ACOUSTICS OF RECTANGULAR FLAT PLATES WITH FREE-SIMPLY SUPPORTED AND CLAMPED-SIMPLY SUPPORTED EDGE CONDITIONS.

BY

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RECOMMENDATION OF THE BOARD OF EXAMINERS.

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ABSTRACT

The total average acoustic power radiation from one side of a baffled rectangular flat plate,vibrating in its natural modes under certain mixed boundary conditions,has been investigated. The mixed boundary conditions which have been investigated are :(1)two opposing ends free and the other two ends simply supported ; (2) one end simply supported while the other three ends clamped. The radiation efficiencies of these plates have also been found for verifying the method of the present investigation by comparing the efficiencies with those available in the literature. Numerical results on the acoustic power radiation by the plate have been obtained for various mode orders, aspect ratios, and thickness ratios.

Expressions for the acoustic power radiation by plates under specific edge- restrains have been derived in terms of the farfield acoustic pressure distribution . The characteristic beam functions developed by Warburton have been used in these derivations for representing the vibration modes of the rectangular flat plates.

At low mode orders, the power radiation from the plates under investigation are observed to increase with the increase of mode orders . But at higher mode orders , the acoustic power radiation is observed to fluctuate, maintaining nearly the same average magnitude. With further increase of mode orders, the waviness is observed to reduce gradually .

The effects of aspect ratio and thickness ratio on the average acoustic power radiation have also been studied. It has been found that the variation of power radiation with the change of these plate parameters is similar for both the boundary conditions. The variation of acoustic power radiation for two different materials have also been studied.

It is observed that at low mode orders, power radiation from a steel plate is higher than that from a plastic plate. But at high mode orders,a plastic plate radiates more power.

The power radiations from plates of different boundary conditions are also compared with that obtained by Ahmed(53) and Mandal(54) over identical range of mode orders. Power radiations are observed to vary with boundary conditions but the difference from one another is not significantly high and the nature of variation for individual plate is more or less similar.

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CONTENTS

NOTATIONS

a	Half of the length of the plate
b	Half of the width of the plate
c	Velocity of sound in air
E	Modulus of elasticity of the plate material
f	Frequency of vibration of the plate
g	Acceleration due to gravity
G_x	A function of m, factor of the dimensionless frequency factor
G_y	A function of n, factor of the dimensionless frequency factor
H_x	A function of m, factor of the dimensionless frequency factor
H_y	A function of n, factor of the dimensionless frequency factor
J_x	A function of m, factor of the dimensionless frequency factor
J_y	A function of n, factor of the dimensionless frequency factor
h	thickness of the plate
k	Acoustic wave number
k_p	Plate wave number
m	Number of nodal lines in x-direction
n	Number of nodal lines in y-directions
P_w	Total acoustic power radiation
p	Acoustic pressure distribution
R	Distance of the receiving point from the center of the plate
R_a	Aspect ratio of the plate , b/a

R_t	Thickness ratio of the plate, h/a
r	Distance of the receiving point from the elemental area
s_{mn}	Radiation efficiency
T	Kinetic energy
t	time
U	Surface velocity distribution,potential energy
U_o	Amplitude of velocity distribution
W	Amplitude of vibration
w	Instantaneous displacement of the plate surface
(x,y,z)	Cartesian coordinate system
γ	Roots of the equations, $\tan(\gamma/2) \pm \tanh(\gamma/2) = 0$; ratio of specific heat at constant pressure to that at constant volume
$\theta(x)$	Displacement function in x-direction
$\theta(y)$	Displacement function in y-direction
λ	Wave length of sound
λ_f	Dimensionless frequency factor
ρ	Density of the surrounding medium
ρ_m	Density of the plate material
σ	Poison's ratio
ϕ	Velocity potential
Ψ	wave number ratio, k/k_p
(R,θ,α)	Spherical coordinate system
w	Circular frequency of plate vibration

CHAPTER -1

INTRODUCTION

1.1.GENERAL:

With the growth of mechanization in modern life,the problem of noise is also growing steadily. The mechanical and electrical equipment which are an integral part of modern living are also the sources of many acoustical problems.Nearly every piece of mechanical equipment in a building is a source of sound and vibration.The size and complexity of machines , increasing road traffic,etc.,are contributing to the ever increasing level of noise. The quality of life is being degraded by these noise sources .The problem is already recognized as one of primary importance and demands immediate attention.

Engineers and scientists are increasingly concentrating on the acoustic design of theater and concert halls, quieter mechanical equipment , reduction of aircraft noise and on such specialized fields as under water-sound and medical application of ultrasonic.One of the most important and extensive application of acoustics and noise control relates to mechanical systems of building.Here attention is focused on ensuring such things as adequate speech privacy ,provision for acceptable sleeping habits and little or no annoyance from equipment noise.The cost of insulating against noise must also be considered. Therefore ,the control of noise is important for providing acceptable acoustic environment as well as for economic reasons.

1.2 MECHANISM OF PRODUCTION AND PROPAGATION OF SOUND:

An understanding of the theory of acoustics is very important and necessary to identify the sources of sound and to realize the mechanism by which it is produced. Noise is caused by the vibration of solid, liquid or gaseous medium.When this vibration is within the range of audible frequency of the human beings, a human ear can perceive the noise ; otherwise ,they go unnoticed . When a solid body vibrates at a frequency within the audible frequency range , a part of the energy dissipated is transmitted to the surrounding environment as perceivable sound.Energy is transferred from one vibrating particle to the next and the acoustic energy travels through the surrounding medium as longitudinal waves. In this study , our interest is to find the amount of acoustic energy dissipated as noise due to the natural vibration of rectangular plates with certain mixed boundary conditions.

1.3 MOTIVATION TO SELECT THE PROBLEM:

Sound produced due to mechanical vibration of solid structures is transferred to the surrounding medium in the form of acoustic energy.

The noise generated by a machine has considerable influence on its salability.Machines which vibrate excessively and generate an unwarranted amount of noise are no longer accepted by either the industrial concern or the home owner.Everyday ,many problems are encountered with unwanted sound.Many of these noise sources are in the form of flat plates.

The familiar flat plate noise sources are windows, walls, floors and roofs of buildings, many enclosed surfaces and integrated parts of machine, the walls of air-conditioning ducts, the air plane wings, etc. Numerous attempts were made in the past to evaluate the noise characteristics of different vibrating sources by direct measurement. Continued efforts in this direction has led to the development of very sophisticated sound measuring devices. Engineers always endeavor to eliminate acoustical noise from the mechanical systems and buildings. In order to determine the noise dissipation from the rectangular flat plates under various mixed boundary conditions, vibrating in one of their natural modes, knowledge of acoustics of these plates is very essential.

1.4 OBJECTIVES OF THE STUDY:

In this analysis, attempt is made to study the noise generating characteristics of a rectangular flat plate in an infinite baffle. The plate is subjected to two mixed boundary conditions. These are : (a) two opposing ends free and other two ends simply supported , (b) one end simply supported and other three ends fixed. The objectives of the study can be outlined as follows.

(1) The development of appropriate displacement functions that satisfy the boundary conditions of the rectangular flat plates vibrating in flexure.

(2) Derivation of expressions for the natural frequencies of vibrating plates for different mode shapes.

(3) Evaluation of the frequency of vibration for a particular case and study its variation with the mode shape.

(4) Derivation of an expression for the power radiation due to natural vibration of the plate under consideration.

(5) Evaluation of the magnitude of the average power radiated from one side of the baffled plates by the method of numerical integration of the expression for the power radiation.

(6) To study the variation in the radiated power from the plate with various mode orders,aspect ratios ,and thickness ratios under different boundary conditions.

(7) Determination of the radiation efficiencies of the plates under different boundary conditions and plotting the same against the wave number ratio for comparing the results of the current investigations with those of the previous works(51,53,54)in order to verify the reliability of the present analysis and to ascertain the absence of any error in the numerical procedure employed.

1.5.DEFINITION OF TERMS:

Some of the terms used in this thesis are defined in order to remove ambiguity in their use and to attach precise meaning to them.

(a)Acoustic Pressure:

Sound is a disturbance that propagates through an elastic medium at a speed characteristics of that medium. Sound travels as a wave of alternate compression and rarefaction with an associated pressure variation. The pressure variation during sound traveling is of great importance in most practical problems. The acoustic at any point is the difference between the actual pressure at that point in the presence of the sound and the pressure that would exist at that point, under identical conditions, in the absence of sound. This is often called excess pressure or instantaneous sound pressure.It varies sinusoidally with time.

(b)Effective Sound Pressure:

Sound consists of a rapid irregular series of positive pressure disturbances(compression) and negative pressure disturbances(rarefaction) measured from the equilibrium pressure value. Therefore ,the mean value of sound pressure disturbance is zero,because there are as many positive compressions as negative rarefactions. An attempt at using the mean value conveys no useful information. Thus , when dealing with pressure , it is convenient to use the root mean square , or r.m.s. ,value.This is defined as the square root of the average of the squares of the instantaneous sound pressure. In mathematical notation,

$P_{rms} = P_o / \sqrt{2}$,where P_o is the peak value of sound wave.The rms value of sound pressure is often called effective sound pressure.

(c)Acoustic Intensity:

Acoustic power is transferred to the surrounding medium in the form of sound due to the vibration of a body. The most basic quantity in understanding the sound power is acoustic intensity. For defining it, a point at some distance from a source of sound and a small area perpendicular to the line joining the point to the source has to be considered. Some of the power generated by the source will be transmitted through the area.The exact amount of power transmitted depends not only on the sound power of the source but also on its directional properties, the distance of the area from the source and the presence of sound absorbing or sound reflecting materials.If the power passing through an area A is W, then the acoustic intensity I is given by

$$I=W/A.$$

CHAPTER - 2

LITERATURE REVIEW

2.1. Introduction :

Work on the study of the vibration characteristics of rectangular plates began in the last part of 19th century when it was realized that a large number of structural systems in civil, mechanical and aerospace engineering are composed of these plate components.The dynamical behabiour of these structures was first studied by Rayleigh.He developed the fundamental equations governing the vibration of plates. Since the middle of the 20th century, noise has been recognized as a source of environmental pollution which ultimately affects human comfort.From that time, research in this field attracted the attention of investigators. The study of acoustic radiation characteristics of flat plates has not yet ascended to a level of satisfaction.

2.2. VIBRATION OF PLATES:

The vibration of plates was studied by many investigators in the past. Most of the initiators confined their study to the determination of the frequency of vibrating plates.

In 1954,Warburton (1) derived approximate expressions for the frequencies of all the modes of vibration of isotropic plates subjected to any combination of free, simply supported or clamped edges.He applied the Rayleigh method, assuming that the deflections of the plates could be represented by suitable characteristic functions satisfying the boundary conditions(2).In his analysis , Warburton first developed a set of beam functions satisfying the edge conditions of beams and used those functions to the vibration of plates.He expressed the frequency in terms of a dimensionless frequency factor which in its turn is a function of the mode shapes of the vibrating plates.

In 1959,Hearmon(3) applied the Rayleigh method to derive closed formulas for the frequency of vibration of orthotropic plates under any combination of clamped or supported edges. He used the characteristic beam functions , developed by Warburton(1) , appropriate to the boundary conditions.In 1962, Claassen and Thorne(4) studied the vibration of thin rectangular isotropic plates and presented four graphs giving the first ten vibration frequencies of a clamped rectangular plate as a function of the ratio of sides, and one graph of nodal lines to illustrate the transition from one mode of vibration to another.

Approximate values of the natural frequency of an isotropic rectangular plate with thickness varying in one direction were reported by Apple and Byers(5). They calculated the upper and lower bounds for the fundamental natural frequency of the simply supported plates. Ueng(6) derived the natural frequencies of vibration of an all-clamped rectangular sandwich panel in 1966. Dickinson(7) extended the sine series solution, previously used for the study of the flexural vibration of rectangular isotropic plates , to freely vibrating orthotropic plates. He presented the application of the method to three particular plates with different support conditions and also the numerical results of two of these examples. Vibration of rib-stiffened elliptical and circular plates were analyzed by Desiderati and Laura(8) in 1970. They dealt with the determination of the fundamental frequency of vibration of simply-supported and clamped rib-stiffened plates of elliptical boundary. They presented calculations of the fundamental frequency of vibration of circular and elliptical plates with stiffeners in one direction , for both the clamped and simply supported cases.

Hoppmann(9)in a comprehensive experimental study measured the resonant frequencies and mode shapes of both circular and elliptical plates with and without stiffeners for the clamped case and calculated the fundamental frequency for the stiffened circular plate using the "orthotropic plate" approximation and the Rayleigh-Ritz(10) method.Leissa(11) studied the vibration of an unstiffened simply supported elliptical plate using the Rayleigh-Ritz method with a three-term deflection function and McNitt(12)obtained results for an unstiffened clamped elliptical plate using Galerkin's method and a two-term deflection function. An analytical study of the natural frequencies and mode shapes of anisotropic thin plates was presented by Mohan and Kingsbury(13)in 1971. They considered certain mixed and unmixed boundary conditions.

Toshiyuki Sakata(14) derived an approximate formula for the estimation of the fundamental frequency of the simply supported orthotropic rectangular plate with thickness varying linearly in one direction. The accuracy of the formula and the influence of the flexural rigidity on the natural frequency was also discussed. Gorman(15) conducted free vibration analysis of plates with all combinations of clamped-simply supported edges,except those plates with two opposite edges simply supported. In 1979, Leissa ,Laura and Gutierrez(16) presented a general method for dealing with supports having translational and rotational flexibilities which vary in an arbitrary manner around the boundary. Iyengar and Raman(17) investigated the free vibration of circular plates of arbitrary thickness using the method of initial functions. Numerical results were obtained for flexural vibration of circular plates . These authors (18) also studied the axisymmetric free vibration of thick annular plates by the same method of initial functions and determined numerically the natural frequencies for two typical support conditions.

Chuh Mei,Narayanaswami and Rao(19) developed a finite element formulation for analyzing large amplitude free flexural vibration of elastic plates of arbitrary shape. Nonlinear frequencies for square, rectangular, circular,rhombic and isosceles triangular plates , with edges simply supported or clamped, were obtained. A triangular plate element was developed to investigate large amplitude free flexural vibrations of thin elastic plates of arbitrary shape. Mizusawa, Kajita and Naruoka(20) presented a general procedure for calculating the free vibration of stiffened skew plates by Rayleigh-Ritz method with B-spline functions as coordinate functions.

In 1980, Ali and Atwal(21) studied the simply supported square plates with square and rectangular cutouts. They followed the Rayleigh's principle and predicted the natural frequencies and mode shapes with the help of finite element technique.A finite strip analysis of the vibration of rectangular Mindlin plates with general boundary conditions was described by Roufaeil and Dawe(22) in 1980. They used the normal modes of vibration of Timoshenko beams to represent the spatial variation along a strip of the deflection and two cross-sectional relations. The accuracy of the approach was demonstrated by the results of a number of applications to square plates with combinations of simply supported, clamped and free edges.

In 1981, Toshihiro Irie, Yamada and Yoda(23) derived the frequency equations of membranes and plates under free vibration by the Ritz method . The natural frequencies and mode shapes were calculated numerically upto modes for the membranes and plates symmetrical with respect to the center lines. Lin and Chiang(24) analyzed the steady state vibration of plates by Laser speckle method and determined the frequencies . Gelos and co-authors(25) , in 1981,conducted the analysis of vibration of circular plates with variable profile. They took into account some complicating factors like concentrated masses , in-plane forces, anisotropic characteristics of the plate material, etc. Basci, Toridis and Khozeimeh(26) developed an improved method to study the free vibration of thin rectangular plates. The natural frequencies of stiffened clamped plates were obtained for plates with different sector angles by Srinivasan and Thiruvenkatachari (27) with the help of integral equation technique. Ovunc (28) obtained a general solution for the Helmholtz differential equations in the complex domain and applied to the non-linear , free, bending vibration of plates. The effect of stretching on the natural circular frequencies was illustrated. The non-linear, free vibration of circular plates was investigated by the dynamic deflection function.

In 1985, Mirza and Bizlani(29) solved the problem of the natural frequencies and mode shapes of cantilevered triangular plates with variable thickness using the finite element technique. K. Ohtomi(30)studied analytically the free vibration of a simply supported rectangular plate, stiffened with viscoelastic beam. The effects of the volume and number of stiffening beam on the plate were clarified. M. B. Rubin(31) studied the free vibration of a rectangular parallelopiped composed of a homogeneous linear elastic isotropic material using the theory of a Cosserat point. In 1986, Ganesan and Rao(32) derived the mass and stiffness matrices of an annular element consisting of base plate and unconstrained damping layer assuming a modal solution to the equation of motion of the plate. The complex eigen equations were solved for frequencies by an extension of the simultaneous iteration technique. Free vibration characteristics of a damped stiffened panel with applied viscoelastic damping on the flanges of the stiffeners were studied using finite element method by Gupta and co-workers(33).

In 1987 , Craig and Dawe(34) considered the free vibration of prismatic plate structures of laminated composite material and having diaphragm end supports using the finite strip method. Natural frequencies were calculated by these authors. Srinivason and Babu(35) investigated the free vibration of laminated quadrilateral plates with clamped edges. the behaviour of trapezoidal plates with different number of layers were studied. Chen and Juans(36) obtained the fundamental natural frequencies of axisymmetric circular and annular plates subjected to a combination of a pure bending stress and extensional stress in the plane of the plate. Y. C. Zhang and Xianyuong He(37)used a technique for using discrete least-squares methods to calculate the natural frequency of thin plates. The single or double fifth B-spline functions were used as trial functions and 2nd order eigen equation was established. The results for calculating simply supported or clamped isotropic plates and particularly orthotropic laminated plates were in good agreement with the analytical solutions.

2.3. Acoustic Radiation from plates:

Although vibration analysis of solid structures like rectangular plates is quite numerous , the study of acoustic radiation due to vibration of flat plates is rather lacking.

In 1954, Skudrzyk(38,39) obtained the solution to the radiation of power from an elastic plate for a time harmonic point force. He used the classical plate theory . The solutions for the acoustic radiated pressure from a point excited plate using the classical theory was given by Feit(40) and Skudrzyk(39). The radiation impedance of membranes and plates of finite width , carrying a traveling wave, was calculated by Lowenthal and Tournois(41). The method used led to a relating simple analytic expression. The problem of acoustic coupling of these strips with the fluid propagating medium was then examined.

An expression was derived for the acoustic power radiated by an infinite uniform isotropic plate excited by a time-harmonic moment acting at a point on the plate by Thompson and Rattayya(42)in 1964. The effect of radiation loading was included. Results were obtained that were valid below the coincidence frequency. Tsakonas, Chen and Jacobs(43) determined the acoustic radiation of an infinite plate excited by the field of a ship propeller both by theoretical and numerical methods. In 1967, Maidanik(44) computed the far-field acoustic pressure generated by an infinite plate driven by a point and a line force. The plate considered here was backed by a fluid medium that was terminated by a parallel infinite baffle. He also applied the same methodology to a coated infinite plate. Magrab and Reader (45)obtained an expression for the far-field pressure radiated by a thin elastic plate bounded on one side by a fluid medium when the plate was excited by a time-varying point load.

In order to discuss the sound radiation from a direct-radiator loudspeaker in the high frequency region, an elastically supported circular plate in an infinite baffle was used by Suzuki and Tichy(46). The normal modes of this plate was consistently treated from the free boundary to the simply supported boundary, which were the limiting cases of the elastically supported boundary , by defining a dimensionless parameter representing the ratio of the edge stiffness to the bending stiffness of the plate.To improve the high frequency prediction of elastic plates, Feit(47) employed the Timoshenko-Miudlin theory for such a prediction. This theory adds shear deformation and rotary inertia to the classical flexural theory. There are two dispersion curves for this plate. The flexural branch(acoustic) has phase and group velocities approaching the Rayleigh velocity of the plate material as the frequency increases.

Sound radiation from beam reinforced plate , excited by point or line forces, has been investigated by a few authors. Feit and Saurenman(48) analyzed the acoustic radiation of a point excited plate reinforced with a beam, but confined their interests to high frequencies. Gorman(49) obtained the solution for a plate reinforced by many beams and excited by a line force parallel to the beam.His solution thus makes the beams' reaction on the plate purely as rotary and transverse impedances without flexural wave traveling in the beams.

Wallace(50) found the radiation resistance from the far-field pressure distribution produced by a baffled beam, vibrating with simple harmonic motion in one of its natural modes. He considered beams hinged at each end and clamped at each end. The author derived expressions for the radiation resistance which are asymptotic to the exact solution as the frequency approaches zero. In addition , numerical integration of the farfield acoustic intensity was used to obtain graphs covering the entire frequency range for the first ten modes of the beam. In another paper (51)Wallace determined the radiation resistance corresponding to the natural modes of a finite rectangular panel supported in an infinite baffle. He used the appropriate beam functions given by Warburton(1) . But his analysis was confined to simply supported plates only .

In 1987, Wallace(52) determined theoretically the acoustic damping for single modes of a finite rectangular panel ,simply supported in an infinite baffle from the ratio of the acoustic energy radiated per cycle to the vibratory energy of the panel. Ahmed(53)and Mandal(54)studied the acoustic power radiation from one side of a baffled rectangular isotropic plate for different thickness ratio, aspect ratio and mode orders. Ahmed(53) found the radiation efficiency of the plate with all edges simply supported.Mandal(54)also found the radiation efficiency of rectangular plates for two opposing ends fixed and other two ends free. The problem of acoustic radiation from baffled finite elastic plates driven by multiple random point forces was analyzed by Peng and Keltie(55). Expressions for the surface acoustic intensity and radiated sound power were derived neglecting the fluid loading. By assuming the plates to be lightly damped and neglecting the modal coupling effects, approximate solutions for the acoustic intensity and sound power were obtained. An experimental study was also carried out to measure the surface acoustic intensity patterns and to compare them with the analytical results.

As appears from the foregoing survey, the majority of the research works in the field of noise of vibration are for the natural frequency of rectangular plates. Not much work has been done to analyze the acoustic radiation characteristics of rectangular flat plates vibrating in its natural modes. Works on natural frequency and acoustic radiation of rectangular plates are continuing and have to go a long way before full comprehension is achieved.

CHAPTER 3

FORMULATION OF THE PROBLEM

3.1. Introduction :

Sound is usually produced by some vibrating object which is in contact with a fluid. In this analysis , the vibrating object is a rectangular plate of uniform thickness. It is assumed that the plate is confined in an infinite baffle. The baffle prevents the movement of air around the edges of the plate and permits radiation into the spaces in front of either of the surfaces of the plate. Since air is an elastic medium ,the disturbed portion transmits its motion to the surrounding air so that the disturbance is propagated in all directions from the source.Energy is transferred from the plate to the fluid due to vibration of the plate.

The properties of sound wave depend on speed characteristics of the medium which in turn depend on the characteristics of the source.Our keen interest here is to determine the generated sound energy and radiation efficiency. Fluid medium outside the source is assumed to be initially uniform and at rest. The physical quantity that is generally of interest is sound pressure. We assume that the acoustic pressure generated outside the source is small enough so that the first order equations of sound are valid in the region outside the source. The plate considered here may have any combination of simply-supported, clamped or freely suspended condition at the edges. The present analysis is centered to certain mixed boundary conditions. They are (1) two opposing ends simply-supported and the other two opposing ends freely suspended, (2) one end simply supported and the other three ends clamped. The plate, along with the coordinate system used in the analysis is shown in fig.-1. Each of the boundary conditions is considered in separate sections later. But only a general mathematical model is presented in this chapter.

3.2. ASSUMPTIONS :

The mathematical analysis of vibrating plates is extremely complex. Due to this, it is necessary to make certain assumptions which will simplify the problem. These are :

(1) The thickness of the plate is small compared with other dimensions and the thickness is uniform.

(2) The plate is free from applied load.

(3) No strain is suffered by the middle plane of the plate. this plane remains neutral during bending.

(4) cross-sectional plane before strain remains plane after strain.

(5) The plate is isotropic , elastic and homogeneous.

(6) Deflections are small in comparison with the thickness of the plate.

(7) The normal stresses in the direction transverse to the plate can be neglected. These stresses must be zero at the free surfaces and, provided that the plate is thin, it is reasonable to assume that it is zero at any section.

(8) The influence of shear and rotating inertia is neglected.

(9) Only transverse displacement is considered.

3.3.MATHEMATICAL MODELING:

The vibrating form of a rectangular plate of dimension 2a 2b must satisfy the boundary conditions at the edges and also satisfy the plate equation of dynamical equilibrium. Due to excitation , the plate vibrates in simple harmonic motion in one of its natural modes. The transverse displacement at a point (x,y) on the surface of the plate at any time t corresponding to the (m,n)th mode of vibration is given by (53)

$$w_{mn} = W_{mn} \quad \theta(x) \ \theta(y) \ e^{i\omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

where,

W_{mn}=amplitude of transverse displacement .

ω=natural angular frequency of vibration of the plate corresponding to the (m,n)th mode.

t= time

$\theta(x),\theta(y)$ = displacement functions describing the wave form of the vibrating plate in conformity with the conditions at the edges.

The motion of the plate surface which generates the acoustic radiation is given by the normal velocity distribution,

$$U = \frac{dw_{mn}}{dt} = i\omega_{mn} W_{mn} \theta(x)\theta(y)e^{i\omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

$$= U_o e^{i\omega t}$$

Due to the vibration of the plate ,a disturbance is created in the fluid surrounding the plate. This disturbance propagates through out the fluid if the fluid is compressible. It is desired to study the characteristics of this propagation . Some relevant necessary notations are needed to be included here for the aforementioned purpose.

The notations are -

x,y,z coordinates of a particle of the medium.

u,v,w component velocities of a particle in the medium,

ρ' instantaneous density of the medium.

c velocity of propagation of the disturbance,

s' the condensation of the medium at any point due to the passage of sound wave and is defined as the ratio of the change in density to the original density,

i.e. $s' = \dfrac{(\rho' - \rho_0)}{\rho_0}$

P' instantaneous sound pressure of the medium at any point,

ρ_0 undisturbed density of the surrounding medium,

ϕ the velocity potential.

c - the velocity of propagation of the disturbance, independent of x,y,z and t. All the other quantities are functions of x,y,z and t. Now, we have to define the velocity potential which is the most important single quantity in the study of the irrotational motion of fluids.

Velocity potential(ϕ) is defined as a scalar function of space and time i.e., $\phi = f(x,y,z,t)$, such that its negative derivative with respect to any direction is the fluid velocity in that direction, i. e.,

$$u = -\frac{\partial \varphi}{\partial x}$$
$$v = -\frac{\partial \varphi}{\partial y}$$
$$w = -\frac{\partial \varphi}{\partial z}$$

where u ,v, w are the velocity components along x, y,z directions ,respectively. The negative sign in the above equations for φ has no physical significance. It is a conventional adoption here which means the fluid moves in the direction in which the velocity potential decreases and agrees with the convention adopted for the direction of other quantities for the potential function in mathematical physics.

Considering a small volume element of the fluid, the difference between the efflux and influx of the medium in this element is equal , by the principle of continuity, to the time rate of growth of mass in the element.

For developing the mathematical expression of this principle , the mass flow through the faces of a small paralellopiped with edges parallel to a x,y,z co-ordinate system have to be considered. Let the center of the body be at (x,y,z) and the center is fixed in space(Fig.3.1).

Now, the mass flow rate through the paralellopiped in the x-direction $= \rho' u \, \partial y \, \partial z$

The in-flow rate of mass through the left x- face of the paralellopiped $= \rho' u - \frac{\partial}{\partial x}(\rho' u)\frac{\partial x}{2} \, \partial y \, \partial z$.

Similarly, the out-flow rate through the right x-face =

$$\rho' u + \frac{\partial}{\partial x}(\rho' u)\frac{\partial x}{2} \, \partial y \, \partial z.$$

Therefore , the net mass inflow rate to the paralellopiped through the two x-faces= $\rho' u - \frac{\partial}{\partial x}(\rho' u)\frac{\partial x}{2}\partial y \partial z - \rho' u + \frac{\partial}{\partial x}(\rho' u)\frac{\partial x}{2}\partial y\, \partial z.$

$$= -\frac{\partial}{\partial x}(\rho' u)\, \partial x\, \partial y\, \partial z.$$

Similarly, the net mass flow rate through the two y-face

$$= -\frac{\partial}{\partial y}(\rho' v)\, \partial x\, \partial y\, \partial z \quad \text{and}$$

through z-faces $= -\frac{\partial}{\partial z}(\rho' w)\partial x\, \partial y\, \partial z.$

Therefore, total mass inflow through x,y,z directions $= -\left[\frac{\partial}{\partial x}(\rho' u)+\frac{\partial}{\partial y}(\rho' v)+\frac{\partial}{\partial z}(\rho' w)\right]\partial x\, \partial y\, \partial z.$

Net increase in mass in the paralellopiped per unit of time =

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t}(\rho' \partial x\, \partial y\, \partial z)$$

But $\partial x, \partial y, \partial z$ are independent of time.

So we can write,

$$\frac{\partial m}{\partial t} = \frac{\partial \rho'}{\partial t}\partial x\, \partial y\, \partial z\ .$$

But the net increase in mass =mass inflow -mass outflow.

Therefore, $-\left[\frac{\partial}{\partial x}(\rho' u)+\frac{\partial}{\partial y}(\rho' v)+\frac{\partial}{\partial z}(\rho' w)\right]\partial x\, \partial y\, \partial z$

$$= \frac{\partial \rho'}{\partial t}\partial x\, \partial y\, \partial z.$$

or, $-\left[+\frac{\partial}{\partial x}(\rho' u)+\frac{\partial}{\partial y}(\rho' v)+\frac{\partial}{\partial z}(\rho' w)\right] = \frac{\partial \rho'}{\partial t}$

or, $\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x}(\rho' u)+\frac{\partial}{\partial y}(\rho' v)+\frac{\partial}{\partial z}(\rho' w) = 0$ .... (3.3)

Substituting $\rho' = \rho_0(1+s')$ in the above equation ,

$\rho_0\frac{\partial s'}{\partial t} + \rho_0(1+s')\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + \rho_0\left(u\frac{\partial s'}{\partial x}+v\frac{\partial s'}{\partial y}+w\frac{\partial s'}{\partial z}\right) = 0$ ....(3.4)

Compared with unity the condensation is a very small quantity in acoustics. Moreover, acoustical wave lengths are so long that u,v,w,and s change very little with x,y,z. Hence $\frac{\partial u}{\partial x}$, $\frac{\partial s'}{\partial x}$,etc. are very small quantities. Therefore, we can neglect terms like $s'\frac{\partial u}{\partial x}$ and $u\frac{\partial s'}{\partial x}$ in comparison to $\frac{\partial u}{\partial x}$ and to this approximation the

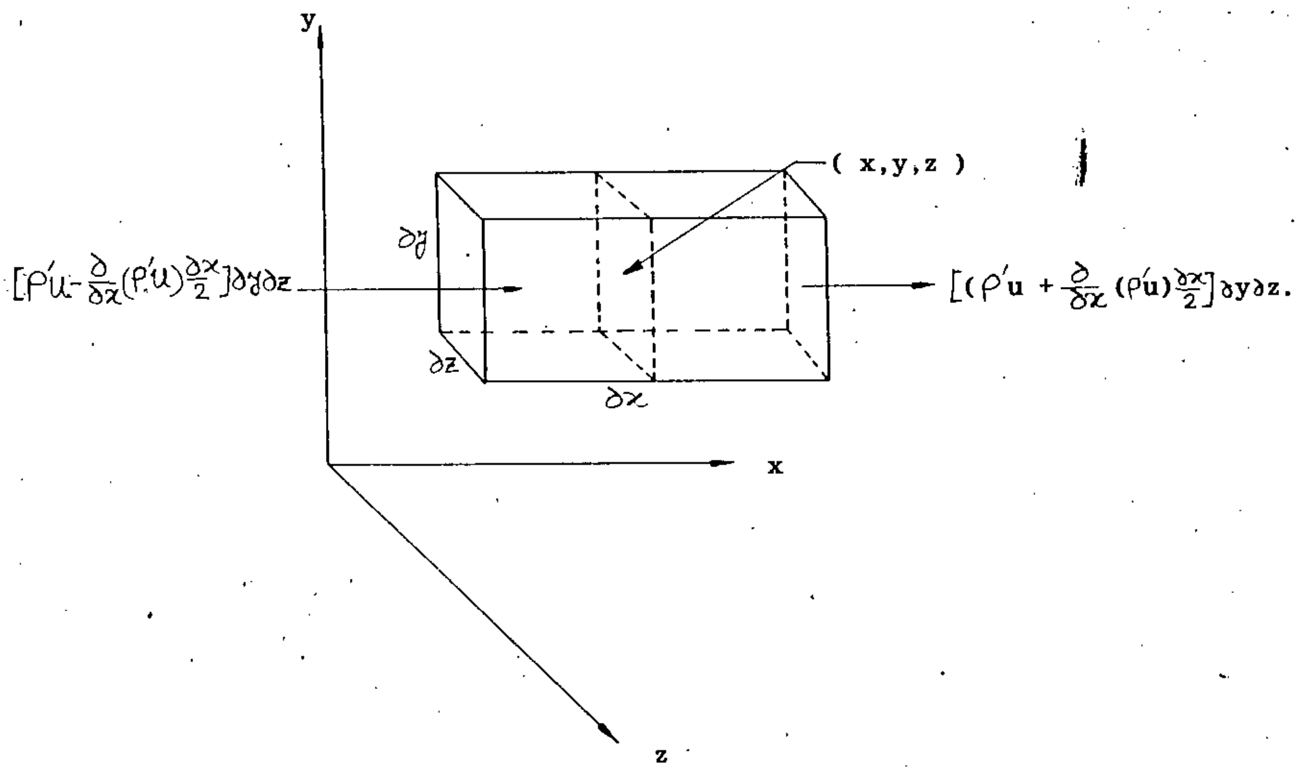

Fig.3.1.

continuity equation becomes

$$\frac{\partial s'}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

Substituting the values of u,v,w in terms of the potential function,

$$\frac{\partial s'}{\partial t} = \nabla^2 \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.6)$$

To replace (3.3) by an equation in which ϕ is the only independent variable,the hydrodynamical equations of motion(65) have to be considered.

these are,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho'}\frac{\partial p'}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

The left-hand sides of the above equations represent the total acceleration (summation of convective and local acceleration) and the right- hand sides, the force per unit mass. There are no body forces acting here.

Reasoning that the change of u, v, and w with (x, y,z) is very small, equations of motion take the simple form :

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho'}\frac{\partial p'}{\partial x}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y}$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

Substitution of $u=-\frac{\partial \phi}{\partial x}$, $v=-\frac{\partial \phi}{\partial y}$ $w=-\frac{\partial \phi}{\partial z}$ into these equations gives

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial x}\right) = -\frac{1}{\rho'}\frac{\partial p'}{\partial x}$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial y}\right) = -\frac{1}{\rho'}\frac{\partial p'}{\partial y}$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial z}\right) = -\frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

Multiplying these three equations by dx, dy ,dz, respectively and adding ,

$$\frac{\partial}{\partial t} d\phi = \frac{1}{\rho'} dp'$$

or integrating :

$$-\frac{\partial \phi}{\partial t} = \int \frac{dp'}{\rho'} \qquad \text{.....(3.7)}$$

Since the density changes very little , the mean density , ρ , may be used. Then $\int dp'$ reduces simply to the excess pressure, so,

$$-\frac{\partial \phi}{\partial t} = P/\rho \qquad \text{.....(3.8)}$$

Where P is the excess pressure.

The next property of a gas which is used to derive the wave equation depends upon the thermodynamic properties of gases. The compressions and rarefactions in a sound wave are too rapid for the temperature of the gas to remain constant . The changes in pressure and density are so rapid that practically no heat energy has time to flow away from the compressed part of the gas before this part is no longer compressed. when the gas temperature changes, but its heat energy does not, the compression is termed adiabatic.

In case of an adiabatic process,

$P'V'^{\gamma} = P_o V_o^{\gamma}$ =constant,

Where V' = volume at pressure P',

V_o = volume at pressure P_o,

P_o = static pressure . The static pressure is the pressure that would exist in the medium with no sound waves present,

γ = ratio of specific heat at constant pressure to that at constant volume.

Now , $V' = \dfrac{m}{\rho'}$ and $V_o = \dfrac{m}{\rho}$

Therefore,

$$\frac{P'}{P_o} = \left(\frac{\rho'}{\rho}\right)^{\gamma} \qquad \text{.........................(3.9)}$$

Again , $P_o V_o^{\gamma}$=constant,

or , $P_o \gamma V_o^{\gamma-1} dV_o + V_o^{\gamma} dP_o = 0$ ....(3.10)

or , $\gamma P_o = -\dfrac{dP_o}{\dfrac{dV_o}{V_o}} = E$, bulk modulus .

Therefore, velocity of propagation ,

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P_o}{\rho}}$$

or $$c^2 = \frac{\gamma P_o}{\rho} \qquad \text{.......................................(3.11)}$$

Now, $\frac{P'}{P_o} = \left(\frac{\rho'}{\rho}\right)^{\gamma} = (1+s')^{\gamma}$, since $s' = \frac{\rho' - \rho}{\rho}$

or, $$\frac{P'}{P_o} = 1 + \gamma s' + \frac{\gamma(\gamma-1)}{\underline{|2}} s'^2 + \text{-----------}$$

Neglecting the higher- order terms since s' is very small,

$$\frac{P'}{P_o} = 1 + \gamma s' \qquad \text{or, } P' = P_o + \gamma P_o s'$$

or, $P' - P_o = \gamma P_o s'$

or, $P = \gamma P_o s'$

or, $$s' = \frac{P}{\gamma P_o} \qquad \text{.......................................(3.12)}$$

Substitution of equation (3.12) and $u = -\frac{\partial \varphi}{\partial x}$, $v = -\frac{\partial \varphi}{\partial y}$, $w = -\frac{\partial \varphi}{\partial z}$ into equation (3.5) gives

$$\frac{\partial}{\partial t}\left(\frac{P}{\gamma P_o}\right) - \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = 0$$

or, $$\frac{\partial P}{\partial t} - \gamma P_o \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = 0 \qquad \text{.......................(3.13)}$$

Again , substituting equation (3.8) into equation (3.13.),

$$\frac{\partial}{\partial t}\left(\rho \frac{\partial \varphi}{\partial t}\right) - \gamma P_o \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = 0$$

or, $$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\gamma P_o}{\rho}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = 0$$

Using equation (3.11) the above expression can be written as,

$$\frac{\partial^2 \varphi}{\partial t^2} - c^2 \nabla^2 \varphi = 0$$

or, $\frac{\partial^2 \varphi}{\partial t^2} = c^2 \nabla^2 \varphi$ ....(3.14)

Which is the familiar equation of wave motion . The general solution of (3.14) shows that the influence of any value of is propagated with velocity c .

The spherical wave which originates at a point and spread out in all directions from a source , is considered to be centrally symmetric. Thus , all the properties of the wave at a point in space depend only on time and on the distance to the origin of a system of coordinates. in spherical polar coordinates, the wave equation can be written as (60),

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \left[\frac{1}{R^2} \frac{\partial}{\partial R}\left(R^2 \frac{\partial \varphi}{\partial R}\right) + \frac{1}{R^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \varphi}{\partial \theta}\right) + \frac{1}{R^2 \sin\theta} \frac{\partial^2 \varphi}{\partial \alpha^2}\right]$$

......................(3.15)

Where, $x = R \sin\theta \cos\alpha$

$y = R \sin\theta \sin\alpha$

$z = R \cos\theta$

The angles θ and α are the polar and the azimuthal angles respectively . For the centrally symmetric case , the velocity potential is only a function of R and t , so that equation(3.15) reduces to

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \left(\frac{\partial^2 \varphi}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi}{\partial R}\right)$$ ....(3.16)

However , since

$$\frac{\partial^2 (R\varphi)}{\partial R^2} = R\left(\frac{\partial^2 \varphi}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi}{\partial R}\right)$$

equation (3.16) may be written as

$$\frac{\partial^2(\varphi R)}{\partial t^2} = c^2 \frac{\partial^2(\varphi R)}{\partial R^2} \qquad \text{.......(3.17)}$$

This is the general wave equation for spherical wave propagation .

Let $\varphi = \left(\frac{1}{R}\right) F(ct+R)$, where F is any arbitrary function .

Clearly , satisfies the general wave equation (3.17) , so it is the solution of the above equation .

Now, postulating a harmonic wave and, for convenience , employing the complex notation for of the plate problem ,

$$\varphi = \frac{\alpha' A}{r} e^{ik(ct+r)} \qquad \text{.......(3.18)}$$

where r is the distance from the source dA to the receiving point ,

α' is the amplitude of the velocity potential produced per unit area of the vibrating surface,

A is the area of the rectangular plate on which disturbances are created and propagated into the surrounding medium ,

$k = 2\pi/\lambda = \omega/\lambda f = \omega/c$, acoustic wave number.

ω is the angular frequency of vibration,

c is the velocity of sound in air,

f is the frequency of harmonic vibration , and

λ is the wave length of sound wave.

Equation (3.18) can be written in differential form as

$$d\varphi = \frac{\alpha' dA}{r} e^{ik(ct+r)}, \text{ where } dA = dxdy.$$

The total velocity potential at the receiving point (R, Θ, α) will be obtained by integration of the contributions from over the whole plane surface of the rectangular flat plate and so

$$\varphi_\gamma = \int \frac{\alpha'}{r} e^{ik(ct+r)} \, dA \quad \text{..........................(3.18a)}$$

$$= \int \frac{\alpha'}{r} e^{i(\omega t+kr)} \, dA$$

In order to evaluate α', the source of hemispherical waves at dA is considered to be a pulsating hemisphere of radius r_o , which is finally to be made vanishingly small. Hence area = dxdy, and the velocity potential at the receiving point is

$$\varphi_\gamma = \frac{2\pi r_o^2 \alpha'}{r} e^{i(\omega t+kr)} \quad \text{..........................(3.19)}$$

The particle velocity at the surface of the hemisphere will be given by

$$U_{r=r_o} = -\left(\frac{\partial\varphi}{\partial r}\right)_{r=r_o}$$

$$= -2\pi r_o^2 \alpha' e^{i(\omega t+kr_o)} \left\{\frac{ik}{r_o} - \frac{1}{r_o^2}\right\} \text{....................(3.20)}$$

where U is the linear surface velocity which follows the harmonic variation . According to equation(3.2)

$$U = U_o e^{i\omega t} \quad \text{..................................(3.21)}$$

where U_o is the amplitude of velocity distribution.

From equations (3.20) and (3.21) it is evident that if $r_o \to 0$,

$$U_o e^{i\omega t} = -\left(\frac{\partial\varphi}{\partial r}\right)_{r_o\to 0} = 2\pi\alpha' e^{i\omega t}$$

$$\text{or, } \alpha' = \frac{U_o}{2\pi} \quad \text{....................................(3.18b)}$$

Therefore, $d\phi = \frac{U_0}{2\pi r} e^{i(\omega t+kr)} dA$

$$= \frac{U}{2\pi r} e^{ikr} dA \quad \text{.............................(3.22)}$$

From the expression (3.8) we have the acoustic excess pressure $P = \rho \frac{\partial \phi}{\partial t}$ for area dA =dxdy,

Then, from equation (3.18a) & (3.18b)

$$dP = \frac{i\omega \rho U_0}{2\pi r} e^{i(\omega t+kr)} dA$$

$$\text{or, } dP = ik\rho c \, d\phi \quad \text{.......................................(3.23)}$$

Substituting $d\phi$ from equation (3.22) into equation (3.23) we have,

$$dP = \frac{ik\rho c U}{2\pi r} e^{ikr} dA \quad \text{.......................................(3.24)}$$

If the receiving point (R, θ, α) is located in the farfield, then the distance r of the receiving point from the elemental area can be expressed with the first order approximation as (53),

$$r \simeq R - (x\cos\alpha + y\sin\alpha)\sin\theta \quad \text{...........................(3.25)}$$

The second term $(x\cos\alpha+y\sin\alpha)$ $\sin\theta$ in equation (3.25) is very small compared to the first and its effect can be neglected in the amplitude factor of equation (3.24). However, both terms are equally significant as far as the phase factor of equation (3.5) is concerned.

Substituting equation (3.2) and (3.25) in equation (3.24) and neglecting the second term in the expression of r in the amplitude factor of equation (3.24), it can be shown that,

$$dP = -\frac{k\rho c \omega_{mn} W_{mn}}{2\pi R}\theta(x)\theta(y) \; e^{i(\omega t+kr)} dA$$

$$= -\frac{k\rho c \omega_{mn} W_{mn}}{2\pi R}\theta(x)\theta(y) \; e^{i\{\omega t+kR-k(x\cos\alpha+y\sin\alpha)\sin\theta\}} dA$$

Thus the net acoustic pressure at the point (R,θ, α) due to (m,n)th mode of vibration of the plate is given by

$$P = -\frac{\rho c k \omega W}{2R\pi} e^{i(\omega t+kR)} \int_0^a \int_0^b \theta(x)\theta(y)$$

$$e^{-ik(x\cos\alpha + y\sin\alpha)\sin\theta}\, dxdy.$$

$$= -\frac{\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \int_0^a \int_0^b \theta(x)\theta(y)\, e^{-i(lx/2a+sy/2b)}\, dxdy \qquad \text{.......(3.26)}$$

where $l = 2ak\cos\alpha\sin\theta$

and $s = 2bk\sin\alpha\sin\theta$.

Acoustic intensity of a sound wave is defined as the average rate of flow of energy through a unit area normal to the direction of wave propagation.

Instantaneous power per unit area is equal to the product of instantaneous pressure (P) and instantaneous particle velocity (V). Average power per unit area measures the acoustic intensity. Therefore, acoustic intensity,

$$I = \frac{1}{T}\int_0^T P\, v\, dt.$$

Now , let us consider a longitudinal compression wave traveling in the x-direction through an infinitesimal element having dimensions $\partial x \partial y \partial z$. If the center of gravity is displaced in the +ve x-direction by a distance w, then the boundaries will be displaced by $(w - \frac{\partial w}{\partial x}\cdot\frac{\partial x}{2})$ and $(w + \frac{\partial w}{\partial x}\cdot\frac{\partial x}{2})$ respectively. The volume therefore decreases by

$$\left[(w - \frac{\partial w}{\partial x}\frac{\partial x}{2}) - (w + \frac{\partial w}{\partial x}\frac{\partial x}{2})\right]\partial y\, \partial z$$

$$= -\frac{\partial w}{\partial x}\partial y\, \partial z\, \partial x.$$

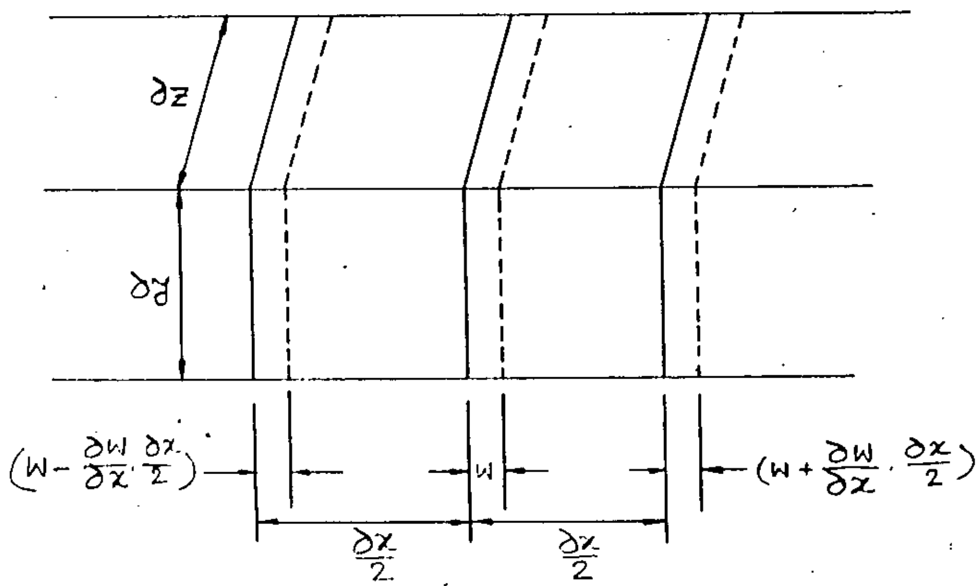
∂z
∂y
$(w - \frac{\partial w}{\partial x} \cdot \frac{\partial x}{2})$
w
$(w + \frac{\partial w}{\partial x} \cdot \frac{\partial x}{2})$
$\frac{\partial x}{2}$
$\frac{\partial x}{2}$

Fig.3.2

The volumetric strain is defined as the ratio of the decrease in volume to the original volume, so we have

$$\text{strain} = -\frac{\partial w}{\partial x}\,\frac{\partial x\,\partial y\,\partial z}{\partial x\,\partial y\,\partial z} = -\frac{\partial w}{\partial x}$$

If the increase in pressure which brings about this strain, or which is associated with it, is P, then by definition the bulk modulus of elasticity is $E = \frac{\text{stress}}{\text{strain}} = \frac{P}{\partial w/\partial x}$

$$\text{or}\ ,\ P = -E\,\frac{\partial w}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(C)}$$

We know that $c=\sqrt{\frac{E}{\rho}}$

Therefore, $E=\rho c^2$. Substituting this value into equation (C), we have,

$$P = -\rho c^2 \left(\frac{dw}{dx}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(D)}$$

If $w = W\cos(\omega t-kx)$, where W is the amplitude of displacement w,

$$V = \frac{dw}{dt} = -\omega W\sin(\omega t-kx)$$

$$\frac{dw}{dx} = -\ Wk\sin(\omega t-kx)$$

If T is the time period,

and $kc = \omega$ then,

$P= -\rho cW\omega\sin(\omega t-kx)$

$$\text{Therefore, } I= (1/T)\int^{T}\left[-\rho cW\sin(\omega t-kx)\right].\left[-\omega W\sin(\omega t-kx)\right]\ dt$$

$$=(1/2)\ \rho c\ \omega^2 W^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(3.27)}$$

Since $P = -\rho cW\omega\sin(\omega t-kx)$,

$P_{max} = -\rho cW\omega$. Therefore, $P = P_{max}\sin(\omega t-kx)$.

The root mean square value of sound pressure is defined as the square root of the average of the square of the instantaneous pressure. In mathematical notation,

$$P^2_{r.m.s.} = (1/T)\int_0^T P^2 dt.$$

$$= (1/T)\int_0^T P^2_{max} \sin^2(\omega t-kx)\, dt = P^2_{max}/2.$$

Therefore, $P_{rms} = P_{max}/\sqrt{2}$

From equation (3.27),

$$I = \frac{\rho^2 c^2 \; W^2\omega^2}{2\rho c} = (P^2_{max})/2\rho c$$

$$= \frac{|P|^2}{\rho c}$$

Where $|P|^2$ is the square of the root mean square value of the net acoustic pressure.

The total average power radiated from one side of the baffled plate, found by integrating the farfield acoustic intensity over the hemispherical surface is ,

$$P_W = \int_0^{2\pi}\int_0^{\pi/2} \frac{|P|^2}{\rho c} R^2 \sin\theta \, d\theta \, d\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.28a)$$

(B) Determination of natural frequency :

Natural frequency is that frequency at which the plate will continue to vibrate if left undisturbed after being given an excitement. A number of authors have determined the natural frequency of the plate w_{mn} corresponding to the transverse mode of vibration (m,n) in terms of boundary conditions , the nodal patterns, the dimensions of the plate and the constant of the plate material. Warburton (1) derived the expression for the natural frequency from energy equation considering a rectangular plate of sides of length a and b . The vibration form must satisfy the boundary conditions at the edges as well as the governing plate equation(54),

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{12\rho(1-\sigma^2)}{Egh^2}\frac{\partial^2 w}{\partial t^2} = 0 \quad \text{......(3.29)}$$

Here, w, the displacement at a point (x,y) at time t , is given by

$$w = W_{xy}\sin wt = A\,\theta(x)\,\theta(y)\sin wt \quad \text{......................(3.30)}$$

Where W_{xy} is the amplitude of the wave form.

$\theta(x)$ and $\theta(y)$ are space functions and w is the natural frequency.

In general , it is not possible to find a form for w to satisfy equation (3.29) together with the boundary conditions. For these cases, an infinite series can be assumed for W_{xy} , each term of which satisfies equation (3.29) and some of the boundary conditions. By taking suitable values of the coefficient A,the remaining conditions are satisfied.

Frequency can also be obtained accurately by considering energy. For a rectangular plate, the potential energy of bending U is given (54) by

$$U = \int_0^a\int_0^b (1/2)\frac{Eh^3}{12)\,1-\sigma^2)}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\sigma\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\sigma)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]dxdy \quad \text{..............................(3.31)}$$

and the kinetic energy T by

$$T = \int_0^a\int_0^b (1/2) \frac{\rho h}{g} \left(\frac{\partial w}{\partial t}\right)^2 dxdy \quad \ldots\ldots(3.32)$$

Natural frequency may be obtained from maximum values of potential and kinetic energy as

$$\omega_{mn}^2 = \frac{U_{max}}{(\rho_m h/2g)\int_0^a\int_0^b W_{xy}^2 dxdy} \quad \ldots\ldots(3.33)$$

Substituting the expressions for space functions $\theta(x)$, $\theta(y)$ into equation (3.30) and (3.33) for the required boundary condition, the expression for natural frequency can be obtained. Warburton(1) expressed the natural frequency in terms of a dimensionless frequency factor , given by

$$\lambda_f^2 = G_x^4 + G_y^4 (a/b)^4 + 2(a/b)^2 \left[6 H_x\ H_y + (\ -6\ ) J_x\ J_x\right] \ldots\ldots(3.34)$$

He obtained the relationship between w_{mn} and λ_f^2 as

$$\lambda_f^2 = \frac{\rho_m a^4 \omega_{mn} 12(1\ -6^2)}{\pi^4 E h^2 g} \quad \ldots\ldots(3.35)$$

The frequency factor λ_f can be obtained for any ratio a/b. from equation (3.35) the frequency is given by ,

$$\omega_{mn}^2 = \frac{\lambda_f^2\ \pi^4\ E\ h^2 g}{\rho_m a^4\ 12(1\ -6^2)}$$

$$\text{or, } f = \frac{\lambda_f h \pi}{a^2}\left[\frac{Eg}{48\,\rho_m(1-6^2)}\right]^{1/2} \ldots\ldots(3.36)$$

Where, ρ_m=density of the plate màterial ,

6 = Poisson's ratio,

E= modulus of elasticity of the plate material,

h = thickness of the plate , and

g= acceleration due to gravity.

The coefficients G_x, G_y, H_x,H_y,J_x and J_y depend on the boundary conditions and mode shapes. The values of this quantities were given by Warburton(1) in the form of a table. For ready reference, those values are presented here for the boundary conditions under consideration.

Table giving values of the parameters H_x, H_y,G_x,G_y,J_x and J_y of dimensionless frequency factor.

Boundary condition	m	G_x	H_x	J_x
Three ends clamped and one end simply-supported	2,3,4,-	m-0.75	$(m-0.75)^2 \times \left[1-\frac{1}{\pi(m-0.75)}\right]$	$(m-0.75)^2 \times \left[1-\frac{1}{\pi(m-0.75)}\right]$
Two ends free and two ends simply supported	2	1.506	1.248	5.017
	3,4,5,---	m-0.5	$(m-0.5)^2 \times \left[1-\frac{2}{(m-0.5)\pi}\right]$	$(m-0.5)^2 \times \left[1+\frac{6}{(m-0.5)\pi}\right]$
	n	**G_y**	**H_y**	**J_y**
Three ends clamped and one end simply supported	2	1.506	1.248	1.248
	3,4,5, ---	n-0.5	$(n-0.5)^2 \times \left[1-\frac{2}{(n-0.5)\pi}\right]$	$(n-0.5)^2 \times \left[1-\frac{2}{(n-0.5)\pi}\right]$
Two ends free & two ends simply-supported.	2,3,4, --	(n-1)	$(n-1)^2$	$(n-1)^2$

(C) Determination of radiation efficiency :

The concept of radiation resistance is useful in calculating the radiation (acoustic) efficiency of a source . The efficiency of a source of sound is defined as the ratio of the energy radiated as sound to the whole of the vibrational energy applied to the source. Wallace(51) studied theoretically the radiation resistance corresponding to the natural modes of a finite rectangular panel by considering the total energy radiated to the farfield. He , in his papers (50,51) introduced a new term radiation efficiency for simplifying the matters and to eliminate the dependence on the impedence of the acoustic medium and plate size during calculation of acoustic radiation from vibrating plates. The radiation efficiency is defined by Wallace (50,51) as

$$S_{mn} = \frac{P_w}{4 \rho c a b \langle |u_w|^2 \rangle} \quad \ldots\ldots (3.37)$$

Here P_w is the total average power radiation from one side of the baffled plate and $\langle |u_w|^2 \rangle$ is the average of the temporal and spatial factor of the square of the surface velocity, given by

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a} \int_{-b}^{b} (1/2)\, u_w^2 \, dx dy \quad \ldots\ldots (3.38)$$

The expression for u_w can be obtained from equation (3.2) as

$u_w = i w_{mn} W_{mn} \Theta(x) \Theta(y)$

3.4. Displacement functions :

In transverse vibration, points in the plate undergo small displacements in the direction perpendicular to the plane of the plate. Warburton(1) found a number of beam functions for describing the displacement of a vibrating beam under different combinations of end conditions. He also used these functions to represent the vibration of rectangular flat plate by assuming that the wave forms of vibrating plates and beams are similar. The slightly modified beam functions, which are hereby referred to as displacement functions, for different boundary conditions are presented here.

(1) Displacement function for the three ends fixed and one end simply supported plate :

If one of the edges perpendicular to x-direction is fixed and the others are simply supported,then the corresponding beam function is

$$\Theta(x) = \sin \gamma_m\left(\frac{x-a}{4a}\right) + R_m \sinh \gamma_m\left(\frac{x-a}{4a}\right) \text{ for } m = 2,3,4, \ldots \text{ and}$$

$$-a \leq x \leq a.$$

Here, $R_m = \dfrac{\sin \gamma_m/2}{\sinh \gamma_m/2}$ and γ_m are the roots of the equation ,

$$\tan(\gamma/2) - \tanh(\gamma/2) = 0$$

If both the edges of the plate perpendicular to y-axis are fixed , the corresponding beams functions are

$$\Theta(y) = \cos \gamma_n(y/2b) + R_n \cosh \gamma_n(y/2b), \text{for } n = 2,4,6, \ldots \text{ and}$$

$$-b \leq y \leq b$$

$$\Theta(y) = \sin \gamma_n'(y/2b) + R_n' \sinh \gamma_n'(y/2b), \text{for } n = 3,5,7, \ldots \text{ and}$$

$$-b \leq y \leq b$$

where, $R_n = \dfrac{\sin \gamma_n/2}{\sinh \gamma_n/2}$ and are the roots of the equation

$$\tan(\gamma/2) + \tanh(\gamma/2) = 0,$$

and $R_n' = -\dfrac{\sin \gamma_n'/2}{\sinh \gamma_n'/2}$ and are the roots of the equation

$$\tan(\gamma/2) - \tanh(\gamma/2) = 0.$$

(2) Displacement function W_{xy} for the two opposite ends free and other two ends simply supported plate :

If the two opposing edges perpendicular to x-direction are free, the corresponding beam functions are given by Warburton as

$\Theta(x) = \cos(\gamma_m x/2a) + R_m \cosh(\gamma_m x/2a)$, m=2,4,6, ... and $-a \leq x \leq a$.

$\Theta(x) = \sin(\gamma'_m x/2a) + R'_m \sinh(\gamma'_m x/2a)$, m=3,5,7, ... and $-a \leq x \leq a$

where, $R_m = -\dfrac{\sin(\gamma_m/2)}{\sinh(\gamma_m/2)}$ and are the roots of the equation,

$$\tan(\gamma/2) + \tanh(\gamma/2) = 0,$$

$R'_m = \sin(\gamma'_m/2)/\sinh(\gamma'_m/2)$ and are the roots of the equation

$$\tan(\gamma/2) - \tanh(\gamma/2) = 0.$$

If both the edges of the plate perpendicular to y-axis are simply supported , the corresponding beam function is

$$\Theta(y) = \sin\frac{(n-1)\pi(y+b)}{2b}, \quad n=2,3,4,.....\quad -b \leq y \leq b.$$

CHAPTER 4

SOLUTION OF THE PROBLEM

4.1. INTRODUCTION :

Analytical solutions for the total average power radiation from one side of the baffled plate are obtained here for various boundary conditions. The expressions for radiation efficiencies for both the free-simply supported and clamped simply supported plates are also derived and presented in this chapter . In the subsequent sections, the method of integration of the expression for acoustic power and radiation efficiency for various boundary conditions are treated.

4.2. Plate with two opposing ends free and the other two ends simply supported .

(a) Power radiation :

From equation (3.26) the acoustic pressure distribution at a point in the farfield is given by

$$P = \frac{-k\rho c\omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} \Theta(x)\Theta(y)\, e^{-i(lx/2a+sy/2b)}\, dxdy$$

The displacement functions $\Theta(x)$ and $\Theta(y)$ for this plate with free edges perpendicular to x-axis and the simply supported edges perpendicular to y-axis are :

$\Theta(x) = \cos(\gamma_m x/2a) + R_m \cosh(\gamma_m x/2a)$ for m=2,4,6, ... and $-a \le x \le a$,

$\Theta(x) = \sin(\gamma_m' x/2a) + R_m' \sinh(\gamma_m' x/2a)$ for m=3,5,7, ... and $-a \le x \le a$.

Where, $R_m = -\sin(\gamma_m/2)/\sinh(\gamma_m/2)$ and are the roots of the equation, $\tan(\gamma/2) + \tanh(\gamma/2) = 0$,

$R'_m = \sin(\gamma'_m/2)/\sinh(\gamma'_m/2)$ and are the roots of the equation, $\tan(\gamma/2) - \tanh(\gamma/2) = 0$.

$$\Theta(y) = \sin \frac{(n-1)\pi(y+b)}{2b}, \quad n=2,3,4,\ \ldots\ \text{and } -b \le y \le b.$$

As the displacement functions are dependent on the odd-even character of m ,it is now required to consider the even and odd values of m separately.

Case - 1 Even values of m .

Substituting the appropriate displacement functions $\Theta(x)$ and $\Theta(y)$, the acoustic pressure distribution P becomes,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} \left[\left\{ \cos(\gamma_m x/2a) + R_m \cosh(\gamma_m x/2a) \right\} \times e^{-ilx/2a} dx \right] \times \left[\sin \frac{(n-1)\pi(y+b)}{2b} e^{-isy/2b} dy \right] \quad \ldots\ldots\ldots\ldots\ldots(4.1)$$

Integration of equation (4.1) gives,

$$P = \frac{8\rho c k \omega W a b}{\pi^2 (n-1)} e^{i(\omega t + kR)} \left[\left\{ \frac{\gamma_m \sin(\gamma_m/2)\cos(l/2) - l\cos(\gamma_m/2)\sin(l/2)}{(\gamma_m^2 - l^2)} \right\} + R_m \left\{ \frac{\gamma_m \sinh(\gamma_m/2)\cos(l/2) + l\cosh(\gamma_m/2)\sin(l/2)}{(\gamma_m^2 + l^2)} \right\} \right]$$

$$\times \left[\frac{\cos(s/2)/i\sin(s/2)}{\left\{ s^2/\pi^2(n-1)^2 \right\} - 1} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

Where $\cos(s/2)$ is used when n is even integer and $i\sin(s/2)$ is used when n is an odd integer.

The farfield acoustic intensity is given by

$$I = |P|^2/\rho c$$

Substituting P from equation (4.2) , it is found that,

$$I = \frac{32\rho c k\omega^2 W^2 a^2 b^2}{\pi^4 R^2 (n-1)^2}\left[\frac{\gamma_m \sin(\gamma_m/2)\cos(l/2) - l\cos(\gamma_m/2)\sin(l/2)}{(\gamma_m^2 - l^2)} + \frac{R_m}{(\gamma_m^2 + l^2)}\left\{\gamma_m \sinh(\gamma_m/2)\cos(l/2) + l\cosh(\gamma_m/2)\sin(l/2)\right\}\right]^2 \times \left[\frac{\cos\left(\frac{s}{2}\right) \text{ or } \sin\left(\frac{s}{2}\right)}{\{s^2/\pi^2(n-1)^2\} - 1}\right]^2 \quad \ldots\ldots(4.3)$$

where, $l = 2ak\cos\alpha\,\sin\theta = (2a\omega)\cos\alpha\,\sin\theta)/c$

$$= \frac{R_t\,\lambda_f\,\pi^2\sqrt{Eg}}{c\sqrt{3\,\rho_m\,(1-\sigma^2)}}\cos\alpha\,\sin\theta \text{ and}$$

$s = 2bk\sin\alpha\,\sin\theta = (2b\omega)\sin\alpha\,\sin\theta)/c$

$$= \frac{R_a\,R_t\,\lambda_f\,\pi^2\sqrt{Eg}}{c\sqrt{3\,\rho_m\,(1-\sigma^2)}}\sin\alpha\,\sin\theta\,.$$

From equation (3.35) the dimensionless frequency factor λ_f is given by

$$= \frac{\rho_m a^4\omega^2 12(1-\sigma^2)}{\pi^4 E\,h^2 g}. \text{ So, } \omega^2 = \frac{Eh^2 g\,\lambda_f^2\pi^4}{a^4\,12(1-\sigma^2)\,\rho_m}$$

where, $\lambda_f^2 = G_x^4 + (a/b)^4 G_y^4 + 2(a/b)^2\left[\sigma H_x H_y + (1-\sigma)J_x J_y\right]$.

The total average power radiated from one side of the baffled plate is then given by

$$P_w = \int_0^{2\pi}\int_0^{\pi/2}\frac{|P|^2}{\rho c}R^2\sin\theta\,d\theta\,d\alpha = \int_0^{2\pi}\int_0^{\pi/2} IR^2\sin\theta\,d\theta\,d\alpha\,.$$

$$\text{or, } P_w = \frac{128\rho c k^2\omega^2 W^2 a^2 b^2}{\pi^4(n-1)^2}\int_0^{2\pi}\int_0^{\pi/2}\left[\frac{\gamma_m\sin(\gamma_m/2)\cos(l/2) - l\cos(\gamma_m/2)\sin(l/2)}{(\gamma_m^2 - l^2)} + \frac{R_m}{(\gamma_m^2+l^2)}\left\{\gamma_m\sinh(\gamma_m/2)\cos(l/2) + l\cosh(\gamma_m/2)\sin(l/2)\right\}\right]^2 \times \left[\frac{\cos\left(\frac{s}{2}\right) \text{ or } \sin\left(\frac{s}{2}\right)}{\{s^2/\pi^2(n-1)^2\} - 1}\right]^2 \sin\theta\,d\theta\,d\alpha\,. \quad \ldots\ldots(Z)$$

$$= \frac{8\rho W^2 \gamma_f^4 Eg^2 R_a^2 R_l^4 \pi^4}{9 c \rho_m^2 (n-1)^2 (1-\sigma^2)^2} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{\gamma_m \sin(\gamma_m/2)\cos(1/2) - 1\cos(\gamma_m/2)\sin(1/2)}{(\gamma_m^2 - 1^2)}$$

$$+ \frac{R_{\gamma m}}{(\eta_m^2+1^2)} \{\eta_m \sinh(\eta_m/2)\cos(1/2) + 1\cos(\eta_m/2)\sin(1/2)\}\Big]^2 \left[\frac{\begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{s}{2}\right)}{\{s^2/\pi^2(n-1)^2\}-1}\right]^2 \times$$

$$\sin\theta d\theta d\alpha \; . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

It is clear from equation (4.4) that the total average power radiated from one side of the plate does not depend on the distance , R' of the receiving point from the center of the plate. Only the acoustic intensity varies inversely as the square of R as seen in equation (4.3).

(b) Radiation efficiency :

To compare the result of this study with those by the previous workers(51,53,54) , the expression for the radiation efficiency is derived here. The radiation efficiency of a vibrating plate is defined by the equation (3.37) as

$$S = \frac{P_W}{4\rho c a b \langle |u_W|^2 \rangle}$$

From equation (Z),

$$P_W = \frac{128 \rho c k^2 \omega^2 W^2 a^2 b^2}{\pi^4 (n-1)^2} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{\gamma_m \sin(\gamma_m/2)\cos(1/2) - 1\cos(\gamma_m/2)\sin(1/2)}{(\gamma_m^2 - 1^2)} +$$

$$\frac{R_{\gamma m}}{(\eta_m^2+1^2)} \{\eta_m \sinh(\eta_m/2)\cos(1/2) + 1\cosh(\eta_m/2)\sin(1/2)\}\Big]^2 \left[\frac{\begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{s}{2}\right)}{\{s^2/\pi^2(n-1)^2\}-1}\right]^2 \times$$

$$\sin\theta d\theta d\alpha \; .$$

Letting $\left[\frac{\eta_m \sin(\gamma_m/2)\cos(1/2) - 1\cos(\gamma_m/2)\sin(1/2)}{(\gamma_m^2 - 1^2)} + \frac{R_m}{(\eta_m^2+1^2)}\right.$

$$\left\{ \gamma_m \sinh(\gamma_m/2)\cos(1/2)+1\cosh(\gamma_m/2)\sin(1/2)\right\}^2 \left[\frac{\cos\atop\sin \left(\frac{\varsigma}{2}\right)}{\left\{s^2/\pi^2(n-1)^2\right\}-1} \right]^2 = A_1,$$

the expression of total average acoustic power becomes

$$P_W = \frac{128\,\rho c k^2 \omega^2 W^2 a^2 b^2}{\pi^4 (n-1)^2} \int_0^{\pi/2}\int_0^{\pi/2} A_1 \sin\theta \, d\theta \, d\alpha .$$

The average of the temporal and spatial factor of the square of the surface velocity, , is given by

$$\langle |u_W|^2 \rangle = (1/4ab) \int_{-a}^{a}\int_{-b}^{b} (u_W^2/2)\, dxdy \quad \ldots\ldots(4.5)$$

But $u_w = i\omega W\, \theta(x)\theta(y)$

$$= i\,\omega W \left\{\cos(\gamma_m x/2a) + R_m \cosh(\gamma_m x/2a)\right\} \left\{\sin \frac{(n-1)\,\pi\,(y+b)}{2b}\right\}$$

Substituting in equation (4.4) and carrying out the integration,

$$\langle |u_W|^2 \rangle = \frac{\omega^2 W^2}{8} \left[\left(1+ \frac{\sin\gamma_m}{\gamma_m}\right) + \frac{4R_m}{\gamma_m} \left\{ \cos(\gamma_m/2)\sinh(\gamma_m/2) + \sin(\gamma_m/2)\cosh(\gamma_m/2)\right\} + R_m^2 \left(1+ \frac{\sinh\gamma_m}{\gamma_m}\right) \right]$$

$$= \frac{\omega^2 W^2}{8} \times B_1$$

Where $B_1 = 1+\sin\gamma_m/\gamma_m+(4R_m/\gamma_m)\left\{\cos(\gamma_m/2)\sinh(\gamma_m/2)+ \sin(\gamma_m/2)\cosh(\gamma_m/2)\right\} + R_m^2\left(1+ \frac{\sinh\gamma_m}{\gamma_m}\right).$

Substituting $\langle |u_W|^2 \rangle$ in the expression of radiation efficiency,

$$S = \frac{256\, k^2 ab}{\pi^4 (n-1)^2 B_1} \int_0^{\pi/2}\int_0^{\pi/2} A_1 \sin\theta \, d\theta \, d\alpha . \quad \ldots\ldots(4.6)$$

The plate wave number , k_p is now defined as

$k_p = \left[\left\{(m-1)\pi/2a\right\}^2 + \left\{(n-1)\pi/2b\right\}^2\right]^{1/2}$, and the wave number ratio

$\psi = k/k_p$. Thus $k = \psi k_p$ or $k^2 = \psi^2 k_p^2$.

Substituting k^2 in equation (4.6), the expression for the radiation efficiency is obtained as

$$S = \frac{64\psi^2[(m-1)^2 R_a + (n-1)^2/R_a]}{\pi^2 (n-1)^2 B_1} \int_0^{\pi/2}\int_0^{\pi/2} A_1 \sin\theta \, d\theta \, d\alpha \quad \ldots\ldots\ldots \ldots (4.7)$$

Making the same substitution, the expression for l and s become

$$l = \left[(m-1)^2 + \{(n-1)^2/R_a^2\}\right]^{1/2} \pi\psi\cos\alpha \sin\theta \quad \text{and}$$

$$s = \left[R_a^2 (m-1)^2 + (n-1)^2\right]^{1/2} \pi\psi\sin\theta \sin \quad .$$

Case 2 : Odd values of m .

(a) Power radiation ;

For this case, the expression for the farfield pressure becomes,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} \left[\{\sin(\gamma_m' x/2a) + R_m' \sinh(\gamma_m' x/2a)\} \times e^{-ilx/2a} dx\right] \times \left[\sin \frac{(n-1)\pi(y+b)}{2b} e^{-isy/2b} dy\right] .$$

Integrating this expression , we get,

$$P = \frac{8ab\,\rho c k \omega W}{\pi^2 R (n-1)} e^{i(\omega t+kR)} \left[\frac{i\{\gamma_m' \cos(\gamma_m'/2)\sin(l/2) - l\sin(\gamma_m'/2)}{(\gamma_m'^2 - l^2)} \frac{-\cos(l/2)\}}{} + \frac{R_m' i}{(\gamma_m'^2 + l^2)} \{l\sinh(\gamma_m'/2)\cos(l/2) - \gamma_m'\cosh(\gamma_m'/2)\sin(l/2)\}\right] \times$$

$$\left[\frac{\cos(s/2) \; / \; i\sin(s/2)}{\{s^2/\pi^2(n-1)^2\} - 1} \right]$$

where cos(s/2) is used when n is an even integer and isin(s/2) is used when n is an odd integer.

Proceeding the same way as in the case of even values of m , the total average power radiated from one side of free-simply supported plate vibrating with odd values of m and even or odd values of n comes out to be

$$= \frac{8\rho\lambda_f^2 W^2 \pi^4 E^2 g^2 R_a^2 R_t^4}{9\, c\,\rho_m^2 (1-\sigma^2)^2 (n-1)^2} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\Big\{\frac{\gamma_m' \cos(\gamma_m'/2)\sin(l/2) - l\sin(\gamma_m'/2)\cos(l/2)}{(\gamma_m'^2 - l^2)}\Big\} +$$

$$+ R_m'\Big\{\frac{l\sinh(\gamma_m'/2)\cos(l/2) - \cosh(\gamma_m'/2)\sin(l/2)}{(\gamma_m'^2+l^2)}\Big\}\Big]^2 \cdot \Big[\frac{\cos \atop \sin (\frac{s}{2})}{\{s^2/\pi^2(n-1)^2\} - 1}\Big]^2 \times$$

$$\sin\theta\, d\theta\, d\alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.7a.)$$

Where l and s are the same as in case 1.

(b) Radiation Efficiency ;

The expression for total average power radiation from the plate is,

$$P = \frac{128\,\rho\, c k^2 \omega^2 W^2 a^2 b^2}{\pi^4 (n-1)^2} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\Big\{\frac{\gamma_m' \cos(\gamma_m'/2)\sin(l/2) - l\sin(\gamma_m'/2)\cos(l/2)}{(\gamma_m'^2 - l^2)}\Big\} +$$

$$R_m'\Big\{\frac{l\sinh(\gamma_m'/2)\cos(l/2) - \cosh(\gamma_m'/2)\sin(l/2)}{(\gamma_m'^2+l^2)}\Big\}\Big]^2 \cdot \Big[\frac{\cos \atop \sin (\frac{s}{2})}{\{s^2/\pi^2(n-1)^2\} - 1}\Big]^2$$

$$\sin\theta\, d\theta\, d\alpha .$$

The average of the temporal and spatial factor of the square of the surface velocity is given by

$$\langle |u_\omega|^2 \rangle = (1/4ab) \int_{-a}^{a}\int_{-b}^{b} (u_\omega^2/2)\, dxdy \quad \ldots\ldots\ldots\ldots\ldots\ldots (M)$$

where , $u_w = i\omega W\, \theta(x)\theta(y)$

$$= i\, W\omega\big[\sin(\gamma_m' x/2a) + R \sinh(\gamma_m' x/2a)\big] \cdot \Big[\sin \frac{(n-1)\pi\,(y+b)}{2b}\Big]$$

Substituting the value of u_w in equation (M) and integrating ,

$$\langle u_w \rangle^2 = \frac{\omega^2 W''}{8}\left[\left(1-\frac{\sin\gamma_n'}{\gamma_n'}\right)+R_n'^2\frac{\sinh(\gamma_n')}{\gamma_n'} - 1 + \frac{4R_n'}{\gamma_n'}\left\{ (\sin(\gamma_n'/2)\right.\right.$$

$$\left.\left.\cosh(\gamma_n'/2)-\cos(\gamma_n'/2)\sinh(\gamma_n'/2)\right\}\right] = \frac{\omega^2 W''}{8} B_2$$

where $B_2 = \left[\left(1-\frac{\sin\gamma_n'}{\gamma_n'}\right) + R_n'^2\frac{\sinh(\gamma_n')}{\gamma_n'} - 1\right) + \frac{4R_n'}{\gamma_n'}\left\{\sin(\gamma_n'/2)\right.$

$\left.\left.\cosh(\gamma_n'/2)-\cos(\gamma_n'/2)\sinh(\gamma_n'/2)\right\}\right]$

Substituting P_w and $\langle u_w \rangle^2$ into equation (3.37), the radiation efficiency of the plate is obtained as ,

$$S_{mn} = \frac{256\, k^2 ab}{\pi^4 (n-1)^2 B_2}\int_0^{\pi/2}\int_0^{\pi/2} A_2 \sin\theta \, d\theta \, d\alpha .$$

$$\text{where } A_2 = \left[\left\{\frac{\gamma_m'\cos(\gamma_m'/2)\sin(1/2) - 1\sin(\gamma_m'/2)\cos(1/2)}{(\gamma_m'^2 - 1^2)}\right\} + \right.$$

$$\left. R_m'\left\{\frac{1\sinh(\gamma_m'/2)\cos(1/2) - \cosh(\gamma_m'/2)\sin(1/2)}{(\gamma_m'^2+1^2)}\right\}\right]^2 \times$$

$$\left[\frac{\begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{s}{2}\right)}{\left\{s^2/\pi^2(n-1)^2\right\} - 1}\right]^2 .$$

After substitution of k ,

$$S = \frac{256ab\,\psi^2\left[\left\{(m-1)\pi/2a\right\}^2+\left\{(n-1)\pi/2b\right\}^2\right]}{\pi^4 (n-1)^2 B_2}\int_0^{\pi/2}\int_0^{\pi/2} A_2 \sin\theta \, d\theta \, d\alpha .$$

From this equation, the radiation efficiency may be evaluated by numerical integration after the substitution for l and s as in case 1.

4.3. Three ends clamped and one end simply supported plate :

The acoustic pressure distribution , given by equation (3.26) , has to be analytically integrated for a clamped simply supported plate in order to get the total average power radiation from one side of the plate . Radiation efficiency of this plate is also required to be calculated to compare it with the previous works (51,53,54).

The displacement functions satisfying the conditions at the edges of the plate in this case are ;

$$\Theta(x) = \sin \gamma_m\left(\frac{x-a}{4a}\right) + R_{\gamma_m} \sinh \gamma_m\left(\frac{x-a}{4a}\right), \text{ for } m=2,3,4,\ldots \text{ and } -a \leq x \leq a$$

where $R_{\gamma_m} = -\sin(\gamma_m/2)/\sinh(\gamma_m/2)$ and γ_m are the roots of the equation, $\tan(\gamma/2)-\tanh(\gamma/2) = 0$.

$$\Theta(y) = \cos(\gamma_n y/2b) + R_{\gamma_n} \cosh(\gamma_n y/2b), \text{ for } n=2,4,6, \ldots\ldots, \quad -b \leq y \leq b$$

$$\Theta(y) = \sin(\gamma_n' y/2b) + R_{\gamma_n}' \sinh(\gamma_n'/2b), \text{ for } n=3,5,7, \ldots\ldots \quad -b \leq y \leq b$$

where $R_{\gamma_n} = \sin(\gamma_n/2)/\sinh(\gamma_n/2)$ and γ_n are the roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2) = 0$,

$R_{\gamma_n}' = -\sin(\gamma_n'/2)/\sinh(\gamma_n'/2)$ and γ_n' are the roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2) = 0$.

It is observed here that the displacement of this plate is dependent on the odd-even nature of the mode order,n.

Case 1 . Even values of n .

(a) Power Radiation :

From equation (3.26), the net acoustic pressure P at the point (R,Θ,α) due to (m,n)th mode of vibration of the plate is given by

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} \Theta(x)\Theta(y)\, e^{-i(lx/2a+sy/2b)}\, dxdy.$$

Where $l = 2ak\cos\alpha \sin\Theta$

and $s = 2bk\sin\alpha\sin\theta$.

Substituting the appropriate displacement function for the vibrating plate with even values of m and both odd-even values of n,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} \left[\sin\gamma_m\left(\frac{x-a}{4a}\right) + R_{\gamma_m}\sinh\gamma_m\left(\frac{x-a}{4a}\right)\right] \times$$

$$\left[\cos(\gamma_n y/2b) + R_{\gamma_n}\cosh(\gamma_n/2b)\right] e^{-i(lx/2a+sy/2b)}\,dxdy.$$

Integrating the above expression of pressure distribution ,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \times 16ab \Bigg[\frac{1}{(4l^2-\gamma_m^2)}\Big\{\gamma_m\cos(l/2) - \gamma_m\cos(\gamma_m/2)$$

$$\cos(l/2) - 2l\sin(\gamma_m/2)\sin(l/2)\Big\} + \frac{R_m}{(4l^2+\gamma_m^2)}\Big\{\gamma_m\cos(l/2) - \gamma_m\cosh($$

$$\gamma_m/2)\cos(l/2) - 2l\sinh(\gamma_m/2)\sin(l/2)\Big\} + \frac{i}{(4l^2-\gamma_m^2)}\Big\{2l\sin(\gamma_m/2)\cos(l/2) -$$

$$\gamma_m\cos(\gamma_m/2)\sin(l/2) - \gamma_m\sin(l/2)\Big\} + \frac{iR_m}{(4l^2+\gamma_m^2)}\Big\{2l\sinh(\gamma_m/2)\cos(l/2) -$$

$$\gamma_m\sin(l/2) - \gamma_m\cosh(\gamma_m/2)\sin(l/2)\Big\}\Bigg].\Bigg[\frac{1}{(\gamma_n^2-s^2)}\Big\{\gamma_n\sin(\gamma_n/2)\cos(s/2) - s\cos$$

$$(\gamma_n/2)\sin(s/2)\Big\} + \frac{R_\eta}{(\gamma_n^2+s^2)}\Big\{\gamma_n\sinh(\gamma_n/2)\cos(s/2) + s\cosh(\gamma_n/2)\sin(s/2)\Big\}\Bigg].$$

$$= \frac{-\rho c k \omega W \times 16ab}{2\pi R} e^{i(\omega t+kR)}\left[A_3 + iA_4\right].A_5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.8)$$

Where, $A_3 = \frac{1}{(4l^2-\gamma_m^2)}\Big\{\gamma_m\cos(l/2) - \gamma_m\cos\ (\gamma_m/2)\cos(l/2)\ -2l\sin(\gamma_m/2)$

$$\sin(l/2)\Big\} + \frac{R_m}{(4l^2+\gamma_m^2)}\Big\{\gamma_m\cos(l/2)-\gamma_m\cosh(\gamma_m/2)\cos(l/2)-2l\sinh(\gamma_m/2)\sin(l/2)\Big\},$$

$$A_4=\frac{1}{(4l^2-\gamma_m^2)}\Big\{2l\sin(\gamma_m/2)\cos(l/2)-\gamma_m\cos(\gamma_m/2)\sin(l/2)-\gamma_m\sin(l/2)\Big\}+$$

$$\frac{R_m}{(4l^2+\gamma_m^2)}\Big\{2l\sinh(\gamma_m/2)\cos(l/2)-\gamma_m\sin(l/2)-\gamma_m\cosh(\gamma_m/2)\sin(l/2)\Big\},$$

$$A_5=\frac{1}{(\gamma_n^2-s^2)}\Big\{\gamma_n\sin(\gamma_n/2)\cos(s/2)-s\cos(\gamma_n/2)\sin(s/2)\Big\}+$$

$$\frac{R_n}{(\gamma_n^2+s^2)}\Big\{\gamma_n\sinh(\gamma_n/2)\cos(s/2)+s\cosh(\gamma_n/2)\sin(s/2)\Big\}.$$

The farfield acoustic intensity is given by

$$I = |P|^2/\rho c$$

If we substitute P from equation (4.8), we have

$$I = \frac{\rho^2 c^2 k^2 \omega^2 W^2\, 256 a^2 b^2}{8\pi^2 R^2}\Big[A_3^2+A_4^2\Big].A_5^2 .$$

Total average power radiated from one side of the baffled plate is now given by

$$P_{w}=\int_0^{2\pi}\int_0^{\pi/2}(|P|^2/\rho c)R^2\sin\theta\, d\theta\, d\alpha .$$

$$= 4\int_0^{\pi/2}\int_0^{\pi/2} IR^2 \sin\theta\, d\theta\, d\alpha .$$

Therefore, acoustic power radiation for both odd-even values of m and even values of n becomes,

$$P_w = \frac{8\,\rho\, W\, \lambda_f^4 \pi^6 E^2 g^2 R_a^2 R_l^4}{9 c\, \rho_m^2 (1-\sigma^2)^2}\int_0^{\pi/2}\int_0^{\pi/2}\Big[A_3^2+A_4^2\Big]A_5^2\sin\theta\, d\theta\, d\alpha .$$

Here, $\lambda_f^2=G_x^4+(a/b)^4 G_y^4+2(a/b)^2\Big[\sigma H_x H_y+(1-\sigma)J_x J_y\Big]$

$$\omega^2 = \frac{\lambda_f^2 \pi^4 E h^2 g}{\rho_m a^4 12 (1-\sigma^2)}$$

$$l = \frac{R_t \lambda_f \sqrt{Eg}\ \pi^2}{c \sqrt{3 \rho_m (1-\sigma^2)}} \cos\alpha \sin\theta$$

$$s = \frac{R_a R_t \lambda_f \sqrt{Eg}\ \pi^2}{c \sqrt{3 \rho_m (1-\sigma^2)}} \sin\alpha \sin\theta .$$

(b) Radiation Efficiency :

The radiation efficiency of a vibrating plate is defined by the equation (3.37) as

$$S_{mn} = \frac{P_W}{4 \rho c a b \langle |u_W|^2 \rangle}$$

where the expression of total acoustic power P_w , without substituting the value of k , is

$$P = \frac{128 \rho c k^2 \omega^2 W^2 a^2 b^2}{\pi^2} \int_0^{\pi/2}\int_0^{\pi/2} [A_3^2 + A_4^2] A_5^2 \sin\theta \, d\theta \, d\alpha .$$

Letting $(A_3^2 + A_4^2) A_5^2 = A_6$,

$$P_w = \frac{128 \rho c k^2 \omega^2 W^2 a^2 b^2}{\pi^2} \int_0^{\pi/2}\int_0^{\pi/2} A_6 \sin\theta \, d\theta \, d\alpha .$$

The average of the temporal and spatial factor of the square of the surface velocity , $\langle |u_W|^2 \rangle$, is given by

$$\langle |u_W|^2 \rangle = (1/4ab) \int_{-a}^{a}\int_{-b}^{b} (u_W^2/2) dx\, dy .$$

But $u_w = i\omega W\, \theta(x)\theta(y)$

$$= i\omega W \left[\sin \gamma_m \left(\frac{x-a}{4a}\right) + R_m \sinh \gamma_m \left(\frac{x-a}{4a}\right) \right] \left[\cos(\gamma_n y/2b) + R_n \cosh(\gamma_n y/2b) \right]$$

Therefore,
$$\langle |u_W|^2 \rangle = \frac{\omega^2 W^2}{8ab} \int_{-a}^{a}\int_{-b}^{b} \left[\sin \gamma_m \left(\frac{x-a}{4a}\right) + R_m \sinh \gamma_m \left(\frac{x-a}{4a}\right) \right]^2 \cdot \left[\cos(\gamma_n y/2b) + R_n \cosh(\gamma_n y/2b) \right]^2 dx\, dy .$$

Carrying out the integration , we get,

$$\langle |u_w|^2 \rangle = \frac{\omega^2 W^2}{8}\Big[(1-\sin\gamma_m/\gamma_m)+\frac{4R_m}{\gamma_m}\Big\{\sin(\gamma_m/2)\cosh(\gamma_m/2)-\cos(\gamma_m/2)\ \sinh(\gamma_m/2)\Big\} + R_m^2\Big\{\sinh\gamma_m/\gamma_m-1\Big\}\Big]\times\Big[(1+\sin\gamma_n/\gamma_n)+\frac{4R_n}{\gamma_n}\Big\{\cos(\gamma_n/2)\sinh(\gamma_n/2)+\sin(\gamma_n/2)\ \cosh(\gamma_n/2)\Big\} + R_n^2\Big\{1+\ \sinh\gamma_n/\gamma_n\Big\}\Big]= B_3\ \frac{W^2\omega^2}{8}$$

Where,

$$B_3=\Big[(1-\sin\gamma_m/\gamma_m)+\frac{4R_m}{\gamma_m}\Big\{\sin(\gamma_m/2)\ \cosh(\gamma_m/2)\ -\ \cos(\gamma_m/2)\sinh(\gamma_m/2)\Big\} + R_m^2\Big\{\frac{\sinh(\gamma_m)}{\gamma_m}-1\Big\}\Big].\Big[(1+\ \sin\gamma_n/\gamma_n)+\ \frac{4R_n}{\gamma_n}\Big\{\cos(\gamma_n/2)\ \sinh(\gamma_n/2)+\ \sin(\gamma_n/2)\ \cosh(\gamma_n/2)\Big\}+\ R_n^2\Big\{1+\ \frac{\sinh\ (\gamma_n)}{\gamma_n}\Big\}\Big]$$

Substituting $\langle |u_w|^2 \rangle$ in the expression of radiation efficiency ,

$$S\ =\ \frac{256\ k^2 ab}{\pi^2 B_3}\int_0^{\pi/2}\int_0^{\pi/2} A_6 \sin\theta\ d\theta\ d\alpha\ . \qquad \text{(P)}$$

Now , the plate wave number

$$K_p=\Big[\{(m-1)\pi/2a\}^2+\ \{(n-1)\pi/2b\}^2\Big]^{1/2}$$ and the wave number ratio

$$\Psi=k/k_p$$

Therefore, $k^2 = \Psi^2 k_p^2$

Substituting the values of k^2 in equation (P) , we have

$$S_{mn}= \frac{64\,\Psi^2\Big[(m-1)^2 R_o+\ (n-1)^2/R_o\Big]}{B_3}\int_0^{\pi/2}\int_0^{\pi/2} A_6 \sin\theta\ d\theta\ d\alpha\ .$$

Case 2 : Odd values of n .

(a) Power Radiation :

For this case of even-odd values of m and odd values of n, the expression for the farfield acoustic pressure distribution, after substituting the displacement functions, becomes,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} \left[\sin \gamma_m\left(\frac{x-a}{4a}\right) + R_m \sinh\gamma_m\left(\frac{x-a}{4a}\right)\right] \times$$

$$\left[\sin(\gamma_n' y/2b) + R_n' \sinh(\gamma_n' y/2b)\right] e^{-ilx/2a}\, e^{-isy/2b}\, dx\, dy.$$

Where l and s are the same as in case 1 for even values of n and odd-even values of m.

Integrating the above expression, we get,

$$P = \frac{-\rho c k \omega W}{2\pi R} 16ab\, e^{i(\omega t+kR)} \Big[\frac{1}{(4l^2-\gamma_m^2)} \{\gamma_m \cos(l/2) - \gamma_m \cos(\gamma_m/2)$$

$$\cos(l/2) - 2l \sin(\gamma_m/2)\sin(l/2)\} + \frac{R_m}{(4l^2+\gamma_m^2)} \{\gamma_m \cos(l/2) - \gamma_m \cosh$$

$$(\gamma_m/2)\cos(l/2)\ - 2l\sinh(\gamma_m/2)\sin(l/2)\} + \frac{i}{(4l^2-\gamma_m^2)} \{2l\sin(\gamma_m/2)\cos(l/2)$$

$$-\gamma_m \cos(\gamma_m/2)\ \ \sin(l/2) - \gamma_m \sin(l/2)\} + \frac{iR_m}{(4l^2+\gamma_m^2)} \{2l\sinh(\gamma_m/2)$$

$$\cos(l/2)\ -\gamma_m \sin(l/2)\ -\gamma_m \cosh(\gamma_m/2)\ \sin(l/2)\}\Big] i\Big[\frac{1}{(\gamma_n'^2-s^2)}\{\gamma_n' \cos(\gamma_n'/2)\sin(s/2) -$$

$$s\ \sin(\gamma_n'/2)\cos(s/2)\} + \frac{R_n'}{(\gamma_n'^2+s^2)} \{s\sinh(\gamma_n'/2)\cos(s/2)\ -\gamma_n' \cosh(\gamma_n'/2)$$

$$\sin(s/2)\}\Big]$$

Following the same procedure as in case 1, the final form of acoustic power radiation becomes,

$$P = \frac{8\rho W^2 \gamma_f^4 \pi^5 E^2 g^2 R_a^2 R_i^4}{c \rho_m^2 (1-\sigma^2)^2} \int_0^{\pi/2}\int_0^{\pi/2} \left[A_\gamma^2 + A_\beta^2\right] A_\eta^2 \sin\theta\, d\theta\, d\alpha\ .$$

Where, $A_7 = \frac{1}{(41^2-\gamma_m^2)}\left\{\gamma_m\cos(1/2)-\gamma_m\cos(\gamma_m/2)\cos(1/2)\ -21\ \sin(\gamma_m/2)\right.$

$\left.\sin(1/2)\right\} + \frac{R_m}{(41^2+\gamma_m^2)}\left\{\gamma_m\cos(1/2)-\gamma_m\cosh(\gamma_m/2)\ \cos(1/2)\ -21\ \sinh(\gamma_m/2)\right.$

$\left.\sin(1/2)\right\}]$

$A_8 = \frac{1}{(41^2-\gamma_m^2)}\left\{21\sin(\gamma_m/2)\ \cos(1/2)\ -\gamma_m\cos(\gamma_m/2)\ \sin(1/2)\ -\ \sin(1/2)\right\}$

$+ \frac{R_m}{(41^2+\gamma_m^2)}\left\{21\ \sinh(\gamma_m/2)\ \cos\ (1/2)-\gamma_m\sin(1/2)\ -\gamma_m\cosh(\gamma_m/2)\ \sin(1/2)\right\}$

and $A_9 = \frac{1}{(\gamma_n'^2-s^2)}\left\{\gamma_n'\cos(\gamma_n'/2)\ \sin(s/2)\ -s\ \sin(\gamma_n'/2)\cos(s/2)\right\} +$

$\frac{R_n'}{(\gamma_n'^2+s^2)}\left\{s\ \sinh(\gamma_n'/2)\ \cos(s/2)\ -\gamma_n'\cosh(\gamma_n'/2)\ \sin(s/2)\right\}.$

Above equation gives the total average power radiated from one side of the baffled plate with three ends fixed and the other one simply supported, vibrating in its (m,n)th mode with even -odd values of m and odd values of n.

(b) Radiation Efficiency :

Radiation efficiency of a vibrating plate is given by the equation (3.37),

$$S_{mn} = \frac{P_W}{4\rho cab\ \langle|u_W|^2\rangle} \quad \text{.................................}(S)$$

Where, P_w , the total average power radiated from one side of the plate in terms of plate parameter, is

$$P = \frac{128\ \rho ck^2\omega^2 \tilde{W}^2 a^2 b^2}{\pi^2}\int_0^{\pi/2}\int_0^{\pi/2}\left[A_7^2\ +A_8^2\right].A_9^2\sin\theta\ d\theta\ d\alpha\ .$$

Substituting $A_{10} = [A_7^2 + A_9^2] A_7^2$, we have,

$$P = \frac{128 \rho c k^2 \omega^2 W^2 a^2 b^2}{\pi^2} \int_0^{\pi/2} \int_0^{\pi/2} A_{10} \sin\theta \, d\theta \, d\alpha \; . \qquad \text{(Q)}$$

Now , as in the previous cases, the average of the temporal and spatial factor of the square of the surface velocity,

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a} \int_{-b}^{b} (u_w^2/2) \, dx \, dy \qquad \text{(R)}$$

Where $u_w = i\omega W \, \theta(x)\theta(y)$

$$= i\omega W \left[\sin \gamma_m \left(\frac{x-a}{4a}\right) + R_m \sinh \gamma_m \left(\frac{x-a}{4a}\right) \right] . \left[\sin (\gamma_n' y/2b) + R_n' \sinh (\gamma_n' y/2b) \right] .$$

Substituting u_w in equation (R) and carrying out the integration,

$$\langle |u_w|^2 \rangle = \frac{\omega^2 W^2}{8} \Big[(1 - \sin\gamma_m/\gamma_m) + \frac{4R_m}{\gamma_m} \{ \sin(\gamma_m/2) \cosh (\gamma_m/2) - \cos(\gamma_m/2) \sinh(\gamma_m/2) \} + R_m^2 \left\{ \frac{\sinh \gamma_m}{\gamma_m} - 1 \right\} \Big] \Big[\left\{ 1 - \frac{\sin \gamma_n'}{\gamma_n'} \right\} + \frac{4R_n'}{\gamma_n'} \{ \sin(\gamma_n'/2) \cosh(\gamma_n'/2) - \cos(\gamma_n'/2) \sinh(\gamma_n'/2) \} + R_n'^2 \left\{ \frac{\sinh \gamma_n'}{\gamma_n'} - 1 \right\} \Big] = B_4 \frac{\omega^2 W^2}{8}$$

Where,

$$B_4 = \Big[\left(1 - \frac{\sin \gamma_m}{\gamma_m}\right) + \frac{4R_m}{\gamma_m} \{ \sin(\gamma_m/2)\cosh(\gamma_m/2) - \cos(\gamma_m/2) \sinh(\gamma_m/2) \} + R_m^2 \left\{ \frac{\sinh \gamma_m}{\gamma_m} - 1 \right\} \Big] . \Big[\left\{ 1 - \frac{\sin \gamma_n'}{\gamma_n'} \right\} + \frac{4R_n'}{\gamma_n'} \{ \sin(\gamma_n'/2)\cosh(\gamma_n'/2) - \cos(\gamma_n'/2) \sinh(\gamma_n'/2) \} + R_n'^2 \left\{ \frac{\sinh \gamma_n'}{\gamma_n'} - 1 \right\} \Big] .$$

Substituting P_w and $\langle |u_w|^2 \rangle$ into equation (S), the radiation efficiency is found to be ,

$$S = \frac{256 \, k^2 ab}{\pi^2 B_4} \int_0^{\pi/2} \int_0^{\pi/2} A_{10} \sin\theta \, d\theta \, d\alpha \; .$$

Now , substituting $k = \gamma k_p$

where $k_p = \left[\left\{(m-1)\pi/2a\right\}^2 + \left\{(n-1)\pi/2b\right\}^2\right]^{1/2}$, the radiation efficiency is obtained as

$$S_{mn} = \frac{64\,\gamma^2\left[(m-1)^2 R_a + (n-1)^2/R_a\right]}{B_4}\int_0^{\pi/2}\int_0^{\pi/2} A_{10}\sin\theta\, d\theta\, d\alpha .$$

This is the expression of radiation efficiency of the baffled plate with three ends fixed and one end simply supported, vibrating in its (m,n)th mode with odd values of n and even-odd values of m.

4.4. Numerical Solution :

The numerical method is a modern tool to solve any linear or nonlinear equations successfully which are otherwise very difficult. The analytical solutions of the equations for calculating the total average power radiation could not be obtained. The numerical method of solution has thus been developed to solve equation (3.26) with the help of a computer. The equations $\tan(\gamma/2) \pm \tanh(\gamma/2)=0$ are solved by the method of bisection and the integrations of equation (3.26) are performed by applying Simpson's Rule. The results of integration are presented in tabular as well as in graphical forms.

The Method of Bisection :

The method of bisection is very efficient but simple. It may be used to solve the roots of continuous non-linear as well as complicated linear equations. The greatest virtue of the bisection method is that it is virtually assured to converge to a root. In this method, the equation to be solved is expressed in the form, f(x) =0. Then an approximate root is determined either by observation or by graphical method. The values of the function f(x) are then determined for successive regular intervals. If x_o is the approximate value of a root of the equation, f(x) =0 , then, $f(x_o)$, $f(x_o+h)$, $f(x_o+2h)$, ----- , are determined, where h is the successive increment of x . If the product of any two consecutive values of the function becomes negative, it can be concluded that at least one root of the equation lies in that interval h of x.

Then a better approximate value of the root is obtained as the average (arithmetic mean) of these two successive values of x. By taking smaller value of h, the accuracy of the result can be increased to any degree. Also the process stated above can be repeated until desired accuracy is obtained.

The Simpson's Formula :

We now turn to one of the most widely known and used techniques in numerical integration-the Simpson's Rule . It is similar to the trapezoidal rule in dividing the total interval into many smaller intervals and approximating the area under them but different in that a parabola is passed through the three ordinates of two adjoining intervals. We would expect that whereas the trapezoidal rule is exact for first degree polynomials, Simpson's rule would be exact for second degree or lower; actually , it turns out somewhat surprisingly to be exact for third-degree or lower. It is therefore a rather accurate method for the effort required and the formula is not significantly more complex than that for the trapezoidal rule. These characteristics account for the wide usage of the method .

It is a multisegment formula . The accuracy of the results obtained is principally determined by the number of segments taken in the calculation . The formula is obtained by summing the areas under the parabolic arcs. If the range (b-a) of the integration , $I = \int_a^b f(x)dx$, is divided into n even equal divisions, so that $h = (b-a)/n$, then by Simpson's formula, the result of integration is given by

$$I=(h/3)\left[f(a)+4f(a+h)+2f(a+2h)+ \text{----------} +4f(a+(n-1)h)+f(b)\right].$$

In the present solution , the range, 0 to $\pi/2$, was divided into 36 equal intervals for applying the Simpson's formula . The number 36 was taken because of the fact that the accuracy of the result with further increase in the number of divisions increases only slightly , but the computer time required increases proportionately . The number of divisions may be further increased or the method of Romberg integration can be used for improving the accuracy of the results if highly accurate result is desired . The detail procedures of evaluating different quantities are shown in Appendix-C.

CHAPTER-5

Reliability and limitations of the method

5.1. Reliability of the method of solution:

The reliability and validity of any new technique should be verified prior to its acceptance as a genuine tool. This is achieved usually by comparing the results of the new method with the results that are already available in the literature. This approach generally helps to ascertain that no error due to logic is committed in formulating the problem and no mistake is made in the computer programming. Keeping these objectives in mind , a number of standard problems are solved with the present method of solution and the results are compared with those of others, obtained analytically or by some other technique. On the basis of this comparison, the reliability and validity of the present method are verified .

Wallace(51) and a number of other authors(53,54) studied the acoustics of a rectangular panel of uniform thickness. .For defining the deformation modes of rectangular panels,the beam functions developed by Warburton(1), were used by these authors. The radiation resistance and radiation efficiency corresponding to the natural modes of finite rectangular panels were theoretically determined from the total energy radiated to the farfield. Wallace(51) studied the case of a simply supported plate in an infinite baffle. Asymptotic solutions for low frequency region were derived and curves covering the entire frequency range for various mode shapes and aspect ratios were obtained through numerical integration.

Wallace(50) also found the radiation resistance from the far-field pressure distribution produced by a baffled beam, vibrating with simple harmonic motion in one of its natural modes. He considered beams hinged at both ends and clamped at both ends. He derived expressions for the radiation resistance which are asymptotic to the exact solution as the frequency approaches zero. In addition, numerical integration of the far-field acoustic intensity was used to obtain graphs covering the entire frequency range for the first ten modes of the beam. Through these investigations Wallace established that the nature of radiation efficiency of beams is similar and independent of end-fixity. This fact provides an opportunity for comparison of the results of the present analysis with those of Wallace and others towards proving its credibility and soundness.

Ahmed(53) also found the radiation efficiency of vibrating plates with all edges simply supported with a different formulation. Mandal(54) found the radiation efficiency of rectangular plates with two opposing ends fixed and other two ends free. When compared, it is observed that the graphs of Wallace (51), Ahmed(53) and Mandal(54), presented in Fig.54,55,56, respectively, are identical to the radiation efficiency graphs of the present study ,presented in Fig.18 to 21,and 34 to 37 . Earlier Ahmed(53) and Mandal(54) observed that although the graphs of radiation efficiency of a rectangular plate at different combination of mode orders and of different end-fixity but of the same mode order differ from one another at the lower range of mode orders,they reach the same maximum value and the same asymptotic value at increasing wave number ratios.This is verified here by comparing the present results(Fig.18 to 21 and 34 to 37) with those of Ahmed(Fig.55), Wallace(Fig.54) and Mandal(Fig.56). Moreover, the power radiations from different plates of the present analysis are compared with those obtained by Ahmed(53) and Mandal(54) over the identical range of mode orders. As expected,the graphs differ from one another, but they maintain similar nature (Figs.46 and47). The above verification proves that the method of solution employed here is accurate and no error is committed either in formulating the problem or in the computer programming.

5.2. Validity of the beam functions:

Warburton (1) developed a number of beam functions to satisfy the compatibility conditions at the ends of a vibrating beam as imposed by physical restrains. The functions were designed to represent accurately the wave form of a beam vibrating in its natural modes. Warburton's beam functions were used by Wallace(50) in the analysis of acoustics of beams with hinged and clamped ends. The results of the investigations by Wallace proved to be satisfactory as the expressions for the radiation resistance asymptotically approached the exact solution as frequency decreased. Wallace also applied Warburton's beam functions to define the mode of vibration of a vibrating plate.

He derived the expression for the radiation resistance of a simply supported plate in an infinite baffle vibrating in its natural modes. The expression for the radiation resistance was asymptotic to the exact solution at low frequency.

In the present analysis Warburton's beam functions have been used to represent the motion of points on the surfaces of vibrating plates. The functions are combined to meet the requirements of the different boundary conditions of vibrating plates like two opposite ends free while the other two ends simply supported and one end simply supported while the other three ends fixed.The exact representations of these beam functions, as used in the present analysis, are shown in figures 2,3.4 and 5. It is observed from these figures that the beam mode shapes can satisfactorily represent the mode shapes of a vibrating plate under different boundary conditions considered here.

The above discussions prove that the beam functions developed by Warburton are sufficiently valid and that these functions can be applied to the vibrating plate with different pure or mixed boundary conditions.

5.3. Limitations of the method:

There are a number of factors involved in the analysis of any engineering problem. It is very difficult to solve these problems considering the influence of all the factors simultaneously.Thus the end results of the solution of any complex engineering problem are always obtained under certain simplifying assumptions and approximations. The limitations imposed on the results must be stated clearly and should be justified.

The present analysis of the acoustics of vibrating rectangular plates with free-simply supported and clamped-simply supported edge conditions have the following limitations:

1. The method of finding the fundamental frequency of the vibrating plates includes certain approximations as stated in Ref.(1) and thus the present analysis is limited by those approximations.

2. The governing equation used in finding the fundamental frequency of the plates are based on certain assumptions as stated in Ref.(1) and hence this analysis is also limited by those assumptions.

3. The displacement functions used in defining the deformation modes of the vibrating plates satisfy the governing equation only approximately (1). Thus the actual deformation modes of the vibrating plates may differ from what have been used in this analysis.

4. The expressions for acoustic power radiation used in this analysis include certain assumptions as stated in Ref. (59,66) and hence this analysis is limited by those assumptions.

But the nature of approximations and assumptions included in this analysis had been tested in different contexts by different authors and was found to be of not much significance. The errors involved are within reasonable range and thus it can be concluded that the end results of this analysis are good enough for engineering uses.

CHAPTER 6

RESULTS AND DISCUSSIONS

6.1. Results and discussions:

The total average power radiation from one side of a rectangular flat plate vibrating in its natural modes is found under clamped-simply supported and free- simply supported edge conditions. The radiation efficiencies of these plates are also obtained. The appropriate beam functions which define the deformation mode in two perpendicular directions and satisfy the conditions at the edges of the vibrating plate under consideration have been combined for defining the deformation mode of the respective plate under vibration. In this analysis, the boundary conditions considered are: (1) two opposing ends free and the other two ends simply supported, (2) one end simply supported and the other three ends fixed. The beam functions in fact satisfy the boundary conditions for plates with fixed or freely supported edges, but are only approximate for free edges. The input variables for any rectangular flat plate with uniform thickness are the aspect ratios, (b/a), and the thickness ratios, (h/a). The results are found for three values of the aspect ratios and three values of the thickness ratios for each case of the boundary conditions. The results are presented in both tabular and graphical forms. Three values of each of the two plate parameters considered are 0.5, 1.0 and 2.0 for aspect ratio and 0.001,0.002 ,0.004 for thickness ratio. The values of other parameters as required in numerical calculations are : density of the surrounding medium, ρ =1.21 kg/m^3 ; velocity of sound in surrounding medium, c= 341 m/sec ; density of the plate material, ρ_m=7700.0 kg/m^3 and 1400.0 kg/m^3 for steel and plastic(pvc), respectively; modulus of elasticity of the plate material, E = 206 GPa and 4.0 GPa for steel and plastic(pvc), respectively ; amplitude of vibration, W = 0.0001 m ; and poison's ratio, σ=0.4 for steel and 0.45 for plastic (pvc). In the subsequent sections , the results of each of the two problems investigated are discussed separately.

(1) Plates with two opposing ends free and the other two ends simply supported :

The values of acoustic power radiation are obtained for various plate parameters and mode orders in x- and y- directions. Figures 6 , 7 and 10 to 13 show the total average acoustic power radiated from one side of the baffled plate, plotted against the number of nodal lines in the y-direction for a fixed mode order in x- direction. In figures 8 and 9 , the total average acoustic power radiation are plotted against the number of nodal lines in x-direction for fixed mode orders in y-direction . The influence of different aspect ratios, thickness ratios and mode orders are also compared in these figures.

In order to offer a plausible explanation of the nature of curves in Figs. 6,7,8 and 9 , it is necessary to refer to figures 14,15,16 and 17. In figures 14 and 15, it is seen that , with the increase of mode order, n, frequency factor increases. Figs. 16 and 17 show the same trend of λ_f with increasing m, the mode order in x-direction . But from the definition of frequency factor λ_f by Warburton (53), it is observed that the frequency of vibration is directly proportional to the frequency factor λ_f . So it can be concluded that the nature of variation of frequency of the plates will be the same as the frequency factor because they have the same trend. Hence it can be concluded that , with the increase of the values of m or n or both , the frequency of vibration would increase. Figures 6 and 7 show the acoustic power radiation plotted against nodal lines in y- direction while Figs. 8 and 9 show the same in x-direction for constant values of aspect ratio and thickness ratio. In figure 6, it is seen that the acoustic power radiation increases somewhat monotonically at the lower range of n and fluctuates at the higher values of n. This is more obvious from figure 7 . The variations of power radiation follows the same trend with variation of m which is shown in figures 8 and 9. These can be explained by the fact that the power radiation should increase with the increase of the frequency of vibration which, in its turn , increase with the increase of the values of m, n . At lower values of m and n , the rate of increase of frequency is higher. So the power radiation increases rapidly at the lower range of mode orders. But the power radiation does not show significant rise with increasing values of m and n at the higher range . The graphs show a waviness after certain values of m and n . At the higher range of mode orders, the values of m and n have less influence over the acoustic power radiation. The waviness of the graphs after some values of mode orders are due to the interference of the waves from x and y - directions. At the high mode orders , the wave form from the two directions interfere frequently. The acoustic power radiation fluctuates due to this interference. When the two crests from the two directions superimpose, the total average acoustic power radiation increases. On the other hand , when the crests from one direction superimposes on the trough from another , the total average acoustic power radiation decreases as depicted by the roots of the graphs. The frequency of this waviness is dependent on the interferences of the waves. The number of interferences increases with the increase in the mode orders. Thus the frequency of waviness has a lower value at the low range of mode orders and increases with the rise of mode orders.

It is observed in figures 7 and 9 that the rate of increase in the acoustic power radiation approaches zero at very high mode numbers. The variation of the acoustic power radiation can be explained with the help of the variation of acoustic pressure in the surrounding medium due to the vibration of the plate. There are various factors which affect the acoustic power radiation. The frequency of vibration , the effective radiating surface of the plate, and the effective acoustic pressure are the major three factors among them.

It is already known that the frequency of vibration increases significantly with the increase of mode orders at lower range , but at high range , the rate of increase of frequency of vibration falls. At low mode orders, the effective radiating surface is larger because, in this case, the points of zero displacement in the plate are less. But with increasing mode orders , the regions of this zero displacement increases so that effective radiating surface decreases . Again, due to the vibration of the plate , wave forms are generated in the surrounding medium, resulting in its alternate compression and refraction . At very high mode orders, the plate wave number becomes very high. Therefore, alternate crests and roots of the plate wave come closer to one another. At any instant of time, a particular crest produces a compression in the surrounding medium while the neighbouring root produces a rarefaction. These alternate compression and rarefaction causes the acoustic energy to travel through the surrounding medium. When the plate wave number becomes very high, the neighbouring roots and crests come so close to each other that the rarefaction by the root partially neutralizes the compression produced by the crest. The effective pressure variation decreases. Thus the intensity of absolute pressure variation is high at the lower mode orders and low at the higher mode orders. Hence, in the low range of mode orders, the acoustic power radiation from the plate rapidly rises due to the positive contributions of the frequency of vibration , effective pressure variation and effective radiating surface of the plate. But at the high mode orders, the effective acoustic pressure in fact has no role on power variation. The effect of frequency of vibration becomes insignificant at high range of mode orders. Also the effective radiating surface of the plate implies negative effect on power variation . Therefore, the absolute amount of acoustic power radiation does not show any further increase with mode numbers. It rather attains a stable magnitude in this case. Though the power radiation displays a fluctuating characteristics at high modes, the radiation efficiency converges to unity and does not show any change with further increase in the mode numbers, as shown in figures 18, 19, 20 and 21. This is because of the fact that , radiation efficiency presents the power radiated from a plate due to its vibration at certain mode orders in comparison to the power radiated by the same plate vibrating as a solid without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate under consideration. At high mode orders, the interferences of the compressions and rarefactions produced by alternate crests and roots of the plate waves tend to reduce the rate of increase in the total acoustic power radiation. Above the critical frequency of vibration of the plate, the acoustic radiation from the plate becomes stable and does not increase with increasing mode orders. This stable acoustic radiation is equivalent to the acoustic radiation from a plate vibrating as a solid body without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate. The effect of this stable magnitude of the acoustic radiation from a plate converges the radiation efficiency to unity above the critical frequency, the frequency at which the wave length of the standing wave of the plate is equal to the wave length of the radiated acoustic wave of the plate.

Figures 18, 19 , 20 and 21 show the radiation efficiency of the vibrating plate with two opposing ends free and other two ends simply supported . As the wave number ratio increases, the radiation efficiency also increases gradually. When the wave number ratio is unity, the magnitude of radiation efficiency becomes maximum which is about 1.2. In Refs.(51,53,54) , maximum value of radiation efficiency is 1.2 when the wave number ratio reaches unity. With further increase of wave number ratio , radiation efficiency converges to unity and remains constant. It is also true in Figs. 54, 55 and 56.

Figures 10 and 11 represent the variation of acoustic power radiation with variation of thickness ratio of the plate. It is seen from these figures that the acoustic power radiation increases when thickness ratio is increased. This is due to the fact that more power will be needed to excite a plate if its thickness is increased. Figure 10 shows the variation of power radiation with thickness ratio for low values of nodal lines in x-direction and figure 11 for high values of the same nodal line.

In figures 12 and 13, the effect of aspect ratio of the plate over the power radiation is shown. Figure 12 shows the variation of average power with aspect ratio at a lower value of m and figure 13 for higher values of m . It can be seen from figure 12 that the total average acoustic power radiation increases with the decrease in the values of the aspect ratio. But this is not true for higher values of m.It is found that the total acoustic power radiation increases with the increase of the aspect ratio at higher values of m at the lower range of n. This is shown in figure 13. But the trend is gradually reversed with the increase in the values of n. The above variations of acoustic power radiation can be explained with the help of figures 14, 15 ,16 and 17. These figures show the variation of dimensionless frequency factor λ_f with the variation of aspect ratio for different values of m and n . It is observed that, at all mode orders, the dimensionless frequency factor, ,increases with the decrease of aspect ratio. As dimensionless frequency factor increases, frequency of vibration also increases, which in turn increases the total acoustic power. Therefore , average acoustic power radiation increases as the aspect ratio is decreased. At lower mode orders, the amount of power radiation mainly depends on the effective radiating surface of the plate . The influence of frequency over the power radiation is less dominant in this case. Effective radiating surface is more in the case of a plate with lower aspect ratios at low values of mode orders in the x-direction. For this reason, at low mode numbers in the x-direction, the plate with lower aspect ratio radiates more power. This is shown in figure 7. At high values of m in the lower range of n , the plate with higher aspect ratio radiates higher amount of power which is shown in figure 13. This is due to the fact that, at these mode orders, the effective radiating surface is more in the case of a plate with higher aspect ratio as the longer side of the plate is divided into fewer waves. The effective radiating surface decreases with increasing mode orders but the frequency of vibration increases. Here the increase in the frequency of vibration far exceeds the reduction in the effective radiating surface. Thus, the frequency of vibration begins to play a dominating role over the acoustic power radiation from the plate. From figure 15, it is seen that the increase in the frequency of vibration with the increase in the mode numbers is more in the case of a plate with an aspect ratio of 0.50. The amount of acoustic power radiation show a steeper rise in this case as shown in figure 13. The amount of power radiation from the plates with the other two aspect ratios is smaller.

(2) Plates with three ends fixed and one end simply supported:

In figures 22 and 23, the variation of power is plotted against the number of nodal lines in y-direction at a particular value of nodal line in x-direction. Figures 24 and 25 show the variation of power radiation from the plate at a particular value of n and plotted against the number of nodal lines in x-direction. From figure 22, it is observed that , the power radiated from a plate increases rapidly with the increase in the values of n upto a certain limit. after that , with increasing n , power radiation increases slowly with moderate fluctuation. The same trend is seen with increasing values of m in figure 24. At high mode orders, the acoustic power radiation does not increase significantly. It rather attains a stable state which is shown in figures 23 and 25. The power radiation due to the vibration of the plate is largely dependent upon the frequency of vibration and the effective radiating surface. From figures 30, 31,32 and 33, it is evident that dimensionless frequency factor increases with the increase of mode orders. At the lower range of mode orders, increasing tendency is higher than that at the higher mode orders. Again, the variation of frequency of vibration follows the same trend as the dimensionless frequency factor. Therefore, with the increase of dimensionless frequency factor, frequency of vibration increases in the same manner as there exists a linear relation between frequency of vibration of the plate and dimensionless frequency factor. But acoustic power radiation increases if frequency of vibration increases. Hence, from figures 30,31,32 and 33 it may be concluded that the radiation of power should increase with increasing values of m and n. With the increase of mode orders , the effective radiating surface decreases which is defined as the surface of the plate from which acoustic radiation takes place. But frequency of vibration increases with increasing mode orders.

Thus, in the lower range of mode orders, due to the positive effect of both the frequency of vibration of the plate and effective radiating surface, the amount of acoustic power radiation increases with the increase of mode orders. This is shown in figures 22 and 24 . At the higher mode orders, frequency of vibration increases at a decreasing rate with the increase of mode orders. On the other hand , the effective radiating surface decreases significantly. Hence , in this range of mode orders, the amount of power radiated increases with a reduced rate and shows a waviness after certain mode orders. With further increase in mode orders, waviness will be asymptotically zero. At high mode orders, the waviness of the acoustic power radiation is due to the interference of the standing waves coming from x and y-directions. When the two crests coming from the two directions superimpose, net result will produce a sharp crest. On the other hand, if the two crests oppose one another, the total acoustic power radiation decreases as depicted by the roots of the graphs. This fluctuation in the acoustic power radiation constitutes the waviness in the total average acoustic power radiation. The frequency and amplitude of interference increase with the increase of mode orders. This is obvious from figures 23 and 25. Though the average acoustic power radiated increases with increasing mode numbers, the rate of increase gradually decreases and ultimately reaches a stable state showing no further increase in the absolute value. In the higher mode orders, neighbouring compression and rarefaction made by the crest and root of standing waves in the plate partially neutralizes each other. This neutralization , in its turn, reduces the acoustic power radiation from the plate. Though the absolute value of the amount of acoustic power radiation does not increase with increasing mode numbers after a certain mode orders, the waviness of power radiation still remains. This is caused by the fact that the interference of the standing waves from the two directions still exists even if the mode numbers are very high.

Figures 34, 35, 36 and 37 show the radiation efficiency of the vibrating plate with one end simply supported and other three ends fixed. It is observed from these figures that the radiation efficiency converges to unity at high modes and does not show any change with further increase in the mode numbers. This is due to the same reason as discussed in the case of the plate with two opposing ends simply supported and the other two ends free. As the wave number ratio increases , the radiation efficiency also increases . When the wave number ratio is unity, the magnitude of radiation efficiency becomes maximum. With further increase of wave number ratio, radiation efficiency converges to unity and remains constant. This same trend is true in Refs. (51,53,54).

Figures 26 and 27 present the nature of variation of power radiation with the variation of thickness ratio for lower and higher values of m, respectively. As in the other case discussed earlier, acoustic radiation increases with the increase in the value of the thickness ratio of the plate. More power is thus needed to excite a plate with higher thickness as the plates radiates more energy.

In figures 28 and 29 , the amount of acoustic radiation from the plate at different aspect ratios have been compared. Figure 28 presents the power variation with aspect ratio for lower value of m and figure 29 for higher value of m. From figure 28 , it is seen that total acoustic power radiation increases with the decrease of aspect ratio. This may be explained by referring to figures 30,31 32 and 33. In these figures, dimensionless frequency factor is plotted for various aspect ratios against the number of nodal lines. It is clear that, as aspect ratio decreases, , dimensionless frequency factor increases. Frequency of vibration increases with the increase of dimensionless frequency factor as their relationship is linear. Thus in the lower range of mode orders, the plate with lower values of aspect ratio radiates more power than those with higher values of the aspect ratio. In this case , the effective radiating surface of the plate with lower aspect ratio is less but the frequency of vibration is much higher. This makes the acoustic radiation to be more with lower aspect ratio than that with higher aspect ratio . As shown in figure 29, for high values of m in the lower range of n , the plate with an aspect ratio of 2.00 radiates more power than the plates with lower values of the aspect ratio. With increasing values of m, the effective radiating surface decreases. This decrease is more in the case of a plate with an aspect ratio of 0.50, making the power radiation to be less than that at higher aspect ratios. With increase in the values of n , the increase in frequency of vibration takes care of the decrease in the effective radiating surface. As such, at high mode orders, the frequency of vibration begins to play a dominant role over acoustic power radiation.

6.2. COMPARISON OF POWER RADIATION FROM PLATES OF DIFFERENT BOUNDARY CONDITIONS AND OF DIFFERENT MATERIALS:

(1) Comparison of the effect of boundary conditions:

In the present analysis, the power radiations from a rectangular plate , vibrating in its natural modes, are studied with two boundary conditions. These are : (a) two opposing ends simply supported and the other two ends free and (b) one end simply supported while the other three ends fixed. Figure 42 to 45 show the comparison of the average acoustic power radiation from the plate with above boundary conditions. It is evident from figures 42 and 44 that , at lower range of mode orders, the amount of power radiation from the plate for both the boundary conditions are nearly equal in magnitude.It can be explained with the help of figures 38, 39, 40 and 41, which show that the dimensionless frequency factor for a particular mode shape is almost the same for both the boundary conditions. This makes the amount of acoustic power radiation to be nearly equal for both the cases. At high mode orders, the alternate compressions and rarefactions of the surrounding medium of the vibrating plate, produced by the neighbouring crests and roots of the plate waves, begin to partially neutralize each other. Thus , at these mode orders, the effects of the edge conditions on the amount of acoustic power radiation are virtually nullified. This is clear from figures 43 and 45.

(2) comparison of the effect of materials:

Figures 50 to 53 compare the average acoustic power radiation from steel and plastic plates. At low mode orders, total acoustic power radiation from a steel plate is higher than that of a plastic plate . The reasons behind it are as follows : (a) the frequency of a steel plate is slightly higher than a plastic plate which is shown in figures 48 and 49 , (b) the steel plate is more rigid than the plastic one for which naturally more power will be needed to excite it. But at high mode orders , plastic plate is observed to radiate more power .

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS.

7.1. CONCLUSIONS :

In this thesis, the total average acoustic power radiated from one side of a baffled plate , vibrating in its natural modes, have been investigated with two different mixed boundary conditions. The radiation efficiencies of the plates are obtained and compared with the results given by Wallace(51),Ahmed(53) and Mandal(54). The effect of material properties on power radiation has also been investigated.

The following conclusions are made from the investigations of the results and graphs for different boundary conditions and for different values of various parameters.

(1) The average acoustic power radiated from one side of a baffled plate vibrating in its natural modes increases with the increase of mode orders. In the lower range of mode orders, this increasing tendency is high. At the high range of mode orders, the acoustic power radiation is in fact in a stable state with the increase of mode numbers. After a certain mode orders, the acoustic power radiation from the plates begin to show a wavy nature.

(2) The average power radiated from a baffled plate increases with the increase of thickness ratio.

(3) With the increase of aspect ratio, the average power radiated

from a baffled plate decreases at low values of m. For high values of m in the low range of n, the average power radiation increases with increasing aspect ratio. But this tendency gradually reverses at the higher range of n and m.

(4) The total average acoustic power radiated from a baffled plate with two opposite ends simply supported and other two ends free is nearly equal to the power radiated from the plate with one end simply supported and other three ends fixed.

(5) The radiation efficiencies of the plates increase with the increase of wave number ratio γ upto its value of unity. With further increase in the values of wave number ratio, radiation efficiency converges to unity and remains constant.

(6) At low mode orders, power radiation from a steel plate is higher than that from a plastic plate. But at high mode orders. a plastic plate radiates more power.

7.2. RECOMMENDATIONS :

From the knowledge and experience of the present investigation, the following fields on plate vibration and acoustics are recommended as the scope of future research.

(1) Accuracy of the present investigation may be enhanced by adopting more appropriate displacement functions to define correctly the motion of the surface of the vibrating plate.

(2) Plates , reinforced with beams, may be studied and the effects of reinforcement can be investigated.

(3) Plates with variable thickness and other irregularities may also be studied.

(4) Plates having nonuniform edge constraints may be studied by introducing the proper elastic constrains into the boundary conditions.

(5) Displacement functions may be evolved for the study of the effect of holes and cracks in vibrating plates.

(6) The vibration of composite plates may be studied.

(7) The effects of various other mixed boundary conditions may be studied by using Warburton's displacement functions for dealing with practical problems.

(8) Methods should be evolved to investigate these problems experimentally.

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APPENDIX - A

FIGURES

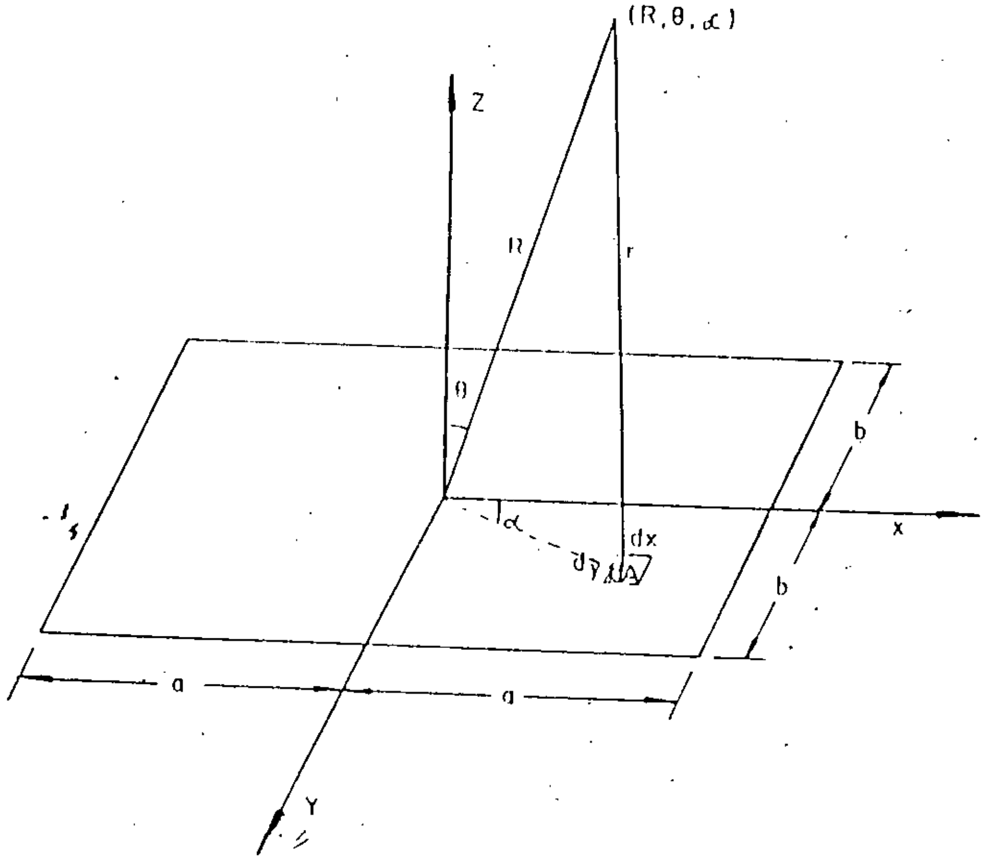


Fig. 1 : Rectangular flat plate with coordinate systems.

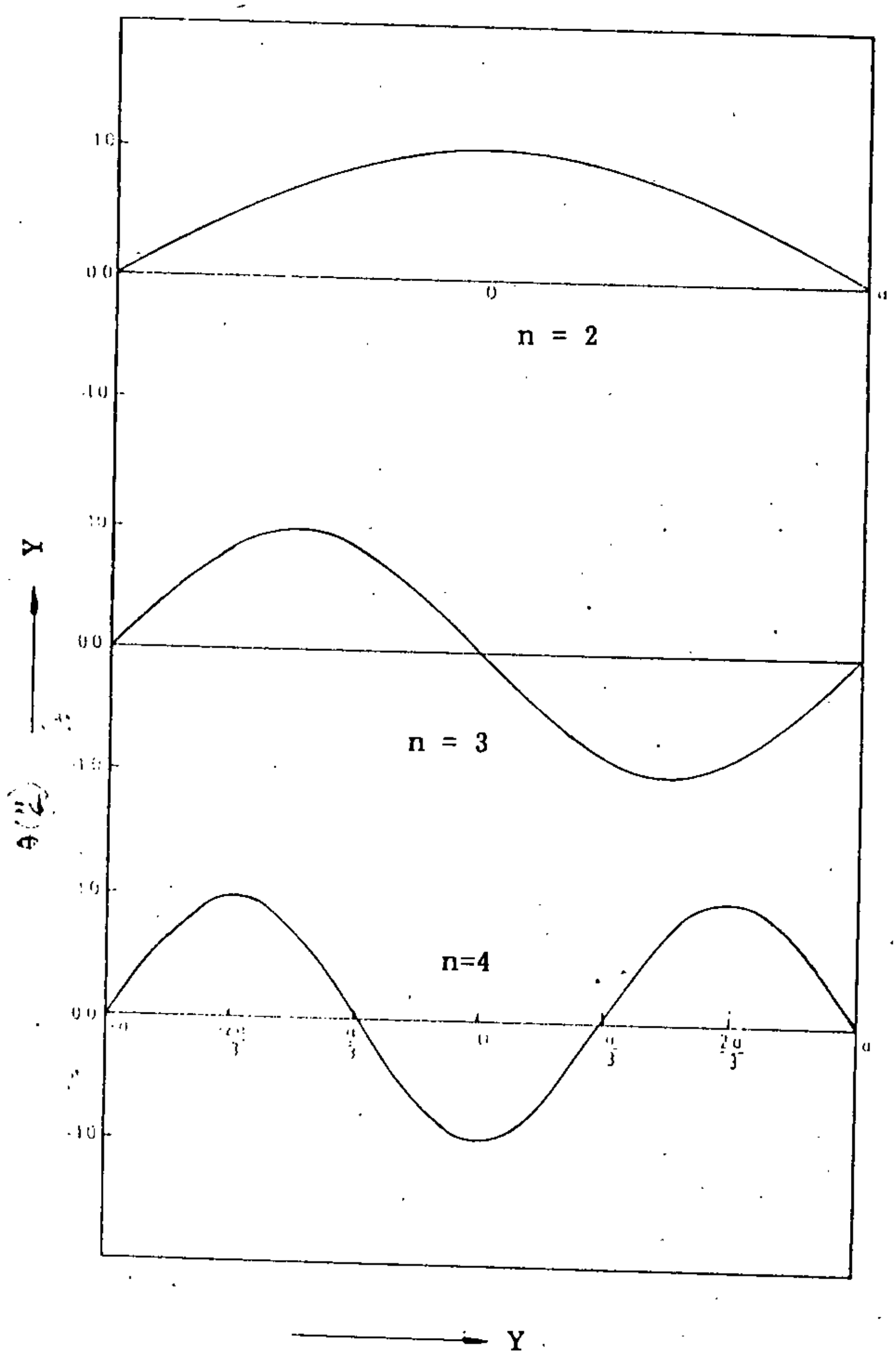


Fig.2: Beam function Θ(y) for the plate with two opposing ends simply-supported for even and odd values of n.

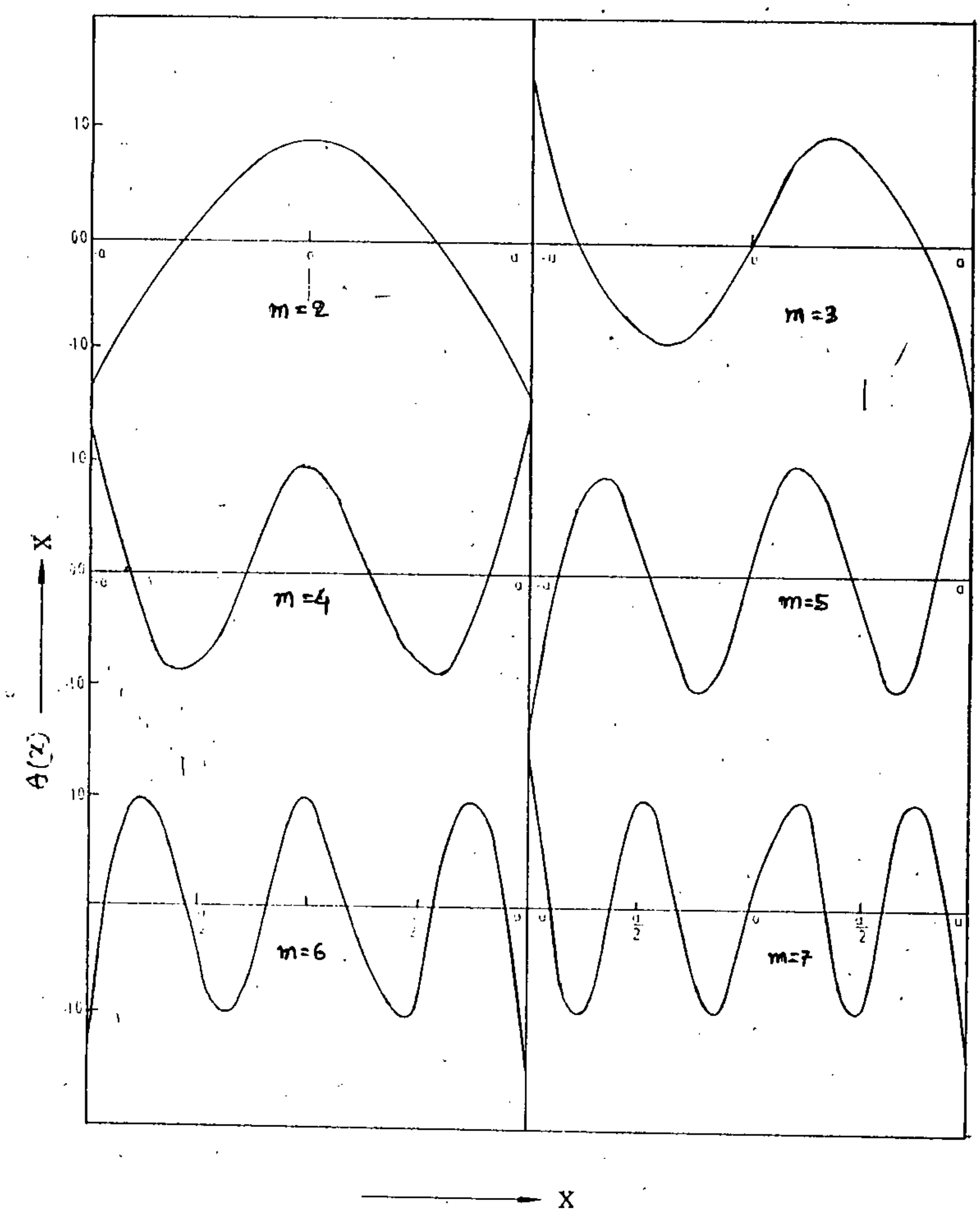


Fig.3: Beam function $\Theta(x)$ for the plate with two opposing ends free for even and odd values of m.

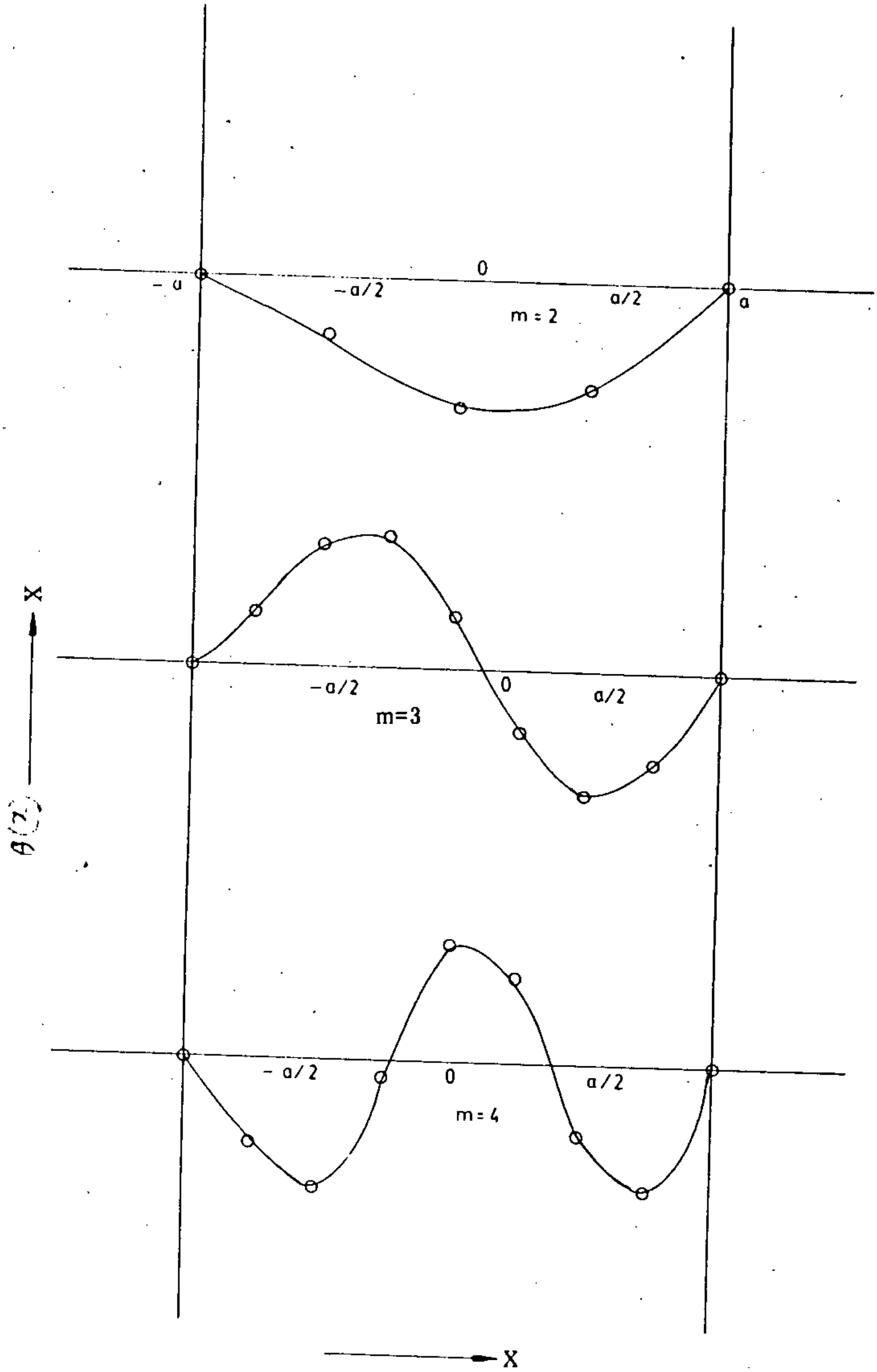


Fig. 4: Beam function θ(x) for the plate with three ends fixed and one end simply-supported. for even and odd values of m.

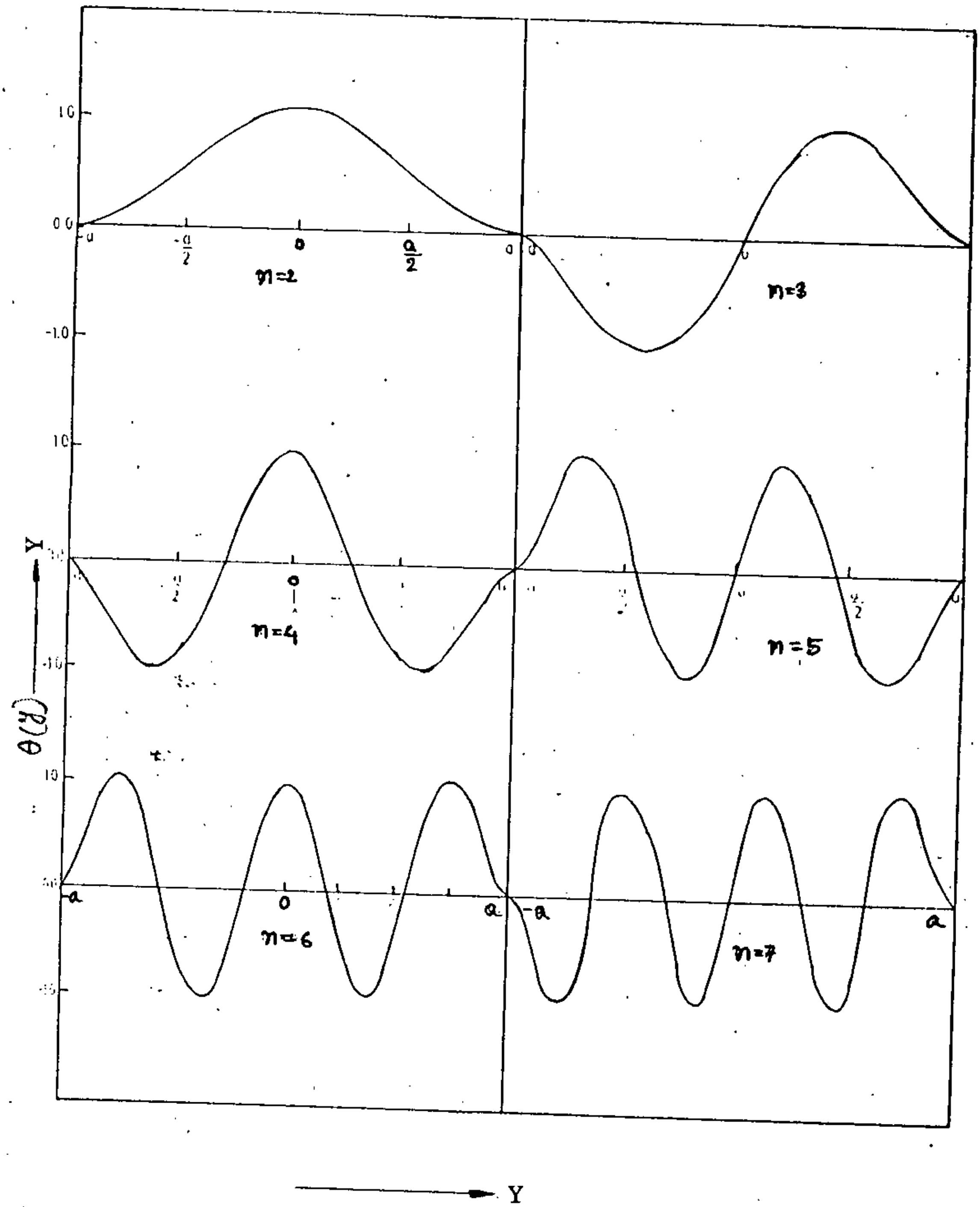


Fig.5: Beam function θ(y) for the plate with three ends fixed and one end simply-supported for even and odd values of n.

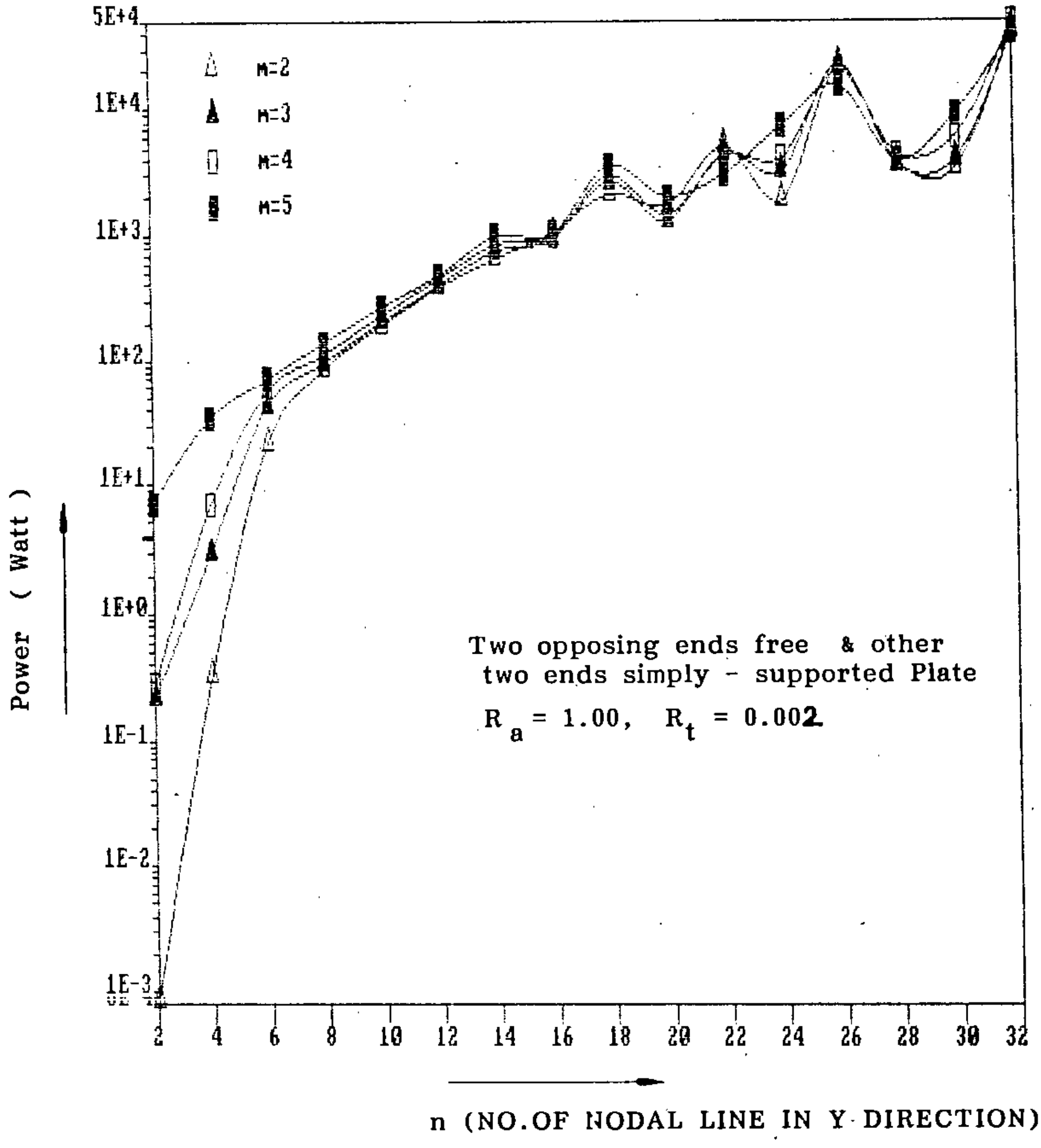


Fig.6: Power radiation from two opposing ends free and other two ends simply - supported Plate at low values of m.

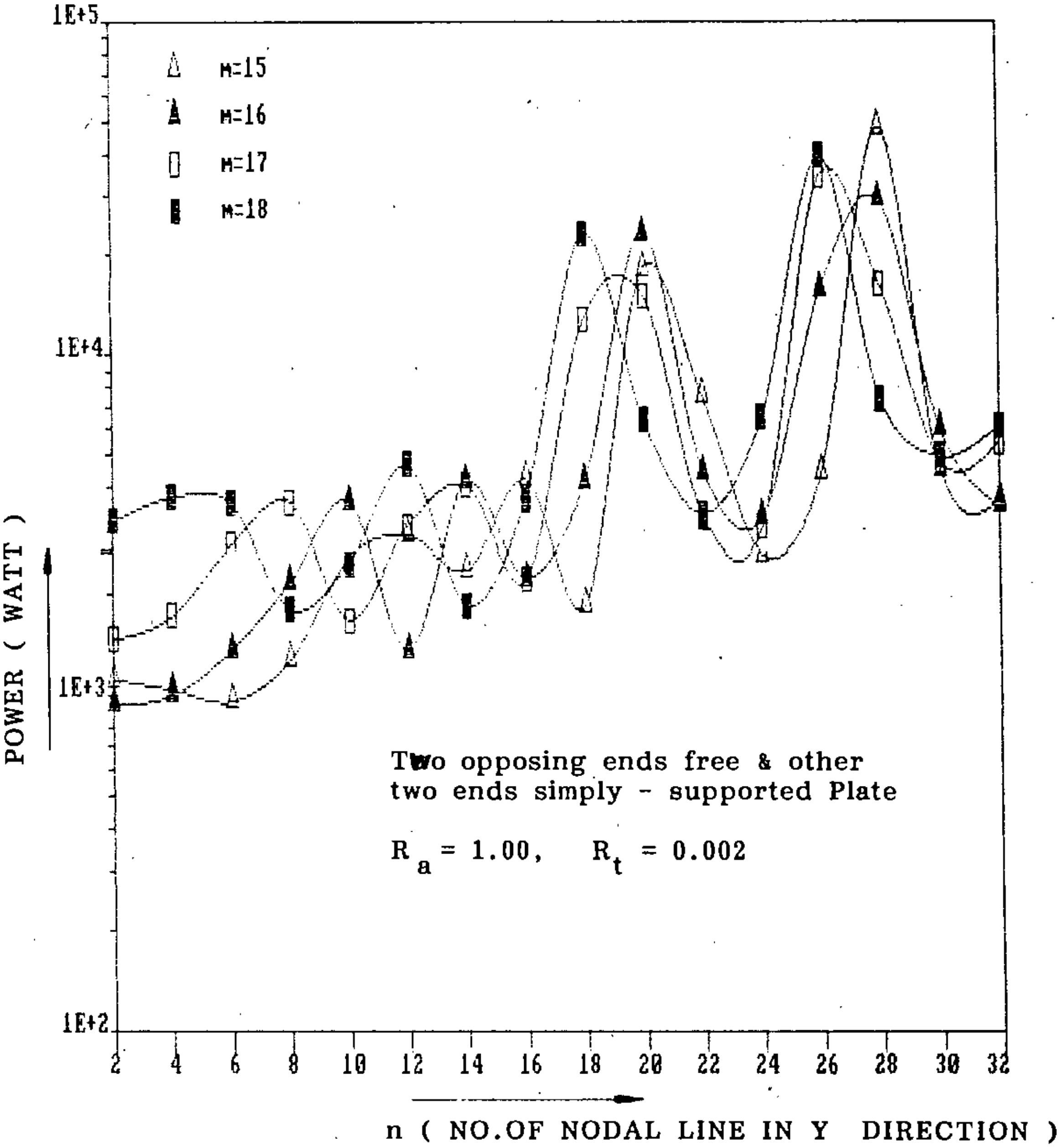


Fig.7: Power radiation from two opposing ends free and other two ends simply - supported Plate at high values of m.

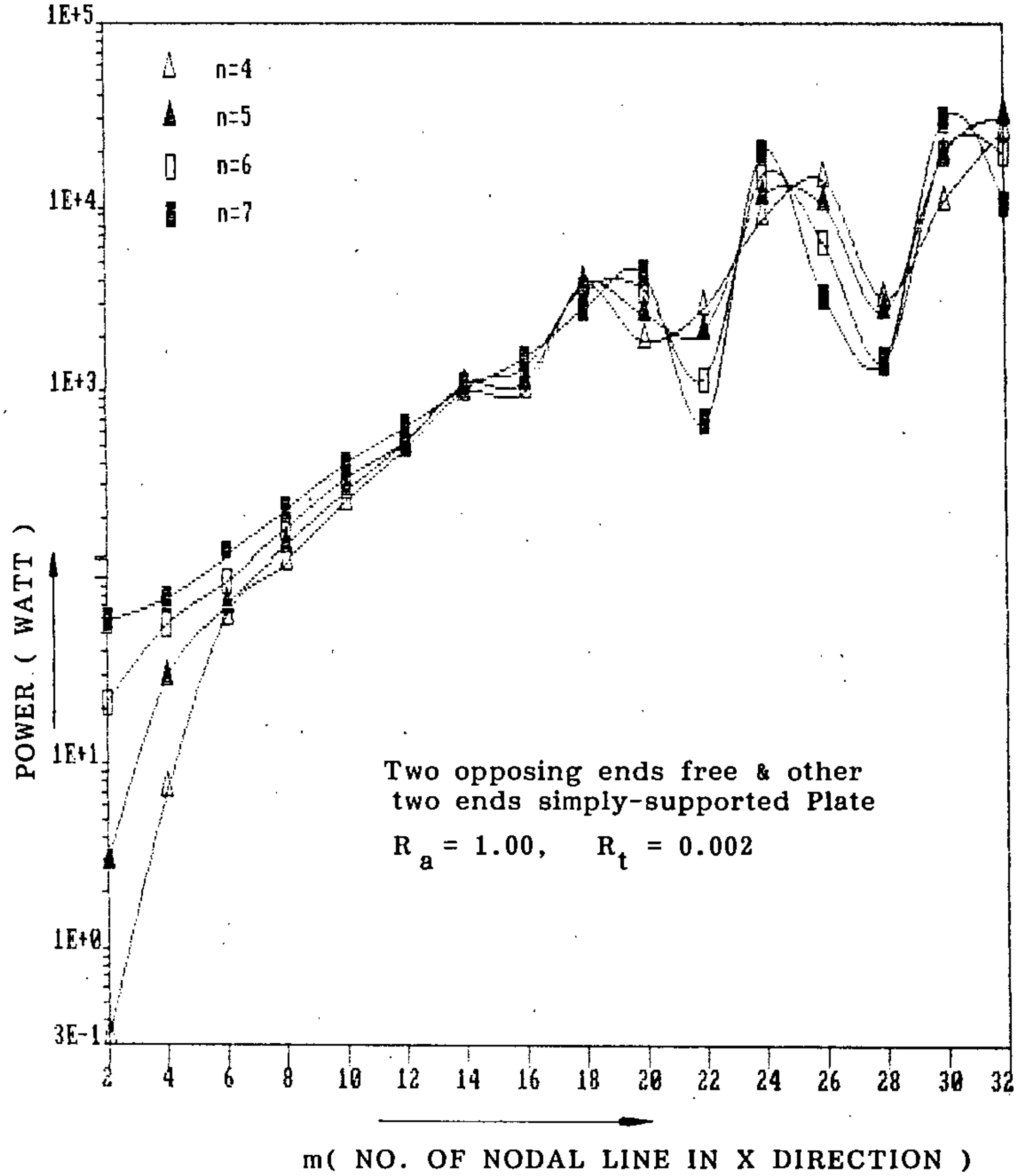


Fig.8: Power radiation from two opposing ends free and other two ends simply-supported Plate at low values of n.

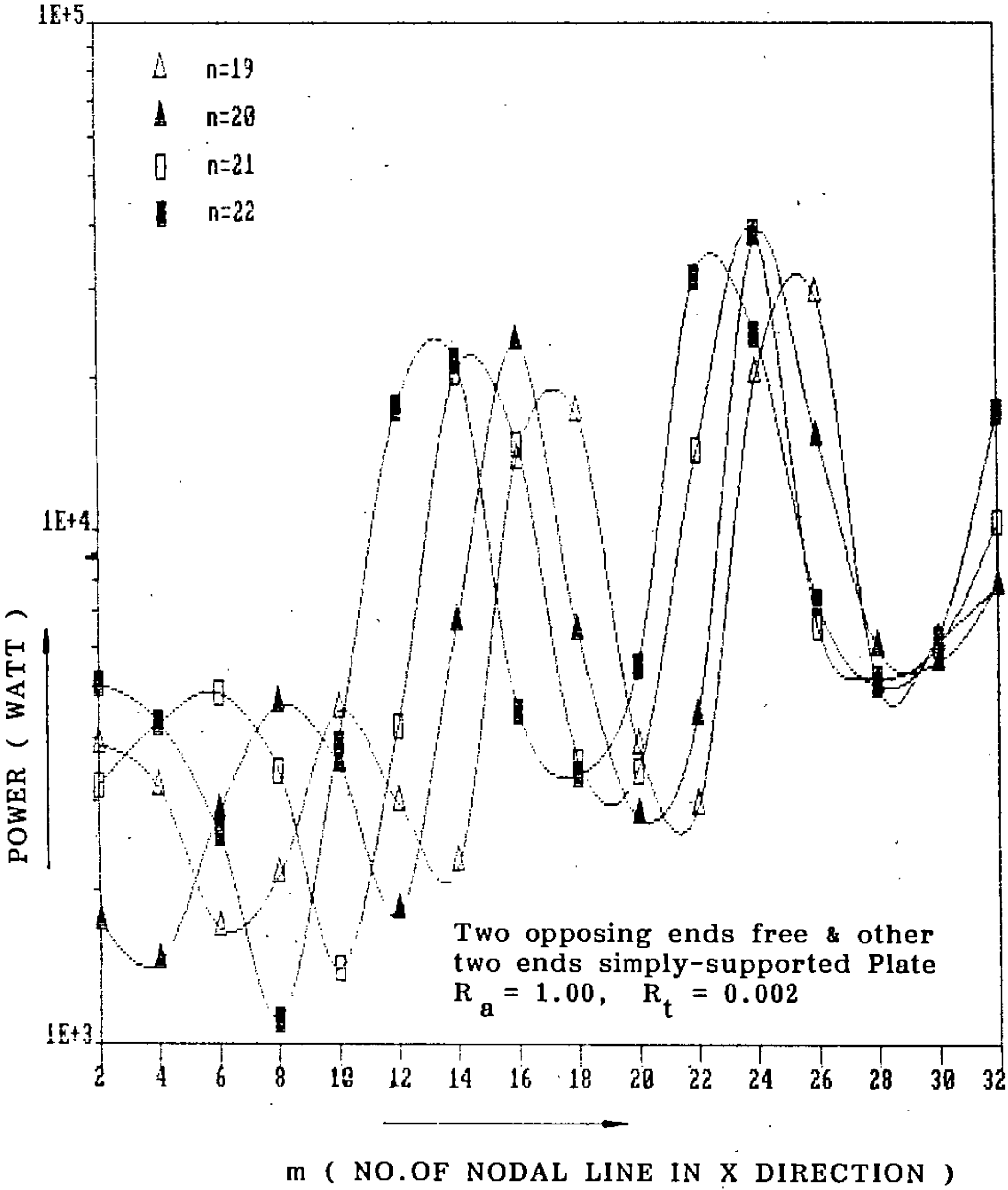


Fig.9: Power radiation from two opposing ends free and other two ends simply-supported Plate at high values of n.

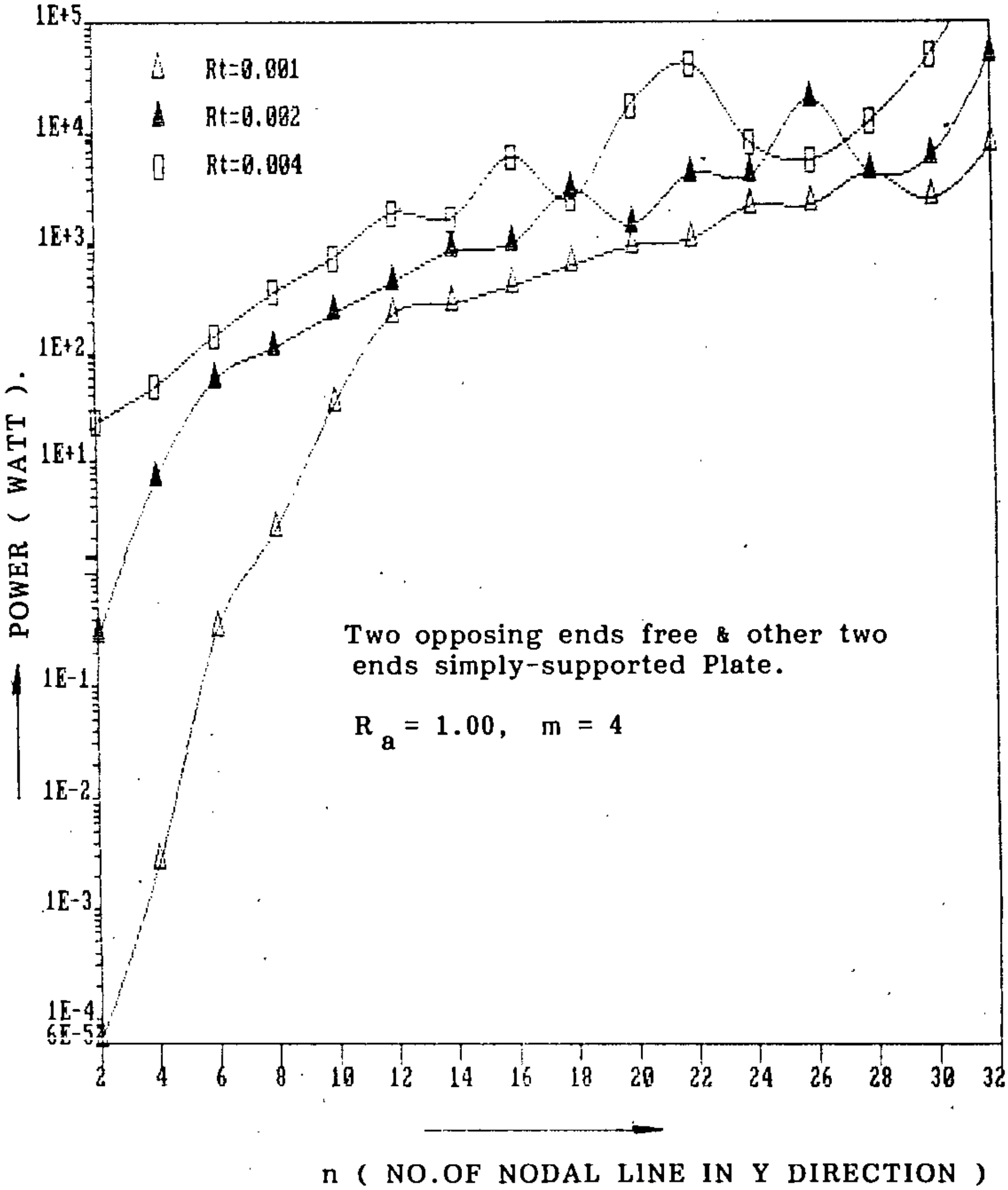


Fig.10: Power radiation from two opposing ends free and other two ends simply-supported Plate for different thickness ratio at low values of m.

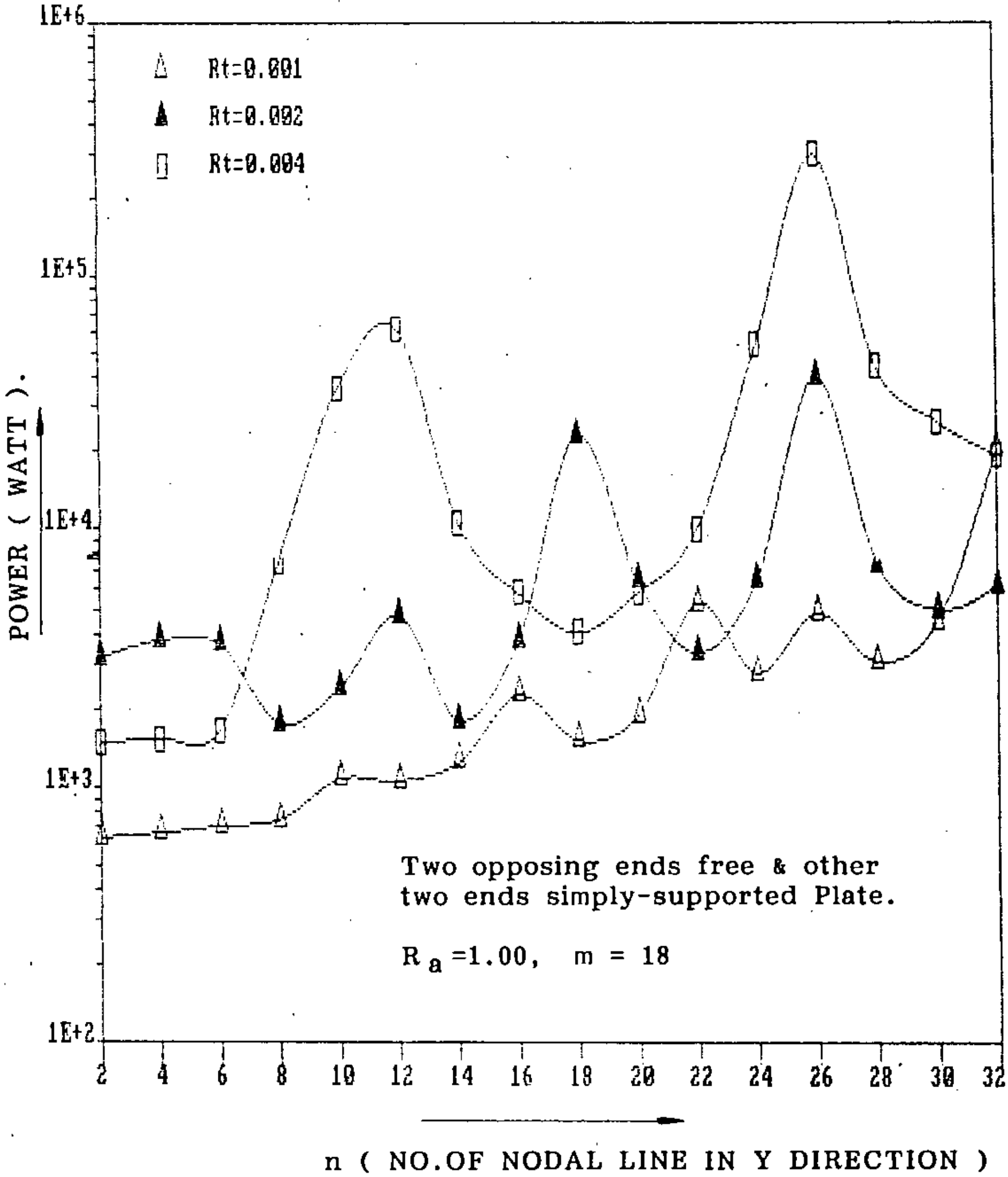


Fig.11: Power radiation from two opposing ends free and other two ends simply-supported Plate for different thickness ratio at high values of m.

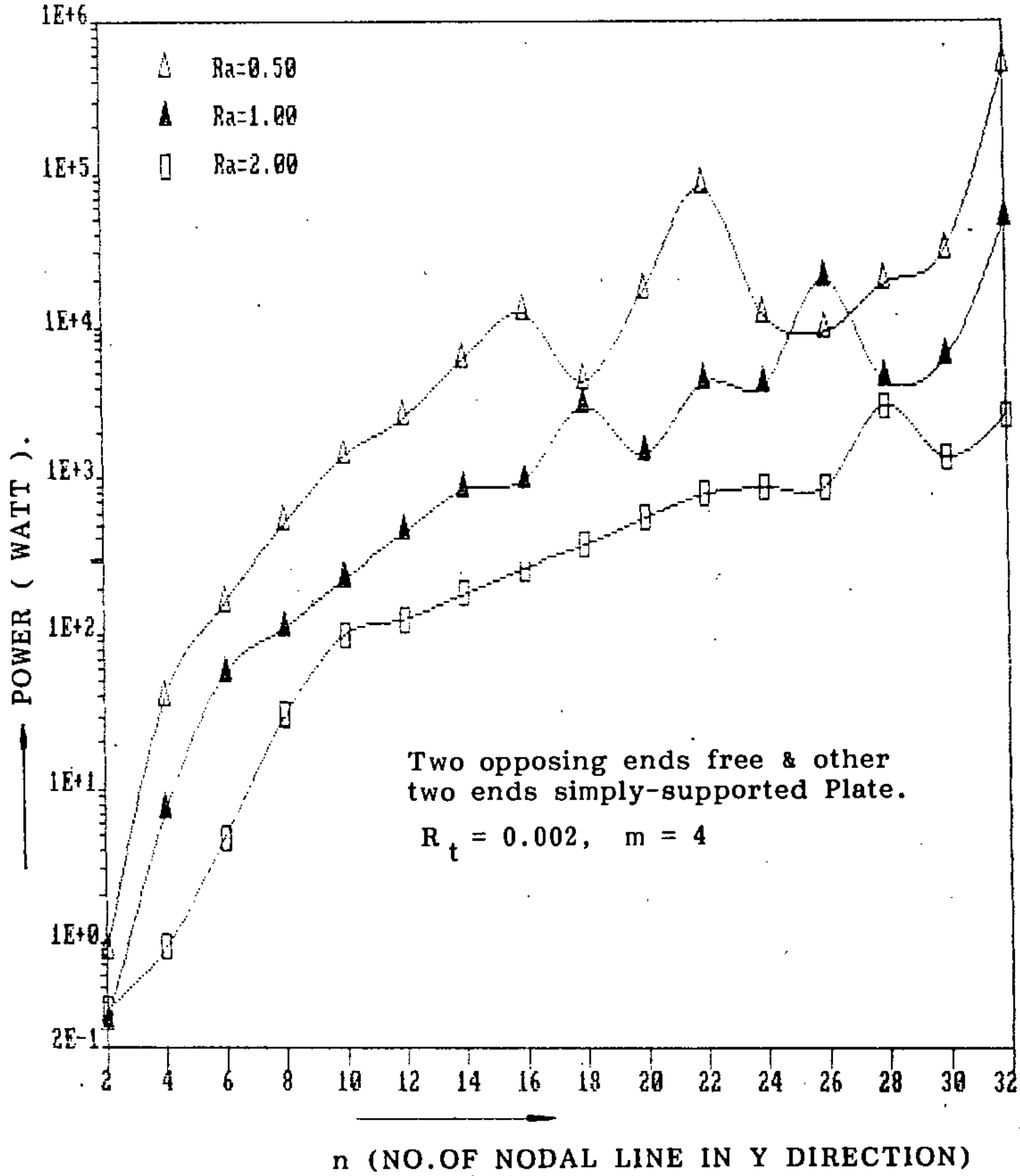


Fig.12: Power radiation from two opposing ends free and other two ends simply-supported Plate for different aspect ratio at low values of m.

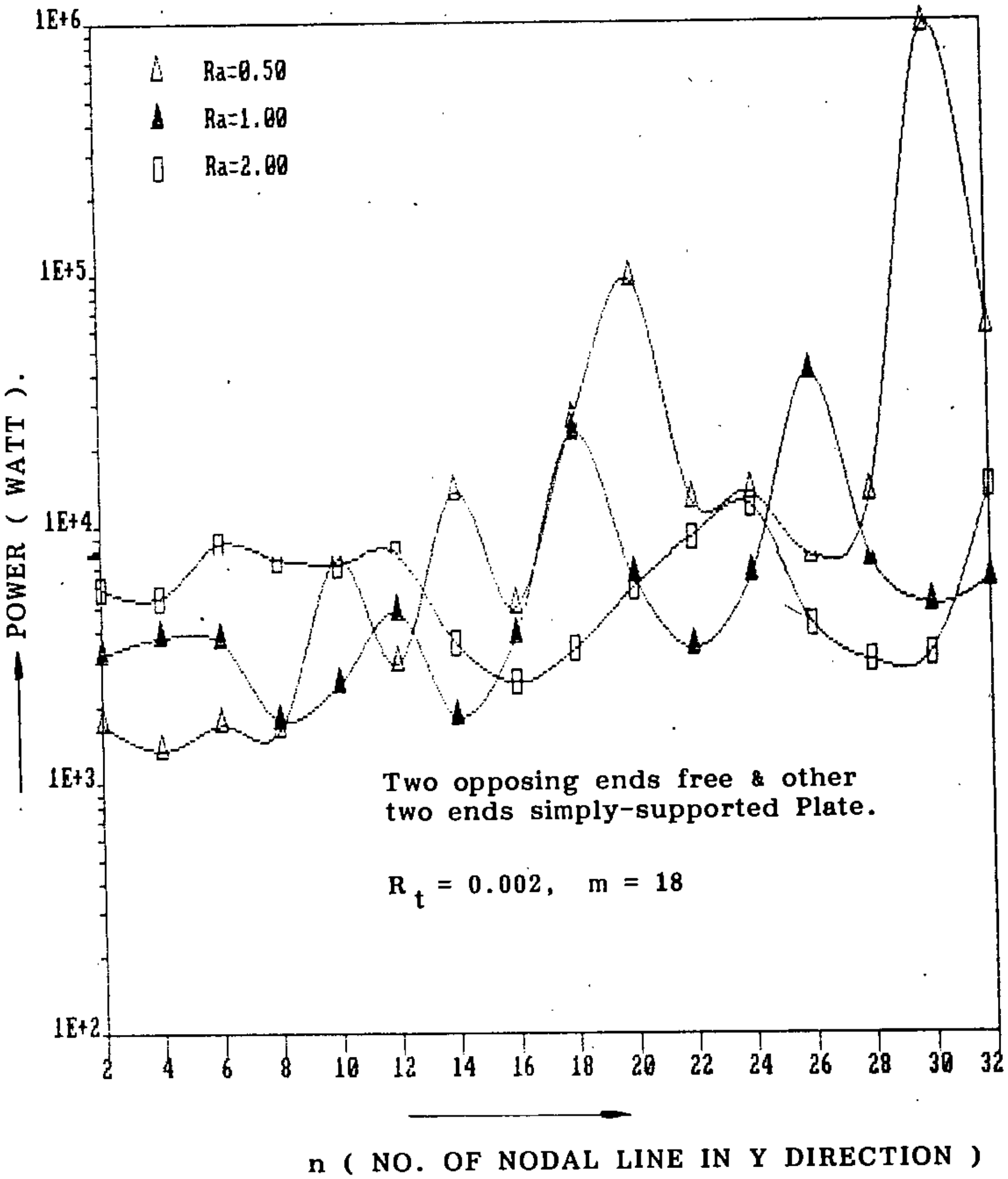


Fig.13: Power radiation from two opposing ends free and other two ends simply-supported Plate for different aspect ratio at high values of m.

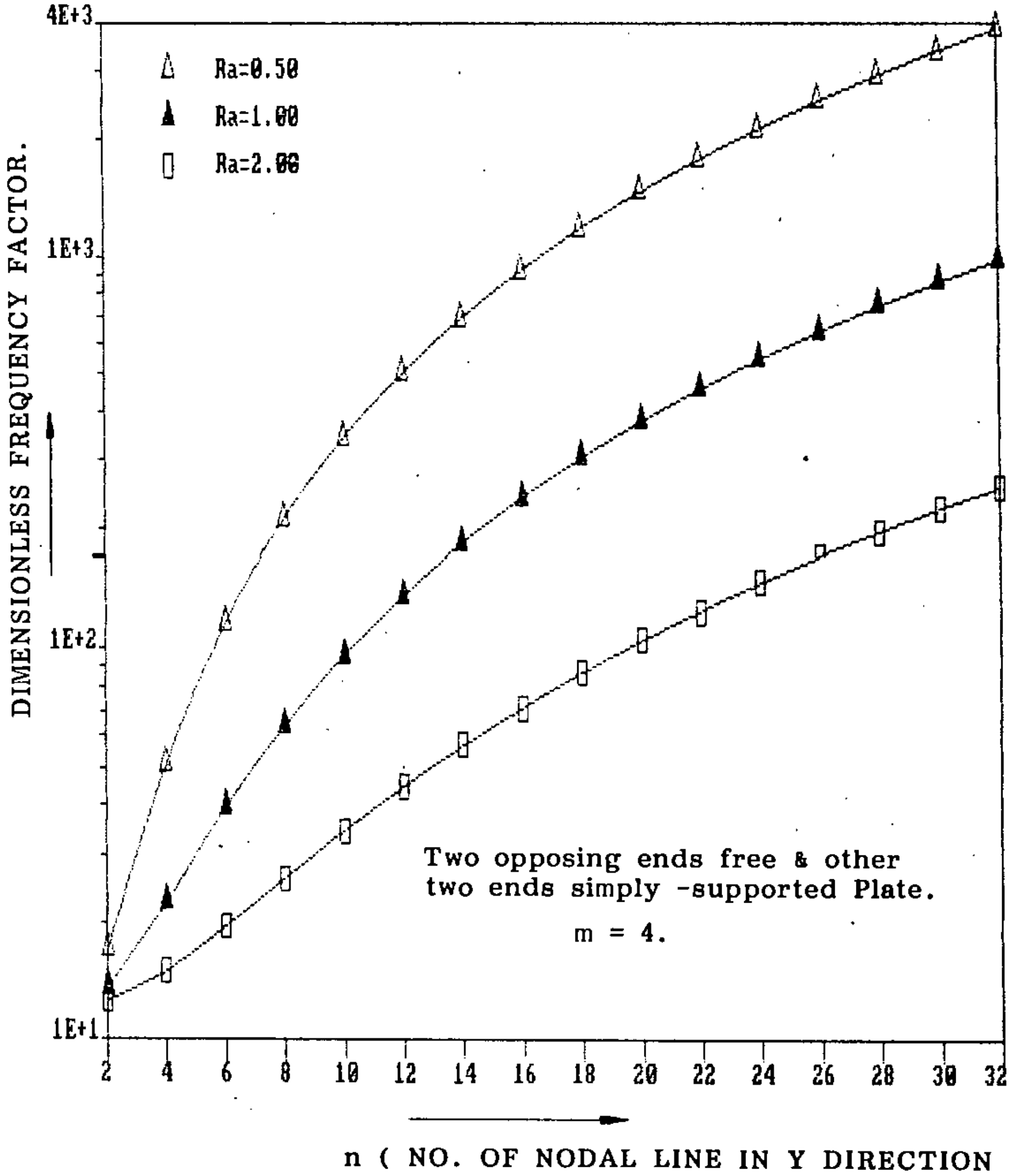


Fig.14: Dimensionless frequency factor for different values of aspect ratio at low values of m of two opposing ends free and other two ends simply-supported Plate.

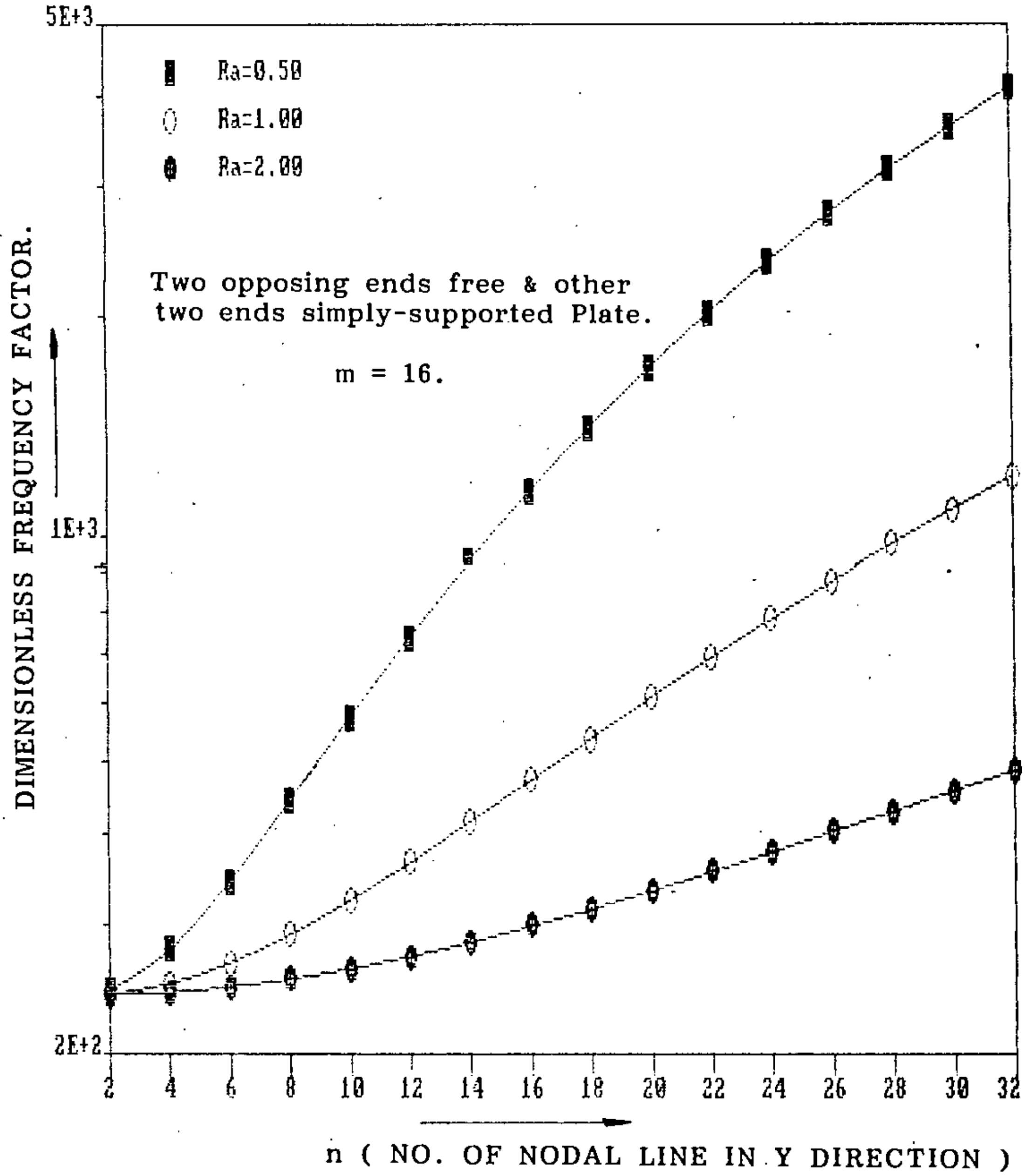


Fig.15: Dimensionless frequency factor for different values of aspect ratio at high of m of two opposing ends free and other two ends simply-supported Plate.

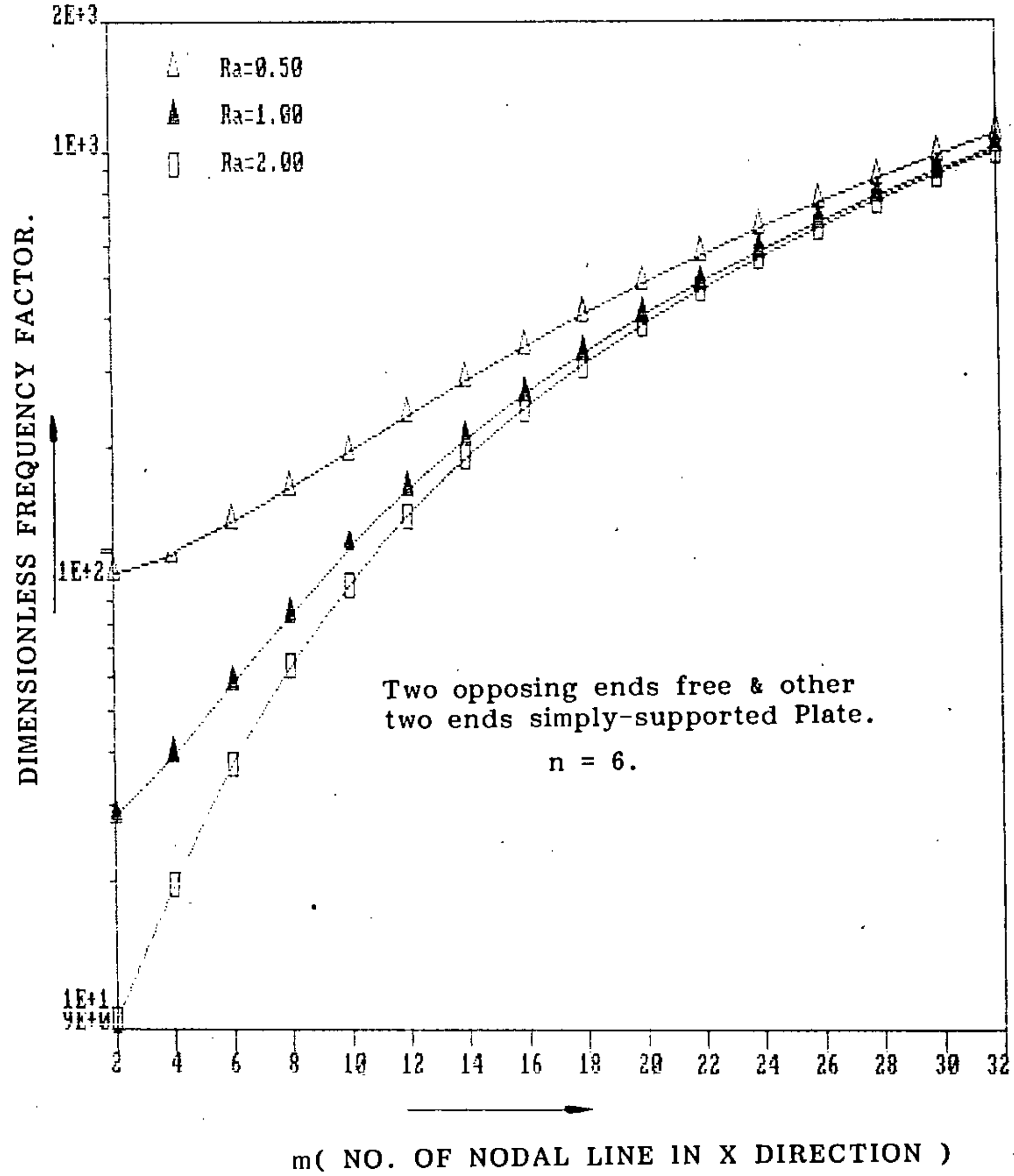


Fig.16: Dimensionless frequency factor for different values of aspect ratio at low values of n of two opposing ends free and other two ends simply-supported Plate.

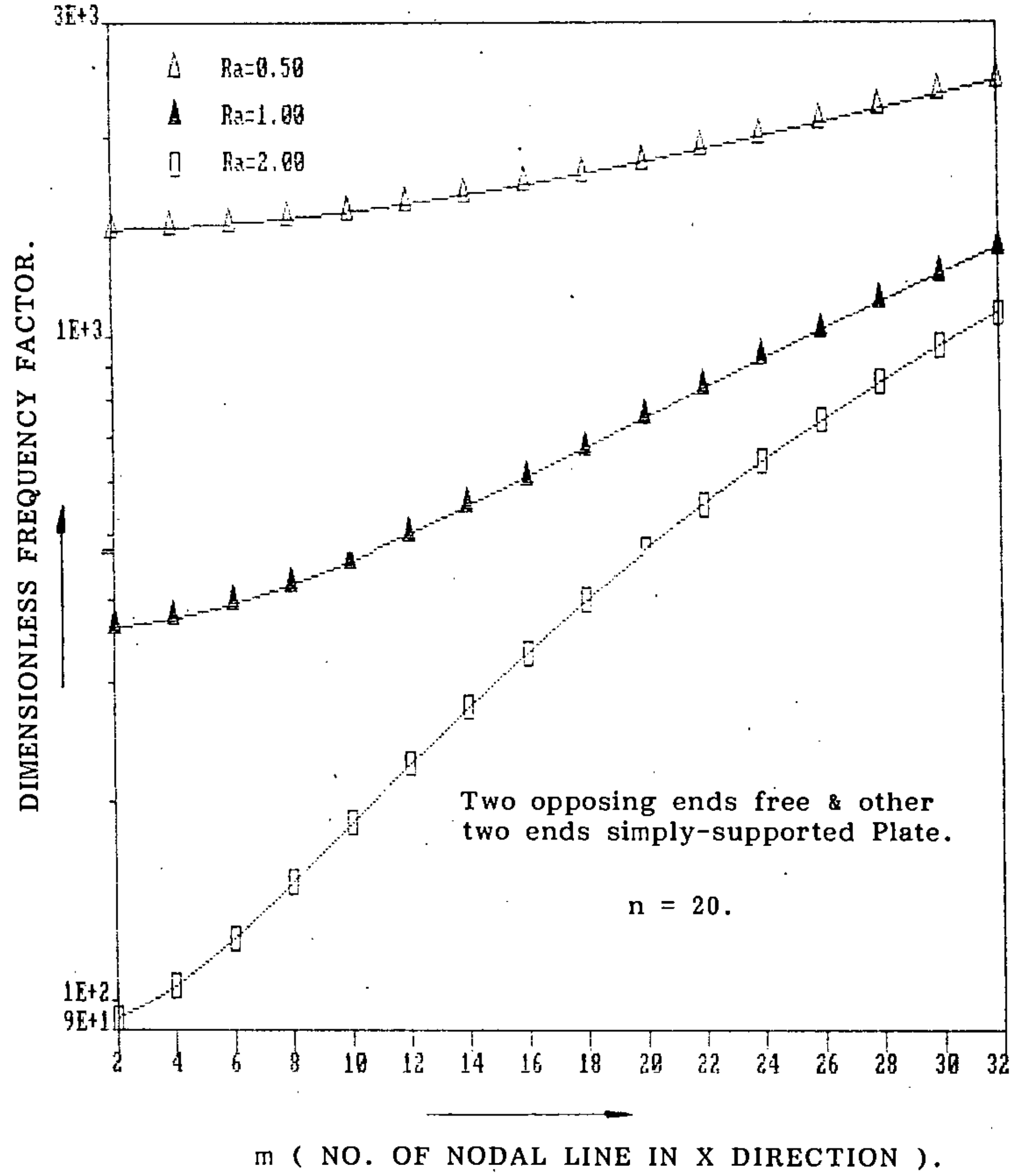


Fig.17: Dimensionless frequency factor for different values of aspect ratio at high values of n of two opposing ends free and other two ends simply-supported Plate.

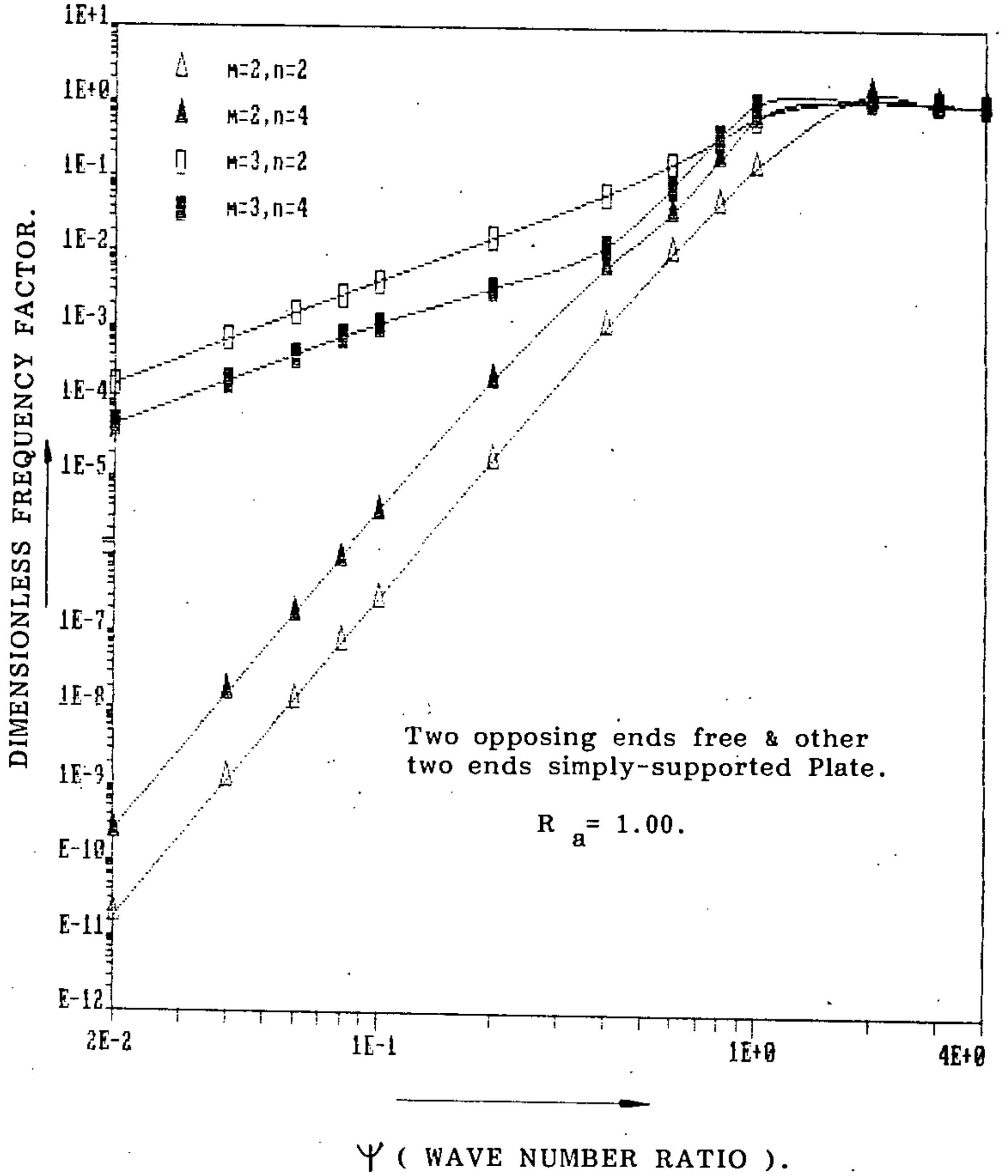


Fig.18: Radiation efficiency for two opposing ends free and other two ends simply-supported plate at low-low mode orders.

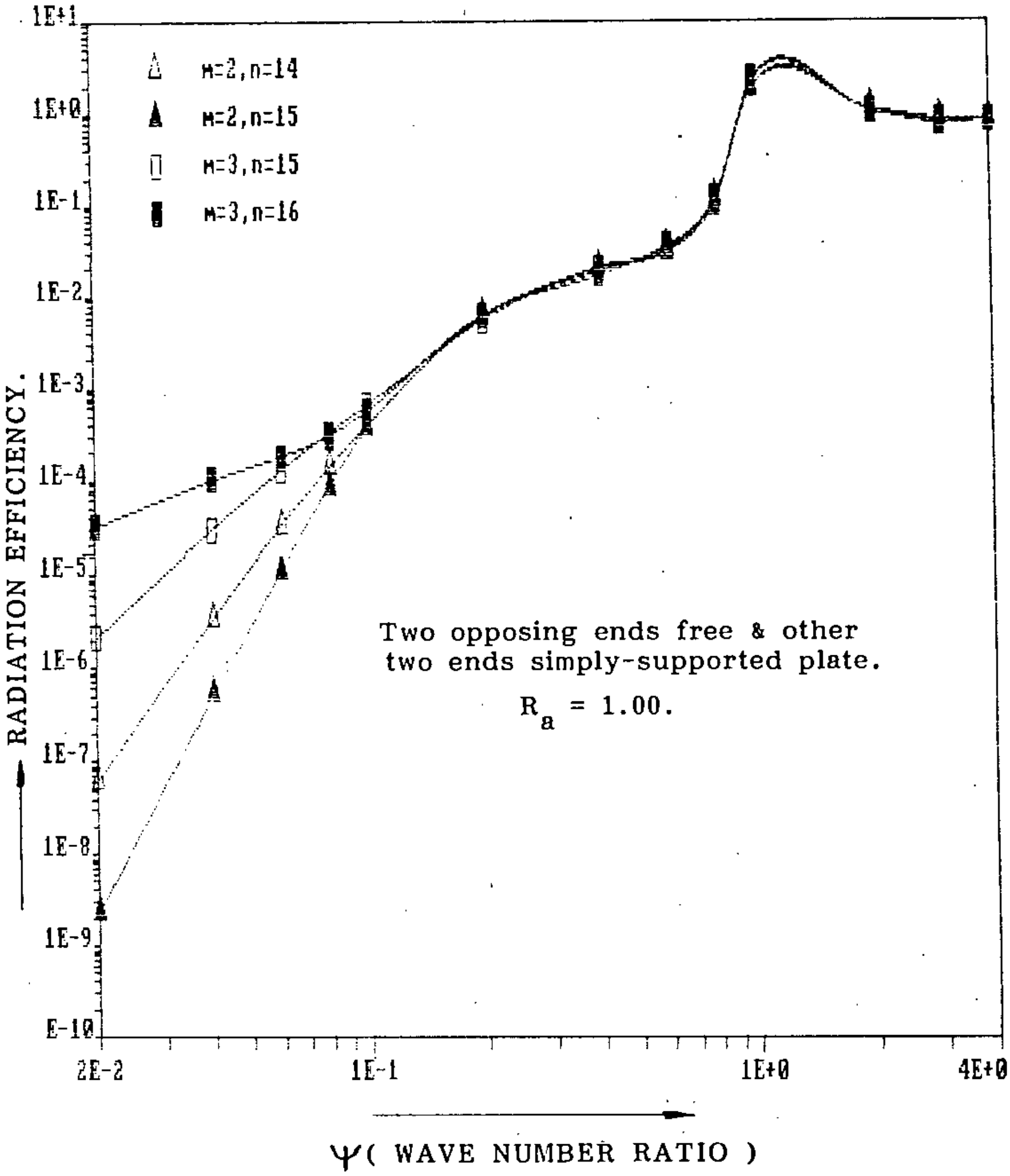


Fig.19: Radiation efficiency for two opposing ends free and other two ends simply-supported plate at low-high mode orders.

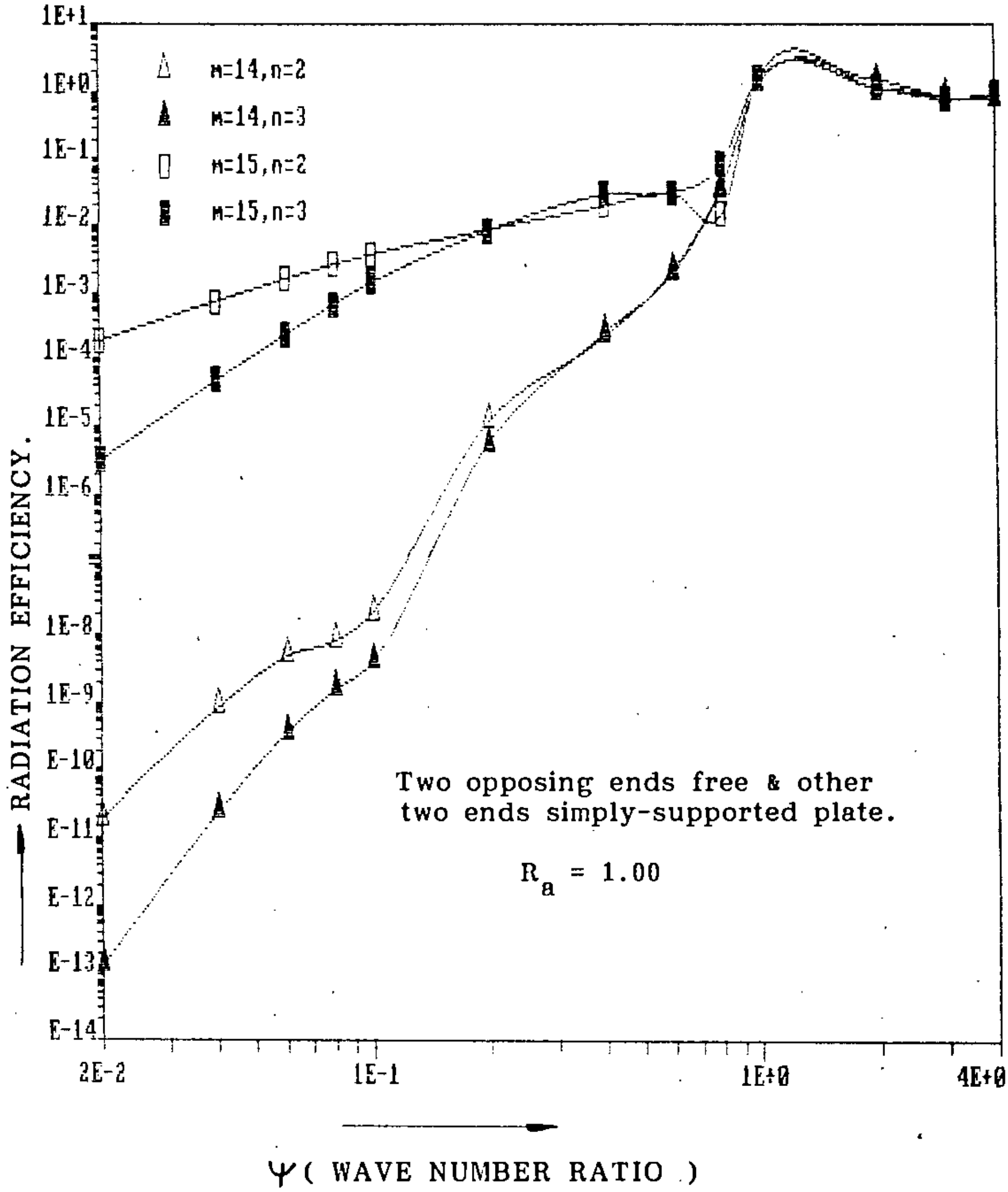


Fig.20: Radiation efficiency for two opposing ends free and other two ends simply-supported plate at high-low mode orders.

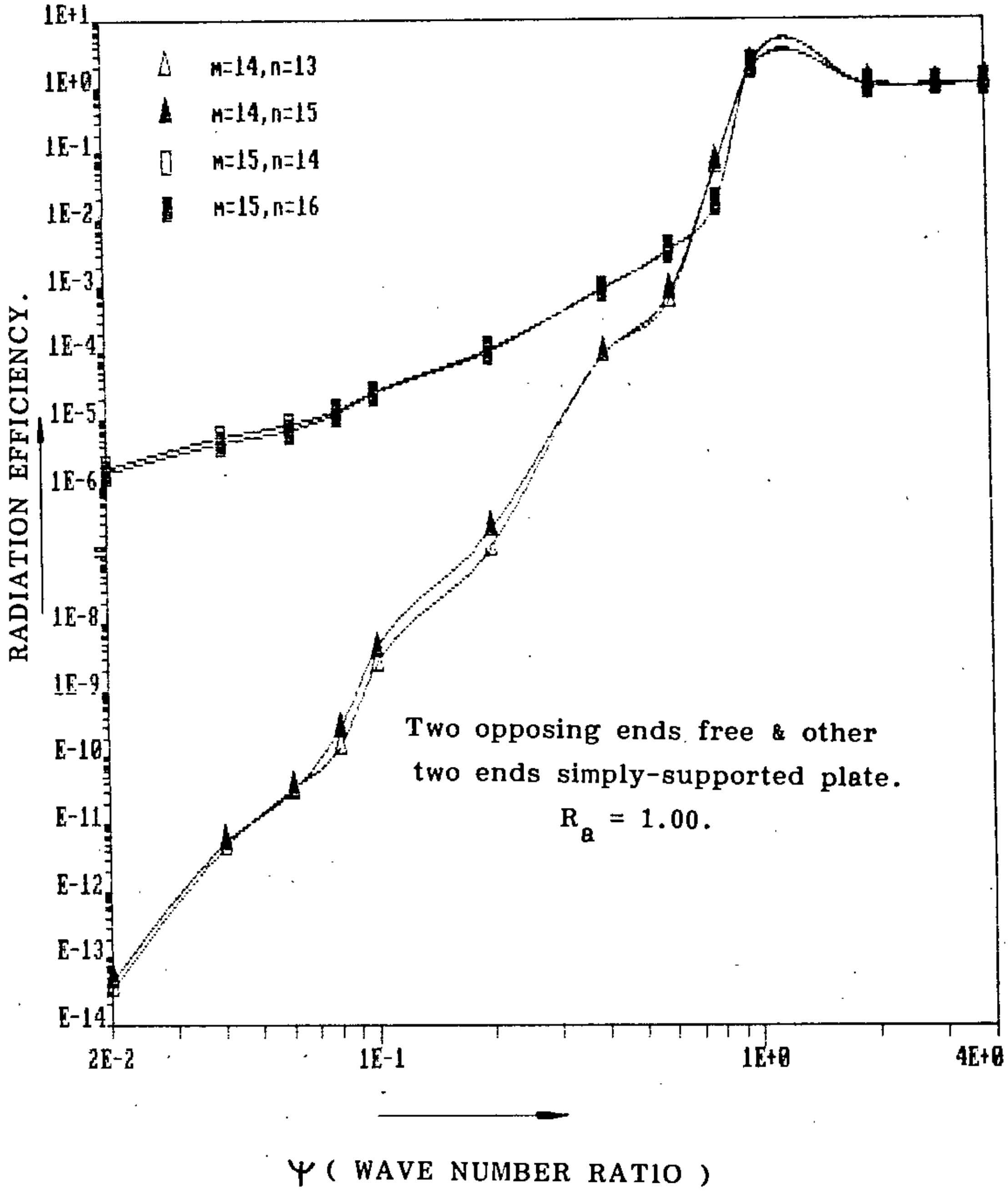


Fig.21: Radiation efficiency for two opposing ends free and other two ends simply- supported plate at high-high mode orders.

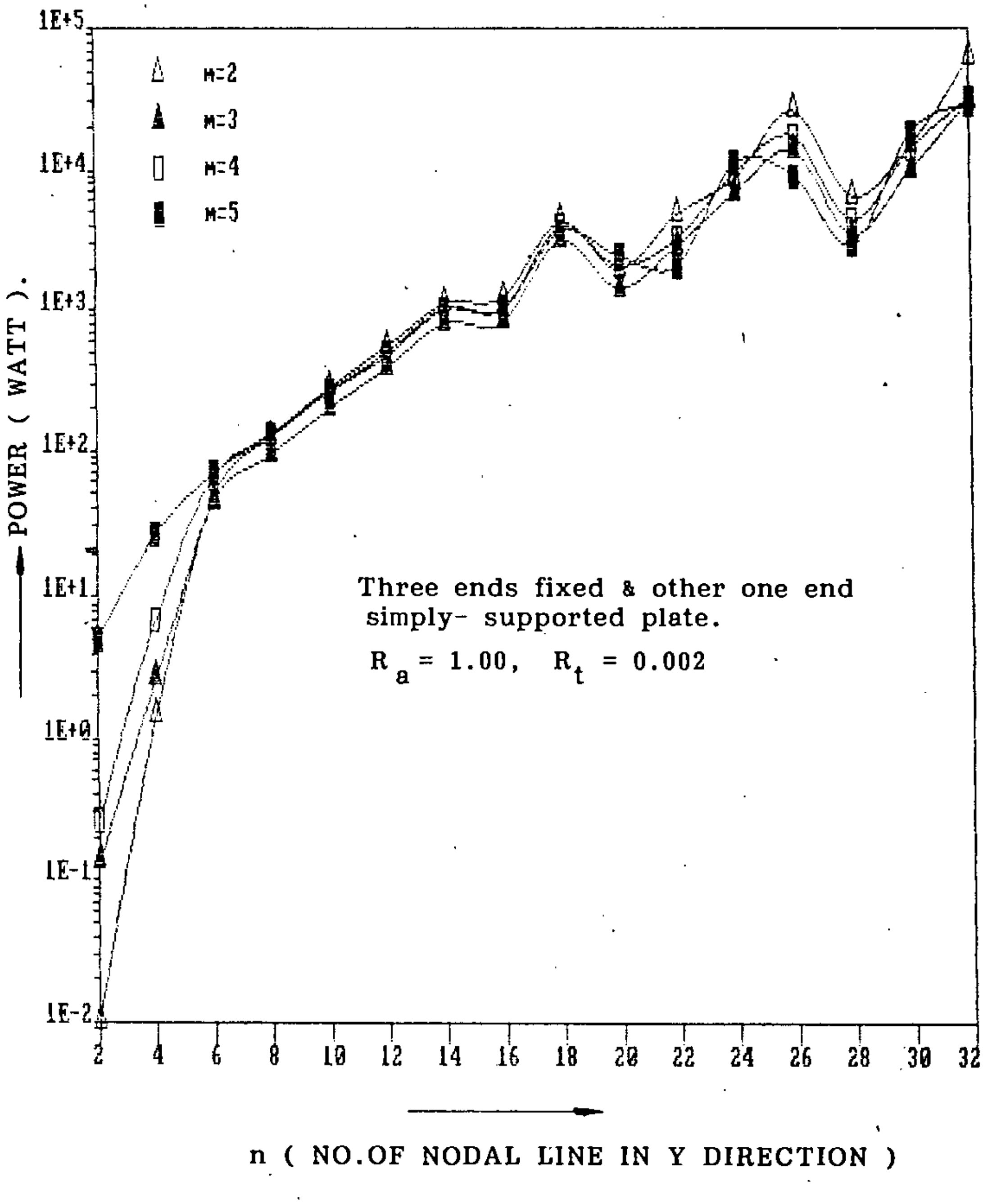


Fig.22: Power radiation from three ends fixed and other one end simply-supported plate at low values of m.

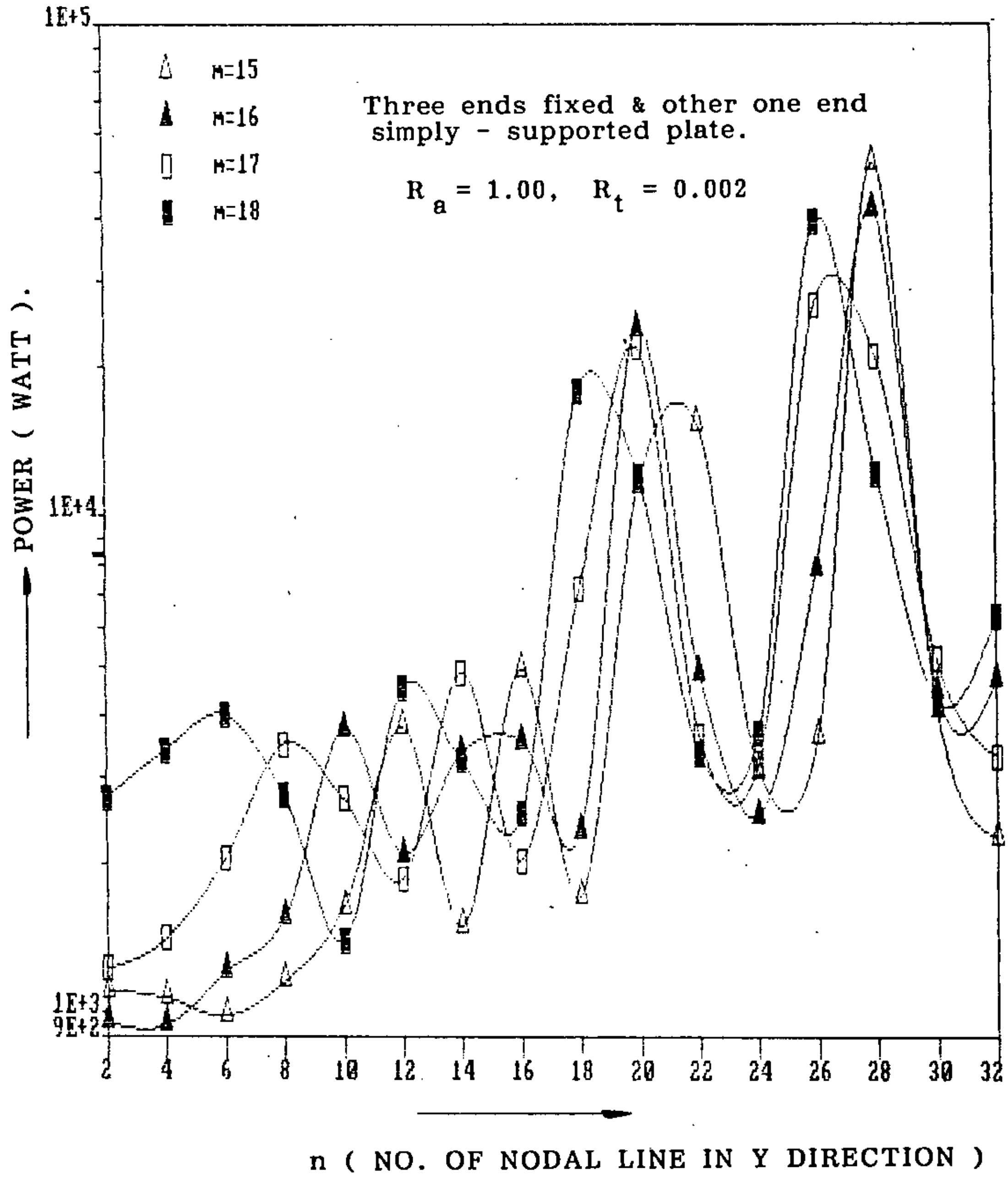


Fig.23: Power radiation from three ends fixed and other one end simply- supported plate at high values of m.

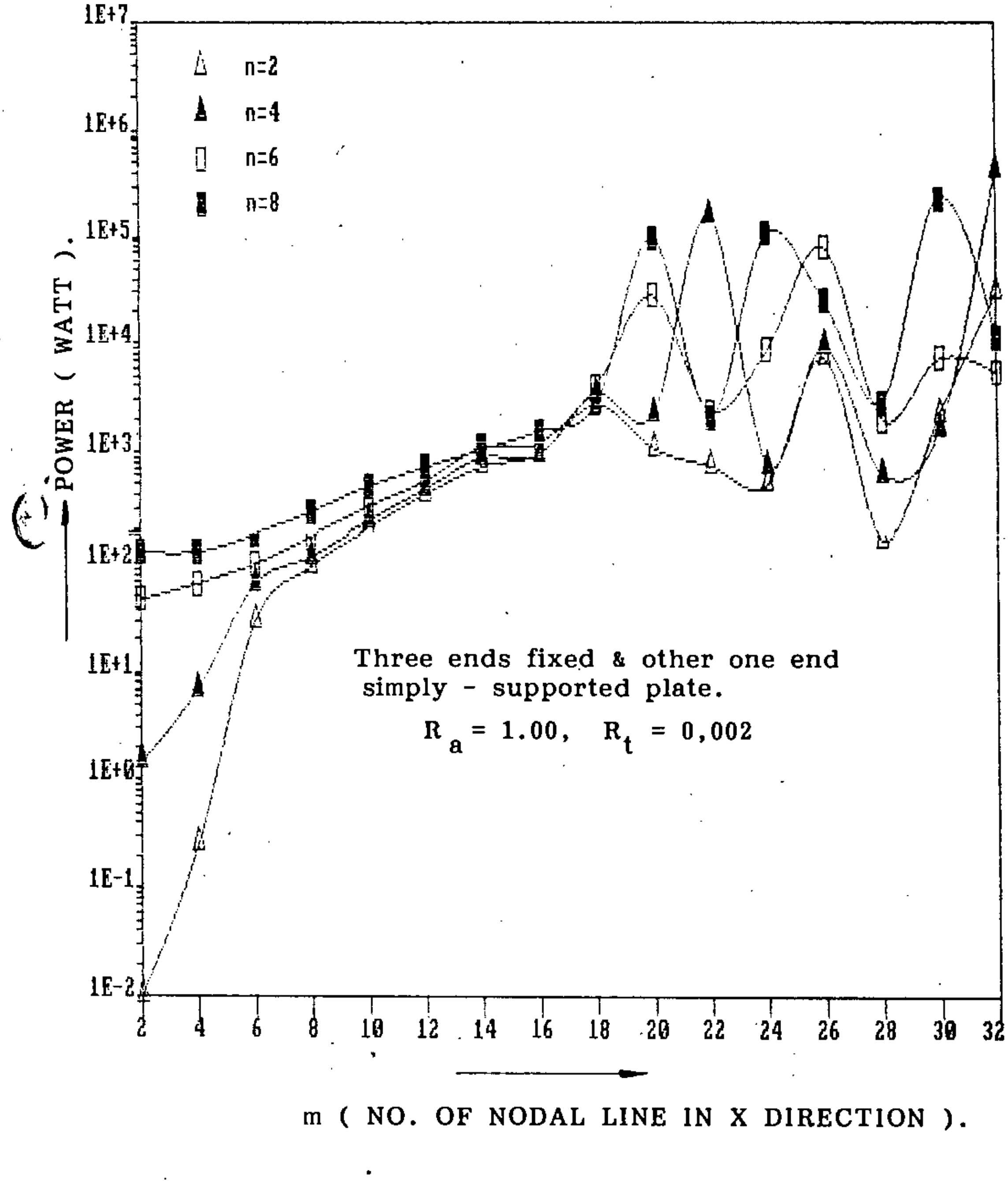


Fig.24: Power radiation from three ends fixed and other one end simply - supported plate at low values of n.

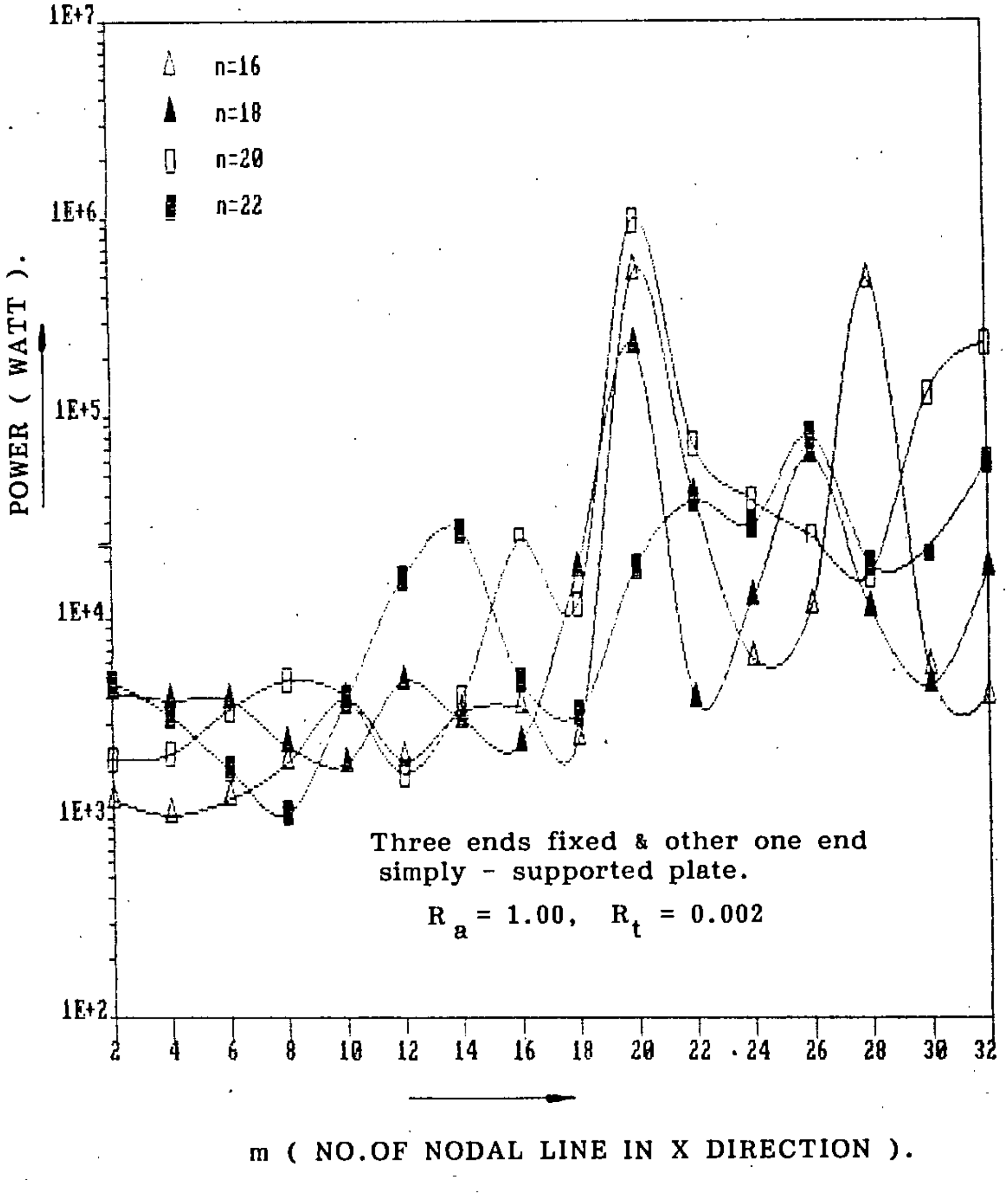


Fig.25: Power radiation from three ends fixed and other one end simply - supported plate at high values of n.

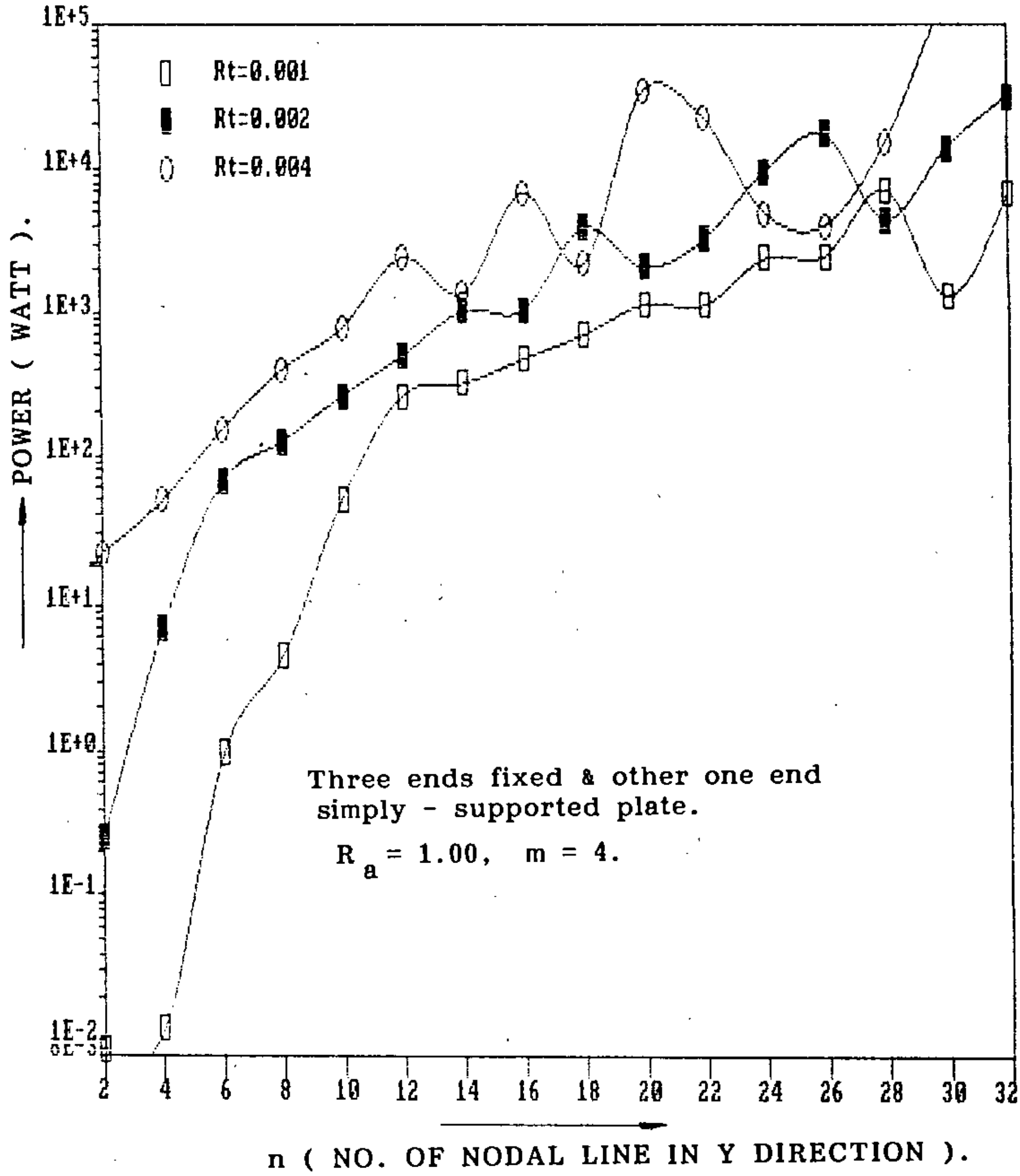


Fig.26: Power radiation from three ends fixed and other one end simply - supported plate for different thickness ratio at low values of m.

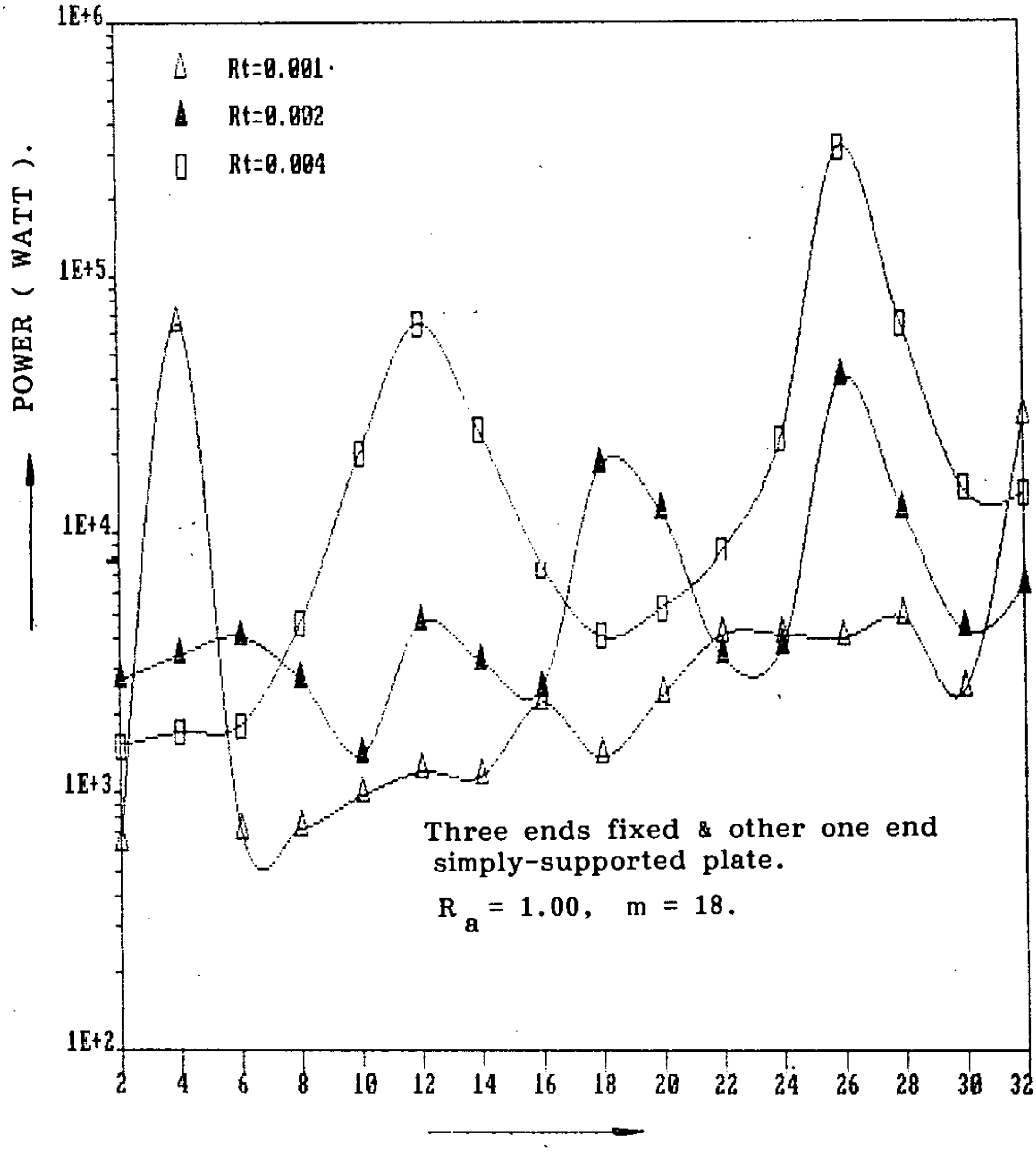


Fig.27: Power radiation from three ends fixed and other one end simply-supported plate for different thickness ratio at high values of m.

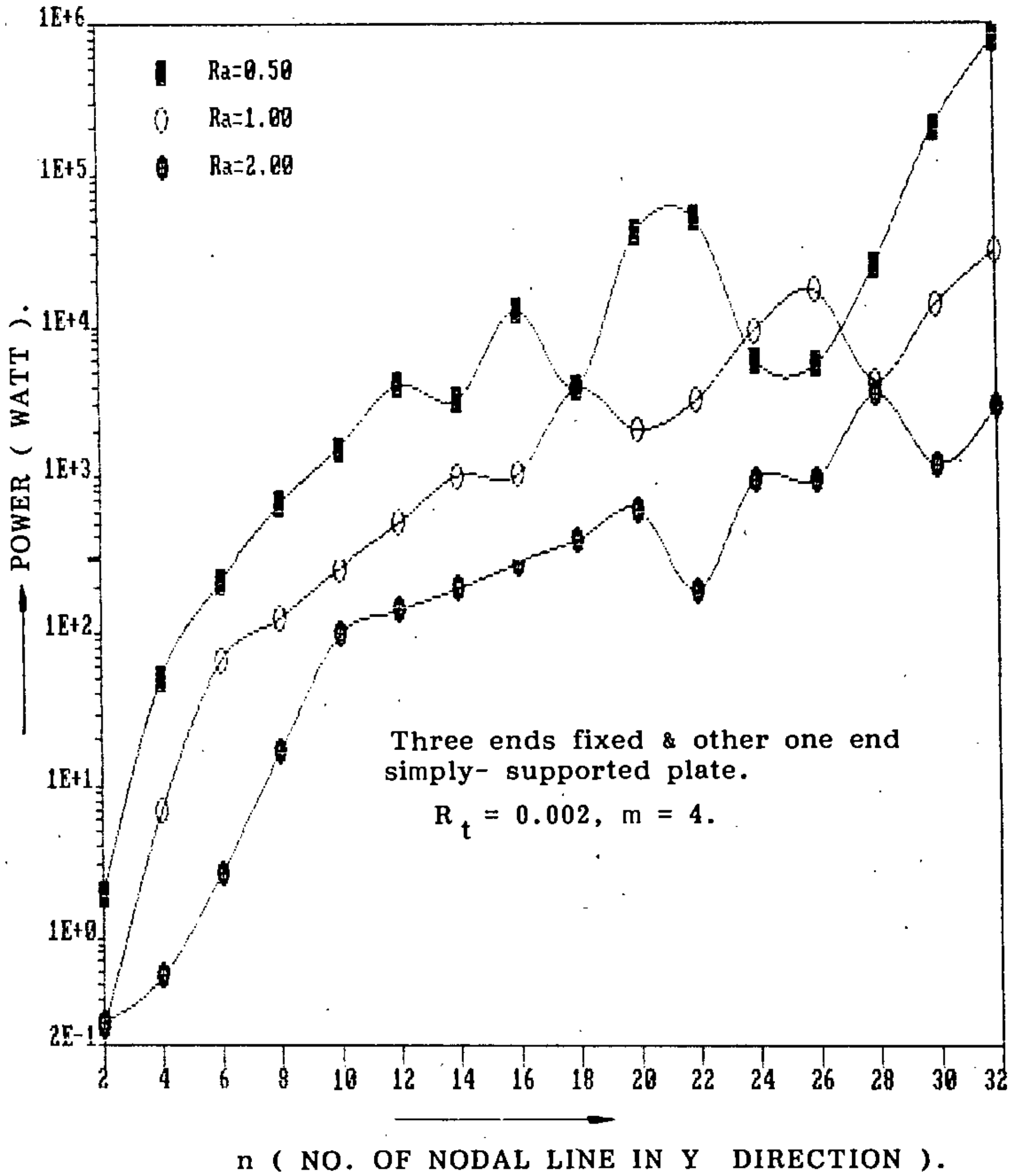


Fig.28: Power radiation from three ends fixed and other one end simply-supported plate for different aspect ratio at low values of m.

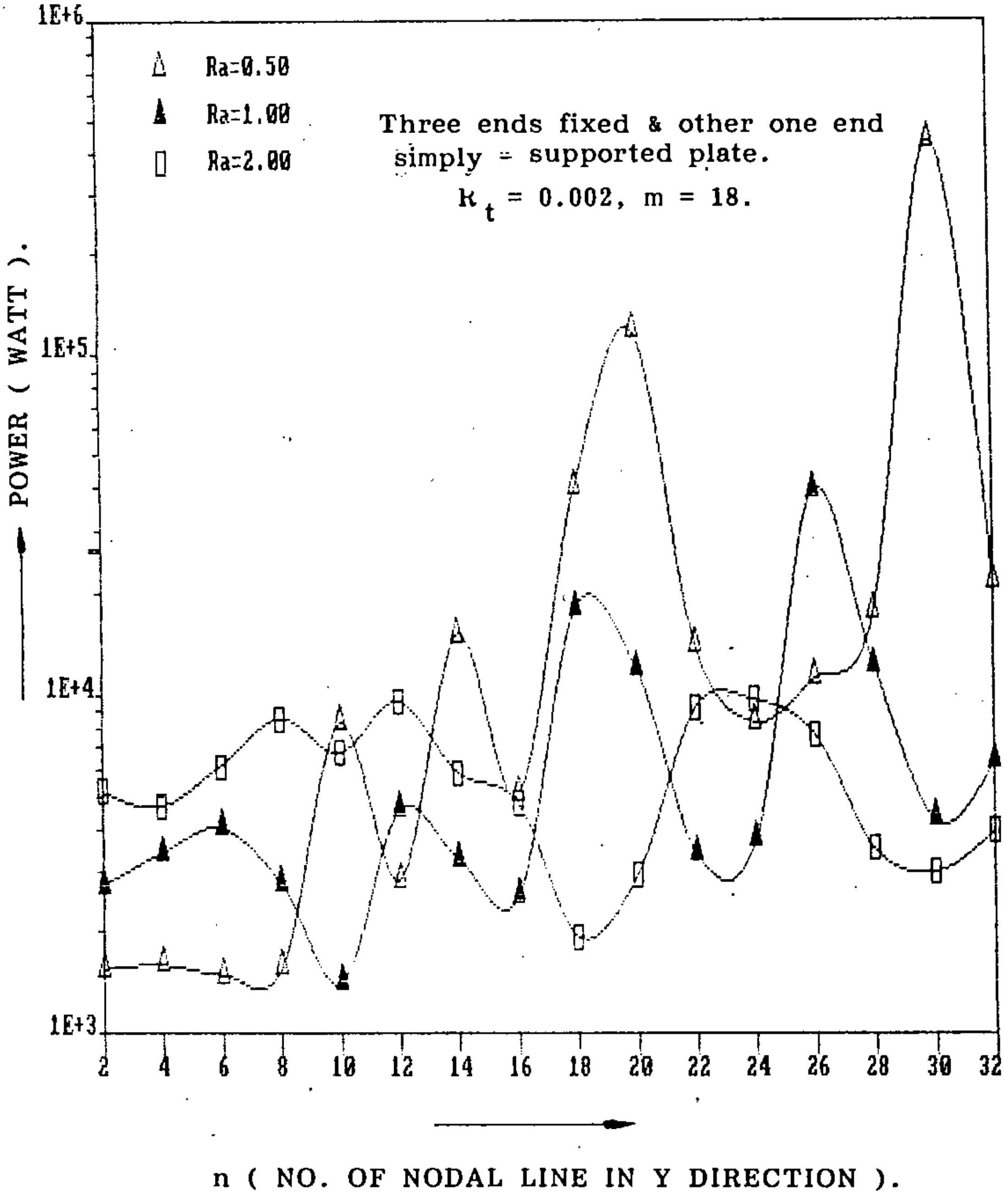


Fig.29: Power radiation from three ends fixed and other one end simply-supported plate for different aspect ratio at high values of m.

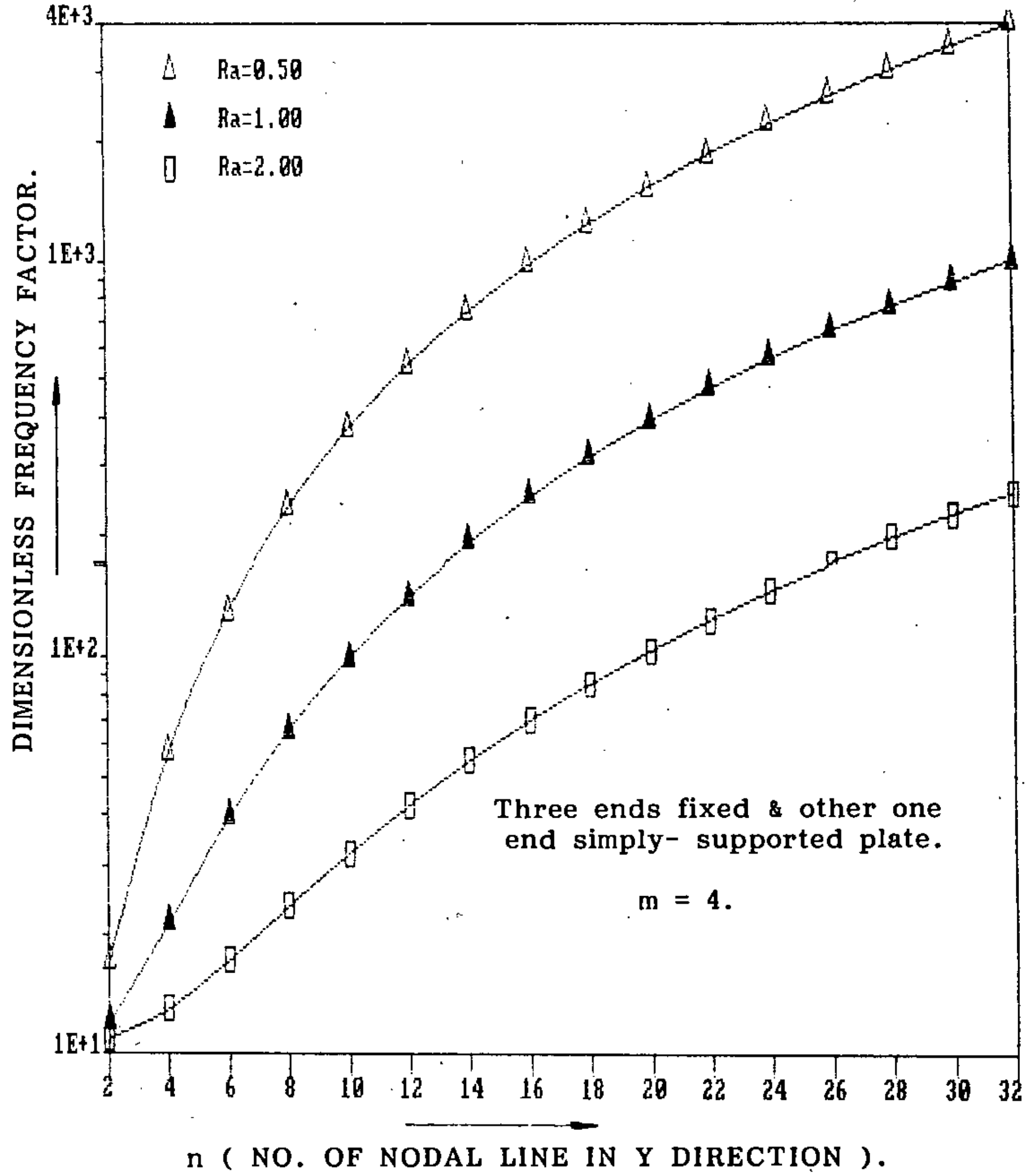


Fig.30: Dimensionless frequency factor for different values of aspect ratio at low values of m of three ends fixed and other one end simply- supported plate.

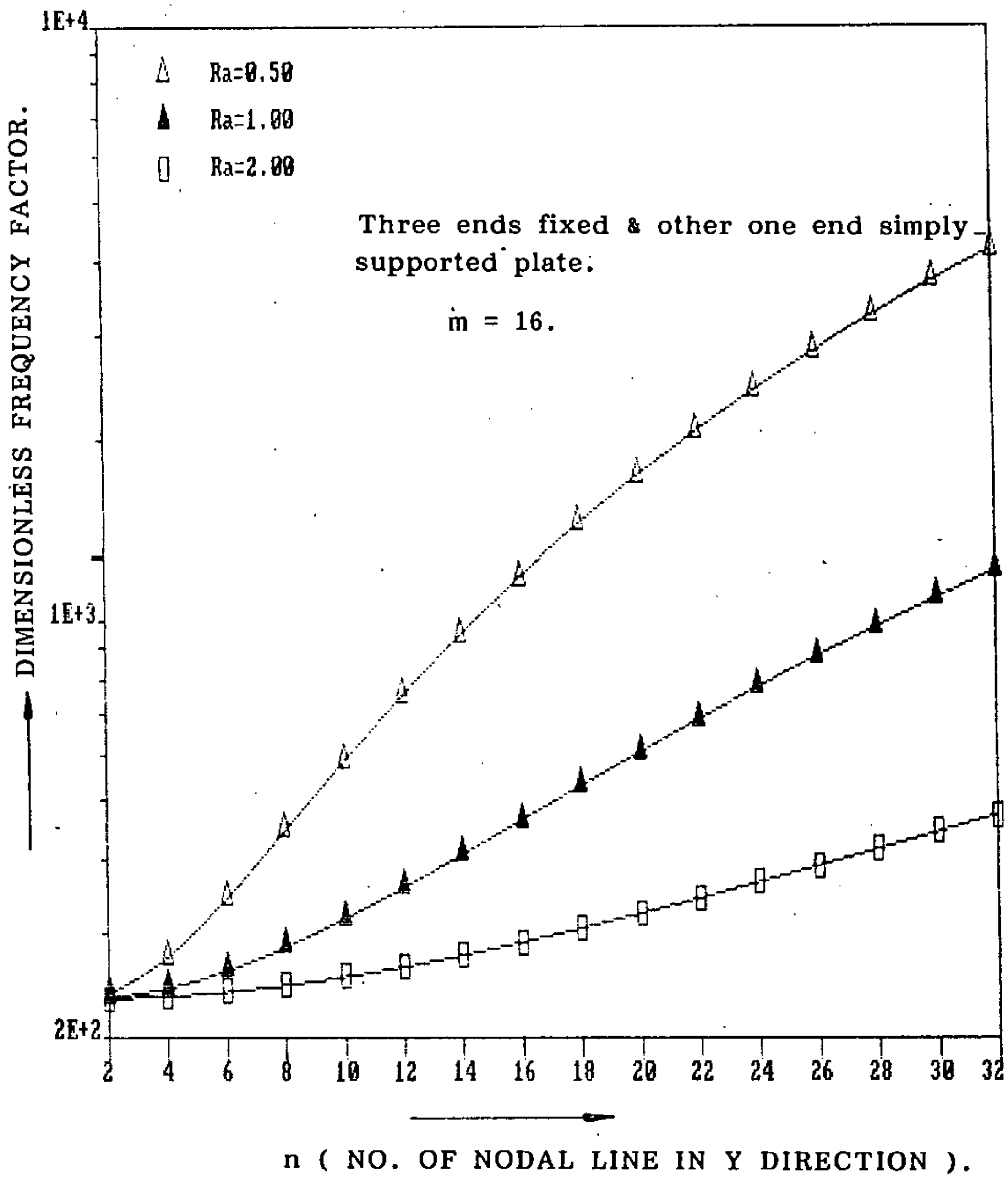


Fig.31: Dimensionless frequency factor for different values of aspect ratio at high values of m of three ends fixed and other one end simply - supported plate.

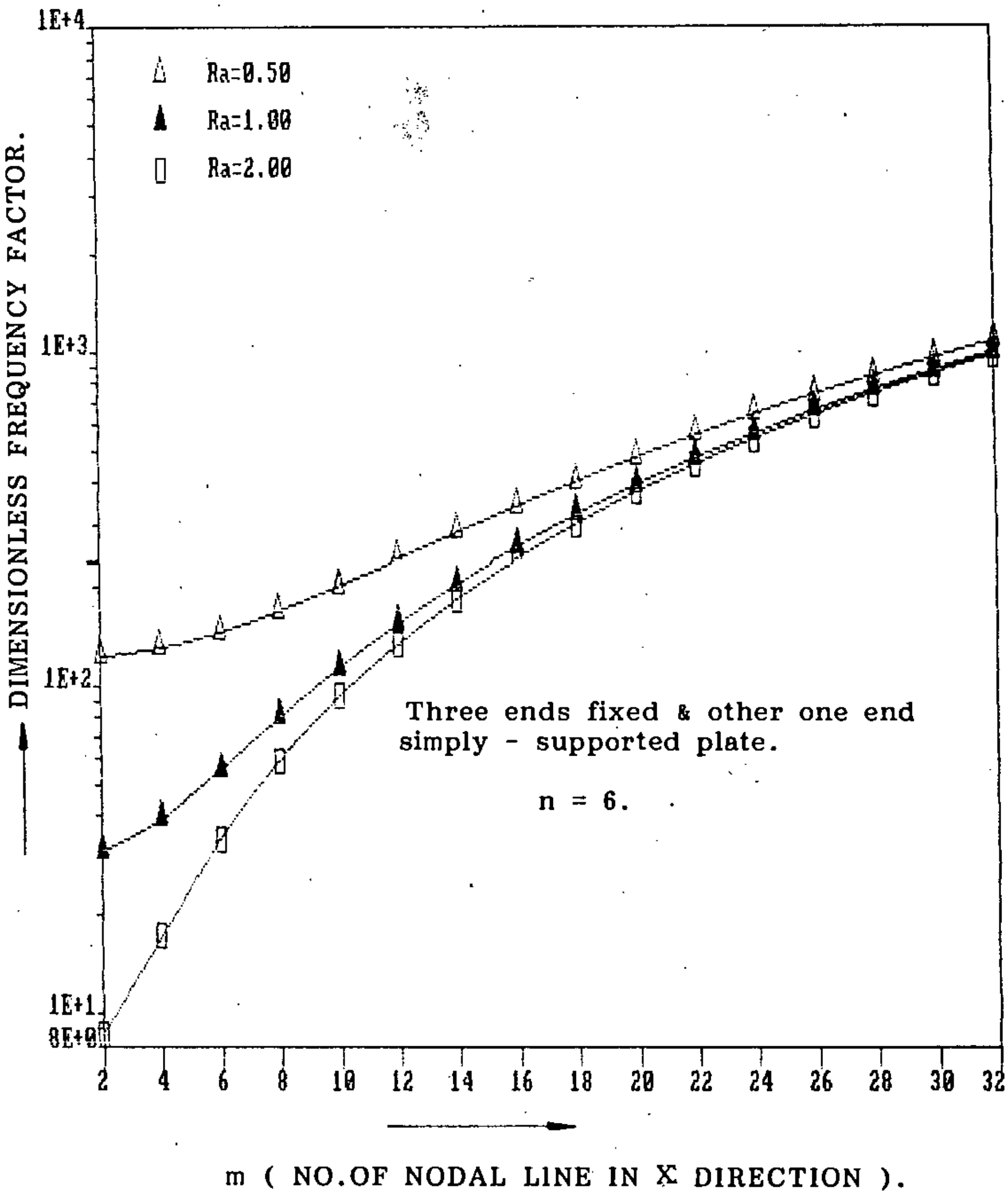


Fig. 32: Dimensionless frequency factor for different values of aspect ratio at low values of n of three ends fixed and other one end simply - supported plate.

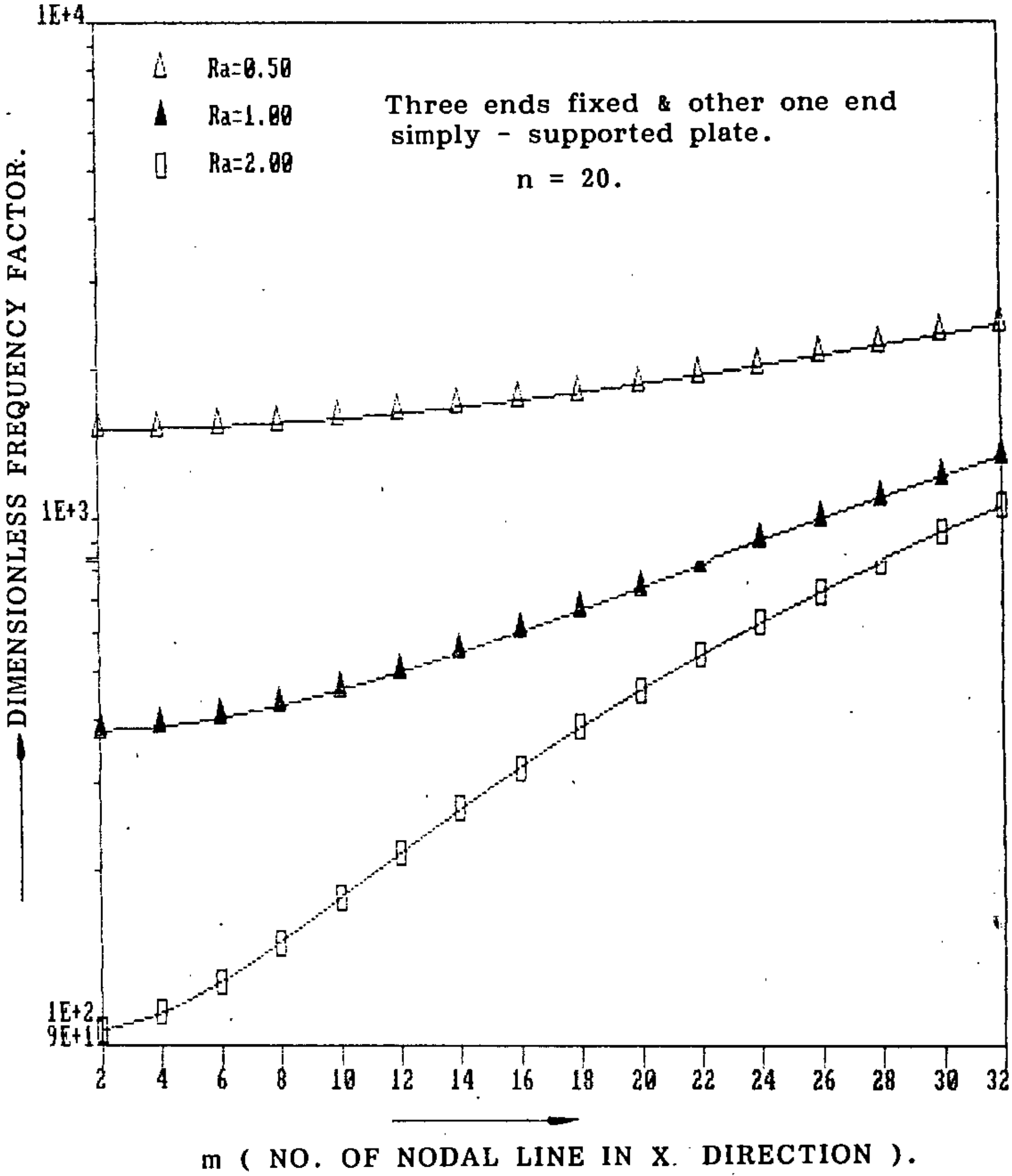


Fig.33: Dimensionless frequency factor for different values of aspect ratio at high values of n of three ends fixed and other one end simply - supported plate.

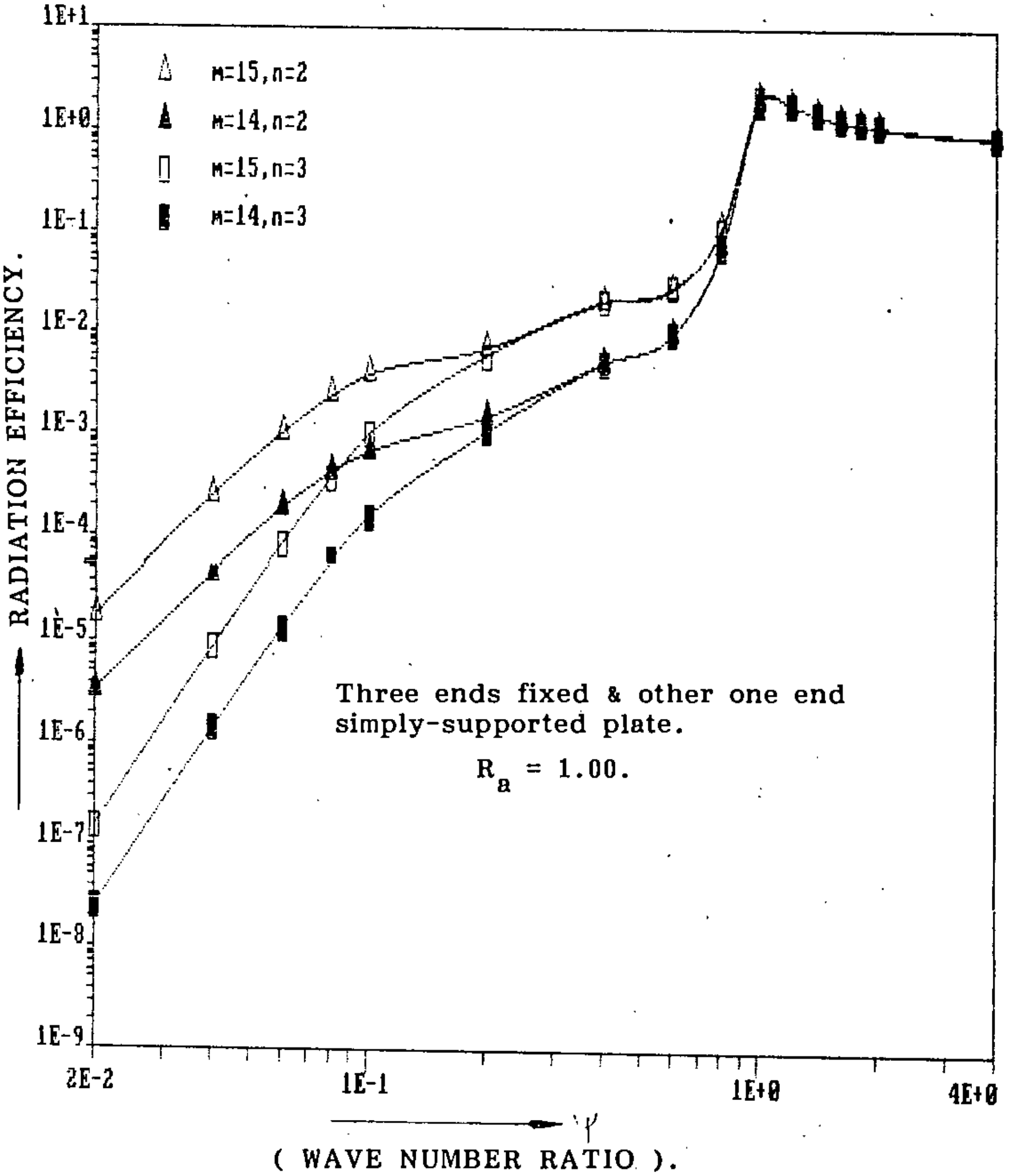


Fig.34: Radiation efficiency for three ends fixed and other one end simply-supported plate at high-low mode orders.

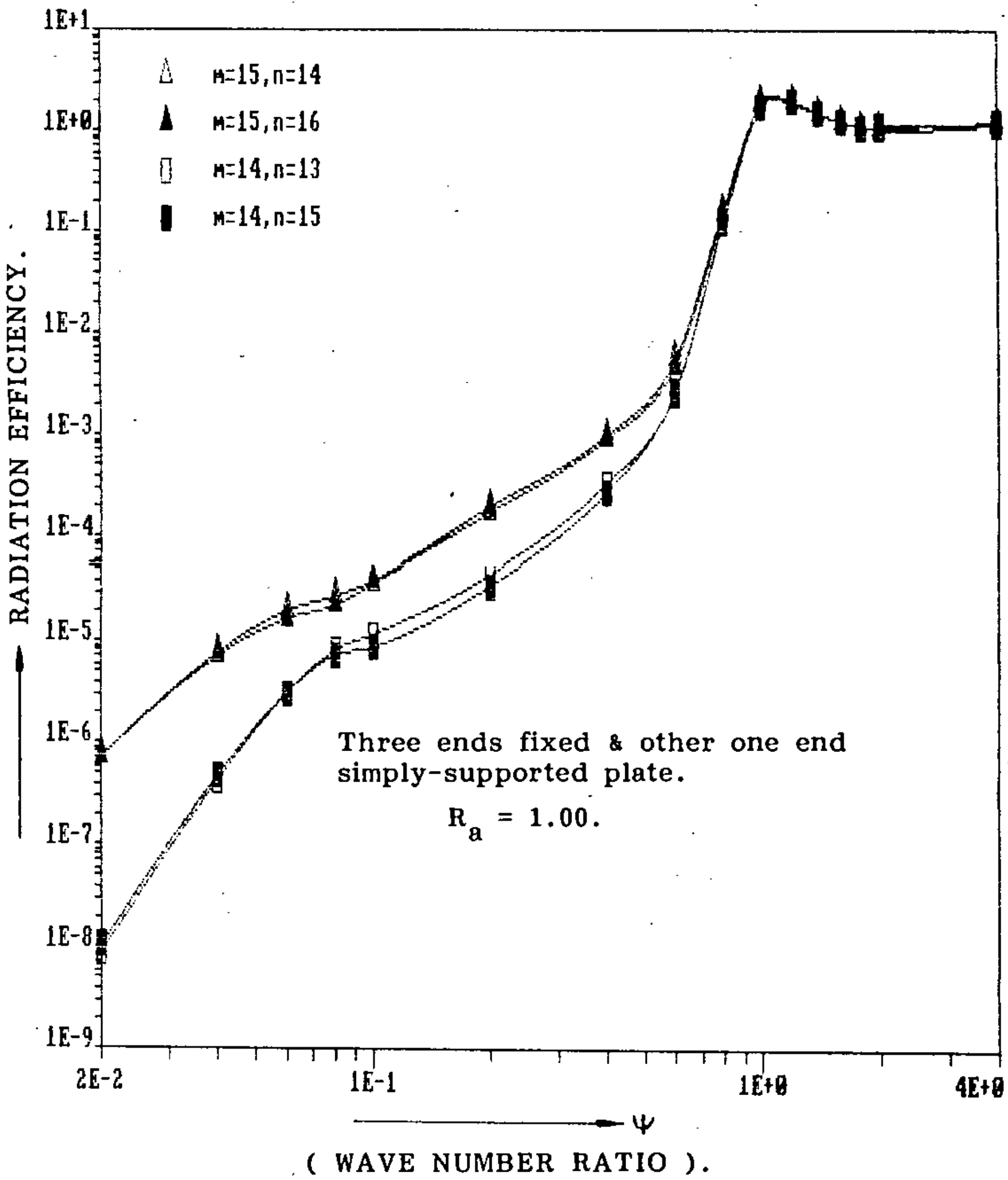


Fig.35: Radiation efficiency for three ends fixed and other one end simply-supported plate at high-high mode orders.

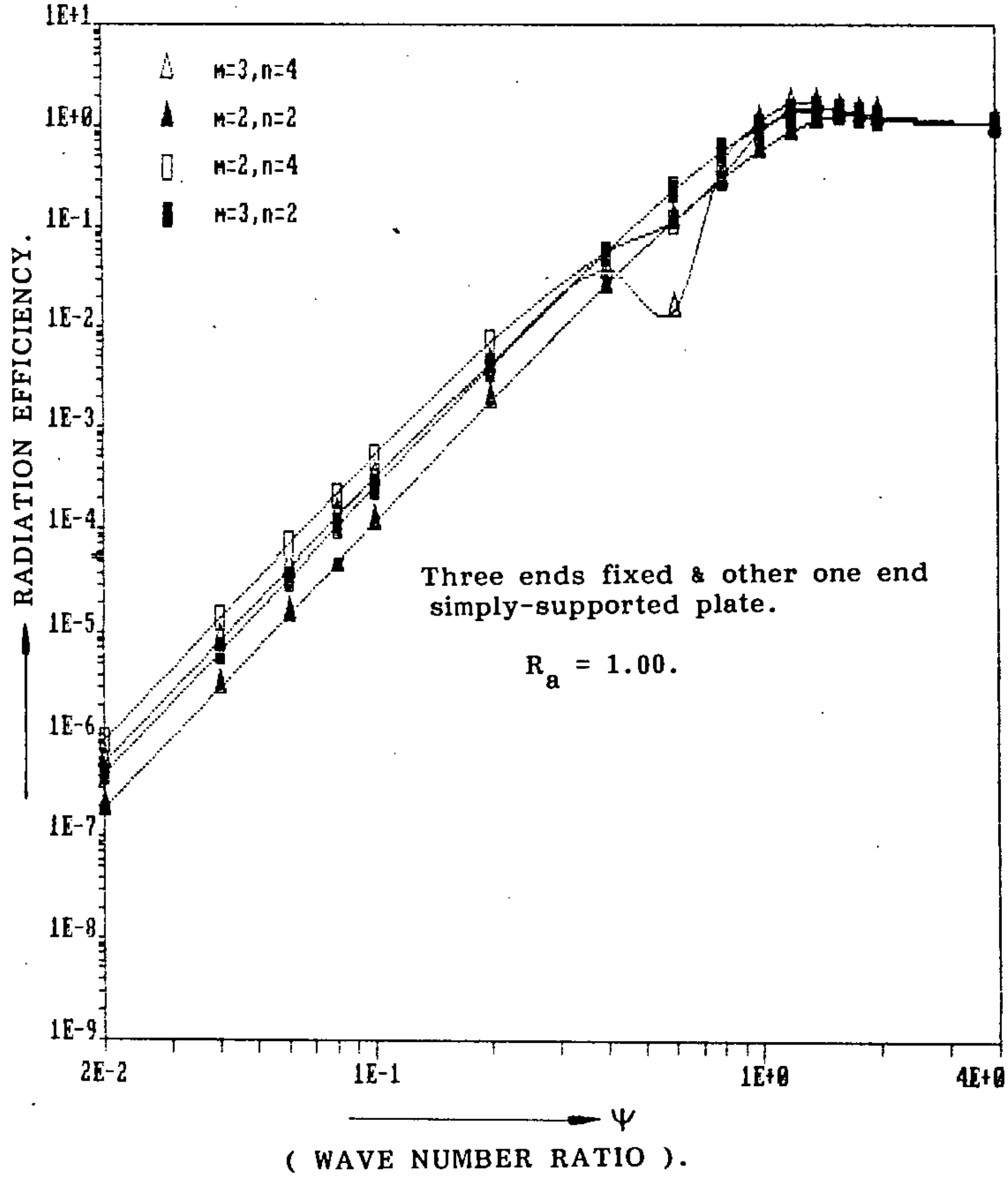


Fig.36. Radiation efficiency for three ends fixed and other one end simply - supported plate at low-low mode orders.

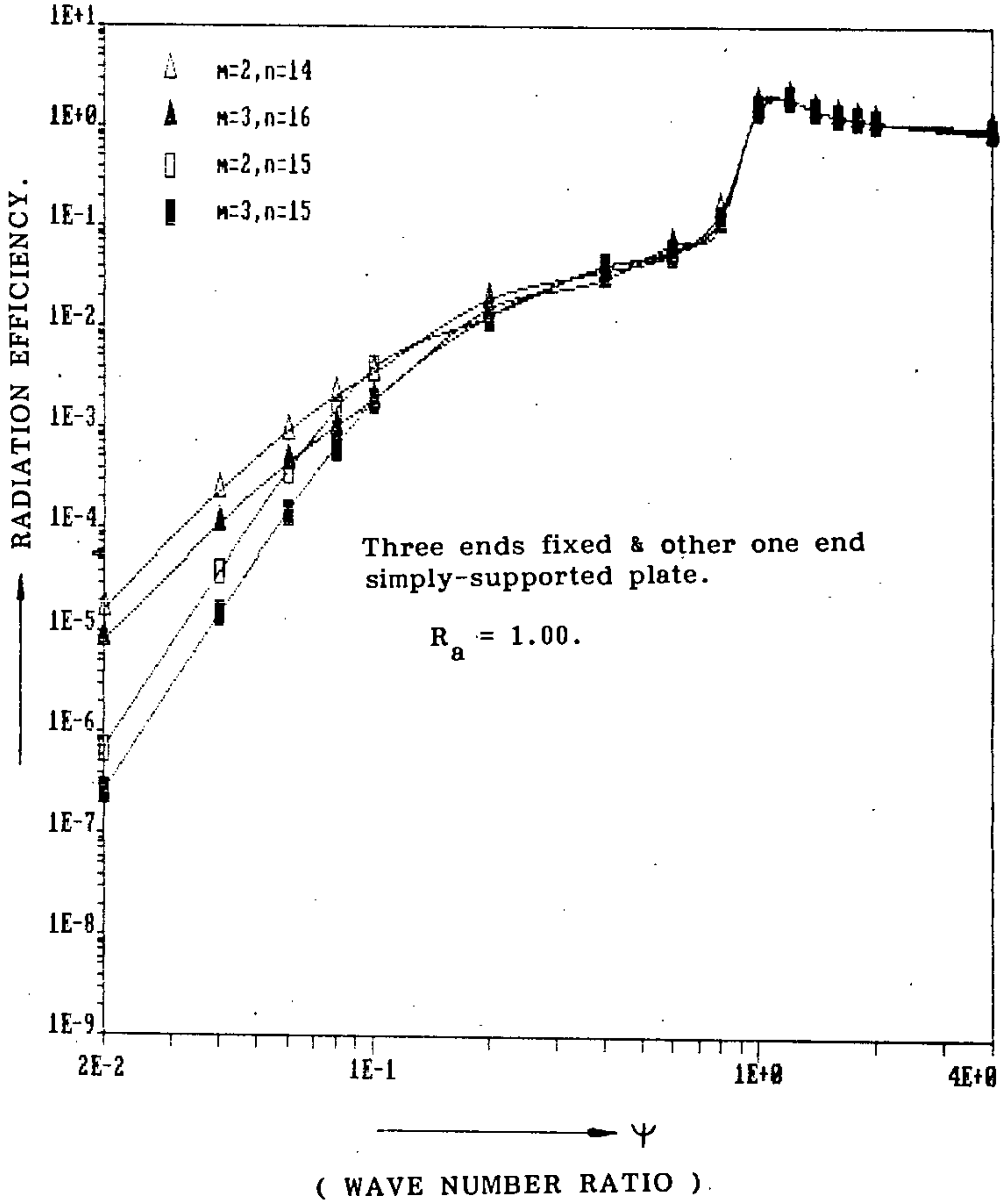


Fig.37: Radiation efficiency σ for three ends fixed and other one end simply-supported plate at low-high mode orders.

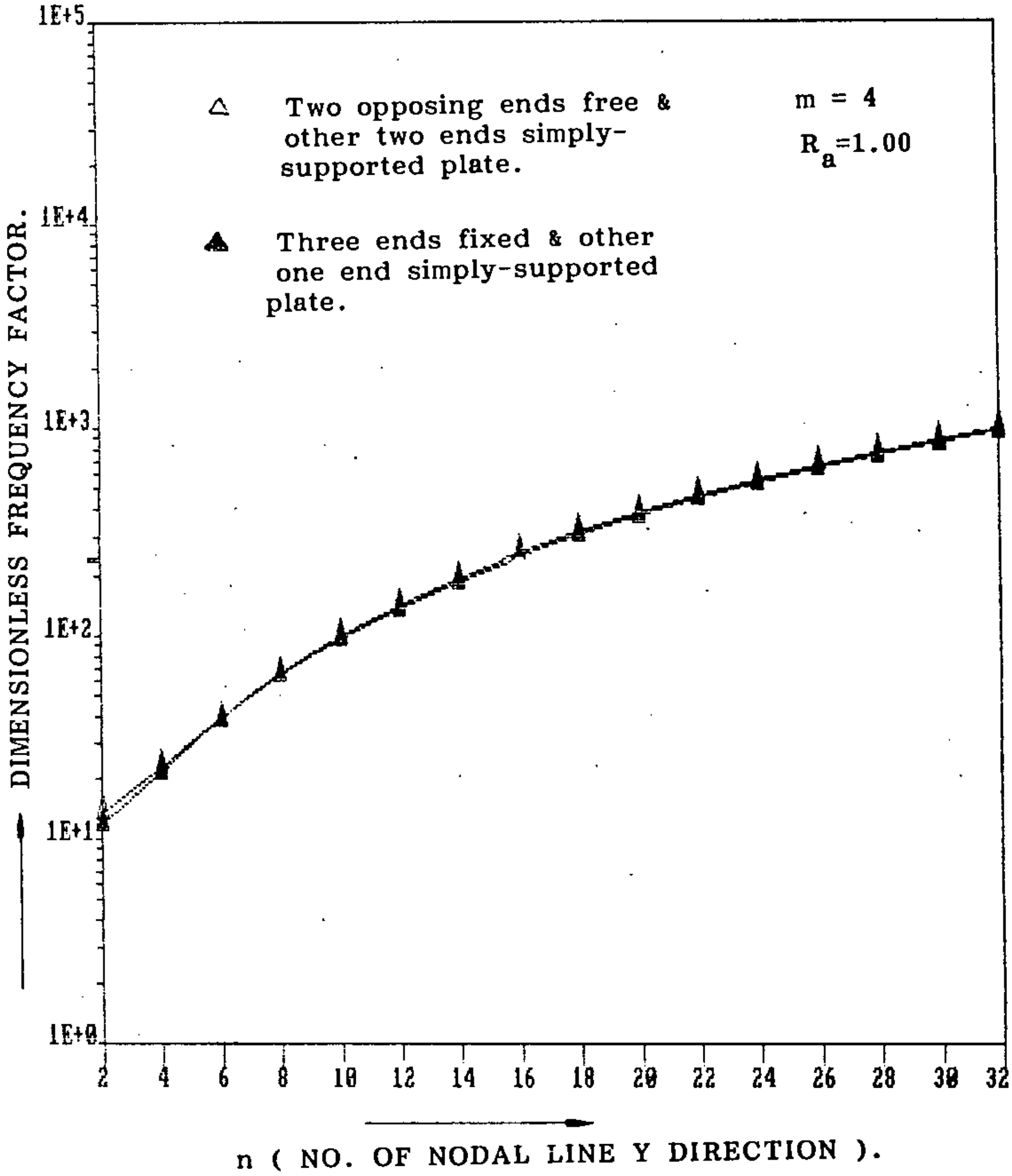


Fig.38: Dimensionless frequency factor for plates with different boundary conditions at low value of m.

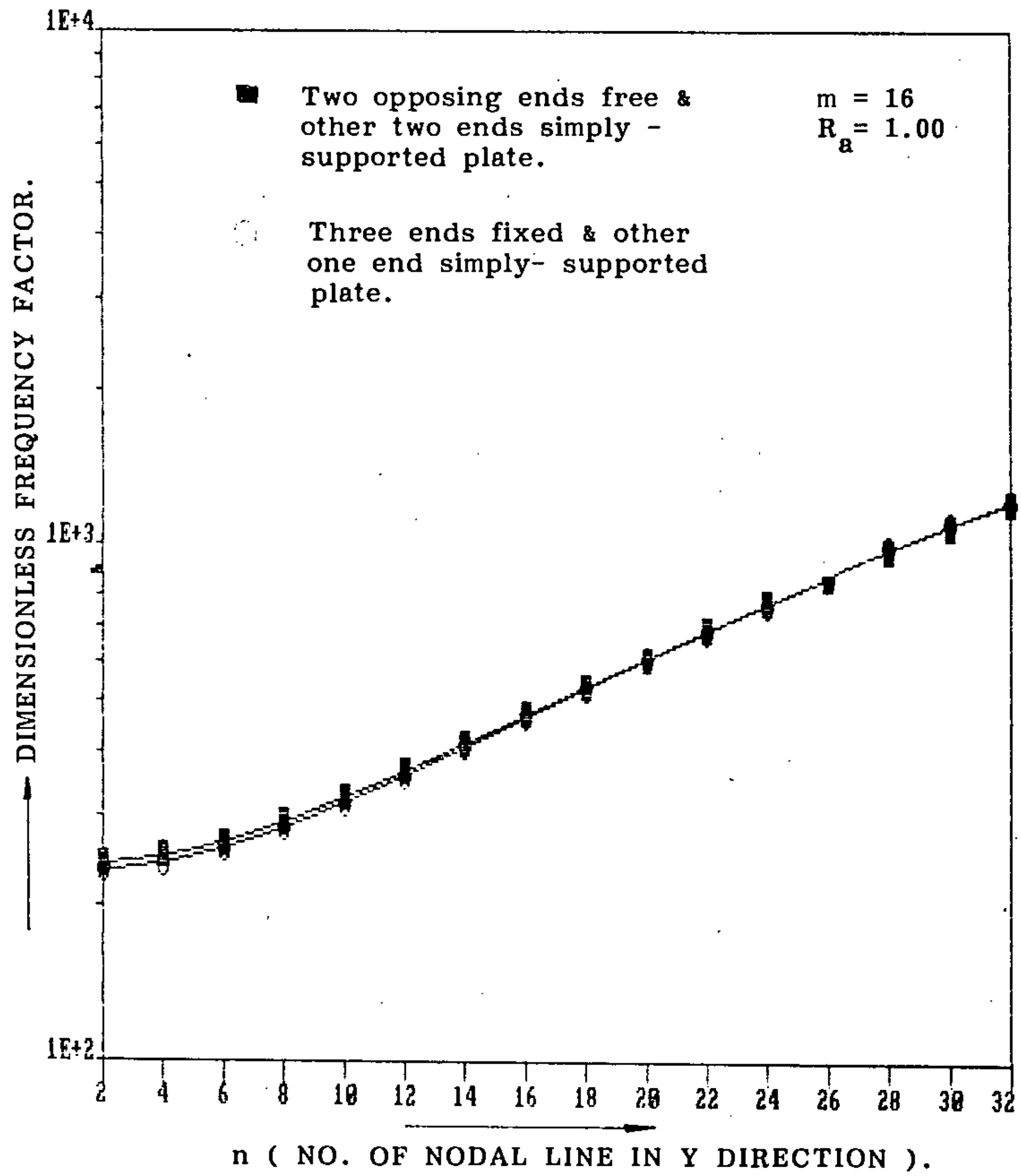


Fig. 39: Dimensionless frequency factor for plates with different boundary conditions at high value of m.

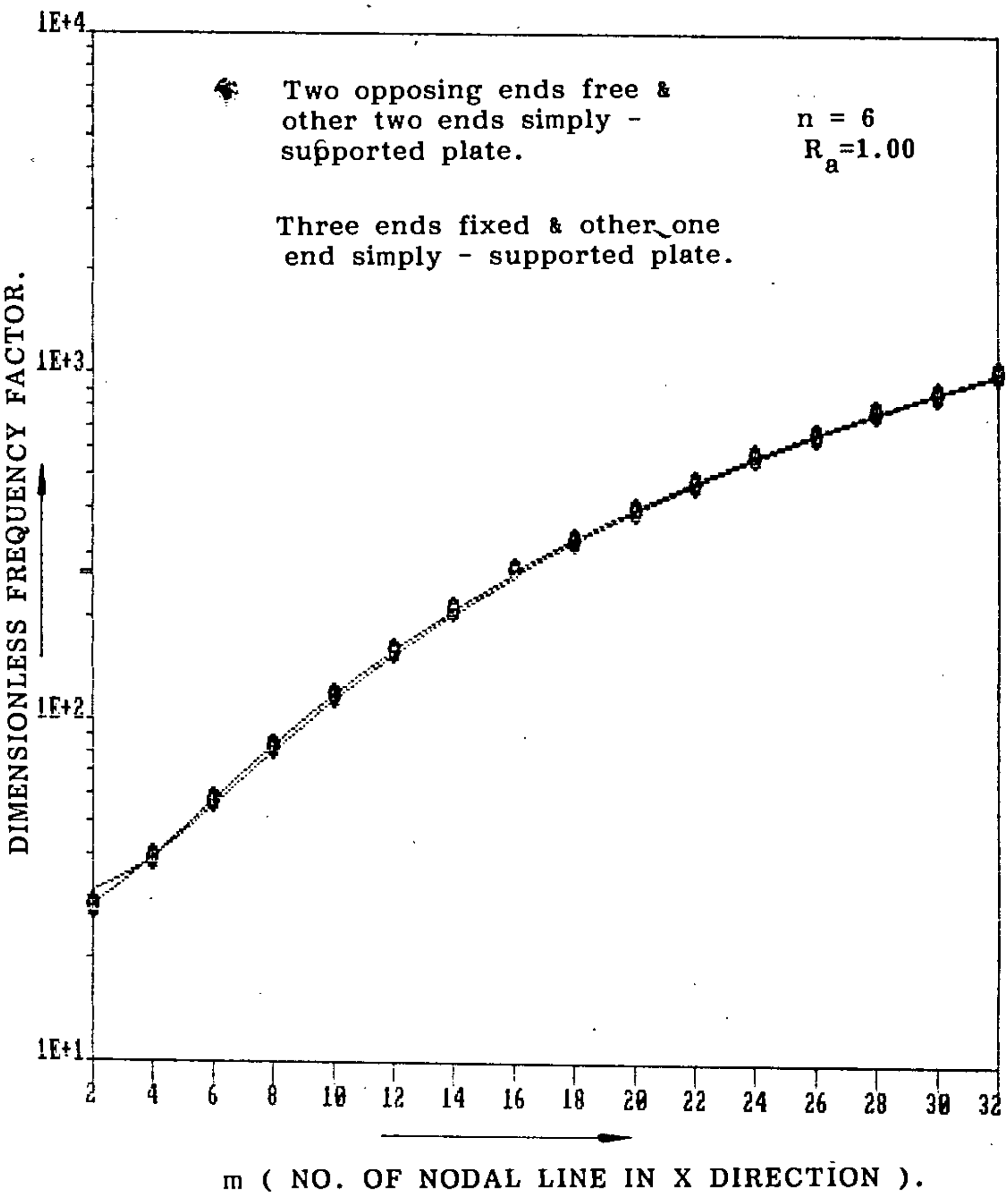


Fig.40: Dimensionless frequency factor for plates with different boundary conditions at low value of n.

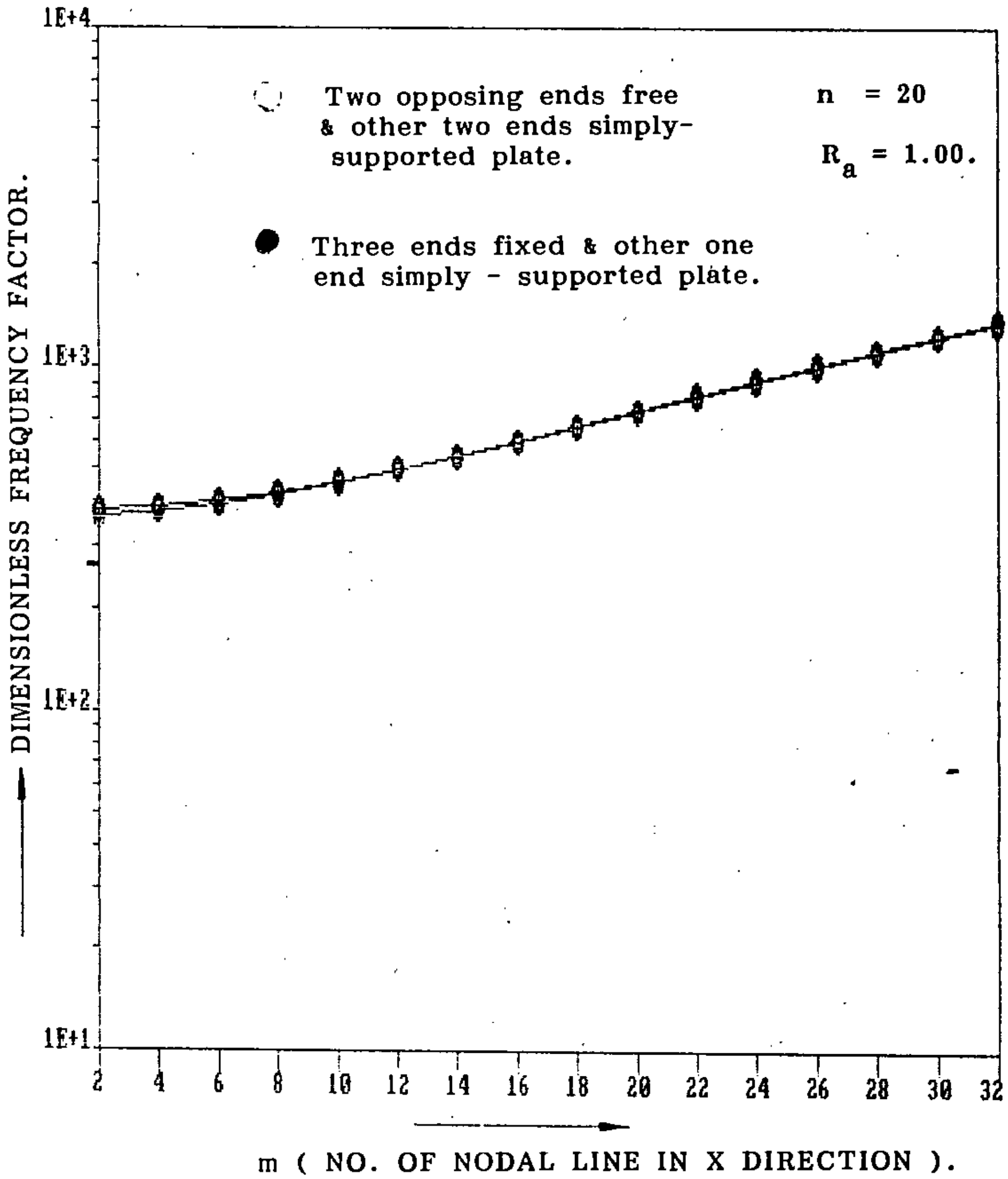


Fig.41: Dimensionless frequency factor for plates with different boundary conditions at high value of n.

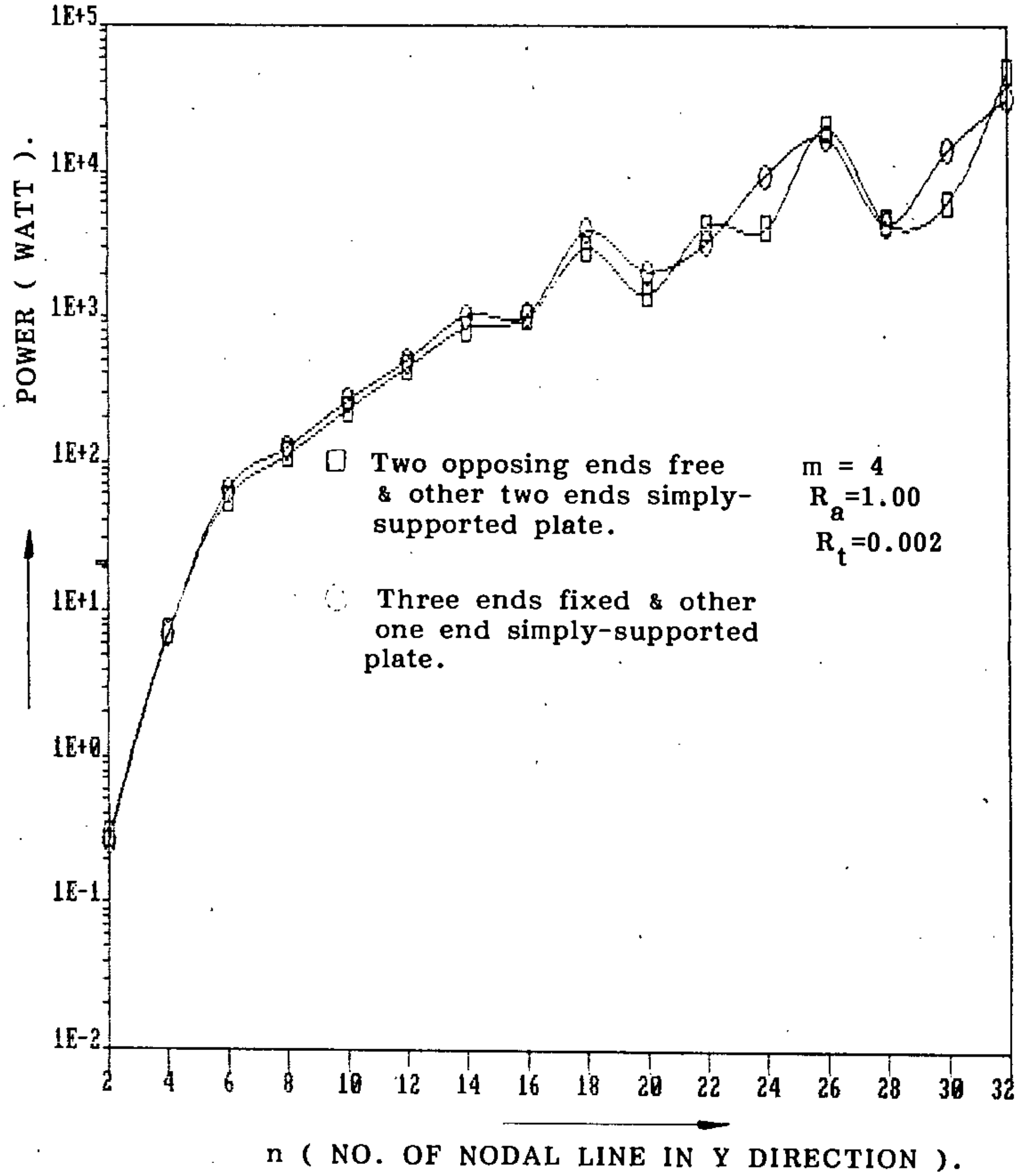


Fig.42: Power radiation from plates with different boundary conditions at low value of m.

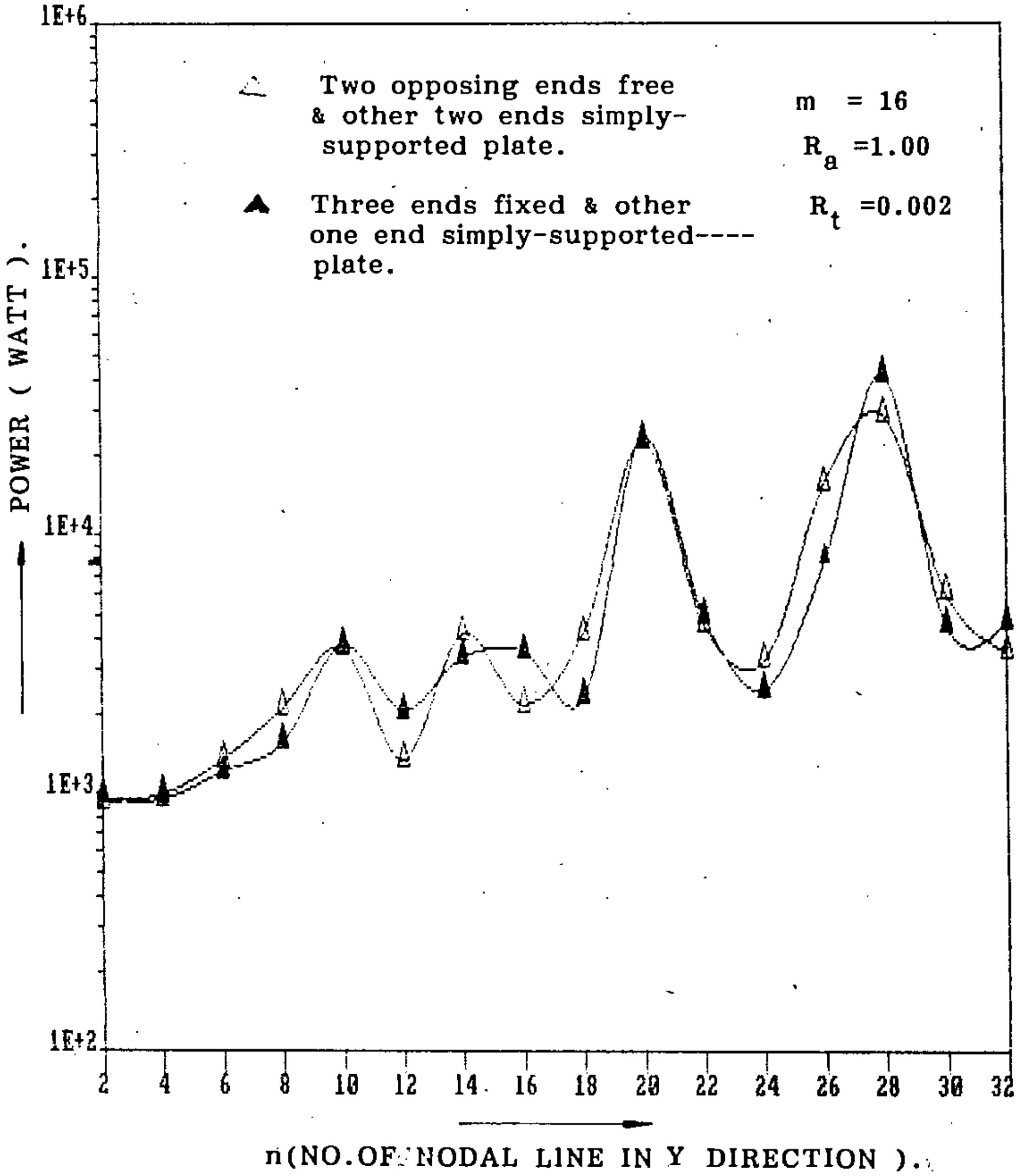


Fig.43: Power radiation from plates with different boundary conditions at high value of m.

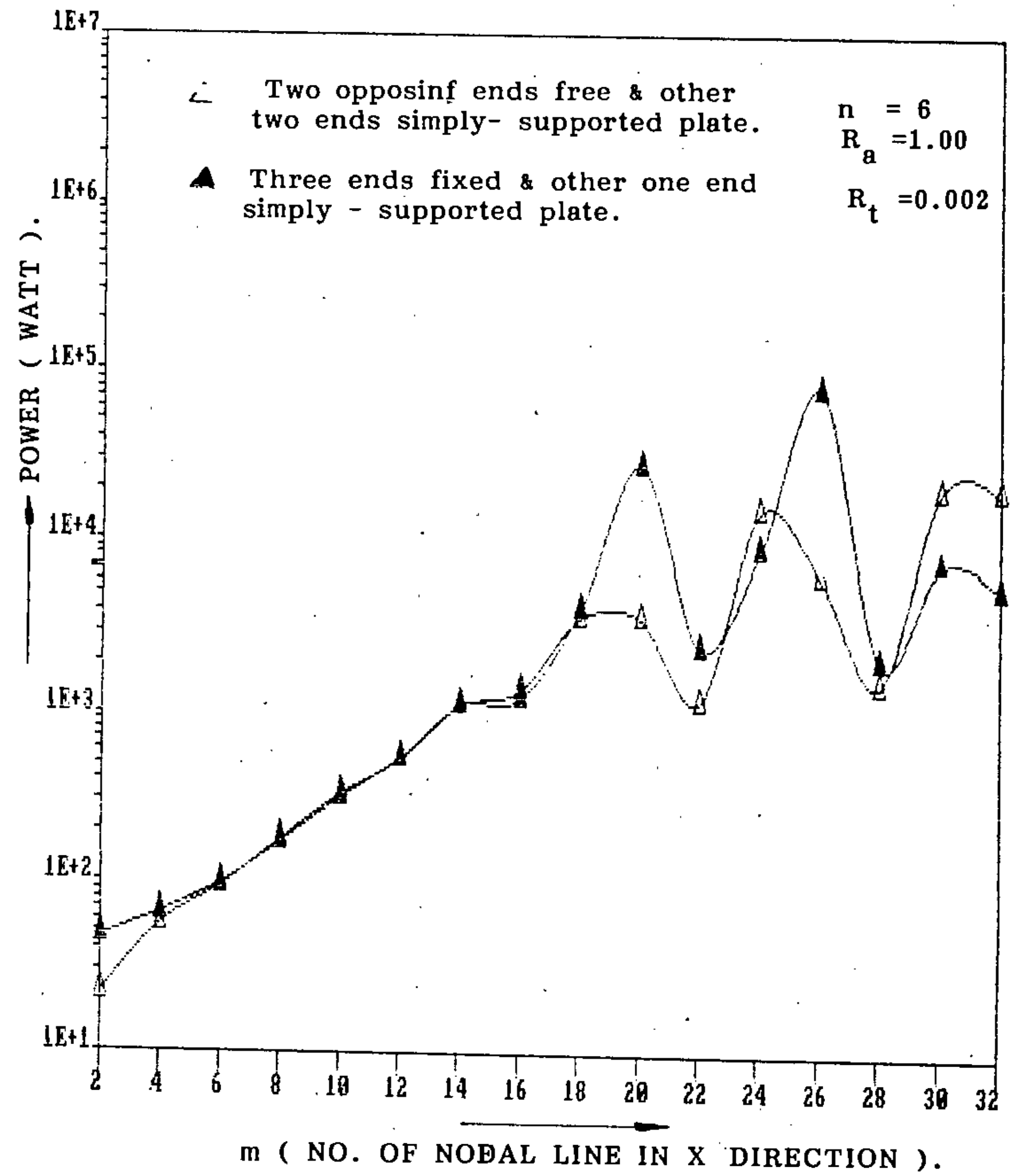


Fig.44: Power radiation from plates with different boundary conditions at low value of n.

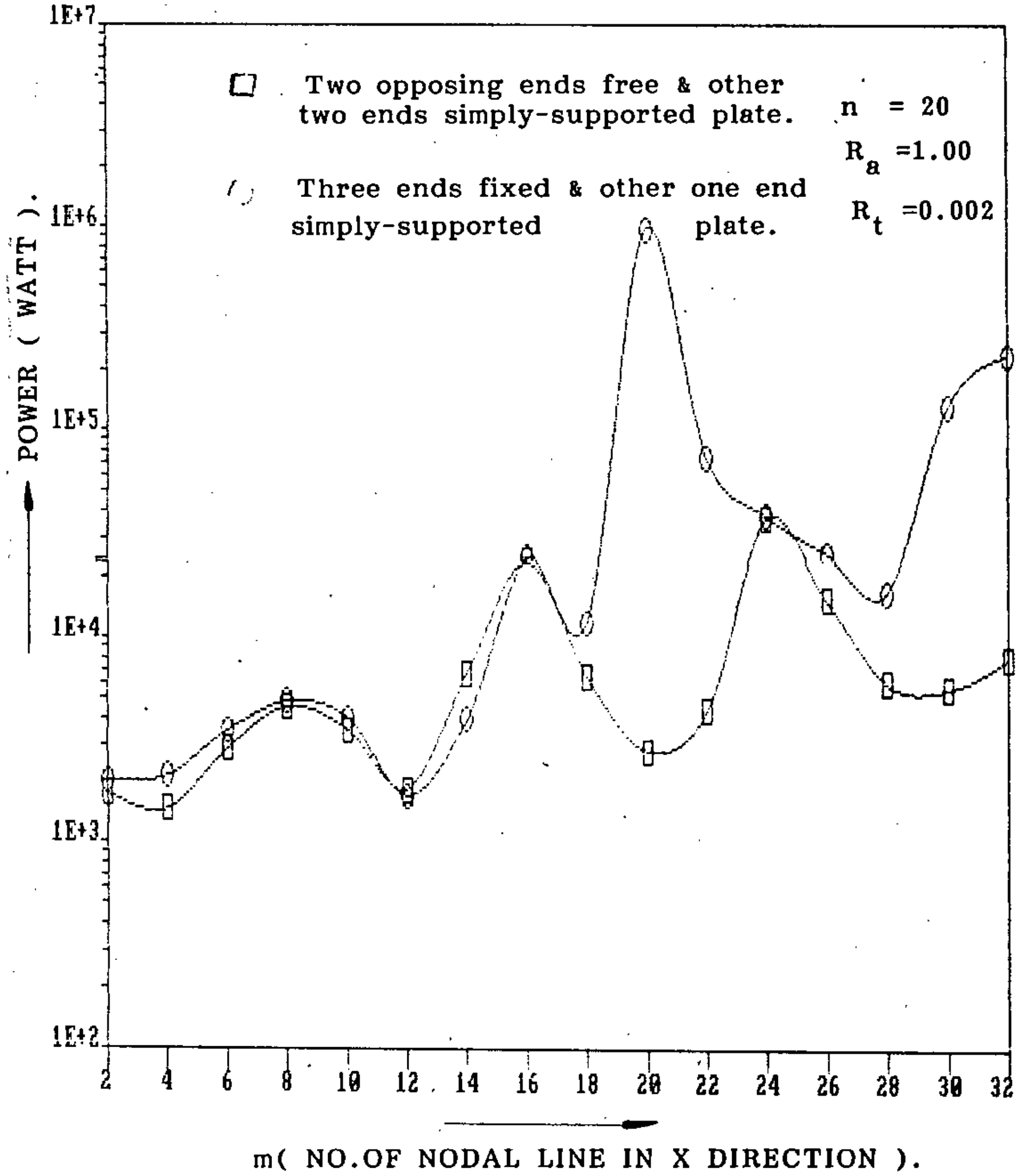


Fig.45: Power radiation from plates with different boundary conditions at high value of n.

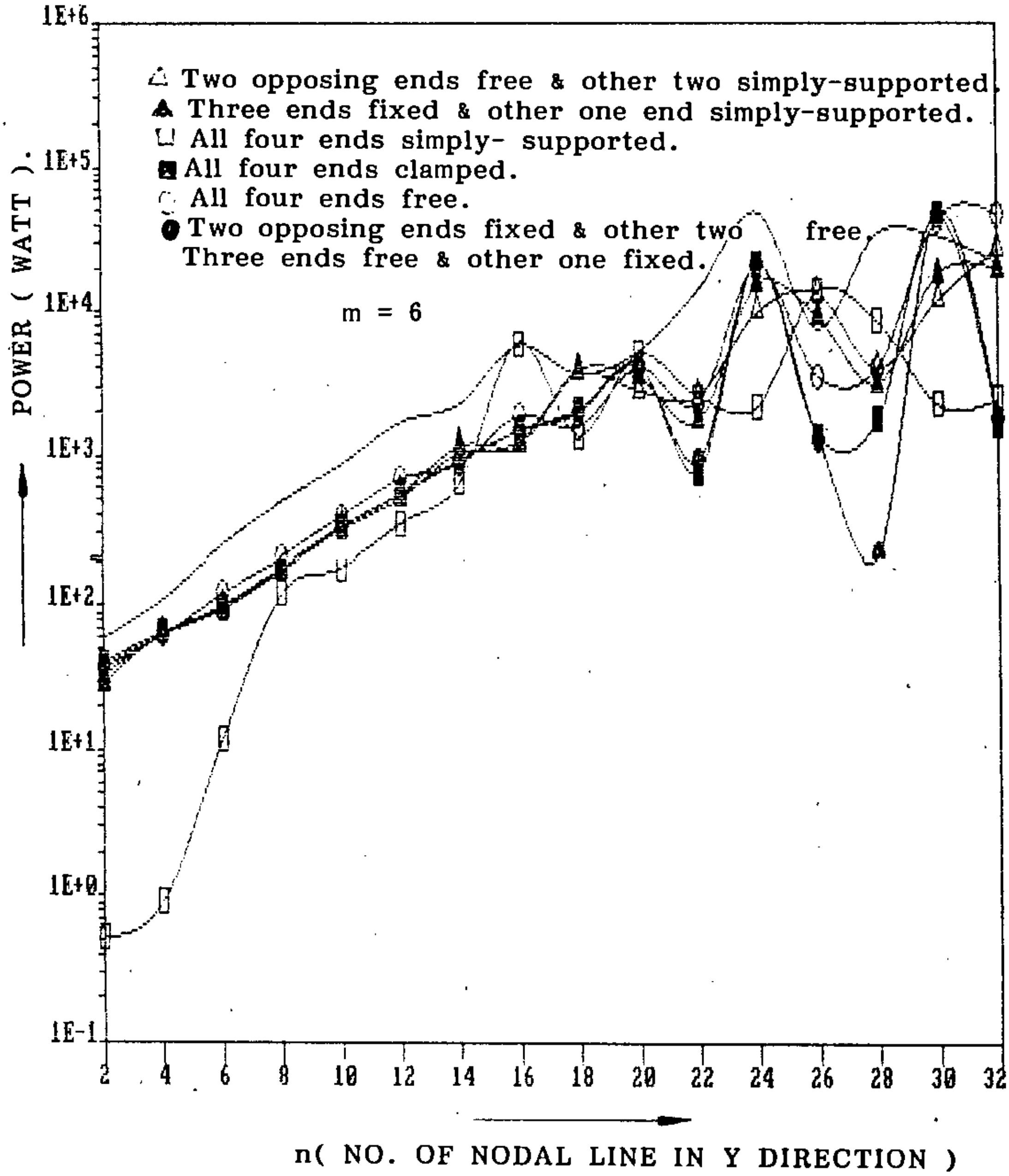


Fig.46: Power radiation from plates with different boundary conditions at low value of m.

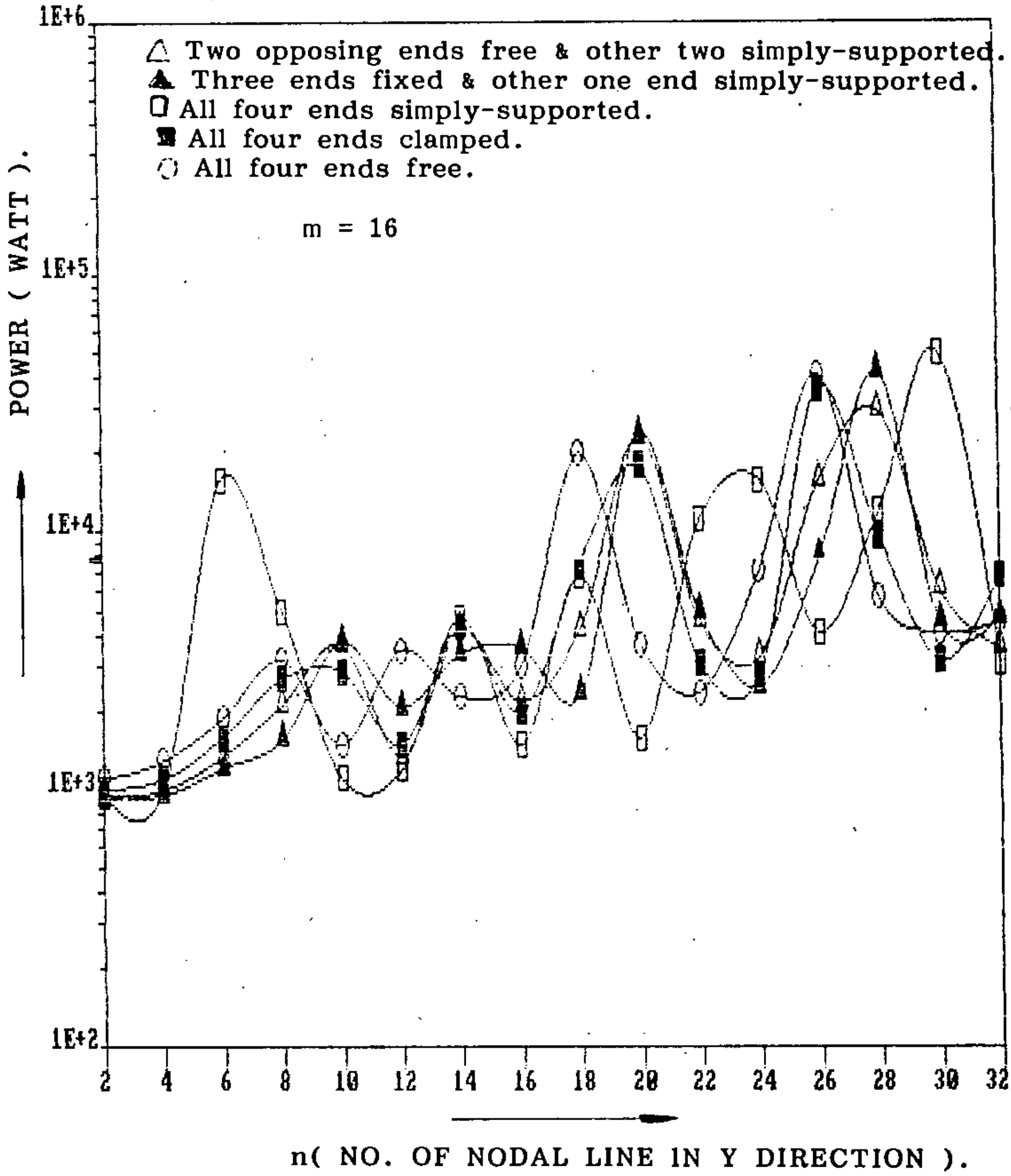


Fig.47: Power radiation from plates with different boundary conditions at high value of m.

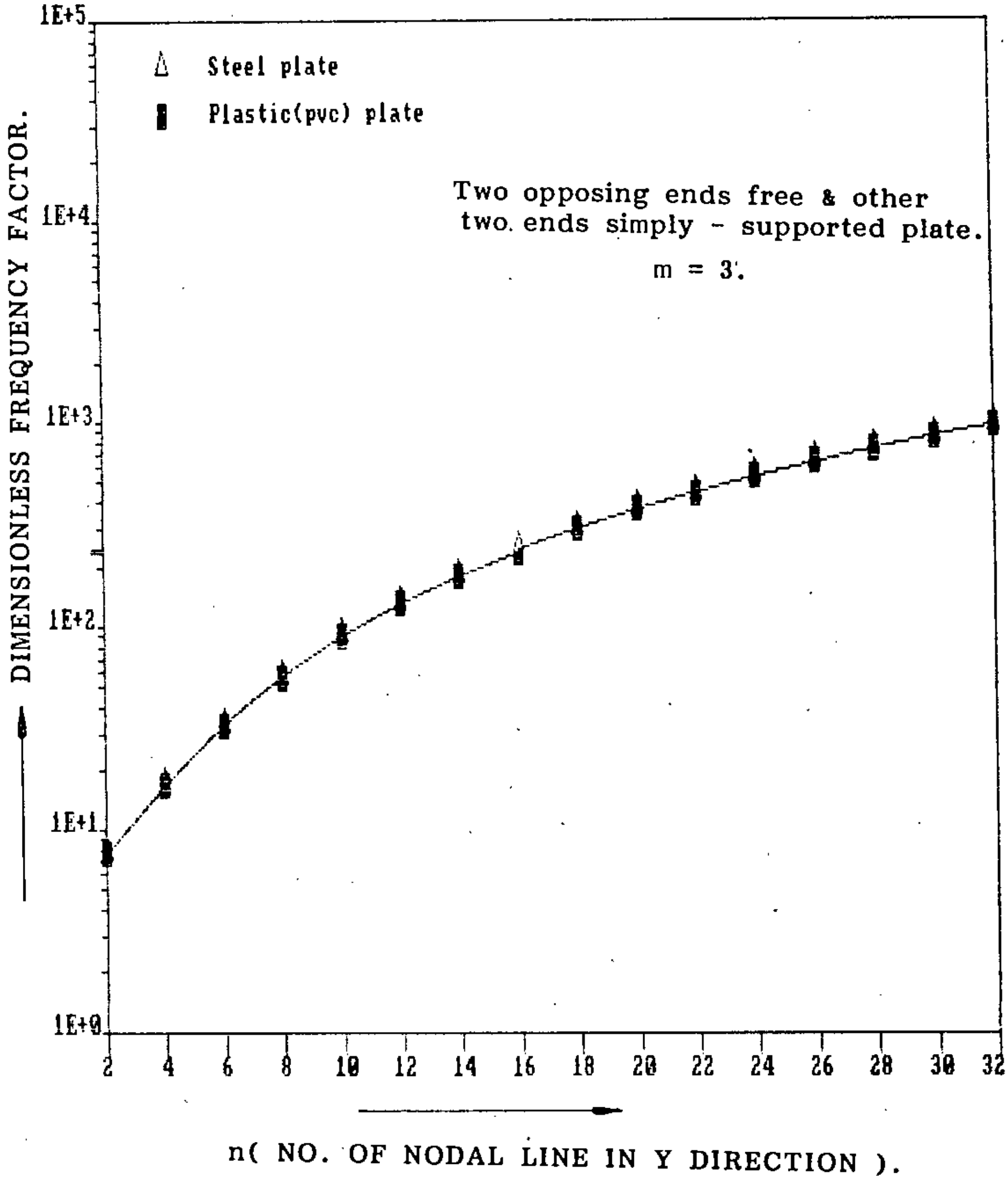


Fig.48: Dimensionless frequency factor for plates with two opposing ends free & other two ends simply-supported for different materials at low values of m.

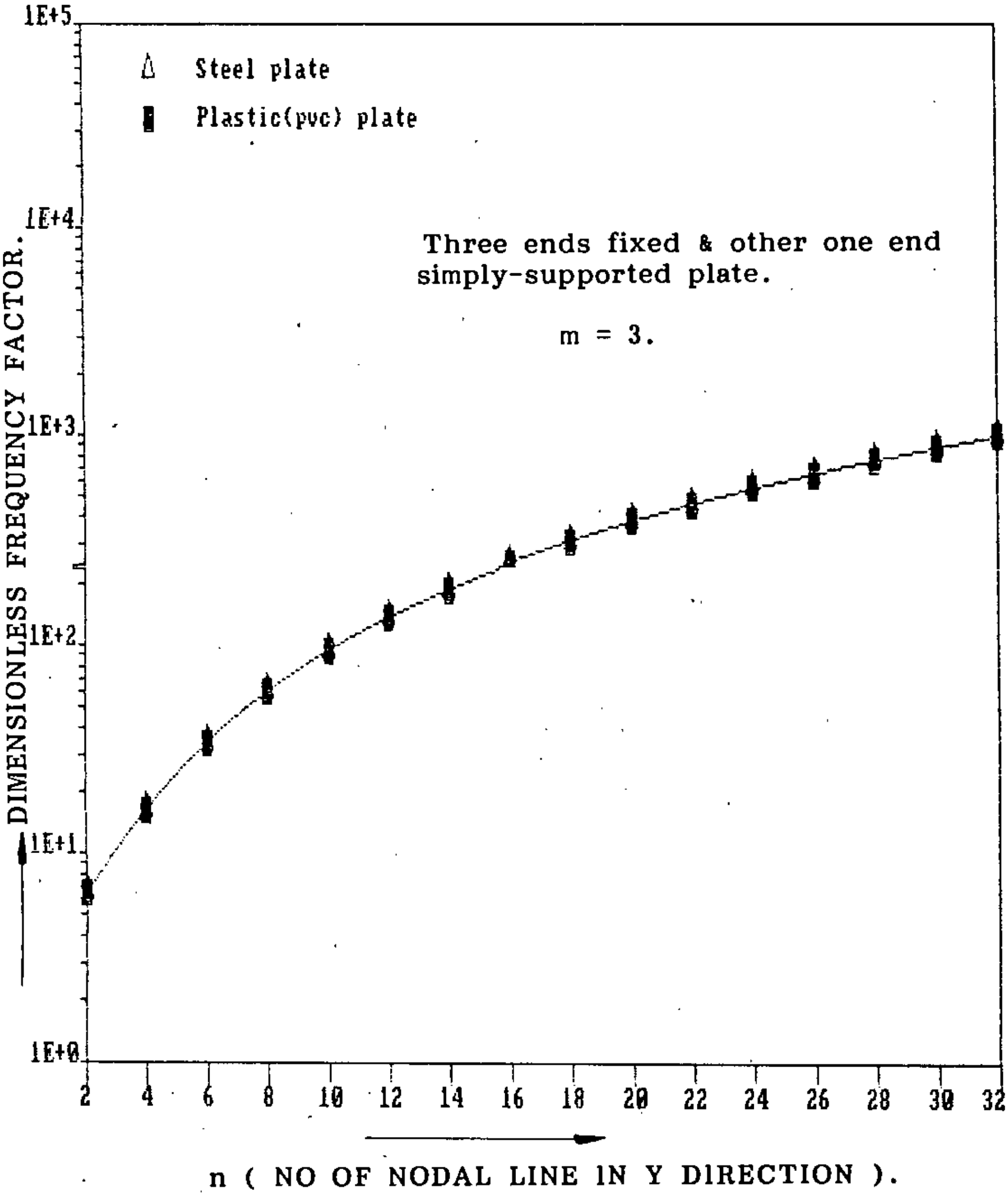


Fig.49: Dimensionless frequency factor for plate with three ends fixed & other one end simply-supported for different materials at low values of m.

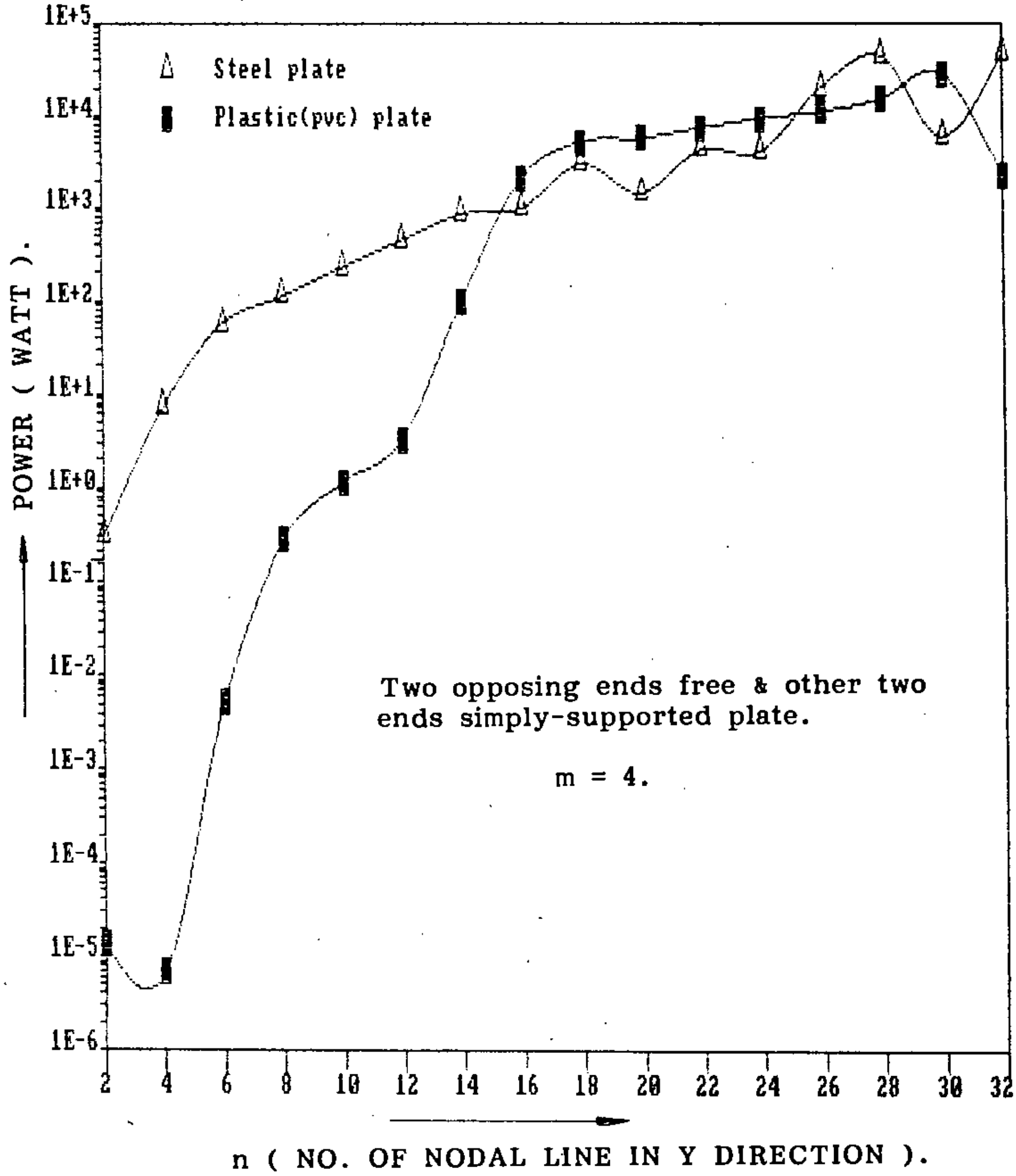


Fig.50: Power radiation from two opposing ends free & other two ends simply - supported plate for different materials at low value of m.

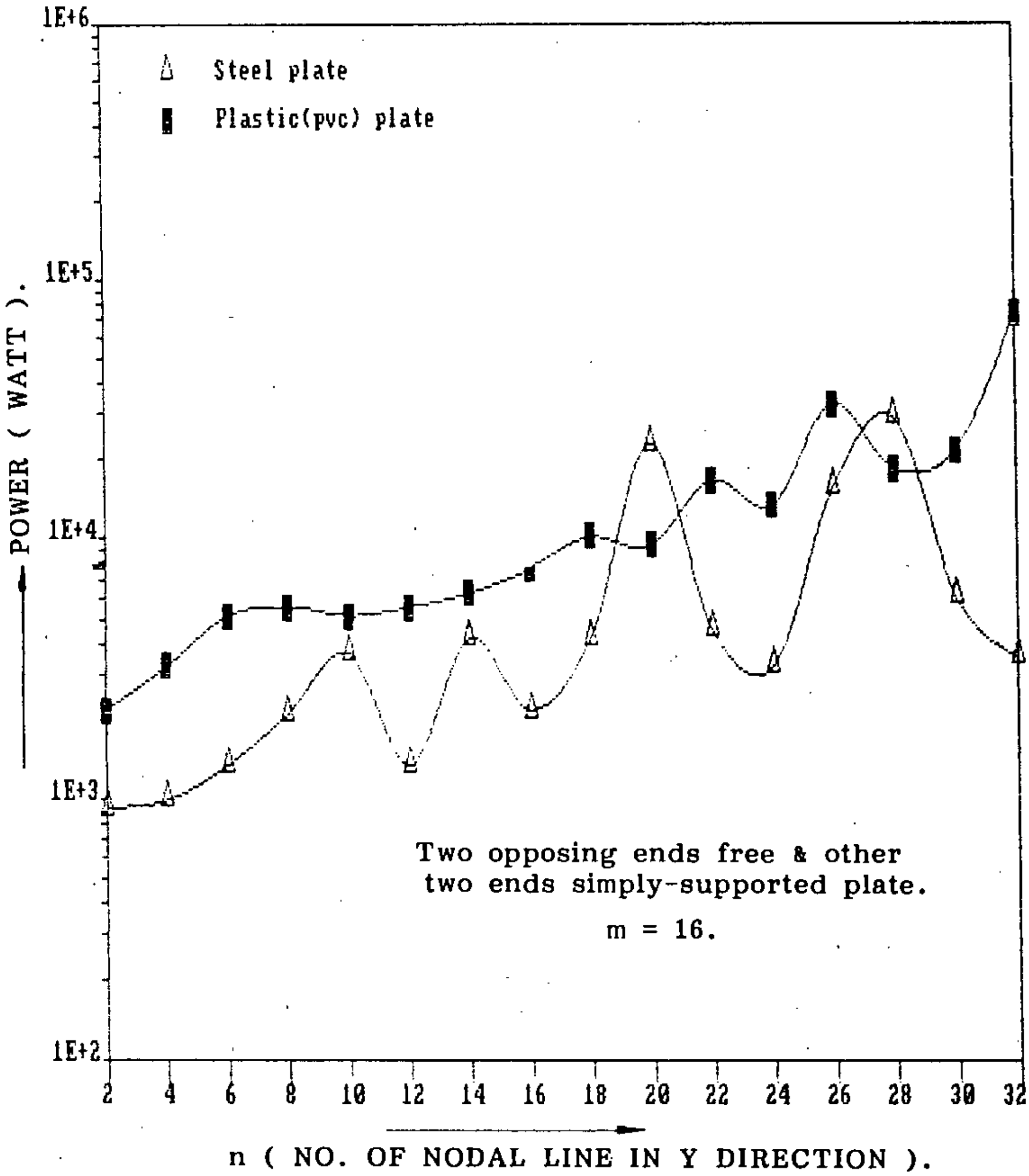


Fig.51: Power radiation from two opposing ends free and other two ends simply-supported plate for different materials at high value of m.

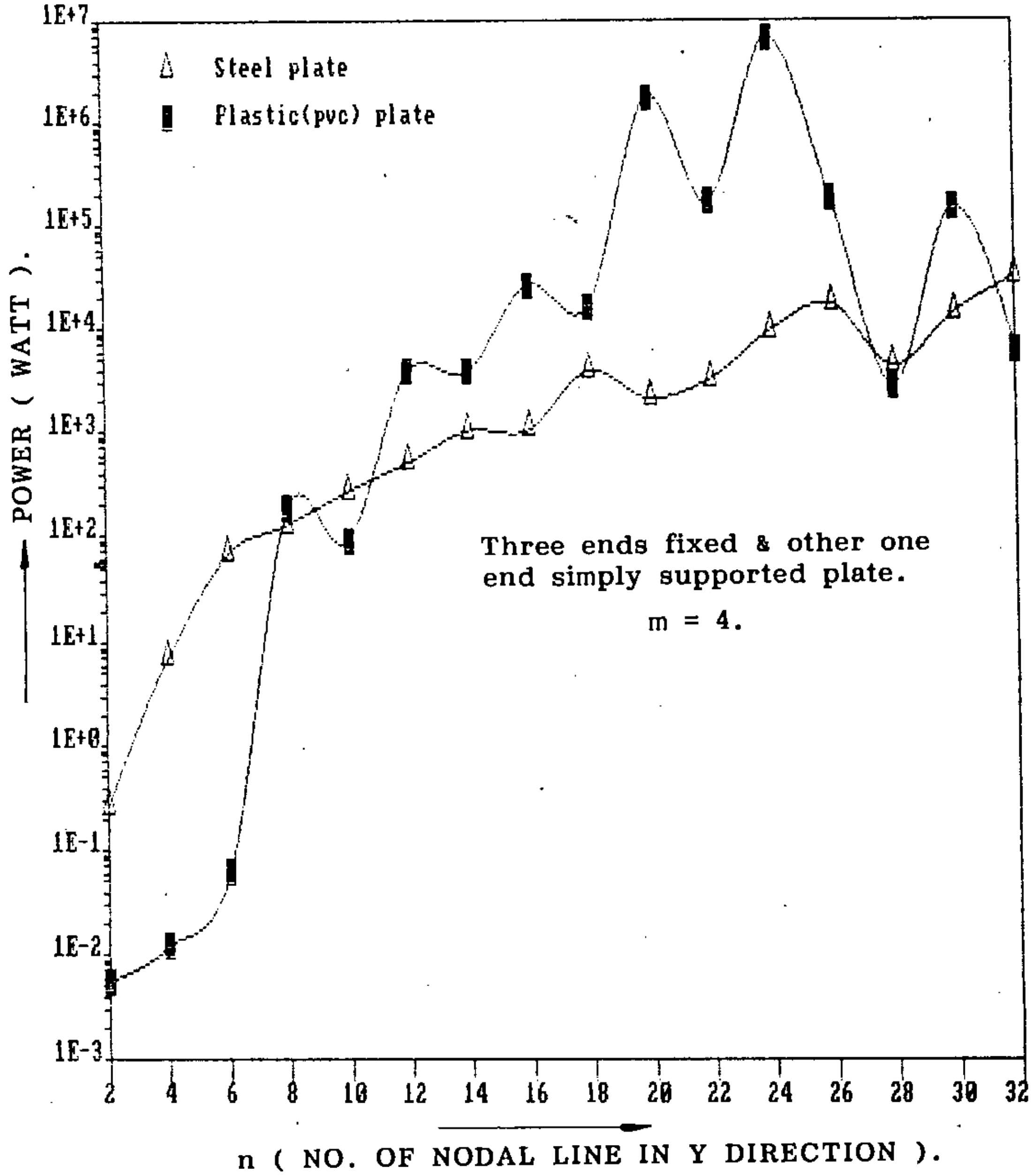


Fig.52: Power radiation from three ends fixed & other one end simply-supported plate for different materials at low value of m.

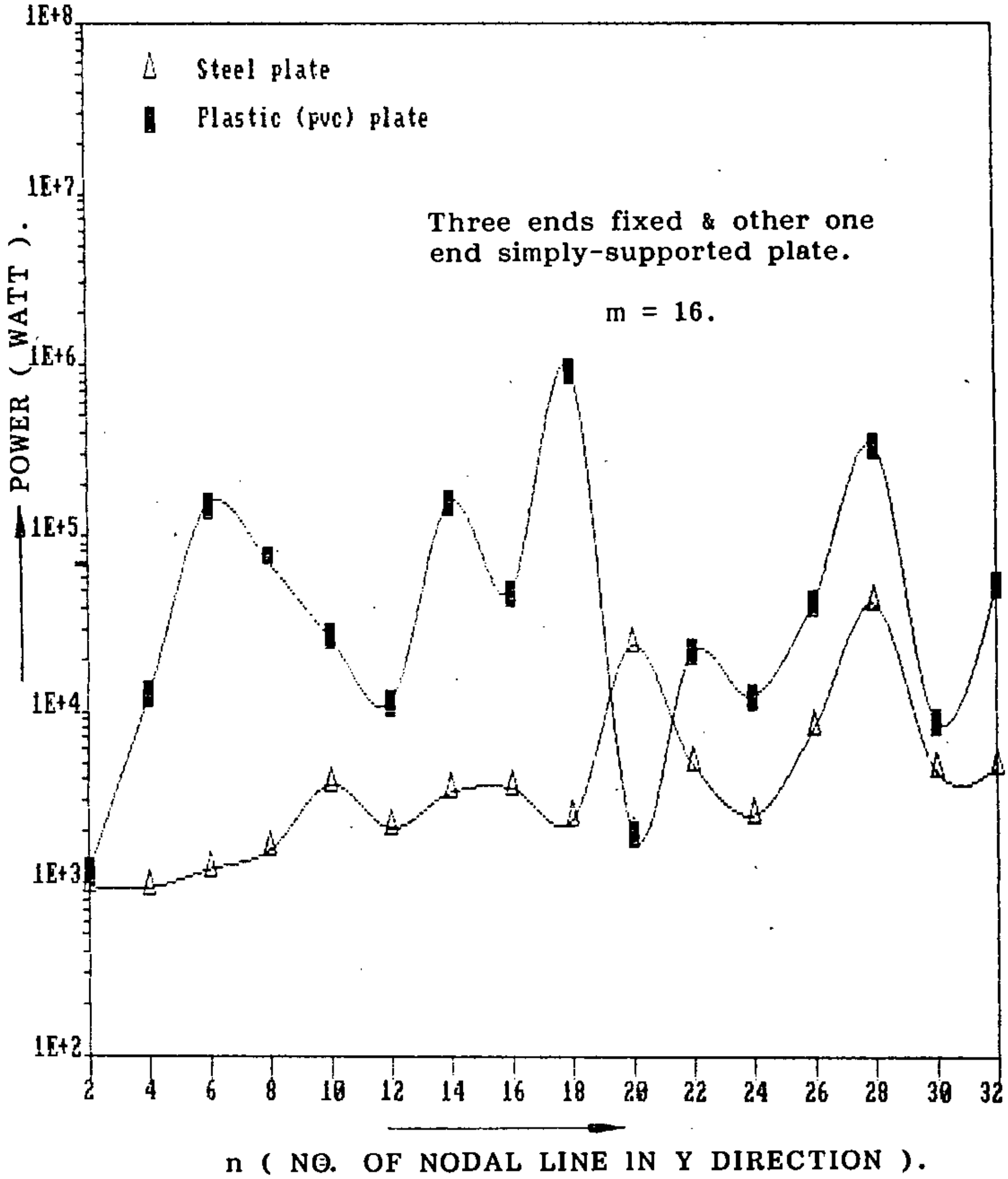


Fig.53: Power radiation from three ends fixed & other one end simply-supported plate for different materials at high value of m.

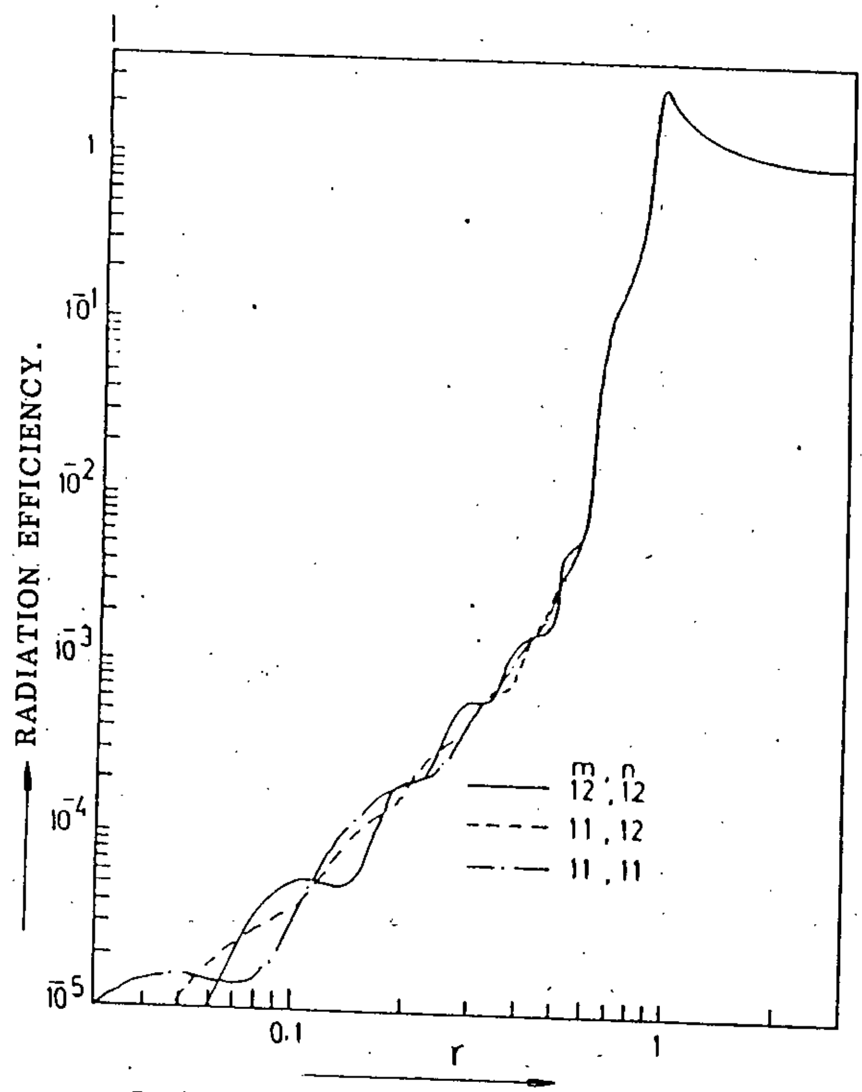


Fig.54: Radiation efficiency for typical high-numbered modes of a square panel ref.(51).

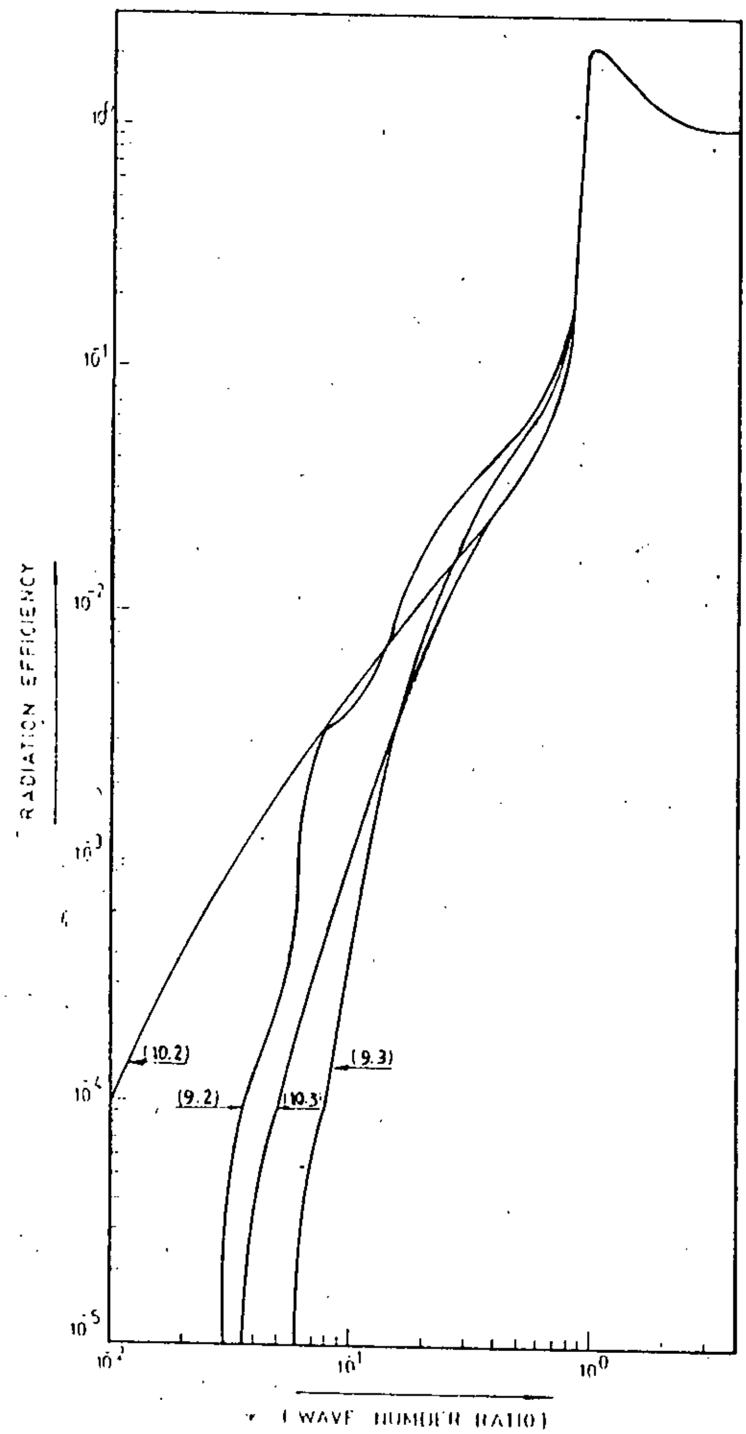


Fig.55: Radiation efficiency of a simply-supported plate for high-low mode orders. Ref.(53).

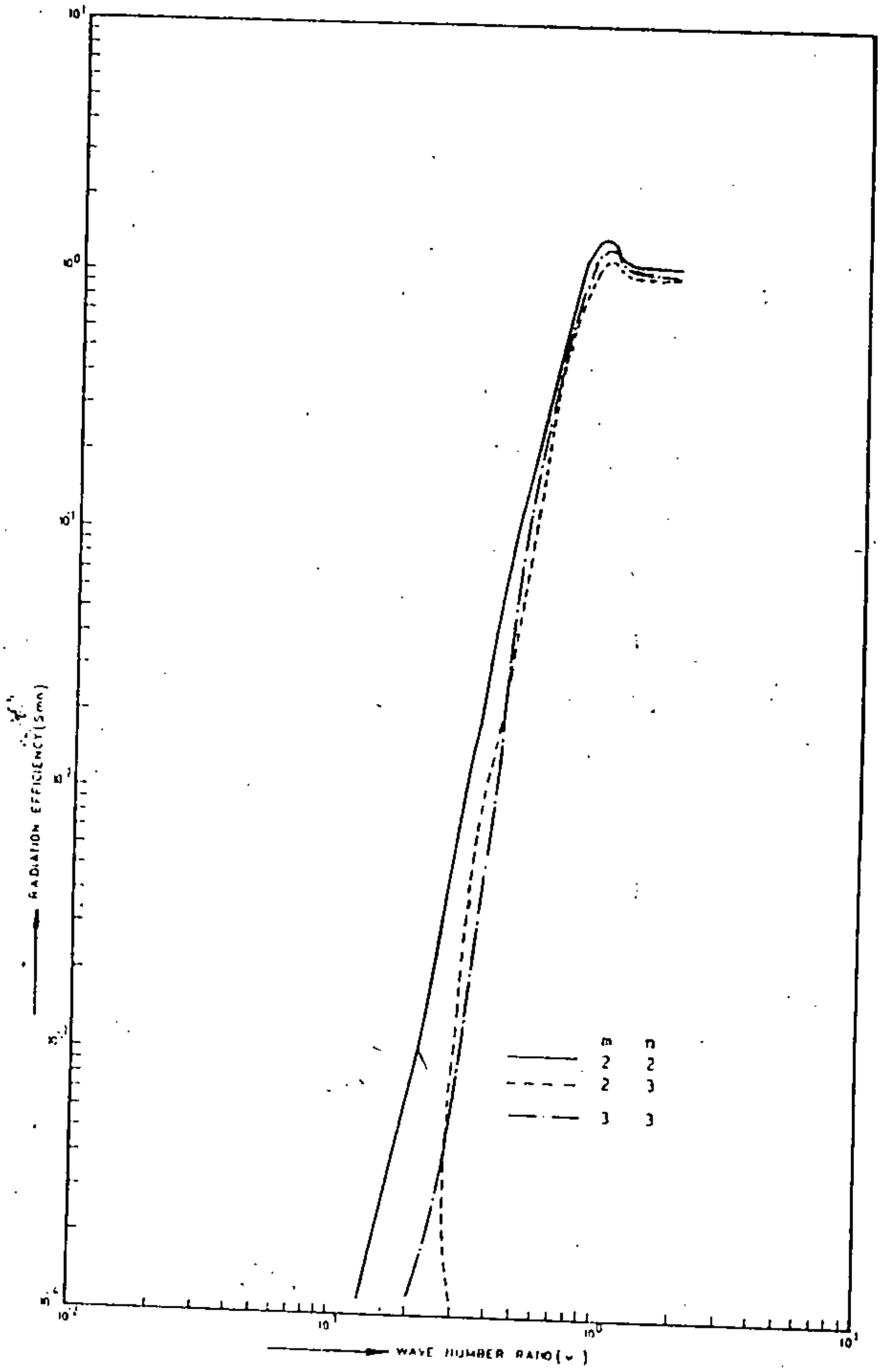


Fig.56: : Radiation efficiency for two opposing ends fixed & other two ends free plate at low-low mode orders. Ref.(54).

APPEMDIX - B

TABLES

Table-1 : Dimensionless frequency factor for two opposing ends free and other two ends simply supported plate for different values of Ra .

Ra	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.5	4	1.7E1	5.1E1	1.2E2	2.1E2	3.4E2	5.0E2	6.9E2	9.2E2	1.2E3	1.5E3	1.8E3	2.1E3	2.5E3	2.9E3	3.4E3	3.9E3
1.0	4	1.3E1	2.3E1	4.0E1	6.4E1	9.6E1	1.4E2	1.8E2	2.4E2	3.0E2	3.8E2	4.6E2	5.4E2	6.4E2	7.4E2	8.6E2	9.8E2
2.0	4	1.3E1	1.5E1	2.0E1	2.6E1	3.4E1	4.5E1	5.7E1	7.1E1	8.7E1	1.1E2	1.3E2	1.5E2	1.7E2	2.0E2	2.3E2	2.6E2
0.5	16	2.4E2	2.8E2	3.4E2	4.4E2	5.7E2	7.3E2	9.3E2	1.2E3	1.4E3	1.7E3	2.0E3	2.4E3	2.8E3	3.2E3	3.6E3	4.1E3
1.0	16	2.4E2	2.5E2	2.7E2	2.9E2	3.2E2	3.7E2	4.1E2	4.7E2	5.4E2	6.1E2	6.9E2	7.8E2	8.8E2	9.8E2	1.1E3	1.2E3
2.0	16	2.4E2	2.4E2	2.5E2	2.5E2	2.6E2	2.7E2	2.8E2	3.0E2	3.2E2	3.3E2	3.5E2	3.8E2	4.0E2	4.3E2	4.6E2	4.9E2
1.0	3	7.6E0	1.7E1	3.3E1	5.7E1	8.9E1	1.3E2	1.8E2	2.3E2	3.0E2	3.7E2	4.5E2	5.4E2	6.3E2	7.4E2	8.5E2	9.7E2

Table-2 : Dimensionless frequency factor for two opposing ends free and other two ends simply supported plate for different values of Ra .

Ra	n/m	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.5	6	1.0E2	1.2E2	1.3E2	1.6E2	1.9E2	2.4E2	2.9E2	3.4E2	4.1E2	4.8E2	5.7E2	6.6E2	7.5E2	8.6E2	9.7E2	1.1E3
1.0	6	2.8E1	3.9E1	5.7E1	8.3E1	1.2E2	1.6E2	2.1E2	2.7E2	3.3E2	4.1E2	4.9E2	5.8E2	6.8E2	7.8E2	9.0E2	1.0E3
2.0	6	9.4E0	2.0E1	3.7E1	6.3E1	9.7E1	1.4E2	1.9E2	2.5E2	3.1E2	3.9E2	4.7E2	5.6E2	6.6E2	7.6E2	8.8E2	10.E2
0.5	20	1.4E3	1.5E3	1.5E3	1.5E3	1.5E3	1.6E3	1.6E3	1.7E3	1.8E3	1.8E3	1.9E3	2.0E3	2.1E3	2.2E3	2.3E3	2.5E3
1.0	20	3.6E2	3.8E2	4.0E2	4.2E2	4.6E2	5.0E2	5.5E2	6.1E2	6.8E2	7.5E2	8.3E2	9.2E2	1.0E3	1.1E3	1.2E3	1.4E3
2.0	20	9.4E1	1.1E2	1.2E2	1.5E2	1.8E2	2.3E2	2.8E2	3.3E2	4.0E2	4.7E2	5.6E2	6.5E2	7.4E2	8.5E2	9.6E2	1.1E3

Table-3 : Power radiation by two opposing ends free and other two ends simply supported plate. Ra =1.0, Rt=0.002

m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	8.3E-4	3.3E-1	2.2E1	8.4E1	1.9E2	3.8E2	6.6E2	1.1E3	2.1E3	1.7E3	5.1E3	1.9E3	2.1E4	3.8E3	3.4E3	4.5E4
3	2.2E-1	3.1E0	4.3E1	9.5E1	2.0E2	4.0E2	7.5E2	9.4E2	2.7E3	1.3E3	4.5E3	3.2E3	2.3E4	3.7E3	4.0E3	4.1E4
4	2.9E-1	7.1E0	5.6E1	1.1E2	2.3E2	4.4E2	8.5E2	9.8E2	3.0E3	1.4E3	4.3E3	4.1E3	2.0E4	4.5E3	6.1E3	4.9E4
5	7.2E0	3.4E1	7.0E1	1.4E2	2.7E2	4.8E2	9.9E2	1.0E3	3.6E3	2.0E3	3.1E3	7.4E3	1.5E4	3.9E3	9.2E3	4.0E4
6	3.3E1	6.2E1	9.4E1	1.7E2	3.2E2	5.1E2	1.1E3	1.2E3	3.7E3	2.8E3	2.6E3	9.9E3	1.3E4	4.0E3	1.2E4	2.7E4
15	1.1E3	1.0E3	9.5E2	1.3E3	2.5E3	2.9E3	2.3E3	4.3E3	1.8E3	1.8E4	7.5E3	2.6E3	4.4E3	4.8E4	4.6E3	3.7E3
16	9.3E2	9.9E2	1.3E3	2.1E3	3.7E3	1.3E3	4.2E3	2.2E3	4.2E3	2.3E4	4.5E3	3.3E3	1.6E4	2.9E4	6.1E3	3.5E3
17	1.5E3	1.7E3	2.7E3	3.7E3	1.7E3	3.1E3	4.1E3	2.2E3	1.3E4	1.5E4	3.3E3	3.1E3	3.4E4	1.6E4	4.9E3	5.5E3
18	3.2E3	3.8E3	3.6E3	1.8E3	2.4E3	4.7E3	1.8E3	3.7E3	2.3E4	6.4E3	3.4E3	6.5E3	4.0E4	7.5E3	5.0E3	6.2E3

Table-4 : Power radiation by two opposing ends free and other two ends simply supported plate. Ra =1.0, Rt=0.002

n/m	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
4	3.3E-1	7.1E0	6.2E1	1.2E2	2.5E2	4.8E2	9.8E2	9.9E2	3.8E3	1.9E3	2.8E3	8.6E3	1.4E4	3.2E3	1.1E4	2.6E4
5	3.0E0	2.9E1	7.1E1	1.4E2	2.8E2	5.2E2	1.1E3	1.1E3	3.9E3	2.7E3	2.1E3	1.2E4	1.0E4	2.7E3	1.9E4	3.1E4
6	2.2E1	5.6E1	9.4E1	1.8E2	3.3E2	5.3E2	1.1E3	1.3E3	3.6E3	3.6E3	1.2E3	1.5E4	6.5E3	1.5E3	2.0E4	2.0E4
7	5.9E1	7.7E1	1.3E2	2.2E2	4.0E2	6.3E2	1.0E3	1.5E3	2.8E3	4.4E3	6.8E2	2.0E4	3.3E3	1.5E3	3.0E4	1.1E4
19	3.8E3	3.2E3	1.7E3	2.1E3	4.5E3	3.0E3	2.2E3	1.4E4	1.7E4	3.8E3	2.9E3	2.0E4	2.9E4	5.1E3	6.2E3	7.9E3
20	1.7E3	1.4E3	2.8E3	4.6E3	3.5E3	1.8E3	6.7E3	2.3E4	6.4E3	2.8E3	4.4E3	3.8E4	1.5E4	6.0E3	5.6E3	7.9E3
21	3.2E3	4.2E3	4.9E3	3.4E3	1.4E3	4.2E3	2.0E4	1.5E4	3.6E3	3.4E3	1.4E4	3.9E4	6.6E3	5.2E3	6.0E3	1.0E4
22	5.1E3	4.3E3	2.6E3	1.1E3	3.9E3	1.7E4	2.2E4	4.5E3	3.4E3	5.5E3	3.2E4	2.4E4	7.4E3	5.0E3	6.2E3	1.7E4

Table-5 : Power radiation by a two opposing ends free and other two ends simply supported plate.

Ra =1.00 and different values of Rt .

Rt	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.001	4	6.5E-5	2.6E-3	3.3E-1	2.4E0	3.4E1	2.3E2	2.8E2	4.1E2	6.3E2	9.3E2	1.1E3	2.1E3	2.2E3	4.3E3	2.5E3	7.5E3
0.002	4	2.9E-1	7.1E0	5.6E1	1.1E2	2.3E2	4.4E2	8.5E2	9.8E2	3.0E3	1.4E3	4.3E3	4.1E3	2.0E4	4.5E3	6.1E3	4.9E4
0.004	4	2.3E1	4.9E1	1.4E2	3.6E2	7.3E2	1.9E3	1.7E3	6.1E3	2.5E3	1.8E4	4.2E4	8.2E3	5.7E3	1.3E4	5.0E4	3.5E5
0.001	18	6.3e2	6.6E2	7.0E2	7.5E2	1.1E3	1.1E3	1.3E3	2.3E3	1.5E3	1.9E3	5.3E3	2.8E3	4.8E3	3.1E3	4.5E3	2.1E4
0.002	18	3.2E3	3.8E3	3.6E3	1.8E3	2.4E3	4.7E3	1.8E3	3.7E3	2.3E4	6.4E3	3.4E3	6.5E3	4.0E4	7.5E3	5.0E3	6.2E3
0.004	18	1.5E3	1.6E3	1.7E3	7.6E3	3.6E4	6.1E4	1.1E4	5.7E3	4.0E3	5.8E3	9.9E3	5.4E4	3.0E5	4.4E4	2.7E4	2.0E4

Table-6 : Power radiation by a two opposing ends free and other two ends simply supported plate.

Rt =0.002 and different values of Ra .

Ra	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.50	4	8.7E-1	3.9E1	1.6E2	5.0E2	1.4E3	2.5E3	6.0E3	1.2E4	4.4E3	1.7E4	8.1E4	2.0E4	9.1E3	1.9E4	3.1E4	5.0E5
1.00	4	2.9E-1	7.1E0	5.6E1	1.1E2	2.3E2	4.4E2	8.5E2	9.8E2	3.0E3	1.4E3	4.3E3	4.1E3	2.0E4	4.5E3	6.1E3	4.9E4
2.00	4	3.6E-1	9.3E-1	4.9E0	3.0E1	1.0E2	1.3E2	1.9E2	2.7E2	3.7E2	5.5E2	7.9E2	8.8E2	8.9E2	3.1E3	1.4E3	2.7e3
0.50	18	1.7E3	1.4E3	1.7E3	1.6E3	7.5E3	2.9E3	1.4E4	4.9E3	2.6E4	9.5E4	1.3E4	1.3E4	7.7E3	1.3E4	9.4E5	6.0e4
1.00	18	3.2E3	3.8E3	3.6E3	1.8E3	2.4E3	4.7E3	1.8E3	3.7E3	2.3E4	6.4E3	3.4E3	6.5E3	4.0E4	7.5E3	5.0E3	6.2e3
2.00	18	5.8E3	5.4E3	8.5E3	7.6E3	7.3E3	7.9E3	3.6E3	2.5E3	3.4E3	5.8E3	9.1E3	1.2E4	4.2E3	3.0E3	3.2E3	1.4e4

Table-7 :Radiation efficiency of two opposing ends free and other two simply supported plate for different mode orders .

Ra = 1.00

m	n	.02	.04	.06	.08	.10	.20	.40	.60	.80	1.0	2.0	3.0	4.0
2	2	1.8E-11	1.1E-9	1.3E-8	7.1E-8	2.7E-7	1.7E-5	1.0E-3	1.0E-2	4.8E-2	1.5E-1	1.4E0	10.E-1	1.0E0
2	4	2.4E-10	1.5E-8	1.7E-7	9.6E-7	3.6E-6	1.9E-4	6.5E-3	3.7E-2	1.8E-1	6.6E-1	1.2E0	1.1E0	1.0E0
3	2	1.6E-4	6.6E-4	1.5E-3	2.6E-3	4.1E-3	1.6E-2	6.2E-2	1.6E-1	3.6E-1	6.8E-1	1.2E0	1.1E0	1.0E0
3	4	4.7E-5	1.9E-4	4.1E-4	7.1E-4	1.1E-3	3.3E-3	1.3E-2	8.0E-2	3.8E-1	1.0E0	1.2E0	1.1E0	1.0E0
2	14	6.1E-8	3.5E-6	3.2E-5	1.4E-4	3.9E-4	6.2E-3	1.9E-2	3.5E-2	1.2E-1	1.9E0	1.1E0	8.6E-1	8.4E-1
2	15	2.3E-9	5.2E-7	1.1E-5	8.6E-5	3.7E-4	5.8E-3	2.1E-2	3.1E-2	1.1E-1	2.0E0	1.1E0	8.4E-1	8.8E-1
3	15	2.1E-6	3.0E-5	1.3E-4	3.3E-4	6.6E-4	5.4E-3	2.1E-2	3.4E-2	1.2E-1	2.3E0	1.1E0	8.3E-1	8.4E-1
3	16	3.1E-5	1.0E-4	1.8E-4	3.0E-4	5.6E-4	6.2E-3	1.7E-2	3.7E-2	1.1E-1	2.3E0	1.1E0	7.9E-1	8.6E-1
14	2	1.9E-11	8.5E-10	4.8E-9	8.1E-9	2.1E-8	1.2E-5	2.3E-4	2.2E-3	3.7E-2	1.3E0	1.5E0	8.4E-1	7.7E-1
14	3	1.3E-13	2.4E-11	3.6E-10	1.5E-9	4.0E-9	5.6E-6	2.6E-4	2.0E-3	3.5E-2	1.3E0	1.2E0	8.4E-1	8.2E-1
15	2	1.9E-4	7.4E-4	1.6E-3	2.6E-3	3.7E-3	8.5E-3	1.9E-2	3.1E-2	1.4E-2	1.5E0	1.1E0	7.9E-1	9.3E-1
15	3	3.2E-6	4.9E-5	2.3E-4	6.5E-4	1.4E-3	8.1E-3	2.9E-2	3.2E-2	8.1E-2	1.6E0	1.1E0	7.9E-1	9.5E-1
14	13	3.4E-14	4.6E-12	3.4E-11	1.4E-10	2.4E-9	1.3E-7	9.1E-5	6.1E-4	6.2E-2	1.8E0	1.1E0	9.9E-1	1.1E0
14	15	4.4E-14	5.4E-12	3.1E-11	2.6E-10	4.2E-9	2.2E-7	9.1E-5	7.6E-4	6.6E-2	1.9E0	1.0E0	1.0E0	1.1E0
15	14	1.9E-6	5.4E-6	8.4E-6	1.4E-5	2.6E-5	1.1E-4	8.6E-4	3.5E-3	1.7E-2	2.1E0	1.0E0	1.0E0	1.1E0
15	16	1.6E-6	4.4E-6	6.9E-6	1.2E-5	2.4E-5	1.0E-4	9.0E-4	3.3E-3	1.8E-2	2.2E0	9.8E-1	1.0E0	1.1E0

Table-8 : Dimensionless frequency factor for three ends fixed and other one end simply supported plate for different values of Ra .

Ra	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.5	4	1.7E1	5.7E1	1.3E2	2.3E2	3.7E2	5.4E2	7.4E2	9.7E2	1.2E3	1.5E3	1.9E3	2.2E3	2.6E3	3.0E3	3.5E3	4.0E3
1.0	4	1.2E1	2.1E1	3.9E1	6.5E1	9.9E1	1.4E2	1.9E2	2.5E2	3.2E2	4.0E2	4.7E2	5.6E2	6.6E2	7.7E2	8.8E2	1.0E3
2.0	4	1.1E1	1.3E1	1.7E1	2.4E1	3.2E1	4.2E1	5.5E1	6.9E1	8.6E1	1.0E2	1.2E2	1.5E2	1.7E2	2.0E2	2.3E2	2.6E2
0.5	16	2.4E2	2.7E2	3.4E2	4.5E2	5.8E2	7.5E2	9.5E2	1.2E3	1.4E3	1.7E3	2.1E3	2.4E3	2.8E3	3.2E3	3.7E3	4.2E3
1.0	16	2.3E2	2.4E2	2.6E2	2.8E2	3.2E2	3.6E2	4.1E2	4.7E2	5.3E2	6.1E2	6.9E2	7.8E2	8.8E2	9.8E2	1.1E3	1.2E3
2.0	16	2.3E2	2.4E2	2.4E2	2.5E2	2.5E2	2.6E2	2.8E2	2.9E2	3.1E2	3.2E2	3.4E2	3.7E2	3.9E2	4.2E2	4.5E2	4.8E2
1.0	3	6.5E0	1.6E1	3.4E1	6.0E1	9.4E1	1.4E2	1.9E2	2.4E2	3.1E2	3.8E2	4.7E2	5.6E2	6.5E2	7.6E2	8.7E2	10.E2

Table-9 : Dimensionless frequency factor for three ends fixed and other one end simply supported plate for different values of Ra .

Ra	n/m	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.5	6	1.2E2	1.3E2	1.4E2	1.7E2	2.0E2	2.4E2	2.9E2	3.4E2	4.1E2	4.8E2	5.6E2	6.5E2	7.5E2	8.5E2	9.6E2	1.1E3
1.0	6	3.1E1	3.9E1	5.5E1	8.0E1	1.1E2	1.5E2	2.0E2	2.6E2	3.2E2	4.0E2	4.8E2	5.7E2	6.6E2	7.7E2	8.8E2	1.0E3
2.0	6	8.7E0	1.7E1	3.4E1	5.9E1	9.2E1	1.3E2	1.8E2	2.4E2	3.0E2	3.8E2	4.6E2	5.5E2	6.4E2	7.5E2	8.6E2	9.8E2
0.5	20	1.5E3	1.5E3	1.5E3	1.6E3	1.6E3	1.6E3	1.7E3	1.7E3	1.8E3	1.9E3	2.0E3	2.0E3	2.1E3	2.2E3	2.4E3	2.5E3
1.0	20	3.8E2	3.9E2	4.1E2	4.3E2	4.6E2	5.0E2	5.5E2	6.1E2	7.0E2	7.4E2	8.2E2	9.1E2	1.0E3	1.1E3	1.2E3	1.3E3
2.0	20	9.6E1	1.0E2	1.2E2	1.5E2	1.8E2	2.2E2	2.7E2	3.2E2	3.9E2	4.6E2	5.4E2	6.3E2	7.3E2	8.3E2	9.5e2	1.1E3

Table-10 : Power radiation by three ends fixed and other one end simply supported plate. Ra =1.0, Rt=0.002

m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	1.0E-2	1.4E0	4.7E1	1.3E2	2.8E2	5.5E2	1.1E3	1.2E3	4.3E3	2.0E3	4.8E3	8.9E3	2.6E4	6.5E3	1.5E4	6.1E4
3	1.3E-1	2.6E0	4.4E1	9.4E1	2.0E2	3.9E2	8.1E2	8.2E2	3.2E3	1.4E3	2.9E3	6.8E3	1.4E4	3.5E3	1.0E4	3.1E4
4	2.6E-1	6.9E0	6.6E1	1.3E2	2.6E2	5.0E2	1.0E3	1.0E3	3.9E3	2.1E3	3.3E3	9.6E3	1.7E4	4.4E3	1.4E4	3.2E4
5	5.0E0	2.6E1	6.9E1	1.3E2	2.6E2	4.7E2	1.0E3	1.0E3	3.6E3	2.5E3	2.1E3	1.1E4	9.1E3	2.9E3	1.8E4	2.9E4
6	2.9E1	6.4E1	9.7E1	1.8E2	3.3E2	5.5E2	1.2E3	1.2E3	3.9E3	3.5E3	1.8E3	1.6E4	9.3E3	3.1E3	1.8E4	2.0E4
15	1.1E3	1.1E3	9.9E2	1.2E3	1.6E3	3.8E3	1.5E3	5.0E3	1.7E3	1.1E4	1.5E4	3.1E3	3.6E3	5.2E4	4.1E3	2.3E3
16	9.6E2	9.5E2	1.2E3	1.6E3	3.7E3	2.1E3	3.4E3	3.6E3	2.4E3	2.4E4	4.9E3	2.5E3	7.9E3	4.2E4	4.5E3	4.7E3
17	1.2E3	1.4E3	2.1E3	3.5E3	2.7E3	1.9E3	4.9E3	2.1E3	7.2E3	2.2E4	3.6E3	3.2E3	2.7E4	2.1E4	5.2E3	3.3E3
18	2.7E3	3.4E3	4.0E3	2.8E3	1.4E3	4.6E3	3.2E3	2.5E3	1.8E4	1.2E4	3.4E3	3.7E3	3.9E4	1.2E4	4.3E3	6.3E3

Table-11 : Power radiation by three ends fixed and other one end simply supported plate. Ra =1.0, Rt=0.002

n/m	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
2	1.0E-2	2.6E-1	2.9e1	9.4E1	2.0E2	4.1E2	7.4E2	9.6E2	2.7E3	1.1E3	7.5E2	5.2E2	7.4E3	1.6E2	2.2E3	3.0E4
4	1.4E0	6.9E0	6.4E1	1.1E2	2.3E2	4.6E2	9.0E2	9.5E2	3.4E3	2.3E3	1.6E5	7.5E2	9.7E3	6.4E2	1.6E3	4.2E5
6	4.7E1	6.6E1	9.7E1	1.7E2	3.2E2	5.4E2	1.1E3	1.2E3	4.0E3	2.8E4	2.4E3	8.8E3	8.1E4	2.0E3	7.3E3	5.5E3
8	1.3E2	1.3E2	1.8E2	2.8E2	4.8e2	7.3E2	1.1E3	1.6E3	2.8E3	9.8E4	2.1E3	1.1E5	2.5E4	2.9E3	2.3E5	1.2e4
16	1.2E3	1.0E3	1.2E3	1.9E3	3.9E3	1.9E3	3.4E3	3.6E3	2.5E3	5.2E5	4.0E4	6.2E3	1.2E4	4.8E5	5.6E3	4.1E3
18	4.3E3	3.9E3	3.9E3	2.3E3	1.9E3	4.8E3	3.1E3	2.4E3	1.8E4	2.3E5	3.9E3	1.3E4	6.4E4	1.1E4	4.6E3	1.8E4
20	2.0E3	2.1E3	3.5E3	4.9E3	4.1E3	1.7E3	4.0E3	2.4E4	1.2E4	9.8E5	7.4E4	3.9E4	2.5E4	1.7E4	1.3E5	2.4E5
22	4.8E3	3.3E3	1.8E3	1.1E3	3.8E3	1.6E4	2.7E4	4.9E3	3.4E3	1.8E4	3.8E4	2.9E4	8.1E4	1.9E4	2.3E4	6.1E4

Table-12 : Power radiation by a three ends fixed and other one end simply supported plate.

Ra =1.00 and different values of Rt .

Rt	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.001	4	8.8E-3	1.2E-2	9.6E-1	4.5E0	5.1E1	2.6E2	3.2E2	4.7E2	7.1E2	1.1E3	1.1E3	2.4E3	2.5E3	7.2E3	1.3E3	6.8E3
0.002	4	2.6E-1	6.9E0	6.6E1	1.3E2	2.6E2	5.0E2	1.0E3	1.0E3	3.9E3	2.1E3	3.3E3	9.6E3	1.7E4	4.4E3	1.4E4	3.2E4
0.004	4	2.2E1	5.0E1	1.5E2	4.0E2	7.8E2	2.4E3	1.4E3	6.8E3	2.2E3	3.4E4	2.3E4	4.9E3	4.1E3	1.5E4	1.4E5	2.0E5
0.001	18	6.3E2	6.5E2	7.1E2	7.3E2	10.E2	1.2E3	1.2E3	2.2E3	1.4E3	2.4E3	4.1E3	4.1E3	4.0E3	4.8E3	2.5E3	2.8E4
0.002	18	2.7E3	3.4E3	4.0E3	2.8E3	1.4E3	4.6E3	3.2E3	2.5E3	1.8E4	1.2E4	3.4E3	3.7E3	3.9E4	1.2E4	4.3E3	6.3E3
0.004	18	1.5E3	1.7E3	1.8E3	4.5E3	2.0E4	6.5E4	2.5E4	7.7E3	4.1E3	5.2E3	8.5E3	2.3E4	3.2E5	6.4E4	1.5E4	1.4E4

Table-13 : Power radiation by a three ends fixed and other one end simply supported plate.

Rt =0.002 and different values of Ra .

Rt	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
0.50	4	2.0E0	5.2E1	2.1E2	6.6E2	1.6E3	4.1E3	3.3E3	1.3E4	4.0E3	4.1E4	5.2E4	6.0E3	5.6E3	2.6E4	2.1E5	7.9E5
1.00	4	2.6E-1	6.9E0	6.6E1	1.3E2	2.6E2	5.0E2	1.0E3	1.0E3	3.9E3	2.1E3	3.3E3	9.6E3	1.7E4	4.4E3	1.4E4	3.2E4
2.00	4	2.8E-1	5.6E-1	2.7E0	1.7E1	9.9E1	1.4E2	2.0E2	2.8E2	3.9E2	6.2E2	7.9E2	10.E2	9.8E2	3.6E3	1.2E3	3.0E3
0.50	18	1.5E3	1.6E3	1.5E3	1.6E3	8.2E3	2.8E3	1.5E4	5.0E3	4.1E4	1.2E5	1.4E4	8.3E3	1.1E4	1.7E4	4.4E5	2.1E4
1.00	18	2.7E3	3.4E3	4.0E3	2.8E3	1.4E3	4.6E3	3.2E3	2.5E3	1.8E4	1.2E4	3.4E3	3.7E3	3.9E4	1.2E4	4.3E3	6.3E3
2.00	18	5.2E3	4.7E3	6.2E3	8.5E3	6.7E3	9.5E3	5.9E3	4.8E3	1.9E3	2.9E3	9.2E3	9.7E3	7.6E3	3.5E3	3.0E3	4.0E3

Table-14 : Radiation efficiency of three ends fixed and other one simply supported plate for different mode orders .

Ra = 1.00

m	n	.02	.04	.06	.08	.10	.20	.40	.60	.80	1.0	1.2	1.4	1.6	1.8	2.0	4.0
2	2	1.8E-7	2.9E-6	1.5E-5	4.7E-5	1.1E-4	1.8E-3	2.6E-2	1.2E-1	3.0E-1	5.8E-1	8.8E-1	1.1E0	1.3E0	1.3E0	1.3E0	1.0E0
2	4	8.8E-7	1.4E-5	7.0E-5	2.2E-4	5.2E-4	7.0E-3	5.6E-2	1.2E-1	3.3E-1	8.7E-1	1.4E0	1.5E0	1.4E0	1.3E0	1.2E0	1.0E0
3	2	4.1E-7	6.5E-6	3.3E-5	1.0E-4	2.5E-4	3.9E-3	5.6E-2	2.4E-1	5.7E-1	9.7E-1	1.3E0	1.5E0	1.5E0	1.3E0	1.2E0	1.0E0
3	4	5.3E-7	8.4E-6	4.2E-5	1.3E-4	3.1E-4	4.2E-3	3.7E-2	1.2E-1	4.2E-1	1.1E0	1.7E0	1.6E0	1.4E0	1.3E0	1.2E0	1.0E0
2	14	1.6E-5	2.3E-4	9.0E-4	2.1E-3	3.5E-3	1.9E-2	3.6E-2	5.5E-2	1.5E-1	1.5E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	8.5E-1
2	15	6.8E-7	3.9E-5	3.6E-4	1.5E-3	3.9E-3	1.3E-2	4.1E-2	5.6E-2	1.2E-1	1.5E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	9.4E-1
3	15	2.6E-7	1.5E-5	1.4E-4	6.4E-4	1.8E-3	1.3E-2	4.0E-2	6.0E-2	1.3E-1	1.7E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	9.5E-1
3	16	8.0E-6	1.1E-4	4.3E-4	1.0E-3	1.9E-3	1.5E-2	3.0E-2	6.8E-2	1.0E-1	1.7E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	1.0E0
4	2	3.3E-6	4.7E-5	1.8E-4	4.2E-4	6.6E-4	1.5E-3	5.2E-3	1.0E-2	7.6E-2	1.9E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	8.8E-1
4	3	2.4E-8	1.4E-6	1.3E-5	5.7E-5	1.6E-4	1.1E-3	5.2E-3	1.0E-2	7.0E-2	1.9E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	9.0E-1
5	2	1.8E-5	2.5E-4	1.0E-3	2.4E-3	3.8E-3	7.1E-3	2.2E-2	2.9E-2	1.2E-1	2.1E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	8.9E-1
5	3	1.5E-7	8.7E-6	8.3E-5	3.6E-4	9.9E-4	6.0E-3	2.1E-2	3.0E-2	1.1E-1	2.1E0	1.9E0	1.5E0	1.3E0	1.2E0	1.1E0	9.0E-1
4	13	8.8E-9	4.3E-7	3.0E-6	8.1E-6	1.2E-5	4.5E-5	3.4E-4	2.7E-3	1.3E-1	1.7E0	1.9E0	1.5E0	1.3E0	1.1E0	1.1E0	1.1E0
4	15	1.0E-8	4.8E-7	3.0E-6	7.2E-6	8.5E-6	3.5E-5	2.8E-4	2.7E-3	1.2E-1	1.7E0	2.0E0	1.5E0	1.2E0	1.1E0	1.1E0	1.2E0
5	14	7.2E-7	7.6E-6	2.0E-5	2.7E-5	3.9E-5	2.0E-4	1.0E-3	5.6E-3	1.7E-1	1.9E0	1.9E0	1.5E0	1.2E0	1.1E0	1.0E0	1.2E0
5	16	7.1E-7	7.0E-6	1.6E-5	2.2E-5	3.6E-5	1.7E-4	9.3E-4	4.9E-3	1.5E-1	1.9E0	2.0E0	1.5E0	1.2E0	1.1E0	1.1E0	1.2E0

Table-15 : Dimensionless frequency factor for a plastic plate under different boundary conditions.

Ra =1.00 .

Two opposing ends free & other two ends simply supported :

Ra	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
1.0	3	7.5E0	1.6E1	3.3E1	5.7E1	8.9E1	1.3E2	1.8E2	2.3E2	3.0E2	3.7E2	4.5E2	5.4E2	6.3E2	7.4E2	8.5E2	9.7E2

Three ends fixed & other one end simply supported :

Ra	m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
1.0	3	6.5E0	1.6E1	3.4E1	6.0E1	9.4E1	1.4E2	1.9E2	2.4E2	3.1E2	3.8E2	4.7E2	5.6E2	6.5E2	7.6E2	8.7E2	10.E2

Table-16 : Power radiation for a plastic plate under different boundary conditions.
Ra =1.00 . Rt= 0.002.

Two opposing ends free & other two ends simply supported :

m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
4	1.3E-5	6.9E-6	5.2E-3	2.7E-1	1.1E0	3.3E0	1.0E1	2.1E2	5.2E2	5.7E2	7.5E2	9.5E2	1.1E3	1.5E3	2.9E3	2.2E3
16	2.2E2	3.3E2	5.1E2	5.6E2	5.1E2	5.5E2	6.3E2	7.7E2	1.0E3	9.4E2	1.7E3	1.3E3	3.3E3	1.9E3	2.2E3	7.5E3

Three ends fixed & other one end simply supported :

m/n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
4	5.6E-3	1.2E-2	6.0E-2	1.8E2	8.5E1	3.8E3	3.8E3	2.7E4	1.6E4	1.8E6	1.7E5	6.8E6	1.9E5	2.8E3	1.5E5	6.3E3
16	1.2E2	1.3E3	1.5E5	7.2E4	2.7E4	1.1E4	1.6E5	4.9E4	9.5E5	1.9E3	2.2E4	1.2E4	4.3E4	3.4E5	8.9E3	5.3E4

APPENDIX-C

PROGRAMMING FEATURES

C. 1. General Features:

The program listings presented in section C.5.have been developed by the author for numerical calculation of total average acoustic power radiation from the plates.The variables used in the program are given in table C.2.

Table- C. 2: Definition of computer variables.

Variable	Symbols used in the text	Description of variables
AR	R_a	Aspect ratio, b/a
TR	R_t	Thickness ratio, h/a
AH		Divisions of the range of integration,
AK		Divisions of the range of integration,
AMP		Amplitude factor of the power radiated,
AL	l	
AS	s	
GR	g	Acceleration due to gravity,
RM	R_m	
RPM	R'_m	
RN	R_n	
RPN	R'_n	
ROA	ρ	Density of the surrounding medium,

RO	ρ_m	Density of the plate material,
SLMB	λ_f^2	Dimensionless frequency factor,
TH	α	
TK	θ	
WNR	γ	Wave number ratio,
GM	γ_m	Roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$,
GPM	γ_m'	Roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$,
GN	γ_n	Roots of the equation $\tan(\gamma/2)+\tanh(\gamma/2)=0$,
GPN	γ_n'	Roots of the equation $\tan(\gamma/2)-\tanh(\gamma/2)=0$,
SUM		The value of the inner integration of equation(4.4),
FUNC		The value of the phase factor of the equation(4.4).

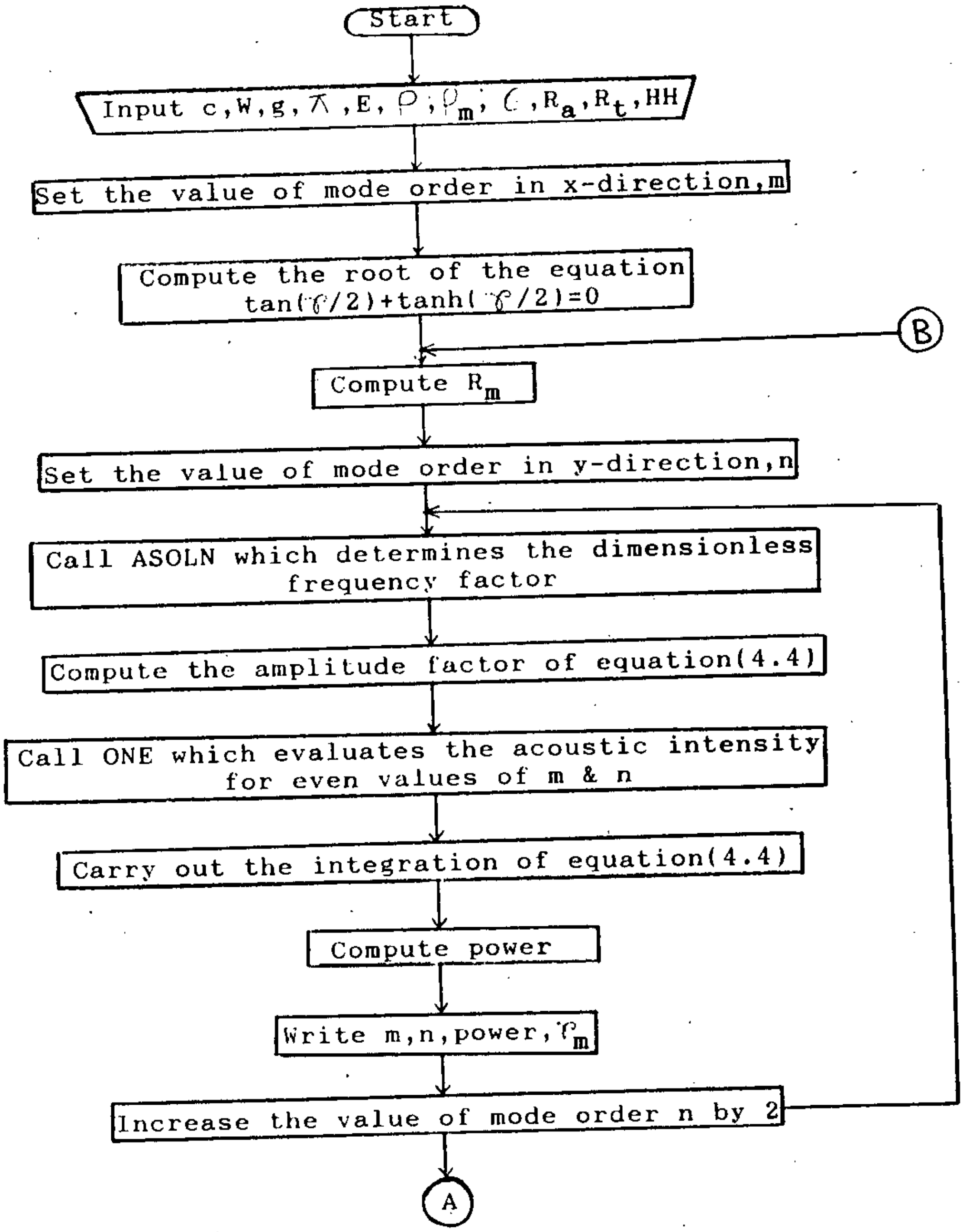
C.3. Procedures for calculating power radiation :

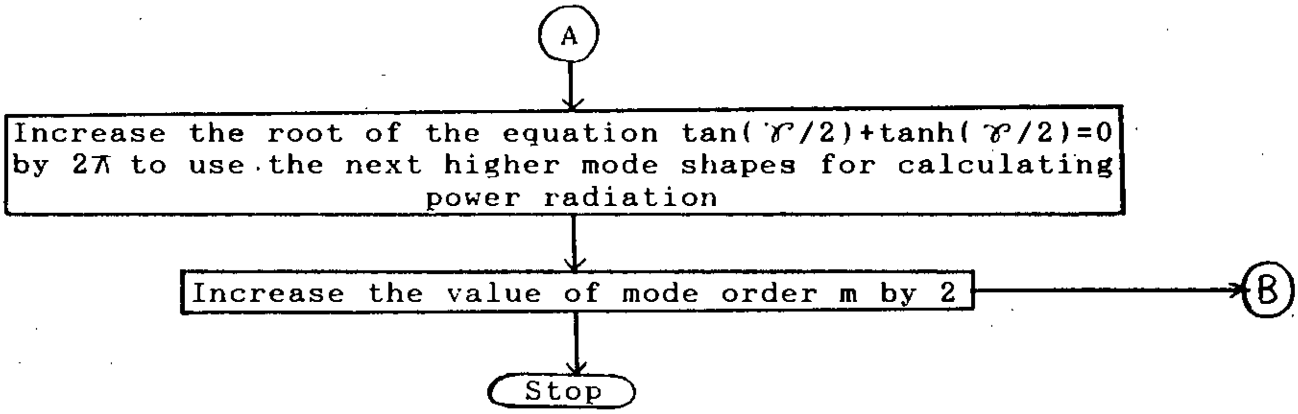
The procedures for evaluating power radiation from a free-simply supported plate have been described here.The program is given in section C.5.1.All other programs presented in section C.5.2.,C.5.3.and C.5.4.are similar in nature.

The list of variables used in the program are given in lines 8 to 20.Input variables c,W,g,π,E,ρ,ρ_m,σ,R_a,R_t & HH are read in lines 24 to 27 and output formats are given in lines 28 to 43.In lines 45 to 54,the roots of the equations $\tan(\gamma/2)\pm\tanh(\gamma/2)=0$ are calculated.Lines 55 to 62 contains the algorithm for calculation of amplitude factor of equation(4.4).Power radiations for even values of mode orders m & n are evaluated by integrating the equation(4.4) in lines 63 to 88.Cos(s/2) of equation(4.4) is used in this case.The output variables m,n,power & γ_m are printed out in lines 89 and 90.The whole procedures are repeated according to the statements shown in lines 91 to 93.In lines 95 to 143,the calculation of power radiations are performed for even values of m & odd values of n by integrating the equation(4.4) & taking sin(s/2) in place of cos(s/2). Lines 145 to 193 & 195 to 243 contain the algorithm for calculating power radiations for odd-odd & odd-even combinations of mode orders m & n respectively by integrating the equation (4.7a). The dimensionless frequency factor is evaluated in lines 248 to 264.Lines 266 to 294,321 to 349,378 to 406 and 434 to 462 are for the integration of acoustic intensity for various combinations of mode orders.The phase factor of the acoustic intensity for various mode orders are evaluated in lines 296 to 319,352 to 375,409 to 432 and 464 to 487.

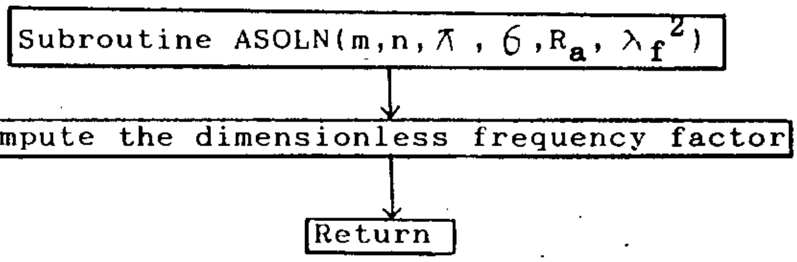
C.4. Flow charts for power radiation from a free-simply suppported plate.

C.4.1. Flow chart for main program.

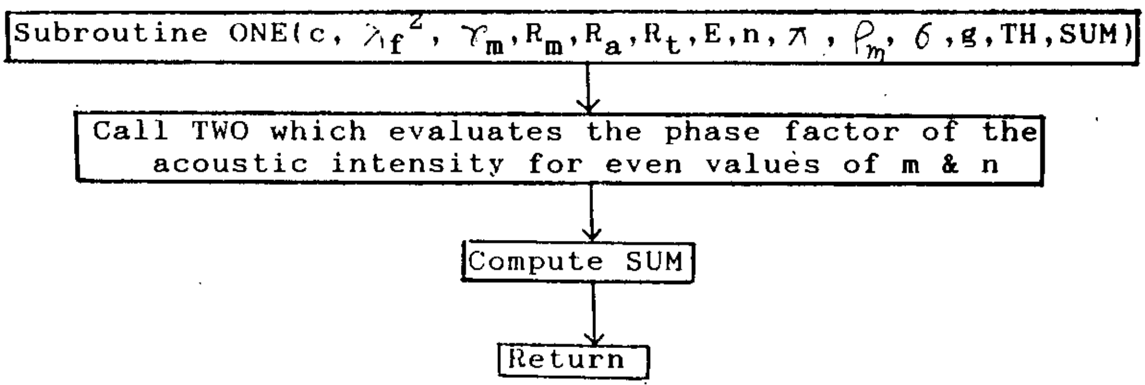




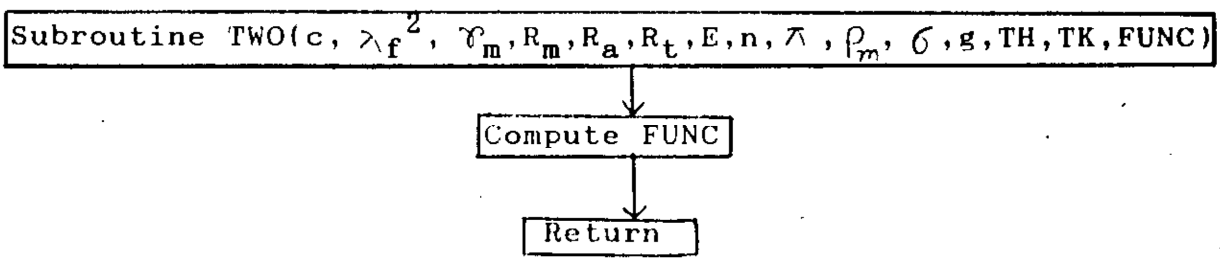
C.4.2. Flow chart for subroutine ASOLN.



C.4.3. Flow chart for subroutine ONE.



C.4.4. Flow chart for subroutine TWO.



Flow charts for subroutines THREE,FIVE and SEVEN are same as subroutine ONE.Also the flow charts for subroutines FOUR,SIX and EIGHT are identical to subroutine TWO.

C.5.1.PROGRAM FOR FREE-SIMPLY SUPPORTED PLATE(POWER RADIATION)

```
*         ***************************************************************
*         ************ LIST OF VARIABLES ********************************
*         ***************************************************************
*         A=LENGTH OF THE PLATE IN METERS.
*         B=WIDTH OF THE PLATE IN METERS.
*         C=VELOCITY OF SOUND IN AIR,METERS PER SECOND.
*         PI=A CONSTANT QUANTITY EQUIVALENT TO 3.1416
*         GR=ACCELARATION DUE TO GRAVITY,METERS PER SECOND SQ.
*         W=AMPLITUDE OF VIBRATION,METERS.
*         H=THICKNESS OF THE PLATE,METERS.
*         E=MODULUS OF ELASTICITY OF THE PLATE MATERIAL,NEWTON PER METER SQ.
*         RO=SPECIFIC WEIGHT OF THE PLATE MATERIAL,KG PER METER CUBE.
*         ROA=DENSITY OF AIR,KG PER METER CUBE.
*         SIG=POISON'S RATIO.
*         AR=ASPECT RATIO,B/A
*         TR=THICKNESS RATIO,H/A
*.        ***************************************************************
          OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
          OPEN(UNIT=3,FILE='IN',STATUS='OLD')
          READ(3,996)C,W,GR,PI,E,ROA,RO,SIG,HH
996       FORMAT(F4.0/F6.4/F4.2/F6.4/E9.3/F4.2/F5.0/F4.2/F6.4)
997       READ(3,998)AR,TR
998       FORMAT(F4.2/F5.3)
          WRITE(9,999)
999       FORMAT(//20X,'****** INPUT DATA ******',//)
          WRITE(9,1000)AR,TR,C,W,GR,PI,E,ROA,RO,SIG,HH
1000      FORMAT(/20X,'AR=',F3.1/20X,'TR=',F5.3/20X,'C=',F4.0/20X,'W=',
         +F6.4/20X,'GR=',F4.2/20X,'PI=',F6.4/20X,'E=',E9.3/20X,'ROA=',
         +F4.2/20X,'RO=',F5.0/20X,'SIG=',F4.2/20X,'HH=',F6.4,/)
          WRITE(9,1001)
1001      FORMAT(10X,'NUMBER OF DIVISIONS=36',/)
          WRITE(9,1002) AR,TR
1002      FORMAT(5X,'ASPECT RATIO= ',F6.4,5X,'THICKNESS RATIO= ',F6.4,/)
          WRITE(9,1003)
1003      FORMAT(1X,131('-'))
          WRITE(9,1004)
1004      FORMAT(3X,'* M *',5X,'* N *',9X,'POWER RADIATED',9X,'GM/GPM')
          WRITE(9,1005)
1005      FORMAT(1X,131('-'),/)
*         ***************************************************************
124       X11=2.2
          DO 1 M=2,32,2
12        X12=X11+HH
          Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
          Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
          Z11=Y11*Y12
          IF(Z11.LT.0.00) GO TO 10
          X11=X12
          GO TO 12
10        GM=X11+X12
          HGM=GM/2.0
```

FILE: FSPR   FORTRAN   A1   BUET COMPUTER CENTRE, DHAKA   VM/SP (4331-L02)

```
      RM=-SIN(HGM)/SINH(HGM)
      DO 2 N=2,32,2
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*RUA*W*W*AR*AR*(PI**4.0)*E*E*(TR**4.0)*SLMB*SLMB*GR*GR
      HN=FLOAT(N)
      AM2=9.*C*RO*RO*((1.-SIG*SIG)**2.0)*(HN-1.0)*(HN-1.0)
      AMP=AM1/AM2
      K=36
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
70    TH=TH+AH
      I=I+1
      CALL ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 70
      TH=TH+AH
      CALL ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GM
      WRITE(9,200) M,N,POWER,GM
200   FORMAT(5X,I3,7X,I3,10X,E10.4,10X,F10.4)
2     CONTINUE
      X11=X11+3.1
1     CONTINUE
*************************************************************************
      X11=2.2
      DO 3 M=2,32,2
52    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 50
      X11=X12
      GO TO 52
50    GM=X11+X12
      HGM=GM/2.0
      RM=-SIN(HGM)/SINH(HGM)
      DO 4 N=3,32,2
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*RUA*W*W*AR*AR*(PI**4.)*E*E*(TR**4.0)*GR*GR*SLMB*SLMB
      HN=FLOAT(N)
```

```
      AM2=9.0*C*RO*RO*((1.0-SIG*SIG)**2.0)*(HN-1.0)*(HN-1.0)
      AMP=AM1/AM2
      K=35
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
270   TH=TH+AH
      I=I+1
      CALL THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 270
      TH=TH+AH
      CALL THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GM
      WRITE(7,400) M,N,POWER,GM
400   FORMAT(5X,I2,7X,I2,10X,E10.4,10X,F10.4)
4     CONTINUE
      X11=X11+3.1
3     CONTINUE
*     ******************************************************************
      X11=3.9
      DO 5 M=3,32,2
32    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 30
      X11=X12
      GO TO 32
30    GPM=X11+X12
      HGPM=GPM/2.0
      RPM=SIN(HGPM)/SINH(HGPM)
      DO 6 N=3,32,2
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*ROA*W*W*AR*AR*(PI**4.0)*E*E*(TR**4.0)*GR*GR*SLMB*SLMB
      HN=FLOAT(N)
      AM2=9.0*C*RO*RO*((1-SIG*SIG)**2.0)*(HN-1)*(HN-1.0)
      AMP=AM1/AM2
      K=35
      AH=(PI/2.0)/K
      KK=K-1
```

```
      GSUM=0.0
      TH=0.0
      I=1
      CALL FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
170   TH=TH+AH
      I=I+1
      CALL FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 170
      TH=TH+AH
      CALL FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GPM
      WRITE(9,300) M,N,POWER,GPM
300   FORMAT(5X,I2,7X,I2,10X,E10.4,10X,F10.4)
6     CONTINUE
      X11=X11+3.1
5     CONTINUE
*     ***************************************************************
      X11=3.9
      DO 7 M=3,32,2
112   X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 110
      X11=X12
      GO TO 112
110   GPM=X11+X12
      HGPM=GPM/2.0
      RPM=SIN(HGPM)/SINH(HGPM)
      DO 8 N=2,32,2
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*ROA*W*W*AR*AR*(PI**4.)*E*E*(TR**4.0)*GR*GR*SLMB*SLMB
      HN=FLOAT(N)
      AM2=9.0*C*RO*RO*((1.0-SIG*SIG)**2.0)*(HN-1.0)*(HN-1.0)
      AMP=AM1/AM2
      K=36
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
```

```
370   TH=TH+AH
      I=I+1
      CALL SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 370
      TH=TH+AH
      CALL SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GPM
      WRITE(9,500) M,N,POWER,GPM
500   FORMAT(5X,I2,7X,I2,10X,E10.4,10X,F10.4)
8     CONTINUE
      X11=X11+3.1
7     CONTINUE
      STOP
      END
*     ********************************************************************
*     ********************************************************************
      SUBROUTINE ASOLN(M,N,PI,SIG,AR,SLMB)
*     THIS SUBROUTINE EVALUATES THE DIMENSIONLESS FREQUENCY FACTOR.
      FM=FLOAT(M)
      GX=FM-0.5
      HX=GX*GX*(1.0-(2.0/(GX*PI)))
      XJ=GX*GX*(1.0+(6.0/(GX*PI)))
      FN=FLOAT(N)
      GY=FN-1.0
      HY=GY*GY
      YJ=HY
      ST1=GX*GX*GX*GX
      ST2=(GY*GY*GY*GY)/(AR*AR*AR*AR)
      ST3=SIG*HX*HY+(1.0-SIG)*XJ*YJ
      ST4=(2.0*ST3)/(AR*AR)
      SLMB=ST1+ST2+ST4
      RETURN
      END
*     ****************************************************************
      SUBROUTINE ONE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR EVEN VALUES OF M AND N.
      K=36
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
```

```
      SUM=SUM+FUNC
90    TK=TK+AK
      I=I+1
      CALL TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 90
      TK=TK+AK
      CALL TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ***************************************************************
      SUBROUTINE TWO(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE IS USED TO DETERMINE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR EVEN VALUES OF M AND N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*AR*TR*SIN(TK)*SIN(TH)/C
      HGM=GM/2.0
      ST5=GM*SIN(HGM)*COS(AL/2.0)
      ST6=AL*COS(HGM)*SIN(AL/2.0)
      ST7=GM*SINH(HGM)*COS(AL/2.0)
      ST8=AL*COSH(HGM)*SIN(AL/2.0)
      ST9=COS(AS/2.0)
      HN=FLOAT(N)
      ST10=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST11=GM*GM-AL*AL
      ST12=GM*GM+AL*AL
      ST13=(ST5-ST6)/ST11+RM*((ST7+ST8)/ST12)
      ST14=ST9/ST10
      FUNC=ST13*ST13*ST14*ST14*SIN(TK)
      RETURN
      END
*     *******************************************************************
      SUBROUTINE THREE(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR EVEN VALUES OF M AND ODD VALUES OF N.
      K=36
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
```

```
      SUM=SUM+FUNC
290   TK=TK+AK
      I=I+1
      CALL FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 290
      TK=TK+AK
      CALL FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE FOUR(C,SLMB,GM,RM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE ACOUSTIC
*     INTENSITY FOR EVEN VALUES OF M AND ODD VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*FR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*AR*TR*SIN(TK)*SIN(TH)/C
      HGM=GM/2.0
      ST25=GM*SIN(HGM)*COS(AL/2.0)
      ST26=AL*COS(HGM)*SIN(AL/2.0)
      ST27=GM*SINH(HGM)*COS(AL/2.0)
      ST28=AL*COSH(HGM)*SIN(AL/2.0)
      ST29=SIN(AS/2.0)
      HN=FLOAT(N)
      ST30=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST31=GM*GM-AL*AL
      ST32=GM*GM+AL*AL
      ST33=(ST25-ST26)/ST31+RM*((ST27+ST28)/ST32)
      ST34=ST29/ST30
      FUNC=ST33*ST33*ST34*ST34*SIN(TK)
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE FIVE(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR ODD VALUES OF M AND N.
      K=36
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
```

```
      I=1
      CALL SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
  190 TK=TK+AK
      I=I+1
      CALL SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 190
      TK=TK+AK
      CALL SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ********************************************************************
*     ********************************************************************
      SUBROUTINE SIX(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE ACOUSTIC
*     INTENSITY FOR ODD VALUES OF M AND N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*TR*PI*PI*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*AR*TR*PI*PI*SIN(TK)*SIN(TH)/C
      HGPM=GPM/2.0
      ST15=GPM*COS(HGPM)*SIN(AL/2.0)
      ST16=AL*SIN(HGPM)*COS(AL/2.0)
      ST17=AL*SINH(HGPM)*COS(AL/2.0)
      ST18=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST19=SIN(AS/2.0)
      HN=FLOAT(N)
      ST20=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST21=GPM*GPM-AL*AL
      ST22=GPM*GPM+AL*AL
      ST23=(ST15-ST16)/ST21+RPM*((ST17-ST18)/ST22)
      ST24=ST19/ST20
      FUNC=ST23*ST23*ST24*ST24*SIN(TK)
      RETURN
      END
*     ********************************************************************
      SUBROUTINE SEVEN(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR ODD VALUES OF M AND EVEN VALUES OF N.
      K=35
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
```

```
      TK=0.0
      I=1
      CALL EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
390   TK=TK+AK
      I=I+1
      CALL EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 390
      TK=TK+AK
      CALL EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ************************************************************
      SUBROUTINE EIGHT(C,SLMB,GPM,RPM,AR,TR,E,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE ACOUSTIC
*     INTENSITY FOR ODD VALUES OF M AND EVEN VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*TR*PI*PI*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*AR*TR*PI*PI*SIN(TK)*SIN(TH)/C
      HGPM=GPM/2.0
      ST35=GPM*COS(HGPM)*SIN(AL/2.0)
      ST36=AL*SIN(HGPM)*COS(AL/2.0)
      ST37=AL*SINH(HGPM)*COS(AL/2.0)
      ST38=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST40=COS(AS/2.0)
      HN=FLOAT(N)
      ST41=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST42=GPM*GPM-AL*AL
      ST43=GPM*GPM+AL*AL
      ST44=(ST35-ST36)/ST42+RPM*((ST37-ST38)/ST43)
      ST45=ST40/ST41
      FUNC=ST44*ST44*ST45*ST45*SIN(TK)
      RETURN
      END
```

```
C.5.2.PROGRAM FOR FREE-SIMPLY SUPPORTED PLATE(RADIATION EFFICIENCY)

*       ************************************************************
*       ********************** LIST OF VARIABLES *******************
*       ************************************************************
*       A=LENGTH OF THE PLATE IN METERS
*       B=WIDTH OF THE PLATE IN METERS
*       AR=ASPECT RATIO, B/A
*       C=VELOCITY OF SOUND IN METERS PER SECOND
*       RO=WEIGHT OF THE PLATE MATERIAL PER UNIT VOLUME IN KG.PER C.M.
*       ROA=DENSITY OF AIR IN KG.PER C.M.
*       W=AMPILTUDE OF VIBRATION
*       PI=A CONSTANT EQ. TO 22.0/7.0
*       GR=ACCELARATION DUE TO GRAVITY IN METERS PER SEC SQ.
*       IN METERS
*       E=MODULUS OF ELASTICITY OF THE PLATE MATERIAL IN NEWTON PER M.SQ.
*       ************************************************************
        OPEN(UNIT=8,FILE='OUT',STATUS='NEW')
        OPEN(UNIT=3,FILE='IN',STATUS='OLD')
        READ(3,996)C,W,GR,PI,E,ROA,RO,SIG,HH
996.    FORMAT(F4.0/F6.4/F4.2/F6.4/E9.3/F4.2/F5.0/F3.1/F6.4)
997     READ(3,998)AR,WNR
998     FORMAT(F4.2/F4.2)
        WRITE(8,999)
999     FORMAT(//20X,'****** INPUT DATA ******',//)
        WRITE(8,1000)AR,C,W,GR,PI,E,ROA,RO,SIG,HH,WNR
1000    FORMAT(20X,'AR=',F3.1/20X,'C=',F4.0/20X,'W=',F6.4/20X,'GR=',F4.2
       +/20X,'PI=',F6.4/20X,'E=',E9.3/20X,'ROA=',F4.2/20X,'RO=',F5.0
       +/20X,'SIG=',F3.1/20X,'HH=',F6.4/20X,'WNR=',F5.3)
        WRITE(8,1001)AR,WNR
1001    FORMAT(5X,'ASPECT RATIO= ',F6.4,5X,'WNR=',F6.4,/)
        WRITE(8,1002)
1002    FORMAT(1X,75('*'),/,1X,20('*'),1X,'OUTPUT RESULTS ',35('*'),
       +/,1X,75('*'),/)
        WRITE(8,1003)
1003    FORMAT(2X,'* M *',5X,'* N *',7X,'GM/GPM',3X,'RADIATION EFFICIENCY'
       +,/)
*       ************************************************************
124     X11=2.2
        DO 1 M=2,16,2
12      X12=X11+HH
        Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
        Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
        Z11=Y11*Y12
        IF(Z11.LT.0.00) GO TO 10
        X11=X12
        GO TO 12
10      GM=X11+X12
        HGM=GM/2.0
        RM=-SIN(HGM)/SINH(HGM)
        DO 2 N=2,16,2
        MP=M-1
        NP=N-1
```

```
      FT1=(64.0*WNR*WNR)*(AR*MP*MP+(NP*NP)/AR)
      FT2=NP*NP*PI*PI
      AMPL=FT1/FT2
      K=36
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
 70   TH=TH+AH
      I=I+1
      CALL ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 70
      TH=TH+AH
      CALL ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      HT1=1.0+(SIN(GM))/GM
      HT2=COS(HGM)*SINH(HGM)
      HT3=SIN(HGM)*COSH(HGM)
      HT4=(4.0*RM)/GM
      HT5=1.0+(SINH(GM))/GM
      HT6=RM*RM
      HT7=HT4*(HT2+HT3)
      HT8=HT5*HT6
      HT9=HT1+HT7+HT8
      SH=(GSUM*AMPL)/HT9
      WRITE(*,*)M,N,GM,SH
      WRITE(8,222)M,N,GM,SH
 222  FORMAT(4X,I2,7X,I2,4X,F10.4,9X,E10.4)
 2    CONTINUE
      X11=X11+3.1
 1    CONTINUE
*     ****************************************************************
      X11=2.2
      DO 3 M=2,16,2
 52   X12=X11+HH
      Y11=(SIN(X11)/COS(X11))+(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))+(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 50
      X11=X12
      GO TO 52
 50   GM=X11+X12
      HGM=GM/2.0
```

```
      RM=-SIN(HGM)/SINH(HGM)
      DO 4 N=3,16,2
      MP=M-1
      NP=N-1
      FT3=(64.0*WNR*WNR)*(AR*MP*MP+(NP*NP)/AR)
      FT4=NP*NP*PI*PI
      AMP2=FT3/FT4
      K=35
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
270   TH=TH+AH
      I=I+1
      CALL THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 270
      TH=TH+AH
      CALL THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      HT1=1.0+(SIN(GM))/GM
      HT2=COS(HGM)*SINH(HGM)
      HT3=SIN(HGM)*COSH(HGM)
      HT4=(4.0*RM)/GM
      HT5=1.0+(SINH(GM))/GM
      HT6=RM*RM
      HT7=HT4*(HT2+HT3)
      HT8=HT5*HT6
      HT9=HT1+HT7+HT8
      SH=(GSUM*AMP2)/HT9
      WRITE(*,*)M,N,GM,SH
      WRITE(8,400) M,N,GM,SH
400   FORMAT(4X,I2,7X,I2,4X,F10.4,9X,E10.4)
4     CONTINUE
      X11=X11+3.1
3     CONTINUE
*     *************************************************************
      X11=3.9
      DO 5 M=3,15,2
32    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 30
```

```
      X11=X12
      GO TO 32
30    GPM=X11+X12
      HGPM=GPM/2.0
      RPM=SIN(HGPM)/SINH(HGPM)
      DO 6 N=3,15,2
      MP=M-1
      NP=N-1
      FT5=(64.0*WNR*WNR)*(AR*MP*MP+(NP*NP)/AR)
      FT6=NP*NP*PI*PI
      AMP3=FT5/FT6
      K=36
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
170   TH=TH+AH
      I=I+1
      CALL FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 170
      TH=TH+AH
      CALL FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      HT1=1.0-(SIN(GPM))/GPM
      HT2=(RPM*RPM)*(((SINH(GPM))/GPM)-1.0)
      HT3=(4.0*RPM)/GPM
      HT4=SIN(HGPM)*COSH(HGPM)-COS(HGPM)*SINH(HGPM)
      HT5=HT1+HT2
      HT6=HT3*HT4
      HT7=HT5+HT6
      SH=(GSUM*AMP3)/HT7
      WRITE(*,*) M,N,GPM,SH
      WRITE(8,300) M,N,GPM,SH
300   FORMAT(4X,I2,7X,I2,4X,F10.4,9X,E10.4)
6     CONTINUE
      X11=X11+3.1
5     CONTINUE
*     ******************************************************************
      X11=3.9
      DO 7 M=3,16,2
112   X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
```

```
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 110
      X11=X12
      GO TO 112
110   GPM=X11+X12
      HGPM=GPM/2.0
      RPM=SIN(HGPM)/SINH(HGPM)
      DO 8 N=2,16,2
      MP=M-1
      NP=N-1
      FT7=(64.0*WMR*WMR)*(AR*MP*MP+(NP*NP)/AR)
      FT8=NP*NP*PI*PI
      AMP4=FT7/FT8
      K=36
      AH=(PI/2.0)/K
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL SEVEN(C,WMR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
370   TH=TH+AH
      I=I+1
      CALL SEVEN(C,WMR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL SEVEN(C,WMR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 370
      TH=TH+AH
      CALL SEVEN(C,WMR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL SEVEN(C,WMR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      HT1=1.0-(SIN(GPM))/GPM
      HT2=(RPM*RPM)*(((SINH(GPM))/GPM)-1.0)
      HT3=(4.0*RPM)/GPM
      HT4=SIN(HGPM)*COSH(HGPM)-COS(HGPM)*SINH(HGPM)
      HT5=HT1+HT2
      HT6=HT3*HT4
      HT7=HT5+HT6
      SH=(GSUM*AMP4)/HT7
      WRITE(*,*) M,N,GPM,SH
      WRITE(8,500) M,N,GPM,SH
500   FORMAT(4X,I2,7X,I2,4X,F10.4,9X,E10.4)
8     CONTINUE
      X11=X11+3.1
7     CONTINUE
      STOP
      END
*     ****************************************************************
*     ****************************************************************
```

```
      SUBROUTINE ONE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR EVEN VALUES OF M AND N.
      K=35
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
90    TK=TK+AK
      I=I+1
      CALL TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 90
      TK=TK+AK
      CALL TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ***********************************************************
      SUBROUTINE TWO(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE IS USED TO DETERMINE THE PHASE FACTOR OF THE
*     RADIATION EFFICIENCY FOR EVEN VALUES OF M AND N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGM=GM/2.0
      ST5=GM*SIN(HGM)*COS(AL/2.0)
      ST6=AL*COS(HGM)*SIN(AL/2.0)
      ST7=GM*SINH(HGM)*COS(AL/2.0)
      ST8=AL*COSH(HGM)*SIN(AL/2.0)
      ST9=COS(AS/2.0)
      HN=FLOAT(N)
      ST10=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST11=GM*GM-AL*AL
      ST12=GM*GM+AL*AL
      ST13=(ST5-ST6)/ST11+RM*((ST7+ST8)/ST12)
      ST14=ST9/ST10
      FUNC=ST13*ST13*ST14*ST14*SIN(TK)
      RETURN
```

```
      END
*     *****************************************************************
      SUBROUTINE THREE(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR EVEN VALUES OF M AND ODD VALUES OF N.
      K=35
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
290   TK=TK+AK
      I=I+1
      CALL FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 290
      TK=TK+AK
      CALL FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     *****************************************************************
*     *****************************************************************
      SUBROUTINE FOUR(C,WNR,GM,RM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE RADIATION
*     EFFICIENCY FOR EVEN VALUES OF M AND ODD VALUES OF N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGM=GM/2.0
      ST25=GM*SIN(HGM)*COS(AL/2.0)
      ST26=AL*COS(HGM)*SIN(AL/2.0)
      ST27=GM*SINH(HGM)*COS(AL/2.0)
      ST28=AL*COSH(HGM)*SIN(AL/2.0)
      ST29=SIN(AS/2.0)
      HN=FLOAT(N)
      ST30=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST31=GM*GM-AL*AL
      ST32=GM*GM+AL*AL
      ST33=(ST25-ST26)/ST31+RM*((ST27+ST28)/ST32)
```

```
      ST34=ST29/ST30
      FUNC=ST33*ST33*ST34*ST34*SIN(TK)
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE FIVE(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR ODD VALUES OF M AND N.
      K=35
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
190   TK=TK+AK
      I=I+1
      CALL SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 190
      TK=TK+AK
      CALL SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE SIX(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE RADIATION
*     EFFICIENCY FOR ODD VALUES OF M AND N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGPM=GPM/2.0
      ST15=GPM*COS(HGPM)*SIN(AL/2.0)
      ST16=AL*SIN(HGPM)*COS(AL/2.0)
      ST17=AL*SINH(HGPM)*COS(AL/2.0)
      ST18=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST19=SIN(AS/2.0)
```

```
      HN=FLOAT(N)
      ST20=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST21=GPM*GPM-AL*AL
      ST22=GPM*GPM+AL*AL
      ST23=((ST15-ST16)/ST21)+RPM*((ST17-ST18)/ST22)
      ST24=ST19/ST20
      FUNC=ST23*ST23*ST24*ST24*SIN(TK)
      RETURN
      END
*     ******************************************************************
      SUBROUTINE SEVEN(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR ODD VALUES OF M AND EVEN VALUES OF N.
      K=36
      AK=(PI/2.0)/K
      KK=K-1
      SUM=0.0
      TK=0.0
      I=1
      CALL EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
390   TK=TK+AK
      I=I+1
      CALL EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 390
      TK=TK+AK
      CALL EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ******************************************************************
      SUBROUTINE EIGHT(C,WNR,GPM,RPM,AR,TR,E,M,N,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE RADIATION
*     EFFICIENCY FOR ODD VALUES OF M AND EVEN VALUES OF N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGPM=GPM/2.0
      ST35=GPM*COS(HGPM)*SIN(AL/2.0)
      ST36=AL*SIN(HGPM)*COS(AL/2.0)
```

```
      ST37=AL*SINH(HGPM)*COS(AL/2.0)
      ST38=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST40=COS(AS/2.0)
      HN=FLOAT(N)
      ST41=(AS*AS)/((PI*PI)*(HN-1.0)*(HN-1.0))-1.0
      ST42=GPM*GPM-AL*AL
      ST43=GPM*GPM+AL*AL
      ST44=((ST35-ST36)/ST42)+RPM*((ST37-ST38)/ST43)
      ST45=ST40/ST41
      FUNC=ST44*ST44*ST45*ST45*SIN(TK)
      RETURN
      END
```

```
C.5.3.PROGRAM FOR CLAMPED-SIMPLY SUPPORTED PLATE(POWER RADIATION)

*      ****************************************************************
*      ******************** LIST OF VARIABLES ************************
*      ****************************************************************
*      A=LENGTH OF THE PLATE IN METERS.
*      B=WIDTH OF THE PLATE IN METERS.
*      C=VELOCITY OF SOUND IN AIR,METERS PER SECOND.
*      PI=A CONSTANT QUANTITY EQUIVALENT TO 3.1416
*      GR=ACCELARATION DUE TO GRAVITY,METERS PER SECOND SQ.
*      W=AMPLITUDE OF VIBRATION,METERS.
*      H=THICKNESS OF THE PLATE,METERS.
*      E=MODULUS OF ELASTICITY OF THE PLATE MATERIAL,NEWTON PER METER SQ.
*      RO=SPECIFIC WEIGHT OF THE PLATE MATERIAL,KG PER METER CUBE.
*      ROA=DENSITY OF AIR,KG PER METER CUBE.
*      SIG=POISON'S RATIO.
*      AR=ASPECT RATIO,B/A
*      TR=THICKNESS RATIO,H/A
*      *****************************************************************
*      *****************************************************************
       OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
       OPEN(UNIT=3,FILE='IN',STATUS='OLD')
       READ(3,996)C,W,GR,PI,E,ROA,RO,SIG,HH
996    FORMAT(F4.0/F6.4/F4.2/F6.4/E9.3/F4.2/F5.0/F3.1/F6.4)
997    READ(3,998)AR,TR
998    FORMAT(F4.2/F5.3)
       WRITE(9,999)
999    FORMAT(//20X,'****** INPUT DATA ******',//)
       WRITE(9,1000)AR,TR,C,W,GR,PI,E,ROA,RO,SIG,HH
1000   FORMAT(/20X,'AR=',F3.1/20X,'TR=',F5.3/20X,'C=',F4.0/20X,'W=',
      +F6.4/20X,'GR=',F4.2/20X,'PI=',F6.4/20X,'E=',E9.3/20X,'ROA=',
      +F4.2/20X,'RO=',F5.0/20X,'SIG=',F3.1/20X,'HH=',F6.4,/)
       WRITE(9,1001)
1001   FORMAT(10X,'NUMBER OF DIVISIONS=35',/)
       WRITE(9,1002) AR,TR
1002   FORMAT(5X,'ASPECT RATIO= ',F6.4,5X,'THICKNESS RATIO= ',F6.4,/)
       WRITE(9,1003)
1003   FORMAT(1X,131('-'))
       WRITE(9,1004)
1004   FORMAT(3X,'* M *',5X,'* N *',5X,'POWER RADIATED',5X,'GPM/GN',
      +5X,'GPM/GPN')
       WRITE(9,1005)
1005   FORMAT(1X,131('-'),/)
*      *****************************************************************
124    X11=3.9
       DO 1 M=2,32
12     X12=X11+HH
       Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
       Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
       Z11=Y11*Y12
       IF(Z11.LT.0.00) GO TO 10
       X11=X12
       GO TO 12
```

```
10    GPM=X11+X12
      X21=2.2
      DO 2 N=2,32,2
22    X22=X21+HH
      Y21=(SIN(X21)/COS(X21))+(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))+(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.00) GO TO 20
      X21=X22
      GO TO 22
20    GN=X21+X22
      HGPM=GPM/2.0
      HGN=GN/2.0
      RPM=-SIN(HGPM)/SINH(HGPM)
      RN=SIN(HGN)/SINH(HGN)
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*ROA*W*W*AR*AR*(PI**6.0)*E*E*TR*TR*TR*TR*GR*GR*SLMB*SLMB
      AM2=9.*C*RO*RO*(1.-SIG*SIG)*(1.-SIG*SIG)
      AMP=AM1/AM2
      K=35
      KK=K-1
      AH=(PI/2)/K
      GSUM=0.0
      TH=0.0
      I=1
      CALL ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
70    TH=TH+AH
      I=I+1
      CALL ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 70
      TH=TH+AH
      CALL ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GPM,GN
      WRITE(9,200) M,N,POWER,GPM,GN
200   FORMAT(3X,I2,4X,I2,7X,E10.4,7X,F10.4,7X,F10.4)
      X21=X21+3.1
2     CONTINUE
*     ************************************************************************
      X11=X11+3.1
*     ************************************************************************
1     CONTINUE
*     ************************************************************************
      X11=3.9
```

```
      DO 3 M=2,32
32    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.0) GO TO 30
      X11=X12
      GO TO 32
30    GPM=X11+X12
      X21=3.9
      DO 4 N=3,32,2
42    X22=X21+HH
      Y21=(SIN(X21)/COS(X21))-(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))-(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.0) GO TO 40
      X21=X22
      GO TO 42
40    GPN=X21+X22
      HGPM=GPM/2.0
      HGPN=GPN/2.0
      RPM=-SIN(HGPM)/SINH(HGPM)
      RPN=-SIN(HGPN)/SINH(HGPN)
      CALL ASOLN(M,N,PI,SIG,AR,SLMB)
      AM1=2.0*RUA*W*W*AR*AR*(PI**6.0)*C*E*TR*TR*TR*TR*GR*GR*SLMB*SLMB
      AM2=9.0*C*RO*RO*(1.-SIG*SIG)*(1.-SIG*SIG)
      AMP=AM1/AM2
      K=36
      AH=PI/(2.0*K)
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
170   TH=TH+AH
      I=I+1
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 170
      TH=TH+AH
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      POWER=4.0*GSUM*AMP
      WRITE(*,*) M,N,POWER,GPM,GPN
      WRITE(7,300) M,N,POWER,GPM,GPN
300   FORMAT(3X,I2,4X,I2,7X,E10.4,7X,F10.4,7X,F10.4)
```

```
      X21=X21+3.1
4     CONTINUE
      X11=X11+3.1
3     CONTINUE
      STOP
      END
*     ****************************************************************
      SUBROUTINE ASOLN(M,N,PI,SIG,AR,SLMB)
*     THIS SUBROUTINE EVALUATES THE DIMENSIONLESS FREQUENCY FACTOR.
      FM=FLOAT(M)
      GX=FM-0.75
      HX=GX*GX*(1.0-(1.0/(GX*PI)))
      XJ=HX
      FN=FLOAT(N)
      GY=FN-0.5
      HY=GY*GY*(1.0-(2.0/(GY*PI)))
      YJ=HY
      ST1=GX*GX*GX*GX
      ST2=(GY*GY*GY*GY)/(AR*AR*AR*AR)
      ST3=SIG*HX*HY+(1.0-SIG)*XJ*YJ
      ST4=(2.0*ST3)/(AR*AR)
      SLMB=ST1+ST2+ST4
      RETURN
      END
*     ****************************************************************
      SUBROUTINE ONE(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR EVEN VALUES OF N.
      K=36
      AK=PI/(2.0*K)
      KK=K-1
      TK=0.0
      I=1
      SUM=0.0
      CALL TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
90    TK=TK+AK
      I=I+1
      CALL TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 90
      TK=TK+AK
      CALL TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
```

```
*     ******************************************************************
      SUBROUTINE TWO(C,SLMB,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
*     THIS SUBROUTINE IS USED TO DETERMINE THE PHASE FACTOR OF THE
*     ACOUSTIC INTENSITY FOR EVEN VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*PI*PI*TR*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*PI*PI*AR*TR*SIN(TK)*SIN(TH)/C
      HGN=GN/2.0
      ST6=GPM*COS(AL/2.0)
      ST7=GPM*COS(HGPM)*COS(AL/2.0)
      ST8=2.0*AL*SIN(HGPM)*SIN(AL/2.0)
      ST9=GPM*COS(AL/2.0)
      ST10=GPM*COSH(HGPM)*COS(AL/2.0)
      ST11=2.0*AL*SINH(HGPM)*SIN(AL/2.0)
      ST12=2.0*AL*SIN(HGPM)*COS(AL/2.0)
      ST13=GPM*COS(HGPM)*SIN(AL/2.0)
      ST14=GPM*SIN(AL/2.0)
      ST15=2.0*AL*SINH(HGPM)*COS(AL/2.0)
      ST16=GPM*SIN(AL/2.0)
      ST17=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST18=4.0*AL*AL-GPM*GPM
      ST19=4.0*AL*AL+GPM*GPM
      ST20=GN*SIN(HGN)*COS(AS/2.0)
      ST21=AS*COS(HGN)*SIN(AS/2.0)
      ST22=GN*SINH(HGN)*COS(AS/2.0)
      ST23=AS*COSH(HGN)*SIN(AS/2.0)
      ST24=GN*GN-AS*AS
      ST25=GN*GN+AS*AS
      ST26=ST6-ST7-ST8
      ST27=ST9-ST10-ST11
      ST28=ST12-ST13-ST14
      ST29=ST15-ST16-ST17
      ST30=ST26/ST18
      ST31=ST27/ST19
      ST32=ST30+RPM*ST31
      ST33=ST28/ST18
      ST34=ST29/ST19
      ST35=ST33+RPM*ST34
      ST36=ST32*ST32+ST35*ST35
      ST37=ST20-ST21
      ST38=ST22+ST23
      ST39=ST37/ST24
      ST40=ST38/ST25
      ST41=ST39+RN*ST40
      ST42=ST41*ST41
      FUNC=ST36*ST42*SIN(TK)
      RETURN
      END
*     ******************************************************************
*     ******************************************************************
      SUBROUTINE THREE(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
```

```
     +SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE ACOUSTIC INTENSITY
*     FOR ODD VALUES OF N.
      K=36
      KK=K-1
      AK=PI/(2.0*K)
      TK=0.0
      I=1
      SUM=0.0
      CALL FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
 190  TK=TK+AK
      I=I+1
      CALL FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 190
      TK=TK+AK
      CALL FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ***************************************************************
*     ***************************************************************
      SUBROUTINE FOUR(C,SLMB,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE ACOUSTIC
*     INTENSITY FOR ODD VALUES OF N.
      AL1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AL2=SQRT(AL1)
      AL=AL2*TR*PI*PI*COS(TH)*SIN(TK)/C
      AS1=(SLMB*E*GR)/(3.0*RO*(1.0-(SIG*SIG)))
      AS2=SQRT(AS1)
      AS=AS2*AR*TR*PI*PI*SIN(TK)*SIN(TH)/C
      HGPM=GPM/2.0
      ST43=GPM*COS(AL/2.0)
      ST44=GPM*COS(HGPM)*COS(AL/2.0)
      ST45=2.0*AL*SIN(HGPM)*SIN(AL/2.0)
      ST46=GPM*COS(AL/2.0)
      ST47=GPM*COSH(HGPM)*COS(AL/2.0)
      ST48=2.0*AL*SINH(HGPM)*SIN(AL/2.0)
      ST49=2.0*AL*SIN(HGPM)*COS(AL/2.0)
      ST50=GPM*COS(HGPM)*SIN(AL/2.0)
      ST51=GPM*SIN(AL/2.0)
      ST52=2.0*AL*SINH(HGPM)*COS(AL/2.0)
      ST53=GPM*SIN(AL/2.0)
      ST54=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST55=4.0*AL*AL-GPM*GPM
```

```
      ST56=4.0*AL*AL+GPN*GPN
      ST57=GPN*COS(HGPN)*SIN(AS/2.0)
      ST58=AS*SIN(HGPN)*COS(AS/2.0)
      ST59=AS*SINH(HGPN)*COS(AS/2.0)
      ST60=GPN*COSH(HGPN)*SIN(AS/2.0)
      ST61=GN*GN-AS*AS
      ST62=GN*GN+AS*AS
      ST63=ST43-ST44-ST45
      ST64=ST46-ST47-ST48
      ST65=ST49-ST50-ST51
      ST66=ST52-ST53-ST54
      ST67=ST63/ST55
      ST68=ST64/ST56
      ST69=ST67+RPM*ST68
      ST70=ST65/ST55
      ST71=ST66/ST56
      ST72=ST70+RPM*ST71
      ST73=ST69*ST69+ST72*ST72
      ST74=ST57-ST58
      ST75=ST59-ST60
      ST76=ST74/ST61
      ST77=ST75/ST62
      ST78=ST76+RPM*ST77
      ST79=ST78*ST78
      FUNC=ST73*ST79*SIN(TK)
      RETURN
      END
```

```
C.5.4.PROGRAM FOR CLAMPED-SIMPLY SUPPORTED PLATE(RADIATION EFFICIENCY)
*     ****************************************************************
*     ******************** LIST OF VARIABLES *************************
*     ****************************************************************
*     A=LENGTH OF THE PLATE IN METERS.
*     B=WIDTH OF THE PLATE IN METERS.
*     C=VELOCITY OF SOUND IN AIR,METERS PER SECOND.
*     PI=A CONSTANT QUANTITY EQUIVALENT TO 3.1416
*     GR=ACCELARATION DUE TO GRAVITY,METERS PER SECOND SQ.
*     W=AMPLITUDE OF VIBRATION,METERS.
*     H=THICKNESS OF THE PLATE,METERS.
*     E=MODULUS OF ELASTICITY OF THE PLATE MATERIAL,NEWTON PER METER SQ.
*     RO=SPECIFIC WEIGHT OF THE PLATE MATERIAL,KG PER METER CUBE.
*     ROA=DENSITY OF AIR,KG PER METER CUBE.
*     SIG=POISSON'S RATIO.
*     AR=ASPECT RATIO,B/A
*     ****************************************************************
*     ****************************************************************
      OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
      OPEN(UNIT=3,FILE='IN',STATUS='OLD')
      READ(3,995)C,W,GR,PI,E,ROA,RO,SIG,HH
995   FORMAT(F4.0/F6.4/F4.2/F6.4/E9.3/F4.2/F5.0/F3.1/F6.4)
996   READ(3,997)AR
997   FORMAT(F4.2)
      WRITE(9,998)
998   FORMAT(//20X,'****** INPUT DATA ******',//)
      WRITE(9,999)AR,C,W,GR,PI,E,ROA,RO,SIG,HH
999   FORMAT(20X,'AR=',F3.1/20X,'C=',F4.0/20X,'W=',F6.4/20X,'GR=',F4.2
     +/20X,'PI=',F5.4/20X,'E=',E9.3/20X,'ROA=',F4.2/20X,'RO=',F5.0
     +/20X,'SIG=',F3.1/20X,'HH=',F6.4)
      WRITE(9,1000)AR
1000  FORMAT(5X,'ASPECT RATIO= ',F6.4,/)
      WRITE(9,1001)
1001  FORMAT(5X,'NUMBER OF DIVISIONS=36')
      WRITE(9,1002)
1002  FORMAT(1X,75('*'),/,1X,20('*'),1X,'OUTPUT RESULTS ',35('*'),
     +/,1X,75('*'),/)
      WRITE(9,1003)
1003  FORMAT(1X,131('-'))
      WRITE(9,1004)
1004  FORMAT(3X,' M ',2X,' N ',4X,'RADIATION EFFICIENCY',5X,'GPM/GN'
     +,5X,'GPM/GPN',5X,'WNP'/)
      WRITE(9,1005)
1005  FORMAT(1X,131('-'),/)
*     ****************************************************************
124   X11=3.9
      DO 1 M=2,16
12    X12=X11+HH
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))
      Z11=Y11*Y12
      IF(Z11.LT.0.00) GO TO 10
```

```
      X11=X12
      GO TO 12
10    GPM=X11+X12
      X21=2.2
      DO 2 M=2,16,2
22    X22=X21+HT
      Y21=(SIN(X21)/COS(X21))+(SINH(X21)/COSH(X21))
      Y22=(SIN(X22)/COS(X22))+(SINH(X22)/COSH(X22))
      Z21=Y21*Y22
      IF(Z21.LT.0.00) GO TO 20
      X21=X22
      GO TO 22
20    GN=X21+X22
      HGPM=GPM/2.0
      HGN=GN/2.0
      RPM=-SIN(HGPM)/SINH(HGPM)
      RN=SIN(HGN)/SINH(HGN)
      MP=M-1
      NP=N-1
      WNR=0.20
800   FT1=(64.0*WNR*WNR)*(AR*MP*MP+(NP*NP)/AR)
      HT1=1.0-SIN(GPM)/GPM
      HT2=(4.0*RPM)/GPM
      HT3=SIN(HGPM)*COSH(HGPM)
      HT=COS(HGPM)*SINH(HGPM)
      HT4=SINH(GPM)/GPM-1.0
      HT5=RPM*RPM
      HT6=HT2*(HT3-HT)
      HT7=HT5*HT4
      HT8=HT1+HT6+HT7
      HT9=1.0+SIN(GN)/GN
      HT10=(4.0*RN)/GN
      HT11=COS(HGN)*SINH(HGN)
      HT12=SIN(HGN)*COSH(HGN)
      HT13=RN*RN
      HT14=1.0+SINH(GN)/GN
      HT15=HT10*(HT11+HT12)
      HT16=HT13*HT14
      HT17=HT10+HT15+HT16
      FT2=HT8*HT17
      AMP1=FT1/FT2
      K=36
      KK=K-1
      AH=(PI/2)/K
      GSUM=0.0
      TH=0.0
      I=1
      CALL ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+SUM
70    TH=TH+AH
      I=I+1
      CALL ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
```

```
      CALL ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)     TEF01110
      GSUM=GSUM+2.0*SUM                                                 TEF01120
      IF(I.LT.KK) GO TO 70                                              TEF01130
      TH=TH+AH                                                          TEF01140
      CALL ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)     TEF01150
      GSUM=GSUM+4.0*SUM                                                 TEF01160
      TH=TH+AH                                                          TEF01170
      CALL ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,SUM)     TEF01180
      GSUM=GSUM+SUM                                                     TEF01190
      GSUM=(AH/3.0)*GSUM                                                TEF01200
      SMN=GSUM*AMPL                                                     TEF01210
      WRITE(*,*) M,N,WNR,SMN                                            TEF01220
      WRITE(9,200) M,N,SMN,GPM,GN,WNR                                   TEF01230
  200 FORMAT(3X,I2,4X,I2,7X,E10.4,5X,F10.4,5X,F10.4,5X,F4.2)            TEF01240
      WNR=WNR+0.20                                                      TEF01250
      IF(WNR.GT.2.0)GO TO 700                                           TEF01260
      GO TO 800                                                         TEF01270
  700 X21=X21+3.1                                                       TEF01280
    2 CONTINUE                                                          TEF01290
*     ******************************************************************TEF01300
      X11=X11+3.1                                                       TEF01310
*     ******************************************************************TEF01320
    1 CONTINUE                                                          TEF01330
*     ******************************************************************TEF01340
      X11=3.9                                                           TEF01350
      DO 3 M=2,16                                                       TEF01360
   32 X12=X11+HH                                                        TEF01370
      Y11=(SIN(X11)/COS(X11))-(SINH(X11)/COSH(X11))                     TEF01380
      Y12=(SIN(X12)/COS(X12))-(SINH(X12)/COSH(X12))                     TEF01390
      Z11=Y11*Y12                                                       TEF01400
      IF(Z11.LT.0.0) GO TO 30                                           TEF01410
      X11=X12                                                           TEF01420
      GO TO 32                                                          TEF01430
   30 GPM=X11+X12                                                       TEF01440
      X21=3.9                                                           TEF01450
      DO 4 N=3,15,2                                                     TEF01460
   42 X22=X21+HH                                                        TEF01470
      Y21=(SIN(X21)/COS(X21))-(SINH(X21)/COSH(X21))                     TEF01480
      Y22=(SIN(X22)/COS(X22))-(SINH(X22)/COSH(X22))                     TEF01490
      Z21=Y21*Y22                                                       TEF01500
      IF(Z21.LT.0.0) GO TO 40                                           TEF01510
      X21=X22                                                           TEF01520
      GO TO 42                                                          TEF01530
   40 GPN=X21+X22                                                       TEF01540
      HGPM=GPM/2.0                                                      TEF01550
      HGPN=GPN/2.0                                                      TEF01560
      RPM=-SIN(HGPM)/SINH(HGPM)                                         TEF01570
      RPN=-SIN(HGPN)/SINH(HGPN)                                         TEF01580
      MP=M-1                                                            TEF01590
      NP=N-1                                                            TEF01600
      WNR=0.20                                                          TEF01610
  900 FT3=(64.0*WNR*WNR)*(AR*MP*MP+(NP*NP)/AR)                          TEF01620
      HT18=1.0-SIN(GPM)/GPM                                             TEF01630
      HT19=(4.0*RPM)/GPM                                                TEF01640
      HT20=SIN(HGPM)*COSH(HGPM)                                         TEF01650
```

```
      HT=COS(HGPM)*SINH(HGPM)
      HT21=SINH(GPM)/GPM-1.0
      HT22=RPM*RPM
      HT23=HT19*(HT20-HT)
      HT24=HT22*HT21
      HT25=HT18+HT23+HT24
      HT26=1.0-SIN(GPN)/GPN
      HT27=(4.0*RPN)/GPN
      HT28=SIN(HGPN)*COSH(HGPN)
      HT29=COS(HGPN)*SINH(HGPN)
      HT30=RPN*RPN
      HT31=SINH(GPN)/GPN-1.0
      HT32=HT27*(HT28-HT29)
      HT33=HT30*HT31
      HT34=HT26+HT32+HT33
      FT4=HT25*HT34
      AMP2=FT3/FT4
      K=35
      AH=PI/(2.0*K)
      KK=K-1
      GSUM=0.0
      TH=0.0
      I=1
      CALL THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
      GSUM=GSUM+SUM
170   TH=TH+AH
      I=I+1
      CALL THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      I=I+1
      CALL THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
      GSUM=GSUM+2.0*SUM
      IF(I.LT.KK) GO TO 170
      TH=TH+AH
      CALL THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
      GSUM=GSUM+4.0*SUM
      TH=TH+AH
      CALL THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
      GSUM=GSUM+SUM
      GSUM=(AH/3.0)*GSUM
      SMN=GSUM*AMP2
      WRITE(*,*) M,N,WNR,SMN
      WRITE(9,300) M,N,SMN,GPM,GPN,WNR
300   FORMAT(3X,I2,4X,I2,7X,F10.4,7X,F10.4,7X,F10.4,5X,F4.2)
      WNR=WNR+0.20
      IF(WNR.GT.2.0)GO TO 400
      GO TO 900
400   X21=X21+3.1
4     CONTINUE
```

```
      X11=X11+3.1
3     CONTINUE
      STOP
      END
*     ************************************************************
      SUBROUTINE ONE(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,
     +SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR EVEN VALUES OF N.
      K=36
      AK=PI/(2.0*K)
      KK=K-1
      TK=0.0
      I=1
      SUM=0.0
      CALL TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+FUNC
90    TK=TK+AK
      I=I+1
      CALL TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 90
      TK=TK+AK
      CALL TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ************************************************************
*     ************************************************************
      SUBROUTINE TWO(C,M,N,WNR,GPM,GN,RPM,RN,AR,TR,E,PI,RO,SIG,GR,TH,
     +TK,FUNC)
*     THIS SUBROUTINE IS USED TO DETERMINE THE PHASE FACTOR OF THE
*     RADIATION EFFICIENCY FOR EVEN VALUES N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGN=GN/2.0
```

```
1200  ST6=GPM*COS(AL/2.0)
      ST7=GPM*COS(HGPM)*COS(AL/2.0)
      ST8=2.0*AL*SIN(HGPM)*SIN(AL/2.0)
      ST9=GPM*COS(AL/2.0)
      ST10=GPM*COSH(HGPM)*COS(AL/2.0)
      ST11=2.0*AL*SINH(HGPM)*SIN(AL/2.0)
      ST12=2.0*AL*SIN(HGPM)*COS(AL/2.0)
      ST13=GPM*COS(HGPM)*SIN(AL/2.0)
      ST14=GPM*SIN(AL/2.0)
      ST15=2.0*AL*SINH(HGPM)*COS(AL/2.0)
      ST16=GPM*SIN(AL/2.0)
      ST17=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST18=4.0*AL*AL-GPM*GPM
      ST19=4.0*AL*AL+GPM*GPM
      ST20=GN*SIN(HGN)*COS(AS/2.0)
      ST21=AS*COS(HGN)*SIN(AS/2.0)
      ST22=GN*SINH(HGN)*COS(AS/2.0)
      ST23=AS*COSH(HGN)*SIN(AS/2.0)
      ST24=GN*GN-AS*AS
      ST25=GN*GN+AS*AS
      ST26=ST6-ST7-ST8
      ST27=ST9-ST10-ST11
      ST28=ST12-ST13-ST14
      ST29=ST15-ST16-ST17
      ST30=ST26/ST18
      ST31=ST27/ST19
      ST32=ST30+RPM*ST31
      ST33=ST28/ST18
      ST34=ST29/ST19
      ST35=ST33+RPM*ST34
      ST36=ST32*ST32+ST35*ST35
      ST37=ST20-ST21
      ST38=ST22+ST23
      ST39=ST37/ST24
      ST40=ST38/ST25
      ST41=ST39+RN*ST40
      ST42=ST41*ST41
      FUNC=ST36*ST42*SIN(TK)
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE THREE(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,
     +TH,SUM)
*     THIS SUBROUTINE IS USED TO INTEGRATE THE RADIATION EFFICIENCY
*     FOR ODD VALUES OF N.
      K=36
      KK=K-1
      AK=PI/(2.0*K)
      TK=0.0
      I=1
      SUM=0.0
      CALL FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+FUNC
```

```
190   TK=TK+AK
      I=I+1
      CALL FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      I=I+1
      CALL FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+2.0*FUNC
      IF(I.LT.KK) GO TO 190
      TK=TK+AK
      CALL FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+4.0*FUNC
      TK=TK+AK
      CALL FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,TH,TK,
     +FUNC)
      SUM=SUM+FUNC
      SUM=(AK/3.0)*SUM
      RETURN
      END
*     ****************************************************************
*     ****************************************************************
      SUBROUTINE FOUR(C,M,N,WNR,GPM,GPN,RPM,RPN,AR,TR,E,PI,RO,SIG,GR,
     +TH,TK,FUNC)
*     THIS SUBROUTINE EVALUATES THE PHASE FACTOR OF THE RADIATION
*     EFFICIENCY FOR ODD VALUES OF N.
      MP=M-1
      NP=N-1
      AL1=MP*MP*PI*PI+(NP*NP*PI*PI)/(AR*AR)
      AL2=SQRT(AL1)
      AL=AL2*WNR*COS(TH)*SIN(TK)
      AS1=AR*AR*MP*MP*PI*PI+NP*NP*PI*PI
      AS2=SQRT(AS1)
      AS=AS2*WNR*SIN(TK)*SIN(TH)
      HGPN=GPN/2.0
      ST43=GPM*COS(AL/2.0)
      ST44=GPM*COS(HGPM)*COS(AL/2.0)
      ST45=2.0*AL*SIN(HGPM)*SIN(AL/2.0)
      ST46=GPM*COS(AL/2.0)
      ST47=GPM*COSH(HGPM)*COS(AL/2.0)
      ST48=2.0*AL*SINH(HGPM)*SIN(AL/2.0)
      ST49=2.0*AL*SIN(HGPM)*COS(AL/2.0)
      ST50=GPM*COS(HGPM)*SIN(AL/2.0)
      ST51=GPM*SIN(AL/2.0)
      ST52=2.0*AL*SINH(HGPM)*COS(AL/2.0)
      ST53=GPM*SIN(AL/2.0)
      ST54=GPM*COSH(HGPM)*SIN(AL/2.0)
      ST55=4.0*AL*AL-GPM*GPM
      ST56=4.0*AL*AL+GPM*GPM
      ST57=GPN*COS(HGPN)*SIN(AS/2.0)
      ST58=AS*SIN(HGPN)*COS(AS/2.0)
      ST59=AS*SINH(HGPN)*COS(AS/2.0)
      ST60=GPN*COSH(HGPN)*SIN(AS/2.0)
```

```
      ST61=GN*GN-AS*AS
      ST62=GN*GN+AS*AS
      ST63=ST43-ST44-ST45
      ST64=ST46-ST47-ST48
      ST65=ST49-ST50-ST51
      ST66=ST52-ST53-ST54
      ST67=ST63/ST55
      ST68=ST64/ST56
      ST69=ST67+RPM*ST68
      ST70=ST65/ST55
      ST71=ST66/ST56
      ST72=ST70+RPM*ST71
      ST73=ST69*ST69+ST72*ST72
      ST74=ST57-ST58
      ST75=ST59-ST60
      ST76=ST74/ST61
      ST77=ST75/ST62
      ST78=ST76+RPM*ST77
      ST79=ST78*ST78
      FUNC=ST73*ST79*SIN(TK)
      RETURN
      END
```