

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **IPE 105** (Principles of Cost and Management Accounting)

Full Marks: 210

Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**

There are **FOUR** questions in this section. Answer **Q. No. 1** and any **TWO** from the rest.

Assume reasonable values for any missing data.

1. (a) The production department of Zan Corporation has submitted the following forecast of units to be produced by quarter for the upcoming fiscal year:

(20)  
(CO2)

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Units to be produced	5,000	8,000	7,000	6,000

In addition, the beginning raw materials inventory for the 1st Quarter is budgeted to be 6,000 grams and the beginning accounts payable for the 1st Quarter is budgeted to be \$2,880. Each unit requires 8 grams of raw material that costs \$1.20 per gram. Management desires to end each quarter with an inventory of raw materials equal to 25% of the following quarter's production needs. The desired ending inventory for the 4th Quarter is 8,000 grams. Management plans to pay for 60% of raw material purchase in the quarter acquired and 40% in the following quarter. Each unit requires 0.20 direct labor-hours and direct laborers are paid \$11.50 per hour. Apply appropriate methods to analyze the data and prepare the company's direct materials budget.

- (b) Analyze the data shown in the following table for a distributor of material arts products, and compute the missing values of some financial performance parameters with appropriate techniques.

(15)  
(CO3)

	Division		
	Alpha	Bravo	Charlie
Sales .....	\$ ?	\$11,500,000	\$ ?
Net operating income .....	\$ ?	\$ 920,000	\$210,000
Average operating assets .....	\$800,000	\$ ?	\$ ?
Margin .....	4%	?	7%
Turnover .....	5	?	?
Return on investment (ROI) .....	?	20%	14%

2. (a) Thermal Rising, Inc., makes paragliders for sale through specially sporting goods. Stores. The company has a standard paraglider model, but also makes custom-designed paragliders. Management has designed an activity-based costing system with the following activity cost pools and activity rates:

(20)

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**Contd... Q. No. 2(a)**

Activity Cost Pool	Activity Rate
Supporting direct labor .....	\$26 per direct labor-hour
Order processing .....	\$284 per order
Custom design processing .....	\$186 per custom design
Customer service .....	\$379 per customer

Management would like an analysis of the profitability of a particular customer, Big Sky Outfitters, which has ordered the following products over the last 12 months:

	Standard Model	Custom Design
Number of gliders .....	20	3
Number of orders .....	1	3
Number of custom designs .....	0	3
Direct labor-hours per glider ..	26.35	28.00
Selling price per glider .....	\$1,850	\$2,400
Direct materials cost per glider ..	\$564	\$634

The company's direct labor rate is \$19.50 per hour. Using the company's activity-based costing system, compute the customer margin of Big Sky Outfitters.

(b) Via Gelato is a popular neighborhood gelato shop. The company has provided the following data concerning its operations:

(15)

	Fixed Element per Month	Variable Element per Liter	Actual Total for June
Revenue .....		\$12.00	\$71,540
Raw materials .....		\$4.65	\$29,230
Wages .....	\$5,600	\$1.40	\$13,860
Utilities .....	\$1,630	\$0.20	\$3,270
Rent .....	\$2,600		\$2,600
Insurance .....	\$1,350		\$1,350
Miscellaneous .....	\$650	\$0.35	\$2,590

While gelato is sold by the cone or cup, the shop measures its activity in terms of the total number of liters of gelato sold. For example, wages should be \$5,600 plus \$1.40 per liter of gelato sold and the actual wages for June were \$13,860. Via Gelato expected to sell 6,000 liters in June, but actually sold 6,200 liters.

**Required:** Prepare a report showing Via Gelato revenue and spending variances for June.

- Profits have been decreasing for several years at Pegasus Airlines. In an effort to improve the company's performance, consideration is being given to dropping several flights that appear to be unprofitable.

(36)

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**Contd... Q. No. 2(b)**

A typical income statement for one round-trip of one such flight (flight 482) is as follows:

Ticket revenue (175 seats × 40% occupancy × \$200 ticket price) .....	\$14,000	100.0%
Variable expenses (\$15 per person) .....	<u>1,050</u>	<u>7.5</u>
Contribution margin .....	<u>12,950</u>	<u>92.5%</u>
Flight expenses:		
Salaries, flight crew .....	1,800	
Flight promotion .....	750	
Depreciation of aircraft .....	1,550	
Fuel for aircraft .....	5,800	
Liability insurance .....	4,200	
Salaries, flight assistants .....	1,500	
Baggage loading and flight preparation .....	1,700	
Overnight costs for flight crew and assistants at destination .....	<u>300</u>	
Total flight expenses .....	<u>17,600</u>	
Net operating loss .....	<u>\$ (4,650)</u>	

The following additional information is available about flight 482:

- (i) Members of the flight crew are paid fixed annual salaries, whereas the flight assistants are paid based on the number of round trips they complete.
- (ii) One-third of the liability insurance is a special charge assessed against flight 482 because in the opinion of the insurance company, the destination of the flight is in a "high-risk" area. The remaining two-thirds would be unaffected by a decision to drop flight 482.
- (iii) The baggage loading and flight preparation expense is an allocation of ground crews' salaries and depreciation of ground equipment. Dropping flight 482 would have no effect on the company's total baggage loading and flight preparation expenses.
- (iv) If flight 482 is dropped, Pegasus Airlines has no authorization at present to replace it with another flight.
- (v) Aircraft depreciation is due entirely to obsolescence. Depreciation due to wear and tear is negligible.
- (vi) Dropping flight 482 would not allow Pegasus Airlines to reduce the number of aircraft in its fleet or the number of flight crew on its payroll.

**Required:** Prepare an analysis showing what impact dropping flight 482 would have on the airline's profits.

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4. The Walton Toy Company manufactures a line of dolls and a doll dress sewing kit. Demand for the dolls is increasing, and management requests assistance from you in determining an economical sales and production mix for the coming year. The company has provided the following data:

(20+10+5)

Product	Demand Next Year (units)	Selling Price per Unit	Direct Materials	Direct Labor
Debbie .....	50,000	\$13.50	\$4.30	\$3.20
Trish .....	42,000	\$5.50	\$1.10	\$2.00
Sarah .....	35,000	\$21.00	\$6.44	\$5.60
Mike .....	40,000	\$10.00	\$2.00	\$4.00
Sewing kit .....	325,000	\$8.00	\$3.20	\$1.60

The following additional information is available:

- (i) The company's plant has a capacity of 130,000 direct labor-hours per year on a single-shift basis. The company's present employees and equipment can produce all five products.
- (ii) The direct labor rate of \$8 per hour is expected to remain unchanged during the coming year.
- (iii) Fixed costs total \$520,000 per year. Variable overhead costs are \$2 per direct labor-hour.
- (iv) All of the company's nonmanufacturing costs are fixed.

***Required:***

- (a) Determine the contribution margin per direct labor-hour expended on each product.
- (b) Prepare a schedule showing the total direct labor-hours that will be required to produce the units estimated to be sold during the coming year.
- (c) Examine the data you have computed in (a) and (b) above. How would you allocate the 130,000 direct labor-hours of capacity to Walton Toy Company's various products?

**SECTION - B**

There are **FOUR** questions in this section. Answer **Q. No. 5** and any **TWO** from the rest.

- 5. (a) With suitable example, explain the difference between a product cost and a period cost.
- (b) The Lakeshore Hotel's guest-days of occupancy and custodial supplies expense over the last seven months were:

(6)  
(CO1)

(14)  
(CO1)

Contd ..... P/5

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Contd... Q. No. 5(b)

Month	Guest-Days of Occupancy	Custodial Supplies Expense
March	4,000	\$7,500
April	6,500	\$8,250
May	8,000	\$10,500
June	10,500	\$12,000
July	12,000	\$13,500
August	9,000	\$10,750
September	7,500	\$9,750

Guest-days is a measure of the overall activity at the hotel. For example, a guest who stays at the hotel for three days is counted as three guest-days.

- (i) Using the high-low method, estimate a cost formula for custodial supplies expense.
- (ii) Using the cost formula derived above, what amount of custodial supplies expense would one expect to be incurred at an occupancy level of 11,000 guest-days?

(c) Professor John Morton has just been appointed chairperson of the Finance Department at Westland University. In reviewing the department's cost records, Professor Morton has found the following total cost associated with Finance 101 over the last several terms:

(15)  
(CO1)

Term	Number of Sections Offered	Total Cost
Fall, last year	4	\$10,000
Winter, last year	6	\$14,000
Summer, last year	2	\$7,000
Fall, this year	5	\$13,000
Winter, this year	3	\$9,500

Professor Morton knows that there are some variable costs, such as amounts paid to graduate assistants, associated with the course. He would like to have the variable and fixed costs separated for planning purposes.

Using the least-squares regression method, estimate the variable cost per section and the total fixed cost per term for Finance 101. Express these estimates in the linear equation form  $Y = a + bX$ .

- 6. (a) High Desert Potteryworks makes a variety of pottery products that it sells to retailers such as Home Depot. The company uses a job-order costing system in which

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**Contd... Q. No. 6(a)**

predetermined overhead rates are used to apply manufacture overhead cost to jobs. The predetermined overhead rate in the Molding Department is based on machine-hours, and the rate in the Painting Department is based on direct labor-hours. At the beginning of the year, the company's management made the following estimates:

(20)

	Department	
	Molding	Painting
Direct labor-hours	12,000	60,000
Machine-hours	70,000	8,000
Direct materials cost	\$510,000	\$650,000
Direct labor cost	\$130,000	\$420,000
Fixed manufacturing overhead cost	\$487,000	\$615,000
Variable manufacturing overhead per machine-hour	\$1.50	
Variable manufacturing overhead per direct labor-hour		\$2.00

Job 205 was started on August 1 and completed on August 10. The company's cost records show the following information concerning the job:

	Department	
	Molding	Painting
Direct labor-hours	30	84
Machine-hours	110	20
Materials placed into production	\$470	\$332
Direct labor cost	\$325	\$588

- (i) Compute the predetermined overhead rate used during the year in the Molding Department. Compute the rate used in the Painting Department.
- (ii) Compute the total overhead cost applied to Job 205.
- (iii) What would be the total cost recorded for Job 205? If the job contained 50 units, what would be the unit product cost?
- (iv) At the end of the year, the records of High Desert Potteryworks revealed the following actual cost and operating data for all jobs worked on during the year:

	Department	
	Molding	Painting
Direct labor-hours	10,000	62,000
Machine-hours	65,000	9,000
Direct materials cost	\$430,000	\$680,000
Direct labor cost	\$108,000	\$436,000
Manufacturing overhead cost	\$570,000	\$750,000

What was the amount of underapplied or overapplied overhead in each department at the end of the year?

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**Contd... Q. No. 6**

(b) Cooperative San José of southern Sonora state in Mexico makes a unique syrup using cane sugar and local herbs. The syrup is sold in small bottles and is prized as a flavoring for drinks and for use in desserts. The bottles are sold for \$12 each. The first stage in the production process is carried out in the Mixing Department, which removes foreign matter from the raw materials and mixed them in the proper proportions in large vats. The company uses the weighted-average method in its process costing system.

(15)

A hastily prepared report for the Mixing Department for April appears below:

<b>Units to be accounted for:</b>	
Work in process, April 1 (materials 90% complete; conversion 80% complete)	30,000
Started into production	200,000
<b>Total units to be accounted for</b>	<b>230,000</b>
<b>Units accounted for as follows:</b>	
Transferred to next department	190,000
Work in process, April 30 (materials 75% complete; conversion 60% complete)	40,000
<b>Total units accounted for</b>	<b>230,000</b>
<b>Cost Reconciliation</b>	
<b>Cost to be accounted for:</b>	
Work in process, April 1	\$ 88,000
Cost added during the month	827,000
<b>Total cost to be accounted for</b>	<b>\$925,000</b>
<b>Cost accounted for as follows:</b>	
Work in process, April 30	\$118,400
Transferred to next department	805,600
<b>Total cost accounted for</b>	<b>\$925,000</b>

Management would like some additional information about Cooperative San José's operations.

- (i) What were the equivalent units for the month?
- (ii) What were the costs per equivalent unit for the month? The beginning inventory consisted of the following costs: materials, \$67,800; and conversion cost, \$30,200. The costs added during the month consisted of: materials, \$579,000; and conversion cost, \$248,000.
- (iii) How many of the units transferred to the next department were started and completed during the month?
- (iv) The manager of the Mixing Department stated, "Materials prices jumped from about \$2.50 per unit in March to \$3 per unit in April, but due to good cost control I was able to hold our materials cost to less than \$3 per unit for the month". Should this manager be rewarded for good cost control? Explain.

Contd ..... P/8

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7. (a) Explain why is cost accumulation simpler in a process costing system than it is in a job-order costing system? (8)

(b) Paul Sabin organized Sabin Electronics 10 years ago to produce and sell several electronic devices on which he had secured patents. Although the company has been fairly profitable, it is now experiencing a severe cash shortage. For this reason, it is requesting a \$500,000 long-term from Gulfport State Bank, \$100,000 of which will be used to bolster the Cash account and \$400,000 of which will be used to modernize equipment. The company's financial statements for the two most recent years follow: (22)

	This Year	Last Year
<b>Assets</b>		
<b>Current assets:</b>		
Cash .....	\$ 70,000	\$ 150,000
Marketable securities .....	0	18,000
Accounts receivable, net .....	480,000	300,000
Inventory .....	850,000	600,000
Prepaid expenses .....	20,000	22,000
<b>Total current assets .....</b>	<b>1,520,000</b>	<b>1,090,000</b>
Plant and equipment, net .....	1,480,000	1,370,000
<b>Total assets .....</b>	<b>\$3,000,000</b>	<b>\$2,460,000</b>
<b>Liabilities and Stockholders' Equity</b>		
<b>Liabilities:</b>		
Current liabilities .....	\$ 800,000	\$ 430,000
Bonds payable, 12% .....	600,000	600,000
<b>Total liabilities .....</b>	<b>1,400,000</b>	<b>1,030,000</b>
<b>Stockholders' equity:</b>		
Common stock, \$15 par .....	750,000	750,000
Retained earnings .....	850,000	680,000
<b>Total stockholders' equity .....</b>	<b>1,600,000</b>	<b>1,430,000</b>
<b>Total liabilities and equity .....</b>	<b>\$3,000,000</b>	<b>\$2,460,000</b>

	This Year	Last Year
Sales .....	\$5,000,000	\$4,350,000
Cost of goods sold .....	3,875,000	3,450,000
Gross margin .....	1,125,000	900,000
Selling and administrative expenses .....	653,000	548,000
Net operating income .....	472,000	352,000
Interest expense .....	72,000	72,000
Net income before taxes .....	400,000	280,000
Income taxes (30%) .....	120,000	84,000
Net income .....	280,000	196,000
Common dividends .....	110,000	95,000
Net income retained .....	170,000	101,000
Beginning retained earnings .....	680,000	579,000
Ending retained earnings .....	\$ 850,000	\$ 680,000

During the past year, the company introduced several new product lines and raised the selling prices on a number of old product lines in order to improve its profit margin. The company also hired a new sales manager, who has expanded sales into several new territories. All sales are on account.

To assist in approaching the bank about the loan, Paul has asked the sales manager to compute the following ratios for both this year and last year. So, compute the following:



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**Contd... Q. No. 7(b)**

- (i) The amount of working capital.
- (ii) The current ratio.
- (iii) The acid-test ratio.
- (iv) The average collection period. (The accounts receivable at the beginning of last year totaled \$250,000.)
- (v) The average sale period. (The inventory at the beginning of last year totaled \$500,000.)
- (vi) The operating cycle.
- (vii) The total asset turnover. (The total assets at the beginning of last year were \$2,420,000.)
- (viii) The dept-to-equity ratio.

Paul Sabin has also gathered the following financial data and ratios that are typical of companies in the electronics industry:

Current ratio	2.5
Acid-test ratio	1.3
Average collection period	18 days
Average sale period	60 days
Debt-to-equity ratio	0.90
Times interest earned ratio	6.0

Comment on the results of the analysis in (i-viii) above and compare Sabin Electronics' performance to the benchmarks from the electronics industry. Do you think that the company is likely to get its load application approved?

(c) What are cash equivalents, and why are they included with cash on a statement of cash flows? (5)

8. (a) Feather Friends, Inc., distributes a high-quality wooden birdhouse that sells for \$20 per unit. Variable expenses are \$8 per unit, and fixed expenses total \$180,000 per year.

Answer the following independent questions: (20)

- (i) What is the product's CM ratio?
- (ii) Use the CM ratio to determine the break-even point in dollar sales.
- (iii) Due to an increase in demand, the company estimates that sales will increase by \$75,000 during the next year. By how much should net operating income increase (or net loss decrease) assuming that fixed expenses do not change?
- (iv) Assume that the operating results for last year were:

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**Contd... Q. No. 8(a)**

Sales .....	\$400,000
Variable expenses .....	160,000
Contribution margin .....	240,000
Fixed expenses .....	180,000
Net operating income .....	<u>\$ 60,000</u>

- A. Compute the degree of operating leverage at the current level of sales.
- B. The president expects sales to increase by 20% next year. By what percentage should net operating income increase?

(v) Refer to the original data. Assume that the company sold 18,000 units last year. The sales manager is convinced that a 10% reduction in the selling price, combined with a \$30,000 increase in advertising, would increase annual unit sales by one-third. Prepare two contribution format income statements, one showing the results of last year's operations and one showing the results of operations if these changes are made. Would you recommend that the company do as the sales manager suggests?

(vi) Refer to the original data. Assume again that the company sold 18,000 units last year. The president does not want to change the selling price. Instead, he wants to increase the sale commission by \$1 per unit. He thinks that this move, combined with some increase in advertising, would increase annual sales by 25%. By how much could advertising be increased with profits remaining unchanged? Do not prepare an income statement; use the incremental analysis approach.

(b) During Heaton Company's first two years of operations, the company reported absorption costing net operating income as follows:

(15)

	Year 1	Year 2
Sales (@ \$25 per unit) .....	\$1,000,000	\$1,250,000
Cost of goods sold (@ \$18 per unit) .....	720,000	900,000
Gross margin .....	280,000	350,000
Selling and administrative expenses .....	210,000	230,000
Net operating income .....	<u>\$ 70,000</u>	<u>\$ 120,000</u>

\*\$2 per unit variable; \$130,000 fixed each year.

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Contd... Q. No. 8(b)

The company's \$18 unit product cost is computed as follows:

Direct materials .....	\$ 4
Direct labor .....	7
Variable manufacturing overhead .....	1
Fixed manufacturing overhead (\$270,000 ÷ 45,000 units) .....	.6
Absorption costing unit product cost .....	<u>\$18</u>

Forty percent of fixed manufacture overhead consists of wages and salaries; the remainder consists of depreciation charges on production equipment and buildings. Production and cost data for the two years are:

	Year 1	Year 2
Units produced .....	45,000	45,000
Units sold .....	40,000	50,000

- (i) Prepare a variable costing contribution format income statement for each year.
- (ii) Reconcile the absorption costing and the variable costing net operating income figures for each year.

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L-1/T-1/IPE

25/5/24  
Date: 04/05/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **PHY 117** (Structure of Matter, Electricity and Magnetism, and Modern physics)

Full Marks: 210

Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define electric dipole moment. Give an example of an electric dipole. (5)  
(b) Derive an expression mathematically  $\vec{\tau} = \vec{P} \times \vec{E}$  where  $\vec{\tau}$  is the torque of an electric dipole placed in a uniform electric field  $\vec{E}$  and  $\vec{P}$  is the electric dipole moment. And the potential energy  $U$  stored in the system is  $U = -\vec{P} \cdot \vec{E}$  (20)  
(c) An electric dipole with dipole moment  $4 \times 10^{-9}$  Coul-m is aligned at  $30^\circ$  with the direction of a uniform electric field of magnitude  $5 \times 10^4$  NC<sup>-1</sup>. Calculate the magnitude of the torque acting on the dipole. (10)
2. (a) Discuss the terms self-inductance and mutual-inductance, back  $e, m, f$  in inductor, eddy current. (10)  
(b) If a straight metal wire of length ' $l$ ' carrying a current ' $i$ ' is placed at right angles to a uniform magnetic field  $\vec{B}$  derive the relation of the deflecting force acting on it is given by  $\vec{F} = i\vec{l} \times \vec{B}$  (15)  
(c) A copper wire has a resistance of  $10\Omega$  and an area of cross-section  $1\text{mm}^2$ . A potential difference of  $10\text{V}$  exists across the wire. Calculate the drift speed of electrons if the number of electrons per cubic meter in copper is  $8 \times 10^{28}$ . (10)
3. (a) Discuss the origin of magnetic moment in magnetic materials. Briefly address the classification of magnetic materials with examples. (15)  
(b) Draw and explain a hysteresis loop showing remanence and coercive force. (10)  
(c) Differentiate between soft and hard magnetic materials. Explain why steel is more suitable for making permanent magnets than soft iron for electromagnets? (10)
4. (a) Define point defects in crystal. Write the names of intrinsic and extrinsic point defects. (7)  
(b) Derive the expression of Bragg's law of X-ray diffraction. Explain the advantages of X-ray diffraction techniques over neutron diffraction technique for crystal structure analysis. (18)  
(c) X-rays having a wavelength of  $0.84 \text{ \AA}$  are used to determine the interplanar spacing of (110) plane in a body centered cubic crystal. The Bragg angle for the reflection from the 3<sup>rd</sup> plane is  $33.5^\circ$ . Calculate the lattice constant and atomic radius of the crystal. (10)

**PHY 117****SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Define linear density and planar density of atoms in a crystal. How density of a crystal can be determined theoretically using lattice constant? (12)
- (b) Describe a face centered cubic (fcc) structure and sodium chloride (*NaCl*) structure. How does *NaCl* structure differ from a standard fcc structure? (15)
- (c) Indium (*In*) shows tetragonal-I crystal structure with the lattice constants  $a = 3.25 \text{ \AA}$  and  $c = 4.95 \text{ \AA}$ . Atomic mass of *In* is 114.82 amu. Calculate the volume of the unit cell and theoretical density of *In* crystal. (8)
6. (a) State the differences between p-type and n-type semiconductors in terms of band theory of solid. (12)
- (b) Explain Van der Waals bond in solid and prove that the Van der Waals force of attraction between polar and nonpolar molecules is inversely proportional to  $r^7$ , where  $r$  is the intermolecular distance. (15)
- (c) The general expression of Coulomb potential energy of an ion pair separated by a distance  $r$  in a lithium chloride (*LiCl*) crystal is,  $U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$  (symbols carry their usual meaning). Write down the expression of total Coulomb potential energy and calculate the first two terms in the series for the Madelung constant for *LiCl* crystal. (8)
7. (a) State and describe the basic postulates of special theory of relativity. (10)
- (b) Deduce the mathematical expression for addition of relativistic velocities. Show that no particle can have velocity greater than the velocity of light. (17)
- (c) At what speed does the kinetic energy of a particle equal its rest energy? (8)
8. (a) Define nuclear fission with an example. What is the source of release of energy in nuclear fission? (8)
- (b) Establish Einstein's photo-electric effect equation. Describe with suitable diagram what might be the effects on a photoelectric effect experiment if (i) the frequency of the incident light is doubled and (ii) the intensity is doubled. (19)
- (c) The stopping potential is  $4.6V$  for light of frequency  $2 \times 10^{15} \text{ Hz}$ . When light of frequency  $4 \times 10^{15} \text{ Hz}$  is used, the stopping potential is  $12.9V$ . Calculate the value of Planck's constant. (8)
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The figures in the margin indicate full marks

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Determine the value of constant  $a$  and  $b$  so that the function  $f(x)$  is continuous everywhere and hence test the differentiability at  $x = 1$  of the function: (20)

$$f(x) = \begin{cases} -x & \text{for } x \leq 0 \\ ax + b & \text{for } 0 < x \leq 1 \\ 2 - x & \text{for } 1 < x \leq 2 \end{cases}$$

- (b) Evaluate  $\lim_{x \rightarrow 0} \frac{3 \tan x - 3x - x^3}{x^5}$  (10)

- (c) If  $y = \frac{1}{2}(\tan^{-1} x)^2$ , apply Leibnitz's theorem to find the value of (16 $\frac{2}{3}$ )

$$y_{n+2}(0) + 2n^2 y_n(0) + n(n-1)^2 (n-2) y_{n-2}(0).$$

2. (a) State Rolle's theorem and verify it for the function  $f(x) = 2x^3 + x^2 - 4x - 2$ . (15 $\frac{2}{3}$ )

- (b) Use Maclaurin's theorem to expand  $f(x) = \frac{1}{(1-2x)^2}$  in powers of  $x$  in infinite series. (15)

- (c) If  $V$  is a function of  $r$  alone, where  $r^2 = x^2 + y^2 + z^2$ , show that (16)

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{dr^2} + \frac{2}{r} \frac{dV}{dr}$$

3. (a) A conical tent of given capacity has to be constructed. Find the ratio of the height to the radius of the base for the minimum amount of the canvas required for the tent. (16 $\frac{2}{3}$ )

- (b) If  $x \cos \alpha + y \sin \alpha = p$  touches the curve  $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$ , show that (15)

$$(a \cos \alpha)^{\frac{m}{m-1}} + (b \sin \alpha)^{\frac{m}{m-1}} = (p)^{\frac{m}{m-1}}.$$

- (c) Show that in curve  $by^2 = (x+a)^3$ , the square of the subtangent varies as the subnormal. (15)

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4. (a) Find the pedal equation of the curve  $r^m = a^m \cos m\theta$ . (16)  
 (b) Find the radius of curvature at the point  $\theta$  on the cycloid (15)  
 $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$ .  
 (c) Find the asymptotes of the curve  $x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$ . (15 $\frac{2}{3}$ )

**SECTION - B**

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. Evaluate the following integrals:
- (a)  $\int \frac{(x+2)\sqrt{x+4}}{\sqrt{x-4}} dx$  (15)  
 (b)  $\int \frac{1}{x^4+1} dx$  (15)  
 (c)  $\int \cot^{-1}(1-x+x^2) dx$ . (16 $\frac{2}{3}$ )
6. (a) Derive the reduction formula for  $\int_0^{\pi/2} \sin^m x \cos^n x dx$ . Use the above formula to evaluate  $\int_0^{\pi/2} \sin^{10} x \cos^8 x dx$ . (16 $\frac{2}{3}$ )  
 (b) Using the properties of definite integral, show that (15)  
 $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \ln(\sqrt{2} + 1)$ .  
 (c) Calculate the value of the definite integral:  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ . (15)
7. (a) Define Gamma and Beta functions. Prove that (15)  

$$\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta, (m, n > 0)$$
 and hence find the value of  $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ .  
 (b) Evaluate:  $\int_0^1 \frac{x}{(1-x^6)^{1/6}} dx$ . (15)  
 (c) Find the area of a loop of the curve  $x(x^2 + y^2) = a(x^2 - y^2)$ . (16 $\frac{2}{3}$ )
8. (a) Determine the perimeter of the loop of  $3ay^2 = x^2(a - x)$ . (16)  
 (b) Find the volume and surface area of the solid formed by revolving the curve  $r^2 = a^2 \cos 2\theta$  about the initial line. (30 $\frac{2}{3}$ )

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