Date: ~~18/04/2024~~ 19/05/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **CSE 103** (Discrete Mathematics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer **Q. No. 1** and any **TWO** from the rest.

Question No. 1 is compulsory.

1. (a) While checking some very old questions you inherited from your parents (who are BUET CSE graduates), you found one interesting multiple choice question. However, the question part is not fully legible. After some try you could identify the following facts from the question (some facts may be missing). **(11)**

- It is talking about a fictitious game titled "Game87" created as part of a term assignment.
- If you get more points than any other players, then you will lose.
- If you lose then you must have received the most number of cards.
- This is a 3 players game.

Now the question asks, which of the following statements is true given that only one statement can be true:

A. Player A wins
B. Player A wins or Player B wins
C. Player B wins or Player C wins

Now you have been trying to determine which could be the correct answer of this question with the facts you have been able to identify but almost reaching the conclusion that without the full question this cannot be determined. Your genius brother had a look at this and after some thought confidently says that Ans. C is correct.

You asked: How did you determine this?

He winked and replied: using discrete maths on the three answer choices.

Do you agree with your brother? Justify your answer.

(b) Assuming that there is at least one person in Bangladesh who is bald, we want to prove the statement "All persons in Bangladesh are bald". Now consider the following inductive proof. **(12)**

Proof. We will first prove *P(n)* where *P(n)* = For all groups of *n* persons, whenever one person is bald, then all persons are bald. We proceed as follows.

Base case: $P(1)$ is clearly true.

Contd P/2

CSE 103

Contd... Q. No. 1(a)

Induction step: Suppose *P(k)* is true for all $1 \le k \le n$; we wish to show $P(n+1)$ is true as well. Given a set W of $n + 1$ persons in which $x \in W$ is bald, take any two proper subsets A, B $\subset$ W (in particular $|A|, |B| < n+1$) such that they both contain the bald person $(x \in A;\ x \in B)$, and A$\cup$B = W (no one is left out). Applying the induction hypothesis to A and B, we conclude that all the persons in A and B are bald, and so everyone is W is bald.

Finally, if we now consider W to be the set of all persons in Bangladesh, the result follows. *QED*

Determine what is wrong with the above inductive proof.

(c) The juice corner of the ECE café offers juice of 5 fruits: apple, mango, lemon, papaya and pineapple. For each fruit you have two options: with sugar or without sugar. Now after a tiresome class test, all 180 CSE 103 course students went to the juice corner to get refreshed. Knowing that these are the students of CSE 103, the owner (who is a math enthusiast) offered a 50% discount, if the students could determine how many different ways the juice can be selected for them. Can you get the 50% discount? You need to count using a suitable mapping and provide detailed explanation. **(12)**

2. (a) In an island, thee are three types of inhabitants, knights that always tell the truth, knaves that always lie, and spies that may tell the truth or lie. You encounter three people, A, B, and C and you know that one is a knight, one is a knave, and one is a spy. Determine, if possible, what A, B, and C are based on the statements (S1, S2, S3) given below. If you cannot determine what these three people are, can you draw any conclusions? **(11)**

S1: A says C is the knave

S2: B says A is the knight

S3: C says I am the spy.

(b) Prove or disprove that $2^{999} + 1$ is a Prime **(6)**

(c) Prove that $\forall$ natural number n, $n^2 + 3n + 2$ is composite. **(6)**

(d) Let $n \in Z$. Prove that if $n^2 - 6n + 5$ is even, then n is odd. **(6)**

(e) Suppose $p \in N$ and $p \ge 2$. Then prove by contradiction that if $2^p - 1$ is prime, then p is prime. **(6)**

Contd P/3

3. (a) Prove that both $\sqrt{2}$ and $\sqrt{3}$ are irrational. Then prove that an irrational raised to an irrational power may be rational. **(15)**

(b) There are two piles of books. Two players take turns removing any positive number of books they want from one of the two piles. The player who removes the last book wins the game. Prove by induction that if the two piles have the same number of books initially, the second player will always win. **(10)**

(c) Prove by weak induction that postage of $n \geq 18$ Taka can be formed using just 4 Taka and 7 Taka stamps. **(10)**

4. (a) If every people in a group has met, then we call the group a club. If every people in a group has not met, then call them a group of strangers. Using pigeon hole principal, prove that every collection of 6 people includes a club of 3 people, or a group of 3 strangers. **(15)**

(b) In your C/C++ sessional you have been given the following assignment (Assignment A) which will be evaluated automatically using an online judge. **(10)**

Assignment A: subset sum

You will be given ninety 25-digit numbers. You will need to check whether there exist two different subsets with the same sum. Your input will be ninety 25-digit numbers, one per line and you will have to output "Yes" if there exist two different subsets with the same sum or "No" otherwise. Your solution must be as efficient as possible. Your genius little brother suggested to write a simple code that output "Yes" for all input without checking anything else. Do you think his solution is correct? Justify your answer.

(c) Prove that in a room of 23 people there is a 50-50 chance of two people having the same birthday. What will be the scenario if there are 75 people in the room? **(10)**

SECTION – B

There are **FOUR** questions in this section. **Q. No. 5** and any **TWO** from the rest.

Question No. 5 is compulsory.

5. (a) A number of processors are interconnected by a cubic structure. That means processors are the vertices of the cube. The processors will run in such a way that adjacent processors will run in different time slots. What will be the minimum number of slots required in this system? Justify your answer. How many neighbors will each processor have? **(10+2)**

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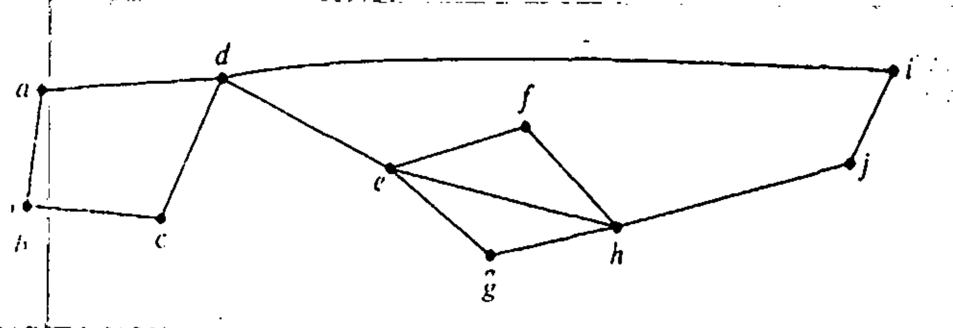
Contd... Q. No. 5

(b) There is a road network among *a, b, c, d,e, f, g, h, i, j*. If you start traveling from *j* determine how can you visit all places such a way that you travel one road just only once and ultimately come back to *j*? Show the steps **(13)**



(c) Represent an organization's organogram with a suitable graph where it is led by the president Rahima Sultana, and she leads 3 vice presidents. Each of the vice president leads 2 directors. Each director leads 3 team leaders where each team has 4 members. If the president wants to broadcast a message determine the number of levels she needs to pass? What will be the adjacency list to represent this organogram? **(5+1+4)**

6. (a) What will be the maximal and minimal elements from the POSET of the divisibility of {2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}? What will be the topological sorted order? **(15)**

(b) If *M* is set of all non singular matrices of order $n \times n$, then find if *M* form a *group* with respect to matrix multiplication *. Is (*M*, *) an Abelian group? Justify your answer. **(12+1)**

(c) Justify that any integer $n > 1$ has unique factorization. **(7)**

7. (a) What will be the solution of the following recurrence relation? **(15)**

$$r_n = r_{n-1} + 2r_{n-2} + 1$$
$$r_0 = 0; r_1 = 1$$

(b) You are working with a population of rabbits. Each mature rabbit pair gives birth to two new pairs each month. Each pair becomes mature after two months. Now, determine the following quantities using a recursive approach: **(12)**

(i) Number of newborn pairs at the end of month n, w_n

(ii) Number of mature pairs at the end of month n, m_n

Also, mention the order of each recurrence relation.

(c) Justify the following statements regarding functions: **(4×2=8)**

(i) A function *f* can have an inverse only if *f* is injective and surjective.

(ii) *f* and *g* are two functions defined by $f : X -> Y$ and $g : Y' -> Z$. $g(f(x))$ is defined only if *Y* is a subset of *Y'*.

Contd P/5

8. (a) Two dice have been rolled together. Determine the probability of the sum of the values being eight when: **(3+7)**

   (i) Both the dices are fair.

   (ii) The first dice is fair. For the second dice, rolling a six is twice as likely as rolling any other value.

(b) There are k varieties of fruits. Determine the number of ways you can choose n fruits from this k varieties so that at least one fruit from each flavor is chosen. You must use generating functions to solve this problem. **(12)**

(c) A survey shows that around 35 out of every 100 people are overweight, and 30 out of 100 are underweight. Another study shows that, overweight people have a 70% chance of contracting heart disease, whereas regular people and underweight people have chances of 20% and 10% respectively. Now, if a particular person does not have any heart disease, identify his most probable weight group. You need to provide probabilities reasoning behind your answer. **(13)**

L-1/T-1/CSE Date: 23/04/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **MATH 141** (Calculus I)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Find the values of the constants k and m, if possible, that will make the function **(15)**

$$f(x)=\begin{cases} 2x^3+x+7 & \text{when} \quad x\le -1 \\ m(x+1)+k & \text{when} \quad -1<x\le 2 \\ x^2+5 & \text{when} \quad x>2 \end{cases}$$

continuous everywhere. Also discuss the differentiability of $f(x)$ at $x=2$.

(b) Evaluate the limit $\lim_{x\to 1}(1-x^2)^{1/\log(1-x)}$ using L' Hospital rule. **(8)**

(c) For the function $y=\frac{1}{2}(\tan^{-1}x)^2$ prove that **(12)**

$$y_{n+2}(0)+2n^2y_n(0)+n(n-1)^2(n-2)y_{n-2}(0)=0$$

2. (a) Discuss the applicability of Rolle's theorem to $f(x)=\ln\left(\frac{x^2+ab}{x(a+b)}\right)$ in the interval $[a, b]$. **(10)**

(b) In the Mean value theorem $f(a+h)-f(a)=hf'(a+\theta h), 0<\theta<1$, if **(10)**

$f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2+2x$ and $a=0, h=3$, show that θ has two values and find them.

(c) If $V=V(r)$ and $r^2=x^2+y^2+z^2$ then show that $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}$. **(15)**

3. (a) A metal Can with a volume of 60 in^3 is to be constructed in the shape of a right circular cylinder. If the cost of the material for the side is \$0.05/in^2 and the cost for the top and bottom is \$0.03/in^2, then find the dimensions of the Can that will minimize the cost. **(13)**

(b) Show that in the curve $a^2y^5=k(bx+c)^4$, the cube of the subtangent varies as the fifth power of the subnormal. **(11)**

(c) Find the equation of the tangent lines at the inflection points of the function **(11)**

$$y=x^4-6x^3+12x^2-8x.$$

4. (a) Find the reduction formula for $\int e^{ax}\cos^n x\,dx$. Also verify that the reduction formula for $\int \cos^n x\,dx$ is a special case of the above-mentioned formula. **(12)**

(b) Find $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\cdots\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$ using integral formula. **(11)**

(c) Evaluate $\int_0^1 x^5\sqrt{\frac{1+x^2}{1-x^2}}dx$. **(12)**

Contd P/2

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Find the exact arc length of the curve **(10)**

$$x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2} \text{ from } y = 1 \text{ to } y = 4.$$

(b) Find the area of the surface generated by revolving the curve $x = 2\sqrt{1-y}, 0 \le y \le 2$ about the y-axis. **(10)**

(c) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval [0,2] is revolved about the x- axis. **(15)**

6. (a) Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$. **(15)**

(b) Use triple integration in cylindrical coordinates to find the volume of the solid that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy-plane, and laterally by the cylinder $x^2 + y^2 = 9$. Sketch the graph of the volume. **(20)**

7. (a) Sketch the slope field of the following differential equation: **(10)**

$$x\frac{dy}{dx} = x/y^2$$

Hence show the solution passing through $y(0) = 1$.

(b) What is the condition for a differential equation to be exact? Determine whether the given differential equation is exact. If it is not, make it exact. Then solve it. **(15)**

$$\left(2y\sin x\cos x - y + 2y^2e^{xy^2}\right)dx - \left(x - \sin^2 x - 4xye^{xy^2}\right)dy = 0$$

(c) A 200-volt electromotive force is applied to an RC-series circuit in which the resistance is 1000 ohms and the capacitance is 5×10^{-6} farad. Find the charge $q(t)$ on the capacitor if $i(0) = 0.4$. Determine the charge and current at $t = 0.05$ sec. **(10)**

8. (a) Discuss about the Cauchy-Euler Equation. Solve the following differential equation by converting it to constant coefficients: **(15)**

$$x^2y'' + xy' - y = \frac{1}{x+1}.$$

(b) A mass of 1 slug, when attached to a spring, stretches it 2 feet and then comes to rest in the equilibrium position. Starting at t = 0, an external force equal to $f(t) = e^{-t}\sin 4t$ is applied to the system. Find the equation of motion if the surrounding medium offers a damping force that is numerically equal to 8 times the initial velocity. Analyze the displacement for $t \to \infty$. **(12)**

(c) Solve: $\frac{d^2y}{dx^2} + y = e^x \sin x$. **(8)**

L-1/T-1/CSE **Date: ~~04/05/2024~~** 25/5/24

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **EEE 163** (Introduction to Electrical Engineering)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

The corresponding Course Outcomes **(COs)** of each part of Questions 1 and 5 are mentioned on the right most column. The **COs** of the Course are mentioned at the end of the question paper.

SECTION – A

There are **FOUR** questions in this section. Answer to **Question no. 1 is Compulsory**.

Answer any **TWO** questions from Questions 2-4.

The symbols have their usual meanings

1. (a) For the circuit shown in Figure for Question 1(a), use mesh analysis techniques to find the power associated with the 40 Ω resistor. **(20)** (CO1)

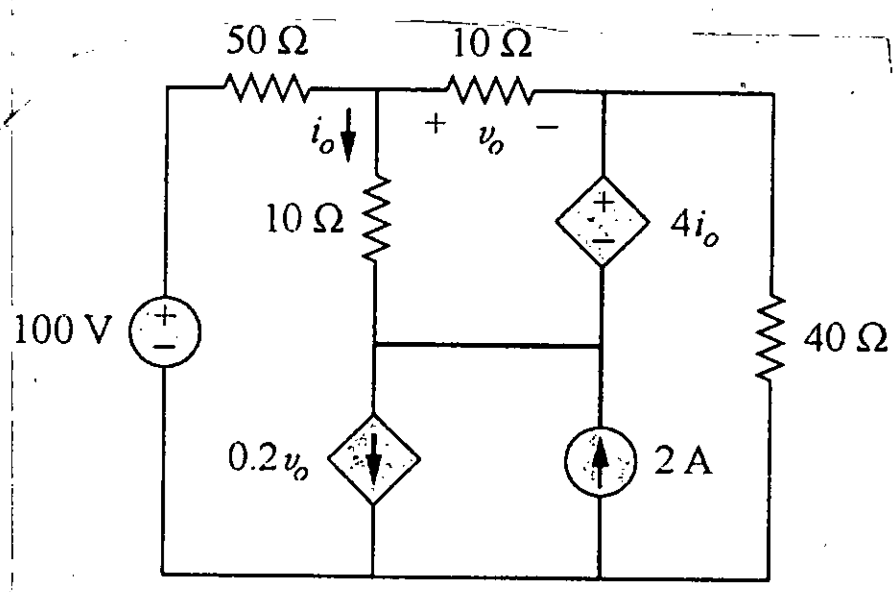


Figure for Question 1(a)

(b) The following three parallel-connected three-phase loads are fed by a balanced three-phase source: **(15)** (CO4)

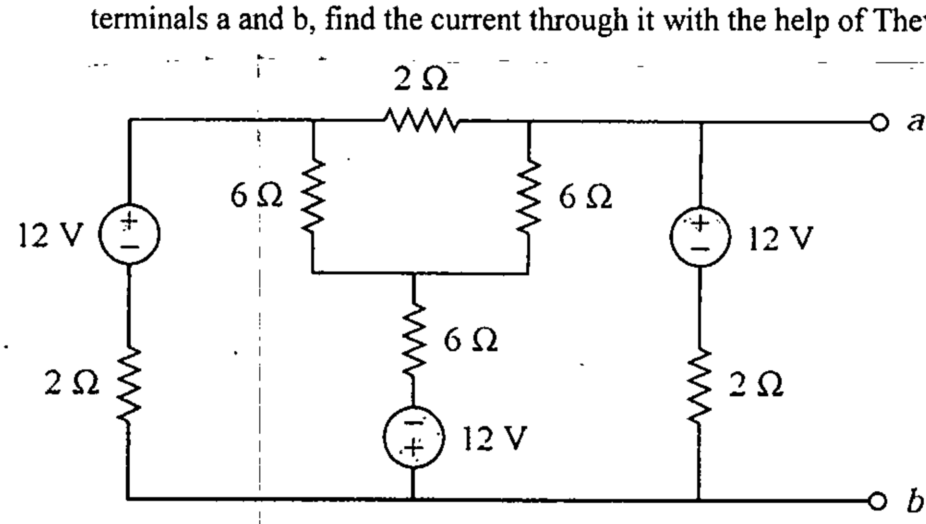
Load 1: 250 kVA, 0.8 pf lagging

Load 2: 300 kVA, 0.95 pf leading

Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line currents and the power factor of the source. Assume that the line impedance is zero.

2. (a) For the circuit in Figure for Question 2(a), if a 5Ω resistor is connected between terminals a and b, find the current through it with the help of Thevenin's theorem. **(18)**



Contd P/2

Figure for Question 2(a)

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Contd ... Q. No. 2(b)

(b) Use an Node analysis technique to find the power dissipated in the 300 Ω resistor in the circuit shown in Figure for Question 2(b). **(17)**

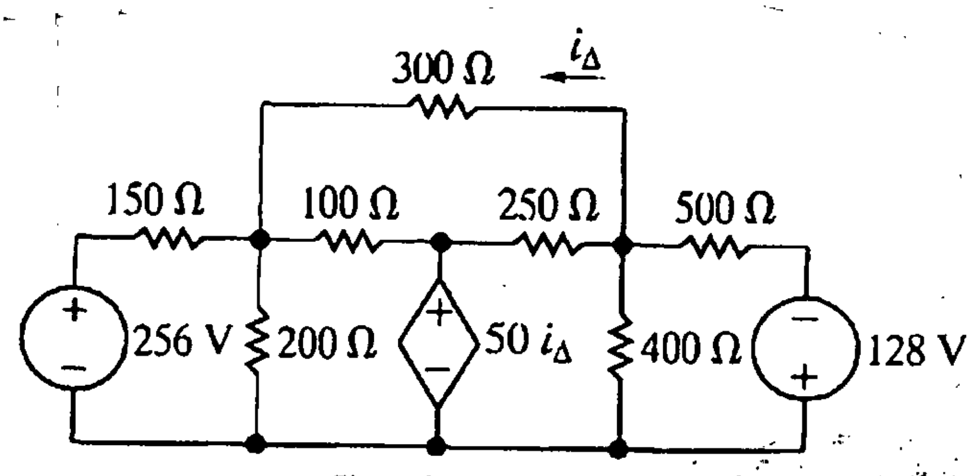

Figure for Question 2(b)

3. (a) For a balanced Wye-Delta system, derive the necessary equations that illustrate the phase relationships between its line currents and phase currents. Also, derive the system's per-phase equivalent circuit. **(15)**

(b) Determine the maximum power delivered to the variable resistor R shown in the circuit of Figure for Question 3(b). Also, determine the percentage change in the delivered maximum power if the 4 V source is replaced by a 5 V source. **(20)**

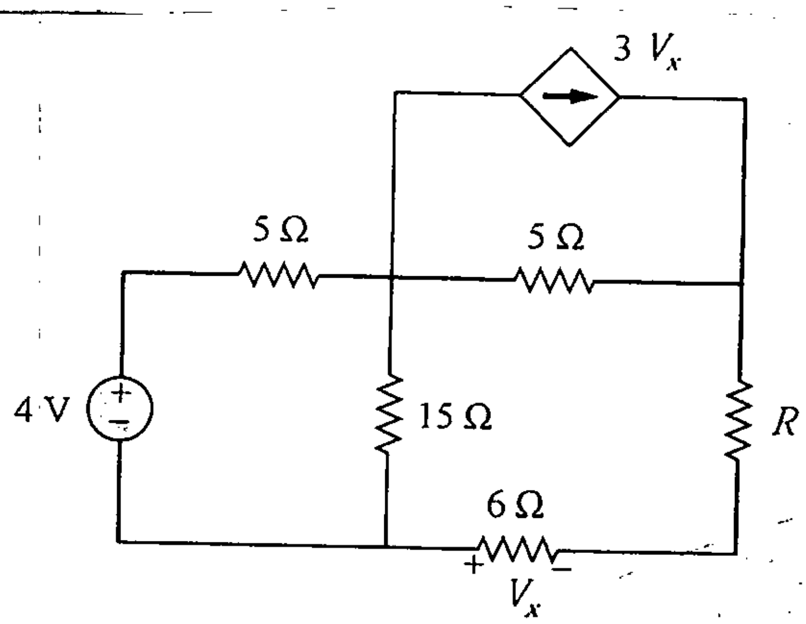

Figure for Question 3(b)

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4. (a) Obtain the Norton equivalent circuit at terminals a-b of the network as shown in Figure for Question 4(a). **(17)**

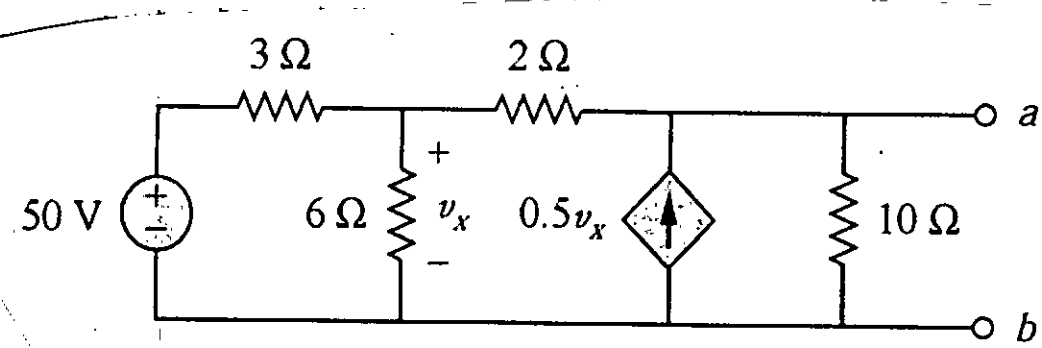

Figure for Question 4(a)

(b) For the circuit in Figure for Question 4(b), use the superposition principle to calculate the power dissipated by the 4 Ω resistor. **(18)**

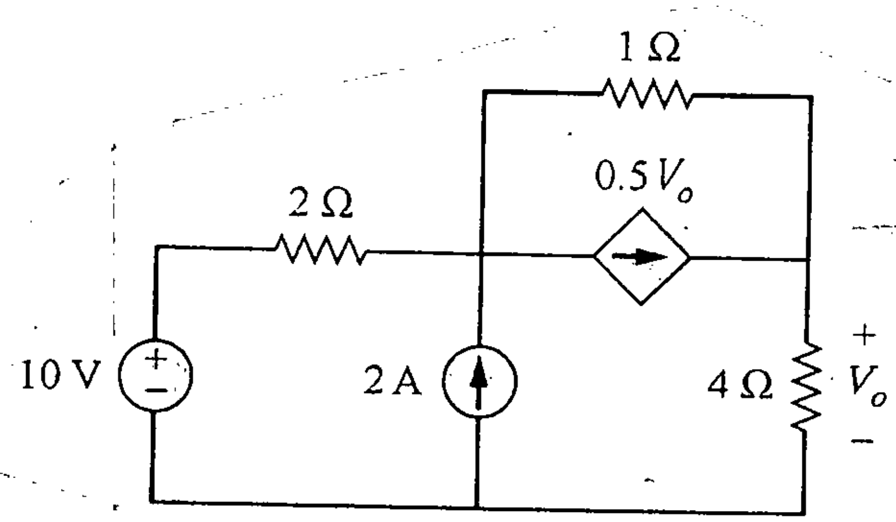

Figure for Question 4(b)

SECTION – B

There are **FOUR** questions in this section. Answer to **Question no. 5 is Compulsory.**
Answer any **TWO** questions from Questions 6-8.

5. (a) If a current, $i(t) = I_m \sin(\omega t)$, is flowing through an RL series circuit, derive the expression of instantaneous voltage and instantaneous power of this circuit. Also, illustrate the waveshapes of the instantaneous voltage and the instantaneous power of the series circuit. **(12)**
(CO2)

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Contd ... Q. No. 5(b)

(b) Applying mesh analysis method, determine $\mathbf{V_0}$ and $\mathbf{I_0}$ in the circuit shown in Fig. for Q. No. 5(b). **(15)** (CO3)

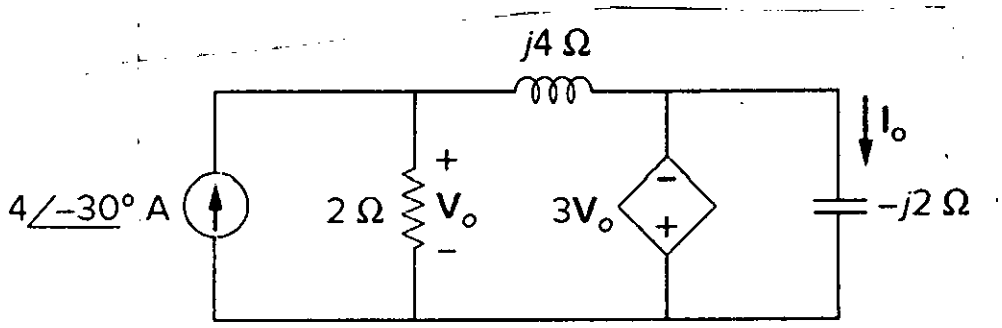

Fig. for Q. No. 5(b)

(c) Explain, with the output voltage equation, what is the function of the circuit shown in Fig. for Q. No.5(c). **(8)** (CO5)

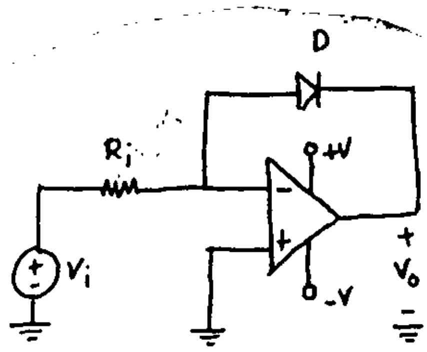

Fig. for Q. No. 5(c)

6. (a) Calculate Z_T for the circuit if Fig. for Q. No. 6(a). Also, compute the real and reactive power supplied by $100\angle 60°$ V voltage source, if Z_T is connected to this source as load. **(18)**

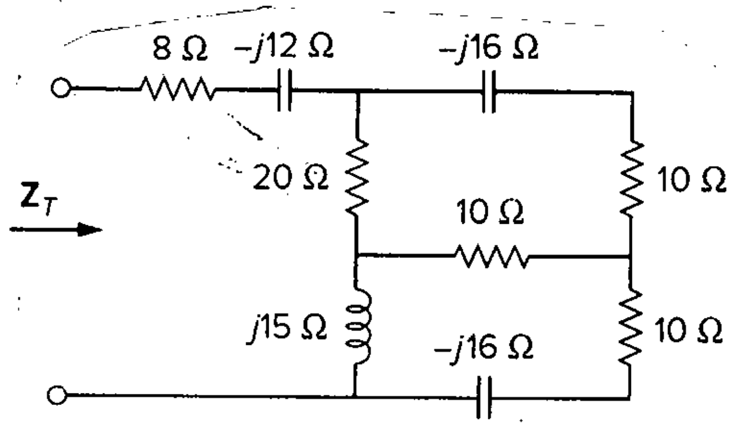

Fig. for Q. No. 6(a)

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Contd ... Q. No. 6(b)

(b) Find the value of Z in the circuit shown in Fig. for Q. 6(b), if $\mathbf{V}_g = 100 - j50$ V, $\mathbf{I}_g = 30 + j20$ A, and $\mathbf{V}_1 = 140 + j30$ V. **(17)**

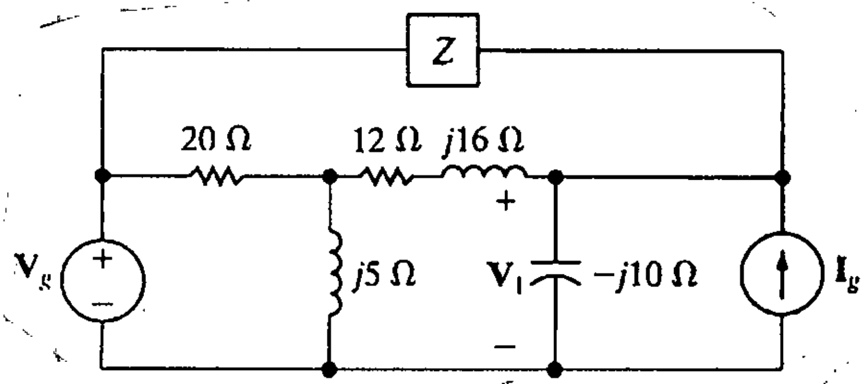

Fig. for Q. 6 (b)

7. (a) In the circuit of Fig. for Q. 7(a), find the value of load impedance, Z_L that will absorb the maximum power and the value of the maximum power. Also, determine the power factor of this load. **(23)**

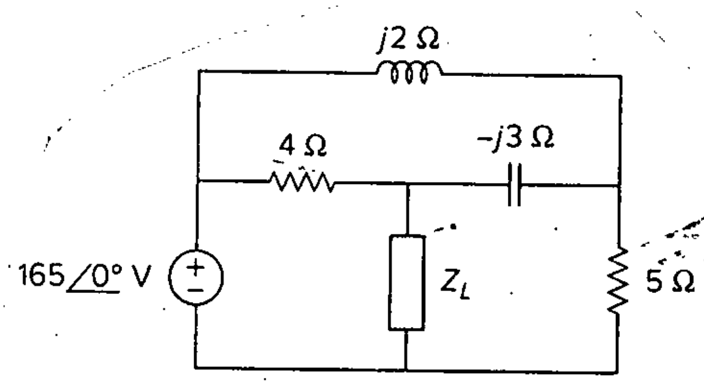

Fig. for Q. 7 (a)

(b) Find the RMS and average values of the signal shown in Fig. for Q. 7(b). **(12)**

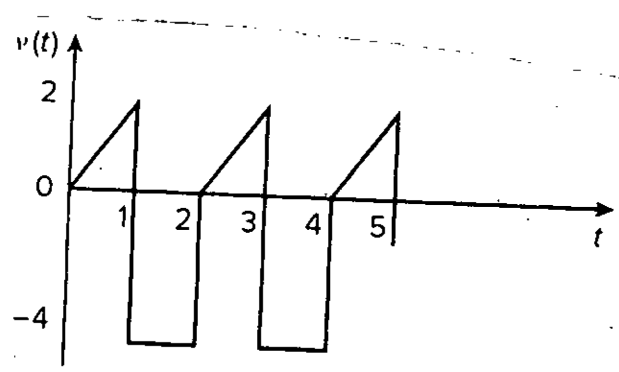

Fig. for Q. 7 (b)

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8. (a) Write down the characteristics of the ideal OP-AMP. Explain the operation and deduce the equation of closed-loop gain of an OP-AMP noninverting amplifier with the necessary diagram. Discuss the applications of this circuit. **(16)**

(b) Design a circuit using OP-AMP that can give the solution to the following equation. **(19)**

$$5\frac{d^2v}{dt^2}-3\frac{dv}{dt}+v-2=0$$

Course Outcomes of EEE 163

CO No.	CO Statement	Corresponding PO(s)
CO1	Apply the concepts of circuit elements, circuit, circuit variables, direct current, voltage dependent and independent sources, circuit laws, analysis methods, theorems to solve various circuits.	PO(a)
CO2	Derive the expressions of voltage, current and power/energy of RL, RC and RLC circuits based on the concepts of phasors	PO(a)
CO3	Employ circuit laws, analysis methods, theorems to solve various AC circuits.	PO(b)
CO4	Analyze the 3-phase circuits with different combination of sources and loads that are used in power systems.	PO(b)
CO5	Explain the operation of Op-Amp and its applications in mathematical and filtering circuits	PO(a)