L-2/T-1/CSE **Date : 22/04/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub : **CSE 205** (Digital Logic Design)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer **Q. No. 1** and any other **TWO**.

1. (a) Describe the steps to find M-N using r-1's complement, where M and N are two n-digit unsigned numbers and r is the base. **(13)**

   (b) Explain the difference between binary and gray encoding and the benefits of each. **(9)**

   (c) Implement the following Boolean function with a 4 × 1 multiplexer and external gates. **(13)**

   $$F(A, B, C, D) = \sum (1, 2, 4, 7, 8, 9, 10, 11, 13, 15)$$

2. (a) Design a combinational circuit that converts a three-bit gray code to a three-bit binary number. Use only a minimum number of gates in your design. **(20)**

   (b) Tabulate the PLA programming table for the two Boolean functions listed below. **(15)**

   $$F_1(A, B, C) = \sum(0, 1, 2, 4); F_2(A, B, C) = \sum(0, 5, 6, 7)$$

   Minimize the numbers of product terms.

3. (a) What do you mean by Universal gate? Show that both NAND gate and NOR gate are universal gates. **(14)**

   (b) Simplify the following Boolean function into (a) sum-of-products form and (b) product-of-sums form: **(14)**

   $$F(A, B, C, D) = \sum (0, 1, 2, 5, 8, 9, 10)$$

   (c) Show that the dual of the exclusive-OR is equal to its complement. **(7)**

4. (a) Convert the Boolean function $F = xy + x'z$ as a product of maxterms using the distributive law. **(10)**

   (b) Design a full adder circuit using a 3 × 8 decoder and two OR gates. **(10)**

   (c) Using tabular method, find the prime implicants and essential prime implicants of the following Boolean function: **(15)**

   $$F(A, B, C, D) = \sum (0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

Contd P/2

CSE 205

SECTION – B

There are **FOUR** questions in this section. Answer **Q. No. 5** and any other **TWO** from the Questions 6, 7, and 8.

5. (a) Develop a sequential circuit that detects a sequence of three or more consecutive 1's in a string of bits coming through an input line. Use D flip-flops. Show the state diagram, the state table and simplified logic equations. You do not need to show the logic diagram. **(12)**

   (b) Construct a JK-flip flop from a T-flip-flop and necessary gates. Show all the steps of this construction process. **(11)**

   (c) Design a sequential circuit to copy the content of shift register A to shift register B. Use a block diagram to represent the shift register in the diagram. **(12)**

6. (a) Draw the block diagram of a negative edge triggered Master-Slave D flip-flop using D latches. **(10)**

   (b) Write a short note about a ring counter. **(5)**

   (c) Design an automatic vending machine. The candy bars inside the machine cost 15 Tk, and the machine accepts 5 Tk and 10 Tk only. An electromechanical system accepts the money sequentially. The circuit produces a pulse output that delivers the candy whenever the amount received by the machine is 15 Tk. or more. The excess amount is kept deposited for the next candy. Design a pulse mode circuit for the vending machine using S-R latches. **(20)**

7. (a) Show the logic diagram of a four-bit binary ripple (asynchronous) countdown counter using T flip-flops that trigger on the negative edge of the clock. **(12)**

   (b) Using J-K flip-flops, design a synchronous counter that counts in the following sequence: **(13)**

   1, 2, 5, 7, 1, 2, 5, 7, 1, 2, ...

   Analyze the final circuit to ensure that of the circuit enters in any of the unused states, after finite number of clock cycles it comes to a used state.

   (c) Differentiate between Mealy and Moore machines with block diagrams. **(10)**

Contd P/3

CSE 205

8. (a) What is a race in fundamental mode circuits? Explain critical and non-critical races with examples. **(12)**

(b) An asynchronous sequential circuit is described by the excitation function **(12)**

$$Y = x_1x'_2 + (x_1 + x'_2)y$$

and the output function

$$z = y$$

(i) Draw the logic diagram of the circuit.

(ii) Derive the transition table and output map.

(iii) Obtain a two-state flow table.

(c) Reduce the number of states of the following state table using implication table. Show the reduced state table. **(11)**

State	Next State		Output	
	Input X = 0	Input X = 1	Input X = 0	Input X = 1
A	A	B	0	0
B	C	D	0	1
C	A	B	0	0
D	E	D	0	1
E	A	D	0	1

18/05/2024

L-2/T-1/CSE Date: ~~17/04/2024~~

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examination 2022-2023

Sub: **CSE 207 (Data Structures and Algorithms II)**

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. You **must answer Question 1** and any 2 questions form Questions 2,3, and 4.

1. (a) Analyze the complexity of the Johnson's algorithm for all pair shortest paths problem. **(12)**

   (b) Consider a peculiar city with N towns and M bidirectional roads (connecting towns) classified into three types. This city is peculiar due to unconventional regulations regarding road usage: men are permitted to travel only on roads of types 1 and 3, while women are permitted to travel only on roads of types 2 and 3 exclusively. The Mayor of the city intends to renovate the city by reconstructing some roads, with the condition that the normal routine of the citizens should not be hampered.

   Now, you are hired as a computer scientist to answer an important question: What is the maximum number of roads that can be renovated (remain closed for traffic during renovation) while ensuring that the city remains connected for both men and women? The city is said to be connected if it is still possible to travel form any town X to any town Y using the existing roads. **(23)**

   For the above problem description:

   i) Formulate the problem as a graph problem,

   ii) Develop the solution using a well-known graph algorithm.

   iii) Write the pseudo-code of the solution.

2. (a) Develop an algorithm for finding the shortest path in a graph where all edge weights are equal. **(8)**

   (b) Devise an efficient algorithm for the same problem (2(a)), where edge weights of the graph are non-equal but positive. **(12)**

   (c) Develop an efficient algorithm for the above problem, when the edge weights can be negative and the graph can have negative edge-weight cycles. **(15)**

3. (a) How the concept of potential function is used to design a splay-tree - Explain. **(8)**

   (b) Formulate the problem of maximum bi-partite matching. Give two examples of real-life problems that can be solved using maximum bi-partite matching. **(6+6)**

Continued --- P/2

CSE 207

(Contd ... Q. No. 3)

(c) Develop a solution for the maximum bi-partite matching problem. What is the Edmond-Karp optimization for the Ford-Fulkerson algorithm? **(10+5)**

4. (a) Prove the max-flow min-cut theorem. **(8)**

(b) Write down the properties of an AVL-tree, and prove that it is a balanced binary search tree. **(12)**

(c) Write the pseudo-code of an insertion operation in a Red-Black tree and explain (graphically) different forms of transformation necessary to maintain the RB property while inserting an item. Briefly compare RB-tree and AVL-tree. **(15)**

SECTION – B

There are **FOUR** questions in this section. **Answering question 5 is compulsory**. Answer any **TWO** questions form Questions 6, 7, and 8.

5. (a) **Differentiate** among NP, NP-hard, and NP-complete classes of problems. **(6)**

(b) Imagine you are managing a team of employees, each with their own set of tasks to complete. However, some tasks cannot be worked on simultaneously due to dependencies or resource constraints. Your goal is to maximize productivity by scheduling as many tasks as possible while ensuring that no two conflicting tasks are assigned to the same team member simultaneously. You described the problem to your friend. He commented, "The problem can be mapped to a popular NP-complete problem and can be proved to be NP-hard by reduction technique." **(17)**

Justify your friend's comment by mapping the problem to an NP-complete problem and reducing it to another known NP-complete problem.

(c) Recall the deletion operation of the Fibonacci Heap data structure. Professor Rahsut has proposed variant of the deletion operation (Figure for Q 5(c)) **(12)**

Figure for Q 5(c): RAHSUT-DELETE for question 5(c)

```
1   RAHSUT-DELETE(H, x)
2       if x == H.min
3           FIB-HEAP-EXTRACT-MIN(H)
4       else
5           y = x.p
6           if y != NIL
7               CUT(H, x, y) // cuts the link between x and its ↩
                   parent y, making x a root.
8               CASCADING-CUT(H, y) // recursively marks and ↩
                   cuts the parents if has to
9           add x's child to the root list of H
10          remove x from the root list of H
```

CSE 207

Contd ... Q. No. 5 (c)

i. Construct a good upper bound on the actual time of RAHSUT-DELETE when x is not *H.min.* Your bound should be in terms of *x.degree* and the number *c* of calls to the CASCADING-CUT procedure.

ii. Suppose that we call RAHSUT-DELETE(H. x) and let *H'* be the Fibonacci heap that results. Assuming that node *x* is not a root, **find** the potential of *H'* in terms of *x.degree*, the number *c* of calls to the CASCADING-CUT procedure, *t(H),* and *m(H).*

6. (a) Suppose you implemented a hashtable ADT that supports constant time insertion, deletion, and search. You used the hash function h1. On the contrary, your friend implemented another hashtable ADT using the hash function h2. The h1 hashes the keys 100, 130, 110, 150, 160, 200, 700, 800, 300 into 0, 3, 2, 0, 3, 1, 2, 5, and 8 respectively. The h2 hashes the same keys into 1, 4, 7, 10, 13, 4, 7, 10, 13. You made the hashtable size 16 but your friend made it to 17. For collision resolution, you used the quadratic probing function $f(i)=2*i^2+3$. But your friend used the double-bashing technique and the second hash function was $h_2(x)=19-(x\%17)$. You inserted all the keys in the same order mentioned before to the hashtable. Then you deleted the key 100 and then searched for the key 150. But your hashtable implementation was showing "key 150 not found". Now, answer the following questions:

i. Compare your implementation with your friend's by analyzing the total unsuccessful probes during insertion in both cases. **(15)**

ii. Identify the possible core reason for not finding the key 150 during the search and provide a solution. **(5)**

(b) Give a 2-approximation algorithm for the vertex cover problem and power that the algorithm computes the vertex cover at most twice the size of the actual vertex cover size. **(15)**

7. (a) There are 4 places in your village. You want to start from a place, then visit all other places and return to the first place. Also, you don't want to visit the same place twice other than the starting place. You want to finish the visit with minimum cost. The cost of the roads between these places are given below: **(20)**

From/To	1	2	3	4
1	0	10	15	20
2	5	0	12	18
3	8	9	0	17
4	11	14	16	0

Now give an exponential time algorithm to solve this problem. The running time of the algorithm has to be $O(n^2 2^n)$, find the optimal tour using your algorithm.

CSE 207

Contd ... Q. No. 7

(b) Suppose you want to fill up your language with the required items for your upcoming tour. But the luggage has a capacity of 60 kg. The items have different values and weights which are given below: **(15)**

Item	Value	Weight
1	120	15
2	110	10
3	95	25
4	135	35
5	80	20
6	150	30

You want to fill up your luggage so that the total values of the items are maximized. Now, solve his problem using branch and bound technique.

8. (a) Answer the following questions for two **max** priority queues pq1 and pq2. The priority queue is implemented using the binomial heap data structure. Both the priority queues are initially empty. **(23)**

i. Insert - 5, 20, 4, 19, 1, -3, 6, 12, 3, 2, 18 in pq1 and 25, 21, 10, 13, 17, 9, 8, 11, 7 in pq2. Draw the two binomial heaps after the insertion.

ii. Meld pq1 and pq2 and then decrease the key of node 20 to -20 from the melded priority queue. Show the necessary steps.

iii. What will be the total actual cost in credits if we call the **Extract-Max** operation two times consecutively on the resultant priority queue of the previous question? Assume, that unlinking a node form its children takes one credit, two node addition operation takes one credit, an insertion of a node takes one credit, and a two-key comparison takes one credit. All other operations take zero credit.

(b) Prove that in a binomial heap, the amortized cost of **INSERT** *is O(1)* and the worst-case cost of **EXTRACT-MIN** and **DECREASE-KEY** *is O(1ogn).* **(12)**

- - - - - - - - - - - - -

L-2/T-1/CSE **Date: 27/03/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub: **CSE 215** (Databases)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. **Question No. 1 (one) is Compulsory**. Answer any **TWO** of the remaining **THREE** questions.

1. (a) Construct a one and a half pass algorithm for computing R ⟕ S natural join operation where ⟕ denotes the left outer join operation. Assume the R is the smaller relation. Compute the memory and disk I/O requirements. **(10)**

   (b) Assume R and S are two database relations, the size of R and S in disk are $B(R)$ = 8000, $B(S)$ = 4000, and the number of available memory buffers is M = 101. Compute the I/O cost of the natural join operation for the following algorithms: (i) block based nested loop join, (ii) sort-based nested loop join, and (iii) hash-based nested-loop join. **(8)**

   (c) Assume you are given the following schedules involving four transactions T_1, T_2, T_3, and T_4: **(12)**

   S_1: $r_1(A)$; $w_2(A)$; $r_2(B)$; $r_4(C)$; $r_3(B)$; $w_2(B)$; $w_3(B)$; $r_4(A)$; $w_4(C)$; $r_1(C)$;

   S_2: $r_1(A)$; $w_2(A)$; $r_2(B)$; $r_4(C)$; $w_2(B)$; $r_3(B)$; $w_3(B)$; $r_4(A)$; $r_1(C)$; $w_4(C)$;

   Construct precedence graphs for the given schedules S_1 and S_2. Are the schedules conflict-serializable? Justify your answer.

   (d) Show that a schedule satisfying strict 2PL (two-phase locking) is ACR (avoids cascading rollback). **(5)**

2. (a) Consider the following two transactions: **(9)**

   T_1: $r_1(A)$; $w_1(A)$;

   T_2: $w_2(A)$;

   Answer the following questions:

   (i) Construct a concurrent schedule of T_1 and T_2 that utilizes "Thomas write rule". Show a serial schedule which is equivalent to your given concurrent schedule.

   (ii) What is the problem of allowing Thomas write rule?

   (b) Give a two-pass hash-based algorithm for computing the set difference $R - S$. Analyze the memory and disk I/O requirements of your algorithm. **(10)**

   (c) The following is a sequence of undo-log records written by two transactions T and U: **(9)**

   <START T>; <T, A, 10>; <START U>; <U, B, 20>; <T, C, 30>; <U, D, 40>; <COMMIT U>; <T, E, 50>; >COMMIT T>.

Contd P/2

CSE 215

Contd ... Q. No. 2(c)

Assume that the recovery manager takes an action <X, V> meaning database element X is restored to the value V in the disk. Determine the actions that will be taken by the recovery manager, if there is a crash and the last log record to appear on disk is one of the following:

(i) <COMMIT U> (ii) <T, E, 50> (iii) <COMMIT T>

(d) Suppose you are given the following sequence of log records representing the actions of a transaction T: <START T>; <T, A, 10>; <T, B, 20>; <COMMIT T>. **(7)**

Answer the following questions:

(i) For undo logging, what is the last log record that must be sent to disk before updated A is sent to disk?

(ii) For undo logging, what is the last log record that must be sent to disk before updated B is sent to disk?

(iii) For redo logging, what is the last log record that must be sent to disk before updated A and B are sent to disk?

3. (a) Consider the following sequence of undo log records: **(7)**

<START S>; <S, A, 60>; <COMMIT S>; <START T>; <T, A, 10>; <START U>; <U, B, 20>; <T, C, 30>; <START V>; <U, D, 40>; <V, F, 70>; <COMMIT U>; <T, E, 50>; <COMMIT T>; <V, B, 80>; <COMMIT V>.

Suppose that we begin a nonquiescent checkpoint immediately after the <U, B, 20> log record has been written (in memory). Determine the last log record that must be sent to disk before each of the following events: (i) <START CKPT (...)> is written to disk and (ii) <END CKPT> is written to disk.

(b) For the given undo log records in Q. 3(a), suppose we begin a nonquiescent checkpoint after the log record <U, B, 20> and a system crash occurs after one of the following: (i) <V, B, 80> and (ii) <T, E, 50>. For each case, determine the oldest log record that the recovery manager needs to scan to find all possible incomplete transactions. **(8)**

(c) Consider the following two transactions that use shared ($sl_i(X)$) and exclusive ($xl_i(X)$) locks: **(10)**

$$T_1: sl_1(A);\ r_1(A);\ xl_1(A);\ w_1(A);\ u_1(A);\ c_1;$$

$$T_2: sl_2(A);\ r_2(A);\ xl_2(A);\ w_2(A);\ u_2(A);\ c_2;$$

Answer the following questions:

(i) Construct a concurrent schedule of T_1 and T_2 that results in a deadlock due to lock upgrading.

(ii) Modify the given transactions using update ($ul_i(X)$) lock to prevent deadlocks.

(d) Consider the following schedule of actions involving four transactions T_1, T_2, T_3 and T_4: **(10)**

$$r_1(A);\ r_2(B);\ w_1(C);\ r_3(D);\ w_3(B);\ w_2(C);\ w_4(A);\ w_1(D);\ w_1(B);$$

Contd P/3

CSE 215

Contd ... Q. No. 3(d)

Assume that the deadlock-timestamps of T_1, T_2, T_3 and T_4 are 1, 2, 3 and 4, respectively. For each action of the given schedule, determine what happens ("wound" or "wait") under the wound-wait deadlock avoidance system. If the action is a "wound", then don't restart the transaction.

4. (a) Consider the following schedule of four transactions T_1, T_2, T_3 and T_4 in a system with timestamp-based concurrency control: **(12)**

$$r_1(A);\ w_2(A);\ r_2(B);\ r_4(C);\ r_3(B);\ w_2(B);\ w_3(B);\ r_4(A);\ w_4(C);\ r_1(C);$$

Assume that the start timestamps of the four transactions are $TS(T_1) = 1$, $TS(T_2) = 2$, $TS(T_3) = 3$ and $TS(T_4) = 4$. The read and write timestamps are all initialized to 0 at the start of the schedule, i.e., $RT(X) = 0$ and $WT(X) = 0$ for $X \in \{A, B, C\}$.

Answer the following questions:

(i) For each action of the given schedule, show the timestamp which is updated and show the updated values.

(ii) Show the actions for which the scheduler rollbacks the relevant transaction.

(b) Consider the following two transactions T_1 and T_2: **(11)**

$$T_1: r_1(A);\ w_1(A); c_1:$$

$$T_2: r_2(A); c_2:$$

(i) Construct a concurrent schedule of T_1 and T_2 that is recoverable but not ACR (avoids cascading rollback).

(ii) Construct a concurrent schedule of T_1 and T_2 that is serializable but not recoverable.

(iii) Construct a schedule of T_1 and T_2 that is ACR (avoids cascading rollback).

(c) Consider the following schedule of actions involving four transactions T_1, T_2, T_3 and T_4: **(12)**

$$r_1(A);\ r_2(B);\ w_1(C);\ r_3(D);\ w_3(B);\ w_2(C);\ w_4(A);\ w_1(D);$$

For each of the sequences of actions given above, assume that shared locks are requested immediately before each read action, and exclusive locks are requested immediately before every write action. Answer the following questions:

(i) Show the wait-for graph after the execution of all actions.

(ii) Determine if there is a deadlock. In case of a deadlock, choose the highest numbered (considering $T_4 > T_3 > T_2 > T_1$) transaction to rollback. Show the resulting wait-for-graph after the rollback.

Contd P/4

CSE 215

SECTION – B

There are **FOUR** questions in this section. Answer to **Question no. 5 (five) is Compulsory**.

Answer any **TWO** questions from Questions 6-8.

5. (a) YouTube is a very popular platform for hosting videos. People create channels and upload videos on that. Sometimes the creators can batch up the videos into playlists. Videos can be free to view or behind paywall which requires subscription to YouTube Premium. A free video can be watched with or without signing into YouTube. However, only signed-in users' views are considered when counting the number of viewers for a video. A signed-in account may or may not have corresponding channels. Each video has a comment section where a signed-in user can comment; and comment to other people's comments. Also, videos contain some tags (given by the uploader or YouTube itself) that help to filter videos. Signed-in users can subscribe to other channels. Now, for each of the mentioned features below, design the corresponding portion of ERD for YouTube only. Add attributes as you see fit and mark primary keys. **(20)**

(i) **User management:** Users should be able to create accounts, create channels on top of the accounts (one user can create at most one channel), and subscribe to different channels.

(ii) **Video hosting:** Creators should be able to upload videos with different tags, and create playlists by batching up multiple videos. Users should be able to view the videos, find them in their history when needed and, also, check the number of viewers to the videos.

(iii) **Commenting:** Users should be able to comment on any video any number of times, They should also be able to comment on other peoples comments.

(iv) **Subscription system:** Creators should be able to mark some videos as premium (i.e., only subscribed users can view). Users should be able to buy subscriptions up to some end date.

(b) Implement relational schemas for each ERD you designed in Question 5(a). **(15)**

6. (a) A school has prepared a database to manage admission tests. The examinees are given roll numbers and assigned to specific rooms for the exam. The test contains 10 questions. Each question is marked by two examiners to ensure fairness and correctness. An absentee list is kept to ensure no script goes missing. The schema for the database is as follows: **(10×3=30)**

```
Room (Room No, Floor, Capacity)
Examinee (Roll No, Name, Address, Date of Birth, Room No)
Evaluation (Roll No, Question No, Examiner, Marks)
Absentee (Roll No)
```

Contd P/5

CSE 215

Contd ... Q. No. 6(a)

Now, write queries using **relational algebra** to compute the following.

(i) The students should be assigned to the rooms in a manner that all the "Roll No"s in the room are sequential (e.g., if a room has 10 examinees and the lowest Roll No in that room is 501, then "Roll No"s 502, 503, ...,510 should be assigned to that room). **Find the rooms** that violate this condition.

(ii) We need to check if all scripts have been correctly evaluated, i.e., all examinees' answers to all questions have been evaluated by exactly two teachers. **Find out the Roll No and Question No** of the students that have not got correctly evaluated. Please note that only the examinees that are present in the exam should be considered here.

(iii) The marks given by two examiners can only be at most 2 marks apart (i.e., for a student's answer to a specific question, the two marks by the two examiners should not differ by more than 2). **Find the Roll No and Question No** for which two examiners' marks differ by more than 2.

(b) Give an example of a Non-Binary Relationship. What issues arise for Non-Binary Relationships. **(5)**

7. (a) You have been tasked to prepare a database for a restaurant, The restaurant has provided the following report for you. **(15)**

Food Item	Containing Items (For Packages)		Ingredients (For Simple Items)			Marked Price	Sales		
	Simple Item	Qnty	Ingredient	Unit Cost	Qnty		Buyer	Date	Qnty
Chicken Burger			Chicken Patty Lettuce Pickle Mayonnaise Sause	60 30 5 100 70	1 0.02 4 0.02 0.01	200	C1 C3 C8 C1	01/02/24 02/02/24 02/02/24 03/02/24	2 4 1 3
Fried Rice			Raw Rice Egg Oil	50 7 500	0.1 1 0.005	80	C4 C9	02/02/24 04/02/24	1 2
Set Menu 1	Fried Rice Vegetable Chicken Steak	1 1 1				500	C2 C4 C1 C1	01/02/24 03/02/24 04/02/24 05/02/24	3 1 4 2

In the above table, **"Qnty"** means "quantity of the item".

Now, create a relational schema, find its functional dependencies and normalize the schema. Write down all normalized forms up to BCNF, (1NF, 2NF, 3NF, BCNF).

(b) (i) Using the schema created in question 7(a), write a SQL query to create a view that contains the total cost of production of each food item. Here, cost of production depends on the total cost of ingredients (here, cost of ingredients = unit cost of ingredient* quantity of ingredients used) for simple items and total cost of containing items (that are simple items) for packages. **(10×2=20)**

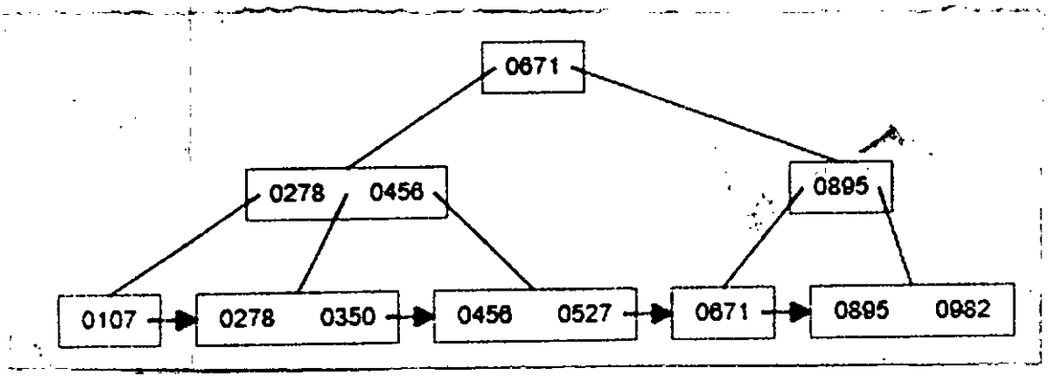
Contd P/6

CSE 215

Contd ... Q. No. 7(b)

(ii) Using the schema created in question 7(a) and view created in question 7(b). i, find monthly profit for each food item. Here, profit = (marketed price - cost of production)* quantity sold in the month.

8. (a) The following is a degree 3 B+ tree created after some insert/delete operations. **(4×5=20)**



Draw the state of this B+ tree after each of the following operations:

(i) Insert 315

(ii) Insert 409 and 513

(iii) Delete 456

(iv) Delete 513

(v) Delete 982 and 895

(b) For a degree 5 B+ tree, what will be the height of the B+ tree when the numbers form 1 to 100 are inserted sequentially. **(5)**

Hint: A tree with t leaves and internal nodes of degree d (some constant number of internal nodes can have degree > d) has height approximately $\log_d t$.

(c) The following table stores information about courses offered by BUET. **(10)**

```
create table course (
    course_code char(10) primary key,
    course_title varchar(255) not null,
    credit_hours numeric not null,
    old_course_code char(10) references course(course_code)
);
```

Here, varchar is variable-length and all other data-types are fixed length, size of `numeric` is 8 bytes, and size of `char(n)` is n bytes.

Draw the **structure of the records**, and the **structure of the page** when the records of this table are stored using **Slotted Page Structure**.

L-2/T-1/CSE **Date : 27/04/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2022-2023

Sub : **MATH 241** (Advanced Calculus)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Find equations for the tangent plane and normal line to the surface $4z = x^2 - y^2$ at the point (3, 1, 2). **(10)**

   (b) Find the directional derivative of $\phi(x, y, z) = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) in the direction of the vector $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. **(10)**

   (c) Prove that $\nabla \times (\nabla \times \mathbf{F}) = -\nabla^2\mathbf{F} + \nabla(\nabla.\mathbf{F})$. **(15)**

2. (a) Show that $\mathbf{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational. Hence find ϕ such that $\mathbf{F} = \nabla\phi$. **(13)**

   (b) Find the work done in moving a particle in the force field **(10)**

   $$\mathbf{F}(x, y, z) = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$$

   along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.

   (c) State Green's theorem. **(2)**

   (d) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$. **(10)**

3. (a) Define Jacobian matrix, Hessian matrix and Laplacian operator. **(5)**

   (b) Evaluate $\iint_R \frac{x - y}{x + y} dA$ where R is the region enclosed by $x - y = 0$, $x - y = 1$, $x + y = 1$ and $x + y = 3$. **(10)**

   (c) Find the image of Im(z) ≥ 0 under the transformation $w = \frac{z - i}{iz - 1}$. Sketch the region Im(z) ≥ 0 and its image. **(8)**

   (d) Test the function $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ for continuity and differentiability at $z = 0$. **(12)**

Contd P/2

MATH 241/CSE

4. (a) Show that $u(x, y)=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic in some domain. Hence find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic in that domain and express $f(z)$ in terms of z. **(15)**

(b) Find the point where the function $f(z) = x^3 + i(1 - y)^3$ is differentiable. Hence find $f'(z)$ at that point. **(6)**

(c) Show that $Log\left(-1+i\sqrt{3}\right)^2 \neq 2Log\left(-1+i\sqrt{3}\right)$. **(6)**

(d) Solve $\cosh z = -2$ by equating the real and imaginary parts in the equation. **(8)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Evaluate $\int_C \left(3xy+iy^2\right)dz$ along the straight line joining $z = i$ and $z = 2 - i$. **(12)**

(b) State Cauchy-Goursat theorem and apply it to find $\int_C \sec hz\, dz$ where C is the circle $|z| = 1$ in either direction. **(10)**

(c) Use Cauchy integral formula to evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is the circle $|z|=\frac{3}{2}$. **(13)**

6. (a) Form a partial differential equation by eliminating arbitrary functions f and g from **(10)**

$$z = f\left(x^2 - y\right)+ g\left(x^2 + y\right).$$

(b) Find the integral surface satisfying $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and passing through $x = 1, y = 0$. **(12)**

(c) Find the complete integral of the following partial differential equation: $2x(z^2q^2 + 1) = pz$. **(13)**

7. Solve the following higher order partial differential equations:

(a) $\left(D_x^3 - 2D_x^2D_y - D_xD_y^2 + 2D_y^3\right)z = \sin(2x + y)$ **(11)**

(b) $\left(D_x^2 - 4D_xD_y + 4D_y^2 + D_x - 2D_y\right)z = e^{x+y} + xy$ **(12)**

(c) $\left(D_x^2D_y + D_y^2 - 2\right)z = 4e^{x+y}\cos(x + y)$ **(12)**

Contd P/3

MATH 241/CSE

8. (a) Reduce the following partial differential equation to linear partial differential equation with constant coefficient and hence solve it: **(10)**

$$\left(x^2D_x^2 - 2xyD_xD_y - 3y^2D_y^2 + xD_x - 3yD_y\right)z = 0$$

(b) Solve the following boundary value problem by the method of separation of variables and interpret it physically: **(25)**

$$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$$

$$u(0, t) = u(2, t) = 0, \ t > 0$$

$$u(x, 0) = x(2 - x) \text{ and } \frac{\partial u}{\partial t} = 0 \text{ when } t = 0, \quad 0 < x < 2.$$
