

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What is a simple pendulum? Discuss that for small angular displacement, the motion of a simple pendulum is simple harmonic. (10)
- (b) Show that in a simple harmonic motion, the average potential energy equals the average kinetic energy when the average is taken with respect to time over one period of motion. (15)
- (c) The amplitude of a simple harmonic oscillator is 10 cm. At what displacement the kinetic energy is $\frac{3}{4}$ th of the total energy of the oscillator? (10)

2. (a) What are the differences between damped and forced oscillations? (5)
- (b) a particle of mass m is executing a damped oscillatory motion. An external force of $F_0 e^{i\omega t}$ is applied to the particle during oscillation. Establish the differential equation for the oscillating particle and then find the general solution of the equation. Obtain the condition of resonance and give some of its practical applications. (25)
- (c) A particle of mass 2 g is free to vibrate under the action of a force of 128 dyne cm^{-1} and a damping force of 8 dyne $\text{cm}^{-1} \text{sec}$. A periodic external force of a maximum value of 256 dynes is applied to the particle. Find the frequency for displacement resonance. (5)

3. (a) Find the relation between particle velocity and wave velocity. (10)
- (b) Show that, in the case of a stationary wave, no energy is transferred across any section of the medium. (15)
- (c) Which of the following is not the solution of the one-dimensional wave equation? (10)
 - (i) $y = x^2 + v^2 t^2$, (ii) $y = 7x - 10t$, (iii) $y = \sin 3x \cos vt$.

The symbols have their usual meaning.

4. (a) Draw the intensity distribution of Young's double slit experiment and write down the condition for bright and dark fringes. (10)
- (b) Derive an expression for the separation between two successive fringes in Young's double slit experiment. (15)
- (c) Two coherent beams with wavelength 520 nm are superposed to form fringes in a Fresnel Biprism. The central fringe shifts by 20 fringes when a thin plate with $\mu = 1.52$ is placed in the path of one of the beams. What is the thickness of the plate? (10)

PHY 123/BME

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Briefly describe the five types of Seidel aberration. (10)

(b) Derive at what distance should two plano-convex lenses be kept to reduce spherical aberration? (15)

(c) If two plano-convex lenses made of same glass and of focal lengths 32 cm and 20 cm, respectively are to be used to exhibit minimum spherical aberration. Find the distance between the two lenses. If $\mu = 1.5$, find the radii of curvature of the lens surfaces. (10)

6. (a) State the differences between Fresnel and Fraunhofer type diffractions. (10)

(b) Obtain an expression for the intensity of light in single slit Fraunhofer diffraction. (15)

(c) Calculate the missing orders for a double slit Fraunhofer diffraction pattern if each slit has width of 0.07 mm and the slits are 0.21 mm apart. (10)

7. (a) State and explain the basic principle of a platinum resistance thermometer. (10)

(b) Write down Maxwell's velocity distribution function for gas molecules and hence obtain the expression of average velocity and root mean square velocity. Draw the Maxwell's velocity distribution curve indicating the variations of different types of velocities. (15)

(c) Calculate the Van der Waals constants for dry air. Given that, $T_c = 132$ K, $P_c = 37.2$ atm, and R per mole = $82.07 \text{ cm}^3 \text{ atm K}^{-1}$. (10)

8. (a) What is meant by thermodynamic equilibrium? Discuss different types of thermodynamic equilibrium. (10)

(b) In case of a real gas, show that (15)

$$C_p - C_v = R \left\{ 1 + \frac{2a}{RTV^3} (V - b)^2 \right\},$$

where the symbols have their usual meaning.

(c) Two Carnot engine of A and B operating in two different temperature regions. For engine A, the temperatures of the two reservoirs are 150°C and 100°C. For engine B, the temperatures of the two reservoirs are 350°C and 300°C. Which engine has lesser efficiency? (10)

SECTION – AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define continuity and differentiability of a function. (5)

- (b) Find values of the constants k and m , if possible, that will make the function $f(x)$ continuous everywhere: (18)

$$f(x) = \begin{cases} x^2 + 5 & , \text{if } x > 2 \\ m(x+1) + k & , \text{if } -1 < x \leq 2 \\ 2x^3 + x + 7 & , \text{if } x \leq -1 \end{cases}$$

- (c) Evaluate: (12)

$$\lim_{x \rightarrow 0} (x \ln \sin x) \text{ using L' Hospital rule.}$$

2. (a) State Leibnitz's theorem. (5)

- (b) If $y = x \cos (\ln x)$, using the hypothesis of Leibnitz's theorem show that (18)

$$x^2 y_{n+2} + (2n-1)xy_{n+1} + (n^2 - 2n + 2)y_n = 0.$$

- (c) Find the n^{th} Taylor polynomial for $1/x$ about $x = 1$ and express it in sigma notation. (12)

3. (a) Define stationary point, critical point, concavity and point of inflection. (8)

- (b) Find the maximum and minimum value of the function $f(x) = x^3 - 3x^2 + x - 2$ and hence find the point of inflection of $f(x)$ (if any). (15)

- (c) Show that the function $f(x) = \frac{1}{4}x^3 + 1$ satisfies the hypotheses of the Mean-Value Theorem over the interval $[0, 2]$, and find all values of c in the interval $(0, 2)$ at which the tangent line to the graph of $f(x)$ is parallel to the secant line joining the points $(0, f(0))$ and $(2, f(2))$. (12)

4. (a) State Euler's Homogeneous theorem. If $u = f(y-z, z-x, x-y)$, deploying the technique of chain rule compute the value of the following expression: (12)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}.$$

- (b) Find the radius of curvature of the cardioid $r = a(1 + \cos \theta)$ at any point (r, θ) . (11)

- (c) What is asymptote of a function? Find all the asymptotes of the curve (12)

$$x^3 + 2x^2y - xy^2 - 2y^3 + x^2 - y^2 - 2x - 3y = 0.$$

MATH 113**SECTION - B**There are **FOUR** questions in this section. Answer any **THREE**.

5. Workout the following:

(a) $\int \frac{dx}{x^2(a+bx)^2}$ (12)

(b) $\int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$ (12)

(c) $\int e^x \frac{1-\sin x}{1-\cos x} dx$ (11)

6. (a) Obtain a reduction formula for $I_m = \int x^m \sin nx dx$ and hence find $\int x^3 \sin 2x dx$. (17)

(b) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$. (18)

7. (a) Evaluate $\lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n^2}\right)^{\frac{2}{n^2}} \left(1 + \frac{2^2}{n^2}\right)^{\frac{4}{n^2}} \left(1 + \frac{3^2}{n^2}\right)^{\frac{6}{n^2}} \dots \left(1 + \frac{n^2}{n^2}\right)^{\frac{2n}{n^2}} \right\}$

using definite integrals. (11)

(b) Sketch the region bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$. Then find the area of the region by integration. (12)

(c) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ then evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx$ using the result of $\Gamma\left(\frac{1}{2}\right)$. (12)

8. (a) Find the area above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. (12)(b) Sketch the graph and find the area bounded by the cardioid $r = 2(1 + \cos\theta)$. (12)(c) Find the volume of the solid formed by revolving the region bounded by the graph of $y = 4 - x^2$ and the x-axis about the line $x = 3$. (11)
