for $\delta < \gamma$. However, when vaccine effectiveness is high (panel c-i), few dilemmas observed that arises for middle cost of C_V values. For comparatively higher and lower C_V, no dilemma situation is detected. We can conclude that when cost is lower people are participating vaccine program without any hesitation (ALLC) and for higher C_V, people are fully avoiding vaccination (ALLD). Meanwhile, when $\delta > \gamma$, dilemma situation arises for higher costly treatment; when $C_T > 0$, we observed $SED_T > 0$. Because of the higher cost of treatment, people will think about whether to participate in treatment or not, which creates a dilemma. On the other hand, for lower treatment costs people are fully cooperative about treatment and willingly go for treatment (ALLC) which displays no dilemma situation at all.

Chapter - 5

Conclusion

5.1 Conclusion

In this thesis work, SVITR epidemic model, integrated with evolutionary game theory, offers a comprehensive understanding of the complex dynamics between disease spread, vaccination, and treatment. This work indicates that ex-post treatment, in certain circumstances, can improve the final epidemic size, depending on factors such as the reliability of vaccination and its cost. These insights provide valuable recommendations for implementing appropriate and careful treatment strategies. By analyzing the interplay of various factors and strategies, this research contributes to the field of epidemiology and provides insights to inform effective public health interventions and decision-making processes. This research developed an SVITR epidemic model for the disease spread and the embedded vaccine and treatment behavioral dynamics by using extensive evolutionary game theory among the individuals in society. The most important contribution is that our new model gives an extensive framework that explains vaccination and treatment strategies considering different effectiveness, associated cost, and payoff structure on local time scales. This model gives a clear context to quantify the social benefit and dilemmas entailed by vaccination and treatment games. Increasing the effectiveness of vaccination and lowering the vaccination cost increased the vaccination coverage, but that's how it reduced the final epidemic size. Lowering the treatment duration and improving treatment costs had a similar effect. Again, improving the vaccine efficacy and reducing treatment duration increased the treatment provision and average social payoff. Lowering vaccine efficacy and recovery rate, on the other hand, hampered treatment-seeking behavior. Besides amplifying the voluntary vaccination game, our model introduced a new game expression with two directions: vaccination as a proactive measure and treatment as a retroactive measure. Thus, by applying proactive vaccination and retroactive treatment, it can investigate and understand the individuals' decisions regarding over-vaccination and perform proper strategies that reduce the divergency of infection and ensure the careful state of both

antiviral treatment and vaccination. As well successfully introduced the concept of SBI to explore the idea of benefitted individuals when they are inclined to a specific strategy. Besides, the SED results also show the social dilemma situation for each strategy (vaccination or treatment). Finally, the results of this model suggest that the ex-post treatment sometimes improves the final epidemic size that depends on other aspects, such as the reliability of vaccination and its cost, which recommend appropriate and careful treatment.

5.2 Future Work

The current research highlights the use of evolutionary game theory to analyze the behavioral dynamics related to vaccination and treatment. This approach should deeper into this aspect by considering different strategies, preferences, and interactions among individuals. Incorporating realistic human behavior will enable a more comprehensive understanding of disease spread and intervention policies. Building upon the insights gained from the SVITRS model, future work should aim to provide policy recommendations for disease control and prevention. These recommendations should consider the model's findings, incorporate societal and ethical considerations, and address the practical challenges of implementing intervention strategies. Meanwhile, this research area will involve refining and validating the SVITRS epidemic model, incorporating real-world data, analyzing behavioral dynamics using game theory, evaluating intervention strategies, conducting sensitivity analysis, validating the model, providing policy recommendations, and exploring its application to other diseases. This iterative process will lead to a more comprehensive understanding of disease dynamics and support evidence-based decision-making for disease control and prevention.

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List of Publications

- **Abu Zobayer**, Mohammad Sharif Ullah, and K. M. Ariful Kabir. "A cyclic behavioral modeling aspect to understand the effects of vaccination and treatment on epidemic transmission dynamics." Scientific Reports 13, no. 1 (2023): 8356. https://doi.org/10.1038/s41598-023-35188-3

- Md. Fahimur Rahman Shuvo, **Abu Zobayer** and K M Ariful Kabir "Assessing the Impact of Health, Economy, and Environment on post-COVID Syndrome: A Study on the Bangladesh Perspective Authors" to the following Journal/Specialty: Heliyon [Submitted on: 24 July 2023]

Conference Proceedings

- Saima Efat, **Abu Zobayer**, K M Ariful Kabir "Optimal control applied to vaccination and treatment strategies for the human papillomavirus (HPV) epidemic model". 1st International conference on frontier in Sciences-2022 organized by Faculty of Science, BUET, Dhaka.

- **Abu Zobayer**, Abhi Chakraborty, K M Ariful Kabir "Analysis of lockdown strategy on epidemic dynamics to understand the multi-wave disease incidence". A F Mujibur Rahman Bangladesh Mathematical Society National Mathematics Conference 2022 Organized by Department of Mathematics, Jahangirnagar University, Savar, Dahaka.