

**SECTION – A**

There are **FOUR** questions in this section. Answer Question 1 and any **TWO** from the rest.

**Question 1 is COMPULSORY.**

1. (a) A ray ( $R_0$ ) originates from a camera at (-2, 3,-1) and is directed toward (4,-5, 4). A sphere with radius 5 is centered at C(1, 8, 0). (10)
  - i. Calculate the intersection point between the ray  $R_0$ , and the sphere.
  - ii. Determine the normal vector at the intersection point facing the camera.
- (b) i. Differentiate between Phong Shading and Phong Lighting Model. (5+5)
  - ii. Compare Phong and Gourad Shading.
- (c) Demonstrate the clipping of PQ and XY lines against the rectangular clipping window ABCD from Figure 1(c)(i) using the Cyrus-Beck parametric line clipping algorithm. The table to find the intersection points is shown in Figure 1(c)(ii). (15)

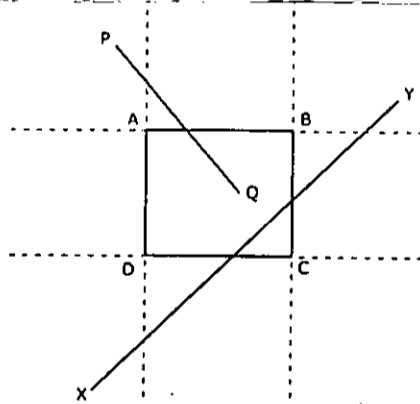


Figure 1(c)(i): Clipping window and lines to clip for Question 1(c)

TABLE 3.1 CALCULATIONS FOR PARAMETRIC LINE CLIPPING ALGORITHM\*

Clip edge <sub>i</sub>	Normal $N_i$	$P_{E_i}$	$P_0 - P_{E_i}$	$t = \frac{N_i \cdot (P_0 - P_{E_i})}{-N_i \cdot D}$
left: $x = x_{min}$	(-1, 0)	( $x_{min}, y$ )	( $x_0 - x_{min}, y_0 - y$ )	$\frac{-(x_0 - x_{min})}{(x_1 - x_0)}$
right: $x = x_{max}$	(1, 0)	( $x_{max}, y$ )	( $x_0 - x_{max}, y_0 - y$ )	$\frac{(x_0 - x_{max})}{-(x_1 - x_0)}$
bottom: $y = y_{min}$	(0, -1)	( $x, y_{min}$ )	( $x_0 - x, y_0 - y_{min}$ )	$\frac{-(y_0 - y_{min})}{(y_1 - y_0)}$
top: $y = y_{max}$	(0, 1)	( $x, y_{max}$ )	( $x_0 - x, y_0 - y_{max}$ )	$\frac{(y_0 - y_{max})}{-(y_1 - y_0)}$

\*The exact coordinates of the point  $P_{E_i}$  on each edge are irrelevant to the computation, so they have been denoted by variables  $x$  and  $y$ . For a point on the right edge,  $x = x_{max}$  as indicated in the first row, third entry.

Figure 1(c)(ii): Intersection points for Question 1(c)

**CSE/409**

2. (a) The Gupta-Sproull algorithm for anti-aliased scan conversion of lines uses the weighting function  $Filter(D, t)$ , where  $D$  is the perpendicular distance between the chosen pixel center and the line center, and  $t$  is the thickness of the line. If the pixel  $P(X_p, Y_p)$  is currently being considered, and the next chosen pixel is  $N$ , show the detailed calculation for finding  $D$  and  $D_{left}$  for  $N$ . See Figure 2(a) for clarification. Here the slope of the line is greater than 1. (17)

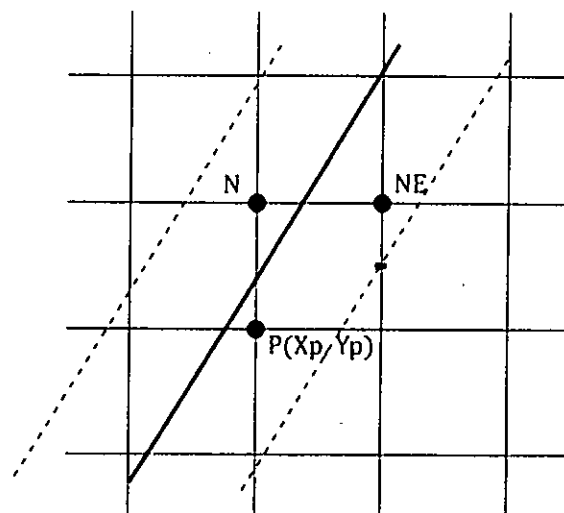


Figure 2(a): Figure for Question 2(a)

- (b) Let  $abcdefghijkl$  and  $WXYZ$  be the polygon to be clipped and viewing window, respectively, as shown in Figure 2 (b). Using the Sutherland-Hodgman polygon clipping algorithm, show the steps of clipping the polygon with respect to the left clip edge  $XY$ . (10)

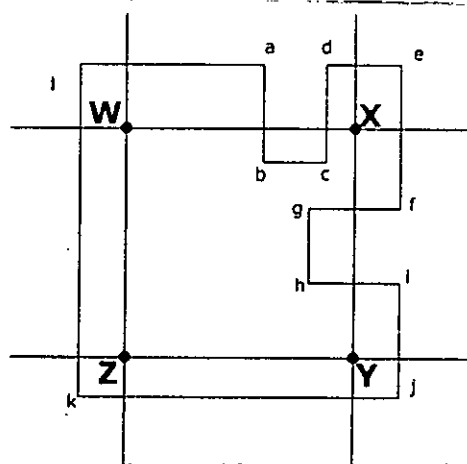
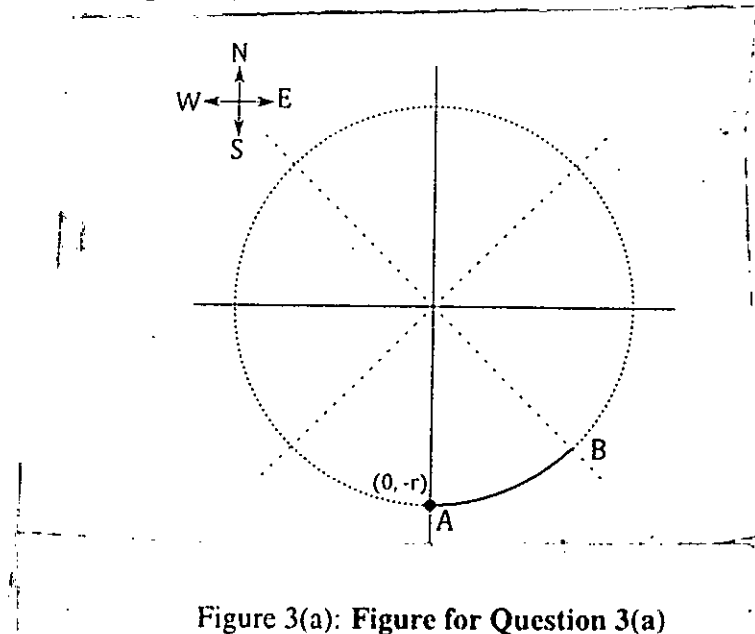


Figure 2(b): Figure for Question 2(b)

- (c) Do you agree with the statement - "The point-to-line error is always  $\leq \frac{1}{2}$  for the midpoint-line scan conversion algorithm"? Justify your answer. (8)

**CSE/409**

3. (a) You have to develop a *Mid-point circle Algorithm* for the circle  $x^2 + y^2 = r^2$ . The algorithm will calculate the pixels to be drawn for the part *AB* only, starting from point  $(0, -r)$ , as shown in Figure 3(a). It will determine the other seven pixels using eight-way symmetry.



(17)

Figure 3(a): Figure for Question 3(a)

To do so, you have to;

- i. Determine which two pixels are of interest in each iteration.
- ii. Propose a decision variable.
- iii. Identify the two cases using the decision variable. In each case, you have to identify the pixel to be populated.
- iv. Derive the value of the decision variable at the start and for the next iteration in terms of the value for the current iteration and other constants.
- v. Reduce the computational load by introducing forward differencing.

- (b) Explain the procedure of computing shadow using shadow buffer. Discuss the limitations of using shadow buffer.

(10)

- (c) Consider the Fractal tree defined by the following rules:

**String Production Rule:**

$0 \rightarrow 1(0)1(0)0$

$1 \rightarrow 11$

**Atom (Generation 0): 0**

**Meaning of the symbols:**

**0** = Draw a line segment ending in a leaf

**1** = Draw a line segment

( = Push current state (position and angle), then turn left  $45^\circ$

) = Pop the last saved state (position and angle), then turn right  $45^\circ$

Draw the second generation of the fractal tree.

(8)

**CSE/409**

4. (a) The Sierpinski triangle is a triangle made up of smaller copies of itself. To get the next generation(s) of the Sierpinski triangle, we need to start with a filled-in triangle, connect the midpoints of each of the sides, remove the middle triangle, and iterate over the remaining three filled-in triangles. Four generations of the Sierpinski triangle are shown in Figure 4(a). Let us assume, we start with a triangle with side length 1. (20)

- i. Calculate the perimeter of the fractal (boundary of the space colored by black) at  $n^{th}$  generation.
- ii. Calculate the area of the Sierpinski triangle (the space colored by black) at  $n^{th}$  generation.
- iii. Calculate the dimension of the fractal.

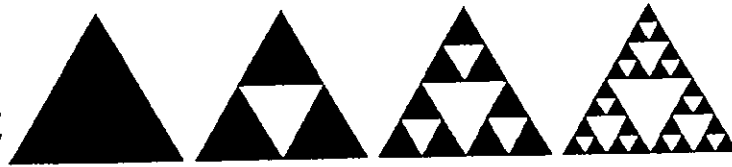


Figure 4(a): First four generations of a fractal

(b) See the top view of a scene in Figure 4(b). The scene has six polygons, *A, B, C, D, E, F*. It also shows the surface normal of each polygon. (15)

- i. Construct a BSP tree by choosing the polygons in alphabetical order.
- ii. For the viewing directions shown in the figure, write down the orders of displaying the polygons.

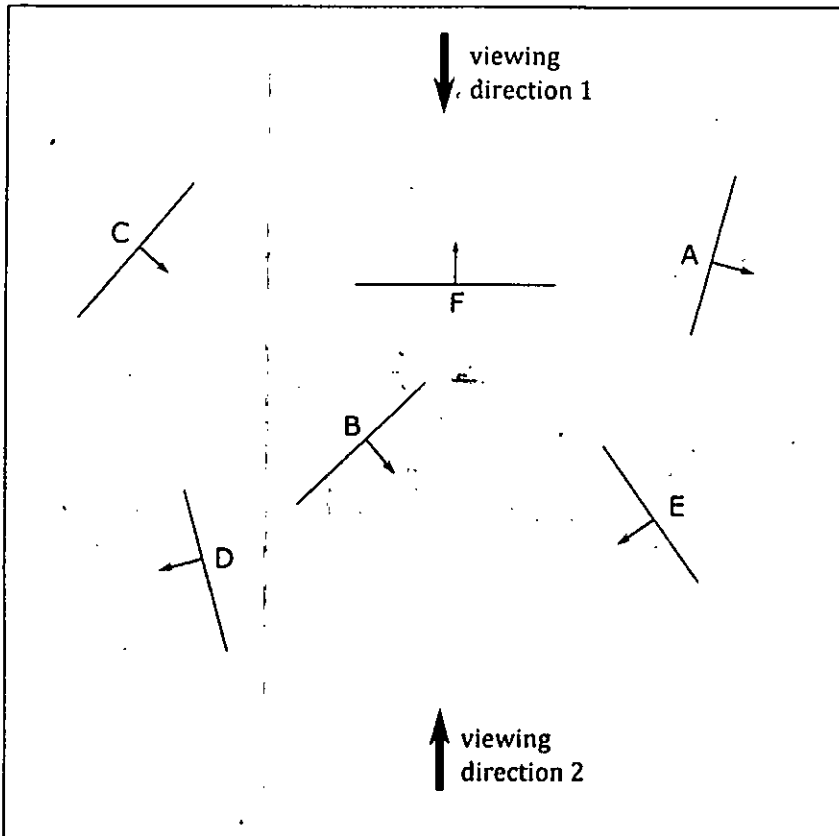


Figure 4(b): Figure for Question 4(b)

**SECTION – B**

There are **FOUR** questions in this section. Answer Question **5** and any **TWO** of the remaining question. **Question 5 is COMPULSORY.**

**5. ANSWERING THIS QUESTION IS MANDATORY**

(a) The formula of reflection is given as,  $\vec{r} = \vec{a} - 2(\vec{a}\hat{n})\hat{n}$ , where  $\vec{a}$  is the incident ray,  $\hat{n}$  is the **unit** normal to the reflection surface and  $\vec{r}$  is the reflected ray. **(5+5)**

i. From the vector formula, **formulate** the matrix equation that transforms  $\vec{a}$  into  $\vec{r}$  given  $\hat{n}$ . Assume 3D space.

ii. Using the matrix equation, determine the direction of the reflected ray when an incident ray along the vector (1,1,1) gets reflected on the plane  $3x + 2y + 6z = 30$

(b) **Explain** why do we prefer cubic polynomials for parametric curve representation? **(5)**

(c) You want to transform the points **(8)**

A (0,0), B(a,0), C(a,a), and D(0,a) to

$\bar{A}$  (a,0),  $\bar{B}$  (0,a),  $\bar{C}$  (-a,0), and  $\bar{D}$  (0,-a)

respectively. You can perform only (not necessarily in the given order):

- One scaling
- One translation
- One rotation

**Design** the corresponding composite transformation matrix. You don't need to multiply the three matrices. Use homogeneous coordinates.

(d) In an oblique parallel projection setting, the projection plane is  $z = -1$  and a point **(6+6)**

$P(\frac{\sqrt{3}}{2}, \frac{3}{2}, 0)$  projects to the point  $\bar{P}(\sqrt{3}, 3, -1)$

**Calculate** the values of  $\lambda$  and  $\phi$ , where  $\tan \alpha = \frac{1}{\lambda}$

Also, **develop** the projection matrix (M) and show that  $\bar{P} = MP$ .

6. (a) Given two lines,  $L_1 = (0,0,0) + t(1,1,1)$  and  $L_2 = (0,0,2) + s(-1,1,1)$ . Determine **the pair of points** ( $P_1, P_2$ ), where  $P_1$  is on  $L_1, P_2$  is on  $L_2$ , and the distance between  $P_1$  and  $P_2$  is **minimum** over all possible pair of points on  $L_1$  and  $L_2$ . **(8)**

(b) i. Prove the Rodrigues formula for rotation. **(5+5+10)**

$$R(\vec{x}) = \cos \theta \vec{x} + (1 - \cos \theta)(\hat{a} \cdot \vec{x})\hat{a} + \sin \theta(\hat{a} \times \vec{x})$$

where  $\theta$  is the angle of rotation and  $\hat{a}$  is the rotation axis

ii. Develop the corresponding matrix form.

iii. Use the following rotation matrix to find out  $\theta$  and  $\hat{a}$

$$R = \begin{bmatrix} \frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{bmatrix}$$

**CSE/409**

(c) In 2D space, you have a line with equation  $L \equiv y = -x + 1$ . Generate the matrix transformation ( $M_l$ ) to reflect any point about the line  $L$ . You don't need to multiply the matrices. Use homogeneous coordinates. (7)

7. (a) A plane is represented in point-normal representation  $\{(1,1,1), (\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}})\}$ . Determine its three point representation and parametric representation **directly from point-normal representation**. (5+5)

(b) i. Derive the **general** perspective projection onto a plane with reference point

$R_0 = (x_0, y_0, z_0)$ , normal vector  $N = n_x \hat{i} + n_y \hat{j} + n_z \hat{k}$  and using  $C(a, b, c)$  as the center of projection. (12+5)

ii. Now using the derived matrix, find where the point (11,22,18) will be projected. Here, the projection plane is  $x + y = 13$  and the center of projection is at  $C(1,2,3)$ .

(c) A camera is located at (1,2,3). Its viewing direction is given by the vector  $\hat{i} + \hat{j}$  and up direction is  $-\hat{i} + \hat{j}$ . (8)

Derive the view transformation matrix ( $V$ ) so that it looks towards the positive Y axis and its up direction remains along the **negative Z** axis.

8. (a) Find the intersection of  $x + 2y - z = 5$  and  $x - 4y + z = 3$ . (7)

(b) A piece-wise function is defined as (8)

$$f(u) = \begin{cases} (\cos(2\pi u), \sin(2\pi u)) & \text{if } 0 \leq u \leq 0.5 \\ (-1, .5 - u) & \text{if } 0.5 < u \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Comment on  $G^{(0)}$ ,  $C^{(0)}$ ,  $G^{(1)}$ ,  $C^{(1)}$  continuity when  $u = 0.5$ .

(c) Suppose you are given a parametric curve  $Q(t)$  of third degree in 2D space, where  $t \in [0, 1]$ . Your friend has supplied you  $Q(0)$ ,  $Q(0.5)$ ,  $Q'(0.5)$  and  $Q(1)$ . Generate the **basis matrix M** for the given curve. (10)

(d) The parametric equation of  $Q(t) = TM_b G$  of the cubic Hermite curve is; (5+5)

$$Q(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q(0) \\ Q(1) \\ Q'(0) \\ Q'(1) \end{bmatrix}$$

i. Show that the cubic Bézier curve is a variation of the cubic Hermite curve.

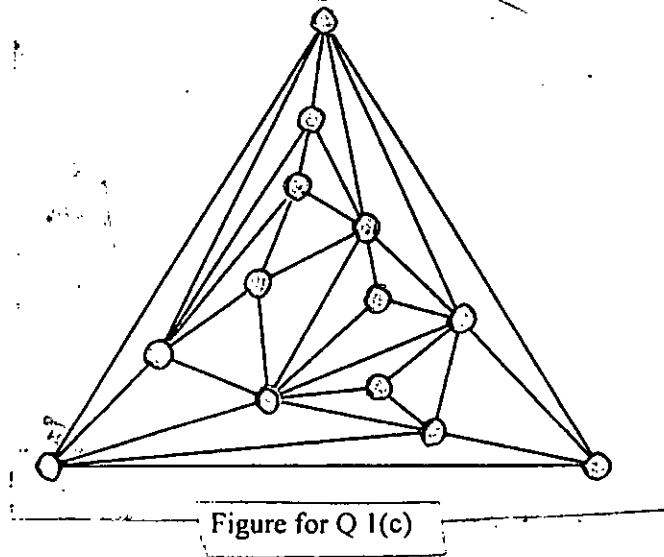
ii. Derive the basis matrix of the Bézier curve  $M_b$  using the given basis of Hermite curve  $M_h$ .

**SECTION – A**

There are **FOUR** questions in this section. Answering **Question 1** is mandatory.

Answer any two from the rest.

1. (a) Define  $k$ -tree. Provide examples of a 4-tree and a partial 4-tree, both having seven vertices. Define simplicial vertex. Provide an illustrative example of simplicial vertices contributing to a perfect elimination scheme in chordal graphs. (13)
- (b) Prove that a graph  $G$  is chordal if and only if every minimal separator of  $G$  is a clique. (10)
- (c) Determine canonical ordering of the vertices of the graph in the figure for 1(c). (8)



- (d) Which class of graphs is useful for solving sequence dating or seriation problems in Archeology? Seven types of jewellery were found in five ancient tombs as shown in the table for Q 1(d). Draw the graph with vertex set  $\{a, b, c, d, e, f, g\}$  in which two vertices are joined when the corresponding types of jewellery are found in the same tomb. Determine a possible chronological ordering of the seven types of jewellery. (8)

		Jewellery						
		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
Tombs	1		✓		✓			
	2	✓						✓
	3		✓			✓		
	4	✓		✓			✓	
	5		✓	✓		✓		

Table for Q1(d)

**CSE 421**

2. (a) Draw the digraph whose incidence matrix is given below. Determine the Laplacian matrix of the underlying graph. (10)

1	-1	1	-1	0	0	0	0
-1	1	0	0	-1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	-1	-1	-1
0	0	-1	0	1	0	1	1

- (b) Prove that every simple planar graph is 5-colorable. (10)
- (c) Define chromatic polynomial  $P_G(k)$  of a graph  $G$ . Explain how  $P_G(k)$  can be computed recursively. Determine the chromatic polynomial for a tree of  $n$  vertices. (13)
3. (a) Show that Petersen graph is nonplanar by using Kuratowski's Theorem. In addition, show that Petersen graph is nonplanar by using Euler's polyhedral formula. (10)
- (b) Demonstrate that there are only five platonic polyhedra as an application of Euler's polyhedral formula. Which polyhedral graph was utilized in Hamilton's Icosian puzzle game? (13)
- (c) Prove that the embedding of a 2-connected planar graph  $G$  is unique if and only if  $G$  is a subdivision of a 3-connected graph. (10)
4. (a) Let  $G$  be a connected simple graph. Explain how the domination number is related to the diameter of  $G$ . Recursively compute the domination number of the graph  $G$ , shown in figure for Q 4(a). (17)

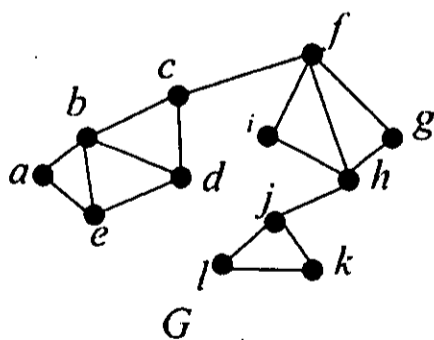


Figure for Q 4(a)

- (b) Prove that an independent set of a graph has a complement relation with a vertex cover of the graph. In addition, prove that if a tree has a 1-factor, then the 1-factor is unique. (16)



**SECTION - B**

There are **FOUR** questions in this section. Answering **Question 5** is mandatory.

Answer any two from the rest.

5. (a) Consider the following matrix of courses (*Table 5a*). You need to schedule the final exam for these courses. The entries marked 'x' on the matrix represent courses that have common students, which means they cannot be scheduled together. (10)

Design a graph theoretic model to find the minimum number of time slots required to schedule all the exams. Can you additionally minimize the number of rooms required to hold the exams?

	101	103	305	311	405	421
101		x				x
103	x		x			
305		x		x	x	
311			x			x
405			x			x
421	x			x	x	

*Table 5a*

- (b) Define "complement" of a graph? Show that for any graph of six vertices, either the graph or its complement contains a triangle. (3+7=10)

- (c) Define the degree sequence of a graph using illustrative examples. When do we call a degree sequence a graphic sequence? (4+3+8=15)

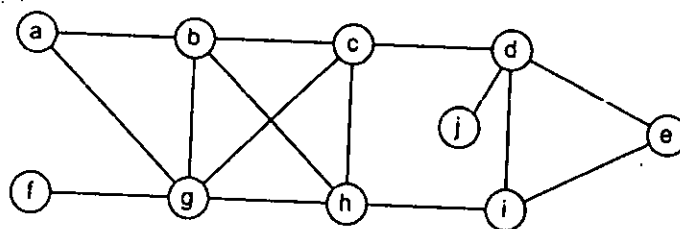
Showing every step, determine whether 5, 4, 4, 3, 2, 2 is a graphic sequence or not.

6. (a) Write short notes on the following topics. Provide examples for each: (3 × 3 = 9)

- i) Vertex cut
- ii) Bridge
- iii) Edge connectivity

- (b) Derive an upper bound and a lower bound on the number  $m$  of edges of a simple graph  $G$  of  $n$  vertices and  $k$  connected components. (13)

- (c) Find the eccentricities of the vertices of the graph in *Figure 6c*. Also find the radius and the center of the graph. (8+2+3=13)



*Figure 6c*

**CSE 421**

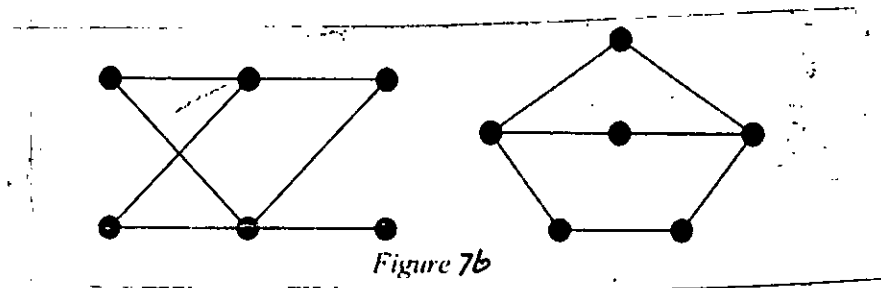
7. (a) Illustrate the following graphs:

(2+2+2+4=10)

- i)  $W_6$
- ii)  $K_{3,2}$
- iii)  $C_6$
- iv) Petersen graph

(b) Describe when a matching is perfect. Find a maximum matching in the two following graphs in *Figure 7b*. Determine whether these graphs have a perfect matching or not.

(2+4+4=10)



(c) Let  $G$  be a graph with  $n$  vertices. Then, prove that any two of the following three: (5×3 = 15)

- i)  $G$  is connected
- ii)  $G$  contains no cycle
- iii)  $G$  has  $n - 1$  edges

8. (a) Construct the tree corresponding to the Pruffer's code 1, 2, 3, 8, 4, 5, 4. (10)

(b) Let  $G$  be a connected simple graph. Show that  $k(G) \leq k'(G) \leq \delta(G)$ ; where  $k(G)$ ,  $k'(G)$ , and  $\delta(G)$  are the connectivity, edge connectivity, and minimum degree of  $G$ , respectively. (10)

(c) Let  $G$  be a bipartite graph with bipartition  $V(G) = X \cup Y$ .

Prove that  $G$  contains a matching that saturates every vertex in  $X$  if and only if

$|N(S)| \geq |S|$  for every subset  $S$  of  $X$ . Here,  $N(S)$  is the neighbor set of  $S$  in  $G$ . (15)

-----

**SECTION - A**

There are **FOUR** questions in this section. **Question 1 is mandatory to answer.**

Answer any **two** questions out of the rest **three**.

1. (a) A hardware designer is designing an electrically-operated switching circuit, where a switching unit generally remains closed and the unit gets open when an electrical signal is applied on it. The designer attempts to build an arbitrarily reliable circuit from unreliable units. To do so, he adopted the following circuit structure. (23)

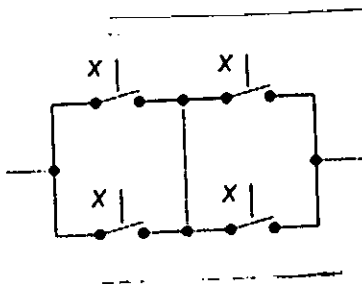


Figure 1(a): An arbitrarily reliable switching circuit from unreliable switching units

You need to investigate reliability of the above design through formulating the probability of the above design to be faulty - assuming the probability of a switching unit to be faulty as  $p$  and the probability of a switching circuit (built from switching units as shown above) to be faulty as  $h(p)$ . Besides, based on your formulation, you need to analyze fault tolerance of the above design in detail.

- (b) In the case of memory systems, as shown below, it is generally practiced to have spare rows and spare columns to avail redundancy for the purpose of overcoming defects and hardening the system. (12)

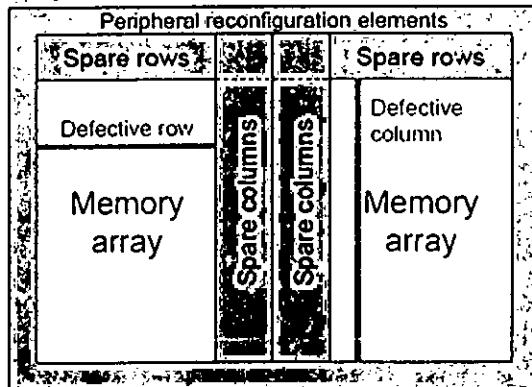


Figure 1(b): A memory system with spare rows and columns

For such systems, you need to design a methodology for replacing faulty parts using spare rows or spare columns. Here, you need to analyze how to distinguish suitable spare rows or spare columns for the purpose of necessary replacements.

**CSE 423**

2. (a) Consider a long-running computation's running time  $T$ , which is twice the MTTF of the machine used to execute it. Ignoring the checkpointing time overhead, formulate and compare the probability of completing the computation in  $2T$  time in the following two cases. (23)
- i. Assuming no checkpointing.
  - ii. Assuming checkpointing at regular intervals of  $T/2$
- Show and elaborate all steps of your formulation, as necessary.
- (b) What could be the problems of radiation? What are the solutions to guard against radiation? (12)
3. (a) Analyze the notion defect modeling. Distinguish the defects and their models from two different perspectives - (23)
- i. Defect size, and
  - ii. Defect type.
- (b) What is Byzantine Fault model? Elaborate with an example. (12)
4. (a) Distinguish among different types of software fault tolerance methods. Formulate relevant model for reliability analysis and present its outcome with a graph. (23)
- (b) Design different data structures for a linked list with varying levels of capabilities for fault detection and fault correction. (12)

**SECTION - B**

There are **FOUR** questions in this section. **Question 5 is mandatory.**

Answer any **two** from questions **6-8**.

5. (a) Suppose you are in charge of designing a distributed file storage system with 5 nodes, all connected in a fully meshed network. Only direct communication is allowed among the nodes. The probability of a node being active is 0.97, and the probability of a link being active is 0.94. **Analyze** the availability for each of the scenarios provided below and **examine** the trade-off between these scenarios. (23)
- i. Single server: Each file is stored on a single designated node (primary server).
  - ii. Duplication: Each file is replicated and stored on two different nodes (primary and mirror).
  - iii. Triplication: Each file is replicated and stored on three different nodes (primary and two mirrors).
  - iv. Dispersion: Data is encoded and divided on all nodes. You are free to assume any method of the encoding. However, you need to mention and describe your assumed encoding.

**CSE 423**

(Contd.....Q. No. 5)

- (b) Imagine you are designing an autonomous drone navigation application that makes real-time decisions based on the decision of a sensor module. Now you are a bit concerned about the reliability of the navigation system. **Design** the system to incorporate Triple Modular Redundancy (TMR) of the sensor module for hardening the system while considering the failure of the voter. (12)
6. (a) In a car assembling industry, the reliability of a safety-critical module is X. The module has a standby spare with reliability Y, which is to be used in case the first one malfunctions. However, the probability that a faulty unit is correctly identified and reconfigured is C. **Analyze** the impact of C on the system performance with necessary derivations and graphs. (20)
- (b) "Amdahl's Law is applicable for reliability while considering the performance metric Reliability Improvement Index, however, not for Reliability Improvement Factor" - **Justify or refute** this statement with necessary arguments and derivations. (15)
7. (a) A data center utilizes a critical sever with two potential failure points: Power supply Unit (PSU) failure that results in a complete server shutdown, requiring a reboot and Hard Disk Drive (HDD) failure that leads to data corruption, requiring data recovery (restraining state). Now there are two sets of PSU and HDD to choose for this server. (20)
- In the first set, PSU and HDD failures occur every 365 days and 100 days, respectively. The replacement time for a faulty PSU or HDD is approximately 1 day.
- For the second set, PSU and HDD failures occur every 335 and 110 days. However, in this case, the replacement time for the PSU is approximately 2 days, and for the HDD, it is 1 day.
- Examine** which set is more favorable by conducting steady-state analysis on the corresponding state-diagram for each scenario.
- (b) **Evaluate and compare** the "2-out-of-3 system" and "1-out-of-1 system" in terms of reliability and Mean Time to Failure (MTTF). Provide an analysis considering the relevant figures. Clearly state any assumptions made during the evaluation. (15)
8. (a) Imagine you are a developer who is developing a mobile banking app for a large financial institution. This app will allow users to check account balances, transfer funds, and pay bills. Answer the following questions based on this scenario. (20)
- i) **Differentiate** between a fault and a failure in the context of the mobile banking app with an example of each.
- ii) **Explain** why reliability, maintainability, and safety are important for a mobile banking app.
- iii) **Discuss** two strategies you would adopt to ensure fault avoidance for the mentioned application.
- (b) **Define** safe failure, fail-soft system, and performability. **Show** the modeling of performability for a three-state dependable system. (15)
- .....

**SECTION – A**

There are **FOUR** questions in this section. Answer to **Question no. 1** is **Compulsory**.

Answer any **TWO** questions from Questions 2-4.

**1. [Mandatory]**

(a) Consider the following DNA sequence (of length 10) of an organism SP1. (8)

1	2	3	4	5	6	7	8	9	10
A	T	C	G	A	T	C	A	C	G

Assume that after 800 years, SP1 has evolved into SP1' through the following evolutionary events:

- (i) 'C' at position 3 is substituted by 'G'
- (ii) The substring "GAT" (starting from 4 to 6) is deleted
- (iii) "GT" is inserted after the 'A' at position 8.

Show the alignment of SP1 and SP1' and indicate the substitutions, insertions and deletions.

(b) Find the optimal global alignment of the following two sequences using -2 as the gap penalty, -1 as the mismatch penalty, and 2 as the score for a match. You have to use the Needleman-Wunsch algorithm, and show the corresponding dynamic programming (DP) table. In case there are multiple optimal alignments, find all of them. Please mark the paths, which correspond to the optimal alignments, in the DP table. (15)

ATCCTGT

TCTCT

(c) Consider the sequences and the scoring scheme mentioned in Q 1(b). Find the optimal local alignment(s) of these two sequences using Smith-Waterman algorithm. Show the DP table and mark the path(s) that corresponds to the alignment. (12)

2. (a) For a given gene tree  $gt = (((((a, b), c), d), e), f)$ , and a species tree  $ST = (((((a, (b,c)), f), e), d)$ , explain the discordance between  $gt$  and  $ST$  using minimum numbers of: (25)

- (i) Deep coalescence (DC),
- (ii) Gene duplications and losses (GDL).

Show the reconciliations with appropriate figures (separate figures for DC and GDL) indicating the deep coalescence, duplication, and loss events. Report the numbers of extra lineages, duplications and losses.

(b) Consider the following set of clusters: {bc, de, fg, abc, defg, bcdefg}. Is this a compatible set of clusters, meaning that can these clusters co-exist in a phylogenetic tree? If yes, show the corresponding tree. If not, explain why. (10)



**CSE 463**

3. (a) Construct a tree on the leaf set {a, b, c, d, e} which is consistent with each of the following triplets. You have to show the intermediate steps of your algorithm. (10)

ab c	ac d
ab d	ac e
ab e	ad c
bc d	bd c
bc e	cd e

- (b) (i) Consider a gene tree  $gt$  and a species tree  $ST$ , both on the same set of 5 taxa. If five quartets in  $gt$  are consistent with  $ST$ , how many triplets in  $gt$  are consistent with  $ST$ ? (6+8=14)

(ii) Consider a set of 5 phylogenetic trees each containing 7 taxa.

1. How many quartets and triplets are there in this set of trees?
2. What are the maximum numbers of unique quartets and triplets in this set of trees?

- (c) (i) Consider a true species tree  $ST = (((a,b),c), (d,(e,f)))$ , and an estimated tree  $ET = (((a,c), b), (f,(e,d)))$ . Draw the unrooted versions of these two trees and indicate the false positive (FP) and false negative (FN) edges. (6+5=11)

(ii) Suppose you are trying to construct a species tree on 20 different species. You have sampled 100 genes from each of these 20 species. Your supervisor has asked you to use a method called GT-est for constructing trees from sequence alignments, and SP-est (which is a summary method) for estimating species trees from gene trees.

1. How many times do you need to run GT-est and SP-est to estimate a species tree by summarizing gene trees?
2. How many times do you need to run GT-est and SP-est to estimate a species tree using "Combined Analyses"?

4. (a) Consider the following five reads. Construct the overlap graph with minimum overlap 2 (meaning that there is an edge between two reads if the overlap between them is at least two). Find a shortest common superstring from that overlap graph. (18)

TAGGCTA  
TAATACTTAGG  
AATTTGCTA  
GCTAGGAAT  
CTAGCTA

- (b) Construct the De Bruijn graph from the following reads using 3-mers. (12)

ATGCCGT  
CCGTAGT  
GTCC

- (c) Suppose you have a set of reads sequenced from a genome. The number of reads is low and the reads are not highly accurate. Which technique would you prefer for assembling a genome from this set of reads: overlap graph or De Bruijn graph? Briefly justify your answer. (5)



**CSE 463**

**SECTION – B**

There are **FOUR** questions in this section. Answer to **Question no. 5 (five) is Compulsory**.  
Answer any **TWO** questions from Questions 6-8.

(If you find any information missing in the question. Just assume a value of your choice and clearly mention your assumption.)

5. [Mandatory]

(a) Construct the suffix array for the DNA string **TGTACGAC\$**. From the suffix array, construct the Burrows-Wheeler transform (BWT). Using these two data structures, identify whether the DNA pattern "GACT" exists in the original string and, if so, in which position. Adequately explain different steps in this process. (20)

(b) Consider the Kullback-Leibler divergence between two probability distributions P and Q is calculated using the following equation. (15)

$$D_{KL}(P|Q) = \sum_i P_i \log\left(\frac{P_i}{Q_i}\right)$$

Explain how this equation can be utilized to find biologically relevant motifs in the presence of skewed background nucleotide distribution. Particularly you must discuss the following points:

- (i) How will you compute P and Q in the given context?
- (ii) Will you use this equation as a minimization score? Justify.

6. (a) Consider the 5 DNA strings, each of length 10, in Table for Q. 6(a). You have to find a 4-mer motif from each string using the GIBBS-SAMPLER method. Suppose you start with the initial motifs marked using underline and then randomly decide to remove the 5<sup>th</sup> string (i.e., CCGGCGTATT). Now execute one iteration of the GIBBS-SAMPLER and show the updated motifs at the end of the iteration. We need to show the calculation for each step and give justification for each decision you make. (20)

T	T	A	T	G	T	G	A	T	
G	T	A	C	G	C	A	A	T	
T	T	A	A	T	A	T	A	C	
G	T	A	C	A	T	A	A	T	
C	C	G	G	C	G	T	A	T	

Table for Q.6(a)

(b) Illustrate, with appropriate figures, how the FM index strikes a balance between time and space while resolving the starting position(s) of a short read within the reference genome? (15)

7. (a) Illustrate, with appropriate figures, how we can get a variable number of clusters from a single run of a hierarchical clustering algorithm. (15)





**CSE 463**

**Contd ... Q. No. 7**

(b) Consider an extended amino acid alphabet where each integer between 57 and 200 represents the mass of an amino acid. Now, you are given the following noisy experimental spectrum of a cyclic peptide:

(20)

0	71	113	129	147	200	218	260	313	331	347	389	460
---	----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

Suggest an approach to identify a reduced set of the most probable amino acid alphabet, which is used to sequence the cyclic peptide based on the given noisy experimental spectrum. Demonstrate your approach for the given spectrum and list the five most probable amino acid masses.

8. (a) Consider a toy gene expression matrix of eight genes shown in Table 1 for Q. 8(a). We want to apply the Soft k-Means Clustering to partition these genes into three clusters with centers denoted by C1, C2, and C3. Suppose a responsibility matrix (also known as hidden matrix) has been computed as shown in Table 2 for Q. 8(a) based on the initial choice of C1, C2 and C3. Calculate the updated values of C1, C2 and C3 from the given information.

(15)

g1	-4	5
g2	4	5
g3	-2	3
g4	3	-4
g5	6	7
g6	-4	2
g7	6	-6
g8	7	-2

	g1	g2	g3	g4	g5	g6	g7	g8
C1	0.70	0.15	0.73	0.40	0.15	0.80	0.05	0.05
C2	0.20	0.80	0.17	0.20	0.80	0.10	0.05	0.20
C3	0.10	0.05	0.10	0.40	0.05	0.10	0.90	0.75

Table 2 for Q.8(a)

Table 1 for Q.8(a)

(b) The noisy spectrum (i.e., contains false/missing masses) of a cyclic peptide is given below:

(20)

0	71	113	129	147	200	218	260	313	331	347	389
---	----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

Determine the amino acid sequence in this peptide using the LEADERBOARD-CYCLOPEPTIDE-SEQUENCING algorithm with N = 3. You can avoid showing the candidate peptides that are removed by the algorithm at each iteration. Consider the amino acid alphabet in the Table for Q. 8(b).

Amino acid	Mass	Amino acid	Mass
G	57	D	115
A	71	K/Q	118
S	87	E	129
P	97	M	131
V	99	H	137
T	101	F	147
C	103	R	156
L/I	113	Y	163
N	114	W	186

Table for Q.8(b)

**SECTION – A**There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Differentiate between CPI-based inflation and GDP deflator. Calculate GDP deflator from the following table 1. (20)

**Table 1**

Commodity	Year 2021		Year 2023	
	Unit produced	Price (Tk)	Unit produced	Price (Tk)
A	250	150	300	160
B	500	750	600	800
C	400	600	500	700
D	300	500	350	550
E	600	200	700	250
F	155	1250	205	1380

- (b) What do you understand by expansionary monetary policy and contractionary monetary policy? Illustrate the effects of monetary policy on real GDP and price level. (15)

2. (a) Explain the expenditure method of measuring 'gross domestic product (GDP)'. Why are 'inflation' and 'exchange rate' taken into consideration for measuring GDP? How would you measure 'gross national income (GNI)', and 'net national income (NNI)' from the gross domestic product (GDP)? (20)

- (b) The equilibrium condition of a closed economy with a constant price level is given by (15)

$$Y = C + I + G \dots\dots\dots (1)$$

Where,

$$C = \bar{C} + MPC (Y - \bar{T}) \dots (2) \text{ is the Consumption function}$$

$$I = \bar{I} \dots (3) \text{ is the Planned investment function}$$

$$G = \bar{G} \dots (4) \text{ is the Government purchases function}$$

$$T = \bar{T} \dots (5) \text{ is the Tax function.}$$

The letters with bars above them represent fixed or autonomous values and do not depend on the values of other variables. Therefore,  $\bar{C}$  represents autonomous consumption.

- (i) Derive the formulae for the Government purchases multiplier and the tax multiplier.  
(ii) What would be the multipliers if MPC is 0.75?  
(iii) Prove that the balanced budget multiplier is equal to one.

**HUM 475/CSE**

3. (a) Clarify the concepts of 'economic growth' and 'economic development'. 'Development is both a physical reality and a state of mind in which society has, through some combinations of social, economic, and institutional processes, secured the means of obtaining a better life'. Explain. (20)
- (b) What are the main challenges that every economy struggles to overcome? Briefly discuss how these challenges are addressed with reference to different economic systems. (15)
4. Write short notes on any THREE of the following: (35)
- (i) Quantity theory of money
  - (ii) Circular flow of income and expenditure
  - (iii) Determination of macroeconomic equilibrium using the 45°-line diagram
  - (iv) Human Development Index (HDI) and its importance

**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) "Scarcity implies choice and choice implies opportunity cost". Justify the statement. (10)
- (b) Suppose the demand and supply functions of pizza are respectively as follows (10)
- $$Q_d = 80 - 5P \text{ and}$$
- $$Q_s = -10 + 10P$$
- Where P is price and Q is quantity of pizza. Determine equilibrium price and quantity of pizza. Show the equilibrium price and quantity in a diagram.
- (c) What would happen to the equilibrium price and quantity of pizza (in Question b) if each of the following occurs, (15)
- (i) An increase in the Price of cheese.
  - (ii) A decrease in the Price of burger.
  - (iii) An increase in consumer income and simultaneously advancement in the technology of pizza production.
  - (iv) An inclusion of sales tax on Pizza.
- [N.B. Explain them in separate diagram].

6. (a) Suppose that the demand schedule for commodity X is as follows: (10)

Price of X (Tk.)	Price of Y (Tk.)	Quantity Demand of X (units)
2	12	20
4	9	15
6	6	10
8	3	5

Calculate the price elasticity of demand as the price of commodity X increases from Tk. 6 to Tk. 8 and calculate the cross-price elasticity of demand as the price of commodity Y falls from Tk. 12 to Tk. 9.

**HUM 475/CSE**

**Contd ... Q. No. 6**

(b) Define indifference curve. Briefly explain the properties of indifference curve. (10)

(c) Given the following utility maximization problem (15)

$$U = x^{0.25} y^{0.4}$$

Subject to  $2x + 8y = 104$

Here, U = Amount of Utility, x = Amount of Orange, y = Amount of Mangoes

(i) Find the Optimal value of x and y at which utility is maximized.

(ii) Find the value of Lagrange multiplier ( $\lambda$ ) and interpret it.

(iii) Find the maximum level of utility.

7. (a) Why does monopoly arise? Explain. How does a monopoly determine price and output in the short run? Explain. (12)

(b) Why will a perfectly competitive firm shut down in the short run? (8)

(c) A producer has the possibility of discriminating price between the domestic and foreign markets for a product where the demands, respectively, are (15)

$$Q_1 = 21 - 0.1P_1$$

$$Q_2 = 50 - 0.4P_2$$

Total cost =  $2000 + 10Q$  where  $Q = Q_1 + Q_2$ . What price will the producer charge in order to maximize profits (i) with discrimination between markets and (ii) without discrimination? (iii) Compare the profit differential between discrimination and nondiscrimination.

8. (a) Distinguish between a monopolistically competitive market and an oligopoly market. (8)

(b) "A monopolistically firm may face positive profit or loss in the short-run but it always faces normal profit (zero profit) in the long run". Justify the statement. (15)

(c) Define Nash equilibrium. Consider trade relations between the United States and Mexico. Assume that the leaders of the two countries believe the payoff to alternative trade policies are as follows (12)

		United States	
		Low Tariffs	High Tariffs
Mexico	Low Tariffs	(\$25 billion, \$25 billion)	(\$10 billion, \$30 billion)
	High Tariffs	(\$30 billion, \$10 billion)	(\$20 billion, \$20 billion)

Find the Nash equilibrium. Under what condition the trade partners can improve their gains.

-----