ACOUSTICS OF RECTANGULAR FLAT PLATES

BY

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The Board of Examiners hereby recommends to the Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka, the acceptance of the thesis, "ACOUSTICS OF RECTANGULAR FLAT PLATES", submitted by Alauddin Ahmed, in partial fulfilment of the requirements for the degree of Master of Science in Mechanical Engineering.

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D E C L A R A T I O N

This is hereby declared that neither this thesis nor any part thereof has been submitted or is being currently submitted elsewhere for the award of any degree or diploma or for any publication.

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ABSTRACT

The acoustic radiation due to natural vibration of baffled rectangular flat plates of constant thickness with three different pure boundary conditions has been investigated in this thesis. The radiation efficiency of a rectangular flat plate with all four edges simply supported, has also been found to check the reliability of the method employed by comparing with the results available in the literature. Extensive numerical results on the acoustic power radiation by plates with different aspect ratios, thickness ratios and mode orders have been presented.

The functions developed by Warburton to represent the vibration of beams, have been used to apply to different cases of vibrating plates. The method of solution involves the formulation of the expression for the acoustic power radiation in terms of the farfield acoustic pressure distribution. Numerical integration of the acoustic intensity have been used to obtain the total average acoustic power radiation from one side of the plate under consideration. The Simpson rule for numerical integration has been used in the computer programming.

It has been found that, at low mode numbers, the power radiation depends upon the boundary conditions, mode numbers, aspect ratios and thickness ratios, but at high mode numbers, the effects of boundary conditions and mode numbers are almost nullified.

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CONTENTS

NOTATIONS

a	Half the length of the plate
b	Half the width of the plate
c	Velocity of sound in air
E	Modulus of elasticity of the plate material
f	Frequency of vibration of the plate
g	Acceleration due to gravity
G_x	A function of m, factor of the dimensionless frequency factor
G_y	A function of n, factor of the dimensionless frequency factor
H_x	A function of m, factor of the dimensionless frequency factor
H_y	A function of n, factor of the dimensionless frequency factor
J_x	A function of m, factor of the dimensionless frequency factor
J_y	A function of n, factor of the dimensionless frequency factor
h	Thickness of the plate
k	Acoustic wave number
k_p	Plate wave number
l	$2ak\cos\alpha\sin\theta$
m	Number of nodal lines in x-direction
n	Number of nodal lines in y-direction
P_w	Total acoustic power radiation
p	Acoustic pressure distribution
R	Distance of the receiving point from the plate center
R_a	Aspect ratio of the plate, b/a
R_t	Thickness ratio of the plate, h/a
$R_{m,n}$	Radiation resistance
$R_{m/n}$	$\pm\sin(\gamma/2)/\sinh(\gamma/2)$

r	Distance of the receiving point from the elemental area
$S_{m,n}$	Radiation efficiency
s	$2bk\sin\alpha\sin\theta$
t	Time
U	Surface velocity distribution
W	Amplitude of vibration
w	Instantaneous displacement of the plate surface
(x,y,z)	Cartesian coordinate system
γ	Roots of the equations $\tan(\gamma/2)\pm\tanh(\gamma/2)=0$
Ψ	Wave number ratio, k/k_p
λ	Wavelength of the sound emmitted
λ_f	Dimensionless frequency factor
π	A constant
ρ	Density of the surrounding medium
ρ_m	Density of the plate material
σ	Poison's ratio of the plate material
ϕ	Velocity potential
$\theta(x)$	Displacement function in x-direction
$\theta(y)$	Displacement function in y-direction
(R,θ,α)	Cylindrical coordinate system
ω	Circular frequency of plate vibration

CHAPTER -1

INTRODUCTION

1.1. GENERAL:

Within the last few years, concern about the protection of environment has grown rapidly as it has become generally recognized that the steady rise in pollution of all kinds can not be allowed to continue indefinitely. With the growth of mechanization in modern life, the problem of noise is also growing steadily. The escalation in number, size and complexity of machines, increasing road traffics etc. are contributing to the higher level of noise and hence degrading the quality of life. The problem is already recognized as one of considerable importance and demands immediate attention. To combat this problem, many countries and communities have recently introduced legislations making it a legal requirement to measure community noise level, to reduce noise from vehicles at source and to maintain acceptable noise levels in factories to prevent hearing loss.

This activity has led to a greater appreciation of the benefits of quiet environment and a preference for quieter domestic equipments, if these are available. The quieter item therefore, often has a sales advantage over its more noisy competitor which may be reflected in the command of a higher price. Economic advantages are also apparent in property values which are lower in noisy areas than in quiet areas.

The cost of insulating against noise must also be considered. The control of noise is therefore, of importance not only in the prevention of hearing damage and in providing an acceptable acoustic environment but also from economic point of view.

1.2. SOURCES AND MECHANISM OF PRODUCTION AND PROPAGATION OF SOUND:

To identify the sources of sound and to understand the mechanism by which it is produced and how it travels from one point to another is the first prerequisite to the study of acoustics. Noise is caused by the vibration of solid, liquid or gaseous medium. When this vibration is within the range of audible frequency of the human beings, a human ear can perceive the noise, otherwise, they go unnoticed. When a solid body vibrates at a frequency within the audible frequency range, a part of the energy dissipated is transmitted to the surrounding environment as perceivable sound. Energy is transferred from one vibrating particle to the next and the acoustic energy travels through the surrounding medium as longitudinal waves. In the current study, attention is concentrated to the amount of acoustic energy dissipated as noise due to natural vibration of plane rectangular plates of constant thickness.

1.3. MOTIVATION BEHIND THE SELECTION OF THE PROBLEM:

In a solid structure, the dominant sound generating mechanism is attributed to the mechanical vibration of the system. The structure in this analysis is taken to be a simple plane rectangular plate of constant thickness vibrating in flexure. In everyday life, many problems are encountered with unwanted sound. Many of these noise sources are in the form of flat plates. The windows, walls and floors of buildings, the exposed surfaces of large machines, the walls of air-conditioning ducts, the air plane wings are some of the examples of such noise radiators. To control noise produced by these structures, it is always very convenient to become familiar with the idealized problems.

Investigations into the interaction of acoustic field and vibrating structure have demonstrated the importance for a greater understanding of energy radiation from vibrating structures. Numerous attempts are made in the past to evaluate the noise characteristics of different vibrating sources by direct measurement. Continued efforts in this direction has also led to the development of very sophisticated sound measuring devices. But an exact analysis of the noise radiation from flexural vibration of a plate vibrating in its natural modes under different edge conditions is yet to be achieved.

1.4. OBJECTIVES OF THE STUDY:

In this analysis, attempt is made to study the noise generating characteristics of a rectangular flat plate in an infinite baffle. The plate is studied under three different boundary conditions. These are: (i) all four edges of the plate are simply supported, (ii) all four edges of the plate are clamped and (iii) all four edges of the plate are freely suspended. These three boundary conditions of the plate are herein referred to as pure boundary conditions in order to differentiate them from large number of mixed boundary conditions. The objectives of the current study can be outlined as follows.

(i) The development of appropriate displacement functions that satisfy the boundary conditions of the rectangular plates vibrating in flexure.

(ii) Derivation of an expression for the natural frequency of the vibrating plate for different mode shapes.

(iii) Evaluation of the frequency of vibration for a particular case and study its variation with the mode numbers.

(iv) Derivation of an expression for the power radiated due to natural vibration of the plate under consideration.

(v) Evaluation of the magnitude of the average power radiated from one side of the baffled plates by the method of numerical integration of the expression for the power radiated. For this purpose, a computer program is to be developed.

(vi) To study the variation in the radiated power from the plate with the variation of mode numbers, aspect ratio, and thickness ratio under different boundary conditions.

(vii) Determination of the radiation efficiency of the simply supported plate and plotting the same against the wave number ratio for comparing the results of the current investigations with those of the previous works in order to verify soundness of the present analysis and to ascertain the absence of any mistake in the numerical procedure employed here.

1.5. DEFINITION OF TERMS:

Some of the terms used in this thesis are defined here in order to remove ambiguity in their use and to attach precise meaning to them.

(a) ACOUSTIC PRESSURE:

Sound travels as a wave of compression and rarefaction with an associated wave of pressure variation. In most practical problems, it is the pressure variation that is of greater importance and greater interest. The acoustic pressure at any point is the difference between the actual pressure at that point in the presence of the sound and the pressure that would exist at that point under identical conditions in the absence of any sound. This acoustic over pressure at any point varies sinusoidally with time exactly as an electrical current and exactly as in electrical measurements, it is convenient to use the r.m.s. value.

(b) ACOUSTIC INTENSITY:

Perhaps the most basic quantity with which one is closely concerned is sound power. This is associated with the actual source of sound. The source radiates power which is transmitted in the form of sound. The sound power of a source is the total power coming from it. It is the rate at which energy in the form of sound leaves the source.

For defining acoustic intensity, a point at some distance from a source of sound and a small area perpendicular to the line joining the point to the source has to be considered. Some of the power being generated by the source will be transmitted through the area; the exact amount depends not only on the sound power of the source, but also on its directional properties, the distance of the area from the source and the presence of sound absorbing and sound reflecting materials. If the power passing through the area A is W, then the acoustic intensity I is the power passing through a unit area or
$I=W/A$.

(c) RADIATION RESISTANCE:

While calculating the displacement of a solid due to an applied force, the effects of the presence of fluid must be taken into consideration. This effect may be represented by introducing the idea of radiation resistance.

The radiation resistance is the equivalent of the frictional forces tending to damp out the vibration. It provides one of the means of removing the energy originally applied to the solid. Some of the applied energy must be used to overcome the purely mechanical resistance to motion and in some cases electrical resistance. This portion of the energy is converted into heat. Only a proportion of the applied energy is finally radiated as acoustic energy. This last part is regarded as overcoming the radiation resistance. The advantages of the concept of radiation resistance is that the whole of the applied energy may be treated as having to overcome a series of resistances, namely, electrical, mechanical and radiation resistance.

The concept of radiation resistance is useful in calculating the acoustic efficiency of a source. The efficiency of a source of sound is defined as the ratio of the energy radiated as sound to the whole of the vibrational energy applied to the source. However in this thesis the radiation resistance of a source is defined as (68),

$R_{mn}=P_w/\langle|u_m|^2\rangle$, where,

P_w is the total average acoustic energy radiated by the solid and $\langle|u_m|^2\rangle$ is the temporal and spatial average of the square of the surface velocity.

CHAPTER -2

LITERATURE REVIEW

2.1. INTRODUCTION:

The history of the study of vibration of flat plates dates back to as early as the 19th century. The first person to begin the study of the dynamical behavior of structures was probably Rayleigh, who, in 1889, developed the fundamental equations governing the vibration of plates. But the problem did not get significant importance until the middle of the 20th century. During the middle of the 20th century, people began to consider noise as a source of environmental pollution and consequently the study of vibration was initiated. Yet, the study of the acoustic radiation characteristics of flat plates has not yet got the full momentum.

2.2. VIBRATION OF PLATES:

Within the last few decades, intensive work has been reported on vibration analysis of plates. Most of the initiators confined their study to the determination of the frequency of vibrating plates.

Toshiyuki Sakata(51) derived an approximate formula for the estimation of the fundamental natural frequency of the simply-supported orthotropic rectangular plates with thickness varying linearly in one direction. The accuracy of the formula and the influence of the flexural rigidity D on the natural frequency was also discussed. Approximate values of the natural frequency of an isotropic rectangular plate with thickness varying in one direction were reported by Apple and Byers(1), Gontkevich(20) and Soni and Rao(59). Apple and Byers calculated the upper and lower bounds for the fundamental natural frequency of the simply-supported plates. Gontkevich derived approximate expression for calculating the natural frequency of the plate with various boundary conditions by use of finite difference method.

John Hunt, Max Knittel and Don Branch(28) jointly reported an approximate method for solving the equations of motion that describe the vibration of an elastic structure immersed in an infinite acoustic fluid medium. The mathematical model that was developed used the finite element method to calculate the vibrational characteristics of the elastic body and the acoustic pressure field of that portion of the fluid which closely surrounds the vibrating structure. Analytical methods were used to obtain the boundary conditions for the mathematical model. Claassen and Thorne(4) presented four graphs giving the first ten vibration frequencies of a clamped rectangular plate as a function of the ratio of sides and one graph of nodal lines to illustrate the transition from one mode of vibration to another. Similar results for a rectangular plate clamped on two opposite edges and free on the other two edges as well as a table, discussions of convergence and complete mathematical model in both boundary value problems were reported by the same authors(5). The results of these investigations (4 and 5) were found to agree upto an accuracy that could be expected with the previous results of similar investigations (65,71), as checked by the authors.

Dickinson(7) extended the sine series solution, previously used for the study of the flexural vibration of rectangular isotropic plates, to freely vibrating orthotropic plates. The author presented the application of the method to three particular plates with different support conditions and also the numerical results of two of these examples. The author used the sine series solution developed by Dill and Pister(8). Warburton(70) derived approximate expressions for the frequencies of all the modes of vibration of isotropic plates subjected to any combination of free, simply supported or clamped edges. He applied the Rayleigh method, assuming that the deflections of the plates could be represented by suitable characteristic functions satisfying the boundary conditions(72). In his analysis, the author first developed a set of beam functions satisfying the edge conditions of beams and applied those results to the vibration of plates. He expressed the frequency in terms of a dimensionless frequency factor which in its turn is a function of the mode shapes. The factors of the frequency factor for different combinations of simply supported, clamped or freely suspended edge conditions are given in the form of a table. He also discussed the accuracy of the approximate frequency expression and the existance of the modes m/n+n/m. Hearmon(23) extended the treatment presented by Warburton to the orthotropic plates with any of its edges either clamped or simply supported, that is to a plate made of a material possessing three mutually perpendicular axes of symmetry, two of which lie in the plane of the plate parallel to the respective sides. The third symmetry axis is normal to the other two and is therefore perpendicular to the plane of the plate. The results of his study (23) were found to be satisfactory except that the exact boundary conditions could seldom be realized in experimental work(24).

Lin (38) studied the free vibration of plates, stiffened with many stingers. The differential equation was solved for an individual plate and the solution for an individual plate were coupled with the boundary conditions. Duffield and Williems (9) analyzed the parametric resonance of a plate using the energy method. Saito and Suzuki (50) performed an analytical evaluation of the viscoelastic beam effect on plate vibration characteristics. Ohtomi (44) reported the analytical study of the free vibration of a simply supported rectangular plate, stiffened with viscoelastic beams. The effects of the volume and number of stiffening beams were clarified.

Goreman (15) conducted free vibration analysis for all plates with combinations of clamped-simply supported edge, excepting those with two opposite edges simply supported. He discussed the vibration of plates with opposite edges simply-supported in a separate paper (16). The author also introduced the analysis of free vibration of rectangular plates in his earlier works (17). A thorough analysis of the cantilever plates by the method of superposition was also reported by the author (18). Laura, Ercoli, Cortinez and Padin de Iriso (35) studied the transverse vibration of rectangular plates, continuous in two directions. The author used the Rayleigh-Schmidt methodology coupled with the use of polynomial coordinate functions. The frequency values obtained by these authors were very close to those obtained by Leissa (36).

For problems of solid circular plates, analytical solutions was presented by Wah (67)and the Ritz and Galekin solution have been obtained by Laura and others (33,34,35) to calculate the fundamental frequency and buckling loads. Pardoen (45,46) dealt with the problem by finite element approach. Jain (30) and Gupta and Lal (21) analyzed circular plates of variable thickness by the Frobenius method and this problem was also solved by the Ritz method (66). Narita (43) made an attempt to develop a general solution procedure for the vibration and stability analysis under arbitrary distribution of inplane forces.

2.3. ACOUSTIC RADIATION FROM PLATES:

Though intensive research works have been performed to study the vibration of plates and plate-like structures, the study of the acoustic radiation due to vibration of flat plates is still in its infancy. The acoustic radiation from elastic structures has occupied acousticians interested in radiated noise. Many structures are either large compared to the wave length or highly damped so that outgoing waves do not reflect from boundaries, effectively making the structure infinite in extent.

The first solution to the radiated power from an elastic plate, modeled by classical plate theory, was obtained by Skudrzyk (56,57) and Heckl (25,26,27) for a time harmonic point force and by Thompson and Rattaya (63) for time harmonic point moment. The solutions for the acoustic radiated pressure from a point excited plate using the classical theory was given by Gutin (22), Feit (10) and Skudrzyk (57). The influence of fluid loading on the radiation from elastic plate was investigated by Maidanik and Kerwin (42). All of these investigations employ classical plate theory. Such theory fails at high frequencies where the phase and group velocities become infinite as the frequency become unbounded. Such a plate theory is useful only for frequencies where the ratio of the wavelength to the plate thickness exceeds eight.

To improve the high frequency prediction of elastic plates, Feit (11) employed the Timoshenko-Miudlin theory for such a prediction. This theory adds shear deformation and rotary inertia to the classical flexural theory. There are two dispersion curves for this plate. The flexural branch (acoustic) has phase and group velocities approaching the Rayleigh velocity of the plate material as the frequency increases. Stuart (60,61,62) explored the solutions for the same plate with new insights into the leaky wave emanating from the plate and obtained a more accurate solution when the angle of observation approaches the coincidence angle. This new solution would be accurate at closer observer distance than Feit's (11).

Sound radiation from beam reinforced plate, excited by point or line forces, has been investigated by a few authors. Romanov (49) obtained the solution of the radiated pressure from a plate reinforced with beam and excited by a line force. Feit and Saurenman (12) analyzed the acoustic radiation of a point excited plate reinforced with a beam, but confined their interests to high frequencies. Gorman (19) obtained the solution for a plate reinforced by many beams and excited by a line force parallel to the beam. His solutions thus makes the beams' reaction on the plate purely as rotary and transverse impedences without flexural wave travelling in the beams. Garrelick and Lin (13) analyzed the radiation from a beam reinforced plate and confined their attention to an on-axis response. Lin and Hayek (37) obtained the exact solution for the radiation from a point excited plate reinforced by one beam, which is valid in the entire acoustic space.

The transient acoustic radiation from a plate under the influence of time dependent point forces was investigated by Magrab and Reader (40) and Stuart (60). Magrab and Reader predicted the time signature of radiated pressure from a sinusoid phase. However, the solution was valid only after the acoustical arrival and there was an analytical error in the choice of the complex poles of the solution. Stuart predicted the impulse response of an elastic plate. His formulation accounts for shear and rotary inertia of the plate as well as the fluid loading of the acoustic medium. The resulting farfield radiated pressure as determined from the standard saddle point method was obtained for times before and after the acoustic arrival. However, the origin of the first arrival was not predictable from his solution. Furthermore, he predicts the solutions for the acoustic pressure after the acoustic arrival time to be a monotonically decaying solution. However, the author supposed that it is more physically reasonable to assume that the plate will vibrate freely, generating a decaying sinusoid time signature. Seroj, Mackertich and Sabih (53) reviewed the acoustic radiation from an infinite elastic plate. The author considered only the infinite elastic plate due to the fact that many structures are large compared to the wavelength or highly damped so that outgoing waves do not reflect from boundaries effectively making the structure infinite in extent.

Wallace (69) found the radiation resistance from the far-field pressure distribution produced by a baffled beam, vibrating with simple harmonic motion in one of its natural modes. He considered beams hinged at each end and clamped at each end. The author derived expressions for the radiation resistance which are assymptotic to the exact solution as the frequency approaches zero. In addition, numerical integration of the farfield acoustic intensity was used to obtain graphs covering the entire frequency range for the first ten modes of the beam. In another paper (68) the same author determined the radiation resistance corresponding to the natural modes of a finite rectangular panel supported in an infinite baffle. He used the appropriate beam functions given by Warburton (70). But his analysis was confined to simply-supported plates only.

It is found that not much work has been done to analyze the acoustic radiation characteristics of rectangular flat plates vibrating in its natural modes. Till now, no development is reported in the literature on the determination of the exact amount of acoustic radiation due to flexural vibration of rectangular flat plates.

CHAPTER -3

FORMULATION OF THE PROBLEM

3.1. INTRODUCTION:

In this analysis, a uniform, elastic, plane rectangular plate of size 2ax2b is assumed to be contained in an infinite baffle. The baffle prevents the movement of air around the edges of the plate and permits radiation into the half spaces in front of either of the surfaces of the plate. The plate may have any combination of simply supported, clamped or freely suspended conditions at the edges. This analysis will be confined only to the pure boundary conditions. The pure boundary conditions are: (i) all four edges of the plate simply supported, (ii) all four edges of the plate clamped and (iii) all four edges of the plate freely suspended. The plate, along with the coordinate systems used in the analysis is shown in figure 1. Each of the boundary conditions will be dealt with in separate sections. But only a general mathematical model will be presented in this chapter.

3.2. ASSUMPTIONS:

The analysis that follows in the subsequent sections is based on the following assumptions:

(i) The plate is thin and of uniform thickness h; thus the free surfaces of the plate are the planes $z=\pm(h/2)$.

(ii) The direct stress in the transverse direction is zero. This stress must be zero at the free surfaces and provided that the plate is thin, it is reasonable to assume that it is zero at any section z.

(iii) The stresses in the middle plane of the plate (membrane stress) are neglected, that is, transverse forces are supported by bending stresses, as in flexure of a beam. For membrane action not to occur, the displacement must be small compared with the thickness of the plate.

(iv) Plane sections that are initially plane and normal to the middle plane remain plane and normal to it.

(v) Only the transverse displacement has to be considered.

3.3. MATHEMATICAL MODELING:

(A). DETERMINATION OF THE POWER RADIATED:

When excited, the plate will vibrate with simple harmonic motion in one of its natural modes. The instantaneous transverse displacement at a point (x,y) on the surface of the plate, corresponding to the (m,n)th mode of vibration, is given by (70)

$$w_{mn}=W_{mn}\ \theta(x)\theta(y)e^{i\omega_{mn}t} \qquad \text{.....................(3.1)}$$

where, W_{mn} is the amplitude of transverse displacement and ω is the natural angular frequency of vibration of the plate corresponding to the (m,n)th mode and t is the time. $\theta(x)$ and $\theta(y)$ are the displacement functions describing the wave form of the vibrating plate and satisfying the conditions at the edges.

The motion of the plate surface which generates the acoustic radiation is given by the normal velocity distribution,

$$U=\frac{dw_{mn}}{dt}=i\omega_{mn}W_{mn}\ \theta(x)\ \theta(y)e^{i\omega_{mn}t} \qquad \text{........................(3.2)}$$

The approximate expression for the velocity potential, $d\Phi$ of the elemental area dA, radiating in only one direction is given by,

$$d\Phi=\frac{U}{2\pi r}e^{ikr}\ dA \qquad \text{..................................(3.3)}$$

where U is the normal vibrational velocity of the element dA, r is the distance of the receiving point (R,θ,α) of sound wave from the element and k is the acoustic wave number, $2\pi/\lambda$, where λ is the wave length of the sound emmitted.

The elemental acoustic pressure, dp at the receiving point (R,θ,α) due to the elemental area dA is given by the relation

$$dp = ik\rho\, c d\phi \qquad \text{.............................(3.4)}$$

where, ρ is the density of the medium surrounding the plate. Substitution of equation (3.3) into equation (3.4) gives,

$$dp = \frac{ik\rho cU}{2\pi r}\, e^{ikr}\, dA \qquad \text{...........................(3.5)}$$

where, c is the velocity of sound waves in the medium surrounding the plate.

If the receiving point (R,θ,α) is located in the farfield, then the distance r of the receiving point from the elemental area dA can be expressed with first approximation as,

$$r \simeq R-(x\cos\alpha+y\sin\alpha)\sin\theta \qquad \text{......................(3.6)}$$

The second term $[x\cos\alpha+y\sin\alpha)\sin\theta]$ in equation (3.6) is very small compared to the first and its effect can be neglected in the amplitude factor of equation (3.5). However, both terms are equally significant as far as the phase factor of equation (3.5) is concerned.

Substituting equations (3.2) and (3.6) in equation (3.5) and neglecting the second term in the expression of r in the amplitude factor of equation (3.5), it can be shown that,

$$dp=\frac{-\rho c k\omega_{mn}W_{mn}}{2\pi R}\,\theta(x)\theta(y)e^{i\{\omega_{mn}t+kR-k(x\cos\alpha+y\sin\alpha)\sin\theta\}}\,dA. \qquad \ldots\ldots(3.7)$$

Thus the net acoustic pressure at the point (R,θ,α) due to the (m,n)th mode of vibration of the plate is given by,

$$p=\frac{-\rho c k\omega_{mn}W_{mn}}{2\pi R}e^{i(\omega_{mn}t+kR)}\int_{-a}^{a}\int_{-b}^{b}\theta(x)\theta(y)e^{-ik(x\cos\alpha+y\sin\alpha)\sin\theta}\,dxdy. \qquad \ldots\ldots(3.8)$$

Substituting $l=2ak\cos\alpha\sin\theta$ and $s=2bk\sin\alpha\sin\theta$, into equation (3.8), it is found that the equation takes the form,

$$p=\frac{-\rho c k\omega_{mn}W_{mn}}{2\pi R}e^{i(\omega_{mn}t+kR)}\int_{-a}^{a}\int_{-b}^{b}\theta(x)\,\theta(y)e^{(-ilx/2a)}e^{(-isy/2b)}\,dxdy. \qquad \ldots\ldots(3.9)$$

The total average power radiated from one side of the baffled plate, found by integrating the farfield acoustic intensity over the hemispherical surface is,

$$P_w=\int_0^{2\pi}\int_0^{\pi/2}\frac{|p|^2}{\rho c}R^2\sin\theta\,d\theta\,d\alpha \qquad \ldots\ldots(3.10)$$

where $|p|^2$ is the square of the root mean square value of the net acoustic pressure.

(B). DETERMINATION OF NATURAL FREQUENCY:

The natural frequency of the plate ω_{mn} that corresponds to the transverse mode of vibration (m,n) can be determined in terms of the boundary conditions, the nodal patterns, the dimensions of the plate and the constants of the plate material. Warburton (70) derived the expression for the natural frequency from the energy equation. He expressed the natural frequency in terms of a dimensionless frequency factor, λ_f, given by,

$$\lambda_f^2 = G_x^4 + G_y^4 (a/b)^4 + 2(a/b)^2 \{\sigma H_x H_y + (1-\sigma) J_x J_y\} \qquad \text{.............(3.11)}$$

and the relation between ω_{mn} and λ_f^2 is given by,

$$\lambda_f^2 = \frac{\rho_m a^4 \omega^2 12(1-\sigma^2)}{\pi^4 E h^2 g} \qquad \text{................................(3.12)}$$

where, ρ_m is the density of the plate material, a,b are the dimensions of the plate, σ is the Poisson's ratio and E is the modulus of elasticity of the plate material, h is the thickness of the plate and g is the acceleration due to gravity. The quantities G_x, G_y, H_x, H_y, J_x and J_y are the functions of boundary conditions and mode shapes. The values of these quantities are given by Warburton (70) in the form of a table. In this thesis, the expression for natural frequency given by Warburton (70) will be used. The values of these quantities for the boundary conditions under consideration, are given in the form of a table in the following page as a ready reference.

Table giving the values of the terms of the frequency factor expression.

Boundary Condition	m	G_x	H_x	J_x
Simply-Supported	2,3,4,..	m-1	$(m-1)^2$	$(m-1)^2$
Clamped	2	1.506	1.248	1.248
	3,4,5,..	m-0.50	$(m-0.5)^2\left[1-\frac{2}{(m-0.5)^2\pi}\right]$	$(m-0.5)^2\left[1-\frac{2}{(m-0.5)^2\pi}\right]$
Freely-Suspended	2	1.506	1.248	5.017
	3,4,5,..	m-0.50	$(m-0.5)^2\left[1-\frac{2}{(m-0.5)^2\pi}\right]$	$(m-0.5)^2\left[1+\frac{6}{(m-0.5)^2\pi}\right]$
	n	G_y	H_y	J_y
Simply-Supported	2,3,4,..	n-1	$(n-1)^2$	$(n-1)^2$
Clamped	2	1.506	1.248	1.248
	3,4,5,..	n-0.50	$(n-0.5)^2\left[1-\frac{2}{(n-0.5)^2\pi}\right]$	$(n-0.5)^2\left[1-\frac{2}{(n-0.5)^2\pi}\right]$
Freely-Suspended	2	1.506	1.248	5.017
	3,4,5,..	n-0.50	$(n-0.5)^2\left[1-\frac{2}{(n-0.5)^2\pi}\right]$	$(n-0.5)^2\left[1+\frac{6}{(n-0.5)^2\pi}\right]$

(C). DETERMINATION OF RADIATION RESISTANCE:

The concept of radiation resistance is very useful in calculating the radiation efficiency of a vibrating system. Though the main objective of the present investigation is to determine the actual amount of acoustic energy radiated from one side of the plate, the radiation efficiency of a plate with simply supported boundary condition is determined here for the purpose of comparing the results of the present study with those of the previous works (68,69). For this purpose, it is required to formulate the expression for radiation resistance.

The radiation resistance is defined by C.E.Wallace (68,69) as,

$$R_w = P_w / \langle |u_w|^2 \rangle \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.13)$$

where P_w is the total average power radiated from one side of the baffled plate and $\langle |u_w|^2 \rangle$ is the average of the temporal and spatial factor of the square of the surface velocity, given by,

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a} \int_{-b}^{b} (1/2) u_w^2 \, dx dy \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.14)$$

The expression for u_w can be obtained from equation (3.2) as,

$$u_w = i \omega_{mn} W_{mn} \ \Theta(x) \ \Theta(y) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.15)$$

(D). DETERMINATION OF RADIATION EFFICIENCY:

C.E.Wallace, in his paper (68,69) introduced the concept of radiation efficiency with a view to simplify the matters. The main idea behind the introduction of the concept of radiation efficiency was to eliminate the dependence on the impedance of the acoustic medium and the plate size while calculating the acoustic radiation from vibrating plates. The radiation efficiency is defined as,

$$S_{mn}=R_{mn}/4\rho cab \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.16)$$

Combining equations (3.15) and (3.16) the expression for radiation efficiency becomes,

$$S_{mn}=P_w/4\rho cab\langle|u_w|^2\rangle \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.17)$$

3.4. DISPLACEMENT FUNCTIONS:

Warburton (70) found a number of beam functions for describing the displacement of a vibrating beam under different combinations of end conditions. He also used the functions to represent the vibration of rectangular flat plates. In this solution, the same beam functions, slightly modified to satisfy the boundary conditions, are used. The modified beam functions, which are henceforth referred to as displacement functions, for different boundary conditions are:

(i) Simply supported Plates:

$$\theta(x)=\mathrm{Sin}\frac{(m-1)\pi(x+a)}{2a}, \quad m=2,3,4,\ldots, \quad -a\leqslant x\leqslant a$$

$$\theta(y)=\mathrm{Sin}\frac{(n-1)\pi(y+b)}{2b}, \quad n=2,3,4,\ldots, \quad -b\leqslant y\leqslant b$$

(ii) Clamped Plates:

$\theta(x)=\mathrm{Cos}(\gamma_m x/2a)+R_m \mathrm{Cosh}(\gamma_m x/2a)$, $m=2,4,6,\ldots$ $-a\leqslant x\leqslant a$

where, $R_m=\mathrm{Sin}(\gamma_m/2)/\mathrm{Sinh}(\gamma_m/2)$ and γ_m are the roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(x)=\mathrm{Sin}(\gamma'_m x/2a)+R'_m \mathrm{Sinh}(\gamma'_m x/2a)$, $m=3,5,7..$ $-a\leqslant x\leqslant a$

where $R'_m=-\mathrm{Sin}(\gamma'_m/2)/\mathrm{Sinh}(\gamma'_m/2)$ and γ'_m are the roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Cos}(\gamma_n y/2b)+R_n \mathrm{Cosh}(\gamma_n y/2b)$, $n=2,4,6,\ldots$ $-b\leqslant y\leqslant b$

where, $R_n=\mathrm{Sin}(\gamma_n/2)/\mathrm{Sinh}(\gamma_n/2)$ and γ_n are the roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Sin}(\gamma'_n y/2b)+R'_n \mathrm{Sinh}(\gamma'_n y/2b)$, $n=3,5,7..$ $-b\leqslant y\leqslant b$

where, $R'_n=-\mathrm{Sin}(\gamma'_n/2)/\mathrm{Sinh}(\gamma'_n/2)$ and γ'_n are the roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$

(iii) Freely Suspended Plates:

$\theta(x)=\mathrm{Cos}(\gamma_m x/2a)+R_m \mathrm{Cosh}(\gamma_m x/2a)$, $m=2,4,6,\ldots$ $-a\leqslant x\leqslant a$

where, $R_m=-\mathrm{Sin}(\gamma_m/2)/\mathrm{Sinh}(\gamma_m/2)$ and γ_m are the roots of the equation $\tan(\gamma/2)+\tan(\gamma/2)=0$

$\theta(x)=\mathrm{Sin}(\gamma'_m x/2a)+R'_m \mathrm{Cosh}(\gamma'_m x/2a)$ $m=3,5,7,\ \ldots.$ $-a\leqslant x\leqslant a$

where, $R'_m=\mathrm{Sin}(\gamma'_m/2)/\mathrm{Sinh}(\gamma'_m/2)$ and γ'_m are the roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Cos}(\gamma_n y/2b)+R_n \mathrm{Cosh}(\gamma_n y/2b)$, $n=2,4,6,....$ $-b\leqslant y\leqslant b$

where, $R_n=-\mathrm{Sin}(\gamma_n/2)/\mathrm{Sinh}(\gamma_n/2)$ and γ_n are the roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Sin}(\gamma_n' y/2b)+R_n' \mathrm{Sinh}(\gamma_n' y/2b)$, $n=3,5,7,.$ $-b\leqslant y\leqslant b$

where, $R_n'=\mathrm{Sin}(\gamma_n'/2)/\mathrm{Sinh}(\gamma_n'/2)$ and γ_n' are the roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$

CHAPTER -4

SOLUTION OF THE PROBLEM

4.1. INTRODUCTION:

The solution of the problem comprises two parts, namely, analytical and numerical. In the analytical part, integration of the expression for the elemental pressure is performed to obtain the farfield pressure distribution. In the second part, numerical integration of the expression for the acoustic intensity is performed in order to get the total average power radiated from one side of the baffled plate. For the plate with simply-supported edge conditions, the values of the radiation efficiency is also found by the method of numerical integration. In the subsequent sections, the method of integration for each set of boundary conditions will be treated individually.

4.2. SIMPLY-SUPPORTED PLATE:

(a) POWER RADIATED:

From equation (3.9), given in chapter -3, the acoustic pressure distribution is given by,

$$p = \frac{-\rho c k \omega_{mn} W_{mn}}{2\pi R} e^{i(\omega_{mn} t + kR)} \int_{-a}^{a}\int_{-b}^{b} \theta(x)\theta(y)\, e^{(-ilx/2a)} e^{(-isy/2b)}\, dx\, dy$$

The displacement functions for the simply-supported plate are,

$$\theta(x) = \frac{(m-1)\pi(x+a)}{2a}, \quad m=2,3,4,\ldots\ldots \qquad -a \leqslant x \leqslant a$$

and

$$\theta(y) = \mathrm{Sin}\frac{(n-1)\pi(y+b)}{2b}, \quad n=2,3,4,\ldots\ldots \qquad -b \leqslant y \leqslant b$$

Substituting in the expression for pressure distribution,

$$p=\frac{-\rho ck\omega_{mn}W_{mn}}{2\pi R}e^{i(\omega_{mn}t+kR)}\int_{-a}^{a}\int_{-b}^{b}\mathrm{Sin}\frac{(m-1)\pi(x+a)}{2a}\,\mathrm{Sin}\frac{(n-1)\pi(y+b)}{2b}\,\times$$

$$e^{(-ilx/2a)}e^{(-isy/2b)}\,dx\,dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

where, $l=2ak\mathrm{Cos}\alpha\,\mathrm{Sin}\theta$ and $s=2bk\mathrm{Sin}\alpha\,\mathrm{Sin}\theta$.

Integration of equation (4.1) gives,

$$p=\frac{-\rho ck\omega_{mn}W_{mn}}{2\pi R}e^{i(\omega_{mn}t+kR)}\left[\frac{\{2a/(m-1)\}\{(-1)^{m-1}e^{(-il/2)}-e^{(-il/2)}\}}{\{l^2/(m-1)^2\pi^2\}-1}\right]\times$$

$$\left[\frac{\{2b/(n-1)\}\{(-1)^{n-1}e^{(-is/2)}-e^{(-is/2)}\}}{\{s^2/(n-1)^2\pi^2\}-1}\right]$$

Further simplification gives,

$$p=\frac{-\rho ck\omega_{mn}W_{mn}}{2\pi R}e^{i(\omega_{mn}t+kR)}\;\frac{16ab}{(m-1)(n-1)\pi^2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

where Cos(l/2) is used when m is an even integer and Sin(l/2) is used when m is an odd integer.

The farfield acoustic intensity is given by,

$$I=|p|^2/\rho c$$

Substituting p from equation (4.2) it is found that,

$$I=\frac{\rho c k^2\omega^2 W^2\,256a^2b^2}{8\pi^2R^2(m-1)^2(n-1)^2\pi^4}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots(4.3)$$

Now $k=2\pi/\lambda$, where λ is the wave length. As $\lambda=c/f$, so $k=2\pi f/c=\omega/c$, where c is the velocity of sound, f is the frequency and ω is the circular frequency. Substituting in equation (4.3),

$$I=\frac{\rho c\omega^4W^2\,256a^2b^2}{8.0\,c^2\pi^2R^2(m-1)^2(n-1)^2\pi^4}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2$$

$$=\frac{\rho\omega^4W^2\,32a^2b^2}{\pi^6R^2c(m-1)^2(n-1)^2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2$$

and l and s become,

$l=2ak\mathrm{Cos}\alpha\ \mathrm{Sin}\theta=(2a\omega/c)\mathrm{Cos}\alpha\ \mathrm{Sin}\theta$

$s=2bk\mathrm{Sin}\alpha\ \mathrm{Sin}\theta=(2b\omega/c)\mathrm{Sin}\alpha\ \mathrm{Sin}\theta$

From equation (3.12) the dimensionless frequency factor λ_f is given by,

$$\lambda_f^2=\frac{\rho_m a^4\omega^2\,12(1-\sigma^2)}{\pi^4Eh^2g}\ ,\ \text{so}\quad \omega^2=\frac{\lambda_f^2\pi^4Eh^2g}{\rho_m a^4\,12(1-\sigma^2)}.$$

Therefore,

$$\omega^4=\frac{\lambda_f^4\pi^8E^2h^4g^2}{a^2\left(12\rho_m(1-\sigma^2)\right)^2}$$

where λ_f is given by,

$$\lambda_f^2=G_x^4+(a/b)^4G_y^4+2(a/b)^2[\sigma H_xH_y+(1-\sigma)J_xJ_y]$$

Substituting for ω^4 in the expression for I,

$$I=\frac{\rho W^2 32 a^2 b^2 \lambda_f^4 \pi^8 E^2 h^4 g^2}{cR^2 (m-1)^2 (n-1)^2 \pi^6 \rho_m^2 a^8 144 (1-\alpha^2)^2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2 x \left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2$$

$$=\frac{2\rho W^2 \pi^2 b^2 h^4 \lambda_f^4 E^2 g^2}{9cR^2 (m-1)^2 a^2 a^4 \rho_m^2 (1-\alpha^2)^2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2$$

$$=\frac{2\rho W^2 \pi^2 \lambda_f^4 E^2 g^2}{9cR^2 (m-1)^2 (n-1)^2 \rho_m^2 (1-\alpha^2)^2} R_a^2 R_t^4 \left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2 \qquad \text{.......(4.4)}$$

where R_a is defined as aspect ratio, b/a, and R_t is defined as thickness ratio, h/a.

l and s in terms of R_a and R_t are given by,

$$l=(2a\omega/c)\,\mathrm{Cos}\alpha\,\mathrm{Sin}\theta=\frac{2a\lambda_f\pi^2 h\sqrt{Eg}}{ca^2\sqrt{12\rho_m(1-\alpha^2)}}\mathrm{Cos}\alpha\,\mathrm{Sin}\theta=\frac{R_t\lambda_f\pi^2\sqrt{Eg}}{c\sqrt{3\rho_m(1-\alpha^2)}}\mathrm{Cos}\alpha\,\mathrm{Sin}\theta$$

and

$$s=(2b\omega/c)\,\mathrm{Cos}\alpha\,\mathrm{Sin}\theta=\frac{R_a R_t\lambda_f\pi^2\sqrt{Eg}}{c\sqrt{3\rho_m(1-\alpha^2)}}\mathrm{Sin}\alpha\,\mathrm{Sin}\theta$$

The total average power radiated from one side of the baffled plate is given by,

$$P_w=\int_0^{2\pi}\int_0^{\pi/2}\frac{|p|^2}{c}R^2\,\mathrm{Sin}\theta\,d\theta\,d\alpha=\int_0^{2\pi}\int_0^{\pi/2} IR^2\,\mathrm{Sin}\theta\,d\theta\,d\alpha$$

Substituting the value of I from equation (4.4), P_w takes the form,

$$P_w = \frac{8\rho W^2\pi^2\gamma^4 E^2 g^2\rho}{9c(m-1)^2(n-1)^2\rho_m^2(1-\alpha^2)^2} R_a^2 R_t^4 \int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2 \times$$

$$\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2 \mathrm{Sin}\theta\, d\theta\, d\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.5)$$

The examination of equation (4.5) shows that P_w is independent of R. This should be so as the total average power radiated from one side of the plate is not a function of R. Only the acoustic intensity varies inversely as the square of R as depicted by equation (4.4).

(b) RADIATION EFFICIENCY:

The objective here is to determine the radiation efficiency of the simply-supported plates in order to compare the results of this study with those by the previous workers (68,69).

Radiation efficiency of a vibrating plate is given in equation (3.17) by,

$$S_{mn} = P_w/\rho cab\langle |u_w|^2\rangle$$

where P_w is the total average power radiated from one side of the baffled plate. But here the original expression has to be used in order to get the radiation efficiency. This is so because, the radiation efficiency obtained by other authors (68,69) are in terms of the parameters other than those used in the determination of P_w.

Taking the expression for p from equation (4.2),

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\,\frac{16ab}{(m-1)(n-1)\pi^2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]$$

Now the acoustic intensity I is given by, $I=|p|^2/\rho c$

So that,

$$I=\frac{32\rho c\omega^2W^2k^2a^2b^2}{\pi^2R^2(m-1)^2(n-1)^2\pi^4}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2$$

where $l=2ak\mathrm{Cos}\alpha\,\mathrm{Sin}\theta$ and $s=2bk\mathrm{Sin}\alpha\,\mathrm{Sin}\theta$

The total average power radiated from one side of the plate becomes,

$$P_w=\frac{128\rho c\omega^2k^2W^2a^2b^2}{\pi^6(m-1)^2(n-1)^2}\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\times\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2\times$$

$$\mathrm{Sin}\theta\,d\theta\,d\alpha \qquad \text{..(4.6)}$$

The average of the temporal and spatial factor of the square of the surface velocity, $\langle|u_w|^2\rangle$ is given by,

$$\langle|u_w|^2\rangle=(1/4ab)\int_{-a}^{a}\int_{-b}^{b}(1/2)u_w^2\,dx\,dy \qquad \text{..................(4.7)}$$

But $u_w=i\omega W\theta(x)\theta(y)=i\omega W\mathrm{Sin}\frac{(m-1)\pi(x+a)}{2a}\,\mathrm{Sin}\frac{(n-1)\pi(y+b)}{2b}$

Substituting in equation (4.7) and carrying out the integration,

$\langle|u_w|^2\rangle=(1/8)\omega^2W^2$

Substitution in the expression for the radiation efficiency gives,

$$S_{mn}=\frac{256k^2ab}{\pi^6(m-1)^2(n-1)^2}\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2\times$$

$\mathrm{Sin}\theta\ d\theta\ d\alpha$ ....(4.8)

The plate wave number, k_p is now defined as,

$k_p=[\{(m-1)\pi/2a\}^2+\{(n-1)\pi/2b\}^2]^{0.5}$, and the wave number ratio ψ as,

$\psi=k/k_p$.

Thus, $k=\psi k_p$ or $k^2=\psi^2\ k_p{}^2$.

Substituting in equation (4.8) the expression for the radiation efficiency is obtained as,

$$S_{mn}=\frac{64[\{(m-1)^2R_a\}+\{(n-1)^2/R_a\}]\psi^2}{\pi^4(m-1)^2(n-1)^2}\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{l}{2}\right)}{\frac{l^2}{(m-1)^2\pi^2}-1}\right]^2\times$$

$$\left[\frac{\begin{matrix}\mathrm{Cos}\\ \mathrm{Sin}\end{matrix}\left(\frac{s}{2}\right)}{\frac{s^2}{(n-1)^2\pi^2}-1}\right]^2\mathrm{Sin}\theta\ d\theta\ d\alpha \qquad ....(4.9)$$

Making the same substitution, the expressions for l and s become,

$l=[(m-1)^2+\{(n-1)^2/R_a{}^2\}]^{0.5}\psi\pi\,\mathrm{Cos}\alpha\,\mathrm{Sin}\theta$ and

$s=[R_a{}^2(m-1)^2+(n-1)^2]^{0.5}\psi\pi\,\mathrm{Sin}\alpha\,\mathrm{Sin}\theta$.

4.3.CLAMPED PLATE:

For a plate with all four edges fixed, only the amount of acoustic power radiation from one side of the plate has to be calculated. The farfield pressure distribution is given by equation (3.9),

$$p = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} \theta(x)\theta(y) e^{(-i\lambda x/2a)} e^{(-i\epsilon y/2b)}\, dx\, dy$$

The displacement functions for the clamped plates are,

$\theta(x)=\mathrm{Cos}(\gamma_m x/2a)+R_m \mathrm{Cosh}(\gamma_m x/2a)$, $m=2,4,6,\ldots\ldots -a\leqslant x\leqslant a$

where $R_m=\mathrm{Sin}(\gamma_m/2)/\mathrm{Sinh}(\gamma_m/2)$ and γ_m are the roots of the equation,

$\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(x)=\mathrm{Sin}(\gamma'_m x/2a)+R'_m \mathrm{Cosh}(\gamma'_m x/2a)$, $m=3,5,7,\ldots$ $-a\leqslant x\leqslant a$

where $R'_m=-\mathrm{Sin}(\gamma'_m/2)/\mathrm{Sinh}(\gamma'_m/2)$ and γ'_m are the roots of the equation,

$\tan(\gamma/2)-\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Cos}(\gamma_n y/2b)+R_n \mathrm{Cosh}(\gamma_n y/2b)$, $n=2,4,6,\ldots\ldots$ $-b\leqslant y\leqslant b$

where $R_n=\mathrm{Sin}(\gamma_n/2)/\mathrm{Sinh}(\gamma_n/2)$ and γ_n are the roots of the equation,

$\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Sin}(\gamma'_n y/2b)+R'_n \mathrm{Sinh}(\gamma'_n y/2b)$, $n=3,5,7,\ldots$; $-b\leqslant y\leqslant b$

where $R'_n=\mathrm{Sin}(\gamma'_n/2)/\mathrm{Sinh}(\gamma'_n/2)$ and γ'_n are the roots of the equation,

$\tan(\gamma/2)-\tanh(\gamma/2)=0$

It is now required to consider each of the combinations of odd and even values of m and n separately.

CASE-I: EVEN VALUES OF m AND n.

Substituting the displacement functions $\theta(x)$ and $\theta(y)$ the acoustic pressure distribution p becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\mathrm{Cos}(\gamma_m x/2a)+R_m\mathrm{Cosh}(\gamma_m x/2a)]\times$$

$$[\mathrm{Cos}(\gamma_n y/2b)+R_n\mathrm{Cosh}(\gamma_n y/2b)]\ e^{(-ilx/2a)}e^{(-isy/2b)}dx\ dy \qquad \ldots\ldots\ldots(4.10)$$

Integration of equation (4.10) gives,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m^2\gamma_n^2}\left[\left\{\frac{l\mathrm{Cos}(\gamma_m/2)\mathrm{Sin}(l/2)-\gamma_m\mathrm{Sin}(\gamma_m/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m^2)-1}\right\}\right.$$

$$\left.+R_m\left\{\frac{l\mathrm{Cosh}(\gamma_m/2)\mathrm{Sin}(l/2)+\gamma_m\mathrm{Sinh}(\gamma_m/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m^2+1)}\right\}\right]\times$$

$$\left[\left\{\frac{s\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sin}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)-1}\right\}+R_n\left\{\frac{s\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n^2)+1}\right.\right.$$

$$\left.\left.+\frac{\gamma_n\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)+1}\right\}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.11)$$

Here $k=\omega/c$ and $\omega^2=\dfrac{\lambda_f^2\pi^4Eh^2g}{\rho_m a^4 12(1-\sigma^2)}$

$l=2ak\mathrm{Cos}\alpha\ \mathrm{Sin}\theta=(2a\omega/c)\mathrm{Cos}\alpha\ \mathrm{Sin}\theta$ and,

$s=2bk\mathrm{Sin}\alpha\ \mathrm{Sin}\theta=(2b\omega/c)\mathrm{Sin}\alpha\ \mathrm{Sin}\theta$.

After making all necessary substitutions and simplifications the amount of power radiated comes out to be,

$$P_w=\frac{8W^2\lambda_f^4\pi^6E^2g^2\rho}{9c\gamma_m^4\gamma_n^4\rho_m^2(1-\sigma^2)^2}R_a^2R_t^4\int_0^{\pi/2}\int_0^{\pi/2}\Bigg[\Big\{\frac{l\cos(\gamma_m/2)\sin(l/2)}{(l^2/\gamma_m^2)-1}-\frac{\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\Big\}+R_m\Big\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\Big\}\Bigg]^2$$

$$\times\Bigg[\Big\{\frac{s\cos(\gamma_n/2)\sin(s/2)-\gamma_n\sin(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)-1}\Big\}+R_n\Big\{\frac{s\cosh(\gamma_n/2)\sin(s/2)}{(s^2/\gamma_n^2)+1}+\frac{\gamma_n\sinh(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)+1}\Big\}\Bigg]^2\sin\theta\,d\theta\,d\alpha \qquad \ldots\ldots\ldots\ldots(4.12)$$

where $l=\dfrac{R_t\lambda_f\pi^2\sqrt{Eg}}{c\sqrt{3\rho_m(1-\sigma^2)}}\cos\alpha\sin\theta$ and $s=\dfrac{R_tR_a\lambda_f\pi^2\sqrt{Eg}}{c\sqrt{3\rho_m(1-\sigma^2)}}\sin\alpha\sin\theta$

Equation (4.12) gives the total average power radiated from one side of a clamped plate vibrating with even values of m and n.

CASE-II: EVEN VALUES OF m AND ODD VALUES OF n.

For this case, the expression for the farfield pressure, after substitution of the appropriate displacement function, becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\cos(\gamma_m x/2a)+R_m\cosh(\gamma_m x/2a)]\times$$

$$[\sin(\gamma_n' y/2b)+R_n'\sinh(\gamma_n' y/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}dx\,dy$$

Integration of the expression for p gives,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m^2\gamma_n'^2}\Bigg[\Big\{\frac{l\cos(\gamma_m/2)\sin(l/2)}{(l^2/\gamma_m^2)-1}-\frac{\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\Big\}+R_m\Big\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\Big\}\Bigg]$$

$$\times\Bigg[\Big\{\frac{s\sin(\gamma_n'/2)\cos(s/2)-\gamma_n'\cos(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)-1}\Big\}+R_n'\times\Big\{\frac{s\sinh(\gamma_n'/2)\cos(s/2)-\gamma_n'\cosh(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)+1}\Big\}\Bigg] \qquad \ldots\ldots\ldots\ldots(4.13)$$

Proceeding in the same way as in the case of even values of m and n, the total average power radiated from one side of a clamped plate vibrating with even values of m and odd values of n comes out to be,

$$P_w=\frac{8W^2\pi^6\lambda_f^4E^2g^2\rho}{9c\gamma_m^4\gamma_n'^4\rho_m^2(1-\alpha^2)^2}R_a^2R_t^4\int_0^{\pi/2}\int_0^{\pi/2}\Big[\Big\{\frac{l\cos(\gamma_m/2)\sin(l/2)}{(l^2/\gamma_m^2)-1}-\frac{\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\Big\}+R_m\Big\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\Big\}\Big]^2$$

$$\times\Big[\Big\{\frac{s\sin(\gamma_n'/2)\cos(s/2)-\gamma_n'\cos(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)-1}\Big\}+R_n'\Big\{\frac{s\sinh(\gamma_n'/2)\cos(s/2)-\gamma_n'\cosh(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)+1}\Big\}\Big]^2\sin\theta\,d\theta\,d\alpha\,. \quad \ldots\ldots(4.14)$$

where l and s are the same as defined earlier.

CASE-III: ODD VALUES OF m AND EVEN VALUES OF m.

For this case, when the displacement functions are substituted into the expression for the farfield pressure distribution, the expression takes the form,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\sin(\gamma_m'x/2a)+R_m'\sinh(\gamma_m'/2a)]\times[\cos(\gamma_ny/2b)+R_n\cosh(\gamma_ny/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}\,dx\,dy.$$

After integration it becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m'^2\gamma_n^2}\Big[\Big\{\frac{l\sin(\gamma_m'/2)\cos(l/2)}{(l^2/\gamma_m'^2)-1}-\frac{\gamma_m'\cos(\gamma_m'/2)\sin(l/2)}{(l^2/\gamma_m'^2)-1}\Big\}+R_m'\Big\{\frac{l\sinh(\gamma_m'/2)\cos(l/2)}{(l^2/\gamma_m'^2)+1}-\frac{\gamma_m'\cosh(\gamma_m'/2)\sin(l/2)}{(l^2/\gamma_m'^2)+1}\Big\}\Big]$$

$$\times\Big[\Big\{\frac{s\cos(\gamma_n/2)\sin(s/2)}{(s^2/\gamma_n^2)-1}-\frac{\gamma_n\sin(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)-1}\Big\}+R_n\Big\{\frac{s\cosh(\gamma_n/2)\sin(s/2)}{(s^2/\gamma_n^2)+1}-\frac{\gamma_n\sinh(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)+1}\Big\}\Big] \quad \ldots\ldots(4.15)$$

Following the same procedure as in CASE-I, the final expression for the acoustic power radiation becomes,

$$P_W = \frac{8W^2\lambda^4\pi^6E^2g^2\rho}{9c\gamma_m'^4\gamma_n^4\rho_m^2(1-\alpha^2)^2} R_a^2 R_t^4 \times$$

$$\int_0^{\pi/2}\int_0^{\pi/2} [\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\}+R_m' \times$$

$$\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\}]^2 \times$$

$$[\{\frac{s\,\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sin}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)-1}\}+R_n \times$$

$$\{\frac{s\,\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(s/2)+\gamma_n\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)+1}\}]^2 \mathrm{Sin}\theta\, d\theta\, d\alpha. \qquad \ldots\ldots\ldots(4.16)$$

where l and s are the same as given in CASE-I.

Equation (4.16) gives the total average power radiated from one side of the baffled plate with clamped edges vibrating in its (m,n)th mode with odd values of m and even values of n.

CASE-IV: ODD VALUES OF m AND n.

In this case when the appropriate displacement functions are substituted in the expression for the farfield pressure distribution, the expression takes the form,

$$p = \frac{-c\rho k\omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} [\mathrm{Sin}(\gamma_m' x/2a)+R_m'\mathrm{Sinh}(\gamma_m' x/2a)]\times$$

$$[\mathrm{Sin}(\gamma_n' y/2b)+R_n'\mathrm{Sinh}(\gamma_n' y/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}\,dx\,dy.$$

After integration it becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}\, e^{i(\omega t+kR)}\, \frac{16ab}{\gamma_m'^2\gamma_n'^2}\Big[\Big\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m'^2)-1} - \frac{\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\Big\}+R_m'\Big\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m'^2)+1} -$$

$$\frac{\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\Big\}\Big]^2\Big[\Big\{\frac{s\,\mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n'^2)-1} - \frac{\gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)-1}\Big\}+R_n'\Big\{\frac{s\,\mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n'^2)+1} -$$

$$\frac{\gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)+1}\Big\}\Big]^2 \mathrm{Sin}\theta\, d\theta\, d\alpha. \qquad \ldots\ldots(4.17)$$

Following the same procedure as in CASE-I,

$$P_w=\frac{8W^2\lambda_f^4\pi^6E^2g^2\rho}{9c\gamma_m'^4\gamma_n'^4\rho_m^2(1-\sigma^2)^2}\,R_a^2R_t^4\times$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\Big[\Big\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\Big\}+R_m'\times$$

$$\Big\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\Big\}\Big]^2\times$$

$$\Big[\Big\{\frac{s\,\mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(s/2)-\gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)-1}\Big\}+R_n'\times$$

$$\Big\{\frac{s\,\mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(s/2)-\gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)+1}\Big\}\Big]^2\mathrm{Sin}\theta\, d\theta\, d\alpha$$

$$\ldots\ldots(4.18)$$

where l and s are the same as given in CASE-I.

Equation (4.18) gives the total average acoustic power radiated from one side of baffled plate with all four edges clamped and vibrating with odd values of m and n.

4.4. FREELY-SUSPENDED PLATE:

For a plate with all four edges freely-suspended, the total average power radiated from one side of the plate is required to be calculated. The total average power radiated from one side of the plate is given by equation (3.10). The analytical integration of equation (3.10) is very difficult and thus numerical integration is performed as discussed in the section of numerical solution. But to obtain the expression for acoustic pressure distribution as used in equation (3.10), equation (3.9) is required to be integrated over the entire surface of the plate with displacement functions satisfying the edge conditions of the freely-suspended plate. This integration is presented here, leading finally to the equation for the total average power radiated from one side of the plate.

The displacement functions satisfying the conditions at the edges of a freely-suspended plate are;

$\theta(x)=\mathrm{Cos}(\gamma_m x/2a)+R_m \mathrm{Cosh}(\gamma_m x/2a)$, $m=2,4,6,\ldots$ $\quad -a\leqslant x\leqslant a$

where $R_m=-\mathrm{Sin}(\gamma_m/2)/\mathrm{Sinh}(\gamma_m/2)$, and γ_m are the roots of the equation,

$\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(x)=\mathrm{Sin}(\gamma'_m x/2a)+R'_m \mathrm{Sinh}(\gamma'_m x/2a)$, $m=3,5,7,\ldots$ $-a\leqslant x\leqslant a$

where, $R'_m=\mathrm{Sin}(\gamma'_m/2)/\mathrm{Sinh}(\gamma'_m/2)$, and γ'_m are the roots of the equation,

$\tan(\gamma/2)-\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Cos}(\gamma_n y/2b)+R_n \mathrm{Cosh}(\gamma_n y/2b)$, $n=2,4,6,$ $\quad -b\leqslant y\leqslant b$

where, $R_n=-\mathrm{Sin}(\gamma_n/2)/\mathrm{Sinh}(\gamma_n/2)$, and γ_n are the roots of the equation,

$\tan(\gamma/2)+\tanh(\gamma/2)=0$

$\theta(y)=\mathrm{Sin}(\gamma_n' y/2b)+R_n'\mathrm{Sinh}(\gamma_n' y/2b)$, $n=3,5,7,\ldots.$ $-b\leqslant y\leqslant b$

where, $R_n'=\mathrm{Sin}(\gamma_n'/2)/\mathrm{Sinh}(\gamma_n'/2)$, and γ_n' are the roots of the equation,

$\tan(\gamma/2)-\tanh(\gamma/2)=0$

Now each of the combinations of even and odd values of m and n will be considered separately.

CASE-1: EVEN VALUES OF m AND n.

For this combination of mode orders, the expression for the farfield acoustic pressure distribution, after making the necessary substitution, comes out to be,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\mathrm{Cos}(\gamma_m x/2a)+R_m\mathrm{Cosh}(\gamma_m x/2a)]\times$$

$$[\mathrm{Cos}(\gamma_n y/2b)+R_n\mathrm{Cosh}(\gamma_n y/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}\,dx\,dy$$

After integration it becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m^2\gamma_n^2}[\{\frac{l\mathrm{Cos}(\gamma_m/2)\mathrm{Sin}(l/2)-\gamma_m\mathrm{Sin}(\gamma_m/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m^2)-1}\}$$

$$+R_m\{\frac{l\mathrm{Cosh}(\gamma_m/2)\sin(l/2)+\gamma_m\mathrm{Sinh}(\gamma_m/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m^2)+1}\}]\times$$

$$[\{\frac{s\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sin}(\gamma_n/2)\mathrm{Cos}(l/2)}{(s^2/\gamma_n^2)-1}\}+R_n\{\frac{s\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n^2)+1}$$

$$+\frac{\gamma_n\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)+1}\}]\qquad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.19)$$

Following the same procedure as followed in the cases of the clamped plates, the expression for the acoustic power radiation becomes,

$$P_w = \frac{8W^2\lambda_f^4\pi^6 E^2 g^2 \rho}{9c\gamma_m^4\gamma_n^4\rho_m^2(1-\sigma^2)^2} R_a^2 R_t^4 x$$

$$\int_0^{\pi/2}\int_0^{\pi/2} [\{\frac{l\cos(\gamma_m/2)\sin(l/2)-\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\}+R_m\ x$$

$$\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\}]^2\ x$$

$$[\{\frac{s\cos(\gamma_n/2)\sin(s/2)-\gamma_n\sin(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)-1}\}+R_n\ x$$

$$\{\frac{s\cosh(\gamma_n/2)\sin(s/2)+\gamma_n\sinh(\gamma_n/2)\cos(s/2)}{(s^2/\gamma_n^2)1}\}]^2 \sin\theta\, d\theta\, d\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.20)$$

where l and s are the same as given in the cases of the clamped plates.

Equation (4.20) gives the total average acoustic power radiated from one side of a freely-suspended plate vibrating with even values of m and n.

CASE-II:EVEN VALUES OF m AND ODD VALUES OF n.

For this particular combination of mode orders, the expression for the farfield acoustic pressure distribution becomes,

$$p = \frac{-\rho c k\omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} [\cos(\gamma_m x/2a)+R_m\cosh(\gamma_m x/2a)]x$$

$$[\sin(\gamma'_n y/2b)+R'_n\sinh(\gamma'_n y/2b)]e^{(-ilx/2a)}e^{(-ilx/2b)}$$

After integration it becomes,

$$p = \frac{-\rho c k\omega W}{2\pi R} e^{i(\omega t+kR)} \frac{16ab}{\gamma_m^2\gamma_n'^2} [\{\frac{l\cos(\gamma_m/2)\sin(l/2)}{(l^2/\gamma_m^2)-1} -$$

$$\frac{\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\}+R_m\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\}]\ x$$

$$[\{\frac{s\sin(\gamma'_n/2)\cos(s/2)-\gamma'_n\cos(\gamma'_n/2)\sin(s/2)}{(s^2/\gamma_n'^2)-1}\}+R'_n\ x$$

$$\{\frac{s\sinh(\gamma'_n/2)\cos(s/2)-\gamma'_n\cosh(\gamma'_n/2)\sin(s/2)}{(s^2/\gamma_n'^2)+1}\}] \quad \ldots\ldots\ldots\ldots(4.21)$$

Following the same procedure as followed in the previous cases, the expression for the power radiated finally becomes,

$$P_w = \frac{8W^2\lambda_f^4\pi^6E^2g^2\rho}{9c\gamma_m^4\gamma_n'^4\rho_m^2(1-\sigma^2)^2} R_a^2R_t^4 \times$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\left[\left\{\frac{l\cos(\gamma_m/2)\sin(l/2)-\gamma_m\sin(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)-1}\right\}+R_m \times\right.$$

$$\left.\left\{\frac{l\cosh(\gamma_m/2)\sin(l/2)+\gamma_m\sinh(\gamma_m/2)\cos(l/2)}{(l^2/\gamma_m^2)+1}\right\}\right]^2 \times$$

$$\left[\left\{\frac{s\sin(\gamma_n'/2)\cos(s/2)-\gamma_n'\cos(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)-1}\right\}+R_n' \times\right.$$

$$\left.\left\{\frac{s\sinh(\gamma_n'/2)\cos(s/2)-\gamma_n'\cosh(\gamma_n'/2)\sin(s/2)}{(s^2/\gamma_n'^2)+1}\right\}\right]^2\sin\theta d\theta\, d\alpha.$$

....................(4.22)

where l and s are the same as defined in the earlier cases.

Equation (4.22) gives the total average power radiated from one side of a freely-suspended plate vibrating with even values of m and odd values of m.

CASE-III: ODD VALUES OF m AND EVEN VALUES OF n.

For this case of odd values of m and even values of n, the expression for the farfield acoustic pressure distribution, after necessary substitution becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\sin(\gamma_m'x/2a)+R_m'\sinh(\gamma_m'x/2a)] \times$$

$$[\cos(\gamma_n y/2b)+R_n\cosh(\gamma_n y/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}dx\, dy$$

After integration it becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m'^2\gamma_n^2}\left[\left\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\right\}\right.$$

$$\left.+R_m'\left\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)+\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\right\}\right]\times$$

$$\left[\left\{\frac{s\,\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sin}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)-1}\right\}+R_n\times\right.$$

$$\left.\left\{\frac{s\,\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)+1}\right\}\right] \qquad \ldots\ldots\ldots\ldots\ldots(4.23)$$

Following the same procedure as in the previous cases, it is found that the expression for the power radiated comes out to be,

$$P_w=\frac{8W^2\lambda_f^4\pi^6E^2g^2\rho}{9c\gamma_m'^4\gamma_n^4\rho_m^2(1-\sigma^2)^2}R_a^2R_t^4\times$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\left[\left\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\right\}+R_m'\times\right.$$

$$\left.\left\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\right\}\right]^2\times$$

$$\left[\left\{\frac{s\,\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(s/2)-\gamma_n\mathrm{Sin}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)-1}\right\}+R_n\times\right.$$

$$\left.\left\{\frac{s\,\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(s/2)+\gamma_n\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n^2)+1}\right\}\right]^2\mathrm{Sin}\theta\,d\theta\,d\alpha$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.24)$$

where l and s are the same as defined in the earlier cases.

Equation (4.24) gives the total average acoustic power radiated from one side of a freely suspended plate in an infinite baffle and vibrating in its (m,n)th mode with odd values of m and even values of n.

CASE-IV: ODD VALUES OF m AND m.

For the case of odd values of m and n, the expression for the farfield pressure distribution after substituting the appropriate displacement functions, becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\int_{-a}^{a}\int_{-b}^{b}[\mathrm{Sin}(\gamma_m' x/2a)+R_m'\mathrm{Sinh}(\gamma_m' x/2a)]\times$$

$$[\mathrm{Sin}(\gamma_n' y/2b)+R_n\mathrm{Sinh}(\gamma_n' y/2b)]e^{(-ilx/2a)}e^{(-isy/2b)}dx\,dy$$

After integration it becomes,

$$p=\frac{-\rho ck\omega W}{2\pi R}e^{i(\omega t+kR)}\frac{16ab}{\gamma_m'^2\gamma_n'^2}\left[\left\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m'^2)-1}-\right.\right.$$

$$\left.\frac{\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\right\}+R_m'\left\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)}{(l^2/\gamma_m'^2)+1}-\right.$$

$$\left.\left.\frac{\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\right\}\right]\times\left[\left\{\frac{s\,\mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n'^2)-1}-\right.\right.$$

$$\left.\frac{\gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)-1}\right\}+R_n'\left\{\frac{s\,\mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(s/2)}{(s^2/\gamma_n'^2)+1}-\right.$$

$$\left.\left.\frac{\gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)+1}\right\}\right] \qquad \text{.............................(4.25)}$$

Following the same procedure as followed in the previous cases, it is found that the expression for the acoustic power radiation becomes,

$$P_w = \frac{8W^2\lambda_f^4\pi^6 E^2 g^2 \rho}{9c\gamma_m'^4\gamma_n'^4\rho_m^2(1-\sigma^2)^2} R_a^2 \ R_t^4 \ \times$$

$$\int_0^{\pi/2}\int_0^{\pi/2} [\{\frac{l\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)-1}\} + R_m' \ \times$$

$$\{\frac{l\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(l/2)-\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(l/2)}{(l^2/\gamma_m'^2)+1}\}]^2 \ \times$$

$$\{\frac{s\,\mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(s/2)-\gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)-1}\} + R_n' \ \times$$

$$\{\frac{s\,\mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(s/2)-\gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(s/2)}{(s^2/\gamma_n'^2)+1}\}]^2 \ \mathrm{Sin}\theta \ d\theta \ d\alpha$$

........................(4.26)

where l and s are the same as defined in the previous cases.

Equation (4.26) gives the total average acoustic power radiated from one side of a plate freely suspended in an infinite baffle and vibrating in its (m,n)th mode with odd values of m and n.

4.5. NUMERICAL SOLUTION:

It has been mentioned earlier that the analytical solution of equation (3.10) for calculating the total average acoustic power radiation, after the necessary substitution, is very cumbersome. Considering the fact that in modern times, the tools and techniques of numerical solution are highly developed, the numerical method of solution has been used to solve equation (3.10) with the help of a computer . The equations $\tan(\gamma/2)\pm\tanh(\gamma/2)=0$ were solved by the method of bisection and the integration was performed by the application of the Simpson Rule. The method involved the preparation of a computer program. The results of integration are presented in tabular as well as in graphical forms.

THE METHOD OF BISECTION:

The method of bisection has been a very efficient method for solving nonlinear as well as complicated linear equations. In fact the method can be applied to solve any kind of equations. The method of bisection is presented here in brief for ready reference.

In this method, the equation to be solved is expressed in the form $f(x)=0$. Then an approximate root is determined, may be by observation or by graphical presentation of the equation. The value of the function is then determined for successive regular intervals. If x_o be the approximate value of one of the roots of the equation $f(x)=0$, then,

$f(x_o), f(x_o+h), f(x_o+2h)\ldots\ldots$, where h is the successive increament in the value of x, are determined. If the product of two consecutive values of the function becomes negative, it can be concluded that at least one root of the equation lies in that interval. Then the average(arithmatic mean) of those two successive values of x gives one of the roots of the equation under consideration. The accuracy of the result can be increased to any degree by taking smaller values of h or by making successive iterations. In a part of the computer program developed for the solution of the whole problem, this method has been introduced to find the values by solving the equations $\tan(\gamma/2)\pm\tanh(\gamma/2)=0$.

THE SIMPSON FORMULA:

The Simpson Formula is an extensively used multisegment formula for integration. The accuracy of the results obtained is principally determined by the number of segments taken in the calculation. In this method, the whole range of integration is divided into a number of equal segments and the sum of the values of the function obtained by successive increment of the independent variable with certain weightage, gives the results of integration. If the range (b−a) of the integral f(x)dx is divided into n equal divisions, so that $h=(b-a)/n$, then by Simpson Formula, the integrated result is given by,

$$I=(h/3)[f(a)+4f(a+h)+2f(a+2h)+.........+4f\{a+(n-1)h\}+f(b)].$$

This method can be employed to calculate the integral of any order.

In the present solution, the range 0 to $\pi/2$ was divided into 35 equal intervals for applying the Simpson Formula. The number 35 was taken because of the fact that the accuracy of the result with further increase in the number of divisions increases only slightly, but the computer time required increases proportionately. When highly accurate results are desired, the number of divisions can be further increased or the method of Romberg integration can be used to improve the accuracy of the results.

CHAPTER -5

RESULTS AND DISCUSSIONS

5.1. RELIABILITY OF THE METHOD:

It is always expected that the efficiency and reliability of any new technique is established first, prior to its acceptance as a genuine tool, by applying the technique to problems for which solutions are already available in the literature. In other words, it should be ascertained that no error due to logic is committed in formulating the problem and no mistake is made in the computer programming. Keeping all these in mind, a number of standard problems are solved with the present method of solution and the results are compared with those of others, obtained analytically or by some other method. On the basis of this comparison, the reliability and efficiency of the method employed here are determined.

Wallace (69) found the radiation resistance and radiation efficiency of a baffled beam from the farfield pressure distribution produced by the beam, vibrating in simple harmonic motion in one of its natural bending modes. He analyzed the beams with hinged ends as well as clamped ends. Wallace derived an expression for the radiation resistance which is assymptotic to the exact solution as the frequency approaches zero. In addition to that, he found the radiation efficiency by numerical integration of the farfield acoustic intensity produced by the vibrating beam, covering the entire frequency range for the first ten modes of vibration of the beam.

Wallace also studied the vibration characteristics of a rectangular plate of uniform thickness (68). He used the same method of solution as of the beam, but confined his attention only to the plates with all the four edges simply-supported. As in the case of the beam, the expression for the radiation resistance proved to be assymptotic to the exact solution. The results of the numerical integration of the farfield acoustic intensity were used to obtain graphs representing the radiation efficiency of the simply-supported plate at different mode orders. When compared, it is observed that these graphs are identical to the radiation efficiency graphs obtained in the present study.

The above developments prove that the method of solution employed here is extremely accurate and no error is committed either in formulating the problem or in the computer programming.

5.2. VALIDITY OF THE BEAM FUNCTIONS:

Warburton (70) developed a number of functions to satisfy the conditions at the ends of a vibrating beam. The functions were designed to represent accurately the wave form of a beam vibrating in its natural modes. Wallace (69) used Warburton's beam functions in his analysis of the acoustics of beams with hinged and clamped ends. The results of the investigations by Wallace proved to be satisfactory as the expression for the radiation resistance assymptotically approached the exact solution as the frequency decreases. The results of the numerical integration of farfield acoustic intensity presented in the form of graphs also proved to be satisfactory. In another investigation (68) Wallace applied the beam functions developed by Warburton to the case of vibration of plates. He studied the case of a simply-supported plate in an infinite baffle vibrating in its natural modes. He derived the expression for the radiation resistance using the beam functions. The expression for the radiation resistance was assymptotic to the exact solution at low frequencies. The results of the numerical integration of the farfield acoustic intensity were presented in the form of graphs. The graphs for different mode orders of a simply-supported rectangular plate proved to be satisfactory when compared with the results of beam investigations.

In the present analysis, the beam functions by Warburton have been used to represent the motions of surfaces of the vibrating plates with different boundary conditions. The functions used are slightly modified to meet the requirements of the different boundary conditions at the edges of simply supported, clamped and freely suspended plates in infinite baffles. The exact representations of the beam mode shapes presented by the modified beam functions are given in figures 2, 3 and 4, respectively, for simply-supported, clamped and freely suspended plates. It is observed from these figures that, the beam mode shapes are satisfactorily able to represent the mode shapes of a vibrating plate with different boundary conditions. Moreover, before attempting the solutions of the actual problem, the expression for the radiation resistance for a vibrating plate with all the four edges simply supported are derived using the modified beam functions. This expression proved to be of the same nature as derived by Wallace (68) using the original beam functions of Warburton. Further, for comparison, the farfield acoustic intensity in the form of radiation efficiency is obtained by integration and plotted in the same way as by Wallace (68). The graphs thus obtained (Figures-5,6,7 and 8) are identical to those of Wallace (68) obtained by using the original beam functions applied to rectangular panel with simply-supported edges.

The above verifications not only prove the validity of the modified beam functions, but also prove that the modified beam functions truly represent the vibration characteristics of a baffled plate.

5.3.RESULTS AND DISCUSSIONS:

The method of investigation employed here is a versatile method for solving any problem of vibrating plates. The input variables for any rectangular flat plate with uniform thickness are the aspect ratios, (b/a) and the thickness ratios, (h/a). In addition, the appropriate beam functions satisfying the conditions at the edges of the plate problem to be solved has to be used. In this analysis, plates with three pure boundary conditions are considered. The boundary conditions are; (i) all four edges of the plate are simply-supported, (ii) all four edges of the plate are clamped and (iii) all four edges of the plate are freely suspended. The results are found for three values of the aspect ratios and three values of the thickness ratios in each case of the three boundary conditions. The results are presented here in the form of graphs. The computer outputs used in plotting the graphs are given in tabular form in the appendix. In the following sections, the results of each of the three boundary conditions are discussed separately. At the end, the results for the three boundary conditions are compared for aspect ratio=1.00 and thickness ratio=0.002. The other aspect ratios considered are, 0.50 and 2.00 and thickness ratios are, 0.001 and 0.004. The values of the different parameters used in the numerical evaluations are: Density of the surrounding medium (ρ)=1.21 kg/m^3. Velocity of sound in the surrounding medium (c)=341 m/sec., density of the plate material (ρ_m)=7700.0 kg/m^3, modulus of elasticity of the plate material (E)=206x10^9 N/m^2 and the amplitude of vibration (W)=0.0001.

(I). PLATES WITH ALL FOUR EDGES SIMPLY-SUPPORTED:

Figures 9 to 16 represent the total average acoustic power radiated from one side of the baffled plate, plotted against the number of nodal lines in the y-direction. The figures compare the influence of different parameters used in the analysis on the power radiated. In figures 9 and 10, the values of the aspect ratio and thickness ratio are kept constant and the variation of the power radiated with the variation of the number of nodal lines in the x-direction are presented. From figure 9, it is seen that, for low values of m, the amount of power radiated increases more or less regularly with the increase of n. This is expected as the power radiated should increase with the increase of the frequency of vibration which, in its turn, increases with the increase in the values of n. It is also observed that, the value of the power radiated is higher for higher values of m following the same logic as above. From figure 10 it is seen that, the power radiated does not show significant rise with the increase in the values of n. Moreover, the graphs show a waviness after certain values of n. With the increase in the mode orders, the frequency of vibration and consequently the frequency of the sound radiated increases. at the same time, the total number of waves in the plate also increases with the increase in the mode orders. At this condition, the effect of the mode orders over the amount of power radiated become insignificant and as such the power radiated does not show significant rise with further increase in the mode orders. But it should be noted that, with the increase in the mode orders, the frequency of vibration still increases making the power radiated to rise slightly. The waviness of the graphs are due to the interference of the waves from the x and y-directions. At high mode orders, the roots and crests of the plate waves from the two directions interfere each other making the acoustic power radiation to fluctuate. When two crests from the two directions superimpose one another, the total average acoustic power radiation increases giving rise to a crest of the wavy graphs. On the other hand, when the crests from the two directions oppose one another, the total acoustic power radiation decreases as depicted by the roots of the graphs. The frequency of this waviness depends upon the number of interferences in the plate. It can be generally concluded that, the number of interferences increases with the increase in the mode orders. Thus, the frequency and amplitude of the waviness increases with increasing mode numbers as shown in figures 11 and 12.

Figures 11 and 12 show the acoustic power radiation with increasing values of n. It is observed from these figures that, the rate of increase in the acoustic power radiation assymptotically reaches zero at very high mode numbers, with waviness still existing. The variation of the acoustic power radiation can be explained with the help of the variation of acoustic pressure in the surrounding medium due to the vibration of the plate. The three major factors are the frequency, effective radiating surface and the effective acoustic pressure. At lower mode orders, the effective radiating surface is high, virtually, the whole of the surface radiates energy, but this is not the case at high mode orders. With increasing mode orders, the regions of zero displacement of the plate increases. These points of zero displacement do not have any role in radiating acoustic power as they can not produce compression or rarefaction in the surrounding medium. Thus, with increasing mode orders, the effective radiating surface of the plate decreases. But the frequency of the vibrating plate increases with increasing mode orders as shown in figures 37 and 38. The rate of increase in the frequency is very high at low mode orders and decreases with increasing mode orders. These effects increases the acoustic power radiation with mode orders but with a decreasing rate. But as can be observed from figures 11 and 12, at very high mode numbers, the absolute amount of acoustic radiation does not show any further increase with mode numbers, rather, it attains a stable magnitude. At very high mode orders, the plate wave numbers become very high and as such the alternate crests and roots of the plate waves come very close to each other. At any instant of time, a particular crest produces a compression in the surrounding medium while the neighbouring root produces a rarefaction. These alternate compression and rarefaction causes the acoustic energy to travel through the surrounding medium. When the plate wave number becomes very high, the neighbouring roots and crests come so close to each other that, the rarefaction by the root partially neutralizes the compression produced by the crest. This neutralization,in its turn, reduces the amount of acoustic power radiation. All these effects, the effective radiating surface, the frequency of vibration and the interference of compressions and rarefactions, when combined at high mode orders, results in the stable magnitude of the acoustic power radiation. Though the power radiated displays a fluctuating characteristic at high modes, the radiation efficiency converges to unity and does not show any change with further increase in the mode numbers, as shown in figures 5,6,7 and 8. This is because of the fact that, radiation efficiency presents the power radiated from a plate due to its vibration at certain mode orders in comparison to the power radiated by the same plate vibrating as a solid without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate under consideration. At high mode orders, the interferences of the compressions and rarefactions produced by alternate roots and crests of the plate waves tends to reduce the rate of increase in the total acoustic power radiation. Above the critical frequency of the plate, the acoustic radiation from the plate become stable and does not increase with increasing mode orders. This stable acoustic radiation is equivalent to the acoustic radiation from a plate vibrating, as a solid body without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate. The effect of this stable magnitude of the acoustic radiation from a plate converges the radiation efficiency to unity above the critical frequency of the plate.

Figures 13 and 14 show the effect of the thickness ratio over the power radiated. The examination of the expression for the power radiated would reveal that the total power radiated must increase with the increase in the values of the thickness ratios. This fact is reflected in figures 13 and 14 showing an increase in the power radiated with increase in the values of the thickness ratio. Figure 13 shows this variation for low values of m and figure

14 for high values of m.

Figures 15 and 16 are plotted with the view to present the variation of the total power radiated due to the variation of the aspect ratio of the plate. The effect of the variation of the aspect ratio over the total power radiated can not be straight way predicted, as in the case of the thickness ratio, from the expression of power. This is clear from the observation of the graphs given in figures 15 and 16. Figure 15 shows the variation of power with aspect ratio at lower values of m and figure 16 shows the same for higher values of m. It can be seen from figure 15 that the value of the total acoustic power radiated increases with the decrease in the values of the aspect ratio. But this is not the case when vibration with higher values of m is considered, as shown in figure 16. From figure 16 it is found that the total acoustic power radiated increases with the increase of the aspect ratio for high values of m and for lower range of the values of n. But the trend is gradually reversed with the increase in the values of n. This variation in the acoustic power radiation is due to the change in the frequency of vibration of the plate at different aspect ratios. Figures 37 and 38 show the variation of the frequency with mode numbers at different aspect ratios of a simply supported plate. It is observed that at all mode orders, the frequency shows an increase with the decreasing aspect ratios. The change of the frequency due to the change of the aspect ratio from 2.00 to 1.00 is smaller than that due to change from 1.00 to 0.50. Moreover, the frequency curve for an aspect ratio of 0.50 is steeper than those for the other two aspect ratios studied. At lower mode orders, the influence of frequency over the amount of acoustic power radiated is insignificant. In this range, the amount of power radiated depends on the effective radiating surface, which is more in the case of a plate with an aspect ratio of 0.50 than those with 1.00 and 2.00. For this reason, at low mode numbers, the plate with an aspect ratio of 0.50

radiates the highest amount of power. The plate with an aspect ratio of 2.00 radiates the lowest amount and the radiation from the plate with an aspect ratio of 1.00 falls in between the two. This is presented in figure 15. At high values of m and lower range of the values of n, as shown in figure 16, the plate with an aspect ratio of 2.00 radiates the highest amount of power, the plate with an aspect ratio of 0.50 radiates the lowest amount and that with 1.00 radiates power falling in between the two. This trend can be explained by considering the fact that, at these mode orders, the effective radiation surface is more in the case of a plate with an aspect ratio of 2.00 as the longer side of the plate is divided into fewer waves. With the increase in the mode orders, the effective radiating surface decreases and the frequency of vibration increases. But the increase in the frequency of vibration far exceeds the reduction in the effective radiating surface. As such, at high mode orders, the frequency of vibration begins to play a dominating role over the acoustic radiation from the plate. Since the increase in the frequency of vibration with the increase in the mode numbers is more in the case of a plate with an aspect ratio of 0.50, the amount of acoustic power radiation show a steeper rise as shown in figure 16. The power radiation from the plates with the other two aspect ratios also increases but the increase is only small.

(II). PLATES WITH ALL FOUR EDGES CLAMPED:

Figures 17 to 24 present the total average acoustic power radiated from one side of the plate clamped in an infinite baffle, plotted against the number of nodal lines in the y-direction. Figures 17,18,19 and 20 compare the amount of acoustic power radiated from a clamped plate at different modes of vibration. figures 21 and 22 present the variation of the power radiated with the variation of the thickness ratio, (h/a), of the plate and figures 23 and

24 give the variation of the power with the aspect ratio of the plate. From figures 17 to 20, it is observed that upto a certain mode orders, the power radiated from a plate increases with the increase in the mode numbers. After that with the increase in the mode numbers the acoustic power radiation does increase significantly, rather, it attains a stable state. The power radiated due to vibration of a plate is largely dependent upon the frequency of vibration and the effective radiating surface of the plate. With increasing mode numbers, the effective radiating surface decreases and the frequency increases. The increase in the frequency associated with the increase in the mode numbers is shown in figures 39 and 40 for different aspect ratios of a clamped plate. It is observed from these figures that the change in the frequency with the change of the mode numbers is more in the lower range of the mode orders than the change in the higher range. In the lower range of the mode orders, the increase in the frequency of vibration exceeds the reduction in the effective radiating surface associated with the increase in the mode numbers.This results in an increase in the amount of acoustic power radiated with the increasing mode orders. This is shown in figure 17. With further increase in the mode numbers, the amount of power radiated still increases but with a reduced rate and shows a waviness after certain mode orders. At high mode orders, the waviness of the acoustic power radiation is caused by the interference of the plate waves from the two directions. At times, one wave peak coincides with the other, increasing the total average acoustic power radiation. On the other hand, for certain mode orders, the wave peaks from the two directions cancel one another, tending to decrease the total average acoustic power radiation. This fluctuation in the acoustic power radiation constitutes the waviness in the total average acoustic power radiation. The interference of the plate waves become more frequent and prominent at higher modes the waviness increases in amplitude and frequency. This wavy charac-

teristic of the acoustic power radiation is shown in figures 18,19 and 20. Though the average acoustic power radiated increases with increasing mode numbers, the rate of increase gradually decreases and ultimately reaches a stable state showing no further increase in the absolute value. With the increase of mode numbers, the effective radiating surface decreases and the frequency increases. As discussed earlier, the rate of increase in the frequency of vibration exceeds the rate of decrease in the effective radiating surface, causing the amount of power radiated to increase. But with further increase in the mode numbers, the rate of increase in the frequency of vibration decreases resulting in a decreases in the rate of increase of the average acoustic power radiation. This trend continues and ultimately comes down to zero, bringing the acoustic power radiation to a stable state. This trend can be explained if the factors influencing the acoustic radiation from the plate are taken into consideration. In addition to the effective radiating surface and frequency of vibration, there is another factor coming to influence the acoustic power radiation only at very high mode orders. This is the interference of the rarefactions and compressions of the surrounding medium produced by the roots and crests of the plate waves. At any instant of time, a crest of the plate wave produces a compression in the surrounding medium, whereas, at the same time, the neighbouring roots produce rarefactions. This alternate compressions and rarefactions causes the acoustic energy to travel through the surrounding medium. At high mode numbers, the plate wave numbers become very high bringing the adjacent roots and crests very close to each other. As the mode number is kept on increasing, the neighbouring roots and crests come so close to each other that, the compression produced by one crest is partially neutralized by the rarefactions produced by the neighbouring roots. This neutralization, in its turn, reduces the acoustic power radiation from the plate. This effect, combined with the effects of the effective radiating surface and the frequency of vibration, brings the acoustic power radiation to a stable state at very high mode numbers. This is shown in figures 19 and 20. Though the absolute value of the amount of acoustic power radiated does not increase with increasing mode numbers after a certain mode orders, the waviness of the power radiated still remains. This is caused by the fact that, the interference of the plate waves from the two directions still exists even if the mode numbers are very high.

Figures 21 and 22 show the variation of the power radiated at different thickness ratios of the plate for constant values of the other parameters, respectively, for lower and higher values of m. The dependence of the acoustic power radiation from a clamped plate upon the the thickness ratio can be predicted by examining the expression for the radiated power. The examination of the expression for the acoustic power radiation from a clamped plate, given in equations (32), (34), (36) and (38) for different combinations of even and odd values of m and n, reveals that the average acoustic power radiation from clamped plate will increase with the increase in its thickness ratio. The increase in the thickness ratio increases the stiffness of the plate making it to require more energy for vibration. As a result, plates with higher thickness ratio radiates more acoustic power than those with lower thickness ratios, as shown in figures 21 and 22.

Figures 23 and 24 present the variation of the acoustic power radiation at different aspect ratios with constant values of the other parameters for a plate with all four edges clamped. The two major factors influencing the acoustic power radiation from a plate are the effective radiating surface and the frequency of vibration of the plate. Since both of these factors vary with varying aspect ratios of a plate, the dependence of the acoustic power radiation upon the aspect ratio of the plate can not be straight way predicted as in the case of the thickness ratio. At lower values of m, the effective surface radiating acoustic power is less in the case of a plate with an aspect ratio of 0.50 than the other two aspect ratios considered. At low mode orders, the effective radiating surface plays a dominating role over the acoustic power radiation and, as such, the plate with an aspect ratio of 0.50 radiates the minimum amount of acoustic power at a particular mode order amongst the three aspect ratios under consideration. The plate with an aspect ratio of 2.00 radiates the maximum amount of energy and the radiation from the plate with an aspect ratio of 1.00 falls in between the two extreme cases. This is shown in figure 23 and partly in figure 24. With increasing mode numbers, The frequency of vibration increases and the effective radiating surface decreases. The result being the increase in the power upto a certain mode numbers and ultimately the effects are nullified with no further rise in the power output. For the plate with an aspect ratio of 2.00, the decrease in the effective radiating surface with increasing mode numbers in the y-direction is less than the lower aspect ratios. Whereas, the increase in the frequency of vibration is more in the case of the plate with an aspect ratio of 0.50 as shown in figure 40. These two effects, when combined, makes the power radiated to vary as shown figure 24. It is seen that the radiation from the plate with an aspect ratio of 2.00 is always higher than the radiation from the plates with the other two aspect ratios. However, With still increasing mode numbers, the difference gradually decreases.

(III). PLATES WITH ALL FOUR EDGES FREELY SUSPENDED:

Figures 25 to 32 present the total average acoustic power radiated from one side of the plate, freely suspended in infinite baffle, plotted against the number of nodal lines in the y-direction. Figures 25,26 27 and 28 compare the amount of acoustic radiation at different mode orders with fixed values of the other parameters. Figures 29 and 30 show the variation of acoustic radiation at different thickness ratios (h/a), at low and high mode orders respectively and figures 31 and 32 show the acoustic radiation at different aspect ratios (b/a), for low and high mode orders respectively, for a plate with all four edges freely suspended. At low mode orders, the amount of acoustic power radiated increases with the increase in the mode numbers. The frequency of vibration of the plate increases with increasing mode numbers, but the effective radiating surface decreases. Since, at low mode orders, the increase in the frequency of vibration is more than the decrease in the effective radiating surface, the amount of acoustic radiation from the plate increases as shown in figure 25. At high mode orders, the rate of increase in the frequency of vibration and the rate of decrease in the effective radiating surface decreases making the rise in the acoustic radiation to be insignificant.But at these conditions, the acoustic radiation shows a waviness due to the interference of the waves from the two directions as shown in figures 26,27 and 28. At still higher modes, the acoustic radiation from the plate does not increase any more with the increasing mode numbers, rather, it attains a stable state. At these higher modes the neighbouring of the compressions and rarefactions of the surrounding medium produced by the crests and roots of the plate waves partially neutralize each other. This neutralization, in its turn, reduces the acoustic power radiation from the plate. This effect at very high mode orders, when combined with the effects of the frequency of vibration and the effective radiating surface, brings the radiation of the acoustic power from the plate to a stable state. Though the radiation of the acoustic power attains a stable state in its magnitude, the waviness, which began earlier still remains with higher amplitude and frequency. At very high mode orders, the interference of the plate waves from the two directions become all the more prominent and frequent. This effect increases the amplitude and frequency of the waviness of the power radiated at very high mode orders.

Figures 29 and 30 show the power radiated from a freely suspended plate at different thickness ratios. As in the other two cases discussed earlier, the amount of acoustic radiation increases with the increase in the value of the thickness ratio of the plate.

In figures 31 and 32, the amount of acoustic radiation from a freely suspended plate at different aspect ratios have been compared. In the lower range of mode orders, the plate with an aspect ratio of 0.50 radiates more power than those with higher values of the aspect ratio. For a plate with an aspect ratio of 0.50, the effective radiating surface is less than the plate with higher aspect ratios, but the frequency of vibration in this case is much higher, as shown in figures 41 and 42. This makes the acoustic radiation to be more with an aspect ratio of 0.50 than the other two. As shown in figure 32, at high values of m and low range of the values of n, the plate with an aspect ratio of 2.00 radiates more power than the plates with lower values of the aspect ratio. With increasing values of m, the effective radiating surface decreases. This decrease is more in the case of a plate with an aspect ratio of 0.50, making the power radiated to be less than those with higher aspect ratios. With increase in the values of n, the increase in frequency of vibration takes care of the decrease in the effective radiating surface and the effects of the aspect ratio of the plate upon the acoustic power radiation from it is almost nullified as shown in figure 32.

(IV). COMPARISON OF BOUNDARY CONDITIONS:

Figures 33 to 36 compare the average acoustic power radiation from plates with three different pure boundary conditions. The boundary conditions being; (i) all four edges simply-supported, (ii) all four edges clamped and (iii) all four edges freely-suspended. In figure 33, the power radiated by plates with different boundary conditions have been presented for lower mode orders. It is observed that, the simply-supported plate radiates the minimum power of the three, the freely-suspended plate radiates the maximum power and the radiation from the clamped plate falls in between the two extremes. As can be observed from the frequency curves given in figures 37 to 42, the frequency of vibration of the simply supported plate is very low as compared with those of clamped and freely-suspended plates. This makes the amount of acoustic radiation to be the minimum in case of a simply-supported plate. Comparing figures 39 and 41, it can be concluded that, the frequency of vibration of a clamped plate and a freely-suspended plate nearly equal over the range of the mode orders. But due to clamping at the edges, the effective radiating surface is less in the case of a clamped plate than that of a freely-suspended plate. As such, the acoustic radiation from a freely-suspended plate is the maximum. Observation of figures 35 and 36 reveal that, at high mode orders, the effects of the edge conditions on the amount of acoustic power radiation are virtually nullified as in these cases, the alternate compressions and rarefactions of the surrounding medium of the vibrating plate, produced by the neighbouring crests and roots of the plate waves, begin to partially neutralize each other. This neutralization, as have been mentioned earlier, reduces the amount of acoustic radiation from the plate and thus brings it to a stable state.

CHAPTER -6

CONCLUSIONS AND RECOMMENDATIONS

6.1. CONCLUSIONS:

The total average acoustic power radiated from one side of a baffled plate due to its natural vibration with three different pure boundary conditions have been investigated in this thesis. The variation of the natural frequency with mode orders have also been investigated. In addition, the radiation efficiency of a plate with all the four edges simply supported has been studied in this thesis and compared with the results given by Wallace(68). Warburton's (70) beam functions have been used to represent the wave motion of the vibrating plate and Simpson rule has been applied for the numerical integration of the final expression of radiated power with the help of a computer program.

In this thesis extensive numerical results on plates vibrating with different boundary conditions,mode shapes, aspect ratios and thickness ratios have been presented.

Based on the extent of this investigation the following conclusions can be drawn..

(i) At low range of the values of mode numbers, the total average acoustic power radiated from one side of the vibrating plate differs with boundary conditions,mode orders, aspect ratios and thickness ratios.For the same mode orders, aspect ratio and thickness ratio, the simply supported plate radiates the minimum amount of power, the freely-suspended plate radiates the maximum amount and the radiation from the clamped plates falls in between the simply-supported and freely-suspended plates. With increasing mode orders, the power radiated increases but the rate of increase decreases and asymptotically reaches zero at very high mode numbers. After a certain mode orders, the acoustic power radiation from the plates begin to show a wavy nature.

(ii) At high mode numbers, the effects of the variation of boundary conditions, mode orders and aspect ratios are almost nullified, but the waviness still remains.

(iii) The natural frequency of vibrating plates varies with boundary conditions, mode orders, and aspect ratios. For the same mode orders and aspect ratio the frequency of the simply supported plate is the minimum, that of the freely-suspended plate is the maximum and that of the clamped plates falls in between the two. For the same boundary condition and mode orders the frequency of vibration increases with decreasing aspect ratio, except at very low mode orders, where the frequency for two aspect ratios may coincide. For the same boundary condition and aspect ratio, the frequency of vibration increases with increasing mode numbers. The rate of increase in the natural frequency is very high at low mode orders and decreases with increasing mode numbers, asymptotically, reaching zero at very high mode orders.

(iv) The radiation efficiency of the simply-supported plate increases for all mode numbers with increasing wave number ratios upto the critical frequency. After the critical frequency the radiation efficiency asymptotically converges to unity and does not show any further variation. For frequency near the critical frequency the radiation efficiency also shows a waviness as in the case of the power radiated, but this waviness does not exist after the critical frequency.

6.2. RECOMMENDATIONS:

From the experience of the present investigation, the following fields on plate vibration and acoustics are recommended as the scopes of future research.

(i) More appropriate displacement functions, than the ones developed by Warburton and used in the present investigation, may be developed for more exact representation of the plate motion to obtain still more accurate results.

(ii) The boundary conditions studied in this thesis are the most idealized cases of plate vibration. Plates with more realistic boundary conditions may be studied to suit the practical cases.

(iii) Plates with variable thickness and irregularities may also be studied.

(iv) Plates, reinforced with beams may be studied and the effects of reinforcement can be investigated.

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REFERENCES

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FIGURES

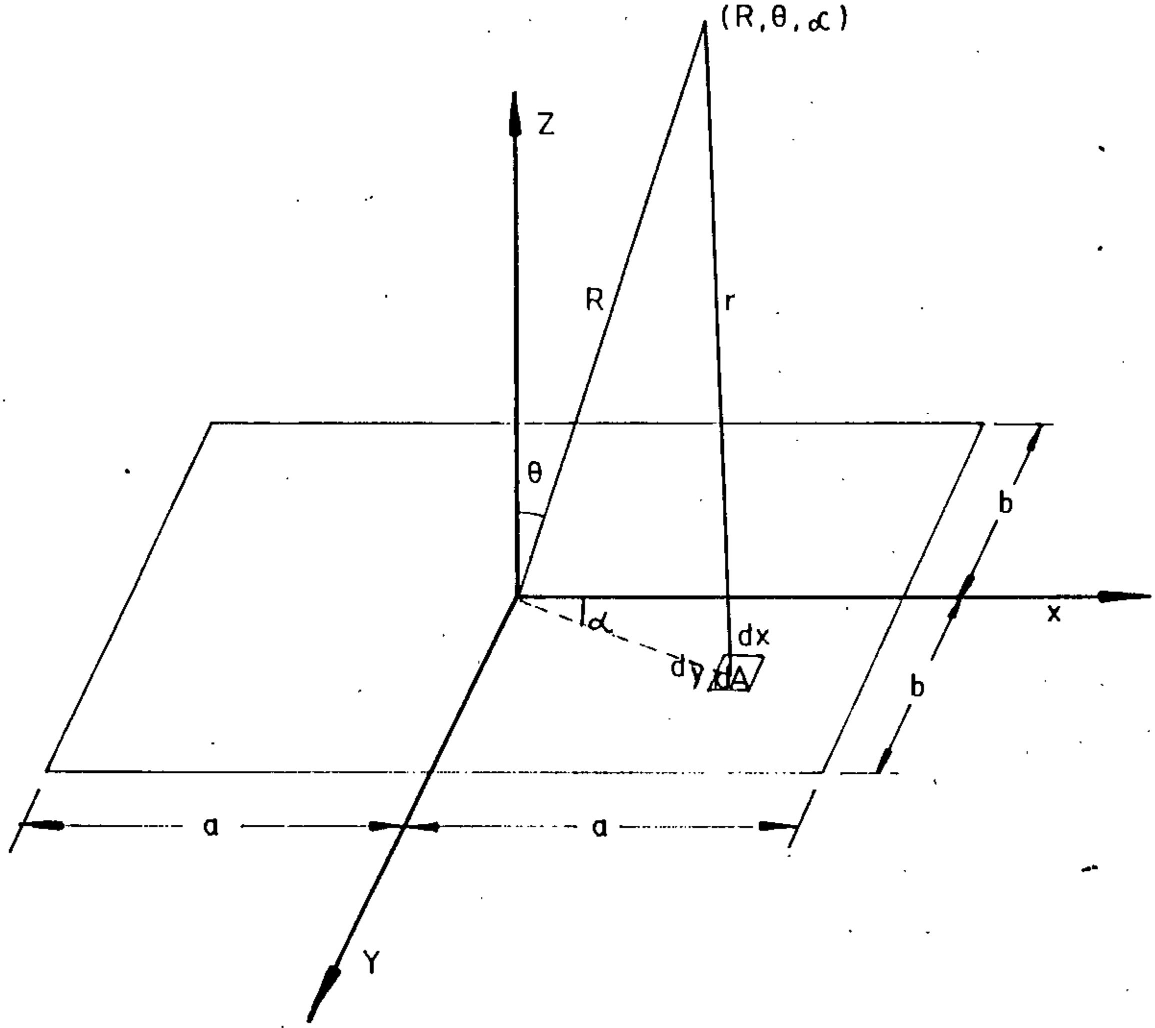

Fig. 1 : Rectangular flat plate with coordinate systems.

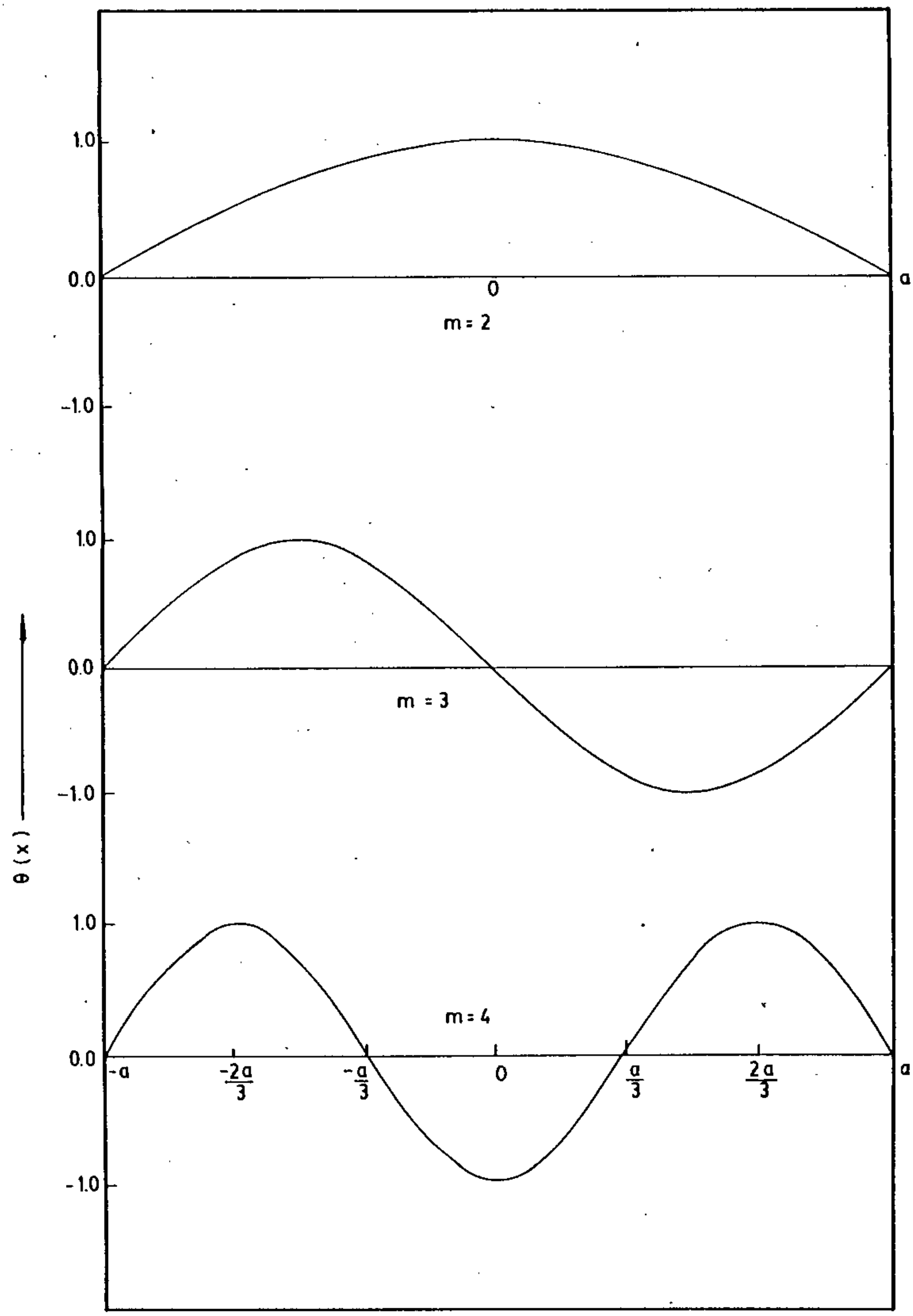

Fig. 2 : Graph of the Beam function for simply supported ends.

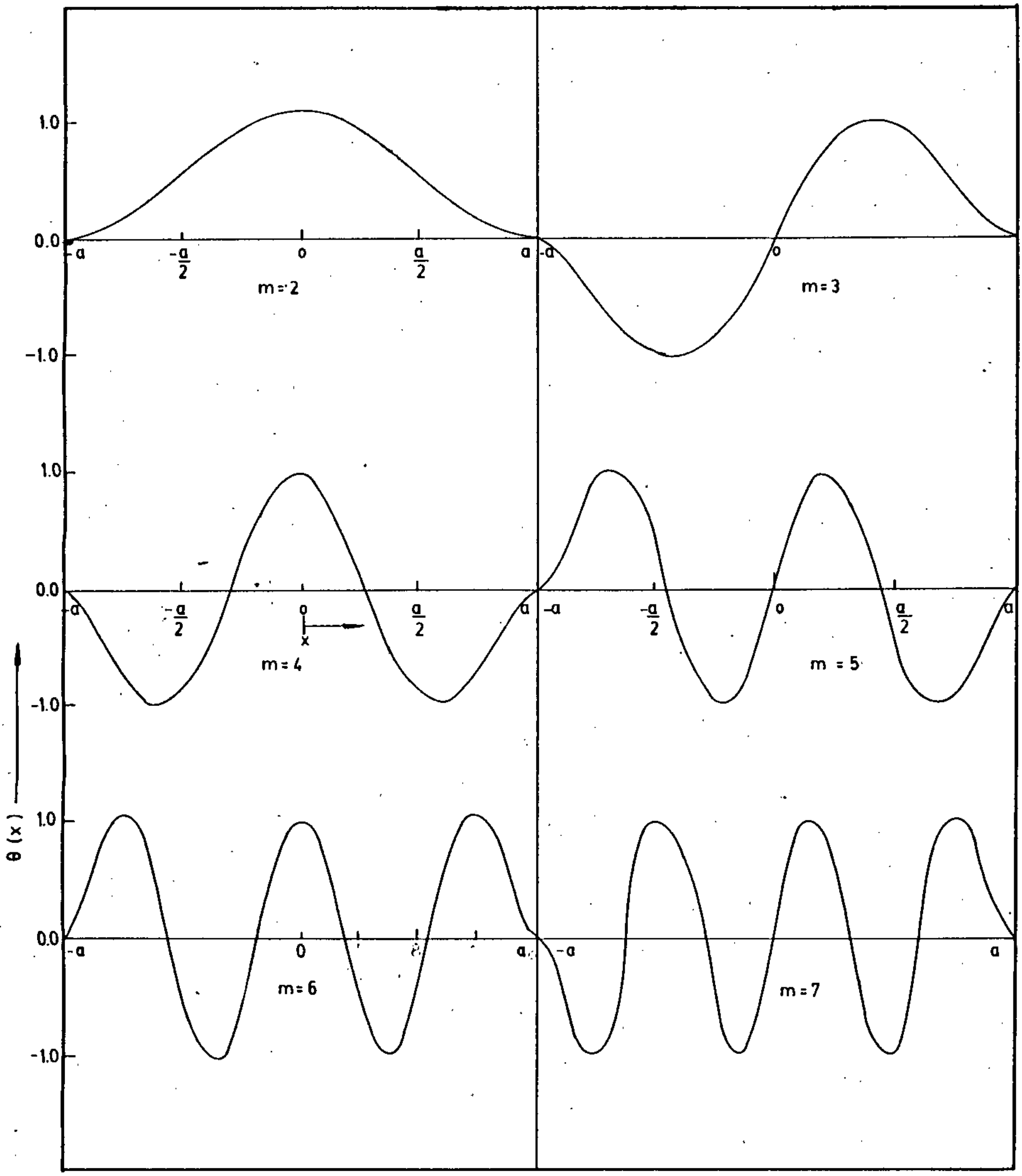

Fig. 3 : Graph of the beam function for clamped ends.

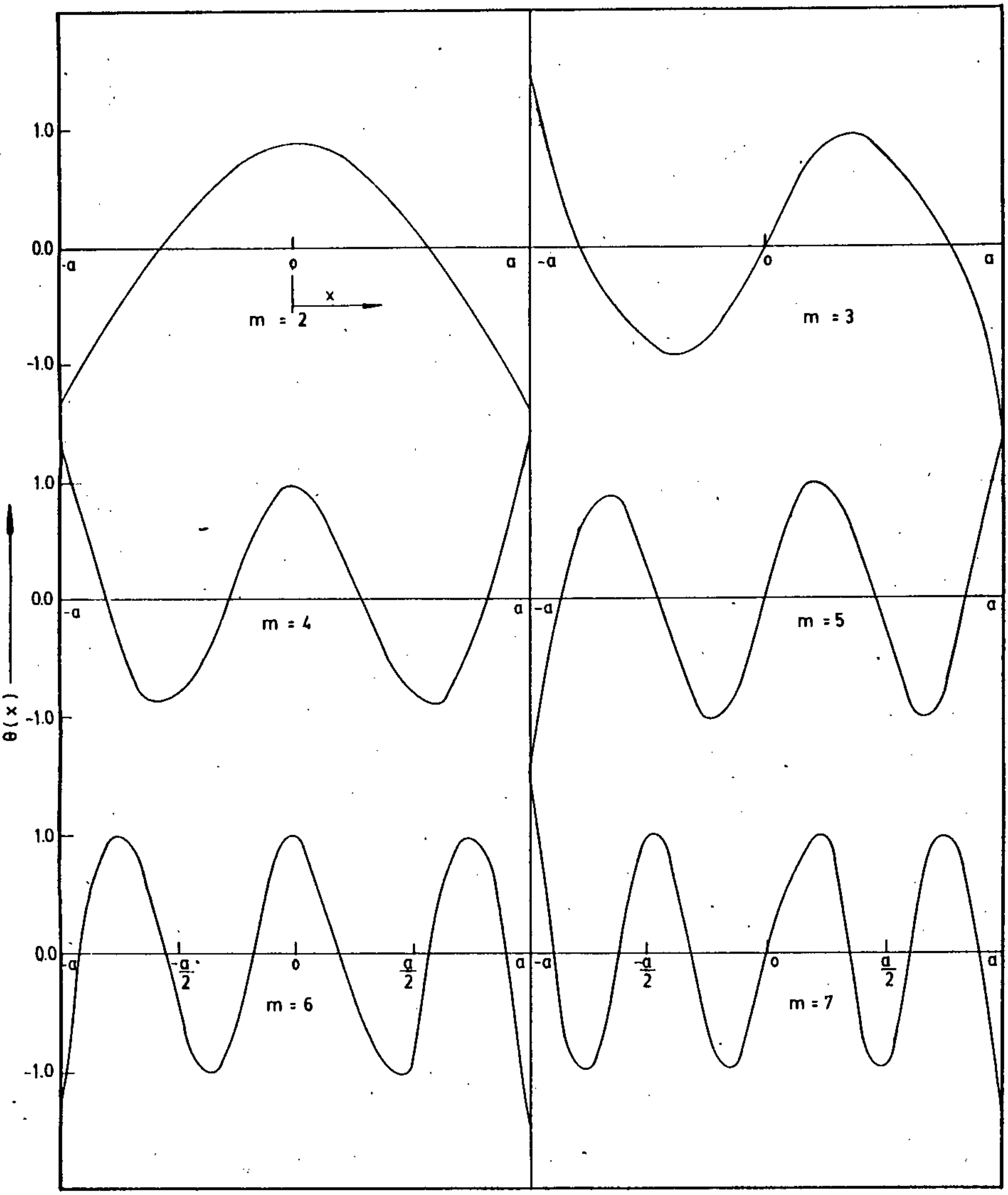

Fig. 4 : Graph of the beam function for freely suspended ends.

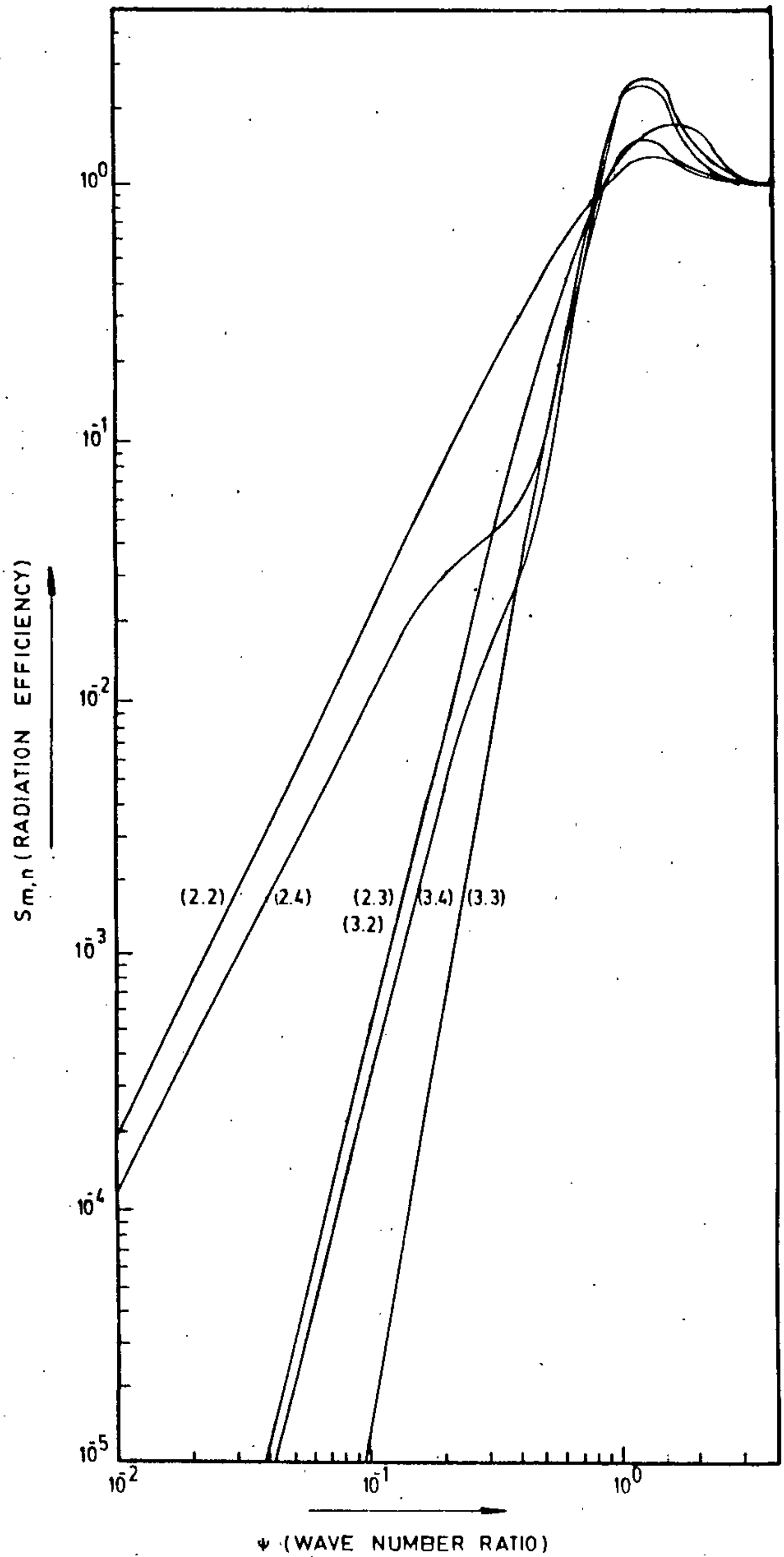

Fig. 5 : Radiation efficiency of a simply supported plate for low-high mode order.

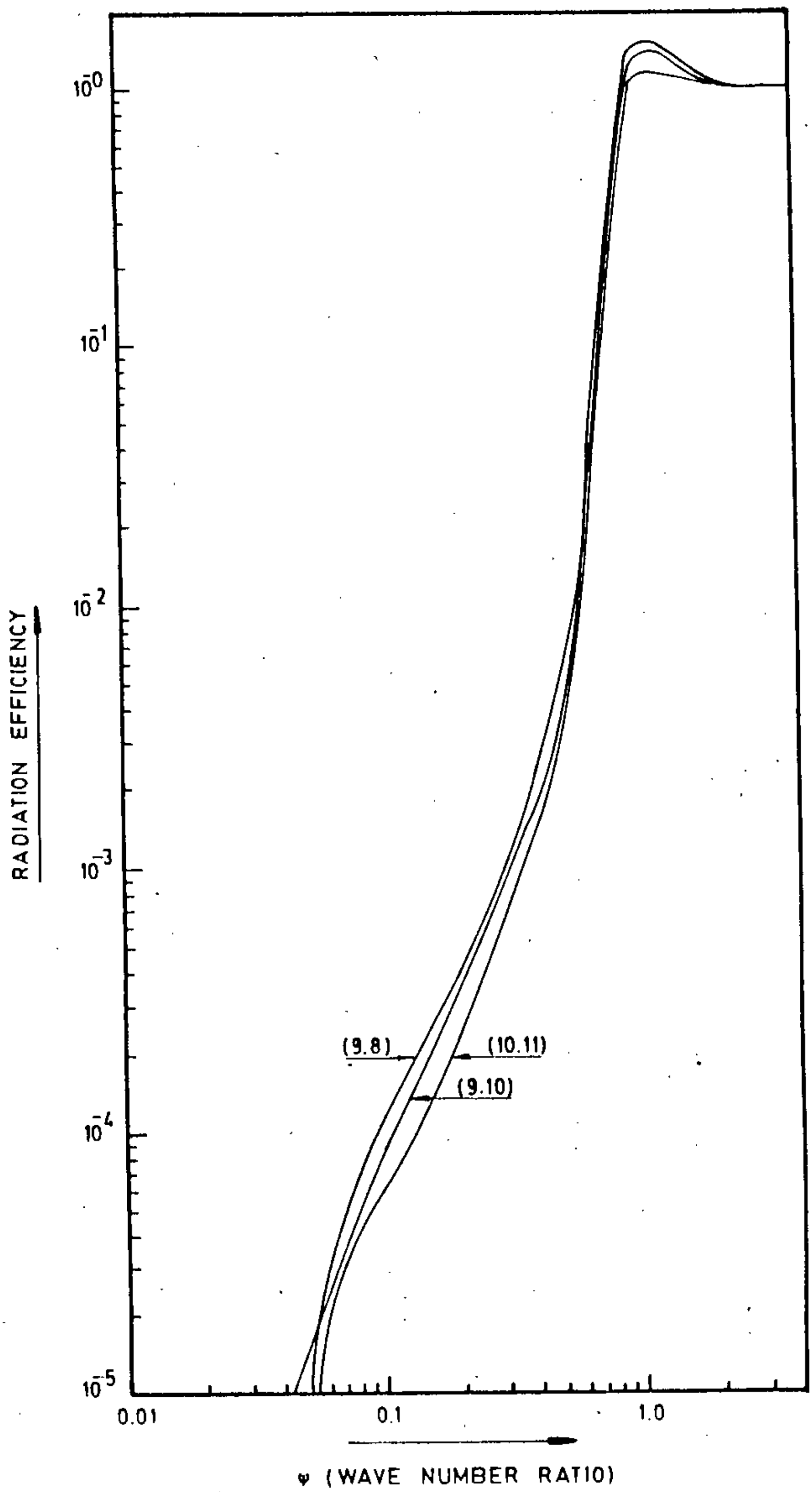

Fig. 6 : Radiati.n efficiency of a simply supported plate for high-high mode orders.

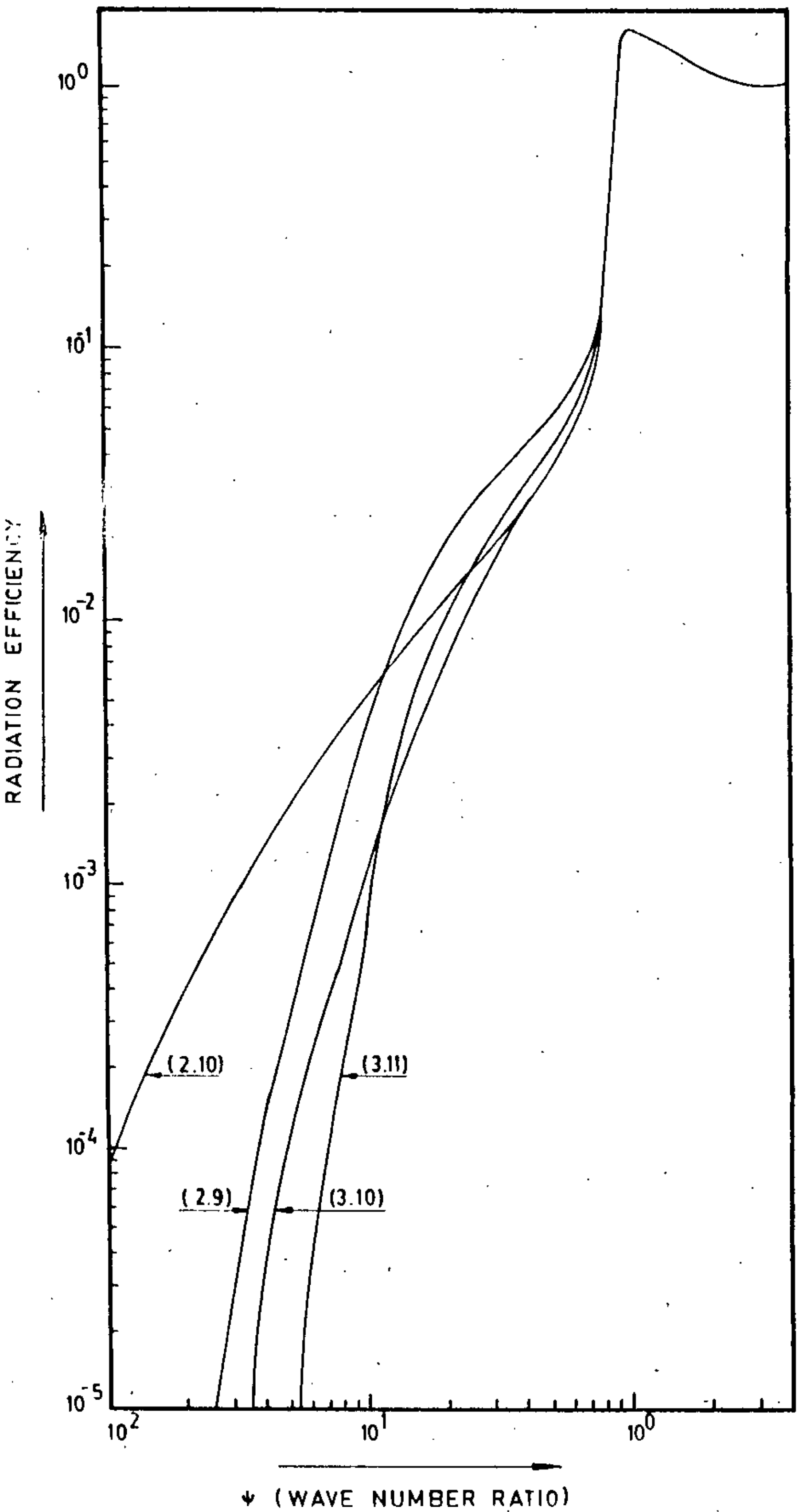

Fig. 7 : Radiation efficiency of a simply-supported plate for low high mode orders.

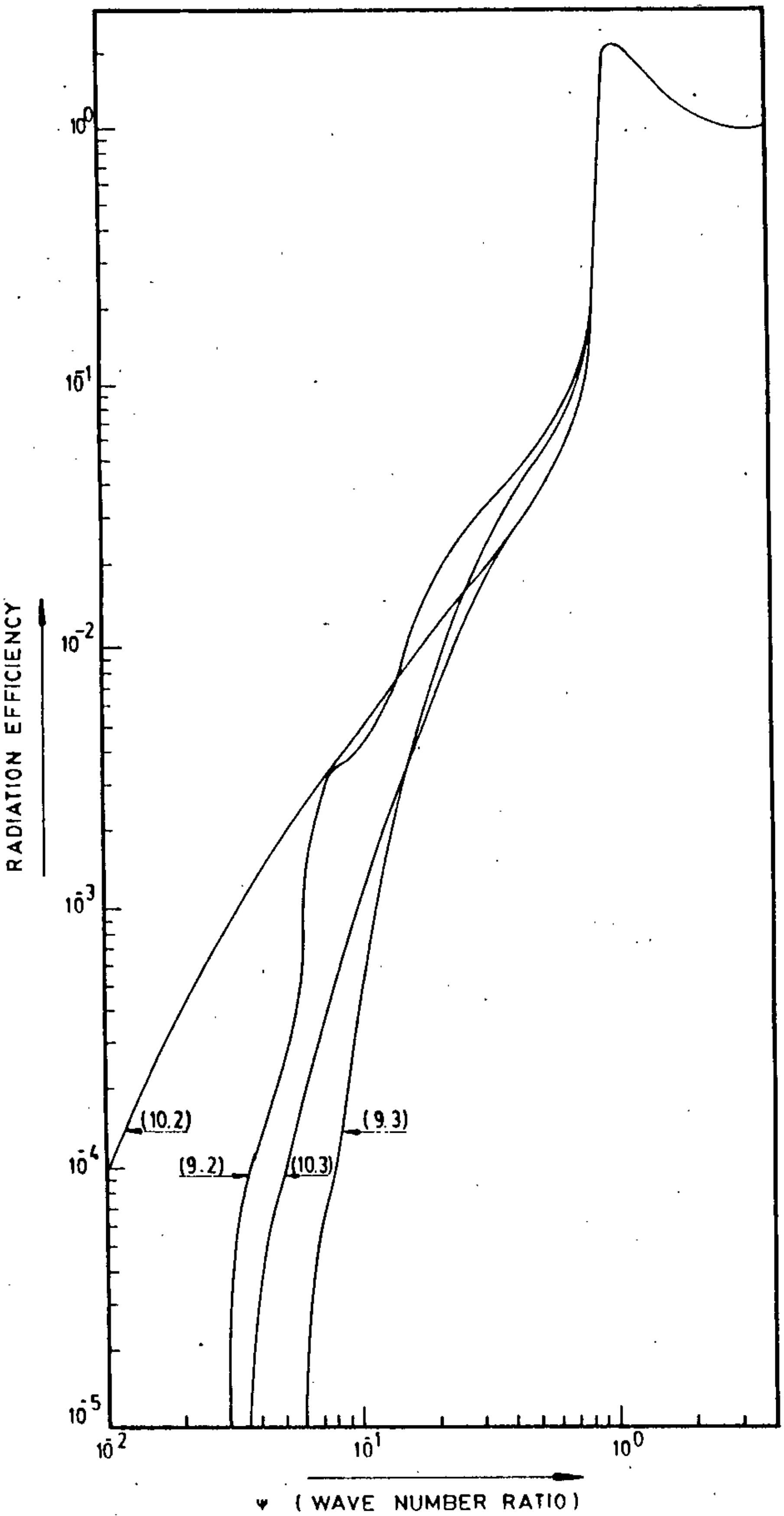

Fig. 8 : Radiation efficiency of a simply-supported plate for high-low mode orders.

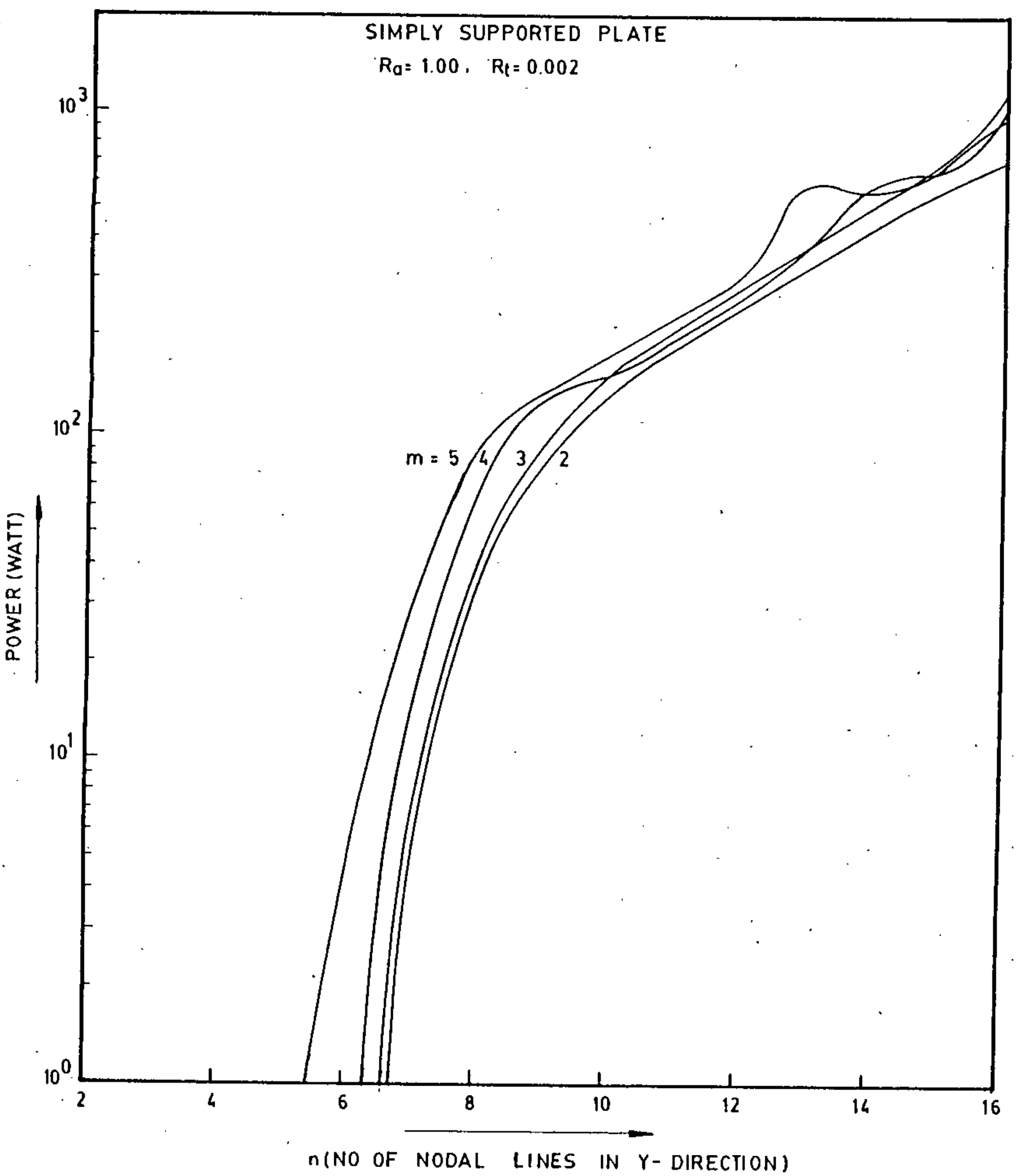

Fig. 9 : Power radiated by a simply-supported plate for low values of m.

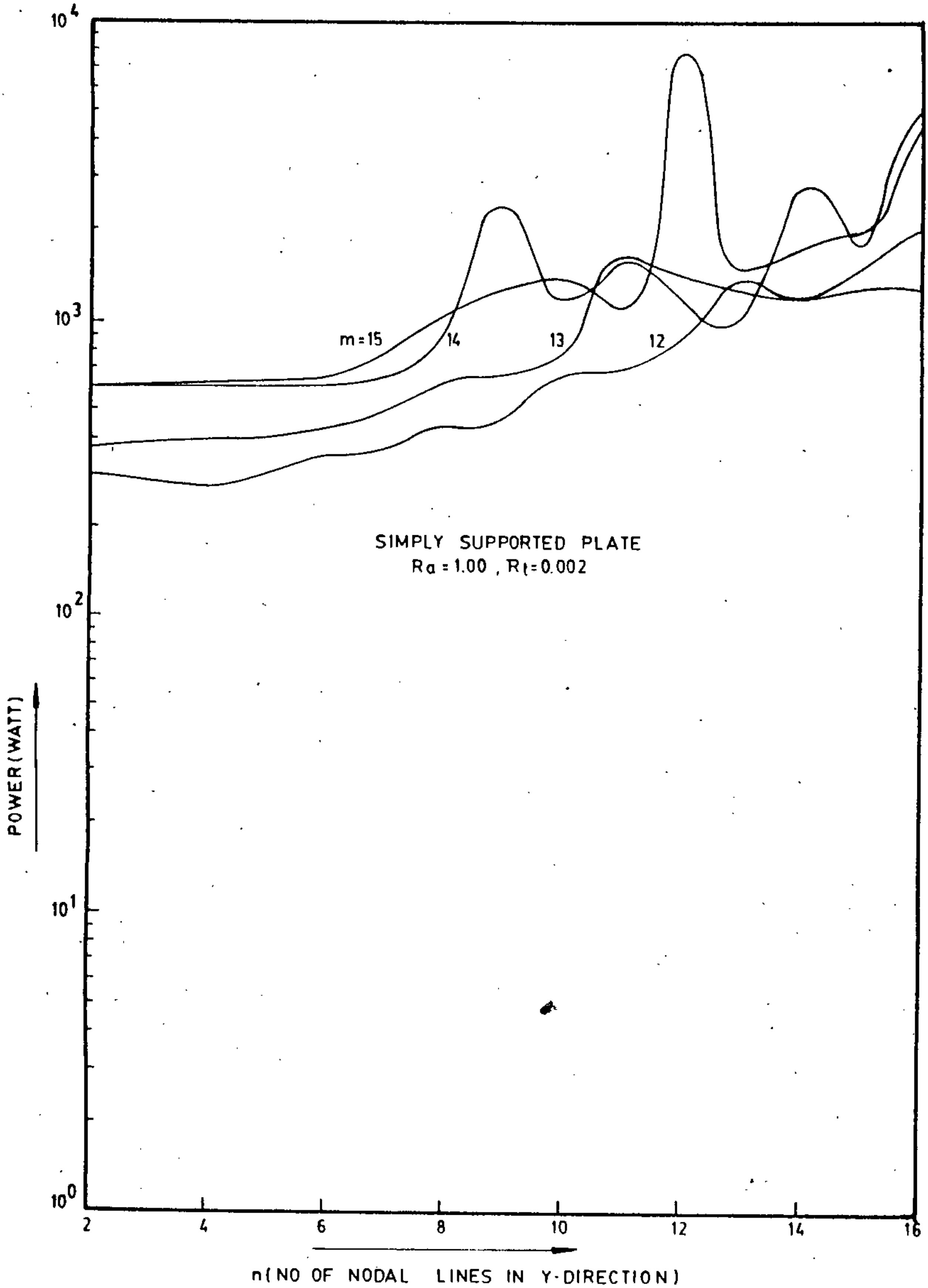

Fig. 10 : Power radiated by a simply-supported plate for high values of m.

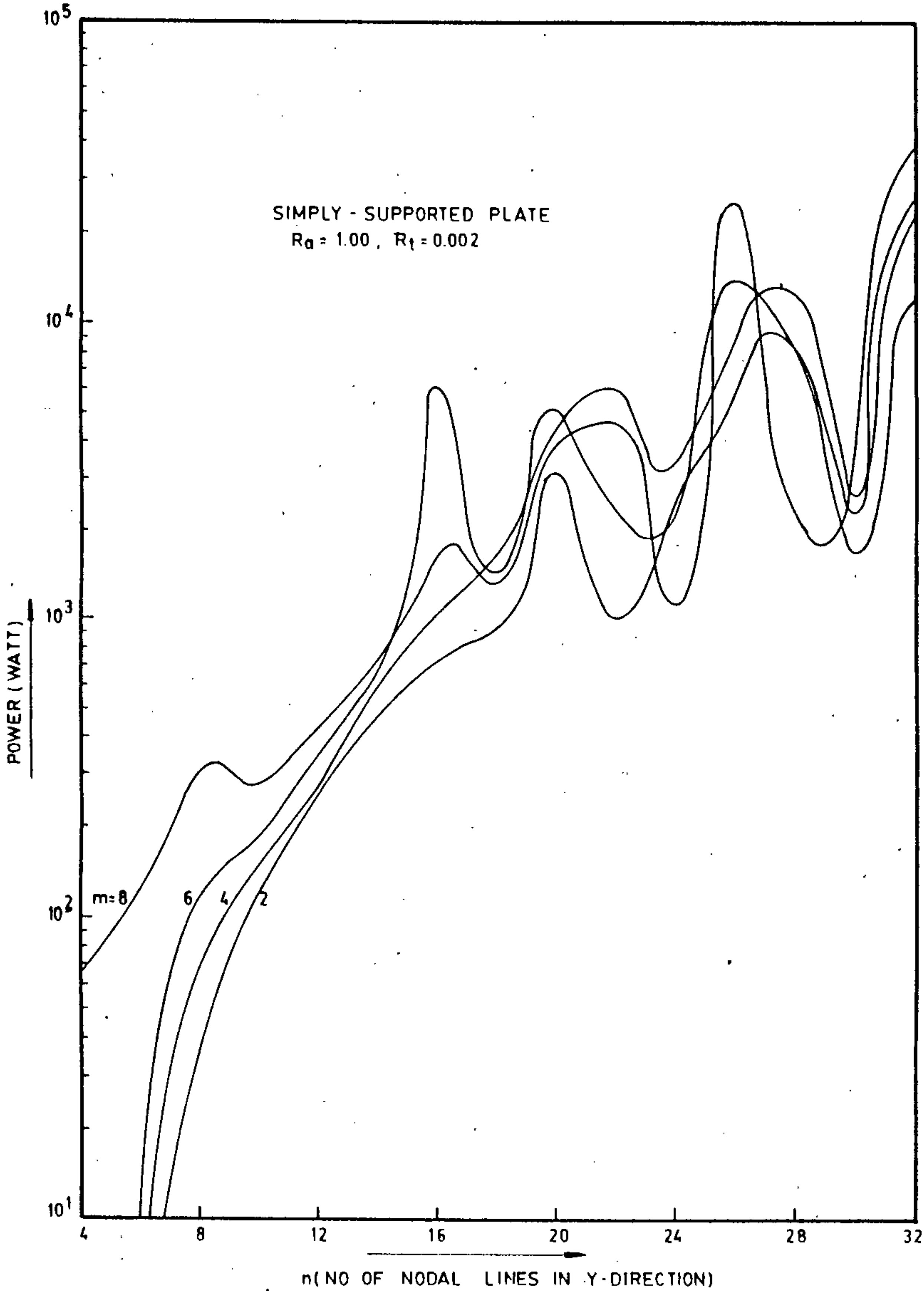

Fig. 11 : Power radiated by a simply-supported plate for low values of m and very high range of the values of n.

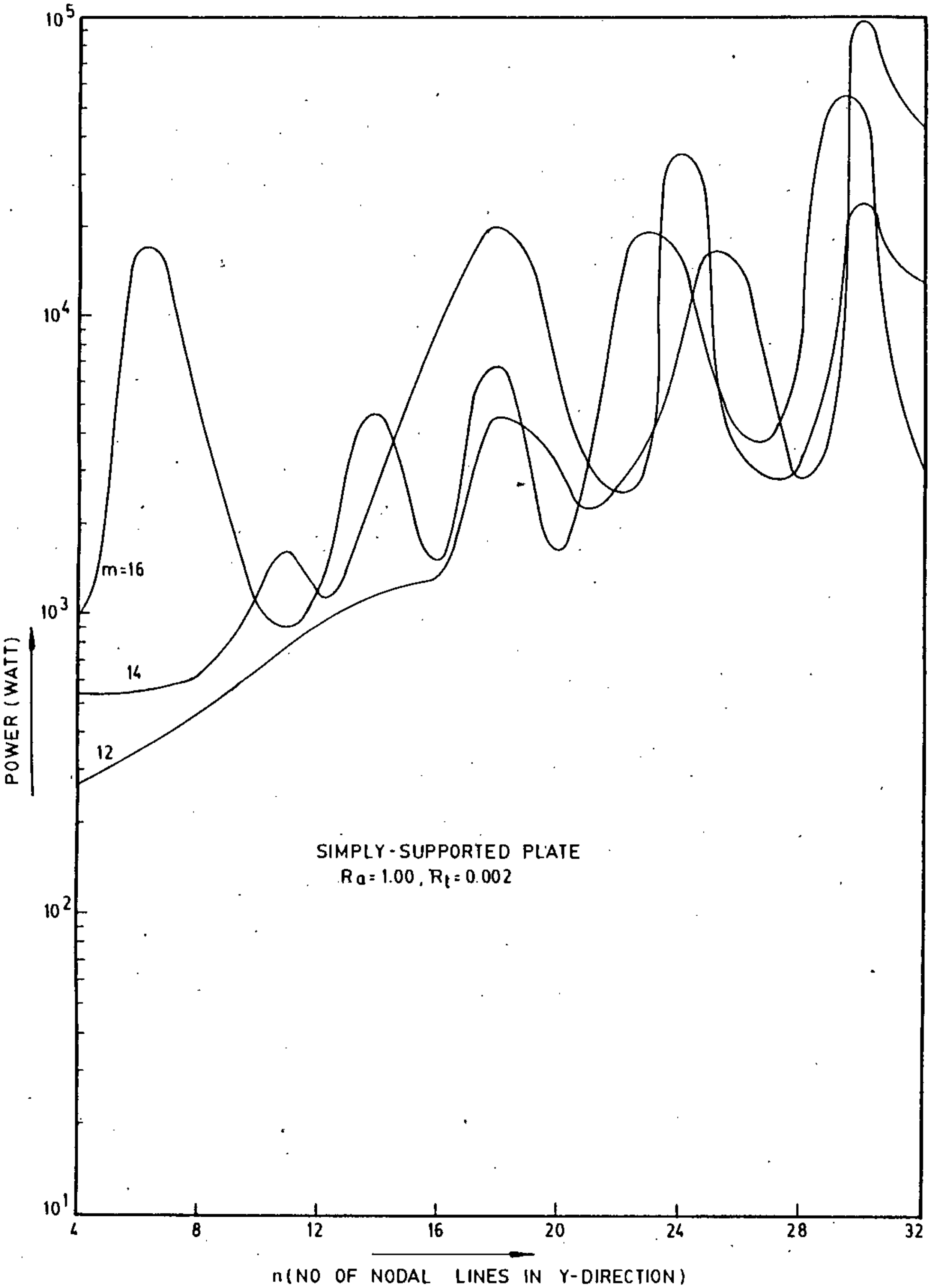

Fig. 12 : Power radiated by a simply-supported plate for high values of m and very high range of the values of n.

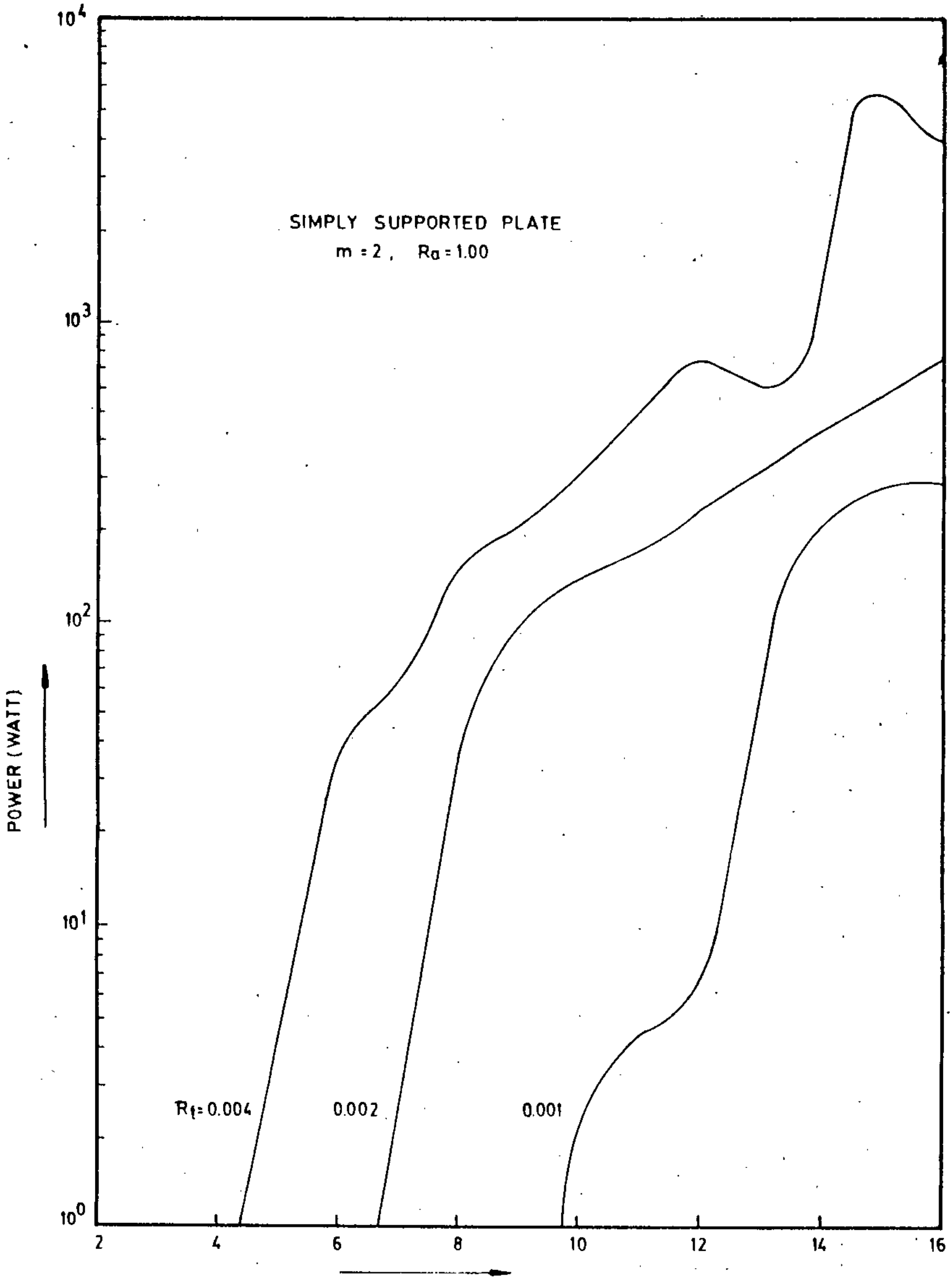

Fig. 13 : Power radiated by a simply-supported plate for different thickness ratios and low values of m.

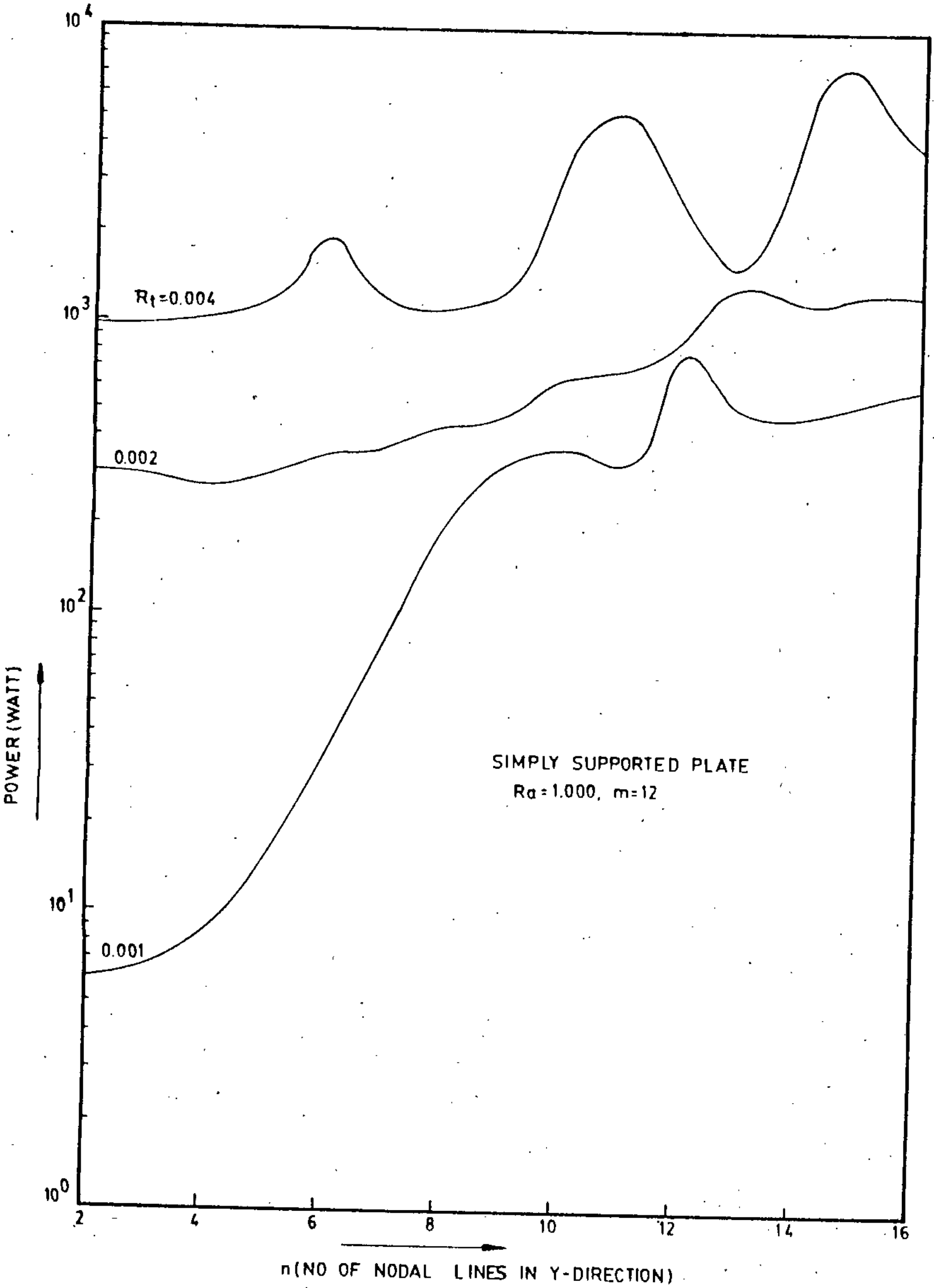

Fig. 14 : Power radiated by a simply-supported plate for different thickness ratios and high values of φ

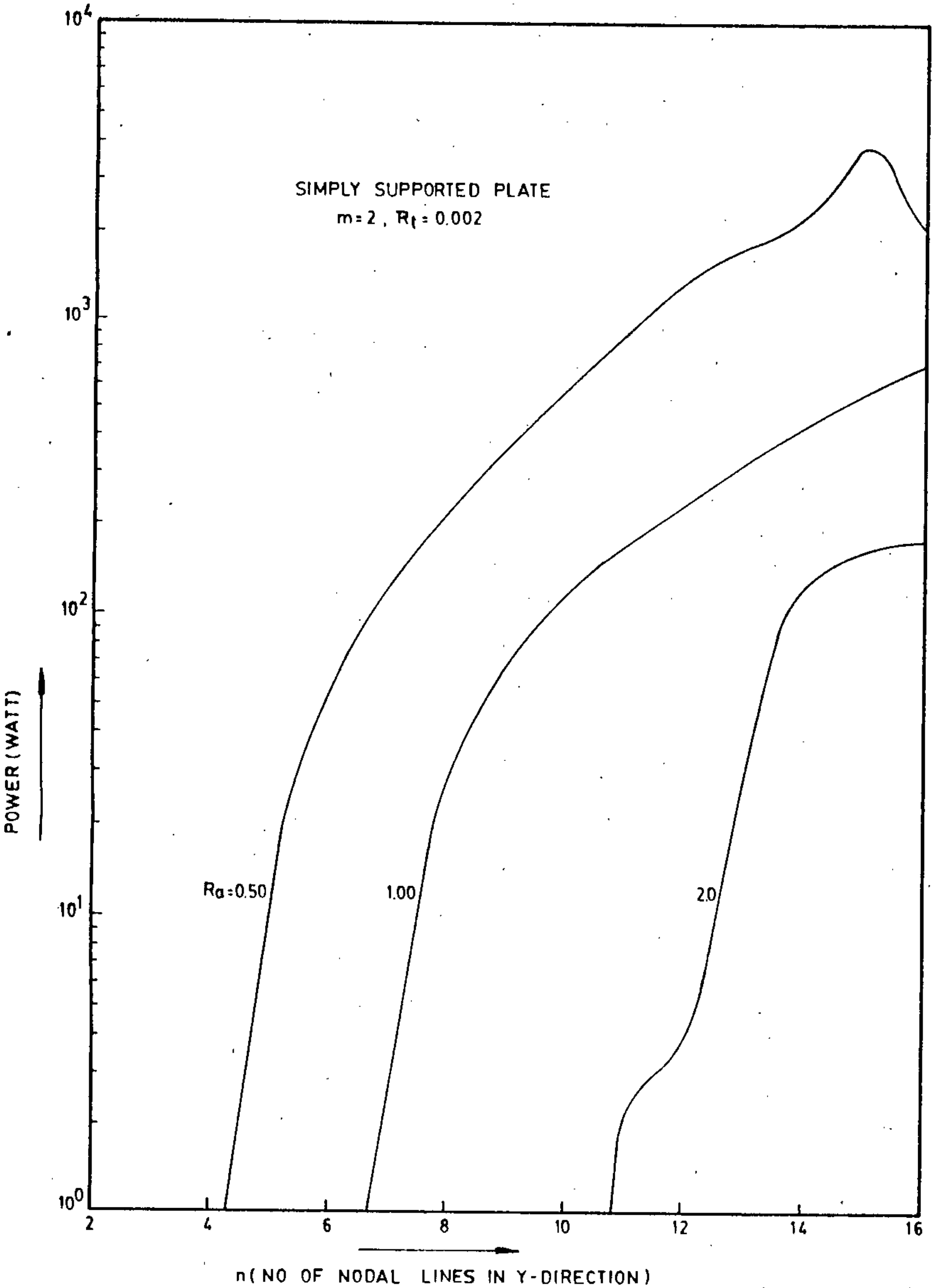

Fig. 15 : Power radiated by a simply-supported plate for different aspect ratios and low values of m.

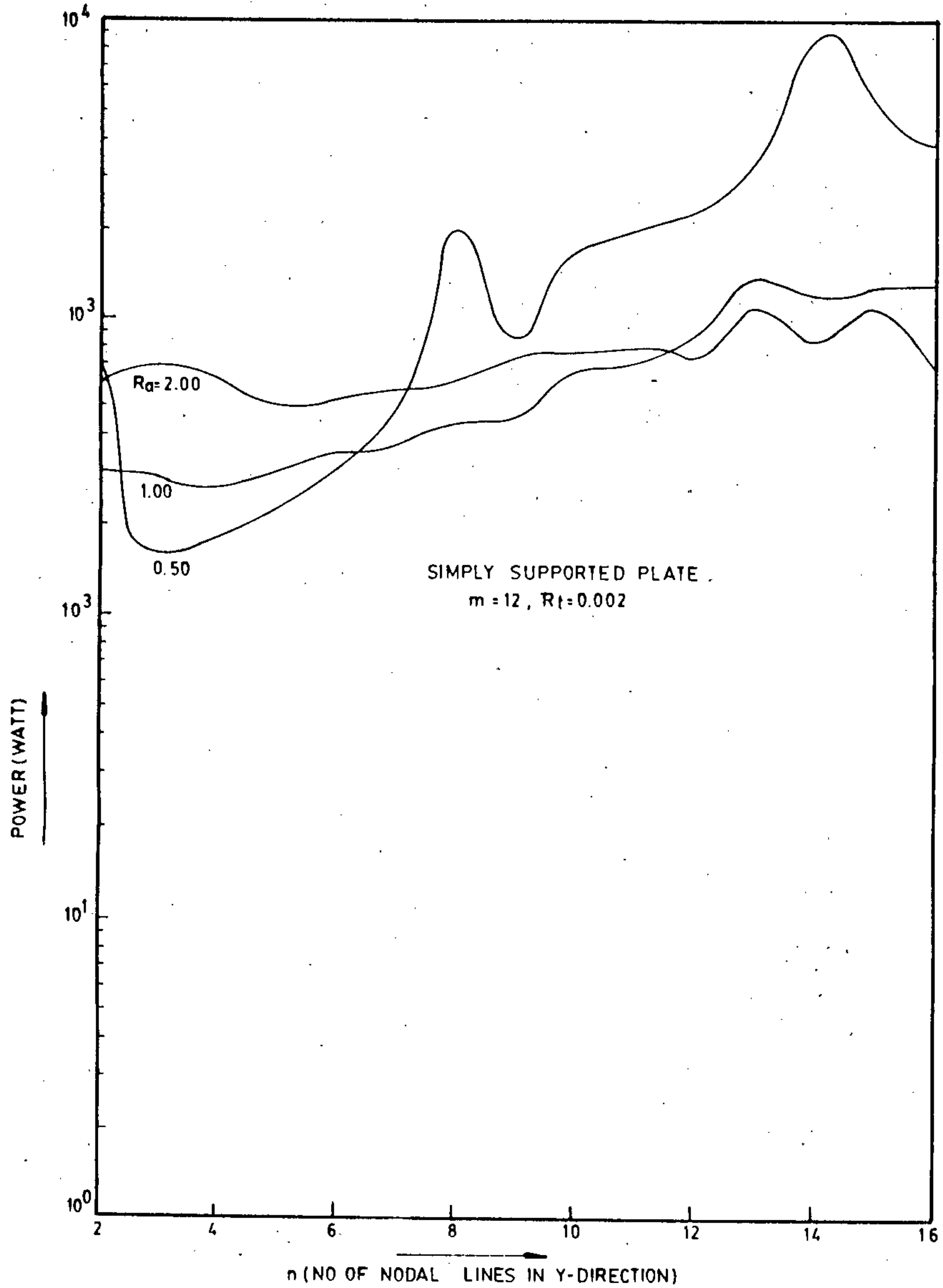

Fig. 16 : Power radiated by a simply-supported plate for different aspect ratios and high values of m.

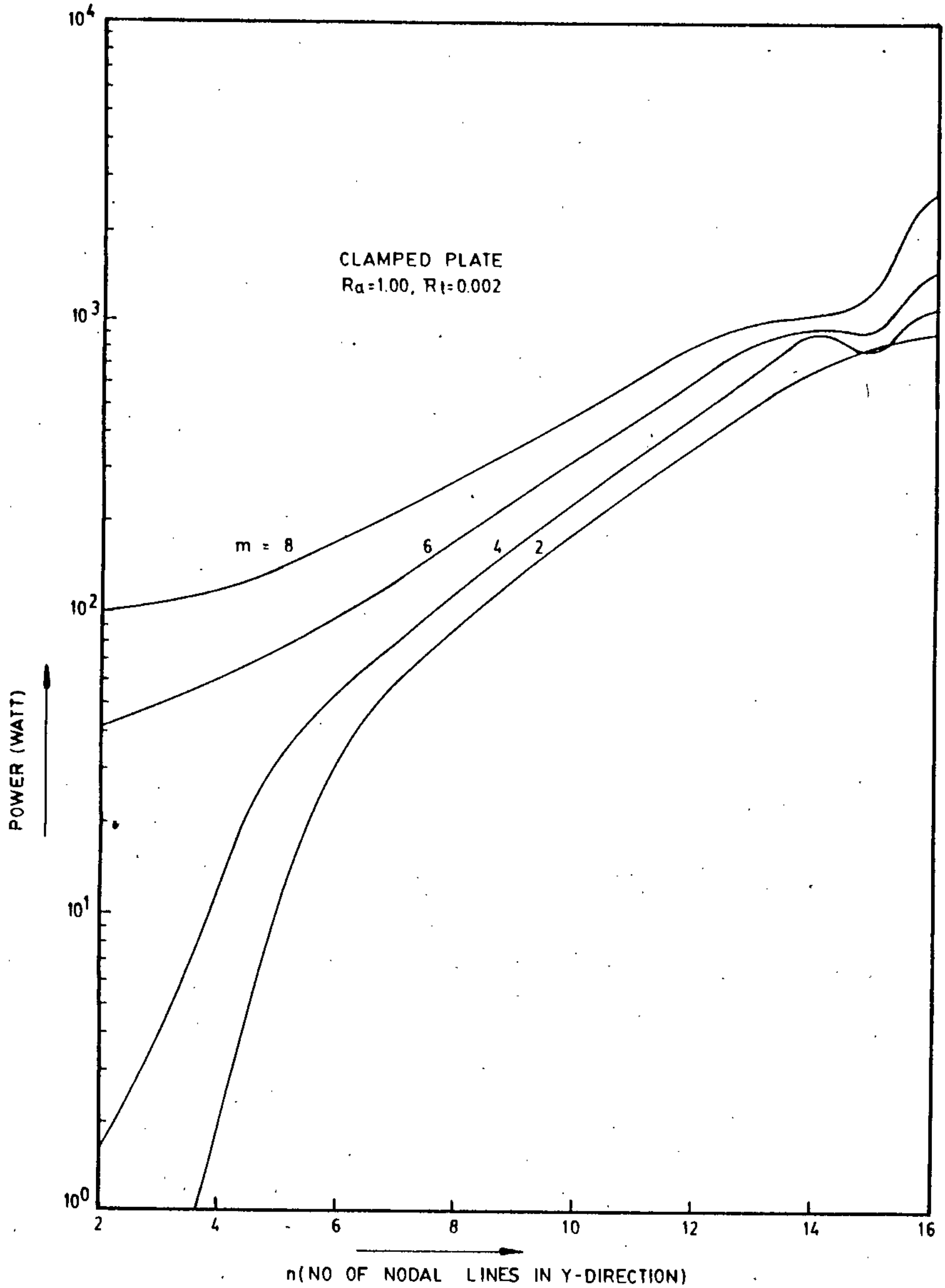

Fig. 17 : Power radiated by a clamped plate for low values of m.

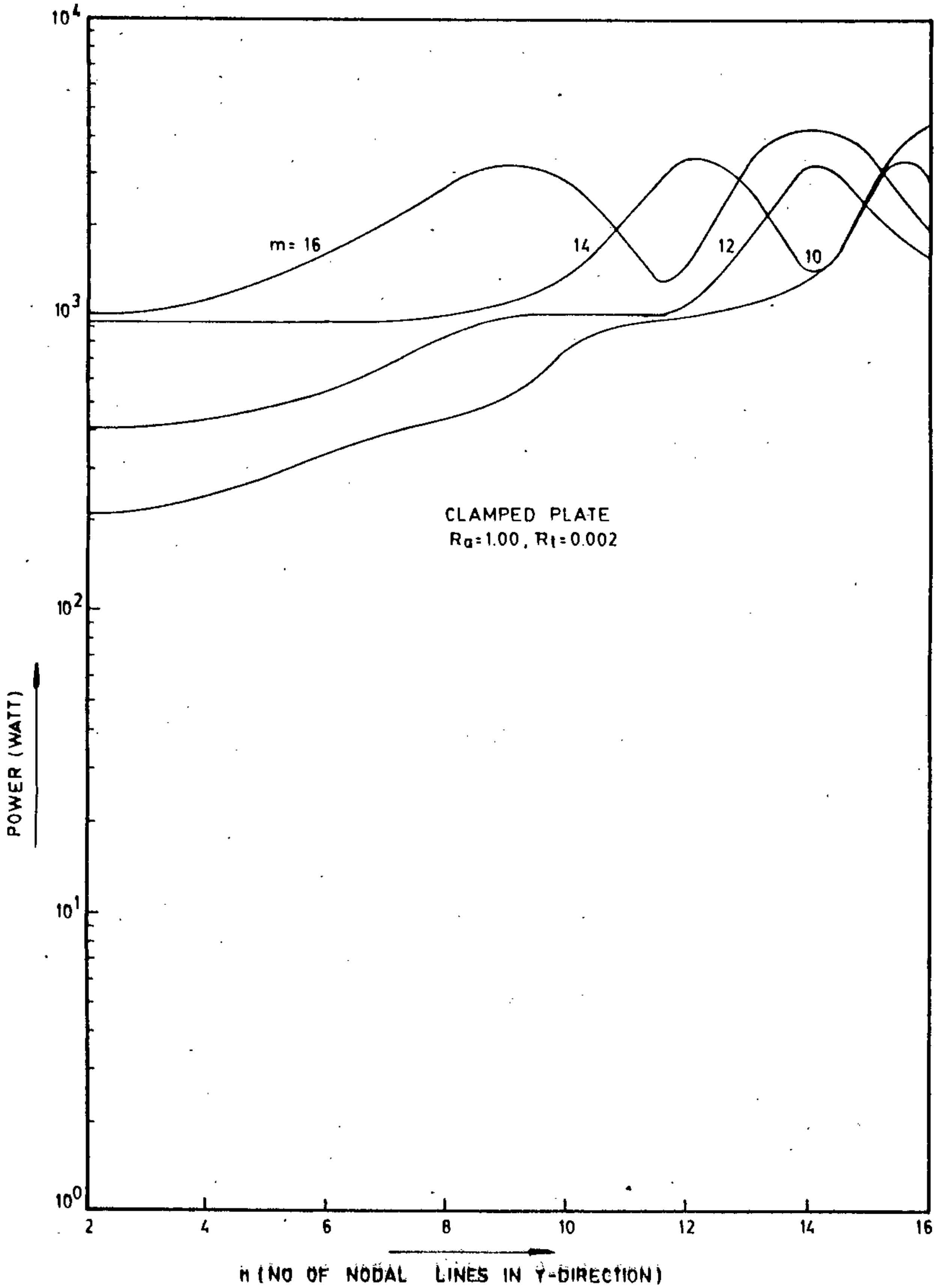

Fig. 18 : Power radiated by a clamped plate for high values of m.

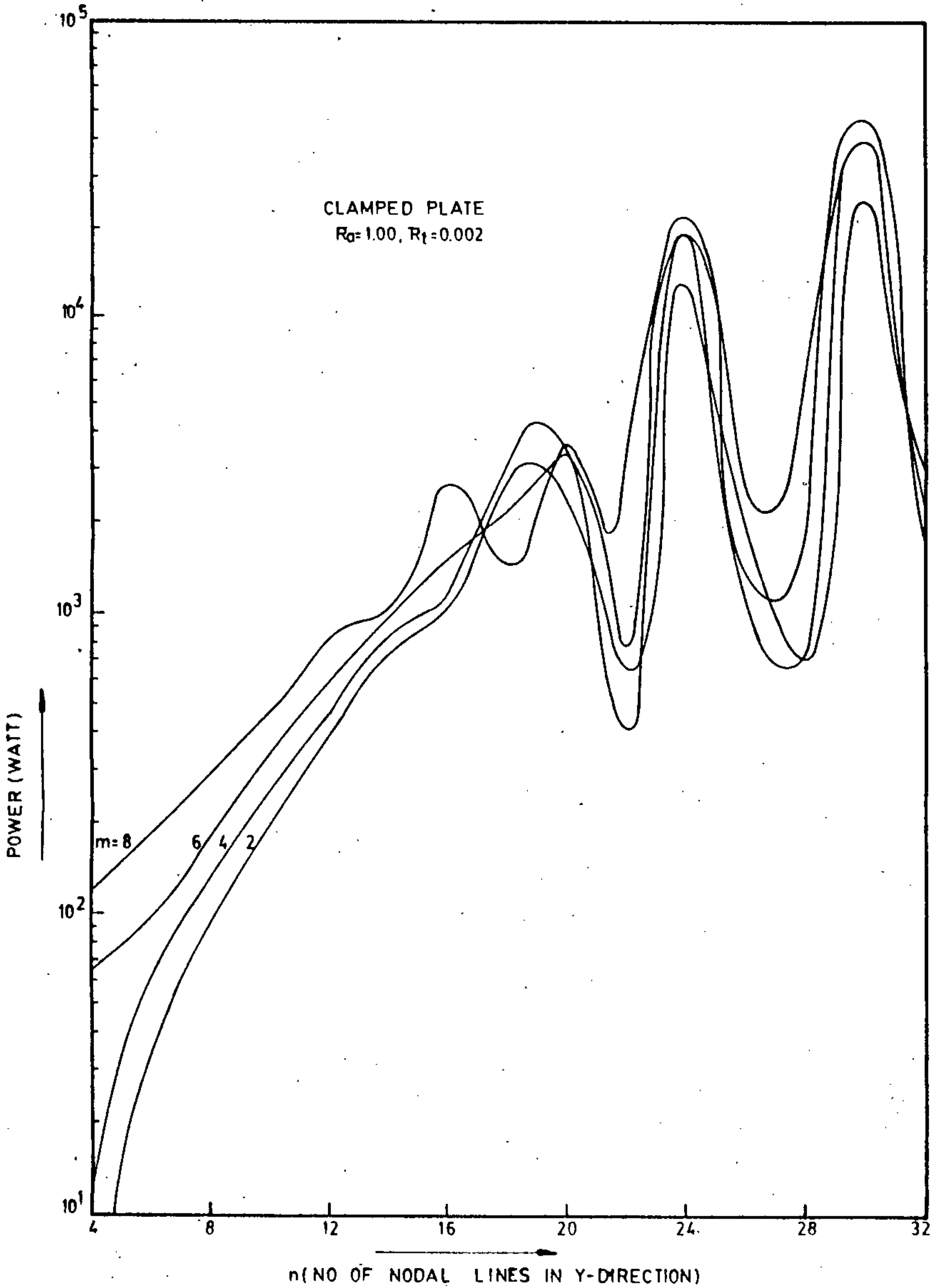

Fig. 19 : Power radiated by a clamped plate for low values of m and very high range of the values of n.

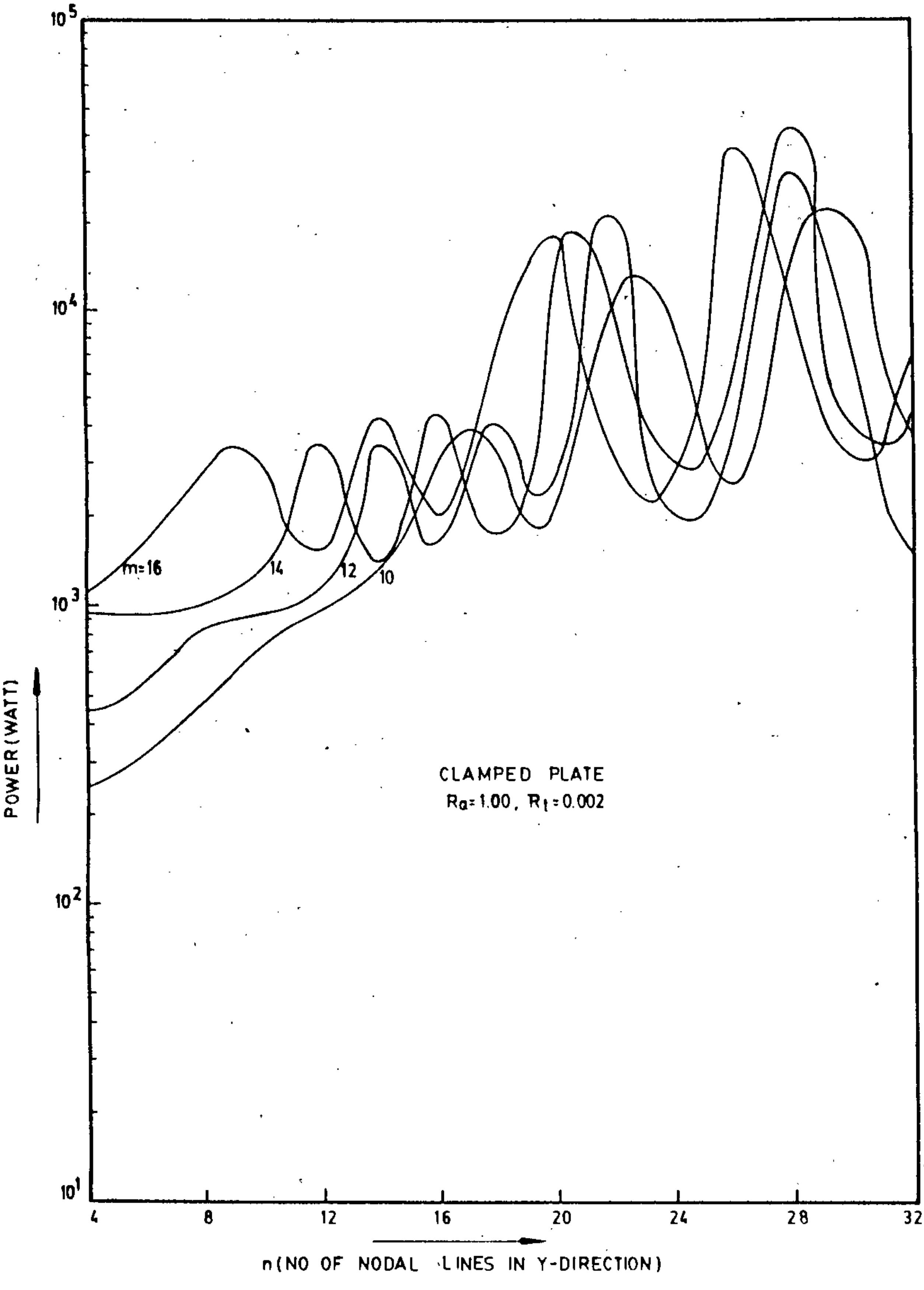

Fig. 20 : Power radiated by a clamped plate for high values of m and very high range of the values of n.

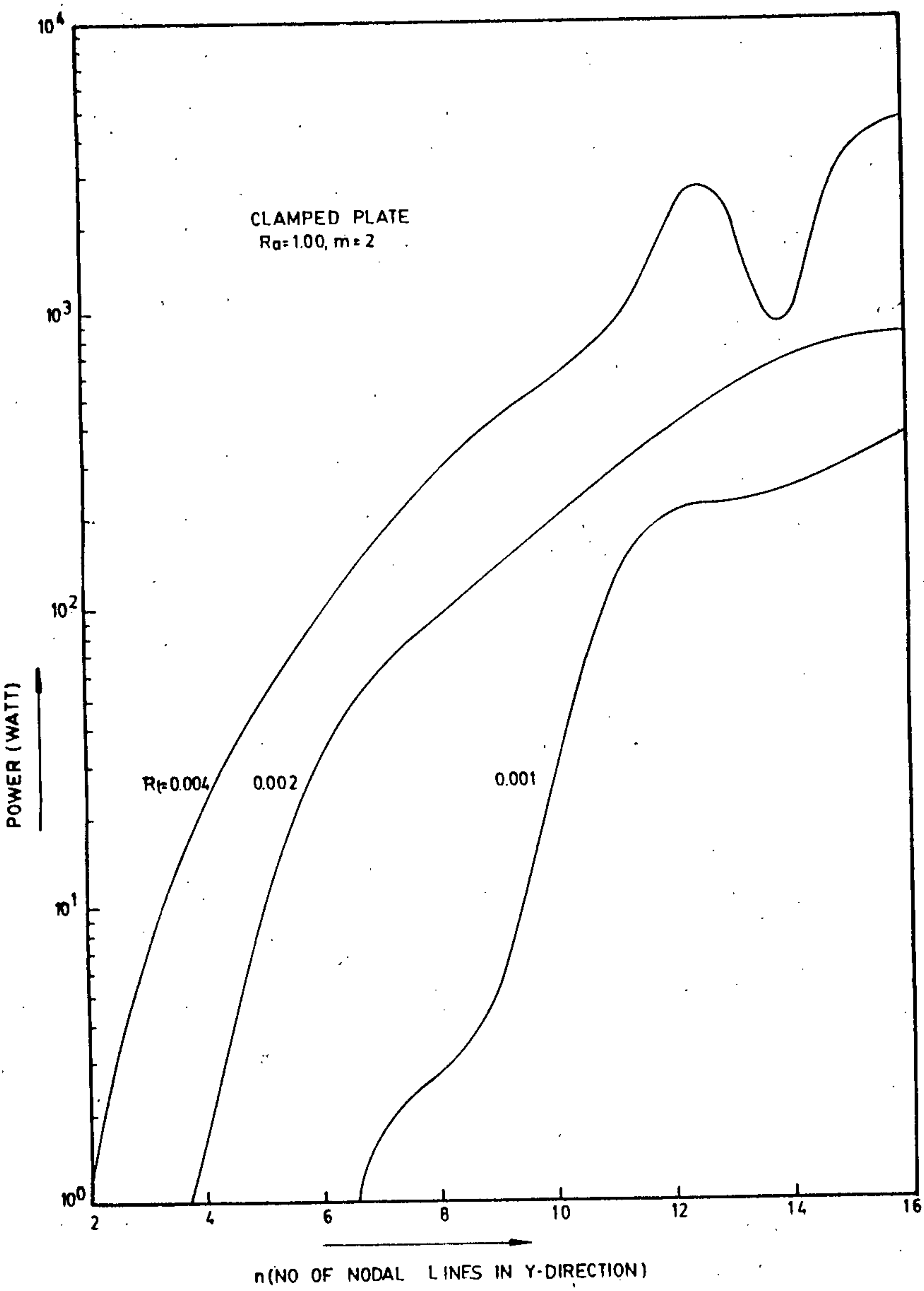

Fig. 21 : Power radiated by a clamped plate for different thickness ratios and low values of m.

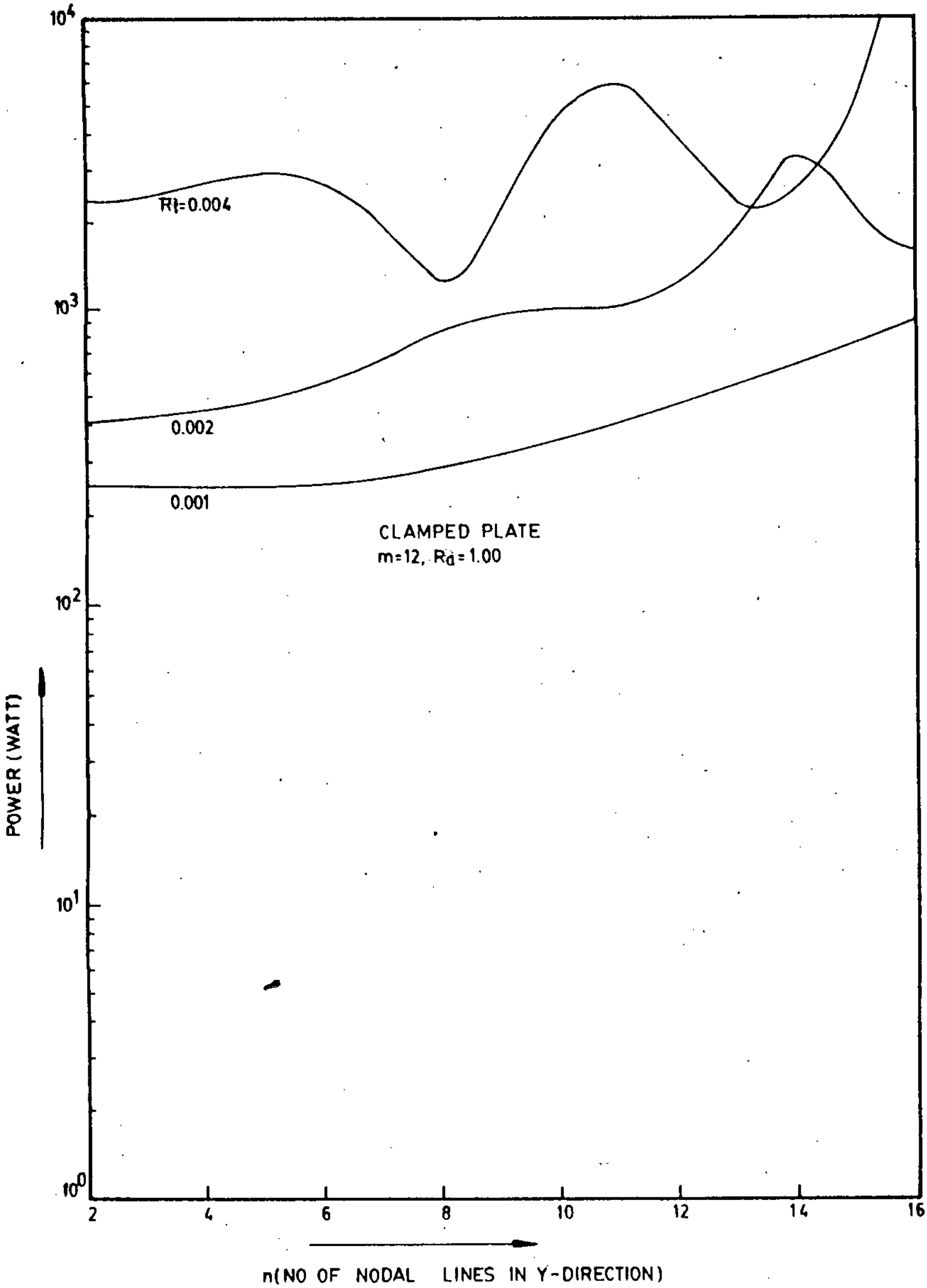

Fig. 22 : Power radiated by a clamped plate for different thickness ratios and high values of m.

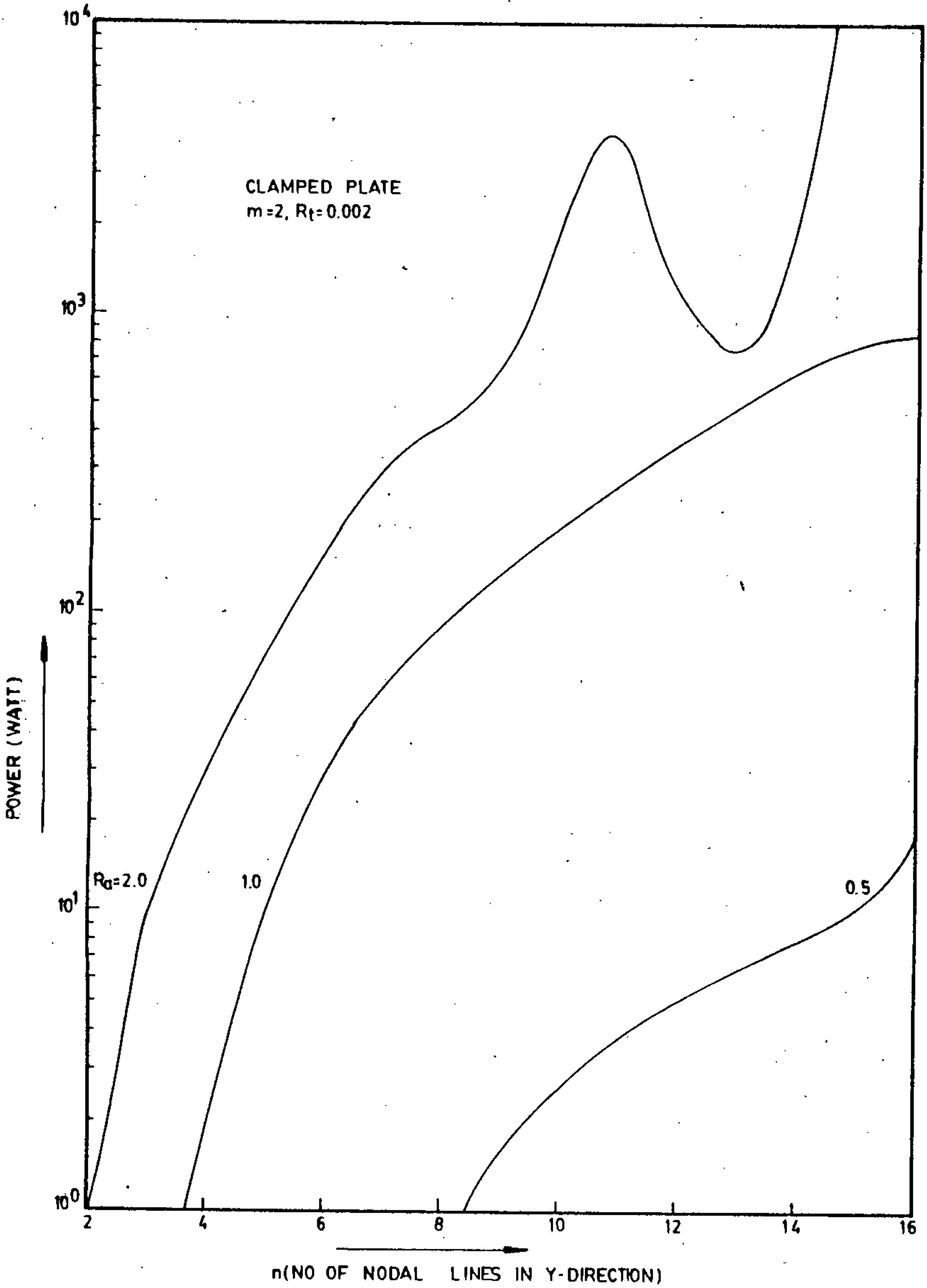

Fig. 23 : Power radiated by a clamped plate for different aspect ratios and low values of m.

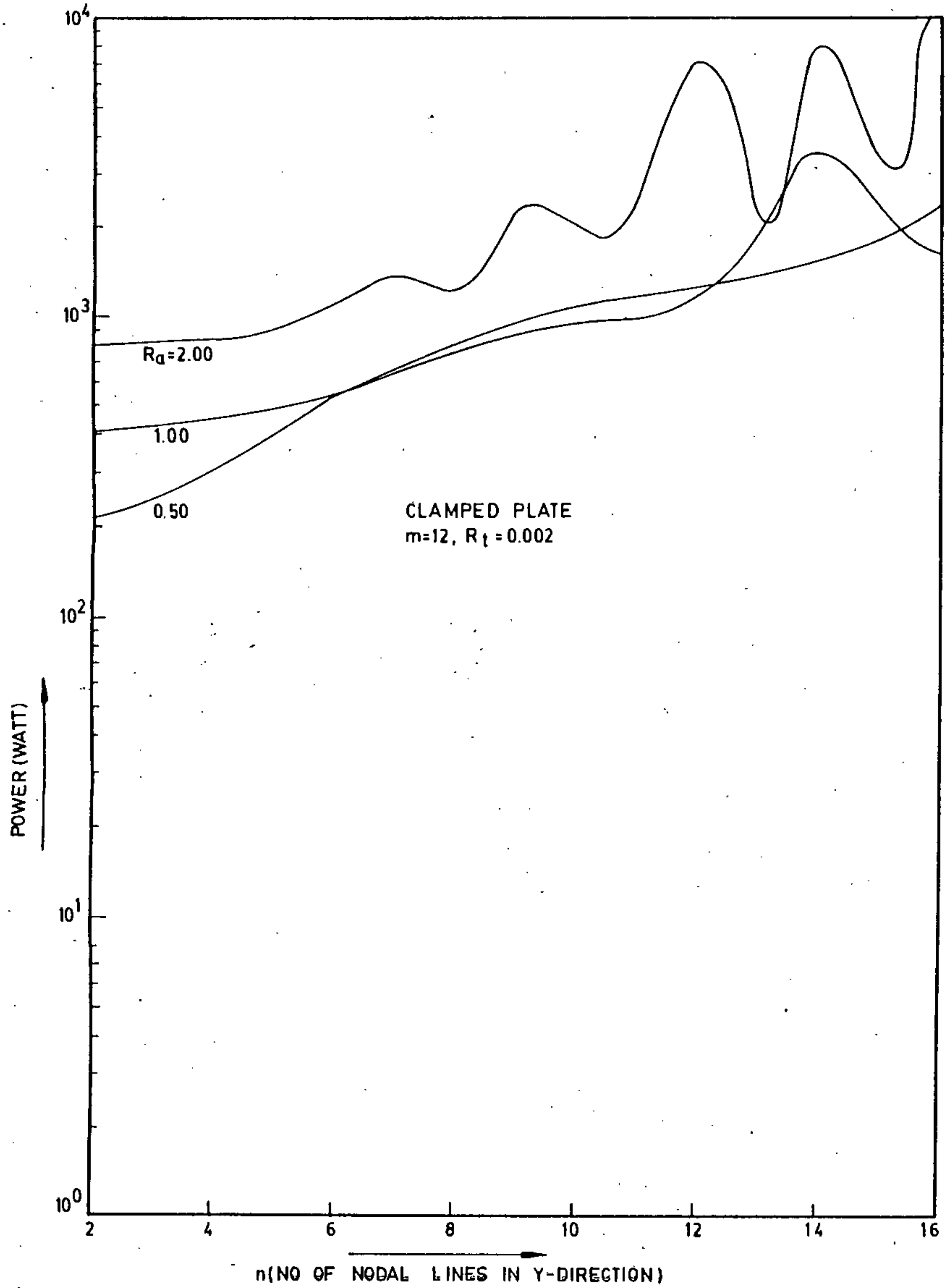

Fig. 24 : Power radiated by clamped plate for different aspect ratios and high values of m.

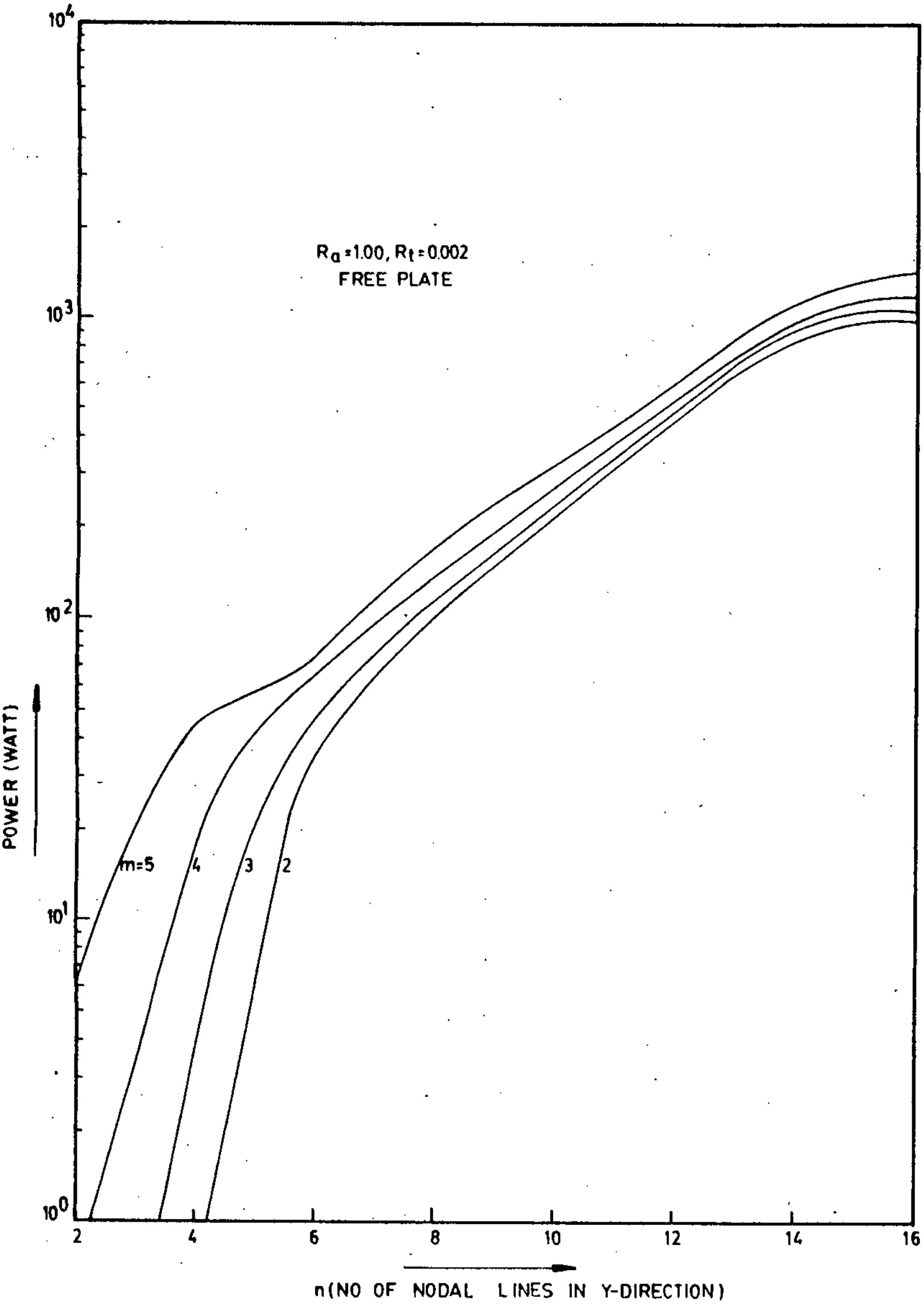

Fig. 25 : Power radiated by a freely-suspended plate for low values of m.

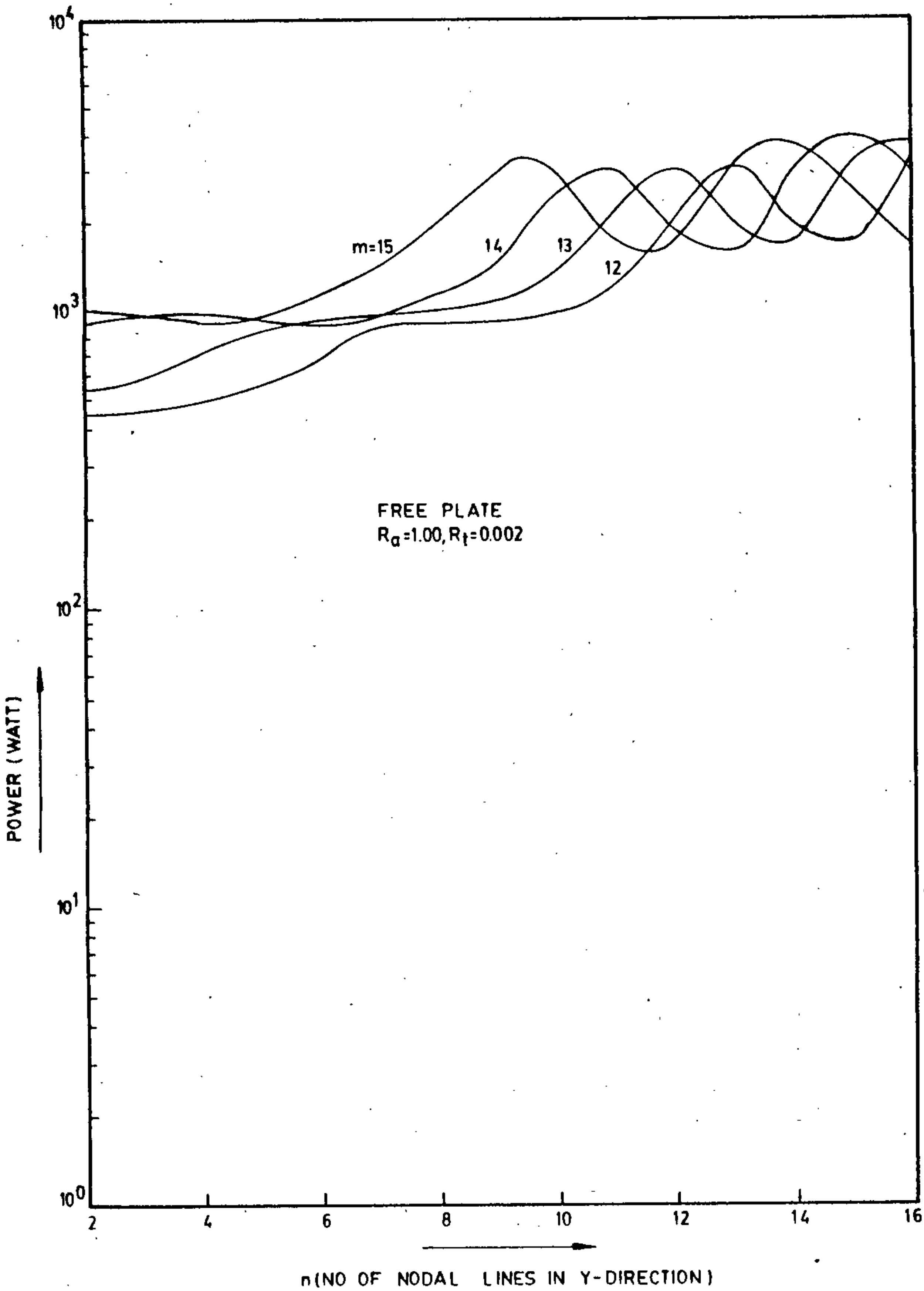

Fig. 26 : Power radiated by a freely-suspended plate for high values of m.

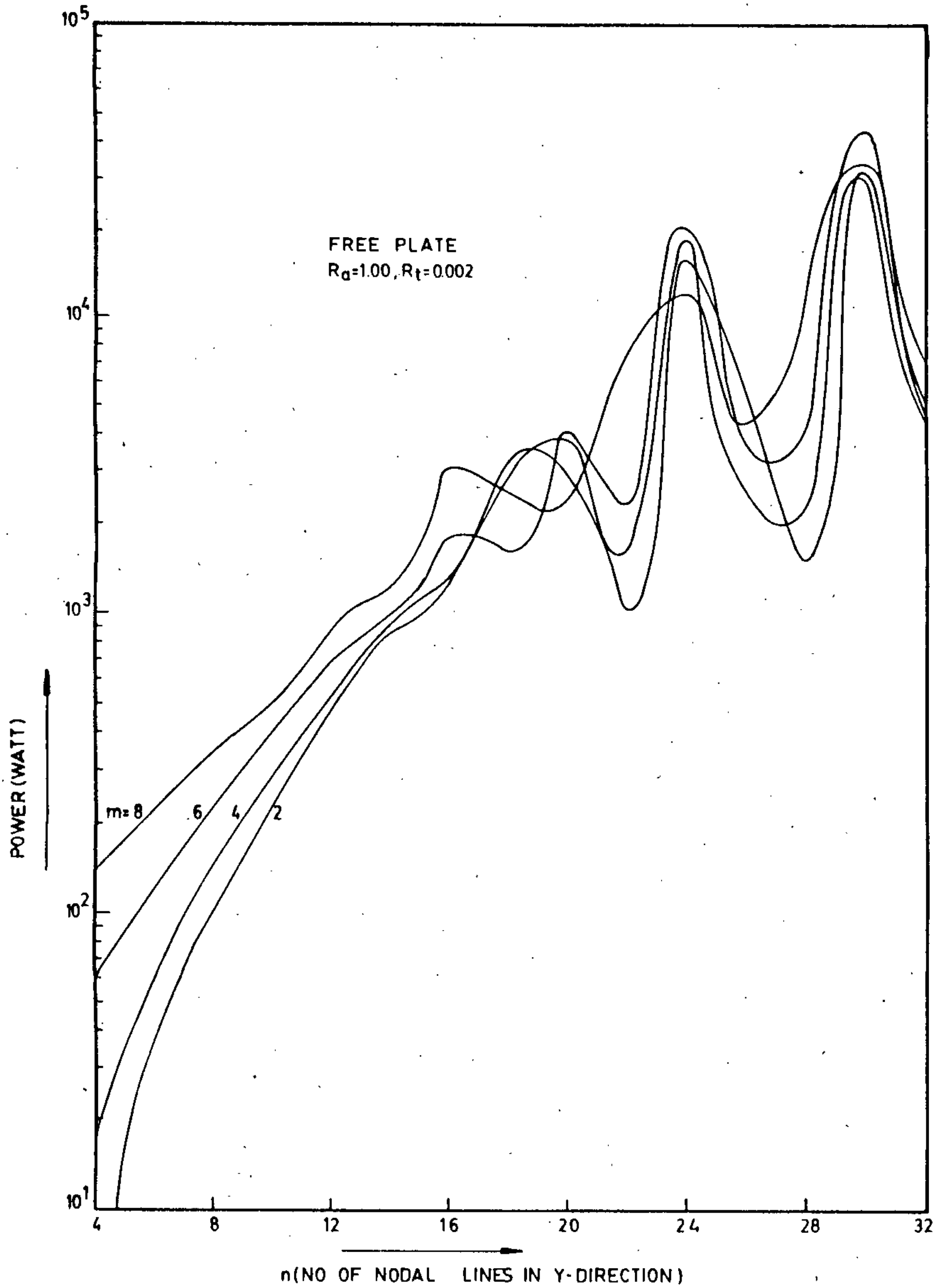

Fig. 27 : Power radiated by a freely-suspended plate for low values of m and very high range of the values of n.

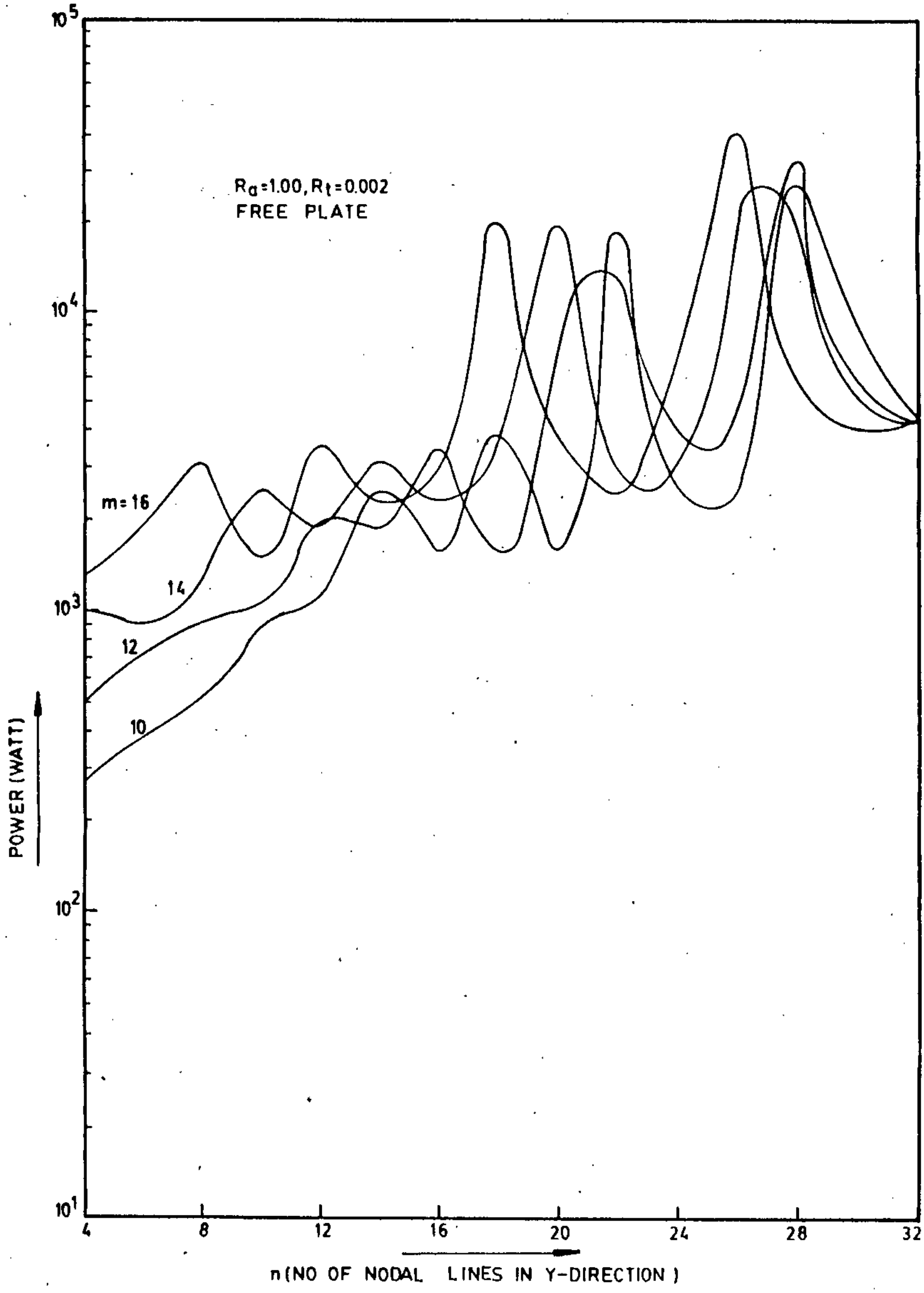

Fig. 28 : Power radiated by a freely-suspended plate for high values of m and very high range of the values of n.

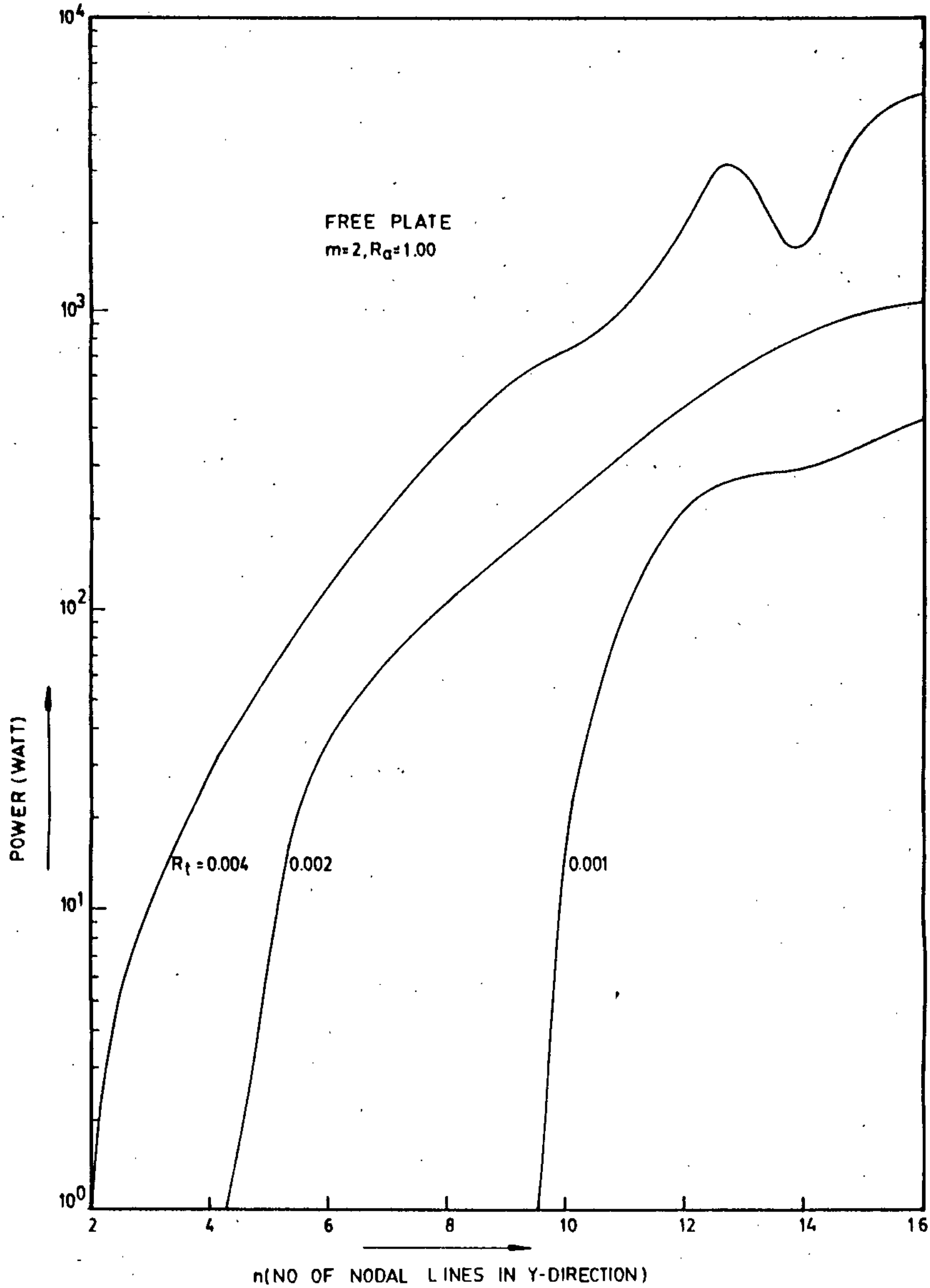

Fig. 29 : Power radiated by a freely-suspended plate for different thickness ratios and low values of m.

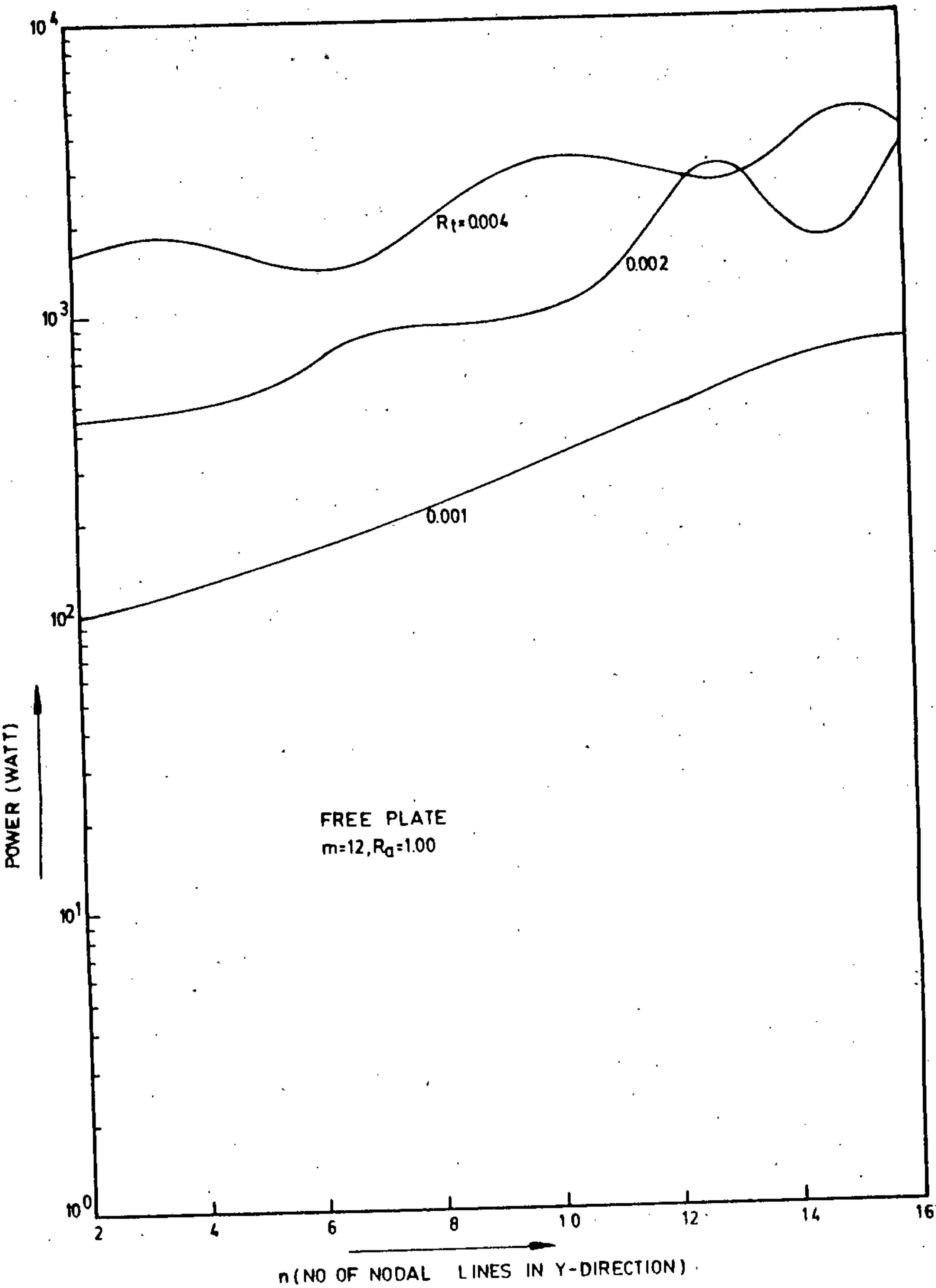

Fig. 30 : Power radiated by a freely-suspended plate for different thickness ratios and high values of m.

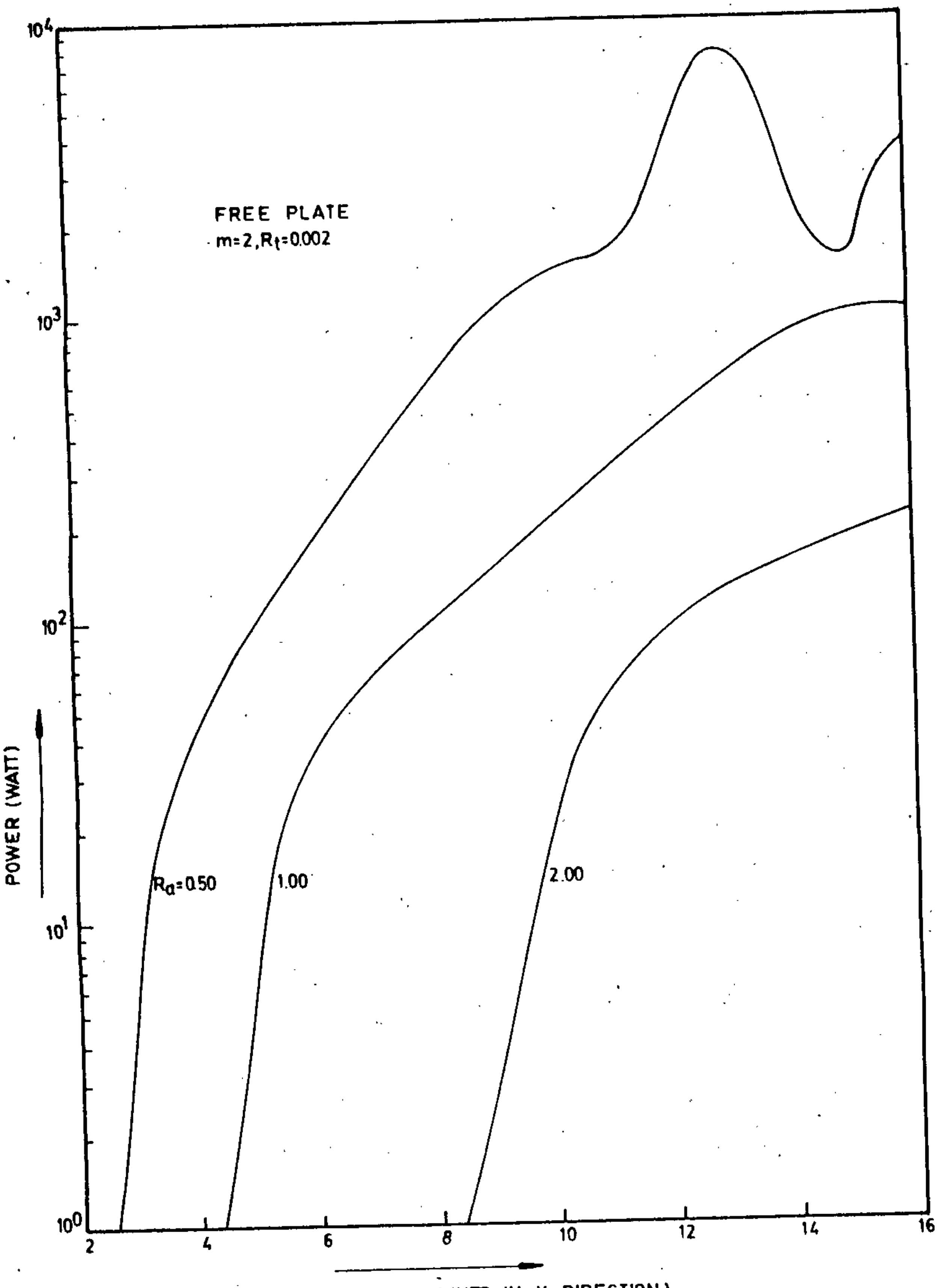

Fig. 31 : Power radiated by a freely-suspended plate for different aspect ratios and low values of m.

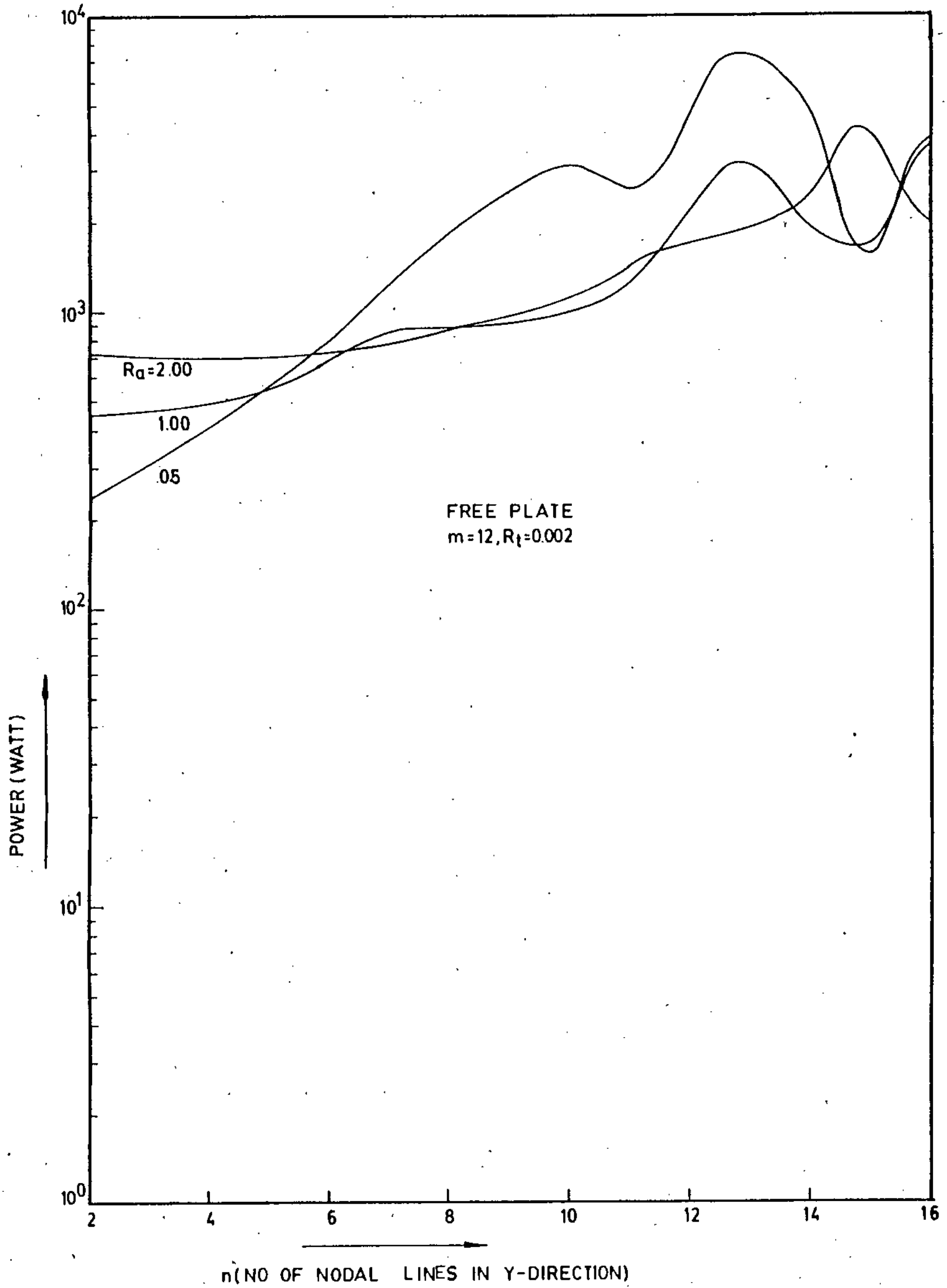

Fig. 32 : Power radiated by a freely-suspended plate for different aspect ratios and high values of m.

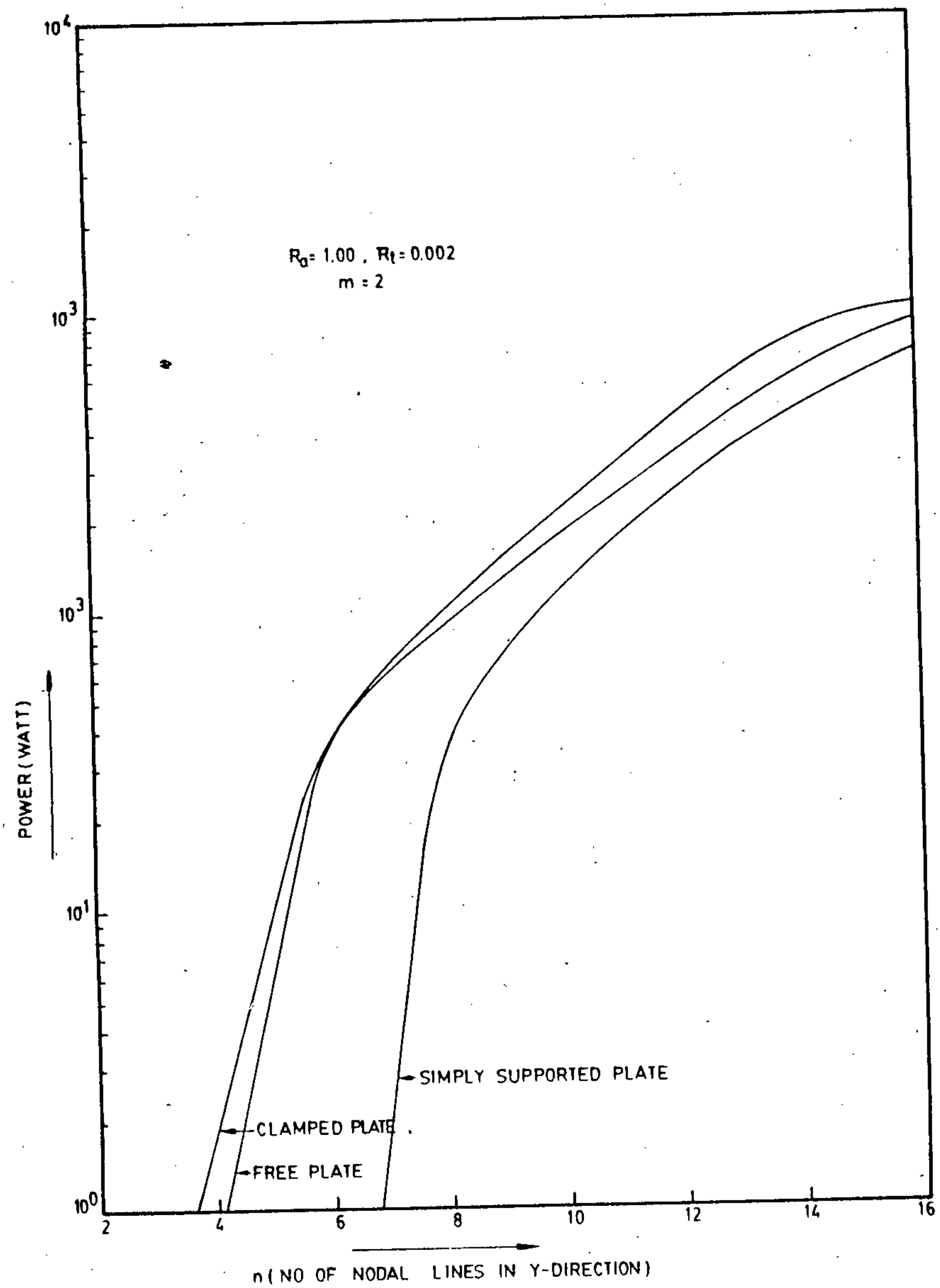

Fig. 33 : Power radiated by plates with different boundary conditions and low values of m

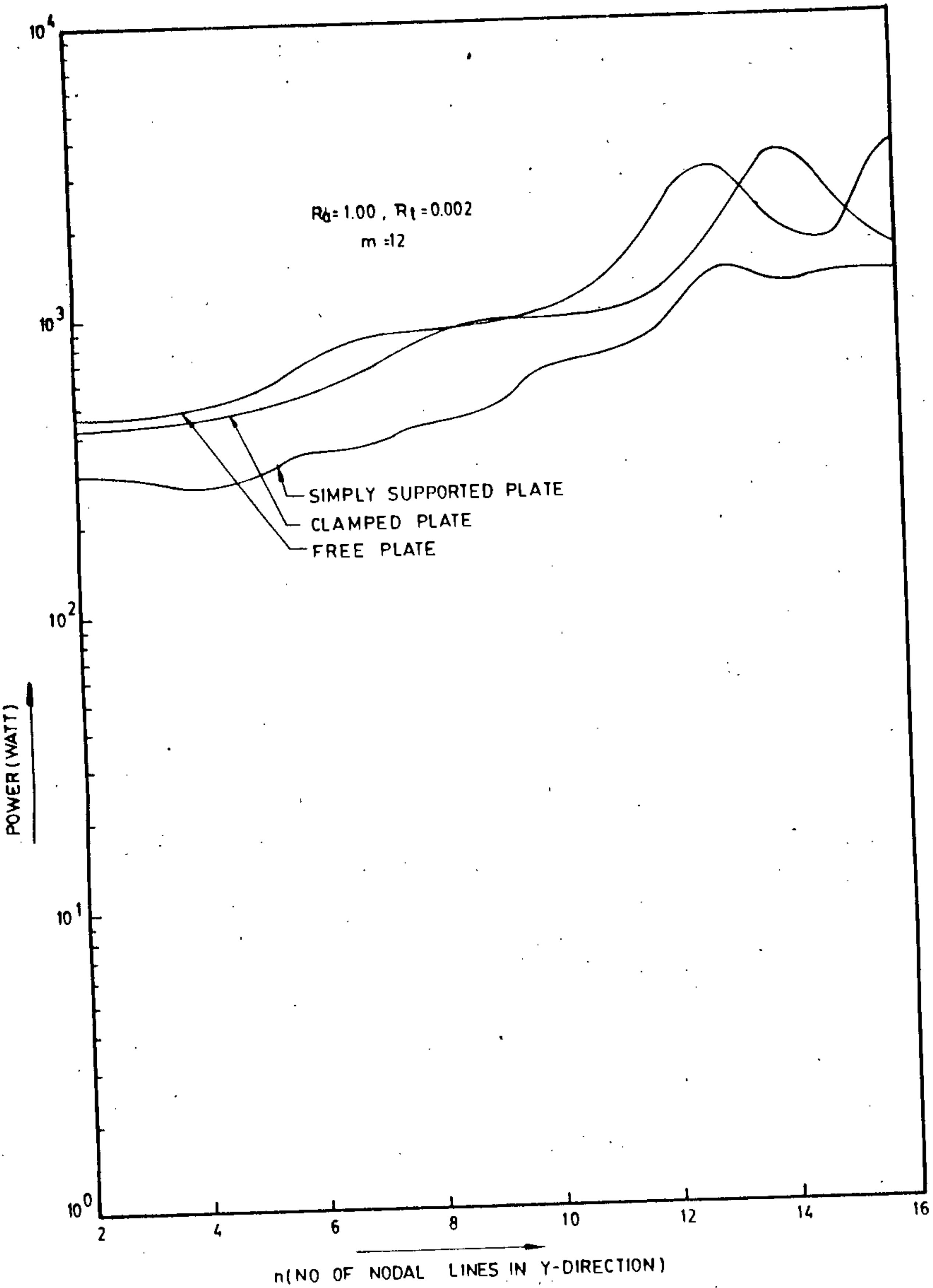

Fig. 34 : Power radiated by plates with different boundary conditions and high values of m.

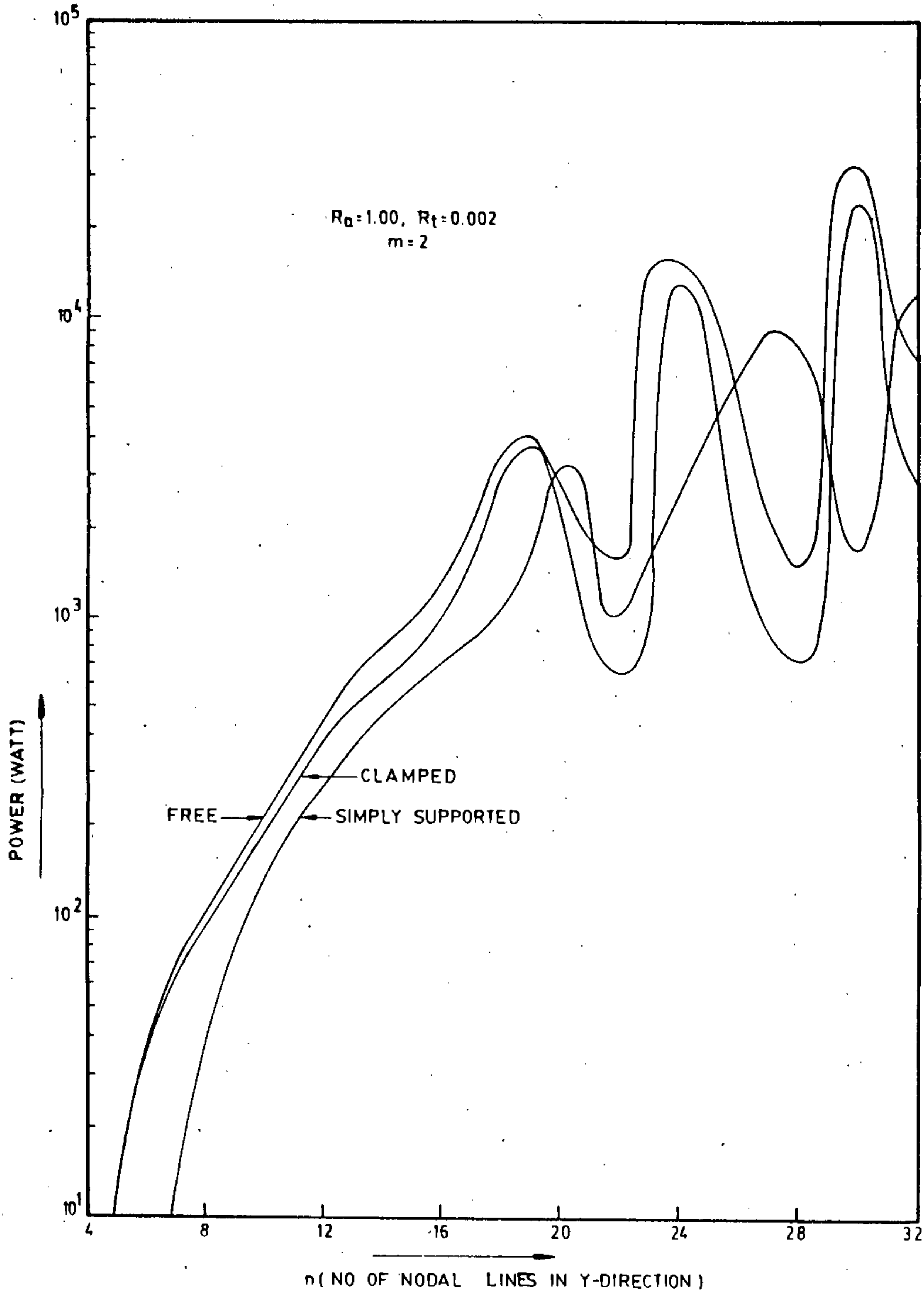

Fig. 35 : Power radiated by plates with different boundary conditions, low values of m and very high range of the values of n.

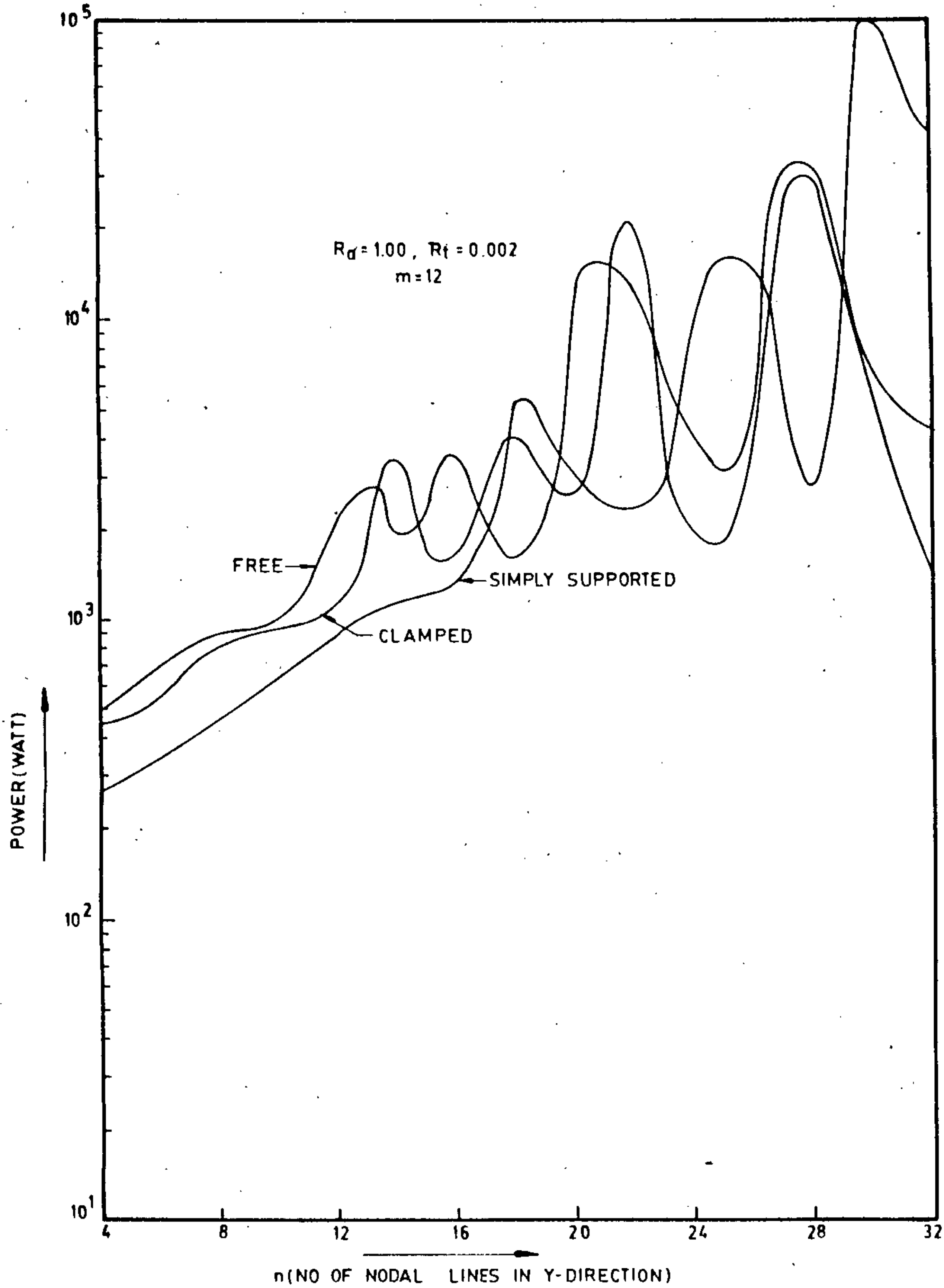

Fig. 36 : Power radiated by plates with different boundary conditions, high values of m and very high range of the values of n.

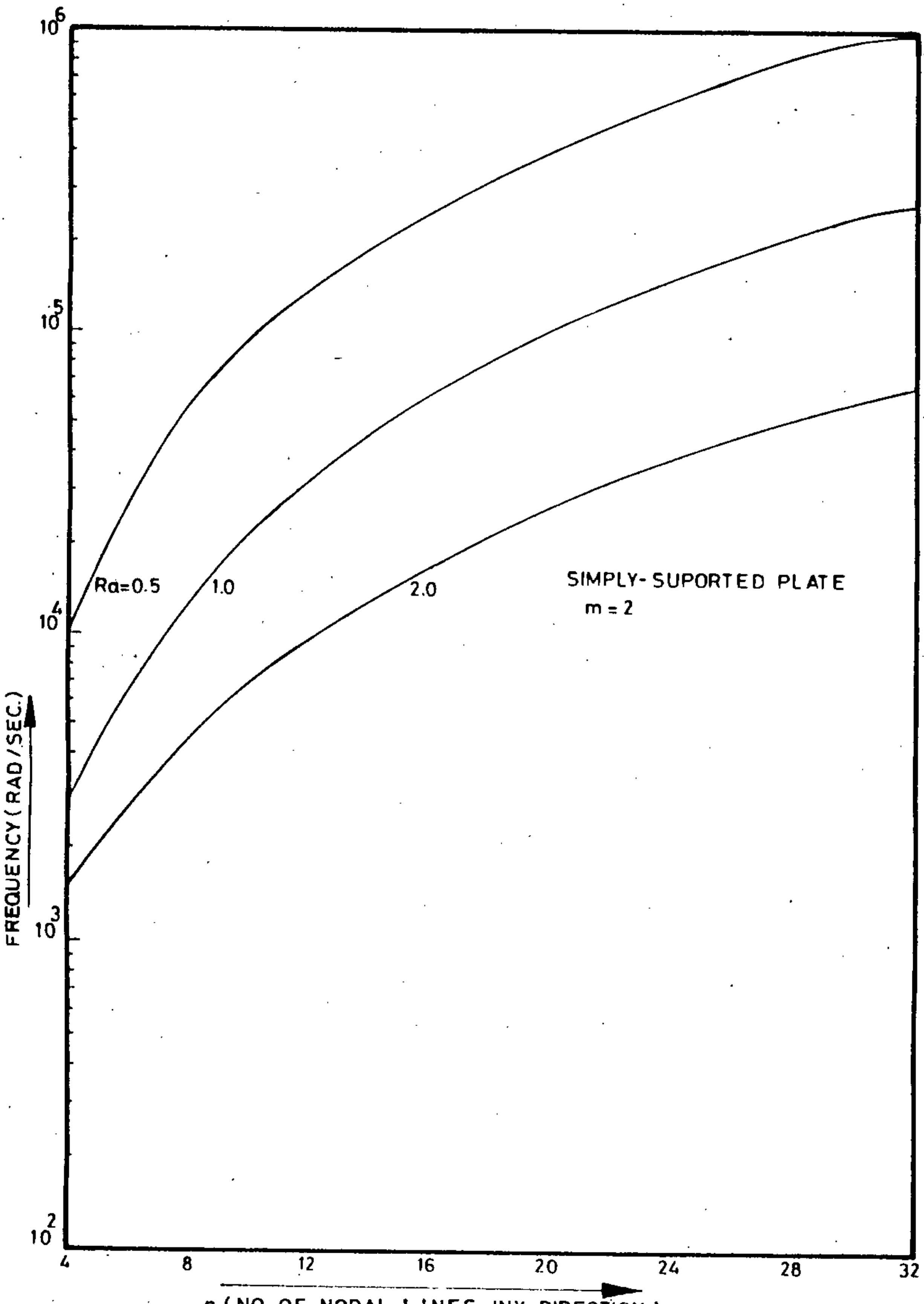

Fig. 37 : Frequency of a simply-supported plate for different aspect ratios and low values of m

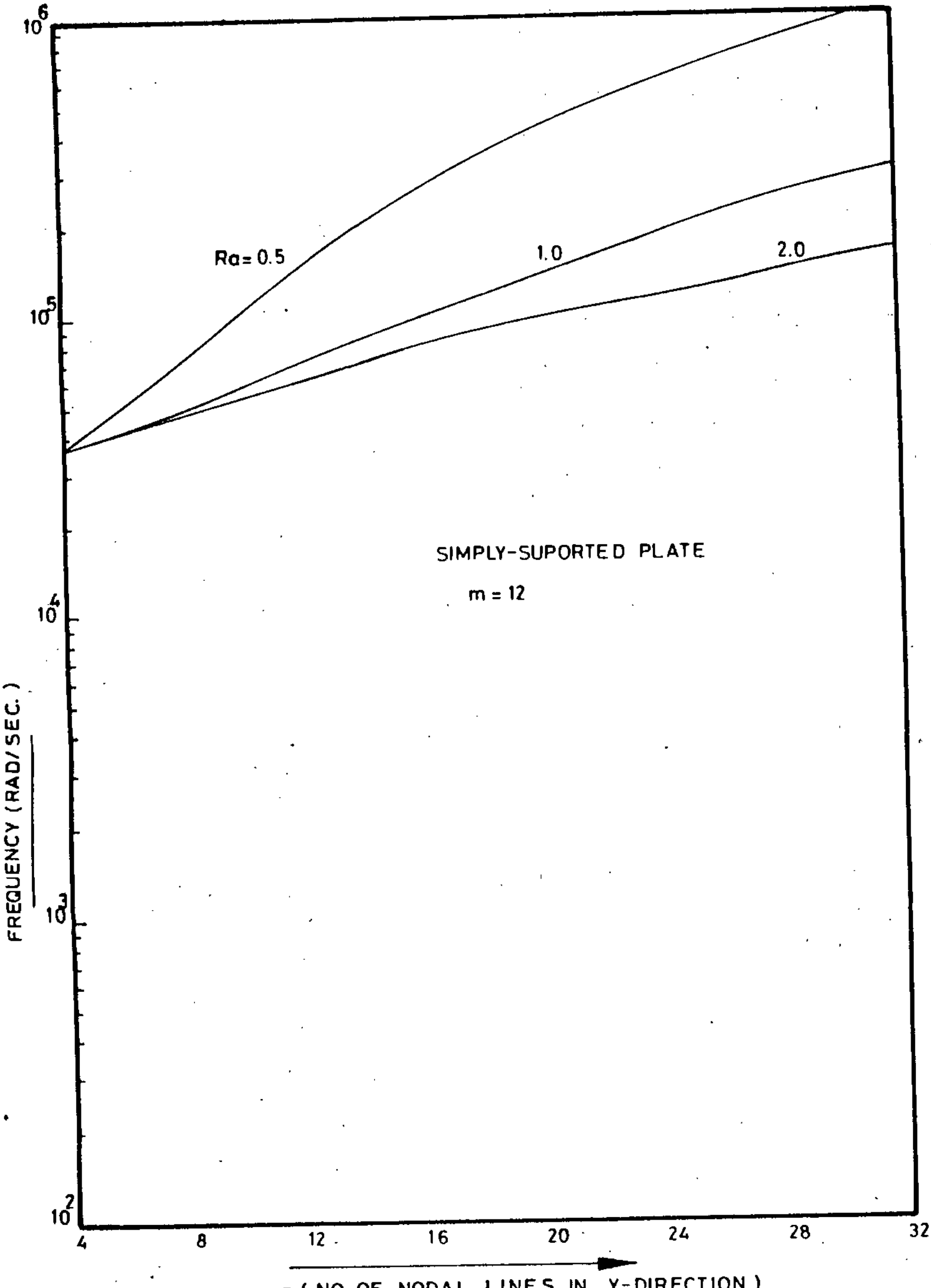

Fig. 38 : Frequency of a simply-supported plate for different aspect ratios and high

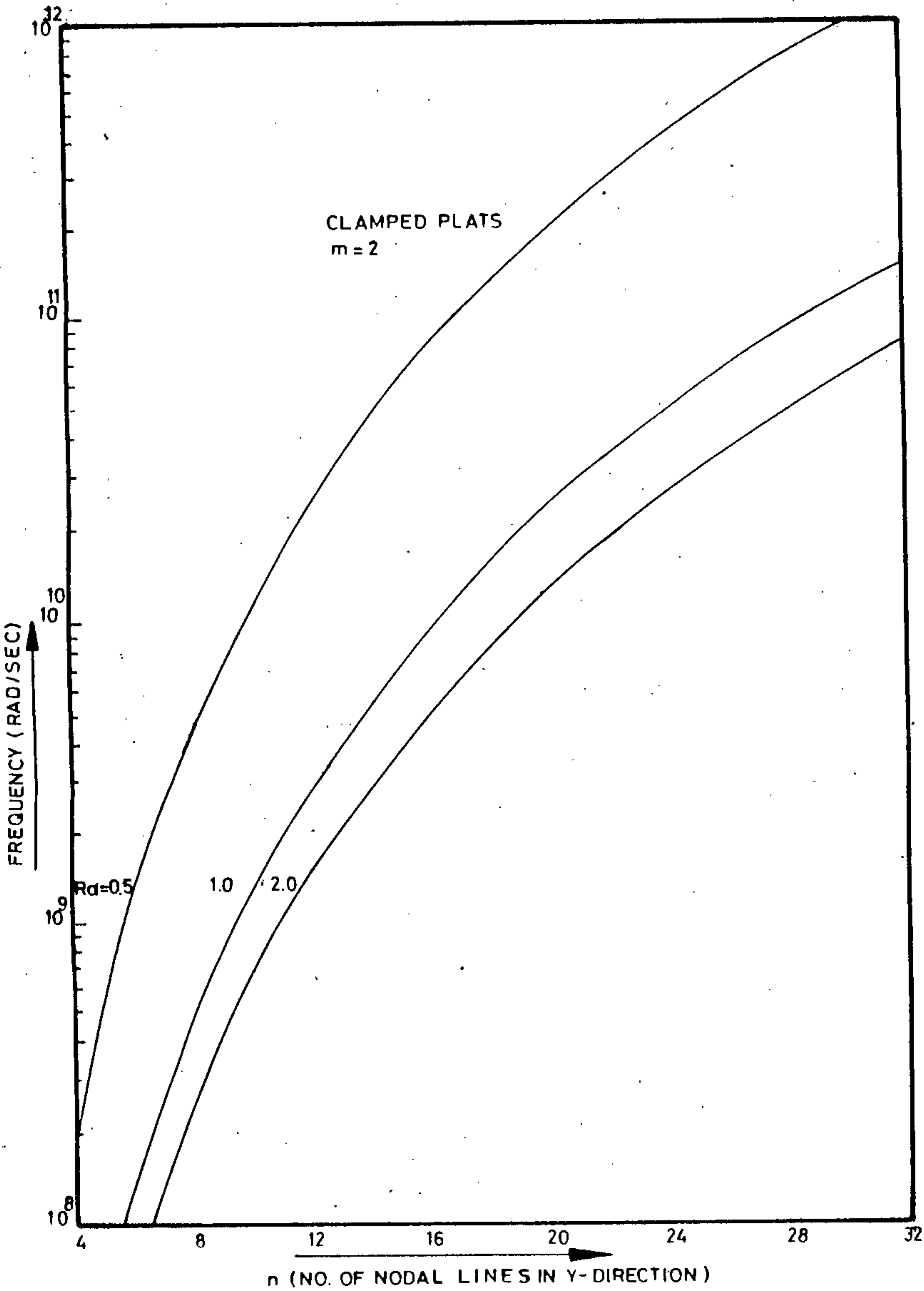

Fig. 39 : Frequency of a clamped plate for different aspect ratios and low values of m.

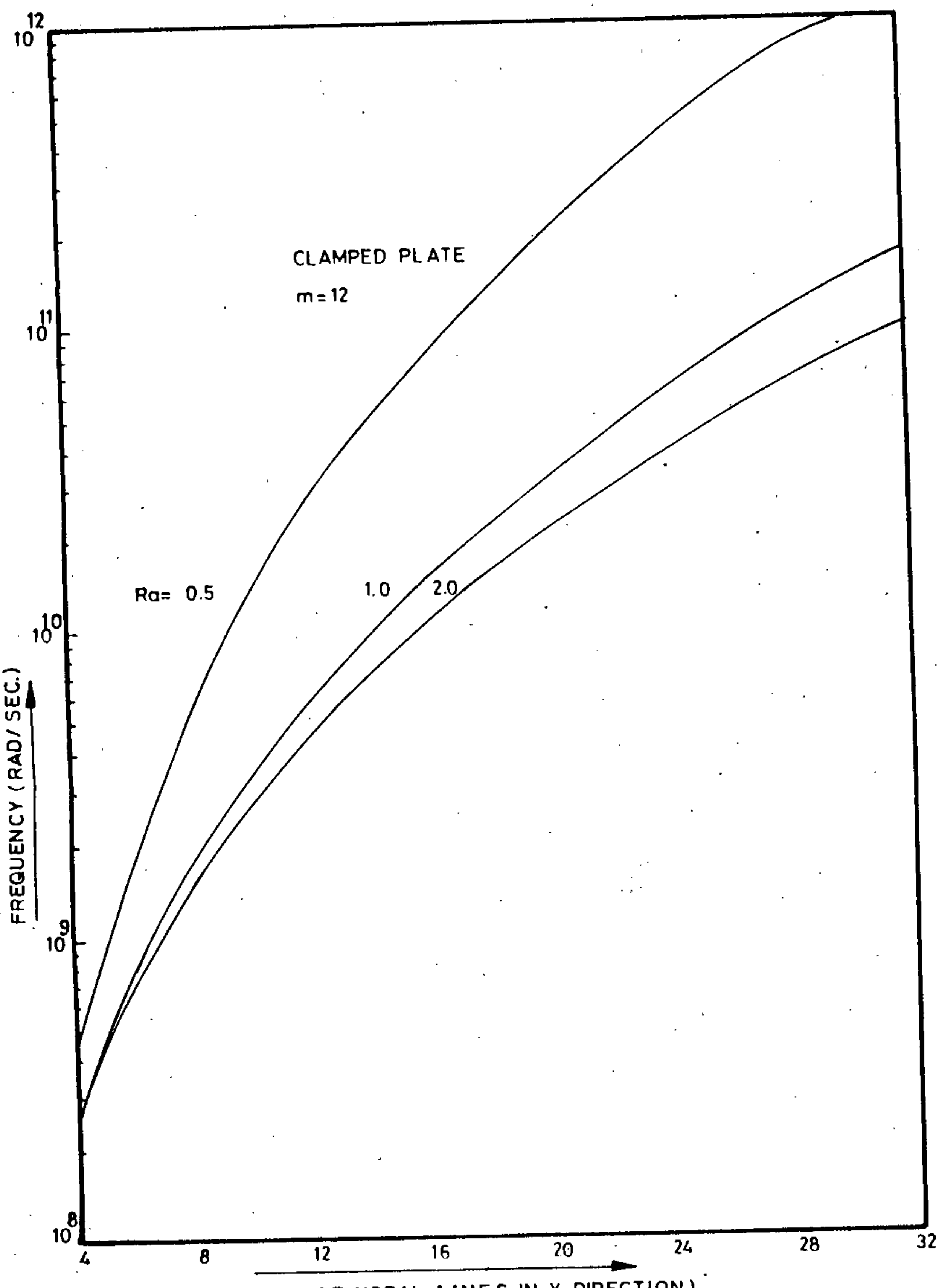

Fig. 40 : Frequency of a clamped plate for different aspect ratios and high values of m.

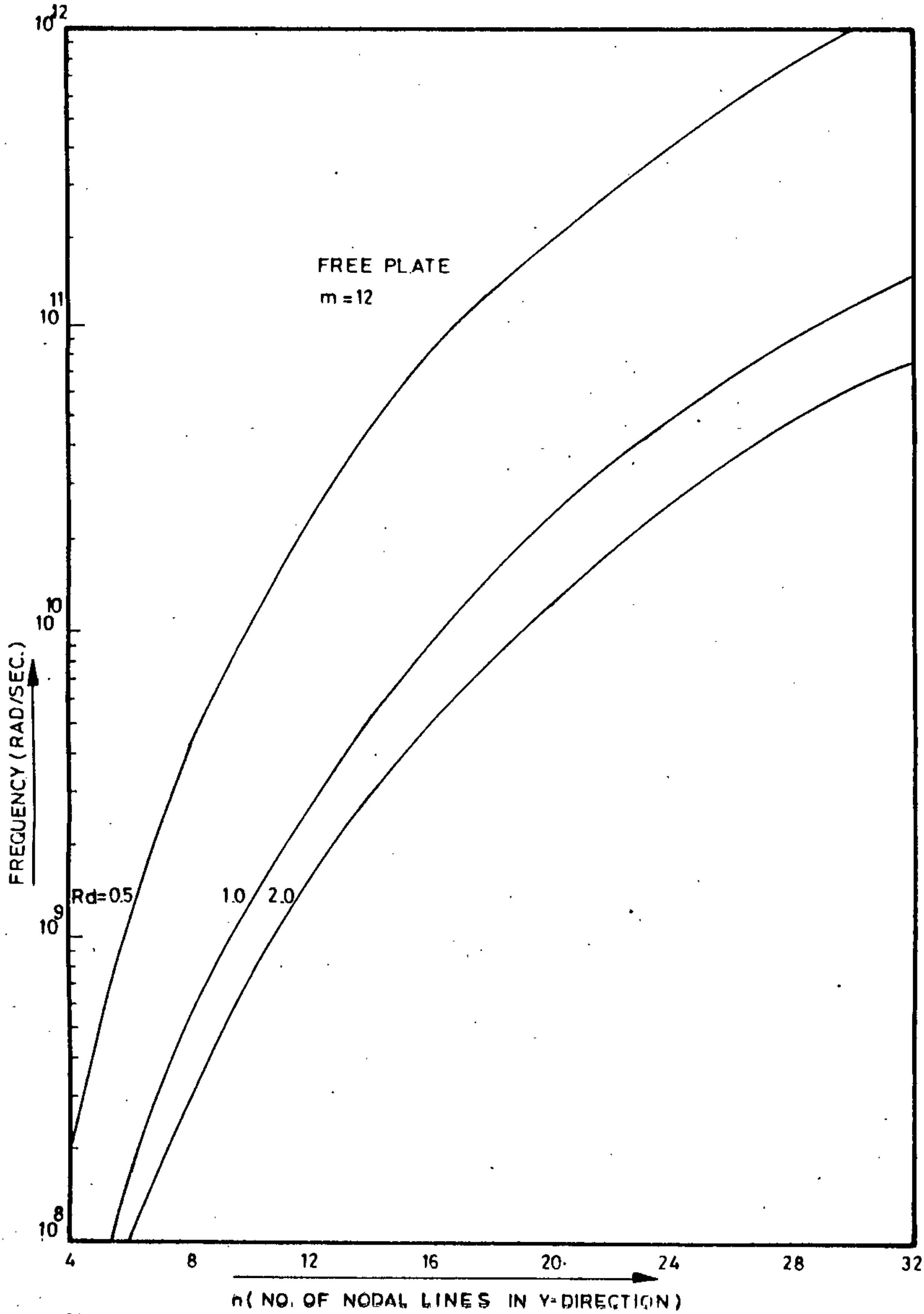

Fig. 41 : Frequency of a freely suspended plate for different aspect ratios and low values of m.

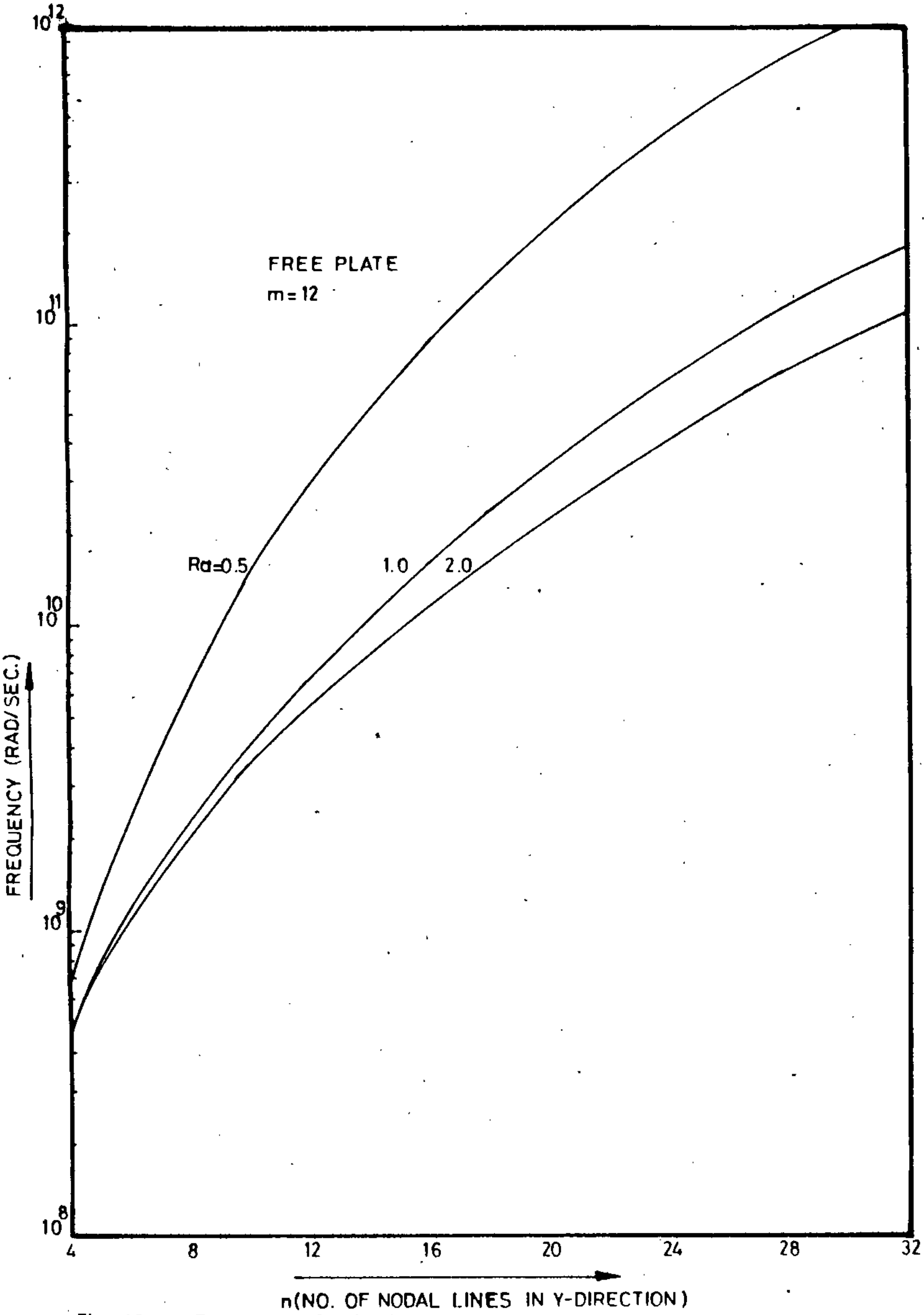

Fig. 42 : Frequency of a freely-suspended plate for different aspect ratios and high values of m.

APPENDICES

APPENDIX A

TABLES

Table-1: Radiation Efficiency of Simply-Supported Plate, For low-low mode orders.

m	n\	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80	1.00	2.00	3.00	4.00
2	2	8.0E-4	3.1E-3	7.0E-3	1.3E-2	2.0E-2	7.8E-2	2.9E-1	5.7E-1	8.6E-1	1.1E+0	1.1E+0	1.0E+0	1.0E+0
2	3	7.8E-7	1.2E-5	6.3E-5	2.0E-4	4.8E-4	7.3E-3	9.9E-2	3.8E-1	8.5E-1	1.2E+0	2.5E+0	1.0E+0	1.0E+0
2	4	4.3E-4	1.7E-3	3.8E-3	6.6E-3	1.0E-2	3.1E-2	5.5E-2	1.9E-1	6.8E-1	1.3E+0	1.1E+0	1.0E+0	1.0E+0
3	2	8.1E-7	1.3E-5	6.5E-5	2.1E-4	5.0E-4	7.7E-3	1.0E-1	4.0E-1	8.4E-1	2.2E+0	1.2E+0	1.1E+0	1.0E+0
3	3	8.1E-9	5.2E-8	6.0E-7	3.3E-6	1.2E-5	7.4E-4	3.6E-2	2.7E-1	8.2E-1	2.3E+0	1.2E+0	1.1E+0	1.0E+0
3	4	6.1E-7	9.6E-6	4.8E-5	1.5E-4	3.5E-4	4.4E-3	3.2E-2	1.6E-1	7.0E-1	2.4E+0	1.4E+0	1.1E+0	1.0E+0

Table-2: Radiation Efficiency of Simply-Supported Plate, For high-high mode orders.

m	n\	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80	1.00	2.00	3.00	4.00
9	10	5.0E-7	6.7E-6	2.6E-5	5.5E-5	8.3E-5	3.4E-4	1.8E-3	1.4E-2	2.3E-1	2.6E+0	1.0E+0	1.0E+0	1.0E+0
10	11	4.7E-7	6.1E-6	2.2E-5	4.2E-5	5.7E-5	2.5E-4	1.3E-3	1.1E-1	2.2E-1	2.6E+0	1.0E+0	1.0E+0	1.0E+0
9	8	5.0E-7	7.1E-6	2.9E-5	6.8E-5	1.1E-4	4.0E-4	2.4E-3	1.9E-2	2.3E-1	2.1E+0	1.0E+0	1.0E+0	1.0E+0
10	9	4.8E-7	6.4E-6	2.5E-5	5.3E-5	7.9E-5	3.3E-4	1.7E-3	1.2E-2	2.0E-1	2.2E+0	1.0E+0	1.0E+0	1.0E+0

Table-3: Radiation Efficiency of Simply-Supported Plate, For low-high mode orders.

m	n\	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80	1.00	2.00	3.00	4.00
2	9	8.0E-6	1.2E-5	5.8E-4	1.7E-3	3.5E-3	1.8E-2	4.2E-2	6.6E-2	1.1E-1	1.7E+0	1.0E+0	1.0E+0	1.0E+0
2	10	3.9E-4	1.4E-3	2.7E-3	3.9E-3	4.8E-3	1.2E-2	2.5E-2	4.5E-2	9.9E-2	1.6E+0	1.0E+0	1.0E+0	1.0E+0
3	10	2.8E-6	4.2E-5	1.9E-4	5.0E-4	1.0E-3	6.8E-3	2.7E-2	5.0E-2	1.1E-1	2.0E+0	1.0E+0	1.0E+0	1.0E+0
3	11	7.0E-8	4.2E-6	4.3E-5	2.1E-4	6.7E-4	9.1E-3	2.9E-2	5.5E-2	1.2E-1	2.3E+0	1.1E+0	1.0E+0	1.0E+0

Table-4: Radiation Efficiency of Simply-Supported Plate, For high-low mode orders.

m	n\	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80	1.00	2.00	3.00	4.00
9	2	8.5E-6	1.3E-4	6.0E-4	3.7E-3	3.7E-3	1.8E-2	4.5E-2	7.4E-2	1.2E-1	2.2E+0	1.1E+0	1.0E+0	1.0E+0
9	3	3.1E-8	1.8E-6	2.0E-5	1.0E-4	3.5E-4	8.2E-3	3.7E-2	6.6E-2	1.3E-1	2.0E+0	1.0E+0	1.0E+0	1.0E+0
10	2	3.9E-4	1.4E-3	2.7E-3	3.9E-3	4.7E-3	1.2E-2	2.7E-2	4.9E-2	1.1E-1	2.1E+0	1.0E+0	1.0E+0	1.0E+0
10	3	2.7E-6	4.0E-5	1.8E-4	4.9E-4	1.0E-3	6.3E-3	2.7E-2	5.0E-2	1.1E-1	2.0E+0	1.1E+0	1.0E+0	1.0E+0

Table-5: Power Radiated by a Simply-Supported Plate, For Low values of m, R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	4.7E-9	5.0E-6	1.3E-2	1.5E-1	5.0E-1	2.3E+0	3.6E+1	1.0E+2	1.4E+2	1.7E+2	2.4E+2	3.2E+2	4.5E+2	5.5E+2	7.5E+2
3	5.5E-6	5.5E-6	5.5E-3	1.1E-1	6.5E-1	2.6E+0	4.1E+1	1.2E+2	1.6E+2	2.0E+2	2.8E+2	3.7E+2	6.0E+2	7.0E+2	1.0E+3
4	1.3E-2	5.0E-3	8.0E-3	1.3E-1	9.5E-1	5.5E+0	6.5E+1	1.4E+2	1.5E+2	2.0E+2	2.6E+2	3.8E+2	6.0E+2	6.5E+2	1.0E+3
5	1.5E-1	1.1E-1	1.3E-1	4.7E-1	2.7E+0	1.9E+1	1.1E+2	1.3E+2	1.6E+2	2.3E+2	2.5E+2	6.0E+2	5.5E+2	6.5E+2	1.1E+3

Table-6: Power Radiated by a Simply-Supported Plate, For High values of m, R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
12	3.0E+2	2.9E+2	2.7E+2	3.0E+2	3.5E+2	3.7E+2	4.4E+2	4.6E+2	6.7E+2	7.0E+2	8.7E+2	1.4E+3	1.2E+3	1.3E+3	1.3E+3
13	3.7E+2	3.7E+2	3.9E+2	4.0E+2	4.4E+2	5.0E+2	6.5E+2	6.5E+2	6.5E+2	1.7E+3	1.4E+3	1.3E+3	1.2E+3	1.5E+3	2.0E+3
14	5.5E+2	5.5E+2	6.5E+2	6.0E+2	6.0E+2	6.5E+2	6.0E+2	2.4E+3	1.2E+3	1.6E+3	1.2E+3	1.0E+3	2.8E+3	1.8E+3	5.0E+3
15	6.0E+2	6.0E+2	6.0E+2	6.5E+2	6.5E+2	8.0E+2	1.1E+3	1.3E+3	1.4E+3	1.1E+3	8.0E+3	1.5E+3	1.8E+3	2.0E+3	4.4E+3

Table-7: Power Radiated by a Simply-Supported Plate For low values of m and very high range of the values of n, R_a=1.00 and R_t=0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	4.7E-8	1.3E-2	5.0E-1	3.6E+1	1.4E+2	2.4E+2	4.5E+2	7.5E+2	9.2E+2	3.2E+3	1.0E+3	2.6E+3	6.2E+3	8.4E+3	1.7E+3	1.2E+3
4	1.3E-2	8.0E-3	9.5E-1	6.5E+1	1.5E+2	2.6E+2	6.0E+2	1.0E+3	1.5E+3	4.4E+3	6.1E+3	3.4E+3	8.7E+3	1.3E+4	2.6E+3	2.2E+4
6	5.2E-1	9.3E-1	1.2E+1	1.2E+2	1.8E+2	3.5E+2	6.6E+2	6.1E+3	1.4E+3	5.1E+3	2.6E+3	2.2E+3	1.4E+4	8.6E+3	2.3E+3	2.5E+3
8	3.9E+1	6.5E+1	1.2E+2	2.9E+2	2.7E+2	4.3E+2	8.2E+2	1.6E+3	1.3E+3	4.0E+3	4.8E+3	1.1E+3	2.5E+4	2.5E+3	2.3E+3	3.8E+3

Table-8: Power Radiated From a Simply-Supported Plate, For high values of m and very high range of the values of n, With R_a=1.00 and R_t=0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
10	1.7E+2	1.5E+2	1.8E+2	2.6E+2	4.0E+2	7.0E+2	1.3E+3	1.2E+3	2.7E+3	1.6E+3	5.0E+3	2.2E+3	3.5E+4	2.2E+3	2.6E+3	1.7E+5
12	3.0E+2	2.7E+2	3.5E+2	4.4E+2	6.7E+2	8.7E+2	1.2E+3	1.3E+3	4.6E+3	3.1E+3	2.8E+3	7.4E+3	1.5E+4	2.8E+3	1.0E+5	4.3E+4
14	5.3E+2	6.7E+2	5.9E+2	6.0E+2	1.2E+3	1.2E+3	2.7E+3	5.2E+3	2.0E+4	6.1E+3	2.5E+3	3.5E+4	3.3E+3	3.5E+3	2.4E+4	1.3E+4
16	9.5E+2	1.0E+3	1.6E+4	5.0E+3	1.1E+3	1.2E+3	4.7E+3	1.5E+3	6.8E+3	1.6E+3	1.1E+4	1.6E+4	4.1E+3	1.2E+4	5.0E+4	3.1E+3

Table-9: Power Radiated by a Simply-Supported Plate, With R_a=1.00 and Different Values of R_t.

R_t	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.001	2	3.2E-9	8.0E-8	1.2E-3	7.5E-3	3.4E-2	1.1E-1	2.7E-1	6.5E-1	2.0E+0	4.5E+0	6.0E+0	6.5E+1	2.7E+2	2.8E+2	3.0E+2
0.002	2	4.7E-8	5.0E-6	1.3E-2	1.5E-1	5.0E-1	2.3E+0	3.6E+1	1.0E+2	1.4E+2	1.7E+2	2.4E+2	3.2E+2	4.5E+2	5.5E+2	7.5E+2
0.004	2	4.8E-6	3.2E-4	1.1E-1	4.3E+0	3.8E+1	6.5E+1	1.5E+2	2.1E+2	3.2E+2	5.0E+2	8.5E+2	6.5E+2	1.2E+3	5.5E+3	1.4E+3
0.001	12	6.5E+0	6.7E+0	8.5E+0	1.6E+1	3.1E+1	7.9E+1	2.0E+2	3.3E+2	3.7E+2	3.2E+2	8.0E+3	5.0E+2	4.9E+2	5.0E+2	6.0E+2
0.002	12	3.0E+2	2.9E+2	2.7E+2	3.0E+2	3.5E+2	3.7E+2	4.4E+2	4.6E+2	6.7E+2	7.0E+2	8.7E+2	1.4E+3	1.2E+3	1.3E+3	1.3E+3
0.004	12	9.7E+2	1.0E+3	1.0E+3	1.0E+3	1.9E+3	1.2E+3	1.3E+3	1.3E+3	3.4E+3	5.5E+3	2.6E+3	1.6E+3	4.7E+3	7.5E+3	4.1E+3

Table-10: Power Radiated by a Simply-Supported Plate, With R_t=.0002 and Different Values of R_a.

R_a	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.50	2	5.6E-2	1.3E-1	2.3E-1	9.5E+0	7.0E+1	1.2E+2	2.1E+2	3.7E+2	5.5E+2	9.0E+2	1.3E+3	6.0E+2	2.1E+3	4.9E+3	2.1E+3
1.00	2	4.7E-8	5.0E-6	1.3E-2	1.5E-1	5.0E-1	2.3E+0	3.6E+1	1.0E+2	1.4E+2	1.7E+2	2.4E+2	3.2E+2	4.5E+2	5.5E+2	7.5E+2
2.00	2	3.6E-9	2.1E-8	3.0E-4	2.0E-3	1.1E-2	4.8E-2	1.4E-1	3.2E-1	8.5E-1	2.4E+0	3.2E+0	3.0E+1	1.5E+2	1.7E+2	1.8E+2
0.50	12	6.8E+2	1.4E+2	1.8E+2	2.4E+2	3.1E+2	4.5E+2	2.0E+3	8.6E+2	1.7E+3	2.0E+3	2.3E+3	2.9E+3	8.6E+3	5.0E+3	3.8E+3
1.00	12	3.0E+2	2.9E+2	2.7E+2	3.0E+2	3.5E+2	3.7E+2	4.4E+2	4.6E+2	6.7E+2	7.0E+2	8.7E+2	1.4E+3	1.2E+3	1.3E+3	1.3E+3
2.00	12	6.0E+2	7.0E+2	6.3E+2	5.0E+2	5.3E+2	5.7E+2	6.2E+2	7.5E+2	7.7E+2	8.1E+2	7.1E+2	1.1E+3	8.4E+2	1.1E+3	7.0E+2

Table-11: Power Radiated by a Clamped Plate, For Low Modes, With R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	9.0E-2	3.2E-1	1.8E+0	9.5E+0	3.8E+1	6.5E+1	9.0E+1	1.3E+2	1.9E+2	2.7E+2	3.7E+2	4.8E+2	8.0E+2	8.0E+2	8.5E+2
4	1.8E+0	3.9E+0	1.2E+1	3.4E+1	6.5E+1	8.0E+1	1.2E+2	1.7E+2	2.4E+2	3.4E+2	4.4E+2	6.0E+2	9.5E+2	8.0E+2	1.1E+3
6	4.0E+1	5.0E+1	6.5E+1	7.5E+1	9.5E+1	1.3E+2	1.8E+2	2.4E+2	3.3E+2	4.4E+2	5.5E+2	8.5E+2	9.5E+2	9.0E+2	1.5E+3
8	1.0E+2	1.1E+2	1.2E+2	1.4E+2	1.8E+2	2.2E+2	2.8E+2	3.7E+2	4.7E+2	5.5E+2	8.5E+2	1.0E+3	1.0E+3	1.3E+3	2.7E+3

Table-12: Power Radiated by a Clamped Plate, For High Modes, With R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	2.1E+2	2.2E+2	2.5E+2	2.8E+2	3.3E+2	4.0E+2	4.7E+2	5.0E+2	7.5E+2	9.5E+2	9.5E+2	1.1E+3	1.3E+3	2.8E+3	2.9E+3
12	4.1E+2	4.3E+2	4.5E+2	4.9E+2	5.5E+2	6.5E+2	8.5E+2	1.0E+3	9.5E+2	1.0E+3	1.1E+3	1.9E+3	3.4E+3	2.3E+3	1.6E+3
14	9.0E+2	9.5E+2	9.5E+2	9.5E+2	9.5E+2	9.5E+2	1.0E+3	1.1E+3	1.3E+3	2.2E+3	3.4E+3	2.7E+3	1.4E+3	2.7E+3	4.4E+3
16	1.0E+3	1.0E+3	1.1E+3	1.3E+3	1.6E+3	2.0E+3	2.7E+3	3.3E+3	2.9E+3	1.7E+3	1.5E+3	3.2E+3	4.3E+3	3.5E+3	2.0E+3

Table-13: Power Radiated by a Clamped Plate, For Low Values of m and Very High Range of the Values of n, R_a =1.00 and R_t =0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	9.0E-2	1.8E+0	3.8E+1	9.0E+1	1.9E+2	3.7E+2	8.0E+2	8.5E+2	2.8E+3	2.4E+3	6.6E+2	1.3E+4	1.9E+3	6.9E+2	2.5E+4	3.0E+3
4	1.8E+0	1.2E+1	6.5E+1	1.2E+2	2.4E+2	4.4E+2	9.5E+2	1.1E+3	3.0E+3	3.5E+3	4.1E+2	1.9E+4	1.4E+3	7.3E+2	4.0E+4	2.3E+3
6	4.0E+1	6.5E+1	9.5E+1	1.8E+2	3.3E+2	5.5E+2	9.5E+2	1.5E+3	2.1E+3	4.3E+3	7.8E+2	2.2E+4	1.4E+3	1.8E+3	4.7E+4	1.7E+3
8	1.0E+2	1.2E+2	1.8E+2	2.8E+2	4.7E+2	8.5E+2	1.0E+3	2.7E+3	1.4E+3	3.7E+3	3.9E+3	1.9E+4	2.5E+3	5.8E+3	3.8E+4	3.2E+3

Table-14: Power Radiated by a Clamped Plate, For High Values of m and Very High Range of the Values of n, R_a =1.00 and R_t =0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
10	2.1E+2	2.5E+2	3.3E+2	4.7E+2	7.5E+2	9.5E+2	1.3E+3	2.9E+3	3.2E+3	2.1E+3	1.1E+4	8.3E+3	2.5E+3	1.4E+4	2.0E+4	3.9E+3
12	4.1E+2	4.5E+2	5.5E+2	8.5E+2	9.5E+2	1.1E+3	3.4E+3	1.6E+3	4.1E+3	2.6E+3	2.1E+4	2.0E+3	3.5E+3	3.0E+4	7.3E+3	1.5E+3
14	9.0E+2	9.5E+2	9.5E+2	1.0E+3	1.3E+3	3.4E+3	1.4E+3	4.4E+3	1.7E+3	1.3E+4	1.0E+4	3.1E+3	6.0E+3	4.3E+4	4.0E+3	4.5E+3
16	1.0E+3	1.1E+3	1.6E+3	2.7E+3	2.9E+3	1.5E+3	4.3E+3	2.0E+3	7.5E+3	1.8E+4	3.1E+3	2.8E+3	3.6E+4	9.6E+3	3.2E+3	6.8E+3

Table-15: Power Radiated by a Clamped Plate, With R_a =1.00 and Different Values of R_t .

R_t	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.001	2	6.0E-3	8.0E-3	8.5E-2	3.5E-1	9.0E-1	1.8E+0	2.8E+0	4.9E+0	2.7E+1	1.2E+2	2.2E+2	2.2E+2	2.5E+2	3.0E+2	3.6E+2
0.002	2	9.0E-2	3.2E-1	1.8E+0	9.5E+0	3.8E+1	6.5E+1	9.0E+1	1.3E+2	1.9E+2	2.7E+2	3.7E+2	4.8E+2	8.0E+2	8.0E+2	8.5E+2
0.004	2	1.1E+0	6.5E+0	2.2E+1	4.6E+1	9.0E+1	1.6E+2	2.8E+2	4.8E+2	6.0E+2	8.5E+2	2.0E+3	2.4E+3	9.0E+2	3.8E+3	4.6E+3
0.001	12	2.5E+2	2.5E+2	2.5E+2	2.5E+2	2.5E+2	2.7E+2	2.9E+2	3.2E+2	3.6E+2	4.1E+2	4.6E+2	5.5E+2	6.5E+2	6.6E+2	9.0E+2
0.002	12	4.1E+2	4.3E+2	4.5E+2	4.9E+2	5.5E+2	6.5E+2	8.5E+2	1.0E+3	9.5E+2	1.0E+3	1.1E+3	1.9E+3	3.4E+3	2.3E+3	1.6E+3
0.004	12	2.3E+3	2.5E+3	2.8E+3	3.0E+3	2.8E+3	2.0E+3	1.3E+3	2.1E+3	4.4E+3	6.0E+3	3.9E+3	2.3E+3	3.2E+3	5.0E+3	2.0E+4

Table-16: Power Radiated by a Clamped Plate, With R_t=0.002 and Different Values of R_a.

R_a	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.50	2	4.9E-2	4.0E-2	1.7E-1	4.4E-1	5.5E-1	6.0E-1	9.0E-1	1.6E+0	2.6E+0	3.8E+0	5.0E+0	6.5E+0	8.0E+0	1.0E+1	1.8E+1
1.00	2	9.0E-2	3.2E-1	1.8E+0	9.5E+0	3.8E+1	6.5E+1	9.0E+1	1.3E+2	1.9E+2	2.7E+2	3.7E+2	4.8E+2	8.0E+2	8.0E+2	8.5E+2
2.00	2	9.5E-1	9.5E+0	2.9E+1	7.5E+1	1.6E+2	3.3E+2	4.0E+2	6.0E+2	2.1E+3	4.0E+3	1.1E+3	7.5E+2	2.2E+3	1.3E+4	1.9E+4
0.50	12	2.2E+2	2.5E+2	3.1E+2	4.1E+2	5.5E+2	6.5E+2	8.0E+2	9.5E+2	1.1E+3	1.2E+3	1.3E+3	1.4E+3	1.5E+3	2.0E+3	2.4E+3
1.00	12	4.1E+2	4.3E+2	4.5E+2	4.9E+2	5.5E+2	6.5E+2	8.5E+2	1.0E+3	9.5E+2	1.0E+3	1.1E+3	1.9E+3	3.4E+3	2.3E+3	1.6E+3
2.00	12	8.0E+2	8.5E+2	8.5E+2	9.0E+2	1.1E+3	1.4E+3	1.2E+3	2.3E+3	2.1E+3	2.3E+3	7.0E+3	2.2E+3	8.0E+3	3.2E+3	1.4E+4

Table-17: Power Radiated by a Freely Suspended Plate, For Low Mode Orders With R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1.1E-3	3.0E-2	4.9E-1	6.4E+0	3.7E+1	6.7E+1	1.0E+2	1.5E+2	2.2E+2	3.2E+2	4.5E+2	5.7E+2	9.0E+2	1.0E+3	1.0E+3
3	3.0E-2	3.8E-1	3.6E+0	2.2E+1	5.3E+1	7.5E+1	1.1E+2	1.7E+2	2.4E+2	3.4E+2	4.7E+2	6.0E+2	9.6E+2	9.5E+2	1.1E+3
4	4.9E-1	3.6E+0	1.8E+1	4.6E+1	6.5E+1	9.5E+1	1.4E+2	2.0E+2	2.7E+2	3.8E+2	4.9E+2	7.4E+2	1.0E+3	9.3E+2	1.3E+3
5	6.4E+0	2.2E+1	4.6E+1	5.8E+1	8.5E+1	1.2E+2	1.7E+2	2.4E+2	3.2E+2	4.3E+2	5.3E+2	8.6E+2	9.5E+2	9.6E+2	1.5E+3

Table-18: Power Radiated by a Freely Suspended Plate, For High Mode Orders With R_a=1.00 and R_t=0.002.

m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
12	4.5E+2	4.7E+2	4.9E+2	5.3E+2	6.8E+2	8.7E+2	9.2E+2	9.0E+2	1.0E+3	1.2E+3	2.1E+3	3.1E+3	1.9E+3	1.7E+3	3.5E+3
13	5.4E+2	5.9E+2	7.2E+2	8.6E+2	9.4E+2	9.1E+2	8.9E+2	1.1E+3	1.3E+3	2.2E+3	3.1E+3	2.0E+3	1.5E+3	3.3E+3	3.9E+3
14	9.1E+2	9.5E+2	9.8E+2	9.5E+2	8.9E+2	9.5E+2	1.2E+3	1.4E+3	2.5E+3	3.0E+3	1.9E+3	1.6E+3	3.1E+3	4.0E+3	2.4E+3
15	1.0E+3	9.4E+2	9.1E+2	9.6E+2	1.2E+3	1.4E+3	2.0E+3	2.9E+3	2.9E+3	1.7E+3	1.7E+3	3.1E+3	4.0E+3	2.5E+3	1.6E+3

Table-19: Power Radiated by a Freely Suspended Plate, For Low Values of m and Very High Range of the Values of n, R_a=1.00 and R_t=0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	1.1E-3	4.9E-1	3.7E+1	1.0E+2	2.2E+2	4.5E+2	9.0E+2	1.0E+3	3.4E+3	2.9E+3	1.6E+3	1.6E+4	5.5E+3	1.5E+3	3.3E+4	7.5E+3
4	4.9E-1	1.8E+1	6.0E+1	1.4E+2	2.7E+2	5.0E+2	1.0E+3	1.3E+3	2.9E+3	3.9E+3	1.0E+3	1.9E+4	2.4E+3	2.3E+3	3.1E+4	4.5E+3
6	3.7E+1	6.2E+1	1.2E+2	2.1E+2	3.9E+2	7.0E+2	8.9E+2	1.9E+3	1.6E+3	4.1E+3	2.3E+3	2.1E+4	3.6E+3	4.1E+3	4.4E+4	4.8E+4
8	1.0E+2	1.4E+2	2.1E+2	3.4E+2	5.0E+2	9.2E+2	1.2E+3	3.1E+3	2.5E+3	2.4E+3	7.5E+3	1.2E+4	4.3E+3	1.0E+4	3.4E+4	5.0E+3

Table-20: Power Radiated by a Freely Suspended Plate, For High Values of m and Very High Range of the Values of n, R_a=1.00 and R_t=0.002.

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
10	2.2E+2	2.7E+2	3.9E+2	5.0E+2	9.1E+2	1.0E+3	2.5E+3	1.4E+3	3.9E+3	1.6E+3	1.9E+4	2.7E+3	2.5E+3	2.7E+4	9.2E+3	4.6E+3
12	4.5E+2	5.0E+2	6.9E+2	9.2E+2	1.0E+3	2.1E+3	1.9E+3	3.5E+3	1.6E+3	8.4E+3	1.3E+4	4.0E+3	4.9E+3	3.9E+4	5.9E+3	4.3E+3
14	9.1E+2	1.0E+3	9.1E+2	1.2E+3	2.5E+3	1.9E+3	3.1E+3	2.4E+3	3.7E+3	2.0E+4	2.9E+3	3.2E+3	2.1E+4	2.0E+4	4.3E+3	4.4E+3
16	1.1E+3	1.3E+3	1.9E+3	3.2E+3	1.5E+3	3.5E+3	2.3E+3	3.0E+3	2.0E+4	3.7E+3	2.4E+3	7.2E+3	4.1E+4	5.6E+3	4.1E+3	4.5E+3

Table-21: Power Radiated by a Freely Suspended Plate For R_a=1.00 and Different Values of R_t.

R_t	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.001	2	4.0E-7	4.3E-6	2.0E-5	1.6E-3	1.8E-2	9.5E-2	2.5E-1	9.6E-1	1.5E+1	1.0E+2	2.2E+2	2.6E+2	2.9E+2	3.6E+2	4.3E+2
0.002	2	1.1E-1	3.0E-2	4.9E-1	6.4E+0	3.7E+1	6.7E+1	1.0E+2	1.5E+2	2.2E+2	3.2E+2	4.5E+2	5.7E+2	9.0E+2	1.0E+3	1.0E+3
0.004	2	1.1E+0	1.1E+1	2.7E+1	5.7E+1	1.1E+2	2.0E+2	3.3E+2	5.7E+2	7.3E+2	1.0E+3	1.9E+3	3.1E+3	1.4E+3	4.3E+3	5.6E+3
0.001	12	1.0E+2	1.1E+2	1.2E+2	1.4E+2	1.6E+2	1.9E+2	2.2E+2	2.7E+2	3.2E+2	3.8E+2	4.6E+2	5.6E+2	6.7E+2	7.5E+2	7.8E+2
0.002	12	4.5E+2	4.7E+2	4.9E+2	5.3E+2	6.8E+2	8.7E+2	9.2E+2	9.0E+2	1.0E+3	1.2E+3	2.1E+3	3.1E+3	1.9E+3	1.7E+3	3.5E+3
0.004	12	1.6E+3	1.8E+3	1.8E+3	1.6E+3	1.4E+3	1.4E+3	2.0E+3	2.6E+3	3.3E+3	3.1E+3	3.1E+3	2.7E+3	3.4E+3	4.8E+3	4.1E+3

Table-22: Power Radiated by a Freely Suspended Plate For R_t=0.002 and Different Values of R_a.

R_a	m\n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.50	2	1.1E-1	5.9E+0	4.3E+1	9.5E+1	1.8E+2	3.5E+2	6.0E+2	1.0E+3	1.4E+3	1.8E+3	3.6E+3	7.6E+3	2.5E+3	8.5E+3	1.1E+4
1.00	2	1.1E-1	3.0E-2	4.9E-1	6.4E+0	3.7E+1	6.7E+1	1.0E+2	1.5E+2	2.2E+2	3.2E+2	4.5E+2	5.7E+2	9.0E+2	1.0E+3	1.0E+3
2.00	2	8.1E-4	3.1E-4	6.8E-4	2.2E-3	1.3E-2	7.1E-2	4.3E-1	3.4E+0	2.2E+1	7.6E+1	1.0E+2	1.3E+2	1.5E+2	1.9E+2	2.3E+2
0.50	12	2.4E+2	3.0E+2	4.0E+2	5.5E+2	7.8E+2	1.2E+3	1.8E+3	2.5E+3	3.1E+3	2.6E+3	4.6E+3	7.3E+3	5.0E+3	1.5E+3	3.8E+3
1.00	12	4.5E+2	4.7E+2	4.9E+2	5.3E+2	6.8E+2	8.7E+2	9.2E+2	9.0E+2	1.0E+3	1.2E+3	2.1E+3	3.1E+3	1.9E+3	1.7E+3	3.5E+3
2.00	12	7.3E+2	7.1E+2	7.1E+2	7.1E+2	7.2E+2	7.8E+2	8.7E+2	9.7E+2	1.1E+3	1.4E+3	1.7E+3	1.9E+3	2.1E+3	4.1E+3	2.0E+3

Table-23: Frequency of a Simply-Supported Plate.

Ra	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.50	2	1.2E+3	1.0E+4	2.8E+4	5.5E+4	9.1E+4	1.4E+5	1.9E+5	2.5E+5	3.2E+5	4.0E+5	4.9E+5	5.9E+5	7.0E+5	8.2E+5	9.4E+5	1.0E+6
1.00	2	5.9E+2	2.9E+3	7.3E+3	1.4E+4	2.3E+4	3.1E+4	4.8E+4	6.3E+4	8.1E+4	1.0E+5	1.2E+5	1.5E+5	1.7E+5	2.0E+5	2.4E+5	2.7E+5
2.00	2	5.2E+2	1.5E+3	2.8E+3	4.6E+3	6.9E+3	9.8E+3	1.3E+4	1.7E+4	2.2E+4	2.7E+4	3.2E+4	3.8E+4	4.5E+4	5.2E+4	6.0E+4	6.9E+4
0.50	12	3.4E+4	3.8E+4	5.0E+4	7.3E+4	1.0E+5	1.5E+5	2.0E+5	2.6E+5	3.4E+5	4.2E+5	5.0E+5	6.0E+5	7.1E+5	8.3E+5	9.5E+5	1.1E+6
1.00	12	3.4E+4	3.7E+4	4.2E+4	5.0E+4	6.0E+4	7.2E+4	8.6E+4	1.0E+5	1.2E+5	1.4E+5	1.6E+5	1.9E+5	2.2E+5	2.4E+5	2.8E+5	3.1E+5
2.00	12	3.4E+4	3.7E+4	4.2E+4	4.8E+4	5.5E+4	6.4E+4	7.2E+4	8.2E+4	9.1E+4	1.0E+5	1.1E+5	1.2E+5	1.3E+5	1.4E+5	1.6E+5	1.7E+5

Table-24: Frequency of a Clamped Plate.

Ra	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.50	2	7.1E+6	2.0E+8	1.2E+9	4.2E+9	1.0E+10	2.3E+10	4.4E+10	7.7E+10	1.2E+11	1.9E+11	2.8E+11	4.1E+11	5.6E+11	7.6E+11	1.0E+12	1.3E+12
1.00	2	1.1E+6	2.6E+7	1.5E+8	1.5E+8	1.3E+9	2.8E+9	5.2E+9	9.1E+9	1.5E+10	2.3E+10	3.4E+10	4.8E+10	6.6E+10	9.0E+10	1.2E+11	1.5E+11
2.00	2	7.5E+5	1.5E+7	8.3E+7	2.8E+8	7.0E+8	1.5E+9	2.8E+9	4.8E+9	7.9E+9	1.2E+10	1.8E+10	2.5E+10	3.5E+10	4.8E+10	6.3E+10	8.2E+10
0.50	12	3.8E+7	4.4E+8	1.9E+9	5.4E+9	1.3E+10	2.6E+10	4.8E+10	8.2E+10	1.3E+11	2.0E+11	2.9E+11	4.2E+11	5.8E+11	7.8E+11	1.0E+12	1.3E+12
1.00	12	3.2E+7	2.7E+8	7.9E+8	1.7E+9	3.3E+9	5.8E+9	9.4E+9	1.5E+10	2.2E+10	3.2E+10	4.4E+10	6.1E+10	8.1E+10	1.0E+11	1.4E+11	1.8E+11
2.00	12	3.2E+7	2.6E+8	7.2E+8	1.5E+9	2.7E+9	4.5E+9	7.0E+9	1.0E+10	1.5E+10	2.1E+10	2.9E+10	3.8E+10	5.0E+10	6.5E+10	8.4E+10	1.0E+11

Table-25: Frequency of a Freely Suspended Plate.

Ra	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.50	2	1.0E+7	2.1E+8	1.2E+9	4.3E+9	1.0E+10	2.3E+10	4.4E+10	7.7E+10	1.3E+11	1.9E+11	2.8E+11	4.1E+11	5.6E+11	7.6E+11	1.0E+12	1.3E+12
1.00	2	4.3E+6	3.7E+7	1.7E+8	5.5E+8	1.3E+9	2.8E+9	5.3E+9	9.2E+9	1.5E+10	2.3E+10	3.4E+10	4.8E+10	6.7E+10	9.0E+10	1.2E+11	1.5E+11
2.00	2	4.0E+6	2.6E+7	1.0E+8	3.1E+8	7.5E+8	1.6E+9	2.9E+9	5.0E+9	8.0E+9	1.2E+10	1.8E+10	2.6E+10	3.6E+10	4.8E+10	6.4E+10	8.3E+10
0.50	12	1.2E+8	6.6E+8	2.2E+9	6.0E+9	1.4E+10	2.7E+10	5.0E+10	8.4E+10	1.3E+11	2.0E+11	3.0E+11	4.2E+11	5.8E+11	7.8E+11	1.0E+12	1.3E+12
1.00	12	1.1E+8	4.9E+8	1.2E+9	2.3E+9	4.1E+9	6.8E+9	1.0E+10	1.6E+10	2.4E+10	3.4E+10	4.7E+10	6.4E+10	8.5E+10	1.1E+11	1.4E+11	1.8E+11
2.00	12	1.1E+8	4.7E+8	1.1E+9	2.1E+9	3.5E+9	5.6E+9	8.3E+9	1.2E+10	1.7E+10	2.3E+10	3.1E+10	4.2E+10	5.4E+10	7.0E+10	8.8E+10	1.1E+11

APPENDIX B

PROGRAMMING FEATURES

B 1. GENERAL FEATURES:

The computer program used in the current investigation has been developed by the author at the Computer Center of Bangladesh University of Engineering and Technology (B.U.E.T.), Dhaka. The Simpson rule for numerical integration has been used to integrate the farfield acoustic intensity and the equations $\tan(\gamma/2)\pm\tanh(\gamma/2)=0$, have been solved by the method if bisection. At first, the program for a certain set of boundary conditions has been developed and subsequently modified for the others. In the program listing section, the computer program for the clamped plate has been presented. This program, with minor modifications, can be applied to any combination of boundary conditions.

B 2. DEFINITION OF COMPUTER VARIABLES:

Variable	Definition
AR	Aspect ratio
TR	Thickness ratio
AH	Divisions of the range of integration
AK	Divisions of the range of integration
AMP	Amplitude factor of the power radiated
AL	l
AS	s
GM	γ_m, Roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$
GPM	γ_m' Roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$

GN	γ_n, Roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$
GPN	γ_n', Roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$
GR	g, Acceleration due to gravity
RM	R_m
RPM	R_m'
RN	R_n
RPN	R_n'
ROA	ρ, Density of the surrounding medium
RO	ρ_m, Density of the plate material
SLMB	λ_f^2, Dimensionless frequency factor
TH	α
TK	θ
WNR	ψ, Wave number ratio

APPENDIX C

PROGRAM LISTING

PROGRAM LISTING

```
*     ****************************************************************
      OPEN(UNIT=5,FILE='OUT',STATUS='NEW')
*     ****************************************************************
*     ************************ INPUT DATA *****************************
*     ****************************************************************
      AR=1.00
      TR=0.002
      C=343.0
      W=0.0001
      GR=9.81
      PI=3.1416
      E=206E+9
      ROA=1.21
      RO=7700.0
      SIG=0.40
      HH=0.0025
*     ****************************************************************
      WRITE(5,1001) AR,TR
1001  FORMAT(5X,'ASPECT RATIO= ',F6.4,5X,'THICKNESS RATIO= ',F6.4,/)
      X11=2.2
      DO 1 M=2,16,2
12    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.00) GO TO 10
      X11=X12
      GO TO 12
10    GM=X11+X12
      X21=2.2
      DO 2 N=2,32,2
22    X22=X21+HH
      Y21=(SIN(X21)/COS(X21))+(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))+(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.00) GO TO 20
      X21=X22
      GO TO 22
20    GN=X21+X22
      HGM=GM/2.0
      HGN=GN/2.0
      RM=SIN(HGM)/SINH(HGM)
      RN=SIN(HGN)/SINH(HGN)
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.*ROA*W*W*AR*AR(PI**6.)*E*E*(TR**4.)*GR*GR*SLMB*SLMB
      AM2=9.*C*(GM**4.)*(GN**4.)*RO*RO*((1.(SIG*SIG))**2.)
      AMP=AM1/AM2
      AH=PI/70
      YY=33.0*AH
      GSUM=0.0
      TH=0.0
      CALL ONE(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
```

```
70     TH=TH+AH
       CALL ONE(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
       GSUM=GSUM+4.0*SUM
       TH=TH+AH
       CALL ONE(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
       GSUM=GSUM+2.0*SUM
       IF(TH.LT.YY) GO TO 70
       TH=TH+AH
       CALL ONE(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
       GSUM=GSUM+SUM
       GSUM=(AH/3.0)*GSUM
       POWER=4.0*GSUM*AMP
       WRITE(5,200) M,N,GM,GN,POWER
200    FORMAT(7X,I2,8X,I2,10X,F10.4,10X,F10.4,10X,E10.4)
       X21=X21+3.1
2      CONTINUE
       X11=X11+3.1
1      CONTINUE
       X11=3.9
       DO 3 M=3,16,2
32     X12=X11+HH
       Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
       Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
       Z11+Y11*Y12
       IF(Z11.LT.0.00) GO TO 30
       X11=X12
       GO TO 32
30     GPM=X11+X12
       X21=3.9
       DO 4 N=3,32,2
42     X22=X21+HH
       Y21=(SIN(X21)/COS(X21))-(SINH(X21)/COSH(X21))
       Y22=(SIN(X22)/COS(X22))-(SINH(X22)/COSH(X22))
       Z21=Y21*Y22
       X21=X22
       GO TO 42
40     GPN=X21+X22
       HGPM=GPM/2
       HGPN=GPN/2
       RPM=-SIN(HGPM)/SINH(HGPM)
       RPN=-SIN(HGPN)/SINH(HGPN)
       CALL ASOLN(M,N,PI,SIG,AR,SLMB)
       AM1=2.*ROA*W*W*AR*AR(PI**6.)*E*E*(TR**4.)*GR*GR*SLMB*SLMB
       AM2=9.*C*(GM**4.)*(GN**4.)*RO*RO*((1.(SIG*SIG))**2.)
       AMP=AM1/AM2
       AH=PI/70.0
       YY=33.0*AH
       GSUM=0.0
       TH=0.0
       CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
       GSUM=GSUM+SUM
170    TH=TH+AH
       CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
       GSUM=GSUM+4.0*SUM
       TH=TH+AH
```

```
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(TH.LT. YY) GO TO 170
      TH=TH+AH
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(5,300) M,N,GPM,GPN,POWER
300   FORMAT(7X,I2,8X,I2,10X,F10.4,10X,F10.4,10X,E10.4)
      X21=X21+3.1
4     CONTINUE
      X11=X11+3.1
3     CONTINUE
      X11=2.2
      DO 5 M=2,16,2
52    X12-X11+HH
      Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.00) GO TO 50
      X11+X12
      GO TO 52
50    GM=X11+X12
      X21=3.9
      DO 6 N=3,32,2
62    X22=X21+HH
      Y21=(SIN(X21)/COS(X21))-(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))-(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.00) GO TO 60
      X21=X22
      GO TO 62
60    GPN=X21+X22
      HGM=GM/2.0
      HGPN=GPN/2.0
      RM=SIN(HGM)/SINH(HGM)
      RPN=-SIN(HGPN)/SINH(HGPN)
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.*ROA*W*W*AR*AR(PI**6.)*E*E*(TR**4.)*GR*GR*SLMB*SLMB
      AM2=9.*C*(GM**4.)*(GN**4.)*RO*RO*((1.(SIG*SIG))**2.)
      AMP=AM1/AM2
      AH=PI/70.0
      YY=33.0*AH
      TH=0.0
      GSUM=0.0
      CALL FIVE(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
270   TH=TH+AH
      CALL FIVE(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL FIVE(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(TH.LT.YY) GO TO 270
```

```
      TH=TH+AH
      CALL FIVE(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(5,400) M,N,GM,GPN,POWER
400   FORMAT(7X,I2,8X,I2.10X,F10.4,10X,F10.4,10X,E10.4)
      X21=X21+3.1
6     CONTINUE
      X11=X11+3.1
5     CONTINUE
      X11=3.9
      DO 7 M=3,16,2
112   X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.00) GO TO 110
      X11=X12
      GO TO 112
110   GPM=X11+X12
      X21=2.2
      DO 8 N=2,32,2
122   X22=X21+HH
      Y21=(SIN(X21)/COS(X21))+(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))+(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.00) GO TO 120
      X21=X22
      GO TO 122
120   GN=X21+X22
      HGPM=GPM/2.0
      HGN=GN/2.0
      RPM=-SIN(HGPM)/SINH(HGPM)
      RN=SIN(HGN)/SINH(HGN)
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.*ROA*W*W*AR*AR(PI**6.)*E*E*(TR**4.)*GR*GR*SLMB*SLMB
      AM2=9.*C*(GM**4.)*(GN**4.)*RO*RO*((1.(SIG*SIG))**2.)
      AMP=AM1/AM2
      AH=PI/70.0
      YY=33.0*AH
      TH=0.0
      GSUM=0.0
      CALL SEVEN(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
370   TH=TH+AH
      CALL SEVEN(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL SEVEN(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(TH.LT.YY) GO TO 370
      TH=TH+AH
      CALL SEVEN(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
```

```
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(5,500) M,N,GPM,GN,POWER
500   FORMAT(7X,I2,8X,I2,10X,F10.4,10X,F10.4,10X,E10.4)
      X21=X21+3.1
8     CONTINUE
      X11=X11+3.1
7     CONTINUE
      STOP
      END
*     ******************************************************************
*     ********************* END OF MAIN PROGRAM ************************
*     ******************************************************************
*     *********************  SUBROUTINE STARTS  ************************
*     ******************************************************************
      SUBROUTINE ASOLN(M,N,PI,SIG,AR,SLMB)
*     THIS SUBROUTINE EVALUATES THE DIMENSIONLESS FREQUENCY FACTOR.
      FM=FLOAT(M)
      GX=FM-0.5
      HX=(FM-0.5)*(FM-0.5)*(1.0-(2.0/((FM-0.5)*PI)))
      XJ=(FM-0.5)*(FM-0.5)*(1.0-(2.0/((FM-0.5)*PI)))
      FN=FLOAT(N)
      GY=FN-.05
      HY=(FN-0.5)*(FN-0.5)*(1.0-(2.0/((FN-0.5)*PI)))
      YJ=(FN-0.5)*(FN-0.5)*(1.0-(2.0/((FN-0.5)*PI)))
      ST1=GX*GX*GX*GX
      ST2=(GY*GY*GY*GY)/(AR*AR*AR*AR)
      ST3=(SIG*HX*HY)+((1.0-SIG)*XJ*YJ)
      ST4=(2.0*ST3)/(AR*AR)
      SLMB=ST1+ST2+ST4
      RETURN
      END
*     ******************************************************************
      SUBROUTINE ONE(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY FOR
*     EVEN VALUES OF M AND N.
      AK=PI/70.0
      XX=33.0*AK
      TK=0.0
      SUM=0.0
      CALL TWO(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
90    TK=TK+AK
      CALL TWO(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL TWO(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(TK.LT.XX) GO TO 90
      TK=TK+AK
      CALL TWO(C,SLMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
```

```
*     *****************************************************************
      SUBROUTINE TWO(C,SIMB,GM,GN,RM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE IS USED TO DETERMINE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR EVEN VALUES OF M AND N.
      AL1=(SIMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SIMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*TR*AR*SIN(TH)*SIN(TK)/C
      HGM=GM/2.0
      HGN=GN/2.0
      ST7=AL*COS(HGM)*SIN(AL/2.0)
      ST8=GM*SIN(HGM)*COS(AL/2.0)
      ST9=AL*COSH(HGM)*SIN(AL/2.0)
      ST10=GM*SINH(HGM)*COS(AL/2.0)
      ST11=AS*COS(HGN)*SIN(AS/2.0)
      ST12=GN*SIN(HGN)*COS(AS/2.0)
      ST13=GN*SINH(HGN)*COS(AS/2.0)
      ST14=AS*COSH(HGN)*SIN(AS/2.0)
      ST15=((AL*AL)/(GM*GM))-1.0
      ST16=((AL*AL)/(GM*GM))+1.0
      ST25=((AS*AS)/(GN*GN))-1.0
      ST26=((AS*AS)/(GN*GN))+1.0
      ST17=((ST7-ST8)/ST15)+RM*((ST9+ST10)/ST16)
      ST27=((ST11-ST12)/ST25)+RN*((ST13+ST14)/ST26)
      FUNC=ST17*ST17*ST27*ST27*SIN(TK)
      RETURN
      END
*     *****************************************************************
      SUBROUTINE THREE(C,SIMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY FOR
*     EVEN VALUES OF M AND N.
      AK=PI/70.0
      XX=33.0*AK
      TK=0.0
      SUM=0.0
      CALL FOUR(C,SIMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
190   TK=TK+AK
      CALL FOUR(C,SIMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL FOUR(C,SIMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(TK.LT.XX) GO TO 190
      TK=TK+AK
      CALL FOUR(C,SIMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     *****************************************************************
```

```
      SUBROUTINE FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE IS USED TO EVALUATE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR ODD VALUES OF M AND N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*TR*AR*SIN(TH)*SIN(TK)/C
      HGPM=GPM/2.0
      HGPN=GPN/2.0
      ST30=AL*SIN(HGPM)*COS(AL/2.0)
      ST31=GPM*COS(HGPM)*SIN(AL/2.0)
      ST33=AL*SINH(HGPM)*COS(AL/2.0)
      ST34=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST35=AS*SIN(HGPN)*COS(AS/2.0)
      ST36=GPN*COS(HGPN)*SIN(AS/2.0)
      ST37=AS*SINH(HGPN)*COS(AS/2.0)
      ST38=GPN*COSH(HGPN)*SIN(AS/2.0)
      ST39=((AL*AL)/(GPM*GPM))-1.0
      ST40=((AL*AL)/(GPM*GPM))+1.0
      ST41=((AS*AS)/(GPN*GPN))-1.0
      ST42=((AS*AS)/(GPN*GPN))+1.0
      ST43=((ST30-ST31)/ST39)+RPM*((ST33-ST40)/ST40)
      ST44=((ST35-ST36)/ST41)+RPN*((ST37-ST38)/ST42)
      FUNC=ST43*ST43*ST44*ST44*SIN(TK)
      RETURN
      END
*     ********************************************************************
      SUBROUTINE FIVE(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY FOR
*     EVEN VALUES OF M AND ODD VALUES OF N.
      AK=PI/70.0
      XX=33.0*AK
      TK=0.0
      SUM=0.0
      CALL SIX(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
290   TK=TK+AK
      CALL SIX(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL SIX(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(TK.LT.XX) GO TO 290
      TK=TK+AK
      CALL SIX(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ********************************************************************
```

```
      SUBROUTINE SIX(C,SLMB,GM,GPN,RM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE IS USED TO EVALUATE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR EVEN VALUES OF M AND ODD VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*TR*AR*SIN(TH)*SIN(TK)/C
      HGM=GM/2.0
      HGPN=GPN/2.0
      ST30=AL*COS(HGM)*SIN(AL/2.0)
      ST31=GM*SIN(HGM)*COS(AL/2.0)
      ST33=AL*COSH(HGM)*SIN(AL/2.0)
      ST34=GM*SINH(HGM)*COS(AL/2.0)
      ST35=AS*SIN(HGPN)*COS(AS/2.0)
      ST36=GPN*COS(HGPN)*SIN(AS/2.0)
      ST37=AS*SINH(HGPN)*COS(AS/2.0)
      ST38=GPN*COSH(HGPN)*SIN(AS/2.0)
      ST39=((AL*AL)/(GM*GM))-1.0
      ST40=((AL*AL)/(GM*GM))+1.0
      ST41=((AS*AS)/(GPN*GPN))-1.0
      ST42=((AS*AS)/(GPN*GPN))+1.0
      ST43=((ST30-ST31)/ST39)+RM*((ST33+ST34)/ST40)
      ST44=((ST35-ST36)/ST41)+RPN*((ST37-ST38)/ST42)
      FUNC=ST43*ST43*ST44*ST44*SIN(TK)
      RETURN
      END
*     ****************************************************************
      SUBROUTINE SEVEN(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY FOR
*     ODD VALUES OF M AND EVEN VALUES OF N.
      AK=PI/70.0
      XX=33.0*AK
      TK=0.0
      SUM=0.
      CALL EIGHT(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
390   TK=TK+AK
      CALL EIGHT(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL EIGHT(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(TK.LT.XX) GO TO 390
      TK=TK+AK
      CALL EIGHT(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ****************************************************************
```

```
      SUBROUTINE EIGHT(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE IS USED TO EVALUATE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR ODD VALUES OF M AND EVEN VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*TR*AR*SIN(TH)*SIN(TK)/C
      HGPM=GPM/2.0
      HGN=GN/2.0
      ST30=AL*SIN(HGPM)*COS(AL/2.0)
      ST31=GPM*COS(HGPM)*SIN(AL/2.0)
      ST33=AL*SINH(HGPM)*COS(AL/2.0)
      ST34=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST35=AS*COS(HGN)*SIN(AS/2.0)
      ST36=GN*SIN(HGN)*COS(AS/2.0)
      ST37=AS*COSH(HGN)*SIN(AS/2.0)
      ST38=GN*SINH(HGN)*COS(AS/2.0)
      ST39=((AL*AL)/(GPM*GPM))-1.0
      ST40=((AL*AL)/(GPM*GPM))+1.0
      ST41=((AS*AS)/(GN*GN))-1.0
      ST42=((AS*AS)/(GN*GN))+1.0
      ST43=((ST31-ST32)/ST39)+RPM*((ST33-ST34)/ST40)
      ST44=((ST35-ST36)/ST41)+RN*((ST37+ST38)/ST42)
      FUNC=ST43*ST43*ST44*ST44*SIN(TK)
      RETURN
      END
*     ******************************************************************
```