L-1/T-1/EEE Date: ~~23/04/2024~~ 25/5/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **PHY 121** (Waves and Oscillations, Optics and Thermal Physics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) State Lissajous figures. Mention a few applications of it. **(8)**

(b) A particle is influenced by two simple harmonic motions acting at right angle to each other and having frequency ratio 2:1. Find out the general equation of the particle motion. Using this equation, formulate the equation of parabola. **(20)**

(c) A simple pendulum of mass m is raised to a height h from the horizontal plane and released (Fig. 1(c)). After hitting a spring of non-linear force law, $F = -kx - bx^3$, calculate the compression, x of the spring. **(7)**

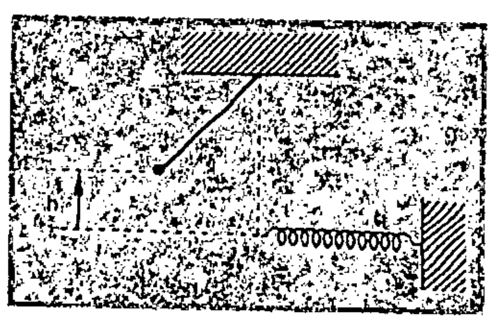

Fig. 1(c)

2. (a) State damped oscillation. Mention a few examples of damped oscillation. **(8)**

(b) Establish a differential equation of damped oscillation and find out the general solution of it. Illustrate different types of damping. **(20)**

(b) A massless spring, suspended from a rigid support, carriers a mass of 600 g at its lower end and the system oscillate with a frequency of 6 Hz. If the amplitude is reduced to half its undamped value in 25 s, calculate the force constant of the spring, relaxation time of the system, and its quality factor. **(7)**

3. (a) State phase velocity, particle velocity, and group velocity. **(8)**

(b) Formulate the relation between group velocity and phase velocity. **(20)**

(c) If the frequency of matter wave is $mv^2/2h$ and the wavelength is h/mv, find out the group velocity, where the symbols have their usual meaning. **(7)**

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4. (a) Briefly describe different thermodynamic processes. **(8)**

(b) What is entropy? Obtain the expression of the change in entropy for the reversible and irreversible process. Also, derive the expression of entropy for Van der Waals gas. **(20)**

(c) 1 g molecule of monoatomic ($\gamma = 5/3$) perfect gas at 21 °C is adiabatically compressed in a reversible process from an initial pressure of 1 atm to a final pressure of 50 atm. Calculate the resulting difference in temperature. **(7)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) State the postulates of the kinetic theory of gases. **(8)**

(b) Define the critical constants of gas. For real gases, obtain the corrected form of the gas equation and hence determine the expression of T_c, P_c and V_c. **(20)**

(c) At normal temperature, a mixture of λ_1 moles of monoatomic gas and λ_2 moles of diatomic gas are prepared. If γ be the adiabatic exponent, then show that, **(7)**

$$\gamma = \frac{5\lambda_1 + 7\lambda_2}{3\lambda_1 + 5\lambda_2}.$$

6. (a) Discuss different types of thermodynamic systems. **(8)**

(b) Write down the first four Maxwell's thermodynamic relation, and **(20)**

(i) Establish the 1st and 2nd TdS equations.

(ii) Show that, $C_P - C_V = -T\left(\frac{\partial V}{\partial T}\right)_P^2\left(\frac{\partial P}{\partial V}\right)_T$, where the symbols have their usual meaning.

(c) Calculate the change in temperature of boiling water when the pressure is increased by 2.712 mm of Hg. The normal boiling point of water at atmospheric pressure is 100 °C. Given that, the latent heat of steam is 537 cal/g, and the volume of 1g of steam and water at 100 °C are 1674 c.c. and 1.000 c.c., respectively. **(7)**

7. (a) Define longitudinal spherical aberration and circle of least spherical aberration. Show them in an optical diagram. **(8)**

(b) Derive and discuss the conditions under which a system of two lenses separated by a finite distance is free from chromatic and spherical aberration. **(20)**

(c)Two thin convex lenses of focal length f_1 and f_2 separated by a distance d have an equivalent focal length of 50 cm. The combination satisfies and conditions for minimum spherical aberration and is also achromatic. Find the values of f_1, f_2, and d. Assume that both the lenses are of the same material. **(7)**

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8. (a) Find a relation among the width of the central maximum, the wavelength of light, and the slit width for Fraunhofer diffraction at circular aperture. **(8)**

(b) Discuss the Fraunhofer diffraction due to double slits and hence obtain an expression for the resultant intensity at a point on the screen. Depict the intensity distribution in a diagram. **(20)**

(c) Calculate the missing orders for a double slit Fraunhofer diffraction pattern, if the slit widths are 8.8×10^{-3} cm and the separation in between the slits are 4.4×10^{-2} cm. **(7)**

L-1/T-1/EEE Date: 04/05/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **MATH 159** (Calculus II)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Calculate the principal value of the complex logarithm *Ln*z for $z = i$. Also show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$ for all complex numbers $z = x + iy$. **(12)**

   (b) An airplane travels 100 miles southeast, 50 miles due west, 200 miles 30° north of east and then 150 miles northeast. Determine (i) analytically and (ii) graphically how far and in what direction it is from its starting point. **(12)**

   (c) Describe Cauchy-Riemann equations in polar form. Test the differentiability of the function $f(z) = \frac{1}{r^4}\cos 4\theta + i\left(-\frac{1}{r^4}\sin 4\theta\right), r > 0,\ 0 < \theta < 2\pi$ in the indicated domain and hence show that $f'(z) = -\frac{4}{r^5 e^{i5\theta}}$. **(11)**

2. (a) Explain that $v(x, y) = \frac{x}{x^2 + y^2} + \cosh x \cos y$ is a harmonic function. Find an analytic function $f(z) = u(x, y) + iv(x, y)$ and express *f (z)* in terms of z. **(12)**

   (b) Let C_R denote the upper half of the circle $|z| = R$ for some $R > 1$. Show that $\left|\frac{e^{iz}}{z^2 + z + 1}\right| \leq \frac{1}{(R-1)^2}$ for all z lying on C_R. **(13)**

   (c) Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 + i\bar{z}_2 = 0$ and arg $(z_1 z_2) = \pi$. Then find arg (z_1). **(10)**

3. (a) Evaluate the integral $\oint_c \frac{e^{3z}}{(z-i)^4} dz$ by Cauchy integral formula, where, $C = \{(x, y) : |x| \leq 2, |y| \leq 2\}$ is positively oriented. **(12)**

   (b) Use Taylor's theorem to find the first three terms of the expansion of $f(z) = \frac{z}{(z+3i)(z-2i)}$ about the given center i and its radius of convergence. **(13)**

   (c) Evaluate $\int_c (x^2 - iy^2)\, dz$, where C is the straight line from $z = 1 + i$ to $z = 1 + 8i$. and then a line parallel to y-axis from $z = 1 + 8i$ to $z = 2 + 8i$. **(10)**

4. (a) Expand $f(z) = \frac{z^2}{(z-1)(2-z)}$ in a Laurent series valid for $0 < |z - 2| < 1$. **(10)**

   (b) Evaluate the integral $\int_c \frac{1}{z(2z-1)(z-4)} dz$ by Cauchy's residue theorem, **(10)**

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where $C = \{z : |z+2| + |z-2| = 6\}$ is positively oriented.

(c) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ using the method of contour integration. **(15)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Express $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$ as a linear combination of $\vec{A}$ and $\vec{B}$. Hence find the volume of the parallelepiped having edges $\vec{B} \times \vec{C}$, $\vec{C} \times \vec{A}$ and $\vec{A} \times \vec{B}$. **(10)**

(b) Develop an equation for the plane determined by the points (2, -1, 1), (3, 2, -1) and (-1, 3 2). How many planes can be constructed using two points? Justify your answer. **(10)**

(c) Classify whether the vectors $\vec{A} = (0, 3, -3, -6)$, $\vec{B} = (-2, 0, 0, -6)$, **(15)**
$\vec{C} = (0, -4, -2, -2)$, $\vec{D} = (0, -8, 4, -4)$ are linearly independent or linearly dependent. Also, establish a relationship among the above vectors (if possible).

6. (a) Define curvature and torsion. Find the curvature for the space curve **(12)**
$x = t - \frac{t^3}{3}$, $y = t^2$, $z = t + \frac{t^3}{3}$ at any point t.

(b) For what value of the constant a will the vector $\vec{A}(x, y, z) = (axy - z^3)\,\hat{i} +$ **(10)**
$(a-2)x^2\,\hat{j} + (1-a)xz^2\,\hat{k}$ have its curl identically equal to zero? Hence briefly discuss the physical significance of curl $\vec{A}$ for that constant a.

(c) Suppose an xyz- coordinate system is located in space such that the temperature T at the point (x, y, z) is given by the formula $T = 100/(x^2 + y^2 + z^2)$. **(13)**

(i) Measure the rate of change of T with respect to distance at the point P(1, 3, -2) in the direction of the vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$.

(ii) In what direction from P does T increase most rapidly? What is the maximum rate of change of T at P ?

7. (a) Let $\vec{F}(x, y, z) = y^2 \cos x\,\hat{i} + (2y \sin x + e^{2z})\hat{j} + 2ye^{2z}\,\hat{k}$ Show that $\int \vec{F} .\, d\vec{r}$ is independent of path, and find a potential function f for $\vec{F}$. If $\vec{F}$ is a force field, calculate the work done by $\vec{F}$ along any curve C from (0, 1, 1/2) to (π/2, 3, 2). **(17)**

(b) Let S be the part of the graph of $z = 9 - x^2 - y^2$ with $z \geq 0$. **(18)**
If $\vec{F}(x, y, z) = 3x\,\hat{i} + 3y\,\hat{j} + z\,\hat{k}$, find the flux of $\vec{F}$ through S.

8. (a) Mention some applications of Green's theorem in the plane. Using Green's theorem find the area bounded by the hypocycloid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}, a > 0$. **(15)**

(b) Verify the divergence theorem for $\vec{A} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and x = 2. **(20)**