L-2/T-1/CHE **Date: 27/03/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **CHE 201** (Material and Energy Balance)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer **Q. No. 1** and any **TWO from the rest.**

Q. NO. 1 is Compulsory.

(A data booklet containing all relevant data is to be provided)

1. (a) Air at 50% relative humidity is cooled isobarically at 1 atm absolute from 90°C to 25°C. **(5+5=10)**

(i) Estimate the dew point and degrees of superheat of the air at 90°C.

(ii) How much water condenses (mol) per cubic meter of feed gas?

(b) Normal heptanes is converted to toluene and hydrogen in a continuous vapor-phase reaction: $C_7H_{16} \rightarrow C_6H_5CH_3 + 4H_2$ **(15)**

Pure heptanes at 400°C is fed to the reactor. The reactor operates isothermally at 400°C and the reaction goes to completion. *Data:* The average heat capacity of *n*-heptane between 25°C and 400°C is 0.2427 kJ/(mol °C).

(i) Calculate the required heat transfer to or from the reactor (state which it is) in kJ.

(ii) What is the heat of the heptane conversion reaction $\Delta\hat{H}_r(400^\circ C)$ at 1 atm?

(c) A continuous stirred-tank reactor is used to produce a compound R in the liquid-phase reaction A → R. Feed enters the reactor at a rate of 1.15 L/s; The rate of product output is 1.15 L/s; The concentration of the reactant in the feed is 10 mol A/L. The volume of the tank contents is 10 L. The vessel may be considered perfectly mixed so that the concentration of A in the product stream equals that in the tank. For this process the rate of consumption of A equals $0.0050C_A$ mol/s-L of reaction volume. The tank is initially filled with a solution that contains 2.00 mol A/L, and the inlet and outlet flows then begin. **(10)**

(i) Write a differential balance on species A in the tank and provide an initial condition.

(ii) Solve the balance equation for $C_A(t)$.

2. Potassium dichromate ($K_2Cr_2O_7$) is to be recovered from a 21 wt% aqueous solution in a continuous crystallization operation. The solution is joined by a recycle stream and fed to a vacuum evaporator where water is removed and the remaining solution is cooled to 30 °C, at which temperature the solubility of the salt is 0.20 kg $K_2Cr_2O_7$/kg H_2O. The solution and suspended potassium dichromate crystals flow to a centrifuge. The crystals and 5.0% of the solution constitute the solid effluent from the centrifuge, and the remaining solution is recycled to the evaporator. The solid effluent, which contains 90 wt% crystals and 10% entrained solution, is fed to a dryer, where it is contacted with hot air. **(35)**

Contd P/2

CHE 201

Contd ... Q. No. 2

The remaining water in the effluent is evaporated, leaving pure potassium dichromate crystals. The air leaves the dryer at 90 °C and 1 atm with a dew point of 39.2 °C. For a production rate of 1000 kg solid $K_2Cr_2O_7$/h, calculate the required feed rate (kg/h) of 21% solution, the rate of evaporation of water in the evaporator (kg/h), the flow rate (kg/h) of the recycle stream, and the feed rate of air (standard liters/h).

3. (a) What is heating value of a fuel? How will you calculate lower heating value of a fuel from the higher heating value and vice-versa? Explain with appropriate example. **(10)**

(b) A natural gas containing 82.0 mole% CH_4 and the balance C_2H_6 is burned with 20% excess air in a boiler furnace. The fuel gas enters the furnace at 298 K, and the air is preheated to 423 K. **(20+5=25)**

(i) Assuming complete combustion of the fuel, calculate the adiabatic flame temperature.

(ii) How would the flame temperature change if the percent excess air were increased? How would it change if the percentage of methane in the fuel increased? Briefly explain both of your answers.

4. (a) A liquid stream consisting of 12.5 mole% n-butane and the balance a heavy nonvolatile hydrocarbon is fed to the top of a stripping column (see Figure for Q.4(a)), where it is contacted with an upward-flowing stream of nitrogen. The residual liquid leaves the bottom of the column containing all of the heavy hydrocarbon, 5% of the butane entering the column, and a negligible amount of dissolved nitrogen. The highest possible butane mole fraction in the exiting gas would be that in equilibrium with the butane in the entering liquid (a condition that would require an infinitely tall column to achieve). Using Raoult's law to relate the mole fractions of butane in the entering liquid and exiting gas, calculate the molar feed stream ratio (mol gas fed/mol liquid fed) corresponding to this limiting condition. **(17)**

(b) Water is added at varying rates to a 300-liter holding tank. When a valve in a discharge line is opened, water flows out at a rate proportional to the height and hence to the volume V of water in the tank. The flow of water into the tank is slowly increased and the level rises in consequence, until at a steady input rate of 60 L/ min the level just reaches the top but does not spill over. The input rate is then abruptly decreased to 20.0 L/min. **(18)**

(i) Write the equation that relates the discharge rate, υ'_{out} (L/min), to the volume of water in the tank, V(L), and use it to calculate the steady-state volume when the input rate is 20 L/min

(ii) Write a differential balance on the water in the tank for the period from the moment the input rate is decreased (t = 0) to the attainment of steady state (t $\rightarrow \infty$). Provide an initial condition.

Contd P/3

CHE 201

Contd ... Q. No. 4(b)

(iii) Without integrating the equation, use it to confirm the steady-state value of V calculated in part (a) and then to predict the shape you would anticipate for a plot of V versus t. Explain your reasoning.

(iv) integrate the differential balance to derive an expression for $V(t)$.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A mixture of methyl isobutyl ketone ($C_6H_{12}O$) vapor and air leaves a solvent recovery unit and flows through a 3 ft-diameter duct at a velocity of 7 m/s. At a sampling point in the duct, the temperature is 107°C, the pressure is 935 mmHg, and the dew point of the gas sample is 84°C. The gas is fed to a condenser in which it is cooled at constant pressure, condensing 55% of the methyl isobutyl ketone in the feed. **(2+4+7+10=23)**

(i) Draw the diagram and label it completely.

(ii) Calculate the molar flow rate of feed stream.

(iii) Determine the molar flow rates and compositions of the streams at the condenser outlet.

(iv) Calculate the cooling rate (kW) at the condenser.

State all necessary assumptions. The properties of methyl isobutyl ketone are given in table for Q5a.

(b) A flow of 1800 kg/h of air initially at a temperature of 18°C, 50% relative humidity is to be used in an air dryer. It is heated to 45°C at constant pressure and passed over a set of trays in an adiabatic shelf dryer, and finally leaves the dryer fully saturated. **(7+5=12)**

(i) Calculate the quantity of water (kg/h) absorbed by the air.

(ii) Determine the quantity of water to be removed from the air, if the air leaves the dryer is to be dehumidified again adiabatically to a relative humidity of 60%.

6. Ethylene oxide (C_2H_4O) is produced by the oxidation of ethylene (C_2H_4) and oxygen (O_2) in the air (21 mole % O_2, 79% N_2) over a catalyst. The conversion per pass is 50%. The ethylene oxide formed is completely condensed out. The uncondensed gases containing 7.8 mole % C_2H_4, 3.9 mole % O_2 and 88.3 mole% N_2 leaving the condenser are then recycled and combined with a fresh ethylene-air mixture prior to feeding into the reactor. For safety considerations, a fraction of gases leaving the condenser has to be purged from the recycle stream to avoid build up of N_2 in the system. **(35)**

For a plant producing 440 kg/hr ethylene oxide, calculate-

(i) the molar flow rates of ethylene and air in the fresh feed to the process.

(ii) the molar flow rate of the purge stream.

(iii) the percentage overall conversion of C_2H_4 and O_2.

(iv) the ratio of the recycle stream to the feed stream to the reactor.

Contd P/4

CHE 201

Contd ... Q. No. 6

(v) For the same production capacity and percent conversion of the reactants, calculate the molar flow rates of ethylene and air to the reactor if all uncondensed gases leaving the condenser are completely purged (i.e. no recycle). Compare them with that calculated in part (i) and part (iii). Give your comments.

7. (a) 10 kmol/h of air-fuel mixture containing 10% (v/v) methane is to be compressed prior to burning in a gas turbine combustor. For safety reasons, the pressure and temperature of the compressed mixture should not be allowed to exceed 72 bar and 210 °C. Calculate the maximum volumetric flow rate (m^3/h) of the mixture leaving the compressor. (Assume the mixture as non-ideal gas and use generalized compressibility charts attached) **(14)**

(b) 100 kmol/min of gas stream containing 45 wt% ethane (C_2H_4) and 55 wt% methane. (CH_4) is fed to a distillation column, where it is separated into an overhead product containing 90 mol% methane and a bottom product containing 98 mol% ethane. However, a customer wants to purchase a product that contains 97 mol% methane. To produce this, the overhead product from the first column is fed to a second column. The overhead from the second column is the desired product and contains 98% of the methane in the feed to this column, while the bottom product from the second column is recycled to mix with the fresh gas stream before feeding back to the first column. **(3+6+6+6=21)**

(i) Draw and label the flowchart from the description of the process.

Calculate-

(ii) The flowrate of the final product in kg/min.

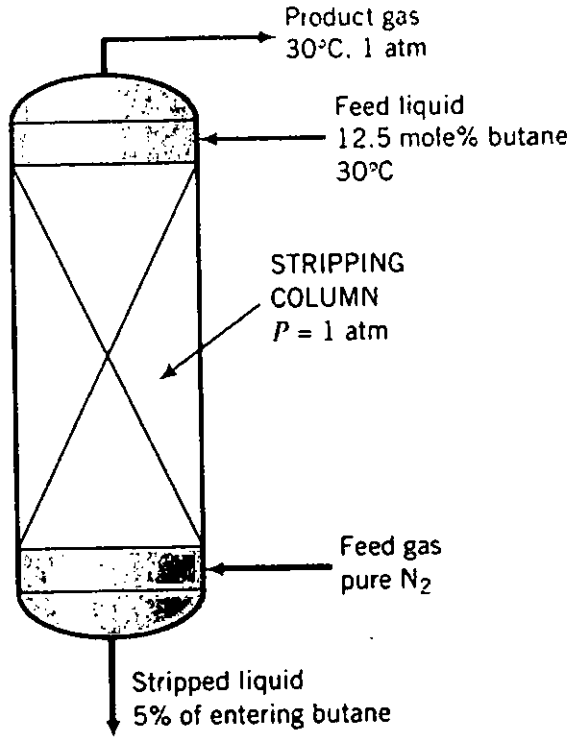
(iii) The flowrate (kmol/min) and the molar composition of the recycle stream.

(iv) The flowrate (kmol/min) and the molar composition of the fresh gas stream.

8. (a) Hydrochloric acid is an important industrial chemical. To make aqueous solutions of it in a commercial grade (known as muriatic acid), purified HCl(g) is absorbed in water in a tantalum absorber in a steady-state continuous process. The feed water can be assumed to be at 25°C, and the exit product HCl(aq) is 25% HCl (by weight) at 35°C. The cooling water does not mix with the HCl solution. **(17)**

How much heat must be removed from the absorber by the cooling water per 100 kg of product if hot HCl(g) at 120°C is fed into water in the absorber as shown in Figure for Q8a?

(b) The principal use of nitric acid (80%) is in the manufacture of fertilizers. Of this 96% is used to make ammonium nitrate and calcium ammonium nitrate. In the manufacture of nitric acid, ammonia (NH_3) and preheated air are mixed to form a feed gas to the reactor containing 10 mole% NH_3 at 600°C. 520 kg/h NH_3 gas enters the mixer at 25°C and the preheated air enters the mixer at 693°C. Calculate the amount of heat transfer (kW) in the mixer. **(18)**



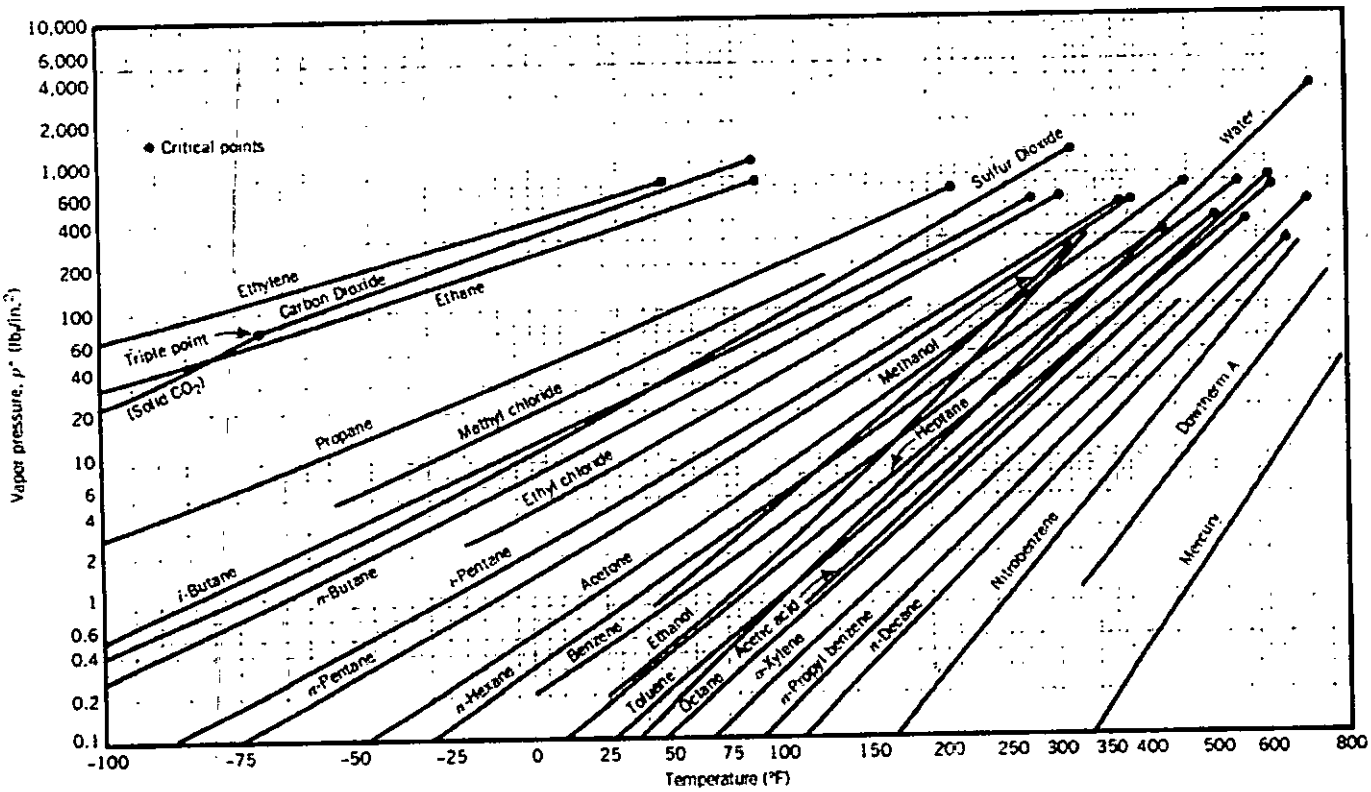


Figure 6.1-4 Cox chart vapor pressure plots. (From A. S. Foust et al., *Principles of Unit Operations*, Wiley, New York, 1960, p. 550.)

Figure for Q. 4(a)

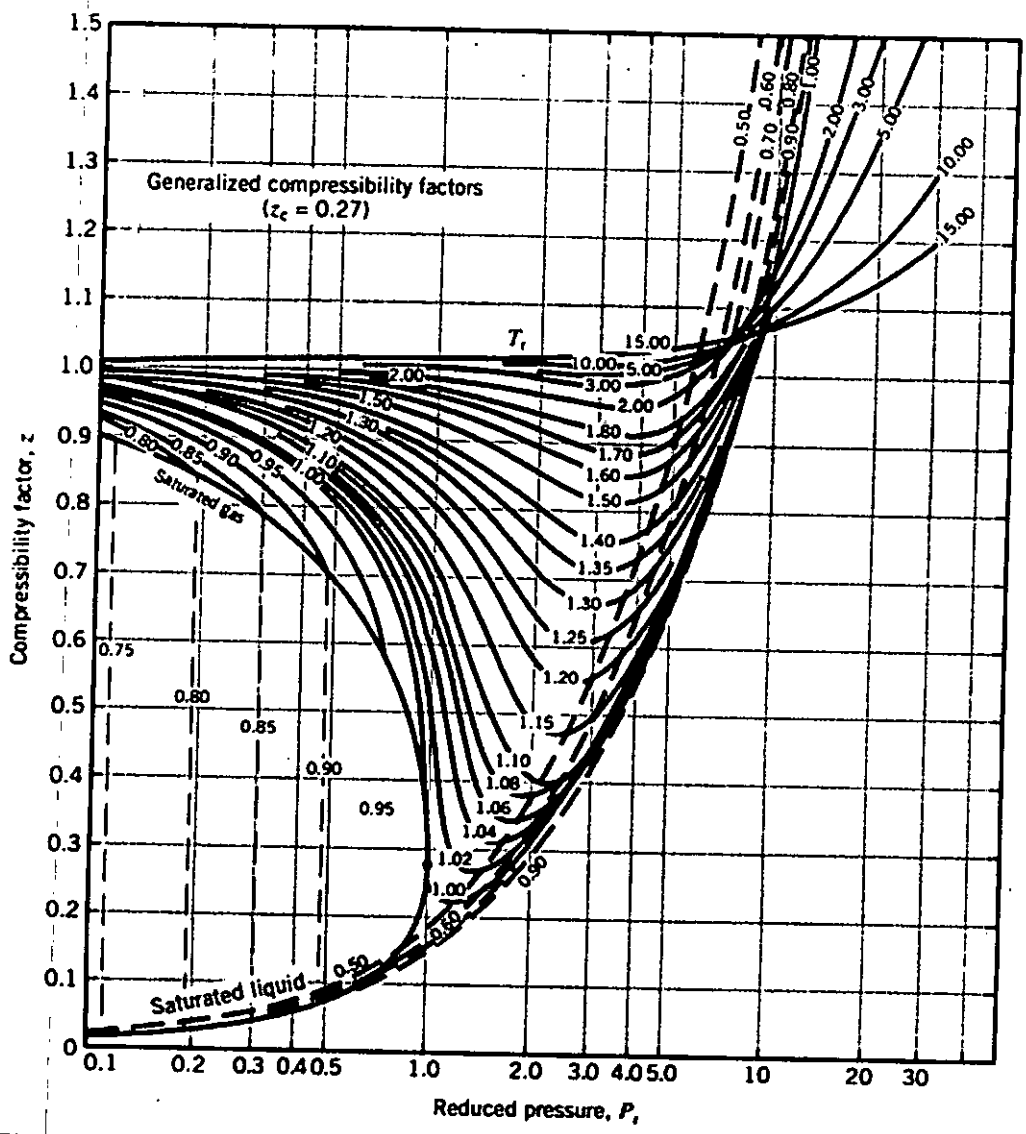


Figure 5.4-1 (Reprinted with permission from *Chemical Process Principles Charts*, 2nd Edition, by O. A. Hougen, K. M. Watson, and R. A. Ragatz, John Wiley & Sons, New York, 1960.)

Figure for Q7a

= 7 c

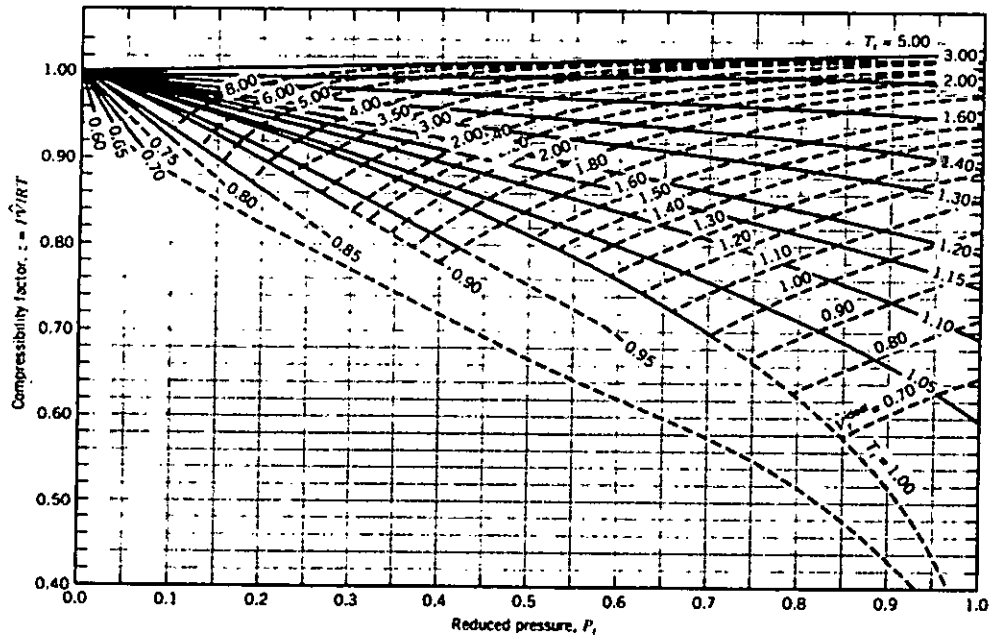


Figure 5.4-2 Generalized compressibility chart, low pressures. (From D. M. Himmelblau, *Basic Principles and Calculations in Chemical Engineering*, 3rd Edition, copyright © 1974, p. 175. Reprinted by permission of Prentice Hall, Inc., Englewood Cliffs, NJ.)

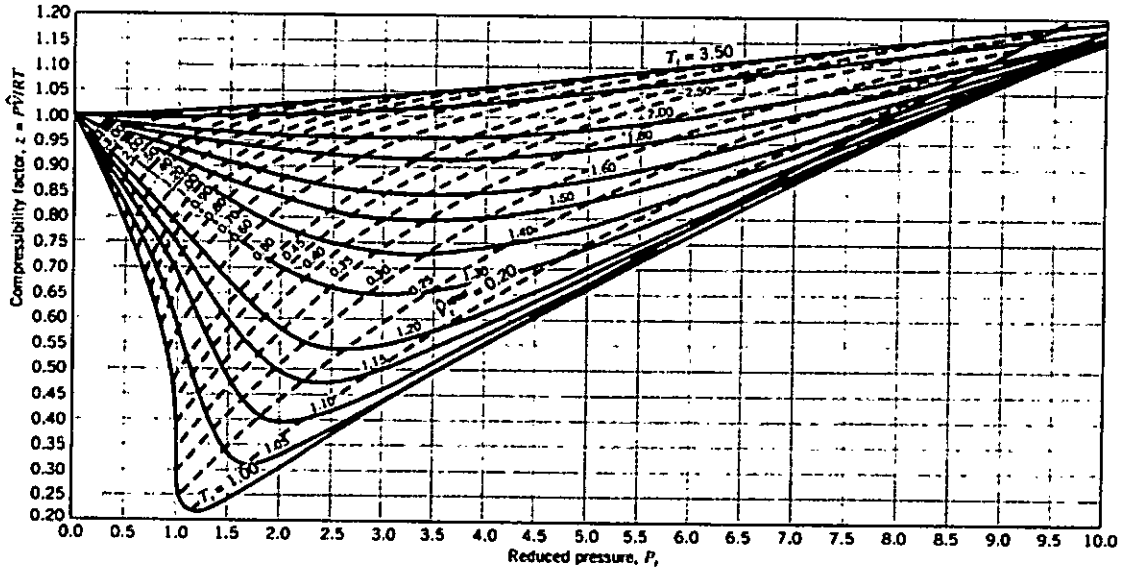


Figure 5.4-3 Generalized compressibility chart, medium pressures. (From D. M. Himmelblau, *Basic Principles and Calculations in Chemical Engineering*, 3rd Edition, copyright © 1974, p. 176. Reprinted by permission of Prentice Hall, Inc., Englewood Cliffs, NJ.)

Figure for Q7a

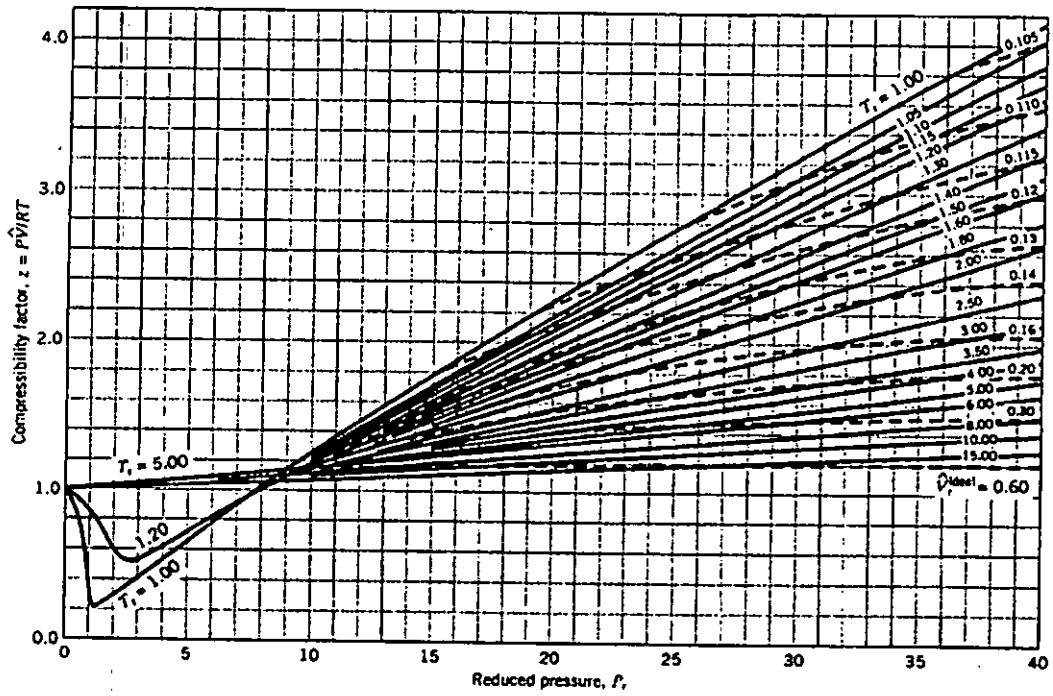


Figure 5.4-4 Generalized compressibility chart, high pressures. (From D. M. Himmelblau, *Basic Principles and Calculations in Chemical Engineering*, 3rd Edition, copyright © 1974, p. 177. Reprinted by permission of Prentice Hall, Inc., Englewood Cliffs, NJ.)

Figure for Q7a

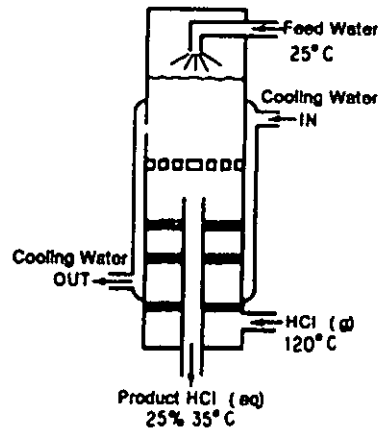


Figure for Q8a

L-2/T-1/CHE Date : ~~27/04/2024~~ 18/05/2024

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub : **CHEM 235** (Physical Chemistry II)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

The corresponding Course Outcomes (COs) of each part of Question 1 and 5 are in the right most column. The COs of the Courses are mentioned in the end of the question paper.

SECTION – A

There are FOUR questions in this section. Answer to Question No. 1 is compulsory. Answer any TWO Questions from Question Nos. 2-4.

1. (a) How hydrogen bonding change the position of absorption in Infrared spectroscopy? **(5)** **(CO1)**

   (b) What do you mean by coupling constant? How can you distinguish between two signals and one doublet from the value of coupling constant? **(10)** **(CO2)**

   (c) Show that half-life for a second order reaction is inversely proportional to the rate constant. **(10)** **(CO3)**

   (d) A 35-mL solution of $K_3Fe(C_2O_4)_3$ is irradiated with monochromatic light at 468 nm for 30 min. The solution is then titrated with 1, 10-phenanthroline to form the red complex of 1, 10-phenanthroline-Fe^{2+}. The absorbance of this complex ion measured in a 1-cm cell at 510 nm is 0.65 ($\varepsilon_{510} = 1.11 \times 10^4$ L mol^{-1} cm^{-1}). Assume that the quantum yield for the decomposition at this wavelength is 0.93, and calculate the number of einsteins absorbed per second and the total energy absorbed. **(10)** **(CO4)**

2. (a) Draw a well labeled phase diagram of sulphur system and discuss its salient features. **(15)**

   (b) Define Congruent Melting Point. Why it is nonvariant for two component system? **(10)**

   (c) Calculate the phase, component and degrees of freedom for the following systems. **(10)**

   (i) $CaCO_3$ (s) $\leftrightarrow$ CaO (s) + CO_2 (g) (ii) S (monoclinic) $\leftrightarrow$ S (rhombic)

3. (a) Explain the effect of polar solvents on (i) $n \rightarrow \pi^*$ and $\pi \rightarrow \pi^*$ transitions. **(15)**

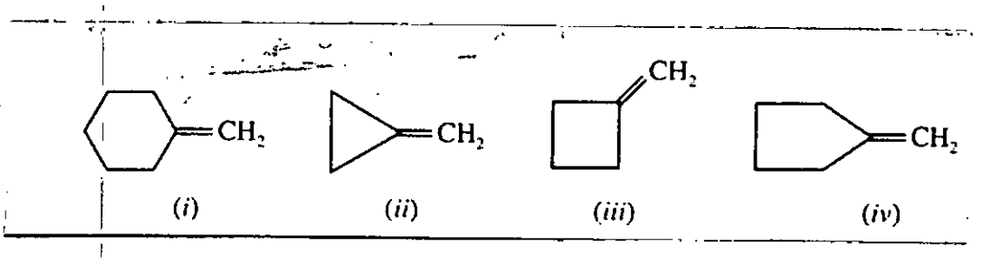
   (b) What are the characteristic differences between Thermochemical and Photochemical reactions? **(10)**

   (c) What do you mean by photochemical smog? Discuss the primary pollutants and secondary pollutants for photochemical smog formation. **(10)**

Contd P/2

CHEM 235

4. (a) Arrange the following compounds in order of their increasing wave number of absorption due to C=C stretching. Explain the reason. **(15)**



(b) Why TMS is used as a reference standard in NMR spectroscopy? **(10)**

(c) Calculate the wave number of stretching vibration of a carbon-carbon double bond (force constant, $k = 10 \times 10^5$ dynes cm^{-1}) **(10)**

SECTION – B

There are FOUR questions in this section. Question No. 5 is compulsory.
Answer any TWO Questions from the rest.

Symbols used here bear usual meanings. Assume reasonable values for any missing data.

5. (a) Explain the term 'ionic strength'. Show that the local charge density due to the presence of excess ions around a central ion is proportional to the ionic strength of the solution. **(11)** **(CO1)**

(b) Use the concept of Q5(a) to derive an expression for the potential produced by a reference charge around a central ion in an electrolyte solution and hence find the form of the Debye-Huckel limiting law applicable to dilute electrolyte solution. **(16)** **(CO2)**

(c) Evaluate γ and $\alpha_{\pm}$ for 0.0120 m solution of Na_3PO_4 at 25°C. How confident are you that the calculated result will agree with the experimental value? **(8)** **(CO3)**

6. (a) With suitable examples, point out the differences between reversible and irreversible cells. Explain how the emf of a reversible cell is affected by the ionic concentration, and hence justify that the formula of mercurous chloride is Hg_2Cl_2 not HgCl. **(11)**

(b) Derive a general expression for the potential at the equivalence point for the redox titration of Fe^{2+} ions with standard $K_2Cr_2O_7$ solution. Using a model calculation show how the sharp change in potential is marked by a very small change in the volume of the titrant (0.0167 M $K_2Cr_2O_7$) near the equivalence point. **(16)**

(c) Evaluate the potential of the indicator electrode at 25°C relative to the saturated calomel electrode (SCE) at the equivalence point for the titration of 10 mL of Fe^{2+} solution (≈0.10 M) with 0.0167 M $K_2Cr_2O_7$ at pH 1.00 (for SCE, $E^0 = 0.2444$V). **(8)**

Contd P/3

CHEM 235

7. (a) Rationalize the followings: **(11)**

(i) Specific conductance increases but molar conductance decreases with increasing the concentration of electrolyte solution.

(ii) The ionic mobilities of the alkali metal ions follows the order $K^+ > Na^+ > Li^+$.

(b) Consider the dissociation of the weak acid HA in aqueous solution. If a is the activity and C is molarity, show how you can evaluate the thermodynamic dissociation constant and the degree of dissociation of the acid. **(16)**

(c) Compute the thermodynamic dissociation constant for 0.01M acetic acid if the degree of dissociation is 0.0417 at 25°C. **(8)**

8. (a) Establish the relationship between the electrical energy and free energy. Explain how the Gibbs-Helmholtz equation can be used to calculate the enthalpy and entropy changes of a reaction occurring in a reversible cell. **(11)**

(b) In what category does the following cell belong to? **(16)**

$$H_2(g, 1\ atm) \mid HCl(a_1) \mid HCl(a_2) \mid H_2(g, 1\ atm)$$

With necessary half-cell reactions, indicate the direction for the flow of electrons and ions, and hence justify the transport number of the ion that should appear in the emf equation of the cell.

(c) If for the cell shown in 8(b), m_1 and m_2 are 0.10 and 0.01, while the $\gamma_\pm$ values are 0.796 and 0.904, respectively, find the emf of the cell 25°C (given t_-=0.1717). **(8)**

Course outcomes of Chem 235

	CO Statement	Corresponding PO(s)*	Domains and Taxonomy level(s)**
CO1	Identify fundamental concepts of reaction rates, adsorption and catalytic, conditions of equilibria between phases, electrolytes in solution and electrochemical reactions	1	C1
CO2	Illustrate the basic principles of rate process, spectroscopic techniques, electrolytic solution, quantification of electrolytes and non-electrolytes in solution	1	C2
CO3	Solve problems associated with chemical and electrochemical reactions, electrolytic conduction and electrochemical changes occurs during cell reactions	2	C3
CO4	Analyze the behavior of materials and ensure quality and purity of materials by spectrophotometric and electroanalytical techniques	3	C4

L-2/T-1/CHE **Date: 02/05/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **HUM 103** (Economics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

Symbols indicate their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) From the following demand function, make a hypothetical demand schedule and plot the curve. **(10)**

$$Q = 100 - 20P + P^2.$$

(b) What are the main causes of shifting of the demand curve? Explain them. **(10)**

(c) Why do demand curves generally slope downward? **(10)**

(d) What are the differences between change in demand and change in quantity demanded? **(5)**

2. (a) How would you measure price elasticity of demand at any point of a straight-line demand curve? Explain graphically. **(15)**

(b) Define cross elasticity of demand and income elasticity of demand. **(10)**

(c) From the following table calculate elasticity of demand if you move from point A to C and explain what you understand from the result. **(10)**

POINT	Y	Q
A	5000	150
B	6000	160
C	7000	170

3. (a) Explain the properties of an indifference curve. **(10)**

(b) Explain consumer's equilibrium with the help of budget line and indifference curve. **(10)**

(c) What are the assumptions of the indifference curve analysis? **(5)**

(d) From the following budget line and the utility function, calculate the amount of two commodities that maximizes satisfaction. What is the maximum amount of satisfaction? **(10)**

$$4000 = 30X + 50Y$$

$$U = 200\ X^{0.6}Y^{0.7}$$

4. (a) In a two country two commodity model, show that both the countries will be benefited after trade if each country has an absolute advantage in the production of one commodity. **(20)**

(b) What will happen if one country can produce both the commodities more efficiently than the other? Explain mathematically and graphically. **(15)**

Contd P/2

HUM 103

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Complete the following table and sketch the graph explaining the relations among the various short run cost curves. **(20)**

Quantity of Output	TFC	TVC	TC	AC	AVC	AFC	MC
1	100	60					
2	100		140				
3		75					
4		85					
5		105					
6	100		210				

(b) The Market price of a product in a perfectly competitive market is Tk. 100. A perfectly competitive firm has a marginal cost given by MC = 10Q, where Q stands for quantity produced. Find the profit maximizing output and revenue. **(15)**

6. A firm producing two goods "A" and "B" has the profit function

$$Z = 1280A - 40A^2 + 80AB - 80B^2 + 640B - 260$$

(a) What are the profit-maximizing level of output for each of the two goods? **(20)**

(b) Test whether profits are maximized. What is the maximized amount of profit? **(15)**

7. (a) Using the following table calculate Consumer Price Index (CPI) in 2020, 2021, 2022 and 2023 (Currency in Taka). **(20)**

Product	**Base Year (2017) Quantity**	**Base Year (2017) Per Unit Price**	**2020 Expenditures (on base-year quantities)**	**2021 Expenditures (on base-year quantities)**	**2022 Expenditures (on base-year quantities)**	**2023 Expenditures (on base-year quantities)**
Vaccination	30	400	14000	15000	18000	20000
Masks	50	40	2200	2300	2700	2800
Hand sanitizer	40	30	1500	1600	1900	2000
Pulse Oximeter	20	700	2000	2100	2150	2700
Books	70	90	6700	7000	8000	10000
Lemons	300	5	1800	1900	2000	2200

(b) Using the CPI you calculated in question 7(a), measure inflation rate from 2020 to 2021, from 2021 to 2022, and from 2022 to 2023. **(15)**

Contd P/3

HUM 103

8. (a) Assume that Gross Domestic Product (GDP)/Total output (Y) is 6,000. Consumption (C) is given by the equation $C = 620 + 0.5(Y - T)$ where T is the tax. Investment (I) is given by the equation $I = 3{,}000 - 100r$, where r is the real rate of interest, in percent. Taxes (T) are 500, and government spending (G) is also 500. What are the equilibrium values of C, I and r? **(20)**

(b) Either coal (C) or gas (G) can be used in the production of fertilizer. The cost of coal is 80, the cost of gas is 400. Draw an isocost curve showing the different combinations of gas and coal that can be purchased (i) with an initial expenditure (E) of 8000, (ii) if expenditures increase by 100 percent, (iii) if the price of gas is reduced by 40 percent, (iv) if the price of coal rises by 50 percent. Always start from the original equation. **(15)**

L-2/T-1/CHE **Date : 22/04/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub : **MATH 225** (Differential Equations and Laplace Transforms)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Formulate the differential equation corresponding to the family of curves $y = c(x - c)^2$, where c is an arbitrary constant. **(11)**

(b) A chemical reaction converts a certain chemical into another chemical, and the rate at which the first chemical is proportional to the amount of this chemical present at any time. At the end of one hour, 50 gm of the first chemical remain, while at the end of three hours, only 25 gm remain. **(12)**

(i) How many grams of the first chemical were present initially?

(ii) How many grams of the first chemical will remain at the end of seven hours?

(iii) In how many hours will only 3 gm of the first chemical remain?

(c) Solve the following differential equation: **(12)**

$$\left(y + \sqrt{x^2 + y^2}\right)dx - xdy = 0, \quad y(1) = 0 \ .$$

2. (a) A circuit in series an electromotive force given by $E = 100 \sin 60t$ V, a resistor of 2 Ω, an inductor 0.1 H, and a capacitor of $\frac{1}{260}$ farads. If the initial current and the initial charge on the capacitor are both zero, form the differential equation and hence find the charge on the capacitor at any time $t > 0$. **(12)**

(b) Explain integrating factor in ordinary differential equations. Check whether the differential equation $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ is exact or not. If not, then reduce it into exact form and hence solve the equation. **(12)**

(c) Solve the following differential equation: **(11)**

$$x\frac{dy}{dx} + y \log y = xye^x \ .$$

Contd P/2

MATH 225/CHE

3. (a) Consider the nonhomogeneous differential equation **(12)**

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$$

(i) Show that e^x and e^{2x} are linearly independent solutions of the corresponding homogeneous equation.

(ii) Obtain the complementary function of the given equation.

(iii) Show that $2x^x + 6x + 7$ is a particular integral of the given equation.

(iv) Write down the general solution of the given equation.

(b) Write down the general form of Cauchy-Euler ordinary differential equation. Reduce the differential equation $\left(x^3D^3 + 3x^2D^2 + xD + 1\right)y = \log x + x, \quad \left(D = \frac{d}{dx}\right)$ into constant coefficient and hence solve. **(12)**

(c) Solve the differential equation **(11)**

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$$

4. (a) Form a partial differential equation by eliminating the arbitrary functions f and g from **(10)**

$z = f(x\cos\alpha + y\sin\alpha - at) + g(x\cos\alpha + y\sin\alpha + at)$.

(b) Find the integral surface of the partial differential equation **(12)**

$(x-y)p + (y-z-x)q = z$ through the circle $z = 1, x^2 + y^2 = 1$.

(c) Using Charpit's method, find the complete integral of the partial differential equation **(13)**

$$\left(x^2 - y^2\right)pq - xy\left(p^2 - q^2\right) = 1.$$

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. Solve the following:

(a) $\left(D_x^2 - D_xD_y - 2D_y^2 + 2D_x + 2D_y\right)z = e^{3x+4y} - 3xy$ **(13)**

(b) $\left(12D_x^2 - 5D_xD_y - 3D_y^2\right)z = 4\sin(x - 3y)$ **(12)**

(c) $\left(D_x^3 - 2D_x^2D_y\right)z = 2x^2y + 2e^x$ **(10)**

6. (a) Solve: $\left(x^2D_x^2 - xyD_xD_y - 2y^2D_y^2 + xD_x - 2yD_y\right)z = \log\left(\frac{y}{x}\right)$. **(15)**

(b) Define Error function $erf(t)$. Hence find $L\left\{erf\left(\sqrt{t}\right)\right\}$. **(10)**

(c) Describe Cosine Integral function. Hence show that $L\left\{\int_t^\infty \frac{\cos\tau}{\tau}d\tau\right\} = \frac{\ln\left(s^2+1\right)}{2s}$. **(10)**

Contd P/3

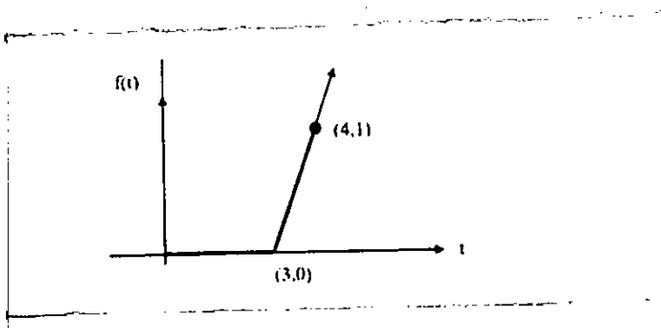
MATH 225/CHE

7. (a) Find $L\{\sin^3 t \cos t\}$ and hence evaluate $\int_0^\infty e^t \, t \sin^3 t \cos t \, dt$. **(15)**

   (b) State Convolution theorem. Apply this theorem to find $L^{-1}\left\{\frac{1}{(s+3)(s^2+4)}\right\}$. **(10)**

   (c) Solve $y'' - 3y' + 2y = 4e^{2t}$ subjected to the initial condition: $y(0) = -3$, $y'(0) = 5$. **(10)**

8. (a) Translate the following graph with the help of a piece-wise function f(t). Hence find Laplace transform of f(t). **(10)**



   (b) Define Periodic function. Find the Laplace transform of the following periodic function: **(10)**

$$f(t) = \begin{cases} 3t\,; & 0 < t < 2 \\ 6\,; & 2 < t < 4 \end{cases}$$

where $f(t + 4) = f(t)$, t > o.

   (c) If $L\{f(t)\} = F(s)$, then show that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(\tau)\, d\tau$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. **(15)**
