BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1 B.Sc. Engineering Examinations 2022-2023

Sub: **CHE 401** (Reaction Engineering)

Full Marks: 210   Time: 3 Hours

The figures in the margin indicate full marks.

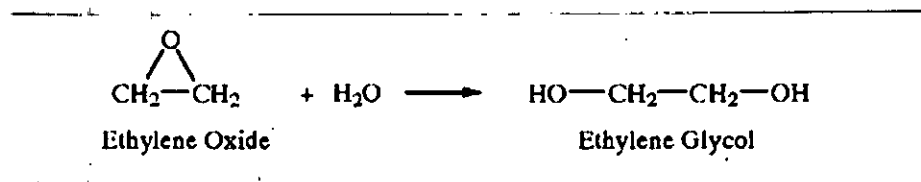USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. You have to answer **THREE**.

The symbols have their usual meaning if not explained.

1. Ethylene glycol is produced form ethylene oxide by reacting it with water. This liquid phase reaction is catalyzed by sulfuric acid.

$$CH_2\text{—}CH_2 \ (\overset{O}{\triangle}) + H_2O \longrightarrow HO\text{—}CH_2\text{—}CH_2\text{—}OH$$

Ethylene Oxide   Ethylene Glycol

The entering concentration of ethylene oxide and water, after mixing the inlet streams, are 18.5 and 60.0 mol/dm^3, respectively. The specific reaction rate is

k = 0.15dm^3/mol.s at 300 K with E = 11500 cal/mol.

(a) Set up a stoichiometric table for the above reaction and express the concentration of each species in the reaction as a function of conversion. **(10)**

(b) Assume the reaction follows an elementary rate law and write the reaction rate solely as a function of conversion. **(7)**

(c) Calculate the CSTR space-time, τ, for 85% conversion at 300 K and 350 K. **(10)**

(d) If the volumetric flow rate is 300 dm^3 per second, what are the corresponding reactor volumes? **(8)**

2. The data below have been reported for the decomposition of diazobenzene chloride.

$$C_6H_5N_2Cl \rightarrow C_6H_5Cl + N_2$$
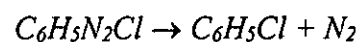
The reaction was followed by measuring the nitrogen evolved from the liquid mixture. The initial concentration diazobenzene chloride was 5.0 kg/m^3. The reaction is believed to be first order and irreversible. **(35)**

| Time, t (s)	N$_2$ evolved (cm^3 at STP)
360	19.3
540	26.0
720	32.6
840	36.0
1080	41.3
1320	45.0
1440	46.5
1560	48.4
1800	50.4
∞	58.3

<u>CHE 401</u>
(Contd ...... Q. No. 2)

(a) Determine if the reaction is indeed first order, and

(b) Evaluate the reaction rate constant.

Hint: You may consider "N$_2$ evolved" as a physical property to determine the extent of reaction.

3. An autocatalytic reaction is to be carried out in aqueous solution in two identical continuous stirred tank reactors operating in series. The reaction stoichiometry is:

$$A \rightarrow B$$
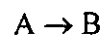
While the reaction rate expression is:

$$r = kC_A C_B$$

(a) If the system is to operate isothermally at 50°C where the reaction rate constant is equal to 0.9 m^3/kmole-ks, determine the reactor volume necessary to achieve an overall fraction conversion of 0.80. Species A is to be fed at a rate of 0.3 mole/sec and initial A concentration of 2 kmoles/m^3. **(20)**

(b) What would be the volume requirement to achieve the same conversion in a PFR? **(5)**

(c) If a recycle reactor with R = 10 is used what would be the volume of the recycle reactor? **(10)**

4. For the homogeneous catalytic reaction

$$A + B \rightarrow B + B, \qquad -r_A = kC_A C_B$$

and with a feed of $C_{A0} = 90$ mol/m^3, $C_{B0} = 10$ mol/m^3:

(a) Draw a Levenspiel plot for this reactions at the given condition. **(15)**

(b) We want about 44% conversion of reactant A. What flow reactor or combination of flow reactors is best in that it gives the smallest total volume of reactors needed? **(7)**

(c) If we need 90% conversion of reactant A, what flow reactor or combination of flow reactors is best in that it gives the smallest total volume of reactors needed? **(7)**

(d) If we only want about 20% conversion of reactant A, what flow reactor or combination of flow reactors is best in that it gives the smallest total volume of reactors needed? **(6)**

SECTION – B

There are **FOUR** questions in this section. You have to answer **THREE** questions.

5. (a) Derive the relationship between Equilibrium constant (K) and Gibb's free energy. **(12)**

(b) Calculate the equilibrium composition of a mixture of the following species. **(23)**

Component	Mole Fraction
N_2	0.15
H_2O	0.60
C_2H_4	0.25

The mixture is maintained at a constant temperature of 527 K and constant pressure of 264.2 bar. Assume that the only significant chemical reaction is

$$H_2O \text{ (g)} + C_2H_4 \text{ (g)} \leftrightarrow C_2H_5OH \text{ (g)}$$
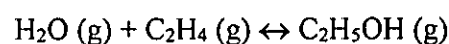
The standard state of each species is taken as the pure material at unit fugacity. Use only the following critical properties, thermochemical data and fugacity data.

Compound	Tc (K)	Pc (bar)
H_2O (g)	647.3	218.2
C_2H_4 (g)	283.1	50.5
C_2H_5OH (g)	516.3	63.0

Compound	$\Delta G^0_{f298.16}(\frac{kJ}{mol})$	$\Delta H^0_{f298.16}(\frac{kJ}{mol})$
H_2O (g)	-228.705	-241.942
C_2H_4 (g)	68.156	53.308
C_2H_5OH (g)	-168.696	-235.421

Fugacity coefficients can be used from the following table:

Species	Reduced temperature, 527 K	Reduced pressure, 264.2 bar	f/P
H_2O (g)	0.814	1.211	0.190
C_2H_4 (g)	1.862	5.232	0.885
C_2H_5OH (g)	1.021	4.194	0.280

CHE 401

6. (a) Derive the equation for internal effectiveness factor for a first order reaction in a spherical catalyst pellet: **(15)**

$$\eta = \frac{3}{\varphi_1^2}(\varphi_1 \coth \varphi_1 - 1)$$

(b) For a single spherical catalyst pellet, derive the differential equation describing diffusion and reaction: **(20)**

$$\frac{d^2 C_A}{dr^2} + \frac{2}{r}\left(\frac{dC_A}{dr}\right) - \frac{k_n}{D_e} C_A^n = 0$$

Mention the boundary condition for the equation after derivation.

7. (a) "Gibb's free energy is a state function, changes in this quantity is independent of whether the reaction takes place in one or, several steps"- Elaborate and clarify statement with reasoning. **(9)**

(b) $CO + S \leftrightarrow CO.S$.................................(R1) **(2+12+12=26)**

$CO + 2S \leftrightarrow C.S + O.S$.....................(R2)
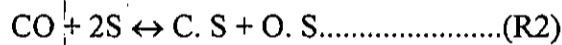
Identify the above reactions whether they could be considered for molecular or dissociative adsorption. Then answer the following questions:

(i) Derive the equation for Langmuir Isotherm for molecular adsorption based on your identification of reactions above.

(ii) Derive the equation for Langmuir Isotherm for dissociative adsorption based on your identification of reactions above.

8. (a) What are the standard states for chemical potential calculation for different phases? **(6)**

(b) What is effective diffusivity? How is it related to the bulk diffusivity (D_{AB}) and Knudsen diffusivity (D_k)? Also, explain how tortuosity and constriction factor affect the effective diffusivity. **(3+4+7=14)**

(c) Consider the following reaction: **(15)**

$CO.S + O.S \leftrightarrow CO_2.S + S$
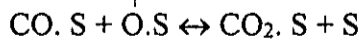
Should you explain this reaction mechanism with single site or, dual site mechanism? Derive the surface reaction rate law based on your decision.

$$t_{1/2} = \frac{(0.5)^{1-n} - 1}{k(n-1)} C_{A0}^{1-n}$$

$$t_F = \frac{F^{1-n} - 1}{k(n-1)} C_{A0}^{1-n}$$

$$N_{A0} \frac{dX_A}{dt} = -r_A V$$

$$\frac{V_i}{F_{A0}} = \frac{\Delta X_{Ai}}{(r_{Ai})} = \frac{\tau_i}{C_{A0}}$$

$$F_{A0} \frac{dX_A}{dV} = -r_A$$

$$F_{A0} \frac{dX_A}{dW} = -r_A'$$

$$p_A = C_A RT$$

$$C_A = C_{A0} e^{-k_1 t}$$

$$C_R = C_{A0} k_1 \left(\frac{e^{-k_1 t}}{k_2 - k_1} + \frac{e^{-k_2 t}}{k_1 - k_2} \right)$$

$$\tau_{p, opt} = \frac{\ln(k_2/k_1)}{k_2 - k_1}$$

$$\frac{C_{R,max}}{C_{A0}} = \left(\frac{k_1}{k_2} \right)^{k_2/(k_2 - k_1)}$$

$$\left[\frac{\partial \ln K_a}{\partial (1/T)} \right]_P = -\frac{\Delta H^0}{R}$$

$$\frac{n_S^s n_T^t}{n_B^b n_C^c} = \frac{K_a}{K_{f/P}} \left(\frac{n_B + n_C + \cdots + n_S + n_T + \cdots + n_I}{P} \right)^{s+t-b-c}$$

$$C_A = \frac{F_A}{v} = \frac{F_{A0}(1-X_A)}{v_0(1 + \varepsilon_A X_A)} \frac{T_0}{T} \frac{P}{P_0} = C_{A0} \frac{(1-X_A)}{(1 + \varepsilon_A X_A)} \frac{T_0}{T} \frac{P}{P_0}$$

$$C_B = \frac{F_B}{v} = \frac{F_{A0}(M_B - \frac{b}{a}X_A)}{v_0(1 + \varepsilon_A X_A)} \frac{T_0}{T} \frac{P}{P_0} = C_{A0} \frac{(M_B - \frac{b}{a}X_A)}{(1 + \varepsilon_A X_A)} \frac{T_0}{T} \frac{P}{P_0}$$

$\Leftarrow 6 \Leftarrow$

$$M_i = \frac{F_{i0}}{F_{A0}} = \frac{C_{i0}}{C_{A0}} = \frac{y_{i0}}{y_{A0}}$$

$$\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$$

$$k = k_1 \exp \exp\left[\frac{E}{R}\left(\frac{1}{T_1} - \frac{1}{T}\right)\right]$$

$$\int_0^x \frac{dx}{1-x} = \ln \ln \frac{1}{1-x}$$

$$\int_{x_1}^{x_2} \frac{dx}{(1-x)^2} = \frac{1}{1-x_2} - \frac{1}{1-x_1}$$

$$\int_0^x \frac{dx}{(1-x)^2} = \frac{x}{1-x}$$

$$\int_0^x \frac{dx}{1+\varepsilon x} = \frac{1}{\varepsilon} \ln(1 + \varepsilon x)$$

$$\int_0^x \frac{(1+\varepsilon x)dx}{1-x} = (1 + \varepsilon)\ln \frac{1}{1-x} - \varepsilon x$$

$$\int_0^x \frac{(1+\varepsilon x)dx}{(1-x)^2} = \frac{(1+\varepsilon)x}{1-x} - \varepsilon \ln \frac{1}{1-x}$$

$$\int_0^x \frac{(1+\varepsilon x)^2 dx}{(1-x)^2} = 2\varepsilon(1 + \varepsilon)\ln(1 - x) + \varepsilon^2 x + \frac{(1+\varepsilon)^2 x}{1-x}$$

Table 5.1 Performance Equations for nth-order Kinetics and $\varepsilon_A = 0$

	Plug Flow or Batch		Mixed Flow	
$n = 0$ $-r_A = k$	$\frac{k\tau}{C_{A0}} = \frac{C_{A0} - C_A}{C_{A0}} = X_A$	(20)	$\frac{k\tau}{C_{A0}} = \frac{C_{A0} - C_A}{C_{A0}} = X_A$	
$n = 1$ $-r_A = kC_A$	$k\tau = \ln \frac{C_{A0}}{C_A} = \ln \frac{1}{1 - X_A}$	(3.12)	$k\tau = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A}$	(14a)
$n = 2$ $-r_A = kC_A^2$	$k\tau C_{A0} = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A}$	(3.16)	$k\tau = \frac{(C_{A0} - C_A)}{C_A^2} = \frac{X_A}{C_{A0}(1 - X_A)^2}$	(15)
any n $-r_A = kC_A^n$	$(n - 1)C_{A0}^{n-1}k\tau = \left(\frac{C_A}{C_{A0}}\right)^{1-n} - 1 = (1 - X_A)^{1-n} - 1$	(3.29)	$k\tau = \frac{C_{A0} - C_A}{C_A^n} = \frac{X_A}{C_{A0}^{n-1}(1 - X_A)^n}$	
$n = 1$ $A \underset{2}{\overset{1}{\rightleftharpoons}} R$ $C_{R0} = 0$	$k_1\tau = \left(1 - \frac{C_{Ae}}{C_{A0}}\right)\ln\left(\frac{C_{A0} - C_{Ae}}{C_A - C_{Ae}}\right) = X_{Ae}\ln\left(\frac{X_{Ae}}{X_{Ae} - X_A}\right)$		$k_1\tau = \frac{(C_{A0} - C_A)(C_{A0} - C_{Ae})}{C_{A0}(C_A - C_{Ae})} = \frac{X_A X_{Ae}}{X_{Ae} - X_A}$	
General rate	$\tau = \int_{C_A}^{C_{A0}} \frac{dC_A}{-r_A} = C_{A0}\int_0^{X_A} \frac{dX_A}{-r_A}$	(19)	$\tau = \frac{C_{A0} - C_A}{-r_N} = \frac{C_{A0}X_A}{-r_N}$	(13)

$= 7 =$

Table 5.2 Performance Equations for nth-order Kinetics and $\varepsilon_A \neq 0$

	Plug Flow		Mixed Flow	
$n = 0$ $-r_A = k$	$\dfrac{k\tau}{C_{A0}} = X_A$	(20)	$\dfrac{k\tau}{C_{A0}} = X_A$	
$n = 1$ $-r_A = kC_A$	$k\tau = (1 + \varepsilon_A)\ln\dfrac{1}{1 - X_A} - \varepsilon_A X_A$	(21)	$k\tau = \dfrac{X_A(1 + \varepsilon_A X_A)}{1 - X_A}$	(14b)
$n = 2$ $-r_A = kC_A^2$	$k\tau C_{A0} = 2\varepsilon_A(1 + \varepsilon_A)\ln(1 - X_A) + \varepsilon_A^2 X_A + (\varepsilon_A + 1)^2 \cdot \dfrac{X_A}{1 - X_A}$	(23)	$k\tau C_{A0} = \dfrac{X_A(1 + \varepsilon_A X_A)^2}{(1 - X_A)^2}$	(15)
any n $-r_A = kC_A^n$			$k\tau C_{A0}^{n-1} = \dfrac{X_A(1 + \varepsilon_A X_A)^n}{(1 - X_A)^n}$	
$n = 1$ $A \underset{\frac{1}{2}}{\rightleftharpoons} rR$ $C_{R0} = 0$	$\dfrac{k\tau}{X_{Ae}} = (1 + \varepsilon_A X_{Ae})\ln\dfrac{X_{Ae}}{X_{Ae} - X_A} - \varepsilon_A X_A$	(22)	$\dfrac{k\tau}{X_{Ae}} = \dfrac{X_A(1 + \varepsilon_A X_A)}{X_{Ae} - X_A}$	
General expression	$\tau = C_{A0}\displaystyle\int_0^{X_A} \dfrac{dX_A}{-r_A}$	(17)	$\tau = \dfrac{C_{A0}X_A}{-r_A}$	(11)

$$\ln\frac{1 - X_B}{1 - X_A} = \ln\frac{M - X_A}{M(1 - X_A)} = \ln\frac{C_B C_{A0}}{C_{B0} C_A} = \ln\frac{C_B}{MC_A} = C_{A0}(M - 1)kt = (C_{B0} - C_{A0})kt, \quad M \neq 1$$

$$\ln\frac{M - 2X_A}{M(1 - X_A)} = \ln\frac{C_B C_{A0}}{C_{B0} C_A} = \ln\frac{C_B}{MC_A} = C_{A0}(M - 2)kt, \quad M \neq 2$$

$$\frac{1}{C_A} - \frac{1}{C_{A0}} = \frac{1}{C_{A0}}\frac{X_A}{1 - X_A} = kt$$

$$\frac{1}{C_A} - \frac{1}{C_{A0}} = \frac{1}{C_{A0}}\frac{X_A}{1 - X_A} = 2kt, \quad M = 2$$

$$\frac{1}{C_A^2} - \frac{1}{C_{A0}^2} = 2kt, \quad M = 1$$

$$\frac{1}{C_A^2} - \frac{1}{C_{A0}^2} = 8kt, \quad M = 2$$
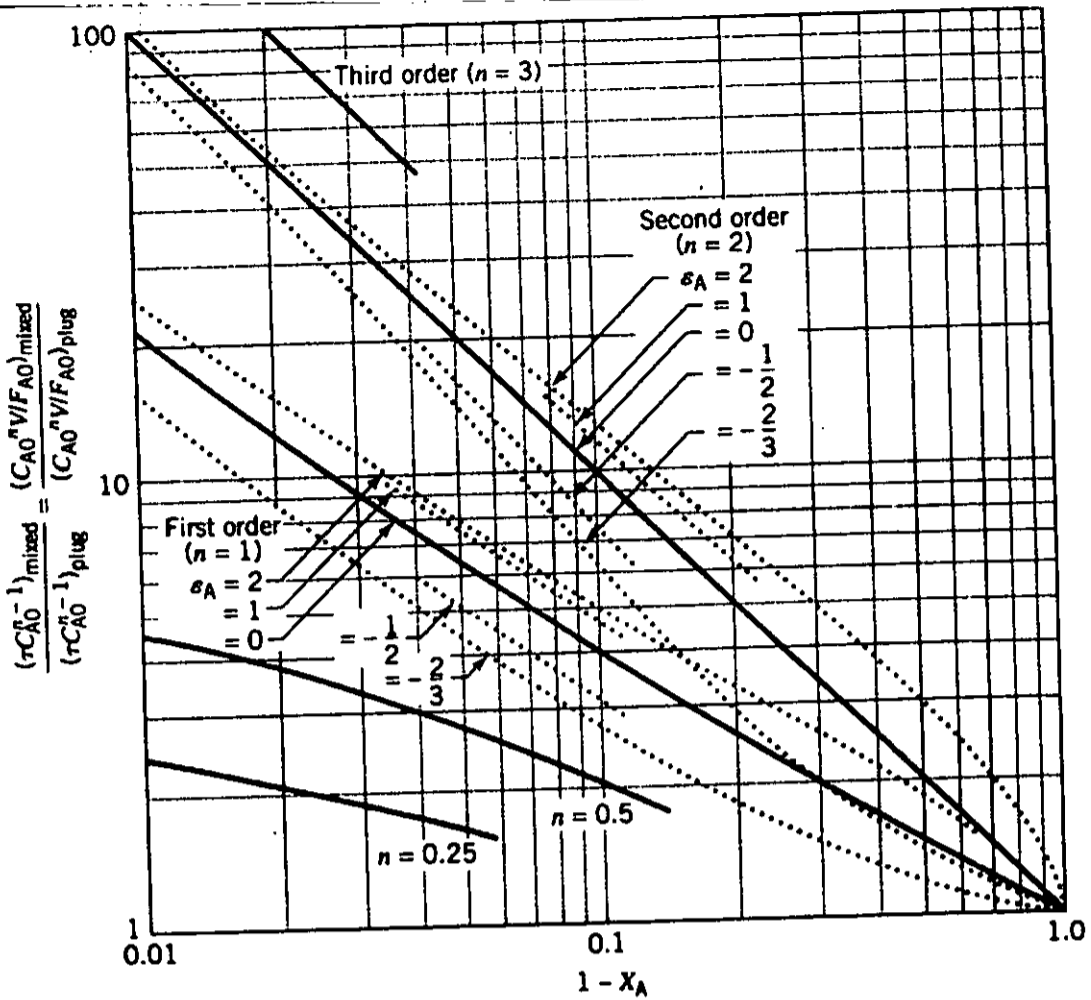
$$C_A^{1-n} - C_{A0}^{1-n} = k(n - 1)t, \quad n \neq 1$$

$$\frac{(\tau C_{A0}^{n-1})_{mixed}}{(\tau C_{A0}^{n-1})_{plug}} = \frac{(C_{A0}^{n}V/F_{A0})_{mixed}}{(C_{A0}^{n}V/F_{A0})_{plug}}$$

Third order $(n = 3)$

Second order $(n = 2)$

$\varepsilon_A = 2$
$= 1$
$= 0$
$= -\frac{1}{2}$
$= -\frac{2}{3}$

First order $(n = 1)$

$\varepsilon_A = 2$
$= 1$
$= 0$
$= -\frac{1}{2}$
$= -\frac{2}{3}$

$n = 0.5$

$n = 0.25$

$1 - X_A$

Figure 6.1 Comparison of performance of single mixed flow and plug flow reactors for the nth-order reactions

$$A \rightarrow \text{products}, \quad -r_A = kC_A^n$$

First-order reaction

$$\frac{\tau_N}{\tau_p} = \frac{\tau_N}{\tau_{N=\infty}} = \frac{(C_{A0}V/F_{A0})_N \text{ reactors}}{(C_{A0}V/F_{A0})_{plug}}$$

$k\tau = 50$

$N = 1$

20

10

5

$N = 2$

$N = 3$
$N = 4$
$N = 6$

2

$k\tau = 1$

$N = \infty$ $N = 10$

$$1 - X_A = \frac{C_A}{C_{A0}}$$
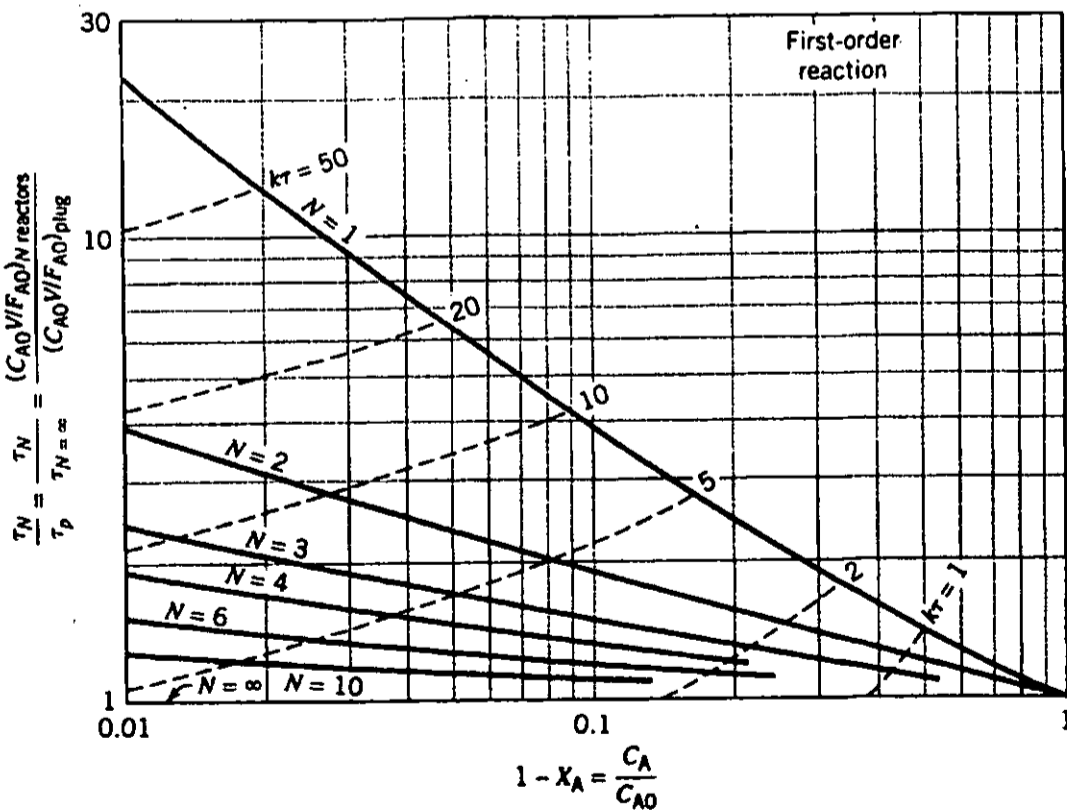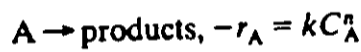
Figure 6.5 Comparison of performance of a series of N equal-size mixed flow reactors with a plug flow reactor for the first-order reaction
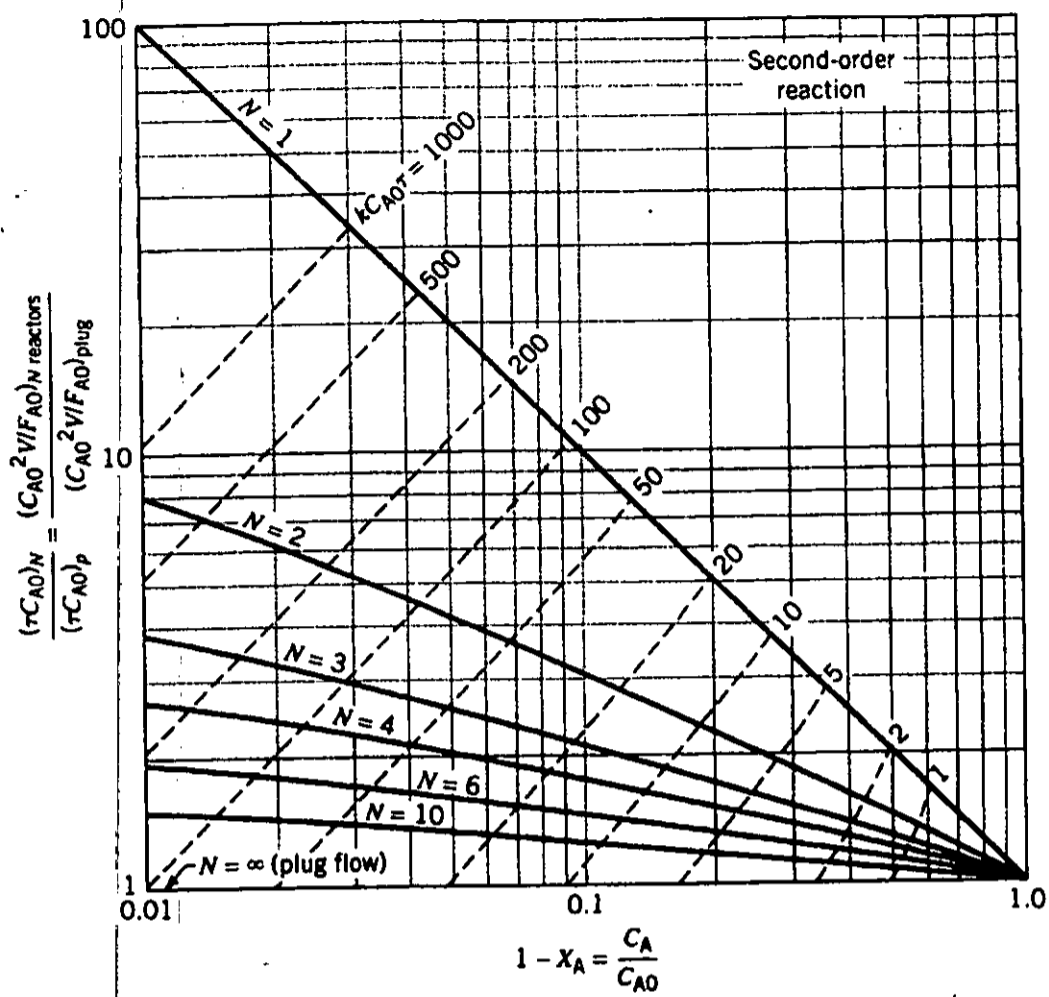
Figure 6.6 Comparison of performance of a series of N equal-size mixed flow reactors with a plug flow reactor for elementary second-order reactions
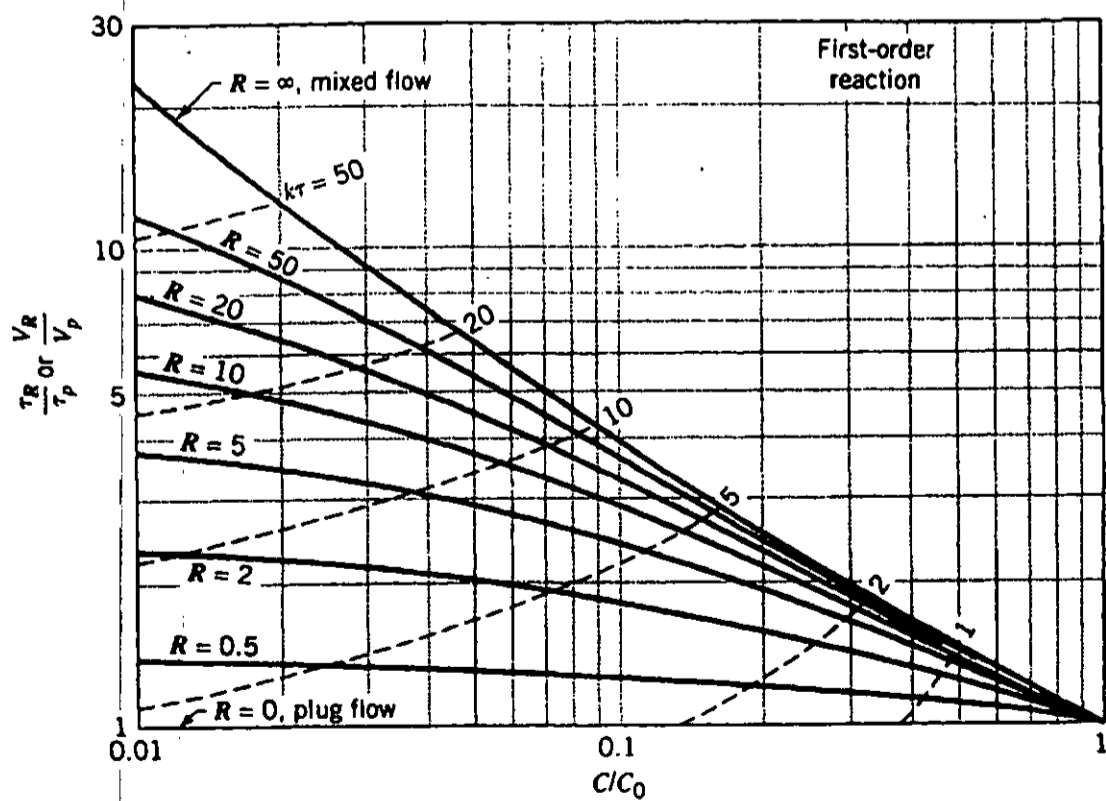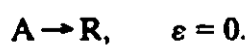
Axis labels:
- Vertical: $(\tau C_{A0})_N = \dfrac{(C_{A0}^2 V/F_{A0})_N \text{ reactors}}{(C_{A0}^2 V/F_{A0})_\text{plug}}$, $\dfrac{(\tau C_{A0})_N}{(\tau C_{A0})_p}$
- Horizontal: $1 - X_A = \dfrac{C_A}{C_{A0}}$

Curves labeled: $N = 1$, $N = 2$, $N = 3$, $N = 4$, $N = 6$, $N = 10$, $N = \infty$ (plug flow); second-order reaction; $kC_{A0}\tau = 1000$, 500, 200, 100, 50, 20, 10, 5, 2, 1.

Figure 6.16 Comparison of performance of recycle and plug flow for first-order reactions

Axis labels:
- Vertical: $\dfrac{\tau_R}{\tau_p}$ or $\dfrac{V_R}{V_p}$
- Horizontal: C/C_0

Curves labeled: $R = \infty$, mixed flow; $R = 50$; $R = 20$; $R = 10$; $R = 5$; $R = 2$; $R = 0.5$; $R = 0$, plug flow; First-order reaction; $k\tau = 50$, 20, 10, 5, 2, 1.

$$A \rightarrow R, \quad \varepsilon = 0.$$
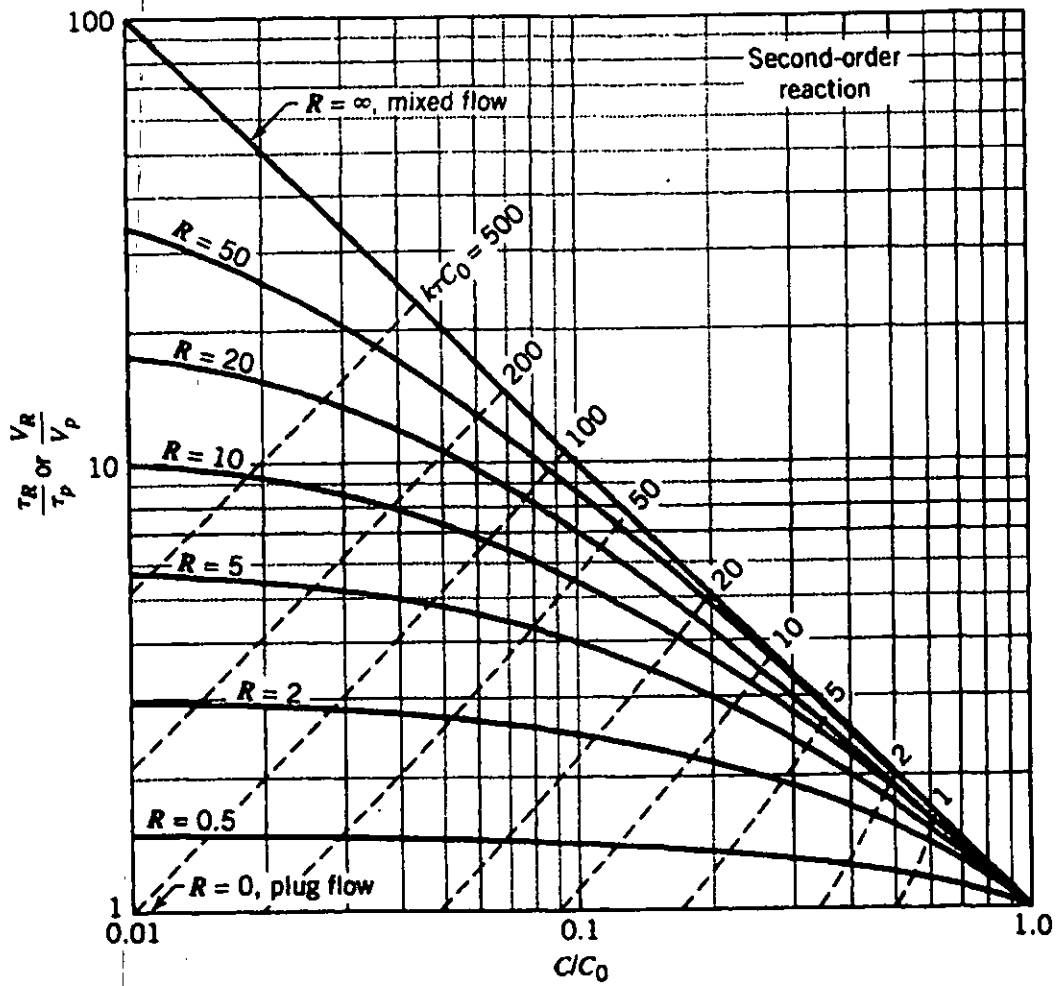
Figure 6.17 Comparison of performance of recycle reactors with plug flow reactors for elementary second-order reactions (Personal communication, from T. J. Fitzgerald and P. Fillesi):

$$2A \rightarrow \text{products}, \qquad \varepsilon = 0$$

$$A + B \rightarrow \text{products}, \qquad C_{A0} = C_{B0} \text{ with } \varepsilon = 0$$

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2022-2023

Sub: **CHE 403** (Mass Transfer II)

Full Marks: 210   Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION
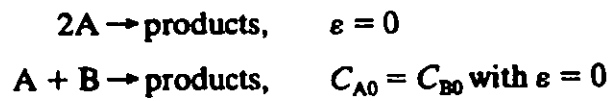
SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

(Symbols indicate their usual meaning)

1. (a) A closed stirred tank reactor with two compartments is shown in figure below.   **(18)**
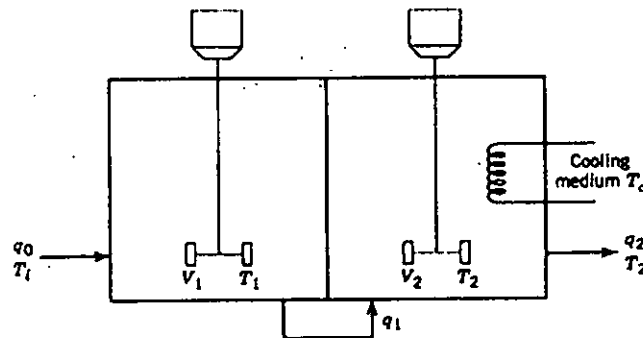
Figure for Question No. 1(a)

The basic idea is to feed the reactants continuously into the first compartment where they will be preheated by energy liberated in the exothermic reaction. Which is anticipated to occur primarily in the second compartment. The wall separating the two compartments is quite thin, thus allowing heat transfer, the outside of the reactor is well insulated; and a cooling coil is built into the second compartment to remove excess energy liberated in the reaction. Tests are to be conducted initially with a single component feed (no reaction) to evaluate the reactor's thermal characteristics.

(i) Develop a dynamic model for this process under the condition of no reaction. Assume that q, T_i, and T_c all may vary.

(ii) make a degree of freedom analysis for the model identifying all parameters, outputs, and inputs that must be known functions of time in order to obtain a solution.

Notes:

U_t, A_t: *Overall heat transfer coefficient and surface area between compartments.*

U_c, A_c: *Overall heat transfer coefficient and surface area of the cooling tube.*

V_1: *Volume of Compartment 1*

V_2: *Volume of Compartment 2*

(b) A process tank has two input streams-Stream 1 at mass flow rate w_1 and Stream 2 at mass flow rate w_2. The tank's effluent stream, at flow rate w, discharges through a fixed valve to atmospheric pressure. Pressure drop across the valve is proportional to the flow rate squared.   **(17)**

So that, Valve equation: $w = C_v' \sqrt{\dfrac{\rho g}{g_c} h}$

CHE 403

Contd ... Q. No. 1(b)

The cross-sectional area of the tank, A, is 5 m^2, and the mass density of all streams is 940 kg/m^3.

(i) Draw a schematic diagram of the process and write an appropriate dynamic model for the tank level. What is the corresponding steady-state model?

(ii) At initial steady-state conditions, with w_1 = 2.0 kg/s and w_2 = 1.2 kg/s, the tank level is 2.25 m. What is the value of the valve constant?

2. (a) Two first-order systems in series results in an overall second-order system. **(20)**
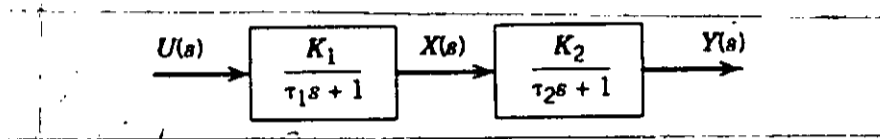
The second-order system yields two alternative forms of equations.

$$\tau_2 + 2\xi\tau s + 1 = (\tau_1 s + 1)(\tau_2 s + 1)$$

For the following underdamped process, draw the performance characteristics for the step response of U(s) by calculating rise time, time to first peak, settling time, overshoot, decay ratio, and period of oscillation.

$$\frac{Y(s)}{U(s)} = \frac{K}{4s^2 + 2s + 1}$$

(b) A continuous reaction (A→B) is taking place in a CSTR reactor. Where, the initial reactant concentration C_A (t=0) and product concentration C_B (t=0) are 2 kgmole/min and 0 kgmole/min respectively. Two material balance equation around the reactor are as follows: **(15)**

$$\frac{dC_A}{dt} + C_A = 6$$

$$4\frac{dC_B}{dt} + C_B = 8$$

If the reactant concentration C_A is changed from 2 to 3 kgmole/min, what will be the product composition C_B at time t = 4 min?

3. (a) A multiple input, multiple output (MIMO) thermal mixing process is shown in Figure for Question No. 3(a). The level h in the stirred tank and the temperature T are to be controlled by adjusting the flow rates of the hot and cold streams and outlet stream, w_h and w_c and w (w=kh) respectively. The temperatures of the inlet streams T_h and T_c are considered to be disturbance variables. Noting that the liquid volume can vary with time and the liquid properties are assumed to be constant (not affected by temperature) in the following derivation. **(20)**
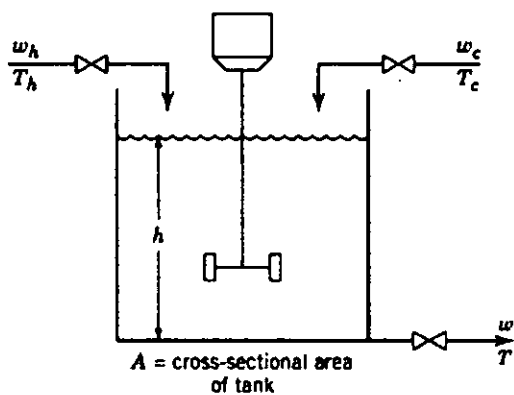
CHE 403

Contd ... Q. No. 3(a)

Figure for Question No. 3(a)

(i) Derive the transfer function of H'(s) and T'(s) with other variables. **(10)**

(ii) Express the transfer functions by means of a *transfer function matrix*. **(5)**

(iii) Draw a block diagram of the MIMO thermal mixing system with variable level. **(5)**

(b) A tank used to dampen liquid flow rate surges is known to exhibit second-order dynamics. The input flow rate changes suddenly from 120 to 140 gal/min. An operator notes that the tank level changes as follows: **(15)**

Before input change: level = 5 ft

Four minutes later: level = 12 ft and maximum

Forty minutes later: level = 9 ft and steady

Graphically show the performance characteristics for the step response of the process.

4. (a) The second order model of flue gas PM concentration $C(s)$ from a brick filed with Coal feed rate $F(s)$ is as follows **(10)**

$$\frac{C(s)}{F(s)} = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1}$$

The output response of PM concentration $C(t)$ has been obtained for the unit step change of Coal feed rate. Use Smith's method as well as nonlinear regression, estimate the parameters (τ_1, τ_2, K, θ) in the second-order model from the experimental step response data shown in below Figure.
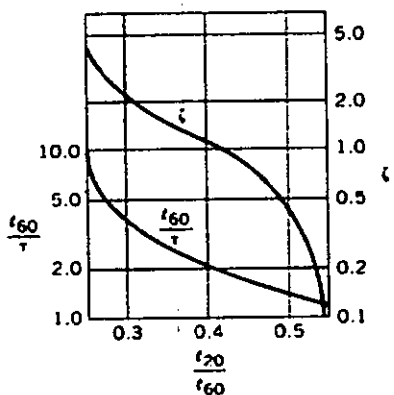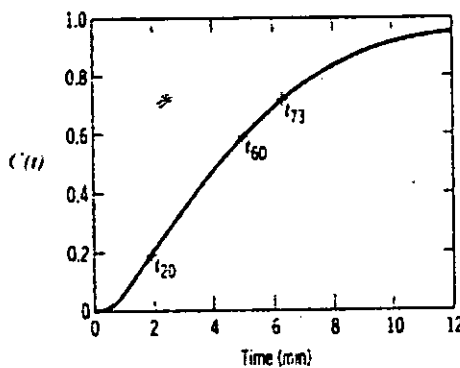

Figure: Smiths Method

Figure: Unit step response

Figure for Question No 4(a)

CHE 403

Contd ... Q. No. 4(a)

Derive an approximate first-order plus time delay model $\dfrac{C(s)}{F(s)} = \dfrac{Ke^{-\theta s}}{\tau s + 1}$ from the estimated second order model using two methods.

(i) The Taylor series expansions

(ii) Skogestad's half rule

You have to draw the three output response in a single graph (two approximate models response and original model response) of C(t) for the unit step change of *F(t)*.
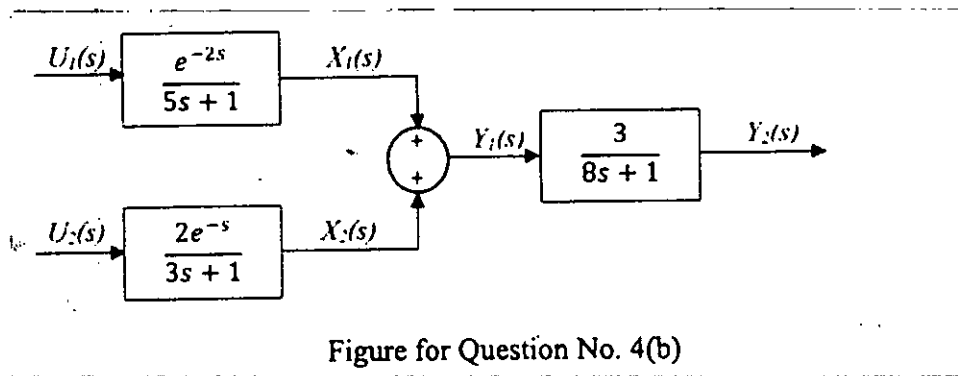
(b) Find the transfer function of $Y_2(s)$ from below Figure (5)

Figure for Question No. 4(b)

SECTION – B

There are **FOUR** questions in this section. **Question No. 5 (five) is Compulsory**. Question 5 and any other **TWO** questions.

5. Compulsory Figure Q. 5(a), shows the top part of a typical distillation column.
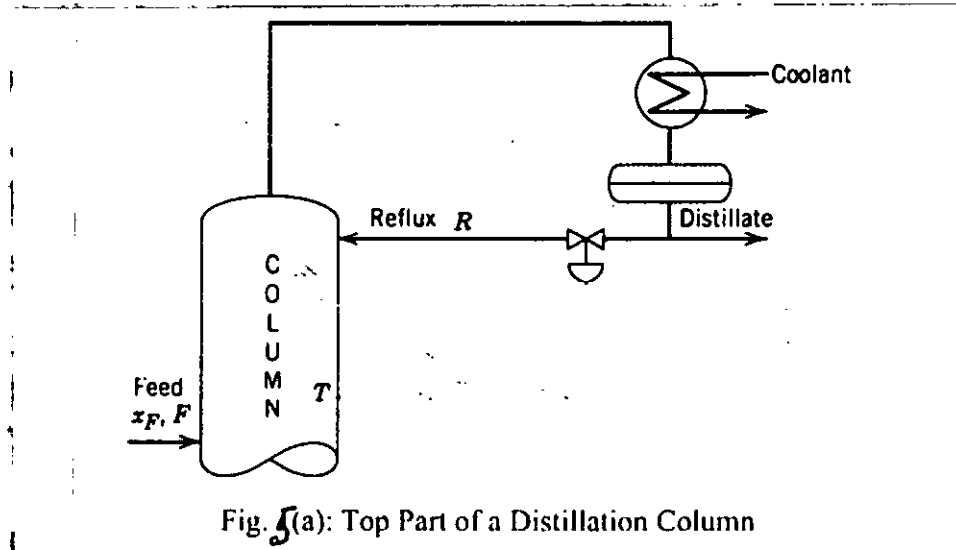

Fig. 5(a): Top Part of a Distillation Column

(a) Identify two important controlled variable(s), two manipulated variable(s) and two disturbance variable(s). (6)

(b) Sketch one important feedback control loop and one feedforward control loop. (6)

(c) A feedback control loop for controlling distillate composition is difficult to implement because online distillate composition measurement is available only in every 5 minutes. Suggest an alternative control strategy to control the distillate composition based on inferential or soft sensor method. Draw the control loop and the corresponding block diagram (label all signals) of the alternative control strategy. (5+3+5=13)

CHE 403

Contd ... Q. No. 5

(d) The transfer functions for controlling pressure of the distillation column by manipulating cooling water flow rate are identified as follows. Is the process, open loop stable? Why or why not? Write down the characteristic equation for a typical feedback control loop and find the range of K_c (where, $G_c = K_c$) for which the system will remain stable. For time delays, use Pade's first order approximation, if needed. **(10)**

$$G_p = \frac{2e^{-4s}}{(10s+1)(-2s+1)}$$

$$G_v = \frac{1}{(s+1)}, \qquad G_m = K_m = 0.5$$

6. (a) Explain the effect of poie-zero locations in s-plane on the stability of a closed loop control system. **(8)**

(b) For a First order Plus Time Delay (FOPTD) process, find the mathematical expression of Amplitude Ratio and phase with respect to frequency for performing frequency response analysis. Sketch qualitatively the Bode plot for such a system. **(10)**

(c) A process instrumentation diagram of a flash drum is shown in Fig. Q 6(c). Steam is condensed in a steam coil to vaporize a portion of the liquid feed, and the liquid product is removed by a pump. The vapor is going to the next equipment for further processing. There are five control valves. What type of valves (A/C or A/O) would you recommend for each of them from safety point of view. Justify your recommendations(s). **(10)**

Fig. Q6(c): Flash drum instrumentation
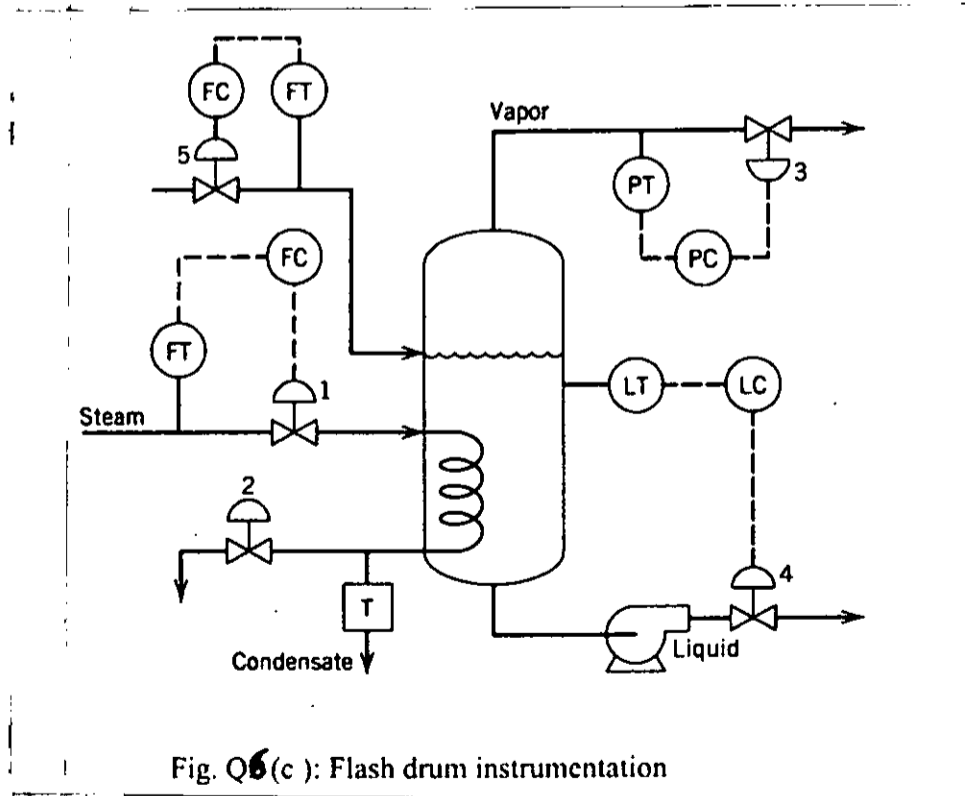
(d) Describe the derivative mode of feedback controllers with their respective merits and demerits. How can the demerits be overcome? **(7)**

CHE 403

7. (a) Describe the IMC method for designing PID controllers (include block diagram, closed loop relations and controller design equations). **(10)**

(b) For a first order plus time delay (FOPTD) process, find the design equations for finding parameters of PID controllers using IMC method (Use first order Pade's approximation for time delay). **(10)**

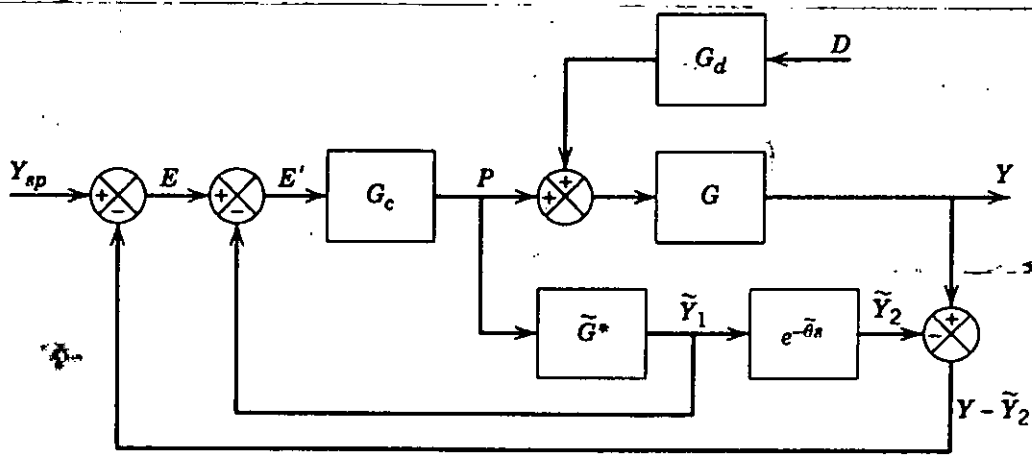(c) Derive the relationship between controlled variable and disturbance variable for the block diagram shown in Fig. 7(c). **(15)**

Fig. 7(c): Figure for Q7(c).

8. Write short notes on **(5×7=35)**

   (a) Cascade control

   (b) Auctioneering control

   (c) Split range control

   (d) Ratio control

   (e) On-off control

Each short note should contain justification of using this control strategy and its working principle with one practical example.

L-4/T-1/CHE   **Date: 23/04/2024**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2022-2023

Sub: **CHE 405** (Process Design I)

Full Marks: 210   Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

(The symbols hare their usual meanings)

CHE 405 data booklet has to be supplied

1. (a) Select the application where following support systems can be used. Justify your answer **(5×3=15)**

   (i)   Bracket support

   (ii) Skirt support

   (iii) Saddle support

   (b) Select best material of construction for the following application. Justify your answer **(5×3=15)**

   (i)   Hydrofloric acid

   (ii) Sulfuric acid, 10-75%

   (iii) Chlorine, wet

   (c) How does the pressure of heating or cooling jacket is accommodated in a jacketed vessel? **(5)**

2. A hydrocarbon vapor is preheated from 10 to 70 °C with a 2.5 kg/s of hot fluid that is to be cooled from 95 to 60 °C. A fin type double-pipe heat exchangers is being considered for this heat-transfer process. **(35)**

   (i) Determine **the area and exchanger length** anticipated for this exchanger fabricated with copper tubes and fins.

   (ii) If the whole operation is shifted to a plate and frame heat exchanger, how many **plates** will be required to achieve same overall heat transfer?

   Following data are available

   Tube dimension: D_1 = 0.0483 m; D_2 = 0.0408 m

   Shell dimension Dos = 0.0889 m;   Dis = 0.0779 m

   Heat transfer co-efficient, h_h = 9.46 × 10^3 W/m^2.K

   Fin height L_f = 0.0127 m

   Fin thickness t_f = 0.0027 m

   Number of fins N_f = 30 longitudinal fins

   Thermal conductivity of copper = 379 J/s-m-K

   Plates are square type having side 1m and spacing between the plates is 0.005 m

CHE 405

Contd ... Q. No. 2

Property	Hydrocarbon vapor (Annular place)	Hot fluid (inner tube)
Heat capacity, C_p	1.622×10^3 J/kg-K	4.198×10^3 J/kg-K
Viscosity, μ	2.70×10^{-3} Pa-s	3.72×10^{-4} Pa-s
Density, ρ	1.044×10^3 kg/m3	9.74×10^2 kg/m3
Thermal conductivity, k	0.138 J/s-m-K	0.668 J/s-m-K

$$h = 0.023 \left(\frac{k}{D}\right) Re^{0.8} Pr^{\frac{1}{3}}$$

$$\eta_f = \frac{\tanh(ML_f)}{ML_f} \quad where, M = \left(\frac{h}{\delta K_f}\right)^{0.5} \quad and \quad \delta = \frac{V_f}{A_f}$$

$$\eta_o A_o = A_p + \eta_f A_f$$

$$\frac{1}{U_1} = \frac{A_1}{h1(A_p + \eta_f A_f)} + \frac{D_1 ln(\frac{D_1}{D_2})}{2K_w} + \frac{D_1}{h_2 D_2}$$

State any reasonable assumptions clearly.

3. (a) What should be your considerations while designing a piping system? **(10)**

(b) What is the pressure loss when 2.14 kg/s of pure benzene at 40°C flows through a 21-m length of straight pipe with an inside diameter of 0.0409 m? The pipeline contains six 90° elbows, one tee used as an elbow (equivalent resistance equal to 60 pipe diameters), one globe valve, and one gate valve. The density of the benzene is 849 kg/m³, and its viscosity at 40°C is 5×10^{-4} Pa-s. See Fig. for Q. 3(b) **(15)**

(c) What do you understand by dead weight of a pressure vessel? List down all the sources of dead weights. **(10)**

4. You are asked to design a packed column for the following distillation process. The distillation column is with a partial reboiler and total condenser and will be used to separate a mixture of benzene, toluene and cumene. The feed consists of 40% benzene, 30% toluene and 30% cumene and will be supplied as saturated vapor at the rate of 100 kg-moles/hr. It is desired to recover 95% toluene at distillate and 95% cumene at the bottoms. The distillation will be carried out at 1 atm. A reflux ratio 2 will be maintained. Equilibrium can be represented as constant relative volatilities. Choosing Toluene as reference element, $\alpha_{BT} = 2.25$ and $\alpha_{CT} = 0.21$. You are supplied with 25 mm plastic pall rings having void fraction 0.9, surface area per volume $a_p = 267$. If pressure drop across the column is 500 pa/m, calculate packing height and diameter of the column. State any diameter of the column. State any reasonable assumption clearly. See Fig. for Q. 4 **(35)**

Following data are available

Liquid density, $\rho_L = 867$ kg/m³

Vapor density, $\rho_v = 56$ kg/m³

Viscosity of liquid, $\mu_L = 5 \times 10^{-4}$ Pa-s

CHE 405
Contd ... Q. No. 4

$$N_{min} = \frac{\ln\left[\left(\frac{x_{LK}}{x_{HK}}\right)_D \left(\frac{x_{HK}}{x_{LK}}\right)_B\right]}{\ln\left(\alpha_{LK/HK}\right)_{av}}$$

$$\left(\alpha_{LK/HK}\right)_{av} = \left[\left(\alpha_{LK/HK}\right)_D\left(\alpha_{LK/HK}\right)_B\right]^{1/2}$$

$$\sum_{i=1}^{n} \frac{\alpha_i x_{F,i}}{\alpha_i - \theta} = 1 - \bar{q}$$

$$R_{min} + 1 = \sum_{i=1}^{n} \frac{\alpha_i x_{D,i}}{\alpha_i - \theta}$$

$$\frac{N - N_{min}}{N + 1} = 0.75\left[1 - \left(\frac{R - R_{min}}{R + 1}\right)^{0.566}\right]$$

$$HETP = \begin{cases} D & \text{for } D \leq 0.5 \text{ m} \\ 0.5D^{0.3} & \text{for } D > 0.5 \text{ m} \\ D^{0.3} & \text{for absorption columns} \\ & \text{with } D > 0.5 \text{ m} \end{cases}$$

SECTION – B

There are **FOUR** questions in this section. Answer to **Question no. 5 (five) is Compulsory.**

Answer any **TWO** questions from Questions 6-8.

5. **This question is compulsory.**

   Acetaldehyde is produced by ethanol dehydrogenation at 7 bar and 330°C via following endothermic reaction. **(35)**

   $CH_3CH_2OH \rightarrow CH_3CHO + H_2$
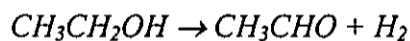
   The feed for this process is 85 wt% of ethanol solution in water. The palladium catalyst used for this process gives a 61% conversion of ethanol with a selectivity of 92% for acetaldehyde. The byproduct formed is mainly acetic acid. The hydrogen separated from the outlet stream of the reactor contains acetaldehyde vapor that needs to be recovered before the gas is vented to the atmosphere.

Table for Question no 5

Component	Boling point	Miscibility in water	Hazard information
Hydrogen	-252.9 °C	Sparingly miscible	Extremely flammable gas
Ethanol	78.37 °C	Highly miscible	Highly flammable liquid and vapor, Causes serious eye irritation
Acetaldehyde	20.2 °C	Highly miscible	Extremely flammable liquid and vapor, Causes serious eye irritation
Acetic acid	117.9 °C	Highly miscible	Substance or mixture corrosive to metals, Causes eye and skin irritation
water	100 °C	-	-

CHE 405

Contd ... Q. No. 5

Develop the following for the above-mentioned process:

(a) Input-output model

(b) Recycle and purge scheme (if any)

(c) Separation scheme: What kind of separating equipment would you choose? Justify. your selection.

(d) Scheme for T, P

(e) Identify the safety and environmental concerns for this production plant and recommend key measures for accident prevention and pollution control.

6. (a) Let us say, Bangladesh Chemical Industry Corporation (BCIC) decides to establish a new Polyester fiber plant in Gazipur. For a quick estimation of Fixed Capital Investment which method of estimation would you recommend? Justify your recommendation. **(10)**

(b) A private company is planning to procure a used sulphuric acid plant with a production capacity of 150 metric tones per day (MTPD) at a price of BDT 100 crore to be set up next to an already existing phosphoric acid plant. The used plant comes with all equipment, instrumentation and utility facilities including an auxiliary boiler (60 ton/hr of steam), steam turbine generator (15 MW, Siemens), and transformers (7 MW each). Estimate the Fixed Capital Investment for this project. (Note: Write down the table number you used for the estimation from the supplied booklet. In your answer include a table listing different components of fixed capital investment, the percentage considered in the cost estimation of each component, the reason for consideration, and the cost of each component). **(25)**

7. (a) Estimate the manufacturing cost of a plant producing 120 million lb/yr of a product with a selling price of $0.20/lb. Given: Fixed Capital: $15,000,000 and Raw Material Cost: $9,600,000/yr. **(15)**

(b) A process plant making 5000 kg/day of a product selling for $1.75/kg has annual variable production costs of $2 Million at 100 percent capacity and fixed costs of $700,000. What is the fixed cost per kilogram at the breakeven point? If the selling price of the product is increased by 10 percent, what is the dollar increase in net profit at full capacity if the income tax rate is 35 percent of gross earnings? **(20)**

8. (a) FAR (fatal accident rate) data show that chemical plants are one of the safest of all manufacturing facilities. Should we be concerned about chemical process safety at all? Support your answer with examples. **(10)**

(b) The Figure for Question no 8 (b) shows a heat exchanger heating a solvent by steam condensation. The solvent is volatile and will vaporize in case of overheating. Prepare a HAZOP table and perform a HAZOP study on the heat exchanger by focusing on the process parameter TEMPERATURE. **(25)**

Fig. for Q. 3(b)

Fittings, valves, etc.	$F = \dfrac{2fV^2 L_e}{D}$		L_e/D per fitting (dimensionless)
		45° elbows	15
		90° elbows, std. radius	32
		90° elbows, medium radius	26
		90° elbows, long sweep	20
		90° square elbows	60
		180° close-return bends	75
		180° medium-radius return bends	50
		Tee (used as elbow, entering run)	60
		Tee (used as elbow, entering branch)	90
		Couplings	Negligible
		Unions	Negligible
		Gate valves, open	7
		Globe valves, open	300
		Angle valves, open	170
		Water meters, disk	400
		Water meters, piston	600
		Water meters, impulse wheel	300

Figure 12-1
Fanning friction factors for long, straight pipes. [Based on L. F. Moody, *Trans. ASME.* 66: 671–684 (1944).]

Packing size, mm	Packing factor F_p
16	100
25	50
38	40
50	25
75	18
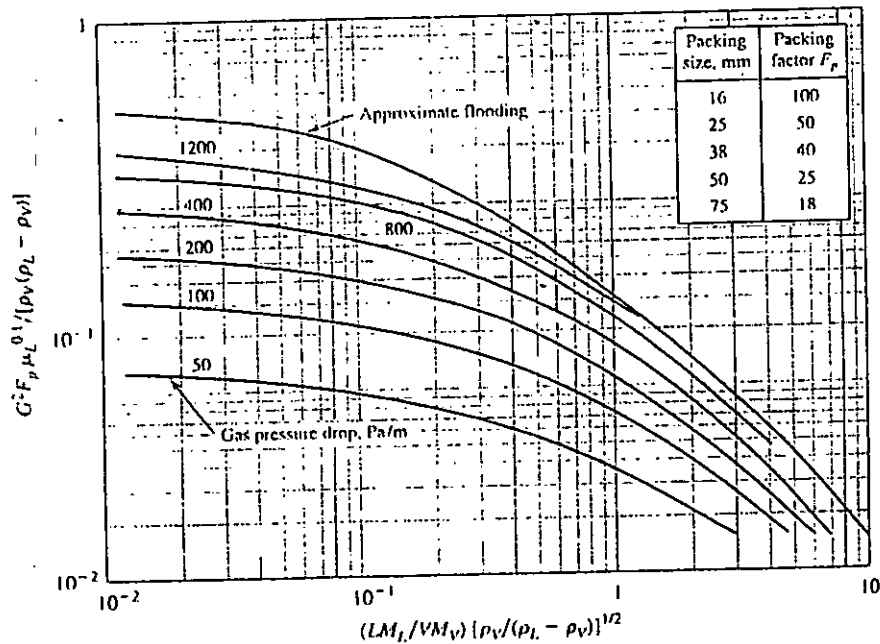
where

$G = VM_V/A_c$, superficial gas mass flux, kg/(s·m²)
F_p = packing factor, dimensionless; see insert
μ_L = liquid viscosity, Pa·s
ρ_V = gas density, kg/m³
ρ_L = liquid density, kg/m³

L = liquid flow rate, kg mol/s
A_c = column cross-sectional area, m²
M_V = molecular weight of vapor, kg/kg mol
M_L = molecular weight of liquid, kg/kg mol
V = gas flow rate, kg mol/s

~~Figure 15-7~~ Fig. 8.4 ₡

Flooding and pressure drop correlations for packed columns. (From R. E. Treybal, *Mass Transport Operations*, 3d ed., McGraw-Hill, New York, 1980, p. 195 with permission of McGraw-Hill.)

Figure for Question no 8 b)

Steam

16

Control Valve (CV2)

17

TC 101

TT 101

Volatile solvent in

3 → Valve (V1) → 4 → Heat Exchanger → 5 → Volatile solvent out

18

Condensate

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
L-4/T-1   B. Sc. Engineering Examinations 2022-2023
Sub : **CHE 453** (Transport Phenomena)
Full Marks : 210   Time : 3 Hours
The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION
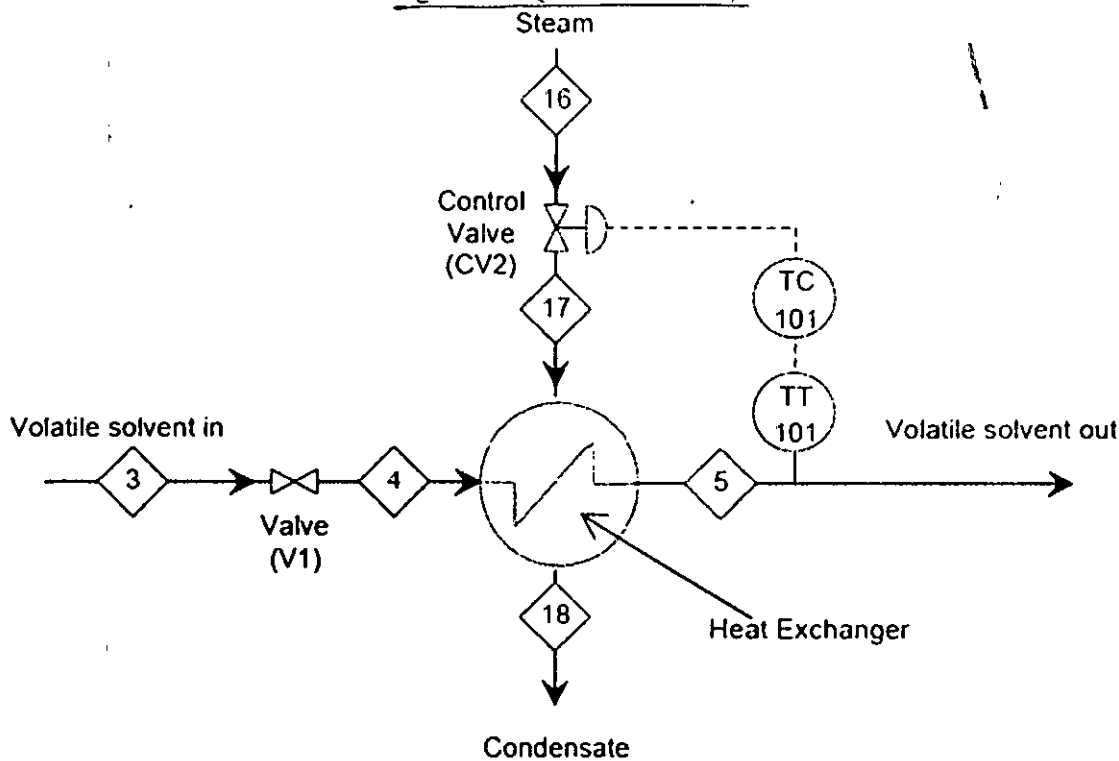
SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.
Symbols have their usual meanings.

1. Chlorine is being absorbed from a gas in a small experimental wetted-wall tower as shown in Figure for Q. 1. The dimensions of the column are given in the Figure for Q. 1. The absorbing fluid is water.

(a) Write down shell mass balance equation for the system. **(5)**

(b) Make necessary assumptions and derive the local mass flux at the gas-liquid interface. **(15)**

(c) Sketch velocity profile and concentration profile of the absorption of chlorine in water. **(5)**

(d) What is the absorption rate in g-mol/hr, if the liquid-phase diffusivity of the chlorine-water system is 1.26×10^{-5} cm^2/s, and if the saturation concentration of chlorine in water is 0.823 g chlorine per 100 g water. Water is moving with an average velocity of 17.7 cm/s. **(10)**

2. A droplet of pure A of initial radius R is suspended in a large body of motionless gas B. The concentration of A in the gas phase is x_{AR} at r = R and zero at an infinite distance from the droplet.

(a) Sketch the system and write down shell balance equation. Assume that R is constant. Show that at steady state: **(15)**

$$R^2 N_{Ar}\Big|_{r = R} = -\frac{cD_{AB}}{1 - x_A} r^2 \frac{dx_A}{dr} \dots \dots \dots \text{(Eq. 1)}$$

where $N_{Ar}\big|_{r = R}$ is the molar flux in the r direction at the droplet surface, c is the total molar concentration in the gas phase, and D_{AB} is the diffusivity in the gas phase. Assume constant temperature and pressure throughout.

(b) Show that integration of Eq. 1 from the droplet surface to infinity gives Eq. 2: **(5)**

$$RN_{Ar}\big|_R = -cD_{AB} \ln\left(1 - x_{AR}\right) \dots \dots \dots \text{(Eq. 2)}$$

(c) Now let the droplet R be a function of time, and treat the problem as a quasi-steady one. Then the rate of decrease of moles of A within the drop can be equated to the instantaneous rate of loss of mass across the liquid-gas interface. **(15)**

Hint: Moles of A can be expressed as volume of the droplet, $\frac{4}{3}\pi R^3$ time the molar density of pure liquid A, $c_A^{(L)}$.

Write down the general expression of the time for complete evaporation of the droplet.

CHE 453

3. (a) A simple rectangular fin is shown in Figure for Q. 3(a). State how you can approximate the true physical situation by a simplified model. Make shell energy balance and derive equation for the "effectiveness" of the fin surface. **(25)**

(b) In Figure for Q. 3(b), a thermocouple is shown in a cylindrical well inserted into a gas stream. Use the concept of cooling fan and estimate the true temperature of the gas stream if temperature indicated by thermocouple and wall temperature are 490 °F and 300 °F, respectively; heat-transfer coefficient and thermal conductivity of well wall are 100 Btu/hr/ft^2.°F and 50 Btu/hr/ft^2.°F, respectively; thickness of well wall and length of well are 0.07 in. and 0.3 ft, respectively. **(10)**

4. In the manufacture of glass-coated steel pipes, it is common practice first to heat the pipe to the melting range of glass and then to contact the hot pipe surface with glass granules (Figure for Q. 4). These granules melt and wet the pipe surface to form a tightly adhering nonporous coat. In one method of preheating the pipe, an electric current is passed along the pipe, with the result that the pipe is heated. For the purpose of this problem make the following assumptions:

- The electrical conductivity of the pipe k_e is constant over the temperature range of interest. The local rate of electrical heat production per unit volume S_e is then uniform throughout the pipe wall.
- The top and bottom of the pipe are capped in such a way that heat losses through them are negligible.
- The heat flux from the outer surface of the pipe to the surroundings is given by Newton's law of cooling: $q_r = h(T_1 - T_a)$. Here h is a suitable heat-transfer coefficient.

(a) Develop shell energy balance equation for this system. **(10)**

(b) State the boundary conditions. **(5)**

(c) Determine how much electrical power is needed to maintain the inner pipe surface at some desired value of the temperature, T_k, for known k, T_a, h, and pipe dimensions? **(20)**

SECTION – B

There are **FOUR** questions in this section. **Question No. 5** is compulsory.
Answer any **TWO** questions from the rest.

5. In a gas absorption experiment, a viscous fluid flows upward through a small circular tube and then downward in laminar flow on the outside. Set up a momentum balance over a shell of thickness Δr in the film, as shown in Fig. for Q. 5.

N.B: The "momentum in" and "momentum out" arrows are always taken in the positive coordinate direction, even though in this problem the momentum is flowing in the negative r direction.

CHE 453

Contd ... Q. No. 5

(a) Applying your knowledge in basic mass and momentum transfer, show that the velocity distribution in the falling film is (you may neglect end effects), **(15)**

$$v_2(r) = \frac{\rho g R^2}{4u}\left[1 - \left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right]$$

(b) Obtain an expression for the mass rate of flow in the film with the help of previously derived equation(s). **(10)**

(c) If the film thickness is very small, what happens to the mass rate of flow derived in Q. 5(b). Explain mathematically with proper explanation. **(10)**

6. A very viscous Newtonian fluid flows in the space between two concentric spheres, as shown in Fig for Q. 6. It is desired to find the rate of flow in the system as a function of the imposed pressure difference. Neglect end effects and postulate that v_θ depends only or r and θ with the other velocity components zero.

(a) Utilizing the knowledge of the equation of continuity, show that $v_\theta\left(r, \theta\right)\sin\theta = u(r)$, where u(r) is a function of r to be determined. **(10)**

(b) Write the θ components of the equation of motion for this system, assuming the flow to be slow enough that the [v. Δv] term is negligible. Then reduced it to the following equation: **(10)**

$$0 = -\frac{1}{r}\frac{\partial P}{\partial \theta} + u\left[\frac{1}{\sin\theta}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{du}{dr}\right)\right]$$

(c) In any case, can the mass flow rate of the above-mentioned scenario be reduced to the following: **(15)**

$$w = \frac{\pi\left(P_1 - P_2\right)R^3\left(1 - k\right)^3\rho}{12\,\mu\,\ln\cot\left(\varepsilon/2\right)}$$

If yes, then explain properly with adequate mathematical derivation.

7. A part of a lubrication system consists of two circular disks between which a lubricant flows radially, as illustrated in Fig. for Q. 7. The flow takes place because of a modified pressure difference $P_1 - P_2$ between the inner and outer radii r_1 and r_2, respectively.

(a) From the basic knowledge of *Transport Phenomena*, write the equation of continuity and motion for this flow system. For your ease, the assumption(s) that can be taken into consideration are steady-state, laminar and incompressible Newtonian flow. Moreover, you can consider only the region $r_1 \le r \le r_2$ and a flow that is radially directed. **(10)**

CHE 453

Contd ... Q. No. 7

(b) Show how the equation of continuity enables one to simplify the equation of motion to give, **(10)**

$$-\rho \frac{\phi^2}{r^3} = -\frac{dP}{dr} + \mu \frac{1}{r} \frac{d^2\phi}{dz^2}$$

in which $\Phi = r v_r$ is a function of z only. Moreover, explain why is Φ independent of r?

(c) With the creeping flow assumption, what will be the form of the previously developed equation? **(5)**

(d) Can the mass flow rate be written as, **(10)**

$$w = \frac{4\pi (P_1 - P_2) b^3 \rho}{3\mu \ln(r_2/r_1)}$$

If not, what will be its form? Derive with proper steps and explain adequately.

8. (a) A catalyst pellet, shown in Figure for Q. 8, has a radius R and a thermal conductivity k (which may be assumed constant). Because of the chemical reaction occurring within the porous pellet, heat is generated as a rate S_c cal/cm^3.s. Heat is lost at the outer surface of the pellet to a gas stream at constant temperature T_g by convective heat transfer with heat-transfer coefficient h. Find the steady-state temperature profile, assuming that Sc is constant throughout. **(25)**

    (i) Set up the differential equation by making a shell energy balance.

    (ii) Integrate the differential equation to get the temperature profile. Sketch the function $T(r)$.

    (iii) What is the limiting form of $T(r)$ when $h \to \infty$.

    (iv) What is the maximum temperature in the system?

(b) Determine the ratio $\mu^{(t)}/\mu$ at $y = R/2$ for water flowing at a steady rate in a long, smooth, round tube under the following conditions, **(10)**

$$R = 7.62 \text{ cm}$$
$$\tau_0 = 0.163 \text{ Pa}$$
$$\rho = 1000 \text{ kg/m}^3$$
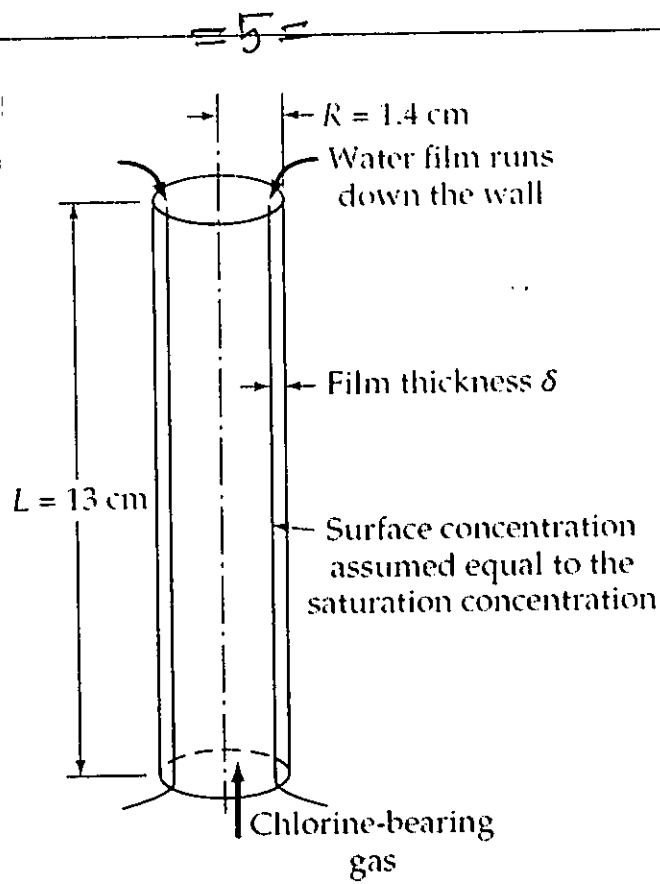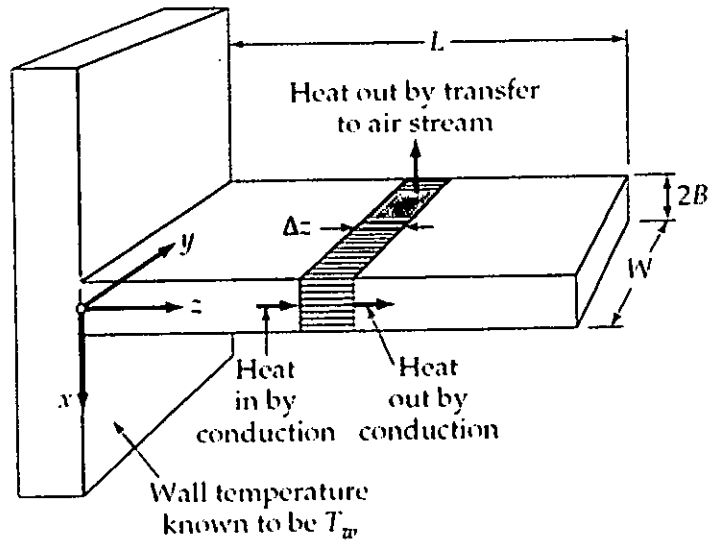$$v = 1.02 \times 10^{-6} \text{ m}^2/\text{s}$$

Fig. for Q.8

R = 1.4 cm

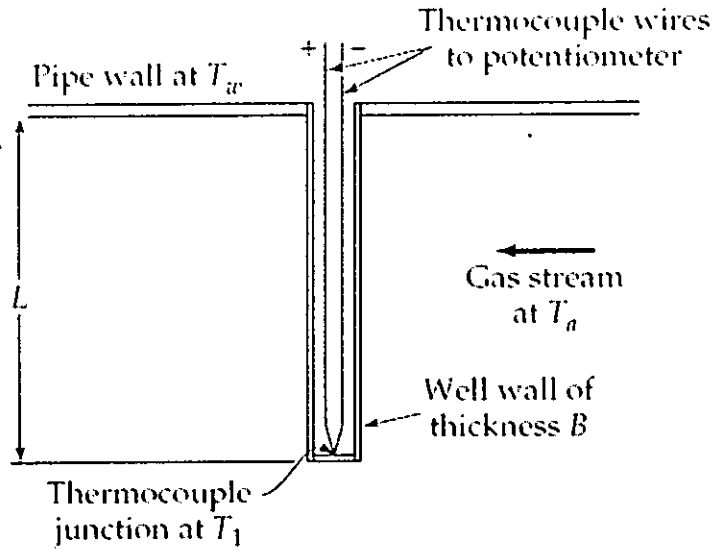Water film runs down the wall

Film thickness δ

L = 13 cm

Surface concentration assumed equal to the saturation concentration

Chlorine-bearing gas

Fig. for Q.1

L

Heat out by transfer to air stream

2B

Δz

W

y

z

Heat in by conduction

Heat out by conduction

x

Wall temperature known to be T_w

Fig. for Q.3(a)

Pipe wall at T_w

Thermocouple wires to potentiometer

+ −

L

Gas stream at T_a

Well wall of thickness B

Thermocouple junction at T_1

Fig. for Q.3(b)

Power supply

T_κ

T_1

Ambient air temperature T_a

L

κR

R

Pipe wall

Fig. for Q.4

Velocity distribution $v_z(r)$ and z momentum balance for the flow of a falling film on the outside of a circular tube.

Fig. for Q. 5
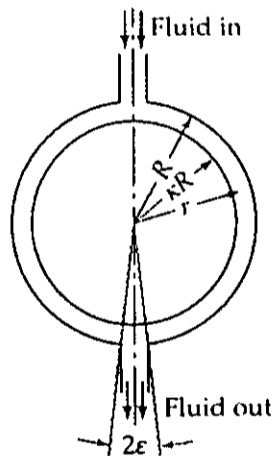
Creeping flow in the region between two stationary concentric spheres.
Fig. for Q. 6
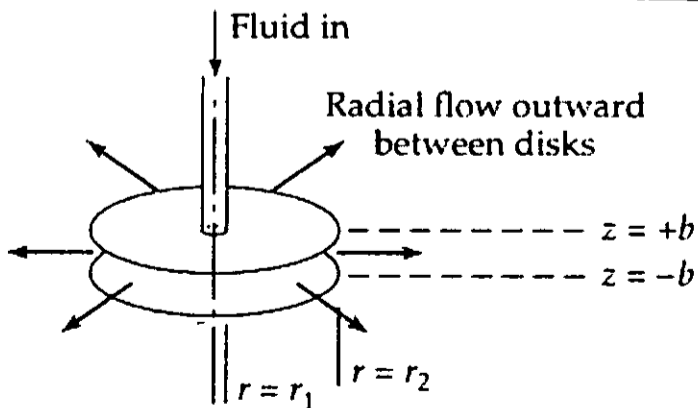
Outward radial flow in the space between two parallel, circular disks
Fig. for Q. 7

Appendix $\mathbb{B}$

The Fluxes and the Equations of Change

§B.1   NEWTON'S LAW OF VISCOSITY

$$[\tau = -\mu(\nabla v + (\nabla v)^\dagger) + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot v)\delta]$$

Cartesian coordinates (x,y,z):

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot v) \tag{B.1-1}^a$$

$$\tau_{yy} = -\mu\left[2\frac{\partial v_y}{\partial y}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot v) \tag{B.1-2}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot v) \tag{B.1-3}^a$$

$$\tau_{xy} = \tau_{yx} = -\mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] \tag{B.1-4}$$

$$\tau_{yz} = \tau_{zy} = -\mu\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right] \tag{B.1-5}$$

$$\tau_{zx} = \tau_{xz} = -\mu\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right] \tag{B.1-6}$$

in which

$$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \tag{B.1-7}$$

^{a}When the fluid is assumed to have constant density, the term containing $(\nabla \cdot v)$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.

§B.1   NEWTON'S LAW OF VISCOSITY (continued)

Cylindrical coordinates (r,θ,z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-8}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-9}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-10}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \tag{B.1-11}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right] \tag{B.1-12}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right] \tag{B.1-13}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \tag{B.1-14}$$

[a]When the fluid is assumed to have constant density, the term containing $(\nabla \cdot \mathbf{v})$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.

Spherical coordinates (r,θ,ϕ):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-15}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-16}^a$$

$$\tau_{\phi\phi} = -\mu\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \tag{B.1-17}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \tag{B.1-18}$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] \tag{B.1-19}$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] \tag{B.1-20}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \tag{B.1-21}$$

[a]When the fluid is assumed to have constant density, the term containing $(\nabla \cdot \mathbf{v})$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.

$-10=$

-10=

§B.2   FOURIER'S LAW OF HEAT CONDUCTION[a]

$$[q = -k\nabla T]$$

Cartesian coordinates (x,y,z):

$$q_x = -k\frac{\partial T}{\partial x} \tag{B.2-1}$$

$$q_y = -k\frac{\partial T}{\partial y} \tag{B.2-2}$$

$$q_z = -k\frac{\partial T}{\partial z} \tag{B.2-3}$$

Cylindrical coordinates (r,θ,z):

$$q_r = -k\frac{\partial T}{\partial r} \tag{B.2-4}$$

$$q_\theta = -k\frac{1}{r}\frac{\partial T}{\partial \theta} \tag{B.2-5}$$

$$q_z = -k\frac{\partial T}{\partial z} \tag{B.2-6}$$

Spherical coordinates (r,θ,ϕ):

$$q_r = -k\frac{\partial T}{\partial r} \tag{B.2-7}$$

$$q_\theta = -k\frac{1}{r}\frac{\partial T}{\partial \theta} \tag{B.2-8}$$

$$q_\phi = -k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi} \tag{B.2-9}$$

[a]For mixtures, the term $\sum_a(\overline{H}_a/M_a)j_a$ must be added to $-k\nabla T$ (see Eq. 19.3-3).

§B.3   FICK'S (FIRST) LAW OF BINARY DIFFUSION[a]

$$[j_A = -\rho \mathscr{D}_{AB} \nabla \omega_A]$$

Cartesian coordinates (x,y,z):

$$j_{Ax} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial x} \tag{B.3-1}$$

$$j_{Ay} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial y} \tag{B.3-2}$$

$$j_{Az} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial z} \tag{B.3-3}$$

Cylindrical coordinates (r,θ,z):

$$j_{Ar} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial r} \tag{B.3-4}$$

$$j_{A\theta} = -\rho \mathscr{D}_{AB} \frac{1}{r} \frac{\partial \omega_A}{\partial \theta} \tag{B.3-5}$$

$$j_{Az} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial z} \tag{B.3-6}$$

Spherical coordinates (r,θ,φ):

$$j_{Ar} = -\rho \mathscr{D}_{AB} \frac{\partial \omega_A}{\partial r} \tag{B.3-7}$$

$$j_{A\theta} = -\rho \mathscr{D}_{AB} \frac{1}{r} \frac{\partial \omega_A}{\partial \theta} \tag{B.3-8}$$

$$j_{A\phi} = -\rho \mathscr{D}_{AB} \frac{1}{r \sin \theta} \frac{\partial \omega_A}{\partial \phi} \tag{B.3-9}$$

[a]To get the molar fluxes with respect to the molar average velocity, replace j_A, ρ, and ω_A by J_A^*, c, and x_A.

§B.4   THE EQUATION OF CONTINUITY[a]

$$[\partial \rho / \partial t + (\nabla \cdot \rho v) = 0] \tag{B.4-1}$$

Cartesian coordinates (x,y,z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{B.4-2}$$

Cylindrical coordinates (r,θ,z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{B.4-3}$$

Spherical coordinates (r,θ,φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \tag{B.4-4}$$

[a]When the fluid is assumed to have constant mass density ρ, the equation simplifies to $(\nabla \cdot v) = 0$.

§B.5   THE EQUATION OF MOTION IN TERMS OF τ

$$[\rho D\mathbf{v}/Dt = -\nabla p - [\nabla \cdot \tau] + \rho \mathbf{g}]$$

Cartesian coordinates (x,y,z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x \qquad (B.5\text{-}1)$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y \qquad (B.5\text{-}2)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z \qquad (B.5\text{-}3)$$

[a] These equations have been written without making the assumption that τ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, τ_{xy} and τ_{yx} may be interchanged.

Cylindrical coordinates (r,θ,z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r \qquad (B.5\text{-}4)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta \qquad (B.5\text{-}5)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z \qquad (B.5\text{-}6)$$

[b] These equations have been written without making the assumption that τ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, $\tau_{r\theta} - \tau_{\theta r} = 0$.

Spherical coordinates (r,θ,ϕ):[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r \qquad (B.5\text{-}7)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta \qquad (B.5\text{-}8)$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi \qquad (B.5\text{-}9)$$

[c] These equations have been written without making the assumption that τ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, $\tau_{r\theta} - \tau_{\theta r} = 0$.

§B.6   THE EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho Dv/Dt = -\nabla p + \mu\nabla^2 v + \rho g]$$

Cartesian coordinates (x,y,z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \qquad (B.6\text{-}1)$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \qquad (B.6\text{-}2)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (B.6\text{-}3)$$

Cylindrical coordinates (r,θ,z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] + \rho g_r \qquad (B.6\text{-}4)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] + \rho g_\theta \qquad (B.6\text{-}5)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (B.6\text{-}6)$$

Spherical coordinates (r,θ,φ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$

$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right] + \rho g_r \qquad (B.6\text{-}7)^a$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta}$$

$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right] + \rho g_\theta \qquad (B.6\text{-}8)$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi}$$

$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \rho g_\phi \qquad (B.6\text{-}9)$$

---

[a] The quantity in the brackets in Eq. B.6-7 is *not* what one would expect from Eq. (M) for $[\nabla \cdot \nabla v]$ in Table A.7-3, because we have added to Eq. (M) the expression for $(2/r)(\nabla \cdot v)$, which is zero for fluids with constant ρ. This gives a much simpler equation.

§B.7 THE DISSIPATION FUNCTION Φ_v FOR NEWTONIAN FLUIDS (SEE EQ. 3.3-3)

Cartesian coordinates (x,y,z):

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2$$

(B.7-1)

Cylindrical coordinates (r,θ,z):

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]^2 + \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2$$

$$-\frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right]^2$$

(B.7-2)

Spherical coordinates (r,θ,φ):

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right)^2\right]$$

$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2$$

$$-\frac{2}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2$$

(B.7-3)

§B.8 THE EQUATION OF ENERGY IN TERMS OF q

$$[\rho\hat{C}_p DT/Dt = -(\nabla\cdot\mathbf{q}) - (\partial\ln\rho/\partial\ln T)_p Dp/Dt - (\tau:\nabla v)]$$

Cartesian coordinates (x,y,z):

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] - \left(\frac{\partial\ln\rho}{\partial\ln T}\right)_p\frac{Dp}{Dt} - (\tau:\nabla v)$$

(B.8-1)[a]

Cylindrical coordinates (r,θ,z):

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}(rq_r) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z}\right] - \left(\frac{\partial\ln\rho}{\partial\ln T}\right)_p\frac{Dp}{Dt} - (\tau:\nabla v)$$

(B.8-2)[a]

Spherical coordinates (r,θ,φ):

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(q_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial q_\phi}{\partial \phi}\right]$$

$$-\left(\frac{\partial\ln\rho}{\partial\ln T}\right)_p\frac{Dp}{Dt} - (\tau:\nabla v)$$

(B.8-3)[a]

[a]The viscous dissipation term, $-(\tau:\nabla v)$, is given in Appendix A, Tables A.7-1, A.7-2, and A.7-3. This term may usually be neglected, except for systems with very large velocity gradients. The term containing $(\partial\ln\rho/\partial\ln T)_p$ is zero for fluids with constant ρ.

§B.9   THE EQUATION OF ENERGY FOR PURE NEWTONIAN FLUIDS WITH CONSTANT^a ρ AND k

$$[\rho \hat{C}_p DT/Dt = k\nabla^2 T + \mu\Phi_v]$$

Cartesian coordinates (x,y,z):

$$\rho\hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu\Phi_v \qquad (B.9\text{-}1)^b$$

Cylindrical coordinates (r,θ,z):

$$\rho\hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r\frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \mu\Phi_v \qquad (B.9\text{-}2)^b$$

Spherical coordinates (r,θ,ϕ):

$$\rho\hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = k\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2\frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right] + \mu\Phi_v \quad (B.9\text{-}3)^b$$

^{a}This form of the energy equation is also valid under the less stringent assumptions k = constant and $(\partial \ln \rho/\partial \ln T)_p Dp/Dt = 0$. The assumption ρ = constant is given in the table heading because it is the assumption more often made.

^{b}The function Φ_v is given in §B.7. The term $\mu\Phi_v$ is usually negligible, except in systems with large velocity gradients.

§B.10   THE EQUATION OF CONTINUITY FOR SPECIES a IN TERMS^a OF j_a

$$[\rho D\omega_a/Dt = -(\nabla \cdot j_a) + r_a]$$

Cartesian coordinates (x,y,z):

$$\rho\left(\frac{\partial \omega_a}{\partial t} + v_x \frac{\partial \omega_a}{\partial x} + v_y \frac{\partial \omega_a}{\partial y} + v_z \frac{\partial \omega_a}{\partial z} \right) = -\left[\frac{\partial j_{ax}}{\partial x} + \frac{\partial j_{ay}}{\partial y} + \frac{\partial j_{az}}{\partial z} \right] + r_a \qquad (B.10\text{-}1)$$

Cylindrical coordinates (r,θ,z):

$$\rho\left(\frac{\partial \omega_a}{\partial t} + v_r \frac{\partial \omega_a}{\partial r} + \frac{v_\theta}{r} \frac{\partial \omega_a}{\partial \theta} + v_z \frac{\partial \omega_a}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} \left(rj_{ar} \right) + \frac{1}{r} \frac{\partial j_{a\theta}}{\partial \theta} + \frac{\partial j_{az}}{\partial z} \right] + r_a \qquad (B.10\text{-}2)$$

Spherical coordinates (r,θ,ϕ):

$$\rho\left(\frac{\partial \omega_a}{\partial t} + v_r \frac{\partial \omega_a}{\partial r} + \frac{v_\theta}{r} \frac{\partial \omega_a}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial \omega_a}{\partial \phi} \right) = -\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 j_{ar} \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(j_{a\theta} \sin\theta) + \frac{1}{r \sin\theta} \frac{\partial j_{a\phi}}{\partial \phi} \right] + r_a \qquad (B.10\text{-}3)$$

^{a}To obtain the corresponding equations in terms of J_a^* make the following replacements:

Replace	ρ	ω_a	j_a	v	r_a
by	c	x_a	J_a^*	v^*	$R_a - x_a \sum_{\beta=1}^{N} R_\beta$

§B.11 THE EQUATION OF CONTINUITY FOR SPECIES A IN TERMS OF ω_A FOR CONSTANT[a] $\rho \mathcal{D}_{AB}$

$$[\rho D\omega_A/Dt = \rho \mathcal{D}_{AB}\nabla^2\omega_A + r_A]$$

Cartesian coordinates (x,y,z):

$$\rho\left(\frac{\partial \omega_A}{\partial t} + v_x\frac{\partial \omega_A}{\partial x} + v_y\frac{\partial \omega_A}{\partial y} + v_z\frac{\partial \omega_A}{\partial z}\right) = \rho\mathcal{D}_{AB}\left[\frac{\partial^2\omega_A}{\partial x^2} + \frac{\partial^2\omega_A}{\partial y^2} + \frac{\partial^2\omega_A}{\partial z^2}\right] + r_A \qquad (B.11\text{-}1)$$

Cylindrical coordinates (r,θ,z):

$$\rho\left(\frac{\partial \omega_A}{\partial t} + v_r\frac{\partial \omega_A}{\partial r} + \frac{v_\theta}{r}\frac{\partial \omega_A}{\partial \theta} + v_z\frac{\partial \omega_A}{\partial z}\right) = \rho\mathcal{D}_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\omega_A}{\partial \theta^2} + \frac{\partial^2\omega_A}{\partial z^2}\right] + r_A \qquad (B.11\text{-}2)$$

Spherical coordinates (r,θ,ϕ):

$$\rho\left(\frac{\partial \omega_A}{\partial t} + v_r\frac{\partial \omega_A}{\partial r} + \frac{v_\theta}{r}\frac{\partial \omega_A}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial \omega_A}{\partial \phi}\right)$$
$$= \rho\mathcal{D}_{AB}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \omega_A}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \omega_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\omega_A}{\partial \phi^2}\right] + r_A \qquad (B.11\text{-}3)$$

[a]To obtain the corresponding equations in terms of x_A, make the following replacements:

Replace	ρ	ω_A	v	r_A
by	c	x_A	v^*	$x_B R_A - x_A R_B$