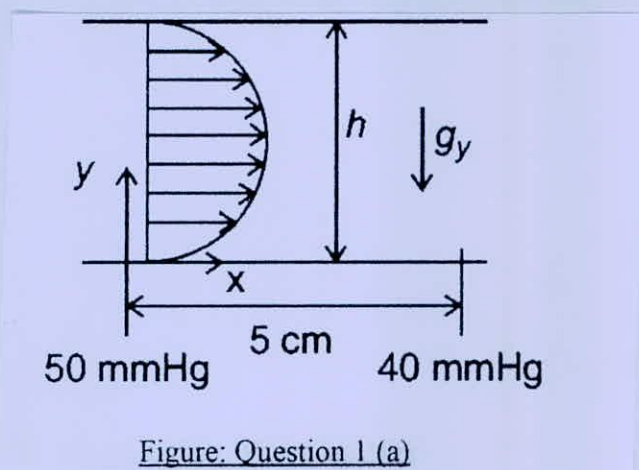


SECTION – A

There are **FOUR** questions in this section. **Question 1 is Mandatory**. Answer any **TWO** of the remaining **THREE** questions.

1. (a) Construct an expression for the velocity profile and the shear stress (τ_{xy}) distribution for blood flowing in an arteriole with a diameter of 500 μm . Use the Navier-Stokes equations for Cartesian coordinates to solve the problem. Make appropriate assumptions where required. The pressure driving this flow is shown in Fig. Question 1(a). Assume that the flow is fully developed. (20)



- (b) For flow in the proximal aorta with a diameter of 2.5 cm and an average velocity of 0.25 m/s, compute the entrance length, and comment on flow development in the aorta. Note that an adult human aorta is < 1 m in length. Assume the aorta to be a circular pipe. Also show a diagram of flow development in the aorta. Clearly indicate inviscid flow region, and boundary layer region in the diagram. (13)
- (c) Explain the physical significance of Navier-Stokes equations, and identify its limitations. Also describe the concept of continuum idealization in fluid mechanics in brief. (12)
2. (a) Suppose a patient is undergoing a cardiac catheterization procedure in which radiopaque dye is injected into his heart through a 2 m long catheter to obtain x-ray images of his left ventricle as shown in Figure Question 2 (a). (10)

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Contd... Q. No. 2(a)

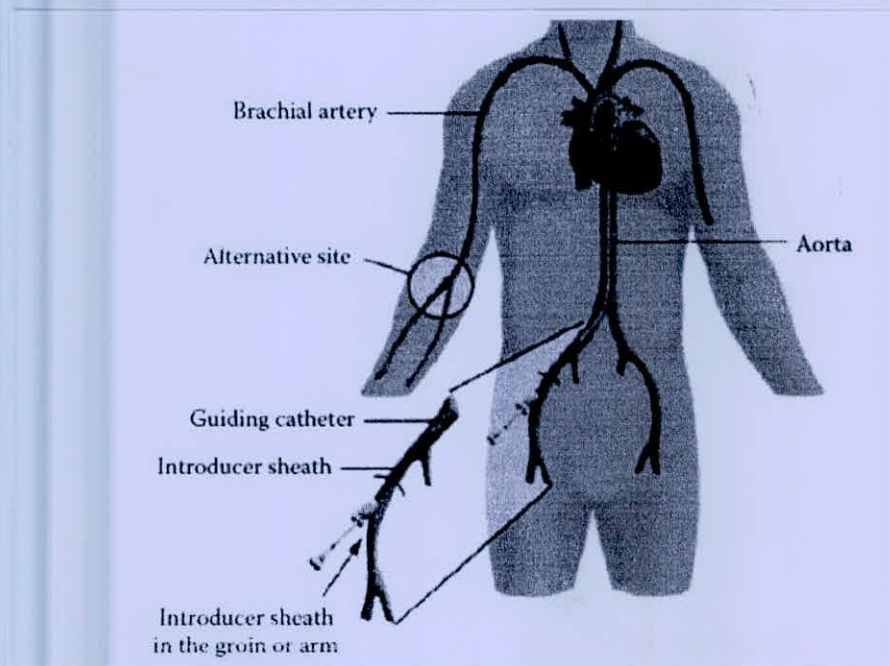


Figure Question 2 (a)

i. If the dye is injected from a syringe outside the body which is 2 cm in diameter, what must be the velocity of the plunger in order to deliver 8.5 cm³ of dye in 1 s?

ii. What would be the average velocity of the dye as it exits the catheter tip if the catheter has a diameter of 2 mm?

(b) Classify blood pumps and explain which one has the main role in blood transportation. Also mention which characteristics are important in selecting a blood pump? (12)

(c) The velocity of a fluid flow field is given by, $\vec{V} = x^3y\hat{i} + y^2z\hat{j} - (x^3yz + yz^2)\hat{k}$. Show that it is a case of possible steady, incompressible fluid flow. Calculate the velocity at point (1,2,3). (8)

3. (a) Calculate the time rate of change of air density during expiration. Assume that the lung has a total volume of 6000 mL, the diameter of the trachea is 18 mm, the airflow velocity out of the trachea is 20 cm/s, and the density of air is 1.225 kg/m³. Also Assume that lung volume is decreasing at a rate of 100 mL/s. (15)

(b) Upon inserting a catheter into the aorta, the blood flow must pass around the catheter. Assume that the catheter is placed directly in the centerline of the flow field and that it's not moving at this instance in time, as shown in Figure Question 3(b). Now derive an expression for the velocity profile assuming that the flow is only pressure driven (dp/dz). The outer radius of the aorta is R and, the outer radius of the catheter is kR. (15)

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Contd... Q. No. 3(b)

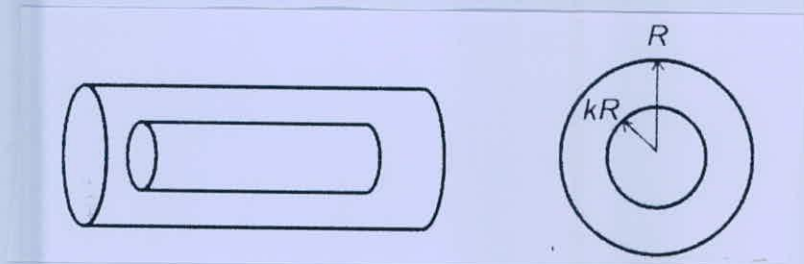


Figure Question 3(b)

4. (a) A syringe is filled with a liquid drug and held vertically as shown in Figure Question 4(a). A pressure of 100 mmHg is applied to the drug by pushing on the plunger. What is the velocity of the drug leaving the syringe tip and how high will the stream go? Assume that the elevation in the barrel is zero and that its velocity is negligible. Take density of the drug to be 1000 kg/m^3 .

(10)

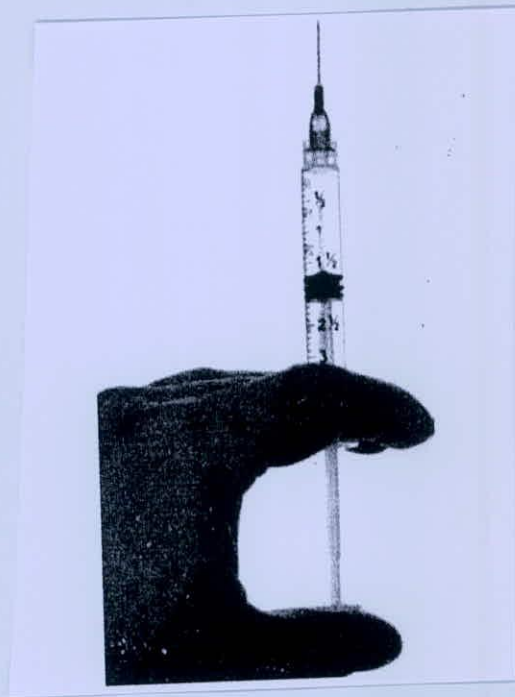


Figure Question 4(a)

- (b) One of the functions of the cardiovascular system is to act as a heat exchanger, to minimum body temperature, as shown in Figure Question 4(b).

(13)

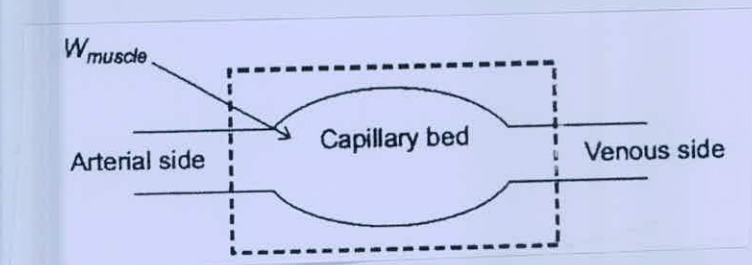


Figure Question 4(b)

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Contd... Q. No. 4(b)

Calculate the rate of heat transfer through a capillary bed, assuming that the blood velocity into the capillary is 100 mm/s and the flow velocity out of the capillary bed is 40 mm/s. The pressure on the arterial side is 20 mmHg, and the pressure on the venous side is 12 mmHg. Assume that the arteriole diameter is 75 μ m and the venule diameter is 50 μ m. The temperature on the arterial side is 35°C, and temperature on the venous side is 33°C. Assume that the power put into the system throughout the muscular system is 15 μ W, and that is no height difference between the arterial side and venous side.

(c) Mathematically show the dependency of total pressure exerted by a liquid on a plane surface on the position of the centroid of surface. (7)

SECTION – B

There are **FOUR** questions in this section. Question No. 5 is **MANDATORY**.

Answer any **TWO** of the remaining **THREE** questions.

5. (a) Compute the average volumetric flow rate for the following blood vessel that is represented as an ellipse in the Figure Question 5(a). The blood vessel has known average velocity values and geometric values for laminae of fluid (r_L is the radius for the long axis and r_S is the radius for the short axis). (15)

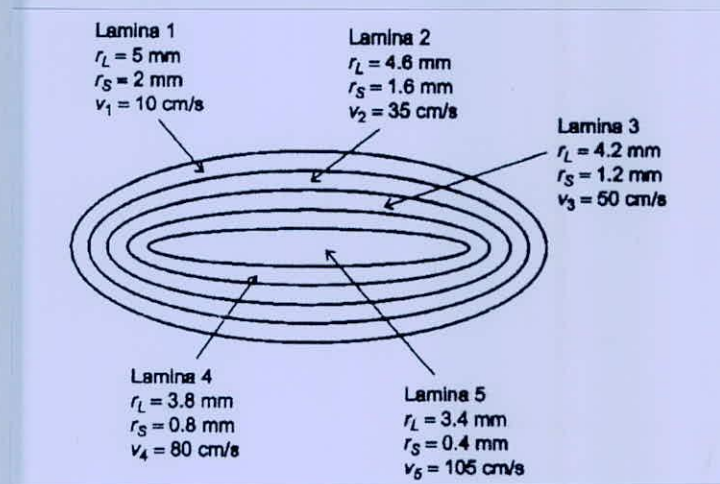


Figure Question 5(a)

(b) Consider that a jelly fish can be approximated as a slab whose total metabolic heat generation in W is given by $0.2 m^{0.85}$ where m is the mass of the fish in kg. Its movement in water leads to a surface heat transfer coefficient of $50 \text{ Wm}^{-2}\text{K}^{-1}$. The jelly fish weighs 2 kg, the thickness of the jelly fish (slab) is 5 cm, and its thermal conductivity is $0.5 \text{ Wm}^{-1}\text{K}^{-1}$. The density of the jelly fish can be assumed to be that of water, 1000 kgm^{-3} . At steady state, investigate the temperature difference between the center (core) and the surface of this fish. (15)

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Contd... Q. No. 5

- (c) Calculate heat flux through 0.3 cm of skin plus subcutaneous fat tissue and contrast this with that through muscle tissue of the same thickness. The effective thermal conductivities of skin plus subcutaneous fat tissue and muscle tissue are $0.28 \text{ Wm}^{-1}\text{K}^{-1}$ and $0.56 \text{ Wm}^{-1}\text{K}^{-1}$, respectively. The temperatures on the two sides of the tissue are 37°C , and 33°C respectively. (10)
- (d) Describe how the blood flow through the aorta arch might be different from the flow in the abdominal aorta. (5)
6. (a) Calculate the radius of one daughter branch knowing that the radius of the parent vessel is $175 \mu\text{m}$ and the radius of the other daughter branch is $125 \mu\text{m}$. Illustrate the bifurcation, to scale, after calculating the bifurcation angles. Also, validate the last equation from Murray's law, which relates both bifurcation angles to the radii of the three blood vessels. (15)
- (b) Determine the volumetric flow rate of water out of the capillary under normal conditions where the permeability constant is $0.25 \text{ mL min}^{-1} \text{ mmHg}^{-1}$. How much fluid leaks out of the capillary within 1 day? Is this reasonable? (10)
- (c) Organize according to the order of magnitude of the thermal conductivity for gases, liquids, and solids at room temperature and 1 atm pressure. (5)
7. (a) Blood is flowing within a small hollow fiber module containing a total of 1,000 fibers. Each fiber has an internal diameter of $60 \mu\text{m}$ and a length of 20 cm. If the total flow rate of the blood is 10 mL min^{-1} , estimate the pressure drop in mmHg across the hollow fibers. Also calculate the tube hematocrit, H_T . Assume the discharge hematocrit of the blood is 0.40 and that the Newtonian viscosity of blood in large tubes is 3 cP. The plasma viscosity is 1.2 cP. (15)

$$\frac{H_T}{0.40} = 0.40 + (1 - 0.40) \left(1 + 1.7e^{-0.415d} - 0.6e^{-0.011d} \right)$$

$$C = \left(0.8 + e^{-0.075d} \right) \left(-1 + \frac{1}{1 + 10^{-11}d^{12}} \right) + \frac{1}{1 + 10^{-11}d^{12}}$$

$$\eta_{0.45}^* = 220e^{-1.3d} + 3.2 - 2.44e^{-0.06d^{0.645}}$$

$$\frac{\mu_{\text{apparent}}}{\mu_{\text{plasma}}} = \left[1 + (\eta_{0.45}^* - 1) \frac{(1 - H_D)^c - 1}{(1 - 0.45)^c - 1} \left(\frac{d}{d - 1.1} \right)^2 \right] \left(\frac{d}{d - 1.1} \right)^2$$

Where, d = diameter (μm), H_D = discharge hematocrit, $\eta_{0.45}^*$ = ratio of apparent viscosity of blood to plasma at a hematocrit of 0.45, C = curve fitting parameter, H_T = tube hematocrit.

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Contd... Q. No. 7

- (b) Consider application of a medication to the surface of skin, maintaining a concentration of $10 \mu\text{g}/\text{cc}$ of the medication at the skin surface. Since the molecules are removed as soon as they reach the microcirculation by a sufficiently high peripheral blood flow through skin, the inner surface of the stratum corneum is assumed to be maintained at essentially zero concentration. The thickness of stratum corneum is $1 \mu\text{m}$. The diffusivity of the medication through the stratum corneum is $10^{-10} \text{cm}^2/\text{s}$. Calculate the total amount of medication in $\mu\text{g}/\text{cm}^2$ of skin that resides in the stratum corneum at steady state. (10)
- (c) Can Murray's law be extended to a more complicated branching pattern (i.e., two daughter branches that branch in three dimensions or three daughter branches)? Explain. (5)
- 8 (a) Calculate the outflow temperature, assuming that there is fully developed blood flow through a $300\text{-}\mu\text{m}$ -long capillary. The capillary can be considered to provide the blood with a constant surface heat flux of $21000 \text{W}/\text{m}^2$. The diameter of the capillary is $15 \mu\text{m}$, and its convection heat transfer coefficient is $2500 \text{W}/\text{m}^2\text{K}$. Blood is flowing with a mass flow rate of $50 \text{g}/\text{s}$ at an inlet temperature of 25°C . (12)
- (b) A particular artery has an internal diameter of 1cm and a wall thickness of 0.75mm at an end diastolic pressure of 85mm Hg . An 8% increase in the diameter was measured for a systolic pressure of 130mm Hg . Compute the circumferential stress in the wall of the artery as well as the elastic modulus of the vessel assuming that the arterial wall is thin and made of linear isotropic elastic material. (10)
- (c) Explain the Kirchhoff's law of radiation and the general heat conduction equation for the following conditions: (8)
- i. Transient, no heat generation
 - ii. Steady state, no heat generation
-

SECTION – A

There are **FOUR** questions in this section. **Question No. 1** is **MANDATORY**.

Answer any **TWO** of the remaining **THREE** questions.

1. (a) Write down the advantages of soft lithography. Can microcontact printing be used to culture cells in specified location? How? **(11)**
- (b) You need to create a bone graft for the supporting structure of a dental implant for a patient who has periodontal disease accompanied by bone loss in the jaw and also suffers from osteoporosis (porous bone) Formulate the design considerations. **(12)**
- (c) How can the signal generated from electron - matter interactions be useful to characterize biomaterials? Illustrate your answer with supporting schematic. **(12)**
- (d) Let's consider differences engendered solely as a result of physical factors. A synthetic hydrogel is fabricated as a solid slab or as a porous structure with 30-40 μm interconnected pores. The chemical compositions of both structures are identical. Also, they have similar mechanical properties, no leachables, and no endotoxin or bacteria. Yet one heals in a capsule with the classic foreign body response (FBR), while the other heals in a vascularized, reconstructed manner with little fibrosis. Will you consider both of these structures "biocompatible"? Justify your answer. **(10)**
2. (a) What are the differences between micro-sized ceramics and nano-sized ceramics? How are these nano-sized fabricated? **(18)**
- (b) Draw the compositional diagram for bone-bonding regions. Mark different reasons like bone bonding, different non-bonding, soft tissue bonding and bioglass. **(12)**
3. (a) Suggest the name of a polymer that can be used for the following medical devices and/or applications (any three): (i) vascular graft, (ii) dialysis tubing (iii) surgical suture, (iv) contact lens, (v) breast implant. **(6)**
- (b) Mention two unconventional application of bioprinting. **(6)**
- (c) You need to manufacturer a collagen based non-woven matrix for potential wound dressing application. How would you manufacturer it? With schematic, describe the working principle of that non-woven technology. **(18)**

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4. (a) Describe *in vitor* cytotoxicity study - elution test according to ISO 10993 and USP 41. (16)
- (b) Describe ethulene oxide sterilization process. Write down the advantages and risk associated with this process. (14)

SECTION – B

There are **FOUR** questions in this section. **Question No. 5** is **MANDATORY**.

Answer any **TWO** of the remaining **THREE** questions.

5. (a) Select the metallic biomaterial that shows better mechanical properties between nanostructured steel and coarse-grained (large grain size) stainless steel and explain your reasonings? (6)
- (b) Although both aluminum and magnesium have the same atomic packing factor theoretically, one of them is more ductile than the other as observed through tensile testing of the materials. Illustrate it mathematically. (15)
- (c) With schematic, correlate the relationship between the mesh size/drug size ratio with drug release from a polymeric hydrogel network which measured by UV-Vis spectroscopy. (11)
- (d) Suppose you have made a total hip joint arthroplasty device. Demonstrate how biomaterials-based factors will affect the reliability of this device with a schematic. (7)
- (e) There are several methods that can be used to crosslink natural polymers. Describe the covalent crosslinking method for alginate and its importance in biomedical applications. (6)
6. (a) Calculate the energy for vacancy formation in nickel (Ni), given that the equilibrium number of vacancies at 850°C is $4.7 \times 10^{22} \text{ m}^{-3}$. The atomic weight and density (at 850°C) for Ni are, respectively, 58.69 g/molecular and 8.80 g/cm^3 . (10)
- (b) Why can Nitinol be used as artificial muscle for robotics applications? (5)
- (c) The density and associated percent crystallinity for two PVA materials are given in Table: Question 6(c) (15)

Table: Question 6(c)

$\rho \text{ (g/cm}^3\text{)}$	Crystallinity (%)
2.144	51.3
2.215	74.3

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Contd... Q. No. 6(c)

Computer the densities of totally crystalline and totally amorphous PVA and determine the percent crystallinity of a specimen having a density of 2.26 g/cm^3 .

7. (a) A polystyrene component must not fail when a tensile stress of 1.25 MPa is applied. Determine the maximum allowable surface crack length if the surface energy of polystyrene is 0.50 J/m^2 . Assume a modulus of elasticity of 3.0 GPa. **(6)**

- (b) The fatigue data for a ductile cast iron are given in Table: Question 7(b): **(8+6=14)**

Table: Question 7(b)

Stress Amplitude (MPa)	Number of Cycles before Failure
248	1×10^5
236	3×10^5
224	1×10^6
213	3×10^6
201	1×10^7
193	3×10^7
193	1×10^8
193	3×10^8

i. Plot the S-N curve (stress amplitude versus logarithm cycles to failure) using these data and determine the fatigue limit.

ii. Determine the fatigue life at stress amplitudes of 230 MPa and the fatigue strength at 6×10^6 cycles.

- (c) With a schematic, explain different transitions in typical polymer rheology when the temperature is increased. **(10)**

8. (a) Using the concept of LCST and UCST phase diagrams, explain why PVA and PNIPAAm can be classified as smart polymers. How can we employ this behavior of PNIPAAm in tissue engineering applications? **(12+6=18)**

- (b) In Table: Question 8(b), the molecular weight (M_i) and the mole fraction (X_i) of a chain are provided for a chitosan sample. Calculate the number average molecular weight, weight average molecular weight, and the PDI of chitosan. **(12)**

Table: Question 8(b)

Molecular weight (M_i)	Mole fraction (X_i)
120000	0.05
200000	0.16
280000	0.24
360000	0.28
440000	0.20
520000	0.07

SECTION – A

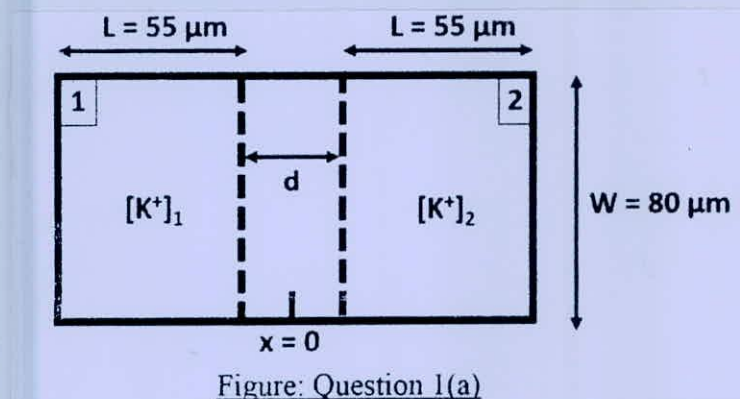
There are **FOUR** questions in this section. Question No. 1 is **MANDATORY**.

Answer any **TWO** of the remaining **THREE** questions.

1. (a) As shown in the Figure for Q. 1(a), a two-chambered monovalent bioelectrical cell is being studied by the Gao Research Group of MIT Bioengineering to gain insights on membrane characteristics. They have summed up the flux due to diffusion and electric field along x and determined that the net potassium current density at $x = 0$ is 536.28 A/m^2 .

(15)

The depth of the cell is $100 \mu\text{m}$ (out of the surface). The cell contains 2 different K^+ solutions, where $[\text{K}^+]_1 = 0.22 \text{ mM}$ and $[\text{K}^+]_2 = 0.02 \text{ mM}$. Within each chamber, the concentration and electric potential are uniform. A linear change occurs in concentration and potential across the transition region (40 \AA thick) which is hypothetically represented by the dotted lines. The only represent the position of the transition region, not a physical boundary. An external voltage, $V = \Phi_1 - \Phi_2 = -0.04 \text{ V}$ is applied between the two chambers. The laboratory temperature is 25°C and the mobility of K^+ (u_{K}) at that temperature is $7.6 \times 10^{-4} \text{ cm}^2/\text{V}/\text{sec}$.



(i) Determine the net current density (at $x = 0$) and its direction. Compare your finding with the results obtained by the Gao group.

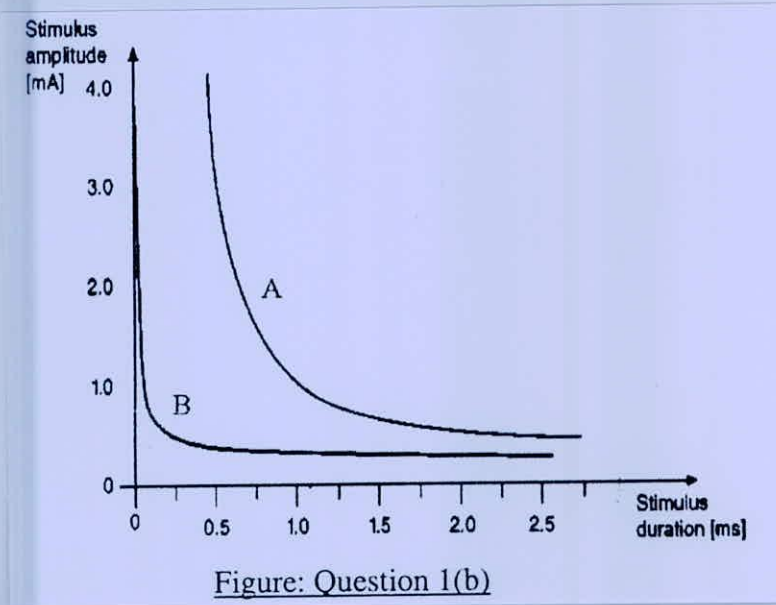
(ii) Current flow changes the concentration in two chambers, Will the system reach an equilibrium state? If so, evaluate the final concentrations in the two chambers.

(iii) Assume, the K^+ in chamber 1 is not fully dissociated ($\alpha = 95\%$). Calculate the equivalent conductance of the solution in the chamber.

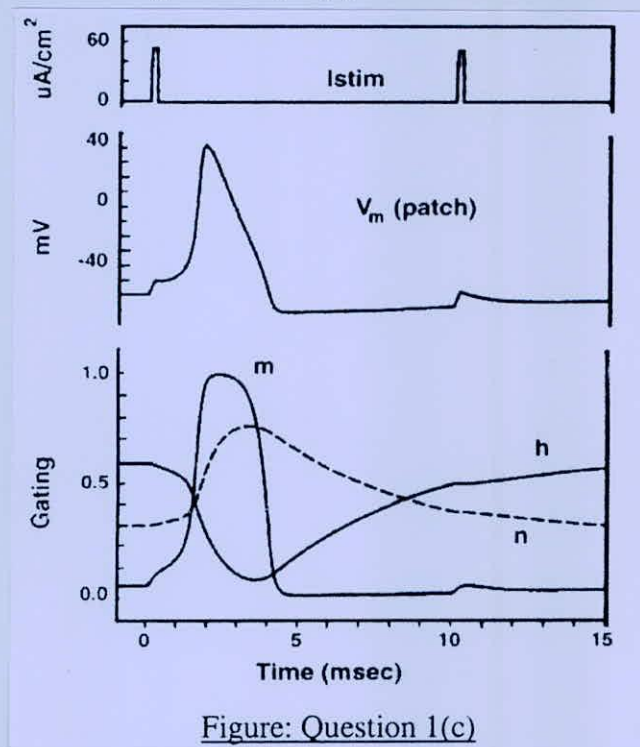
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Contd... Q. No. 1

(b) Define rheobase and chronaxie. Figure for Q. 1(b) shows two strength-duration curves denoted by A and B. A is obtained for a specialized contractile fiber found in the SAFETY node while B is obtained for a Purkinje fiber. Infer which fiber has greater excitability based on mathematical findings from the strength-duration curve. (10)



(c) Applying the knowledge of the Hodgkin-Huxley Membrane Model, explain the refractoriness of action potential with respect to temporal behaviors of the gating variables as depicted in the Figure for Q. 1(c). (10)



(d) Using Boltzmann's distribution, formulate the fraction of open ion channels as: (10)

$$\frac{[open]}{[open + closed]} = \frac{1}{1 + \exp\left[\frac{(w - z_q q_e V_m)}{kT}\right]}$$

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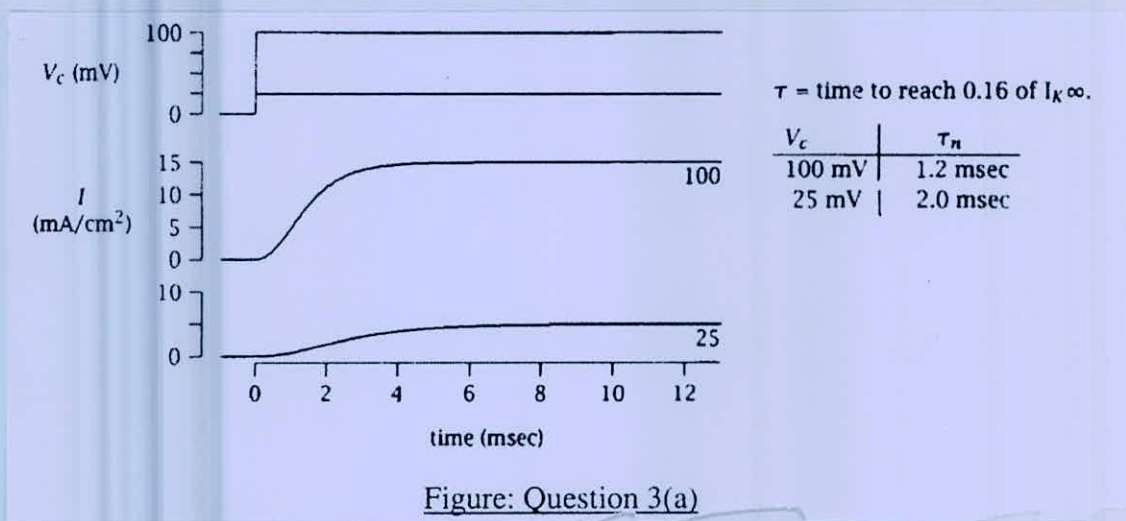
Contd... Q. No. 1(d)

where w is the energy required to open the channel when the transmembrane potential (V_m) is zero, q_e is the charge and z_q is the valence.

2. (a) In complex multi-cellular species, typically, tissues like neurons, muscle fibers etc. demonstrate excitability. Due to geometrical similarity, the response of these excitable tissues to an external stimulus is often modeled with cylindrical fibers. Suppose you are studying the microscopic electrical activity of the heart under subthreshold conditions. For an impulse stimulus to a cardiomyocyte, derive the expression of its steady-state response using the cylindrical fiber model and employing the fiber equations. Mention necessary assumptions. (22)

(b) The Nernst-Palnick Equation predicts the total flux across the cellular membrane. Use the Einstein's Relationship to show that an apparent paradox arises in the equation. Explain how this paradox is resolved. (8)

3. (a) Records of I obtained by voltage-clamping of the squid giant axon in the presence of TTX are shown in the Figure: Question 3(a). The experiment was done at two different control voltages (V_c) 25 mV and 100 mV. It is established that $\bar{g}_K = 83 \text{ mS/cm}^2$. $V_{\text{threshold}} = -20 \text{ mV}$ and $E_K = -80 \text{ mV}$. (22)



(i) From the above data and what you have learned about Hodgkin and Huxley's experiments, estimate the values of $g_{K\infty}$ at each V_c .

(ii) What are the values of n_∞ , α_n , β_n at each V_c ?

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Contd... Q. No. 3(a)

(iii) At $V_c = 25$ mV, the profile of I is defined by: $I = I_{K\infty} \left(1 - e^{\frac{-t}{1.325}} \right)$. Assume, $I_{K\infty}$

is solely associated with the opening of K^+ channels and 5 gating particles (each carries 1 electronic charge, e^-) are required to open a single K^+ channel. What is the density (in channels/cm²) of K^+ channels in this cell membrane?

(iv) Suppose another part of the axon is at a resting state ($V_{rest} = -65$ mV). If a small stimulus burst applied at correct polarity alters V_m to 30 mV, explain the membrane behavior with stimulus intensity fraction vs scaled potential (recorded potential/peak AP) plot. [Mention proper terminologies]

(b) What is core-conductor model? How does it alter for subthreshold and tranthreshold conditions? Illustrate the electrical representations with proper terminologies. (8)

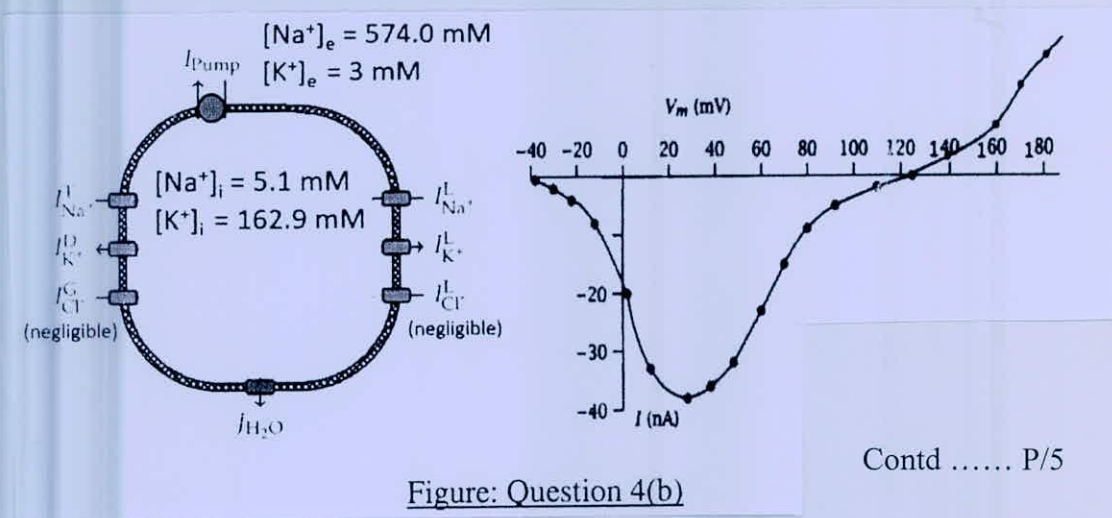
4. (a) An experimental investigation was performed to understand the behavior of Cleanup-under resting conditions by Hodgkn and Horowicz. They placed the prepared frog muscle in a normal extracellular medium. These conditions were applied: (8)

Extracellular region: $[Cl^-]_e = 120$ mM; $[K^+]_e = 2.5$ mM.

Intracellular region: $[Cl^-]_i = 2.4$ mM; $[K^+]_i = 140$ mM.

The resting potential was nominally the chloride equilibrium potential of - 98.5 mV. At $t = 0$, they rapidly reduced the extracellular chloride concentration from 120 mM to 30 mM. Explain, with necessary calculations, what was inferred from this experiment.

(b) The plot shown in the Figure: Question 4(b) is obtained from a voltage-clamped giant *aplysia* muscle cell. The cell is kept in an ionic bath at 30°C for studying the single-channel current-voltage (I-V) characteristics of the GABAA receptor pore. The intracellular and extracellular composition is also given. The receptor channel is only permeable to K^+ and Na^+ ions. The resting membrane potential is -52 mV. (11)



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Contd... Q. No. 4(b)

- (i) Define the reversal potential. Find it using the I-V plot.
 - (ii) Is it possible to determine the reversal potential by only using the information regarding cellular composition? If yes, determine the reversal potential and compare it with the value obtained in the previous question.
 - (iii) Determine the chord conductance at $V_m = +50$ mV.
 - (iv) At the time of peak inward current, calculate the permeability ratio of Na^+ and K^+ .
- (c) Nicholas is a PhD student at JHU BME. In one of his experiments, he took a Na^+ channel in an outside-out patch clamp configuration to investigate the intracellular surfaces of the ion channel. The Na^+ concentration was symmetric, i.e., the Na^+ concentration in the recording pipette and the surrounding bath was the same, 64 mM. The recording bandwidth was 12 kHz, the membrane was clamped at 40 mV and the temperature was 30°C. Experimentally, it is already established that the single-channel conductance of Na^+ channel is 187 pS. (11)

- i. Sketch the proper diagram of the outside-out patch clamp configuration. Was the configuration he chose correct for his purpose? Justify your answer.
- ii. Evaluate the minimum seal resistance between the membrane and the pipette which yields signal to noise ratio of minimum 32.

SECTION – B

There are **FOUR** questions in this section. Question No. 5 is **MANDATORY**.
Answer any **TWO** of the remaining **THREE** questions.

5. (a) Design an Functional Electrical Stimulation (FES) system for rehabilitating small muscle groups in patients with spinal cord injuries, considering all factors, such as electrode type, electrode materials, electrode-tissue interfacing conditions, waveform shape, etc. (20)
- (b) An action potential with the spatial waveform illustrated in Figure: Question 5(b) propagates along a muscle fiber. (20)

The relative transmembrane potential can be described by:

$$v_m = \begin{cases} 138.55x^2 \cdot e^{-2x/x_{peak}}, & \text{for } x \geq 0, \\ 0 & \text{for } x < 0, \end{cases}$$

where x is in units of cm and $x_{peak} = 2$ cm, the fiber has a radius of $a = 6 \mu\text{m}$ and an axoplasmic conductivity of $\sigma_i = 0.02$ S/cm.

Now, mathematically compare the strengths of sources generated by this action potential when modeled as two dipole sources versus when modeled as three monopole sources.

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Contd... Q. No. 5(b)

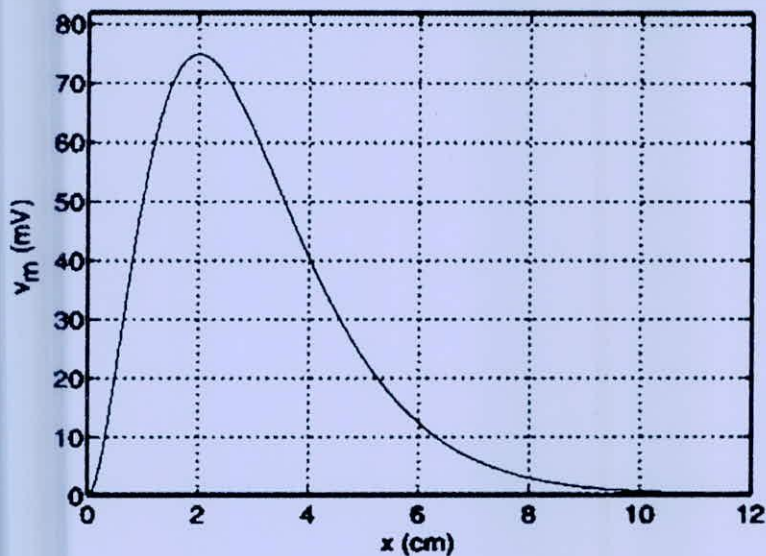


Figure: Question 5(b)

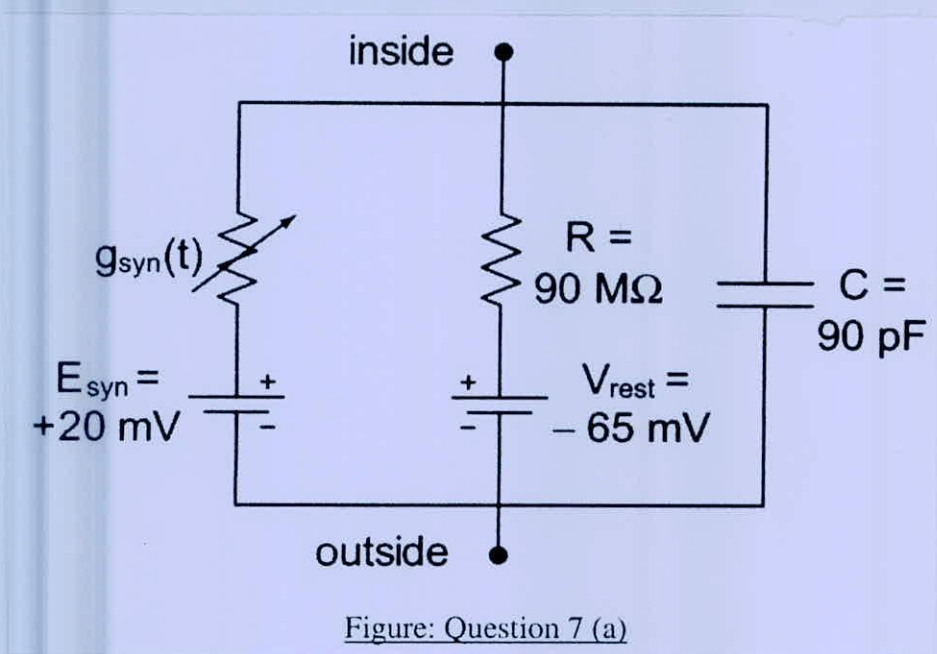
- (c) "Isotropic behavior of the tissues helps us to model the source of biological-signals." - Explain this statement. (5)
6. (a) Suppose the length of a certain cylindrical axon of a nerve cell is 8 cm long and has a radius of 8.0×10^{-6} m. Assume the thickness of the membrane is 10^{-8} m and the dielectric constant is 2.8. (15)
- (i) Calculate the capacitance of the axon of the nerve cell
- (ii) Calculate the electric charge that flows during the depolarization phase of an action potential. Assume, that in the depolarization phase, the voltage rises from -70 mV to 35 mV. You may find the formula of capacitance useful here.
- (iii) Calculate the number of ions that will flow during the depolarization phase of the action potential.
- (b) 'The propagating extracellular electrical potential changes with the distance between the source and point of measurement,' - Provide a critical analysis of this statement. Include mathematical considerations and schematic illustrations to support your analysis. (15)
7. (a) The Figure: Question 7(a) depicts the subthreshold behavior of a postsynaptic membrane. Here conductance $g_{syn}(t)$ is controlled by the reception of neurotransmitter (rather than the transmembrane potential), which has a waveform that is often approximated by a so-called alpha function: (15)

$$g_{syn}(t) = A * t * \exp\left(\frac{-t}{t_{peak}}\right)$$

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Contd... Q. No. 7(a)

- (i) Show that the conductance value of $g_{syn}(t)$ reaches its maximum value of g_{peak} when $t = t_{peak}$.
- (ii) At the time ($t = t_{peak}$), find the value of A with respect to g_{peak} and t_{peak}
- (iii) Calculate the membrane time constant τ when there is no synaptic input, i.e., $g_{syn}(t) = 0$
- (iv) Calculate the membrane time constant τ' when the synaptic input is at its maximum, i.e., $g_{syn}(t) = g_{peak} = 10 \text{ nS}$.



(b) A patch clamp is applied to a preparation consisting of two attached myocytes. The preparation is obtained through an enzymatic technique for isolating cardiac cells, a technique that yields cell pairs as well as single cells. Each cell of a cell pair is connected to a voltage-clamp circuit via a patch electrode in the whole-cell configuration. Separate voltage V_1 , and V_2 can be applied to each cell and the resulting currents, I_1 , I_2 , are measured (subscripts 1 and 2 refer to cell 1 and 2). For adult rat myocytes, the junctional (nexal) resistance is measured as $10 \text{ M}\Omega$, and the access (pipette) resistances are not significant. Calculate the lumped sarcolemmal membrane resistance for cell 2 when V_1 is kept at the holding potential of -42 mV and V_2 is pulsed at 27 mV (relative to the holding potential) for 250 ms . The measured current I_2 is 5 nA .

(15)

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8. (a) Figure: Question 8(a) shows EEG signal patterns from a patient during seizures with motor symptoms. Is it possible to determine the specific region of the brain where the seizure originates, and if so, how can this be accomplished? **(10)**

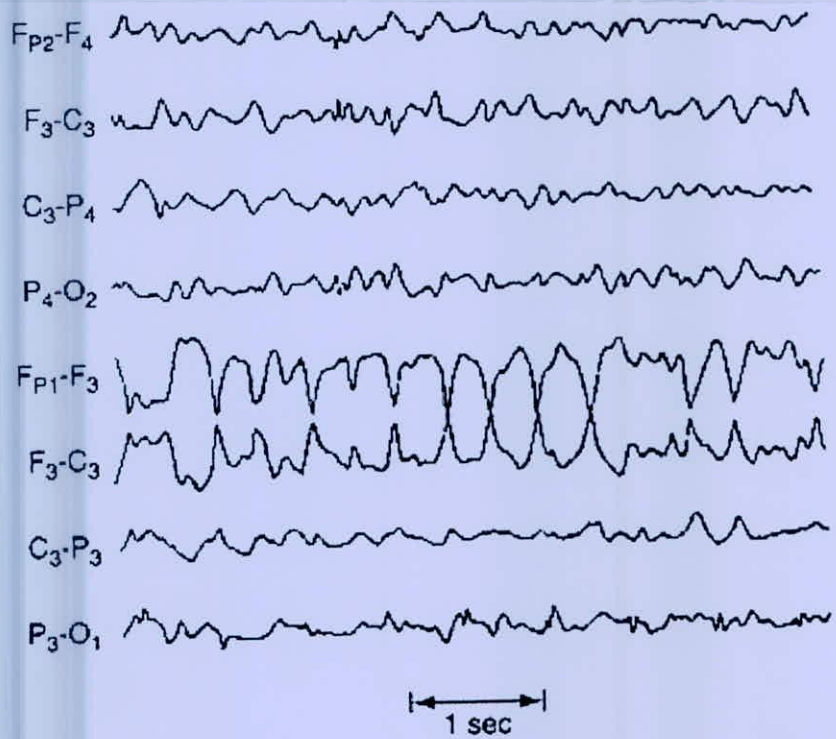


Figure: Question 8 (a)

- (b) Suppose you recorded spontaneous activity from a neuromuscular junction, and you observed the distribution of miniature EPSPs plotted in the top left of the Figure: Question 8(b). Then, under low-probability release conditions (low Ca^{++} , high Mg^{++}), you stimulated the biceps muscle nerve and observed the distribution of evoked EPSPs shown in the top right of Figure: Question 8(b). You also tested the effect of two novel drugs, A and B, and observed new distributions of evoked EPSPs (depicted in the two bottom graphs of Figure: Question 8(b). Here, the quantal content (m) is calculated as the mean EPP size (V_{avg}) divided by the mEPP, or quantal, size (q). **(20)**

Now, address the following questions:

- (i) What was the value of the quantal size according to the first experiment?
- (ii) If you measured evoked EPSP was 1.2 mV, what was the quantal content?
- (iii) Why did you observe "failures" sometimes?
- (iv) Propose potential mechanisms through which drug A and B exerted their effects.

BME 211

Contd... Q. No. 8(b)

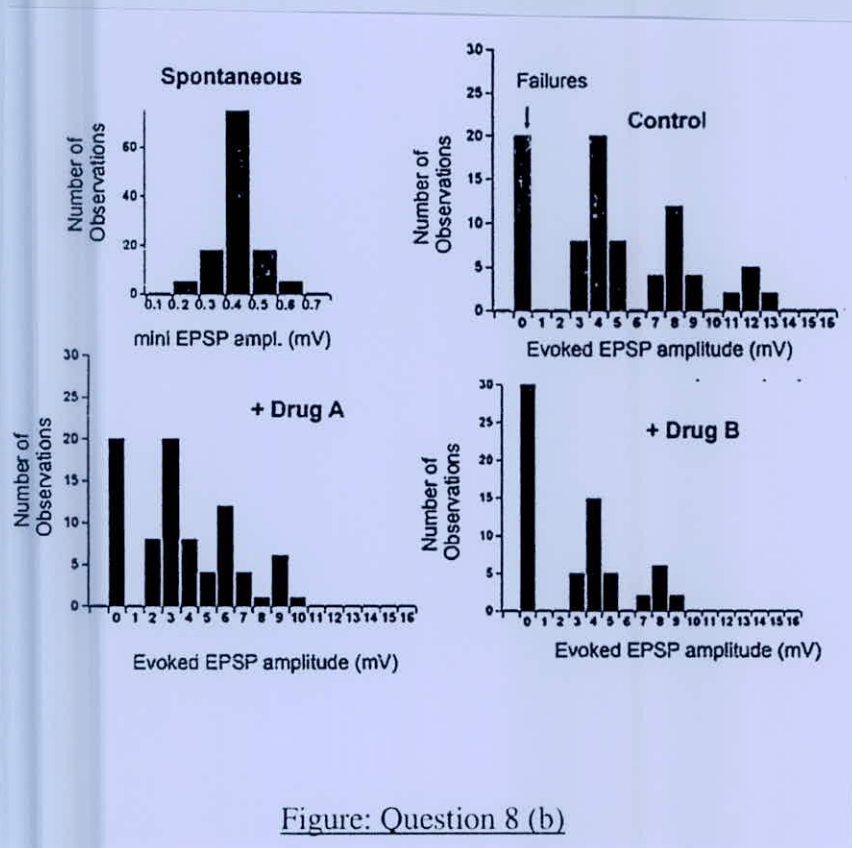


Figure: Question 8 (b)

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** Questions.
Symbols have their usual meaning

1. (a) Define Skew-symmetric, Periodic and Nilpotent matrices with examples. (5)
(b) Find an LU-decomposition of the following matrix, if possible. (15)

$$\begin{pmatrix} 6 & -2 & 0 \\ 9 & -1 & 1 \\ 3 & 7 & 5 \end{pmatrix}.$$

- (c) Find the inverse of the following matrix by using elementary row operations. (15)

$$\begin{pmatrix} 2 & 4 & 1 & 1 \\ 1 & 1 & 5 & 3 \\ 4 & 2 & 0 & 0 \\ 3 & 3 & 2 & 1 \end{pmatrix}.$$

2. (a) Solve the following system of linear equations by reducing the augmented matrix into its reduced row-echelon form and also find its rank: (17)

$$2x + 3y + 5z + t = 3$$

$$3x + 4y + 2z + 3t = -2$$

$$x + 2y + 8z - t = 8$$

$$7x + 9y + z + 8t = 0$$

- (b) Test whether the following matrix A is diagonalizable or not. If yes find a matrix P which diagonalizes A and also find the diagonal matrix. (18)

$$A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$$

Also use the diagonalizability property to find A^{13} .

3. (a) Write down the quadratic form $9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_3 - 8x_1x_2 + x_2x_3$ into matrix form. Reduce this matrix into canonical form. Hence find the rank, index and signature. (17)

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Contd... Q. No. 3

(b) State Cayley-Hamilton theorem. Verify the Cayley-Hamilton theorem and hence find the inverse of the following matrix. (18)

$$\begin{pmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

4. (a) Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ be vectors in \mathfrak{R}^2 . Verify that the weighted Euclidean inner product (17)

$$\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{9}u_1v_1 + \frac{1}{4}u_2v_2$$

satisfies the four inner product axioms. Also sketch the unit circle in an xy -coordinate system in \mathfrak{R}^2 using the weighted Euclidean inner product.

(b) Using matrix method find the currents $i_1(t)$ and $i_2(t)$ in an electrical network containing resistance $R_1 = 8$ ohms, $R_2 = 3$ ohms, inductors $L_1 = 1$ henry, $L_2 = 1$ henry and electromotive force (e.m.f) $E(t) = 100 \sin t$ volts. The currents $i_1(t)$ and $i_2(t)$ are initially zero. (18)

SECTION - B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Determine whether the following subsets are subspaces of \mathfrak{R}^4 . If so, then find a basis in each case and their dimensions. (15)

(i) all vectors of the form (a, b, c, d) , where $b + c + d = 0$.

(ii) all vectors of the form (a, b, c, d) , where $d = 2a + 7c$ and $3c = 2a - 5b$.

(b) Consider the set P_2 of all polynomials of degree 2 and the set $S = \{\underline{p}_1, \underline{p}_2, \underline{p}_3\}$ where $\underline{p}_1 = 1 + x, \underline{p}_2 = 1 + x^2, \underline{p}_3 = x + x^2$. Is the set S a basis for P_2 ? If so, then find the coordinate vector of the polynomial $\underline{p}(x) = 2 - x + x^2$ relative to the basis S . (20)

6. (a) Find the standard matrix for the transformation T on \mathfrak{R}^3 , where T is the composition of a rotation of 45° about the y -axis, followed by a reflection about the yz -plane, followed by a dilation with factor $k = \sqrt{2}$. Then find $T(2, -5, 8)$ using the standard matrix. (18)

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Contd... Q. No. 6

(b) Given $A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$. Find bases for the row space and null space of

A . (17)

Hence, verify the dimension theorem for A .

7. (a) Find a subset of the vectors (17)

$$\underline{u}_1 = (1, -1, 5, 2), \underline{u}_2 = (-2, 3, 1, 0), \underline{u}_3 = (4, -5, 9, 4), \underline{u}_4 = (0, 4, 2, -3), \underline{u}_5 = (-7, 18, 2, -8)$$

that forms a basis for the space spanned by these vectors; then express each vector that is not in the basis as a linear combination of the basis vectors.

(b) Find a basis for the orthogonal complement of the subspace of \mathbb{R}^5 spanned by the vectors $\underline{v}_1 = (1, 4, 5, 6, 9), \underline{v}_2 = (3, -2, 1, 4, -1), \underline{v}_3 = (-1, 0, -1, -2, -1), \underline{v}_4 = (2, 3, 5, 7, 8)$. (18)

8. (a) (i) Apply Gram-Schmidt process to transform the basis vectors $\underline{u}_1 = (1, -1, 0), \underline{u}_2 = (2, 0, -2)$, and $\underline{u}_3 = (3, -3, 3)$ into an orthogonal basis, (ii) Then normalize the orthogonal basis vectors to obtain the orthonormal basis, (iii) Write the QR-decomposition of the matrix A whose columns are the given vectors. (20)

(b) Let $T: P_2 \rightarrow P_3$ be the linear transformation defined by $T(p(x)) = xp(x-3)$. (15)

(i) Find the matrix for T with respect to the standard bases $E = \{1, x, x^2\}$ and $E' = \{1, x, x^2, x^3\}$.

(ii) For any $\underline{x} \in P_2$, verify the relation $A[\underline{x}]_E = [T(x)]_{E'}$, where A is the matrix for T with respect to the bases E and E' .

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** Questions.

1. (a) Explain the differences between: (4+4=8)
- (i) Synchronous and asynchronous sequential circuits
- (ii) Mealy machine and Moore machine
- (b) A sequential circuit has two JK flip-flops A and B, an input x, and an output y. The circuit is described by the following input equations: $J_A = Bx$, $K_A = A'$, $J_B = x$, $K_B = A'x$. The output equation is: $y = AB$. (5+5+4+5=19)
- i. Derive the next state equations $A(t+1)$ and $B(t+1)$.
- ii. Draw the state table
- iii. Draw the state diagram.
- iv. Assuming that the starting state is 00, what functionality is the circuit performing?
- (c) Draw the circuit diagram of a serial adder. What advantages and disadvantages does it have compared to a parallel adder? (8)
2. (a) Explain the differences between a truth table, a characteristic table, a state table, and an excitation table with examples. (10)
- (b) Draw a 3-bit synchronous up-down counter using T flip-flops. The counter should have two control inputs, up and down, to specify whether to count up or down respectively. Assume that down has higher priority than up. (13)
- (c) Design a three-bit register using D flip-flops and 4-to-1-line multiplexers. The register should perform four operations as specified by the following table: (12)

S1	S0	Operation
0	0	Shift left
0	1	No change
1	0	Parallel load
1	1	Rotate right

3. (a) Explain how TTL gates overcome the speed limitations of DTL gates with necessary figures. (8)

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Contd... Q. No. 3

(b) Consider the DTL NAND gate shown in Figure for 3(b). The diode parameters are: cut-in voltage $V_\gamma = 0.6V$, voltage across a conducting diode = $0.7V$. The transistor parameters are: cut-in voltage $V_\gamma = 0.56V$, $V_{BE(sat)} = 0.8V$, $V_{CE(sat)} = 0.2V$, and $h_{FE} = 40$.

(5+5+5=15)

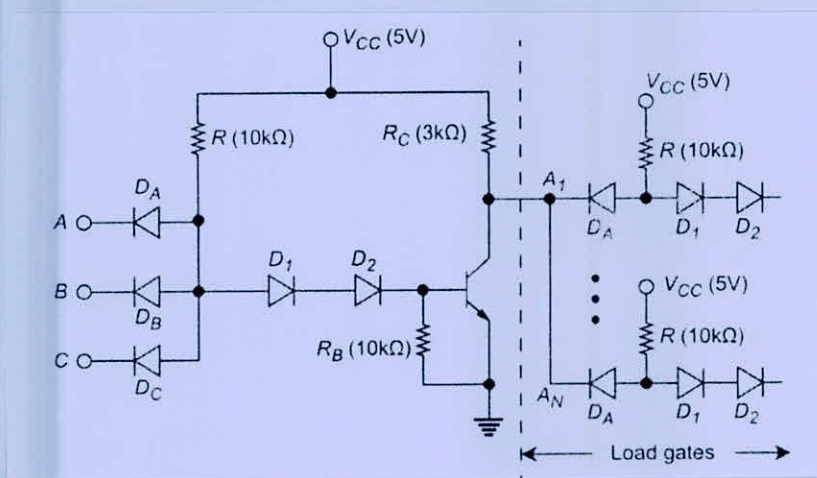


Figure for 3(b)

Now, calculate the following values of the gate:

- i. Fan-out
- ii. Noise margins
- iii. Average power dissipated

(c) Design a mod-6 ripple counter using JK flip-flops and necessary basic gates.

Provide adequate explanations to your design choices.

(12)

4. (a) Design a D flip-flop using only one JK flip-flop and one 1-to-2-line decoder.

(10)

(b) Design a sequential circuit that can detect a sequence of two or more consecutive 0's or two or more consecutive 1's in a string of bits coming through an input line. Derive the state diagram, state table, FF input equations, and the output equation.

You do not need to draw the circuit diagram.

(6+4+5=15)

(c) Consider the state diagram shown in Figure for 4(c) and answer the following questions.

(5+5=10)

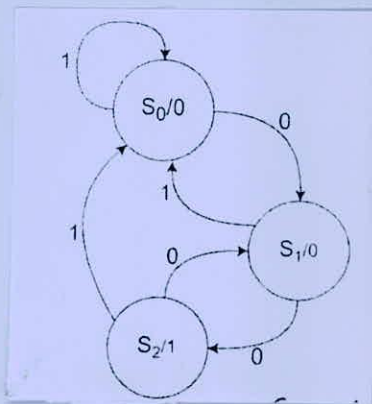


Figure for 4(c)

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Contd... Q. No. 4(c)

- i. Assuming that the machine starts in state S_0 , what functionality does the machine perform?
- ii. Convert the Moore machine to a Mealy machine.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

- 5. (a) Write the following Boolean expression in product of sums form: $a'b + a'c' + abc$ **(5)**
- (b) Show that the dual of the exclusive OR is equal to its complement. **(5)**
- (c) Write Boolean expressions and construct the truth tables describe the outputs of the circuits described by the logic diagrams in Figure for 5(c) **(10)**

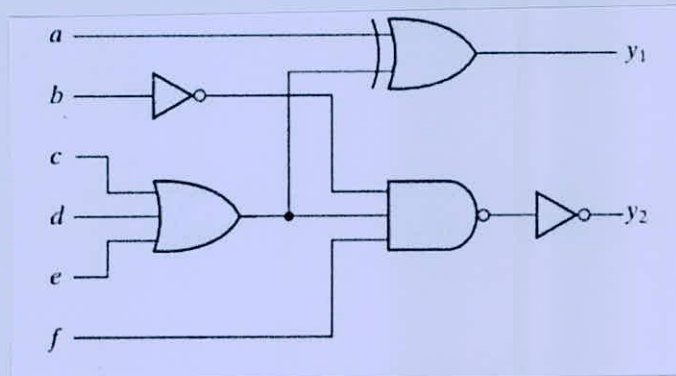


Figure for 5(c)

- (d) For the Boolean function $F = xy'z + x'y'z + w'xy + wx'y + wxy$ **(5+5+5=15)**
 - (i) Obtain the truth table of F.
 - (ii) Use Boolean algebra to simplify the function to a minimum number of literals.
 - (iii) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (i).
- 6. (a) Find the complement of the following expressions: **(10)**
 - (i) $(a+c)(a+b')(a'+b+c')$
 - (ii) $z+z'(v'w+xy)$
- (b) Simplify the following Boolean functions, using Karnaugh maps: **(10)**

$$F(w, x, y, z) = \sum(2, 3, 12, 13, 14, 15)$$
- (c) Find all the prime implicants for the following Boolean functions, and determine which are essential: **(15)**

$$F(A, B, C, D) = \sum(10, 1, 2, 5, 7, 8, 9, 10, 13, 15)$$

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7. (a) Draw a logic diagram using only two-point NOR gates to implement the following function: **(10)**

$$F(A, B, C, D) = (A \oplus B)'(C \oplus D)$$

- (b) Derive the circuits for a three-bit parity generator and four-bit parity checker using an odd parity bit. **(15)**
- (c) Implement the following Boolean expressions with three half adders: $F = ABC' + (A'+B')C$ **(10)**

8. (a) Design a combinational circuit with three inputs, x, y and z, and three outputs, A, B, and C. When the binary input is 0, 1, 2, or 3, the binary output is one greater than the input. When the binary input is 4, 5, 6, or 7, the binary output is two less than the input. **(15)**

- (b) Construct a 5-to-32-line decoder with four 3-to-8-line decoders with enable and a 2-to-4-line decoder. Use block diagrams for the components. **(10)**

- (c) Can decoder work as demultiplexer? Explain how? **(10)**
