

**EFFECTS OF PRESSURE ON NATURAL CONVECTION
FROM
AN INCLINED ISOTHERMAL CYLINDER
IN AN INERT ENVIRONMENT**



BY

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A thesis submitted to the Department of Mechanical Engineering in partial fulfilment of
the requirements for the degree of

Master of Science

in

Mechanical Engineering



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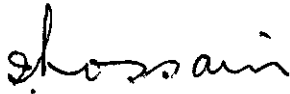
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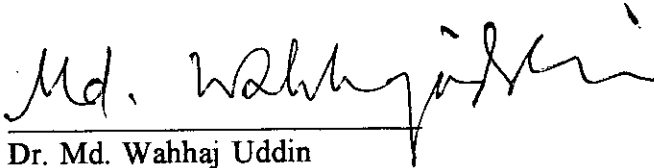
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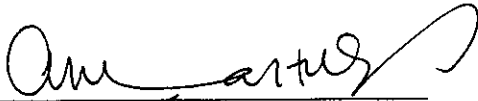
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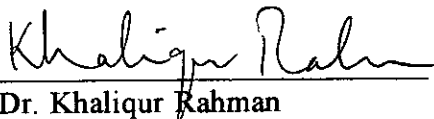
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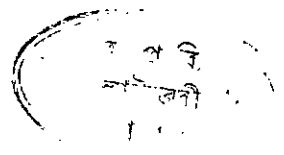
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ABSTRACT

Experimental investigation of natural convection heat transfer from a heated cylinder at different pressure and inclinations to ambient air and argon was carried out. The test cylinder was maintained at a constant temperature of 91.5° c. The measurements covered the fluid pressure ranging from 5 to 1700 mm of Hg absolute. The inclinations of the axis of the cylinder with vertical were 0°, 30°, 45°, 60° and 90°. The range of Rayleigh number covered was $10^3 \leq Ra_L \leq 1.5 \times 10^7$.

The investigations show that heat transfer coefficient vary logarithmically with pressure. The plots of Nusselt number against pressure and against Rayleigh number are both linear on a log-log plot.

From the experimental results, correlations are developed in terms of Nusselt number, Rayleigh number, and inclination of the cylinder. The reliability of these correlations are all within $\pm 10\%$ of the experimental data.

NOMENCLATURE

Symbol	Meaning	Units
A	Effective heat transfer area	m ²
C	Constant for a given inclination	
c	Specific heat for a viscous fluid	kJ/kg-K
c _p	Specific heat at constant pressure	kJ/kg-K
c _v	Specific heat at constant volume	kJ/kg-K
D	Outside diameter of the cylinder	m
D _i	Inside diameter of the cylinder	m
Ec	Eckert number, $U_c^2/(c_p \Delta T)$	Dimensionless
Gr	Grashof number, $g\beta L^3 \Delta T/\nu^2$	Dimensionless
h	Heat transfer coefficient	W/m ² -K
H	Absolute pressure inside the vessel	mm Hg
k	Thermal conductivity	W/m-K
L	Cylinder length	m
m	Exponent	
Nu	Nusselt number, hL/k	Dimensionless
P	Fluid pressure in the enclosure	N/m ²
P _a	Ambient fluid pressure	N/m ²
P _h	Local hydrostatic pressure	N/m ²
P _m	Difference between actual pressure and P _h	N/m ²
Pr	Prandtl number, $\mu c_p/k$	Dimensionless
q	Heat transfer rate	W
q _{cond}	Heat transfer rate by conduction	W
q _r	Heat transfer rate by radiation	W
q _T	Total Heat transfer rate	W
q _{in}	actual heat input to the cylinder	W
q _c	Heat transfer by convection	W

Symbol	Meaning	Units
R	Gas constant	Kj/kg-K
Ra	Rayleigh number, Gr X Pr	Dimensionless
Re	Reynolds number, $\rho VL/\mu$	Dimensionless
t_a	Temperature of ambient fluid	$^{\circ}\text{C}$
T_c	Temperature of the cylinder surface	K
T_v	Vessel temperature	K
ΔT	Temperature difference between the cylinder surface and ambient fluid	K
U_c	Convection velocity	m/s
Suffix		
x	based on variable dimension x	
L	based on length	
D	based on diameter	
Special characters		
α	Thermal diffusivity, $k/\rho c_p$	m^2/s
β	coefficient of expansion, $1/T_a$	/K
μ	absolute viscosity	kg/m-s
ν	kinematic viscosity	m^2/s
ϵ	emissivity	Dimensionless
ρ	density	kg/m^3
σ	Stefan-Boltzman constant, 5.77×10^{-8}	$\text{W}/\text{m}^2\text{K}^4$
τ	time variable	s
θ	inclination with vertical	degree

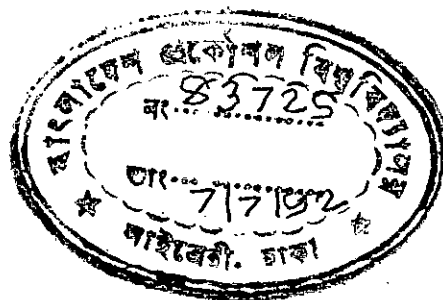
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Chapter -I

Introduction



1.1 General

Heat transfer is a branch of applied thermodynamics. 'Heat Transfer' estimates the rate at which heat is transferred across the system boundaries subjected to specific temperature differences and the temperature distribution of the system during the process. Whereas classical thermodynamics deals with the amount of heat transferred during the process.

Heat transfer is concerned with temperature difference, and we live in a world full of such differences, owing to either natural or artificial causes. Thus an immense variety of heat transfer equipment has been created to carry out these processes; namely boilers, condensers, solar collectors, radiators, compact heat exchangers, combustion chambers, furnaces, driers, distillation columns, refrigerators, insulators, stoves- the list is almost endless. All the above equipments require the application of the first law of thermodynamics, material data and heat transfer relations appropriate to the specific system.

1.2 Natural Convection

When no externally induced flow is provided and the flow arises 'naturally' simply due to the effect of a density difference, resulting from a temperature difference, in a body force field, such as gravitational field, the process is termed as 'natural' or 'free' convection. The density difference gives rise to buoyancy effects due to which the flow is generated. The cooling of a heated body in ambient air generates such a flow in the region surrounding it. Similarly, the buoyant flow arising from heat rejected to atmosphere, in bodies of water and many other

technological applications, is included in the area of natural convection. Flows can be caused by other body forces as well. For instance in a rotating system, centrifugal and coriolis forces exist as body forces. Flow of cooling air through the passage in the rotating blades of a gas turbine is an example of flow under the influence of such forces.

Energy transfer by free convection occurs in many engineering applications. Heat transfer from hot radiators, refrigeration coils, transmission lines, electric transformers, electric heating elements and electronic equipment are typical examples.

Processes involving natural convection are generally much more complex than those in forced convection. Special techniques and methods have, therefore, to be devised to study the process with a view to obtain information on the flow and the heat transfer rate.

1.3 Heat Transfer from Cylinders

Natural convection from the outside surface of cylinders is employed for heating and cooling in many industrial processes. Because of its geometrical relevance, study on cylindrical configuration has great significance in nuclear reactors, heating elements, pipe conveying system, chemical processing plants, refrigeration and air-conditioning plants, etc.

For the large-diameter cylinders, the surface can be approximated to the flat plate to have the simpler governing equations.

1.3.1 Horizontal Cylinders

Cooling or heating of Rolled rods, extruded tubes, horizontal pipes carrying hot feed water or steam are some common examples of heat transfer from horizontal tubes. The most simple case of a horizontal cylinder is the one which is infinitely long and isothermal. For such cases, many investigators have developed different correlations between Nusselt number and Rayleigh number or among Nusselt number, Grashoff number and Prandtl number. A review of previously published empirical and semi-empirical correlation equations reveals that there

are apparent differences in results obtained. These differences in some case extend upto even 50%. But this situation can very well be explained by the different Rayleigh number range employed by the investigators.

Other than isothermal condition, investigation has been carried on other thermal conditions like partly isothermal and partly adiabatic cylindrical surfaces. This situation is related, for example, in heat transfer around metallic tubes partially covered by snow or ice or around pipes where internal layer of deposited salt greatly decreases, in some areas of the cylinders, the heat conducted through the wall.

Almost all investigators in the past have carried out the experiments at atmospheric pressure. The necessity of investigations of the heat transfer behaviour at ambient pressure other than atmospheric lies in the availability of pressure and vacuum producing equipments and the vacuum processes in the food, pharmaceutical, metallurgical, and other industries. The scarcity of heat transfer data at different ambient pressure requires further analysis.

1.3.2 Vertical Cylinders

The natural convection for flow over vertical cylinders is also an important case, due to its relevance to many applied problems, like tubes in nuclear reactors and cylindrical heating elements etc.

For vertical cylinders of small diameter, or for vertical wires, the convection boundary layer may be a substantial fraction of the diameter. In such a circumstance the curvature of the surface has an important effect upon the transport process. The perturbation results of investigators for Prandtl number of 0.72 and 1.0 confirm the importance of curvature.

The problem involving vertical-cylinder case has been addressed in a variety of ways but because of mathematical difficulties, few complete solutions have been obtained.

For vertical cylinders, the data in literatures other than that at atmospheric pressure are very

limited. Although a correlation has been suggested on the basis of wire data at sub-atmospheric pressure in air, it is not applicable to most practical cases due to its low Rayleigh number range. In order to understand the process more convincingly further studies in the field are required.

1.3.3 Inclined Cylinders

Besides horizontal and vertical cylinders, inclined cylinders have extensive application in many industrial plants. Inclined pipes and tubes carrying steam, hot or cold chemicals are common examples of heat transfer from or to the inclined cylindrical surface.

Many investigations have been carried out on natural convection heat transfer from the outside surface of vertical or horizontal tube in both constant temperature and constant heat flux conditions at atmospheric pressure. The data on natural convection from inclined cylinders are, however very limited. Further more, almost very handful investigations have been carried out on heat transfer behaviour in fluid other than air and at higher or sub-atmospheric pressures. The present work has been carried out to fill a part of the existing gap.

1.4 Basis of The Present Work Selection

Most of the studies in the literature on natural convection are on horizontal or vertical cylinders in the environments of air or water at atmospheric status. Studies involving cylinders in the orientations other than horizontal or vertical, and in states other than atmospheric are very few in number, although they are widely used. Experimental studies are therefore essential to observe the variation in heat transfer due to the above physical or environmental changes. The proposed studies are expected to reveal that the heat transfer in such arrangements are different from those above and it will therefore prove useful from the designers point of view in choosing the best physical and environmental condition that suits him.

1.5 Objectives of the Present work

The objectives of the present study on natural convection are as follows:

- a. Designing of an experimental procedure for the determination of average heat transfer coefficient for different inclined cylinders including the horizontal and vertical cylinders.
- b. Comparing the result of study with other relevant works in literatures for establishing reliability of the experimental set up.
- c. Determination of the dependence of average heat transfer coefficient on environmental pressure.
- d. Determination of the effect of inclination on heat transfer coefficient.
- e. Determination of the dependence of heat transfer coefficient on Rayleigh number.
- f. Development of correlation between non-dimensional parameters for achieving the objectives of (c), (d) & (e).

Chapter - II

Literature Review

2.1 General

In the last several years, there is a very rapid increase in research in the field of natural convection. This increased intensity is due to enhanced concerns in the field of science and technology dealing with atmospheric motions, moreover in bodies of water, in quasi-solid bodies such as the earth, and in various devices and process equipment. The natural convection phenomenon involves heat exchange between a fluid and an adjacent boundary where fluid motion occurs due to the density differences that result from energy exchange.

Holman¹³ reported that for air in free convection on a vertical plate, the critical Grashof number has been observed by Eckert and Soehngen⁷ to be approximately 4×10^8 .

Islam¹⁶ reported that Schorr and Gebhart³⁰ investigated natural convection wake arising from a heated horizontal line source in liquids and air. The three dimensional effects for a wire with a length to diameter ratio of 250 were observed in air and water with Schlieren system. The temperature field in the plume above wires with length to diameter ratios of 250 and 1200 in liquid silicone ($Pr=6.7$) was determined using a 20 cm Mach-Zender interferometer. Various wire heating rates were used yielding Grashof numbers, based on vertical distance in the plume, in the range 4×10^3 to 1.7×10^6 . Excellent agreement of the temperature distribution with theory was found for large length to diameter ratios at Grashof number around 10^5 .

2.2 Vertical Plates

Welty³³ reported that according to Schlichting²⁹, for a vertical plate with fluid of $Pr=0.733$

$$Nu_x = 0.359 * Gr_x^{1/4} \quad [2.1]$$

where Gr_x is the local Grashof number, defined for ideal gas behavior as

$$Gr_x = gx^3 \beta \Delta T / \nu^2 \quad [2.2]$$

The mean Nusselt number may be determined in usual fashion from the expression for the local parameter and may be given by

$$Nu_L = 0.478 Gr_L^{1/4}$$

The result of Pohlhausen²⁷ for $Pr=0.733$ were extended to apply to large values of the Prandtl number by Schuh³. The appropriate values of $Nu_L / Gr_L^{1/4}$ and $Nu_L / (Gr_L Pr)^{1/4}$ are listed in Table-1 for Prandtl number up to 1000.

Table-1

Pr	$Nu_L / Gr^{1/4}$	$Nu_L / (Gr_L Pr)^{1/4}$
0.73	0.478	0.517
10	1.09	0.612
100	2.06	0.652
1000	3.67	0.653

The correlating relationship suggested by Eckert and Jackson⁶ for both vertical plates and cylinders in laminar to turbulent boundary layer region are

$$Nu_L = 0.555 (Gr Pr)^{1/4} \quad \text{for } Gr < 10^9 \quad [2.4]$$

$$Nu_L = 0.021 (Gr Pr)^{2/5} \quad \text{for } Gr > 10^9 \quad [2.5]$$

An integral analysis of the laminar boundary layer case is presented by Eckert⁵ for a constant wall temperature. The resulting expression for the local Nusselt number is

$$Nu_x = 0.508 \frac{Pr^{1/2} Gr^{1/4}}{(0.952 + Pr)^{1/4}} \quad [2.6]$$

and mean Nusselt number for a plate of height L is given by

$$Nu_L = 0.678 \frac{Pr^{1/2} Gr_L^{1/4}}{(0.952 + Pr)^{1/4}} \quad [2.7]$$

The case of laminar natural convection adjacent to a vertical plate with uniform surface heat flux was solved by Sparrow and Gregg³¹.

White³⁴ has investigated this problem for the case of low Prandtl no. fluids for both the constant temperature and constant heat flux surface conditions.

Bayazitoglu and Ozisik³ reported that Mc Adams had proposed the following simple correlation for heat transfer by free convection on a vertical wall of height L maintained at uniform temperature :

$$Nu = c (Gr_L Pr)^n = c Ra_L^n \quad [2.8]$$

The values of c and n for various types of flow are given in table-2.

Table-2

Type of flow	Range of $Gr_L Pr$	c	n
Laminar	$10^4 - 10^9$	0.59	1/4
turbulent	$10^9 - 10^{13}$	0.10	1/3

A more elaborate but accurate correlation has been proposed by Churchill and Chu¹⁸ for free convection from an isothermal vertical plate

$$Nu = \left[0.825 + \frac{0.387 Ra_L^{1/6}}{(1 + (0.492/Pr)^{9/16})^{4/27}} \right]^2 \quad [2.9]$$

$$\text{for } 10^{-1} < Ra_L < 10^{12}$$

which is applicable for both laminar and turbulent flow conditions.

Gebhart¹² revealed two traditional correlation equations for the average convection coefficient for vertical plates of height L or for large diameter vertical cylinders in fluids

having Prandtl numbers around 1.0.

$$Nu = 0.59 (Gr Pr)^{1/4} \quad \text{for } 10^4 < Gr Pr < 10^8 \quad [2.10]$$

$$\text{and } Nu = 0.13 (Gr Pr)^{1/3} \quad \text{for } 10^8 < Gr Pr \quad [2.11]$$

Holman¹³ suggested the following correlations with modified Grashof number,

$$Gr_x^* = Gr_x Nu_x = g\beta q_w x^4 / kv^2, \quad [2.12]$$

for laminar range

$$Nu_x = hx/k = 0.60 (Gr_x Pr)^{1/5} \quad [2.13]$$

$$\text{for } 10^5 < Gr_x^* < 10^{11}; q_w = \text{constant}$$

for turbulent range

$$Nu_x = 0.17 (Gr_x^* Pr)^{1/4} \quad [2.14]$$

for $2 \times 10^{13} < Gr_x^* Pr < 10^{16}$; $q_w = \text{constant}$, where q_w is wall heat flux.

2.3 Horizontal Plates

McAdams¹⁹ recommended the following expressions for natural convections adjacent to horizontal plates. For hot plates facing upward or cold plates facing down in the range

$$10^5 < Gr_L Pr < 2 \times 10^7,$$

$$Nu_L = 0.54 (Gr_L Pr)^{1/4} \quad [2.15]$$

and, in the range $2 \times 10^7 < Gr_L Pr < 3 \times 10^{10}$,

$$Nu_L = 0.14 (Gr_L Pr)^{1/3} \quad [2.16]$$

With hot plates facing down and cold plates facing up, the recommended expression, in the range $3 \times 10^5 < Gr_L Pr < 10^{10}$ is

$$Nu_L = 0.27 (Gr_L Pr)^{1/4} \quad [2.17]$$

Bayazitoglu & Ozisik³ reported that the average Nusselt number for uniform surface heat flux q_w has been studied experimentally by Fujii and Imura¹¹, and the following correlations were proposed for the cases in which the heated surface is facing up and facing down.

For horizontal plate with the heated surface facing upward:

$$Nu_L = 0.13 (Gr_L Pr)^{1/3} \quad \text{for } Gr_L Pr < 2 \times 10^8 \quad [2.18]$$

$$Nu_L = 0.16 (Gr_L Pr)^{1/3} \quad \text{for } 5 \times 10^8 < Gr_L Pr < 10^{11} \quad [2.19]$$

For horizontal plate with the heated surface facing downward

$$Nu_L = 0.58 (Gr_L Pr)^{1/5} \quad \text{for } 10^6 < Gr_L Pr < 10^{11} \quad [2.20]$$

The physical properties are to be evaluated at a mean temperature, defined as

$$T_m = T_w - 0.25 (T_w - T_a)$$

T_w = Wall temperature

T_a = Ambient temperature

and thermal expansion co-efficient β at $(T_w + T_a)/2$

The characteristics length L , in the preceding expressions, is the length of a side of a square surface, the mean of the dimension of the rectangular surface, or 0.9 times the diameter of a circular area.

2.4 Inclined Plates

The heat transfer co-efficient for the convection on an inclined plate can be predicted by the vertical plate formulas if the gravitational term g is replaced by $g \cos\theta$, where θ is the angle the plate makes with the vertical. Conventionally the angle that the surface makes with vertical are designated as positive if the hot surface is facing downward and negative if the hot surface is facing upward.

Bayazitoglu & Ozisik³ reported that based on the extensive experimental studies by Fujii and Imura¹¹, the following correlations had been recommended for free convection on an inclined surface subjected to a uniform surface heat flux.

For an inclined plate with heated surface facing downward

$$Nu_L = 0.56 (Gr_L Pr \cos\theta)^{1/5}$$

for $+\theta < 88^\circ$, in the range $10^5 < Gr_L Pr < 10^{11}$

For an inclined plate with the heated surface facing upward, the heat transfer correlation is:

$$Nu_L = 0.145[(Gr_L Pr)^{1/3} - (Gr_c Pr)^{1/3}] + 0.56 (Gr_c Pr \cos\theta)^{1/4} \quad [2.22]$$

for $Gr_L Pr < 10^{11}$, $Gr_L > Gr_c$ and $-15^\circ < \theta < -75^\circ$. Here, the value of the transition Grashof number Gr_c depends on angle of inclination θ , as listed in table-3.

Table-3

θ , degrees	Gr_c
-15	5×10^9
-30	10^9
-60	10^8
-75	10^6

All physical properties are evaluated at the mean temperature $T_m = T_w - 0.25 (T_w - T_a)$ and β is evaluated at $T_a + 0.25 (T_w - T_a)$. For $Gr_L < Gr_c$, the first term of the equation is dropped out.

2.5 Vertical Cylinder

Welty³³ reported that the correlating relationship suggested by Eckert and Jackson⁶ for both vertical cylinders and plates are

$$Nu_L = 0.555 (Gr Pr)^{1/4} \quad \text{for } Gr Pr < 10^9 \quad [2.23]$$

$$\text{and } Nu_L = 0.021 (Gr Pr)^{2/5} \quad \text{for } Gr Pr > 10^9 \quad [2.24]$$

White³⁵ reported that a nonsimilar laminar flow perturbation solution by Mickyava and Sparrow²² for $Pr = 0.733$ (air) suggests the following correlation at constant wall-temperature.

$$Nu_{L,cyl} = Nu_{L,plate} (1 + 1.43 \zeta^{0.9}), \quad [2.25]$$

where $\zeta = (L/D) Gr_L^{-1/4}$ and

$$Nu_{L,plate} = 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \quad [2.26]$$

for $0.1 < Ra_L < 10^{12}$

L and D are the height and diameter of the surface respectively.

Holman¹³ suggested that a vertical cylinder may be treated as a vertical flat plate with an error about 5% when

$$D/L \geq 35/Gr_L^{1/2}$$

Al-Arabi and Salman² developed a correlating equation for constant heat flux condition which is

$$Nu_x = 0.545 (Gr_x Pr)^{1/4} \quad [2.27]$$

and $Nu_L = 0.6 (Gr_L Pr)^{1/4}$ for laminar range. [2.28]

Al-Arabi & Khamis¹ obtained the following correlation

$$Nu_L = 2.9 Gr_D^{-1/12} (Gr_L Pr)^{1/4} \quad [2.29]$$

for $1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $9.88 \times 10^7 \leq Gr_L Pr \leq 2.7 \times 10^9$

and $Nu_L = 0.47 (Gr_D)^{-1/12} (Gr_L Pr)^{1/3}$ [2.30] for

$1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $2.7 \times 10^9 \leq Gr_L Pr \leq 2.95 \times 10^{10}$

For Prandtl numbers 0.733, 1.0, 10 and 100, Gebhart¹² reported that Millsaps and Pohlhausen developed a correlation in the form

$$hD/K = 1.058 (Gr)^{1/4} (Pr^2 / (4+7Pr))^{1/4} \quad [2.31] \text{ for}$$

a surface whose temperature t_w increased linearly with x , i.e. $t_w - t_s = N^*x$; Gr' was defined as temperature gradient Grashof number

$$Gr' = g\beta D^4 N / \nu^2 \quad [2.32]$$

Nagendra, Tirunarayan & Ramacharan²⁴ correlated several simplified form of equation for constant heat flux condition as

$$Nu_D = 0.60 (Ra_D D/L)^{1/4}, \text{ for } Ra.D/L > 10^4, \text{ short cylinder.} \quad [2.33]$$

$$Nu_D = 1.37 (Ra_D D/L)^{1/4}, \text{ for } 0.05 < Ra.D/L < 10^4, \text{ long cylinder} \quad [2.34]$$

$$Nu_D = 0.93 (Ra_D D/L)^{1/20}, \text{ for } Ra.D/L < 0.05, \text{ wires.} \quad [2.35]$$

Where, the average temperature difference was used in Rayleigh number calculation. They claimed that the constant heat flux results differed from constant wall-temperature ones by less than 5%.

2.6 Horizontal Cylinders

White³⁵ reported that a general correlation for mean Nusselt number in free convection over a horizontal cylinder was given by Churchill and Chu⁴ as follows which is valid over a large range of data

$$Nu_D^{1/2} = 0.60 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{4/27}} \quad [2.36]$$

for $10^{-5} < Ra_D < 10^9$

Welty³³ reported that- Natural convection data for the case of heated horizontal cylinders in both liquids and gases had been correlated by McAdams²⁰. In the range $10^4 < Gr_D Pr < 10^9$, McAdams suggested the correlation

$$Nu_D = 0.53 (Gr_D Pr)^{1/4} \quad [2.37]$$

When the cylinder diameter becomes small, as in the case of wire, the Grashof number becomes very small. For cases where $Gr_D Pr < 10^4$, Elenbaas⁸ derived the equation

$$Nu^3 \exp^{-6/Nu} = Gr_D Pr / 235 \quad [2.38]$$

In the case of streamline flow in natural convection of both metallic and nonmetallic fluids adjacent to horizontal cylinders larger than wires, Hsu¹⁴ recommended a correlation in the form

$$Nu_D = 0.53 \left[\frac{Pr}{0.952 + Pr} Gr_D Pr \right]^{1/4} \quad [2.39]$$

Bayazitoglu & Ozisik³ reported that Morgan²³ had presented a simple correlation for free convection from a horizontal isothermal cylinder, covering $10^{-10} < Ra_D < 10^{12}$. It is given in the form

$$Nu_D = C Ra_D^n \quad [2.40]$$

where the constant C and exponent n are listed in Table-4.

Table-4

Ra_D	C	n
$10^{-10} - 10^{-12}$	0.675	0.058
$10^{-12} - 10^2$	1.020	0.148
$10^2 - 10^4$	0.850	0.188
$10^4 - 10^7$	0.480	0.250
$10^7 - 10^{12}$	0.125	0.333

Holman¹³ reported that Churehill and Chu⁴ had provided a simpler equation, but is restricted to the laminar range of $10^{-6} < Gr Pr < 10^9$:

$$Nu_D = 0.36 + \frac{0.518(Gr_D Pr)^{1/4}}{[1 + (0.559/Pr)^{9/16}]^{4/9}} \quad [2.41]$$

Hyman¹⁵ constructed a simpler equation for heat transfer from horizontal cylinders to liquid metals which is

$$Nu_D = 0.53 (Gr_D Pr^2)^{1/4} \quad [2.42]$$

Mikheeva²¹ investigated the process of natural convection from horizontal tubes to air, water and oil and recommended the following correlation:

$$Nu_b = 0.51 X Ra_b^{1/4} (Pr_b/Pr_s)^{1/4} \quad [2.43]$$

Where, the subscripts b and s referred to bulk and surface conditions respectively and the term $(Pr_b / Pr_s)^{1/4}$ was the correction factor intended to account for the influence of property variation with temperature. The empirical study of Fand, Morris and Lum⁹ dealt with the rate of heat transfer by natural convection from horizontal cylinders to air (gases), water and silicone oils (liquids) in the experimental ranges $2.5 X 10^2 < Ra < 1.8 X 10^7$ and $0.7 < Pr < 3090$. Three correlations hypothesis was postulated. Hypothesis I lead to the following correlation equation:

$$Nu_f = 0.474 Ra_f^{0.25} Pr_f^{0.047} \quad [2.44]$$

Where the fluid properties are evaluated at the mean film temperatures.

The purpose of the second hypothesis was to find an optimum reference temperature for evaluating fluid properties as a function of temperature. Hypothesis II lead to the following correlating equation:

$$Nu_j = 0.478 Ra_j^{0.25} Pr_j^{0.050} \quad [2.45]$$

Where the fluid properties were evaluated at the reference temperature defined by

$$t_j = t_b + j (t_s - t_b) \text{ where } J = 0.32$$

Hypothesis III encompassed the so-called "surface correction exponent" method of evaluating fluid properties as a function of temperature. This hypothesis lead to the following correlation:

$$Nu_n = 0.456 Ra_y^{0.25} Pr_p^{0.057} \quad [2.46]$$

where $n = 0.20$; $p = y = 0.50$

with $t_n = t_b + n(t_s - t_b)$

$$t_y = t_b + y(t_s - t_b)$$

$$t_p = t_b + p(t_s - t_b)$$

Al-Arabi & Khamis¹ suggested the following correlation for isothermal cylinders

$$Nu_L = 0.58 Gr_D^{-1/12} (Gr_L Pr)^{1/3} \quad [2.47]$$

for $1.08 \times 10^4 < Gr_D < 6.9 \times 10^5$ and $9.88 \times 10^7 < Gr_L Pr$

Al-Arabi and Salman² developed correlating equations from experimental data for convection heat transfer in the laminar range,

$$Nu_x = 0.158 (Gr_x Pr)^{1/3} \quad [2.48]$$

and $Nu_L = 0.158 (Gr_L Pr)^{1/3}$

Socio¹⁸ investigated on laminar free convection around horizontal cylinders whose surface are partly isothermal and partly adiabatic. For each set of experimental data, a correlation formula of the kind

$$Nu_D = B Ra^m \quad [2.49]$$

for different angles the adiabatic sector made (2ϕ)

ϕ (Deg.)	B	m	
0	0.488	0.246	
45	0.543	0.239	
90	0.581	0.241	Adiabatic sector upward
90	0.569	0.236	Adiabatic sector downward

2.7 Inclined Cylinders

Investigation on natural convection heat transfer from inclined cylinders is very limited. As noted by Farber and Rennant¹⁰, no data were available until 1957 for estimating the loss of heat for an inclined-cylinder.

Al-Arabi and Khamis¹ reported an experiment by Farber and Rennant¹⁰ for the heat loss from a 6 ft long and 0.125 in outside dia cylinder heated electrically from inclined to the vertical at angles using 0° to 90° at constant heat flux condition. The heat transfer co-efficient was found to increase with the angle of inclination. No general correlation of the result was made.

Oosthuizen²⁵ experimented with cylinders of lengths between 152.4 and 304.8mm and outside dia between 19.1 and 25.4mm at angles of inclination from 0 to 90°. The heat transfer was determined by measuring the rate of cooling of the cylinders from 100 to 90°C. The average heat transfer was found to increase with inclination and the results could be correlated in terms of $Nu_D / (Gr_D \sin \theta)^{1/4}$ against $(L/D) \tan \theta$.

Al-Arabi and Khamis² carried out an experiment to determine the average and local heat transfer by natural convection from the outside surface of isothermal cylinders of different diameters (12.75, 19.3, 25, 32, 38.25, 51 mm) and lengths (0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.05, 1.55, 1.65 & 2.0m) at different inclinations (0, 15, 30, 45 & 60) from vertical position in both laminar & turbulent region. The following conclusions were made.

1. For the same cylinder length and inclination, the average heat transfer co-efficient decreases with increase in diameter.
2. For the same cylinder length and diameter, the average heat transfer co-efficient, h_L increases with angle of inclination θ , for the longer lengths. For the shorter lengths it decreases with θ .
3. For the same cylinder diameter and inclination, the average heat transfer co-efficient

decreases gradually with the increase of cylinder length until it becomes constant indicating the beginning of turbulence.

4. Based on average heat transfer the critical transition point from the laminar region to the turbulent region is independent of cylinder diameter. It depends on inclination angle only and increases with the increase of this angle.
5. Based on local heat transfer the critical transition point representing the end of laminar region is independent of cylinder diameter and depends on inclination only.
6. The experimental data could be correlated as followings:

$$Nu_L = [2.9 - 2.32(\sin\theta)^{0.8}](Gr_D)^{-1/12} [Gr_L Pr]^{1/4 + 1/12(\sin\theta)^{1.2}} \quad [2.50]$$

for $1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $9.88 \times 10^7 < Gr_L Pr)_{cr}$

and $Nu_L = [0.47 + 0.11 (\sin\theta)^{0.8}](Gr_D)^{-1/12} (Gr_L Pr)^{1/3} \quad [2.51]$

for $1.08 \times 10^4 \leq Gr_D \leq 6.9 \times 10^5$ and $(Gr_L Pr)_{cr} \leq Gr_L Pr \leq 2.95 \times 10^{10}$

$$Nu_x = [2.3 - 1.72(\sin\theta)^{0.8}](Gr_D)^{-1/12} [Gr_x Pr]^{1/4 + 1/12(\sin\theta)^{1.2}}$$

for $1.08 \times 10^4 < Gr_D < 6.9 \times 10^5$ and $1.63 \times 10^8 < Gr_x Pr < (Gr_x Pr)_{cr-1}$

$$Nu_x = [0.42 + 0.16 (\sin\theta)^{0.8}] (Gr_D)^{-1/12} (Gr_x Pr)^{1/3} \quad [2.52]$$

for $1.08 \times 10^4 < Gr_D < 6.9 \times 10^5$ and $(Gr_x Pr)_{cr-1} < Gr_x Pr < 2.3 \times 10^{10}$

$(Gr_x Pr)_{cr-1}$ represents the end of the laminar region. It can be represented by

$$(Gr_x Pr)_{cr-1} = 2.16 \times 10^9 + 0.283 \times 10^9 \tan\theta$$

$(Gr_x Pr)_{cr-2}$ represents the beginning of the turbulent region. Its value appears to be same for all diameters and angles of inclination. It is about 4.4×10^9 .

$(Gr_L Pr)_{cr}$ is the critical transition point between laminar and turbulent region can be represented by

$$(Gr_L Pr)_{cr} = 2.6 \times 10^9 + 1.1 \times 10^9 \tan\theta$$

Al-Arabi and Salman² carried out experiments on a cylinder of 38mm outside dia and 0.95m length in the constant heat flux condition in the laminar region. The angle of inclination measured from vertical varied between 0 and 90°. The following conclusions were met.

1. For the same heat flux both the local and the average heat transfer co-efficients increase with the angle of inclination.
2. The end of the laminar region, expressed as $Gr_x Pr$, increase with angle of inclination.
3. The local heat transfer results for all the angles can be represented by

$$Nu_x = 0.545 - 0.387(\sin\theta)^{1.462} (Gr_x Pr)^{1/4 + 1/12(\sin\theta)^{1.75}} \quad [2.53]$$

$$\text{for } Gr_x Pr < (Gr_x Pr)_{cr}$$

where $(Gr_x Pr)_{cr} = 1.48 \times 10^5 + 4.5 \times 10^8 \tan\theta$ and the average heat transfer results by

$$Nu_L = 0.60 - 0.488(\sin\theta)^{1.03} (Gr_L Pr)^{1/4 + 1/12(\sin\theta)^{1.75}} \quad [2.54]$$

$$\text{for } Gr_L Pr < 2 \times 10^8$$

Properties are evaluated at the film temperature except β , which is evaluated at ambient conditions.

2.8 At Pressures other than Atmospheric

Research on natural convection at pressures other than atmospheric is very limited. Holman¹³ suggested that the simplified equations from heat transfer co-efficient from various surfaces to air at atmospheric pressure and moderate temperature be extended to higher or lower pressures by multiplying by the following factor:

$$(P/101.32)^{1/2} \text{ for laminar cases}$$

$$(P/101.32)^{2/3} \text{ for turbulent cases}$$

Where P is the pressure in kPa.

Kyte et al¹⁷ investigated heat losses at pressures ranging from 0.1mm to atmospheric from 0.00306 inch dia wire in air and from 0.312 and 1.00 inch diameter spheres in helium, air and argon. Temperatures upto 195°C were employed and radiation loss was taken into account to obtain the net heat transferred by convection.

They observed that at low gas pressures the thickness of the convective boundary layer is large and the effect of free-molecule conduction was important. These effects could be accounted for both spheres and cylinders. This involved two departures from classical practice.

1. The characteristic length used in the dimensionless Grashof number becomes the diameter of the solid plus twice the mean free path length of the gas, and
2. The concept of a conductive film having the same resistance to heat transfer as that of convective boundary layer is used instead of the Nusselt number to effect the single correlation.

They also recommended that, free molecule conduction may be ignored for any gas at atmospheric pressure if the diameter of the solid is 0.001 inch or greater and the solid surface temperature is 200°C.

Islam¹⁶ reported that Warner and Arpachi³² on the basis of their experimental study on turbulent natural convection in air at low pressure along a vertical heated flat plate, suggested the correlation as

$$Nu = 0.10 Ra^{1/3}, Ra \leq 10^{12}$$

2.9 Variable Fluid Properties

As reported by Gebhart¹², Sparrow and Gregg³¹ had shown that for the property variations common to gases, the properties other than β should have been evaluated at the following reference temperature for vertical isothermal plates with laminar boundary layers.

$$t_f = t_w - 0.38 (t_o - t_a)$$

β should be taken as $1/T$. This procedure has been shown to be adequate over the range $0.5 < t_w/t_a < 3.0$, where the temperatures are absolute.

For mercury, the same procedure may be used except that all properties should be evaluated at the reference temperature. For both types of fluids, this reference temperature is a better choice than the film temperature.

Chapter - III

Mathematical Modeling of the Problem

3.1 General

The fundamental physical processes which occur in natural-convection flows are essentially the same as those occurring in other fluid flow and diffusion processes where potential energy and kinetic energy change; momentum, thermal energy, and chemical species diffuse; and viscous and pressure forces oppose or favour the fluid motion. Therefore, the basic equations used to interpret and analyze natural-convection flows, are the same as those applicable to fluid flow and diffusion generally. They are the differential equations which result from the consideration of the conservation of mass (continuity), momentum (Navier-Stokes), and energy (energy equation).

The full set of equations of motion embodying the Boussinesq approximations are written below for constant transport properties μ , k , and D . The relative magnitudes of several possibly small terms are written in parentheses. Two energy equations are given, the first for fluids of otherwise invariant density and the second for ideal gases.

$$\begin{aligned}\nabla \cdot \vec{V} &= 0 \\ \rho \left[\frac{\partial \vec{V}}{\partial \tau} + (\vec{V} \cdot \nabla) \vec{V} \right] &= g \rho \beta (t - t_\infty) \vec{i} + \nabla P_m + \mu \nabla^2 \nabla \\ \rho c_p \left[\frac{\partial t}{\partial \tau} + (\vec{V} \cdot \nabla) t \right] &= k \nabla^2 t + q''' + \mu \phi (R_3) \\ \rho c_p \left[\frac{\partial t}{\partial \tau} + (V \cdot \nabla) t \right] &= k \nabla^2 t + q''' + \frac{DP_h}{D\tau} (R_4) + \frac{DP_m}{D\tau} (R_5) + \mu \phi (R_3)\end{aligned}$$

$$\text{Where } R_0 = \frac{g\beta x}{R}, R_1 = \beta \Delta t, R_3 = O(Pr R_0)$$

$$R_4 = O\left(\frac{\beta T R_0}{R_1}\right), \beta = \frac{1}{T}$$

3.2 Description of the problem

A heating specimen was placed in a leakproof vessel in which undisturbed natural convection is maintained. Heat was transferred from the specimen to the ambient atmosphere of the vessel by natural convection. Energy was supplied to the specimen from an electric source. For a certain ambient pressure and rate of energy input, a set of readings was taken. For the second set of readings, pressure was changed and the rate of energy input was adjusted to keep the specimen temperature same as before. This procedure was repeated 14 times to obtain 14 sets of readings at each inclination of the cylinder. The same sequence was followed for different inclinations of the cylinder and different ambient gases.

3.3 Similarity Analysis

The exact solution of the equations quoted in article 3.1 are very complicated, but the parameters upon which the characteristics of the flow and transport depend may be found by the similarity technique. Considering a vertical surface of height L at a uniform temperature t_w in an extensive uniform medium at t_a , the co-ordinates, velocity components, pressure, and temperature may be normalized by L , U_c , ρU_c^2 , and $t_w - t_a$, respectively. The estimate of convection velocity is given by $U_c = \sqrt{[g\beta(t_w - t_a)]}$. Neglecting energy generation and time dependence, the equations describing the situation in terms of normalized quantities V , $\phi = (t - t_a)/(t_w - t_a)$, P_h , P_m , become

$$\begin{aligned} \nabla \cdot \vec{V} &= 0 \\ (\vec{V} \cdot \nabla) \vec{V} &= \phi \vec{i} - \nabla P_m + \frac{\nu}{U_c L} \nabla^2 \vec{V} \\ \frac{U_c L}{\nu} (\vec{V} \cdot \nabla) \phi &= \frac{k}{C_p \mu} \nabla^2 \phi + \frac{g \beta L}{C_p} \frac{U_c L}{\nu} (\vec{V} \cdot \nabla) (P_h + P_m) + \frac{g \beta L}{C_p} \phi \end{aligned}$$

The boundary conditions are simply

on the surface $V = 0$ and $\phi = 1$

on the distant medium : $V = 0$ $\phi = 0$ and $P_m = 0$

Thus dimensionless parameters which arise in natural convection are

$$\begin{aligned} \frac{U_c L}{\nu} &= \sqrt{\frac{g \beta L^3 \Delta t}{\nu^2}} = \sqrt{Gr} \\ \frac{\mu C_p}{k} &= \frac{\nu}{\alpha} = Pr \\ \frac{g \beta L}{C_p} &= \frac{U_c^2}{C_p \Delta t} = Ec \end{aligned}$$

The quantity $Gr = g \beta L^3 \Delta t / \nu^2$ is called Grashof number and is a measure of the vigor of the flow induced, as indicated by U_c . The Grashof number is essentially the ratio of the relative magnitudes of viscous force and the convection of momentum terms.

The Prandtl number $Pr = \mu C_p / k$, occurs in the energy equation and indicates the steepness of the temperature gradients in the flow field.

The additional quantity $Ec = g \beta L / C_p$ arises as an indication of the relative importance of viscous dissipation.

The above generalization of the equations of motion indicates that the temperature distribution $\phi(x, y, z)$ depends upon Gr , Pr , and Ec . Therefore, heat transfer and the heat-transfer parameter, Nusselt number $Nu = hL/k$, depend only upon these parameter.

$$Nu = F(Gr, Pr, Ec)$$

The above equations and parameters are used in formulating and analyzing various kinds of circumstances.

3.4 Modification of the Similarity Equations for the Present Problem

In the above Nusselt number equation the viscous dissipation factor Eckert number is almost constant since changes in β and C_p are negligible for the present situation. So the Nusselt number equation 3.7 becomes

$$Nu = F (Gr, Pr)$$

Gr can be changed by changing either the fluid properties or the physical dimensions. In the present case physical dimensions remains the same while the changes in density (other properties remaining the same) has been brought about by changing the pressure.

Now for surfaces other than vertical, the inclination θ can be included in the Nusselt number equation 3.8 .

$$\frac{U_c L}{v} = \frac{U_c D \operatorname{Cosec} \theta}{2} = \sqrt{\frac{g\beta D^3 \Delta t}{v^2}} \sqrt{\frac{\operatorname{Cosec}^3 \theta}{8}} = \sqrt{Gr_D} \sqrt{\frac{\operatorname{Cosec}^3 \theta}{8}}$$

As the inclination of the cylinder θ tends to zero, $D/2 \operatorname{Cosec}\theta$ becomes L and $U_c L/v$ becomes $\sqrt{Gr_L}$ as has been discussed earlier. Thus the modified Nusselt number equation becomes

$$Nu = F (Gr, Pr, \theta)$$

For cylindrical surfaces the theoretical analysis is much complicated and is not readily available in the literature. But the vertical cylinders with relatively large diameters can be approximated as vertical flat plates. So instead of deducing separate differential equations, the similarity solution of flat plates will be applied to cylindrical surfaces to correlate the non-dimensional parameters on the basis of the experimental data.

Chapter - IV

Experimental Setup

4.1 General Description

This chapter includes the experimental aspects of the investigation and describes the apparatus used. Figure 18 and 19 show a schematic diagram of the experimental set up and the test cylinder. The apparatus used in the investigation was a " Radiation and Convection Heat Transfer Equipment " , manufactured by Plint and Partners Ltd., England. It consists of an almost spherical steel vessel, within which the test cylinder was suspended. It was equipped with a voltmeter, an ammeter, a vacuum pump, a high pressure connection line and a mercury manometer.

4.2 Test Specimen

The test cylinder in the experiment was made of copper with matt black surface and could be suspended from the top of the vessel at a predetermined inclination. The specimen was 161 mm long, 6.35mm outside diameter and 4.0 mm inside diameter. The test cylinder was heated internally by passing electric current through Ni-Chrome wire. The Ni-Chrome wire was passed through porcelain beads which acted as the insulator between the wire and the cylinder. Beads were approximately 3.85 mm outside diameter, with about 1.0 mm internal hole and 3.5 mm length. At each end of the cylinder a bead extended outside by about 2 mm. The Ni-Chrome wire was fastened to 0.62 mm diameter copper wires at both ends which were used for holding the test cylinder. Holding wires were 90 to 100 mm long. These holding wires could be suspended freely from the cover plate of the vessel top, and in turn supported the test cylinder. The top cover plate had two screws, the inner side of which were connected to the suspending wires and the other side to electric leads.

4.3 Temperature Measuring Devices

The temperature of the cylinder and ambient fluid were measured directly by two independent digital thermometers, fitted with the radiation and convection heat transfer equipment. The difference between the ambient fluid temperature and the outside environment temperature was not much and the outside air movement was also negligible. The vessel size (shown in figure 18) was sufficiently large, about 447 mm in dia. and 465 mm in height, so that the heated fluid rising upward from the test cylinder cooled down to approximately at the bulk fluid temperature as it reached the vessel surface. Since the air outside the vessel was stagnant and both the outside air and inside gas were at equilibrium with the vessel, the vessel temperature might be taken as that of the undisturbed inside ambient fluid. At steady state, the heat generated by the heater in the test cylinder, was equal to the heat that was conducted through the vessel wall to the outside air before appreciable rise in the inside fluid temperature was noticed.

4.4 Pressure Measuring and Control Devices

System pressure (pressure within the vessel) was measured by a Hg manometer provided in the radiation and convective heat transfer equipment. Pressures upto 1000 mm of Hg (gage) could be measured by this manometer. Atmospheric pressure was measured separately by a barometer. To conduct the experiments at high pressure with air, a separate air compressor (1/4 hp, 750 rpm) was connected to the high pressure connection line. By using a pressure release valve, a pressure line disconnection valve and a pressure regulator valve, attached to the apparatus, the desired pressure in the vessel could be maintained. Low pressure could be maintained by operating the vacuum pump (1/4 hp, 822 rpm) of the apparatus. With argon, the experiments were carried out by charging the vessel from a high pressure argon cylinder.

4.5 Input Heat Measuring Devices

The total power input to the apparatus was measured by the voltmeter and the ammeter provided in the apparatus.

Chapter - V

Test Procedure

5.1 Measurement of Temperature

A thermocouple fitted to the mid point of the test cylinder was connected to the digital indicator provided with the experimental set-up. This thermometer directly gives the cylinder surface temperature. It was experimented that the temperature variation along the length of the test cylinder was negligible, so the temperature of middle point was assumed as the average test cylinder surface temperature which will negligibly affect the heat transfer results. The temperature of the vessel was assumed to be same as that of ambient fluid which was measured directly by the thermocouple fixed with the apparatus as mentioned in section 4.3.

5.2 Estimation of Emissivity

The emissivity of cylinder surface which is painted matt black was obtained by an experiment. From this experiment $T_c - T_v$ was plotted against $H^{1/4}$ to facilitate the estimation of $T_c - T_v$ at zero pressure employing the technique of extrapolation. The procedure of estimating emissivity is shown in Appendix B.

5.3 Measurement of Pressure

Pressure in the enclosed vessel was measured by a U-tube mercury manometer. An error of ± 2 mm was observed for the gage pressure reading of ± 750 mm Hg. This might have been due to the non-uniformity of the bore. The error was not significant and as such did not affect the result significantly. Atmospheric pressure was measured once or twice during one set of readings.

5.4 Setting of Inclination of specimen

The top cover-plate of the spherical vessel was fastened to the vessel by twelve bolts fixed on it. To set a given inclination of the test cylinder, the cover plate was opened and the supporting flexible wire length was adjusted. For horizontal position, both supporting wires were of the same length. For 30°, 45° and 60° inclinations one supporting wire length was kept fixed and the other wire length was reduced by $L\cos\theta$ from the length of the wire in the horizontal position. Neglecting the obliquity, the inclination of the wire was taken as 0. For vertical position, the test cylinder was practically suspended from one supporting wire and the other support was replaced by a sufficiently long wire so that it hung freely.

5.5 Determination of Heat Transfer Coefficient

The electrical energy input to the test cylinder was transferred partly to the enclosed vessel by radiation, partly to the thermocouples and holding wires by conduction and the rest to the ambient fluid by convection. The procedure of estimating each of them are discussed below. Having obtained the convective heat transfer rate, the convective heat transfer coefficient was determined by dividing the convective heat transfer rate by the product of the surface area of the test cylinder and the temperature difference between the test specimen and the undisturbed ambient fluid.

5.5.1 Measurement of Input Power

The input voltage and amperage was measured by the voltmeter and ammeter provided in the apparatus. The test cylinder was heated electrically by a 220/230 V ac supply through a transformer and a rheostat, contained in the apparatus. The voltmeter could read upto 15 volt with an accuracy of 0.1 volt. The range of ammeter was 0 to 1.0 ampere and the minimum value that could be read was 0.01 ampere. To get the power input to the test cylinder a separate wattmeter was used to measure the actual power consumed by it. It was seen that an input factor of 0.96 could be integrated which on multiplication to the input volt-ampere gave the heater energy consumption.

5.5.2 Estimation of Conduction Loss

Since thermocouples were attached to the surface of the test cylinder a portion of energy generated in it was conducted through the thermocouples and holding wires to the ambient fluid. Appendix A contains the outline of the procedure for calculating the conduction losses.

5.5.3 Estimation of Radiation from the Cylinder

A substantial part of the heat input to the cylinder was transferred by radiation to the vessel enclosure. This radiation heat transfer was evaluated by using the Stefan-Boltzman equation. The relevant emissivity value used was obtained by using the method discussed in section 5.2.

5.6 Test Procedure

The experiment was conducted with a cylindrical body. The two different conduction fluids used were air and argon. Five orientations of the test cylinder with angles of inclination of 0°, 30°, 45°, 60° and 90° with the vertical were studied at constant surface temperature of 91.5°C. A brief description of the procedure is given below:

1. The two ends of the test cylinder were connected to the holding wires, hanging vertically down from inside face of the cover plate. The thermocouple attached to the mid point of the cylinder was connected to the digital indicator of the set-up.
2. The cover plate was held horizontal and the test cylinder was allowed to hang freely to see if it lay at the desired configuration without any kink or twisting in the supporting wires.
3. The cover plate was set in its seat on the vessel and the nuts were tightened. The leads of the heater line were connected to the screws on the cover plate.

4. To carry on experiments with air as the environmental fluid, the gas release valve was closed, the gas line disconnecting valve was opened, the compressor delivery line was connected to the high pressure connection line and the compressor was switched on. The gas disconnecting valve was closed when the Hg manometer showed a gage pressure of about 900mm. The compressor was switched off and the gage pressure reading was recorded.
5. The heater switch was turned on, the reostat was adjusted to get the test-cylinder temperature (t_c) approximately 91.5°C. When the cylinder surface temperature remained almost unchanged for about half an hour, it was assumed that the system was in steady state condition.
6. The voltmeter and ammeter readings were noted.
7. The vessel temperature was recorded.
8. The manometer reading was taken.
9. To set at different pressure, some amount of air was let out by opening the gas release valve until the pressure dropped by about 175mm, for the next reading. Then the gas release valve was closed.
10. Voltmeter and ammeter were adjusted again to have t_c about 91.5°C. Sufficient time was allowed to attain steady state for the next set of readings and the relevant readings as in steps (6), (7) & (8) were recorded.
11. Steps (9) and (10) were repeated until the vessel pressure became nearly atmospheric.
12. The vacuum pump was switched on and the pump line connecting valve was opened so that the vessel pressure fell below atmospheric. When inside

pressure dropped by about 125mm, the vacuum pump connecting line valve was closed and the pump was turned off.

13. Step (10) was repeated.
14. The vacuum pump was switched on again and the valve in the vacuum pump connecting line was opened. As the inside pressure dropped by about 75 mm, the vacuum line valve was closed and then the vacuum pump was switched off.
15. Step (10) was repeated.
16. Step (14) and (10) were repeated until a vacuum pressure of about 750 mm of mercury attained.
17. To carry out the investigations with argon, the enclosed vessel was filled with argon up to 800 mm gage. The vessel pressure was then reduced to about 750 mm Hg vacuum and then the final charging was made that raised the inside pressure to about 800 mm Hg gage. The recharging of argon was necessary to reduce the dilution of argon with air to about .002%.
18. Steps (5) through (16) was followed to obtain the data with argon for the same inclination.
19. The valves were then opened letting the pressure inside the pressure inside the vessel to be atmospheric. The wires connecting the heater line were disconnected and the cover plate was opened.
20. To align the cylinder at 30° inclination, one of the supporting flexible wire lengths was kept fixed (160 mm) and the other wire length was reduced by an estimated amount of $L\cos 30^\circ$. The supporting wires were then connected to the

screws at the inside face of the cover plate of the vessel.

21. Steps (2) through (18) was repeated.
22. Step (19) was repeated.
23. Two more sets of readings, one for 45° and the other for 60° were completed taking steps similar to (20) and (2) through (18).
24. For conducting the experiment at the vertical position of the test cylinder, one supporting wire length was let loose keeping the other wire length 160 mm. Due to self weight the test cylinder hung vertically from the shorter supporting wire (160 mm). Steps (2) through (18) were repeated.

Chapter - VI

Results And Discussions

In the present study, investigations were carried out for the natural convection heat transfer from a heated inclined cylinder to the enclosed ambient fluids. The investigation was carried out upto a Rayleigh number (based on length) of 1.9×10^7 and hence the convection flow was in the laminar range. The fluids used were either air or argon.

The test cylinder and ambient fluid were contained in a sealed enclosure in such a way that outside fluid did not affect the inside fluid property or heat transfer behaviour directly. Readings were recorded only after acceptable steady state situation of the overall system was attained.

The data were taken at a steady state of wall temperature of 91.5°C . This temperature was selected after conducting a few trial runs at different wall temperatures. It was observed that as the wall temperature changes, the ambient fluid temperature also changes in the same direction causing slight change in convection heat transfer. This observations lead to be chosen the wall temperature of 91.5°C , since the percentage of error in taking readings will be smaller at higher temperature readings.

At a given inclination, a reduction in ambient fluid pressure was resulted in an increase in surface temperature of the test cylinder. To keep the surface temperature constant at 91.5°C , the wattage input was decreased by adjusting the rheostat and hence the voltmeter and ammeter readings. The environment temperature change was small which retained the radiation heat transfer almost constant and with reduced heat input to the system, it ultimately led to a reduction of convective heat transfer.

Figure 1 shows the comparison of heat transfer coefficient in the environment of air and argon.

for an inclination of 45° of the heated body. From this graph it is observed that the heat transfer co-efficient for air is higher than that for argon at this inclination and ambient pressure. Air has a higher value of specific heat and conductivity than argon. Higher value of specific heat helps air to hold more heat and higher value of conductivity means higher molecular vibration which ultimately leads to higher convection heat transfer. Similar nature of the graph was found for all the other inclinations and thus they are not presented.

Figure 2 shows the plot of average heat transfer co-efficient in air for different inclinations. There are five curves in this plot and they correspond to the inclinations of 0° , 30° , 45° , 60° and 90° inclinations with vertical. It reveals that the variations of the observed heat transfer co-efficients with pressure are logarithmic. The heat transfer coefficient approaches zero value as absolute fluid pressure approaches zero. At low pressures the rate of change of heat transfer coefficient is more than that at high pressures. The logarithmic curve fitting gives high correlation co-efficients of almost 0.99. At low pressure the density of the gas decreases and therefore the scope of convection decreases with almost constant radiation. Although with the decrease of density, inter molecular space or meanfree path increases linearly, the molecular velocity distribution is logarithmic with pressure. So heat transfer co-efficient decreases logarithmically with the decrease of pressure. Figure 3 is the plot of heat transfer coefficients obtained for the specimen with argon. Nature of these curves in this plot is similar to those in figure 2 and the heat transfer coefficient varies with the pressure and inclination exactly in the same way as they do in figure 2 for air.

Figure 4 shows the variation of heat transfer co-efficient (h) with the inclination (θ) of the test cylinders in air. It is seen that with the increase of inclination with vertical, the heat transfer rate increases. Figure 5 shows the plot of h vs. θ for argon data. Both the plots show the same nature. With the decrease of inclination, the thickness of the boundary layer on the cylinder surface increases. The pattern of the heat transfer from the upper surface of the cylinder is unaffected by the inclination except at vertical but that of the lower surface is greatly affected. On the upper surface, fluid molecules that come in contact with the cylinder is heated and goes upward without interrupting heat transfer from the cylinder surface. From the lower half of the cylinder, the fluid particles cannot directly move upward, they have to overcome the

cylinder circumference by enveloping it. For the horizontal cylinders the heated fluid rises in the form of a plume and the vertical length travelled by a particle while rising from the lower surface is $D/2$. For inclined cylinder except vertical, this length is $D/2 \operatorname{Cosec}\theta$, $\theta > 0^\circ$, with the constraint that $D\operatorname{cosec}\theta/2$ must be less than cylinder length L . In vertical position (0°), the fluid particles around the test cylinder form an almost semi-paraboloid boundary layer and the extent to which the fluid layers are in contact is the cylinder length L . As the travel of the plume increases, analytically it has been shown in literatures that heat transfer coefficient decreases. That is as the extent to which fluid particles remain in contact with the solid along the direction of flow increases, the thickness of the boundary layer also increases. Inside the boundary layer the bulk fluid cannot come in contact with the heated surface. Moreover for fluid particles inside the thermal boundary layer, the temperature difference between the solid and fluid is lower. As a result lesser amount of heat is transferred to the ambient fluid.

Figure 6 shows the log-log plot of Nusselt number versus normalised pressure for air. There are five curves in the figure corresponding to five inclinations of the test cylinder. Like the heat transfer coefficient as in figure 2, the Nusselt number also increases with the increase of inclination and as well as the increase of pressure. Here the variation of conductivity k is almost insignificant and hence heat transfer coefficient will be almost proportional to the Nusselt number if the length of the cylinder is the same. So, the figure 6 has the same nature of figure 2. Despite the deviation of the data, the linear curve fitting for the log-log plots gives the correlation coefficient of 0.99 or better. Figure 7, for argon as environment, is similar to figure 6.

Figure 8 shows the average plot of Nusselt number versus Rayleigh number for both air and argon for different inclinations of the test cylinder. These curves also shows logarithmic plot like that of figure 6. In the present study the cylinder temperature t_c is constant with small change in ambient temperature. Thus the change in β and ΔT is insignificant. Grashof number and hence Rayleigh number becomes almost proportional to the density i.e. pressure, since the volume in this case is constant and Prandtl number is almost unchanged. From the graphs it is seen that the merging of air and argon data in non-dimensional form is

random. It is observed that the Nusselt number increases logarithmically with Rayleigh number. Figure 9 is the log-log plot of Nusselt number versus Rayleigh number. The fitting of the curves are seen to be linear. These curves also show high correlation coefficient, higher than 0.98.

Figure 10 is the plot of Nusselt number versus the Rayleigh number merging both air and argon data with logarithmic curve fitting. The curves show correlation coefficient of the order of 0.99. The correlation proposed in this thesis has been derived from this plot.

Figure 11 shows the plot of average Nusselt number vs. Rayleigh number obtained from Arabi and Salman¹⁵ data. The nature is similar to the plot of those of the present work.

Figure 12 shows the Nusselt number-Rayleigh number plot that compares the present experimental data and Arabi & Salman¹⁵ data for the inclination of 60° with vertical. It is seen that there is a deviation. This is because Arabi and Salman data are for constant heat flux whereas the present experimental data are for constant surface temperature.

Figure 13 is the comparison of the present experimental data with that of Nagendra et al³⁵ for vertical cylinder. Here also a deviation is observed which evolves probably from the different boundary conditions used by them. Nagendra et al investigation is for constant heat flux condition.

Figure 14 shows the comparison of the data of the present work with that of Socio²⁷ for horizontal cylinder. It is seen that there is a deviation in this graphs also. Here both set of data has been taken at constant surface temperature condition but the present experiment has been carried out at different surface temperature (91.5°C) than that of Socio's. Moreover the present study has been carried out at an ambient pressure other than atmospheric while Socio's experiment was at atmospheric pressure only.

Chapter - VII

Conclusions And Recommendations

7.1 Conclusions

The important conclusions from the present investigations are enlisted below:

1. The natural convection heat transfer coefficient h is a function of both ambient fluid pressure (P/P_a) and inclination of the cylinder (θ).
2. At any inclination of the cylinder and at a given ambient fluid pressure, the natural convection heat transfer coefficient is smaller for argon than that for air.
3. The heat transfer coefficient increases linearly with the inclination of the cylinder. So, heat transfer coefficient is maximum for the horizontal position of the cylinder.
4. The log-log plot of Nusselt number (Nu) vs. normalised pressure (P/P_a) is linear for both air and argon.
5. The Nusselt number (Nu) can be correlated with Rayleigh number (Ra), and inclination (θ) by the empirical formula

$$Nu_L = C (Ra_L)^m$$

$$\text{where } C = 2.776 - 0.4377 \sin^3\theta + 0.9972 \sin^4\theta$$

$$\text{and } m = 0.1913 + 5.914 \times 10^{-4} \sin \theta + 0.0156 \sin^2\theta$$

where θ is the inclination with vertical with $0^\circ \leq \theta \leq 90^\circ$

The fluid properties have been taken at a film temperature $t_f = (t_c + t_a)/2$. The procedure is described in Appendix C.

7.2 Recommendations

1. The variation of Nusselt number for other fluids specially those with better transport properties like helium can be carried out to organize a general correlation.
2. Investigations can be carried out with bodies of other geometries (like wires, different L/D ratios etc.).
3. Investigations may also be carried out by determining the velocity and temperature field experimentally using methods like interferometric technique and Laser doppler anemometry (LDA).
4. By controlling the energy input, the investigation can also be carried out with constant heat flux.
5. From the present experimental data, a computer program can be set up for further development of research so that heat transfer situation of other conditions of operation can be predicted.

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Appendix - A

Correction Factors

As a consequence of the electrical resistance of the leads that supply power to the element and support it, a correction factor must be applied to voltmeter and ammeter readings as suggested by Plint and partners ltd³⁴ which is

$$Q_{\text{corrected}} = 0.96 VI$$

In addition an allowance must be made for heat losses by conduction along the current carrying the thermocouple leads. the effect of these is complex, heat is lead down the conductors and then carried radially to the surface of the insulating sleeving covering the conductor where heat is dissipated by radiation and convection.

It is found that a good approximation to the effect of the supporting leads may be by considering them as equivalent to the combination of a conductor and an increase in surface area of the cylinder.

The conductivity factor is equivalent to a loss of:

$$0.00168 (T_c - T_v) W$$

while the additional surface area is equivalent to:

$$0.02A$$

Combining all these correlations it is found that:

$$Q = 0.96 VI - 0.00168 (T_c - T_v) \text{ and } A = 1.02(\pi d^2/2 + \pi dl)$$

Appendix B

Emissivity Estimation

In order to determine the emissivity of the test cylinder with matt black surface, a test run was made from atmospheric pressure to the lowest attainable pressure which was about 10 mm Hg absolute for the present apparatus. The manufacturer of the apparatus recommended that the temperature between the test cylinder and undisturbed ambient fluid (vessel) varies linearly with fourth root of the absolute pressure inside the vessel. Figure 15 shows the variation of ΔT i.e., $(T_c - T_v)$ with $H^{1/4}$, where H is the absolute pressure inside the vessel in mm of Hg with constant heat input of 2.706 watt.

In figure 15 the linear curve has been extrapolated to reach the absolute zero pressure i.e., vacuum line. The corresponding ordinate represent the temperature difference between the test cylinder and the vessel if the pressure inside the vessel would be reduced to zero. Thus $(T_c - T_v)$ at vacuum can be read from the graph. From figure 15 this value is seen to be approximately 81.63°C. From observation it was seen that the vessel temperature did not vary much, the variation was less than 2°C, The average temperature corresponding to the three lowest pressure was taken as the vessel temperature at vacuum and calculations corresponding to vacuum condition were performed. The relevant data and calculations are shown in table-5.

Table-5

Parameter	Value
$(T_e - T_v), ^\circ K$	81.63
$T_v, ^\circ K$	299.07
$T_e, ^\circ K$	380.7
$q_T, \text{ watt}$	2.706
$L, \text{ mm}$	165
$D, \text{ mm}$	6.35
$CA = \pi DL + \pi/2(D^2), \text{ mm}^2$	3354.94
$A = 1.04 \times CA, \text{ m}^2$	3.422
$q_{\text{cond}} = .00168\Delta T$	0.137
fac	0.96
$q_{\text{in}} = q_T \times \text{fac}, \text{ watt}$	2.598
$q_r = q_{\text{in}} - q_{\text{cond}}, \text{ watt}$	2.461
$\varepsilon = q_r / [A\sigma(T_e^4 - T_v^4)]$	0.9687

Therefore the emissivity for the specimen is 0.9687.

Appendix C

Determination of Coefficient C and Exponent m

From figure 10 five equations of five curves have been obtained. They are

$$\begin{aligned} \text{Nu}_L &= 2.755 (\text{Ra}_L)^{0.1915} && \text{for } \theta=0 \\ \text{Nu}_L &= 2.818 (\text{Ra}_L)^{0.1939} && \text{for } \theta=30^\circ \\ \text{Nu}_L &= 2.870 (\text{Ra}_L)^{0.2021} && \text{for } \theta=45^\circ \\ \text{Nu}_L &= 3.029 (\text{Ra}_L)^{0.2023} && \text{for } \theta=60^\circ \\ \text{Nu}_L &= 3.347 (\text{Ra}_L)^{0.1993} && \text{for } \theta=90^\circ \end{aligned}$$

i.e , in the form $\text{Nu}_L = C (\text{Ra}_L)^m$

Now

At $\text{Sin}\theta=0$	$C=2.755, m=0.1915$
At $\text{Sin}\theta=0.5$	$C=2.818, m=0.1939$
At $\text{Sin}\theta=0.7071$	$C=2.870, m=0.2021$
At $\text{Sin}\theta=0.866$	$C=3.029, m=0.2023$
At $\text{Sin}\theta=1$	$C=3.347, m=0.1993$

For the calculation of C, C against $\text{Sin}\theta$ has been plotted in figure 16 and the relation becomes

$$C = 2.776 + 0.4337 \text{Sin}^3\theta + 0.9972 \text{Sin}^4\theta$$

For the calculation of m, m against $\text{Sin}\theta$ has been plotted in figure 17 and the relation becomes

$$m = 0.1913 + 5.915 \times 10^{-4} \text{Sin } \theta + 0.0156 \text{Sin}^2 \theta$$

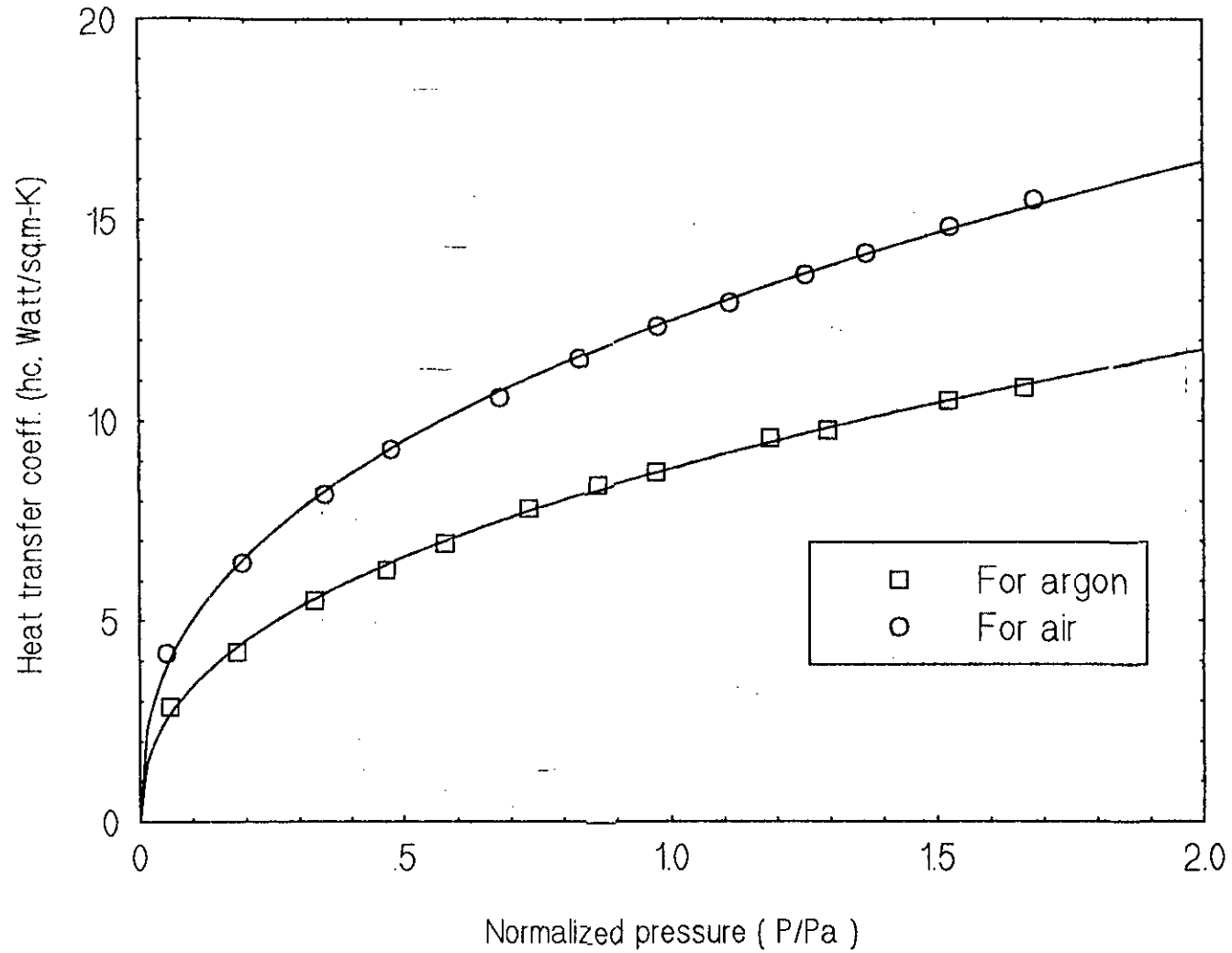


Fig. 1: Comparison of heat transfer coefficient in both air and argon at an inclination of 45° with vertical.

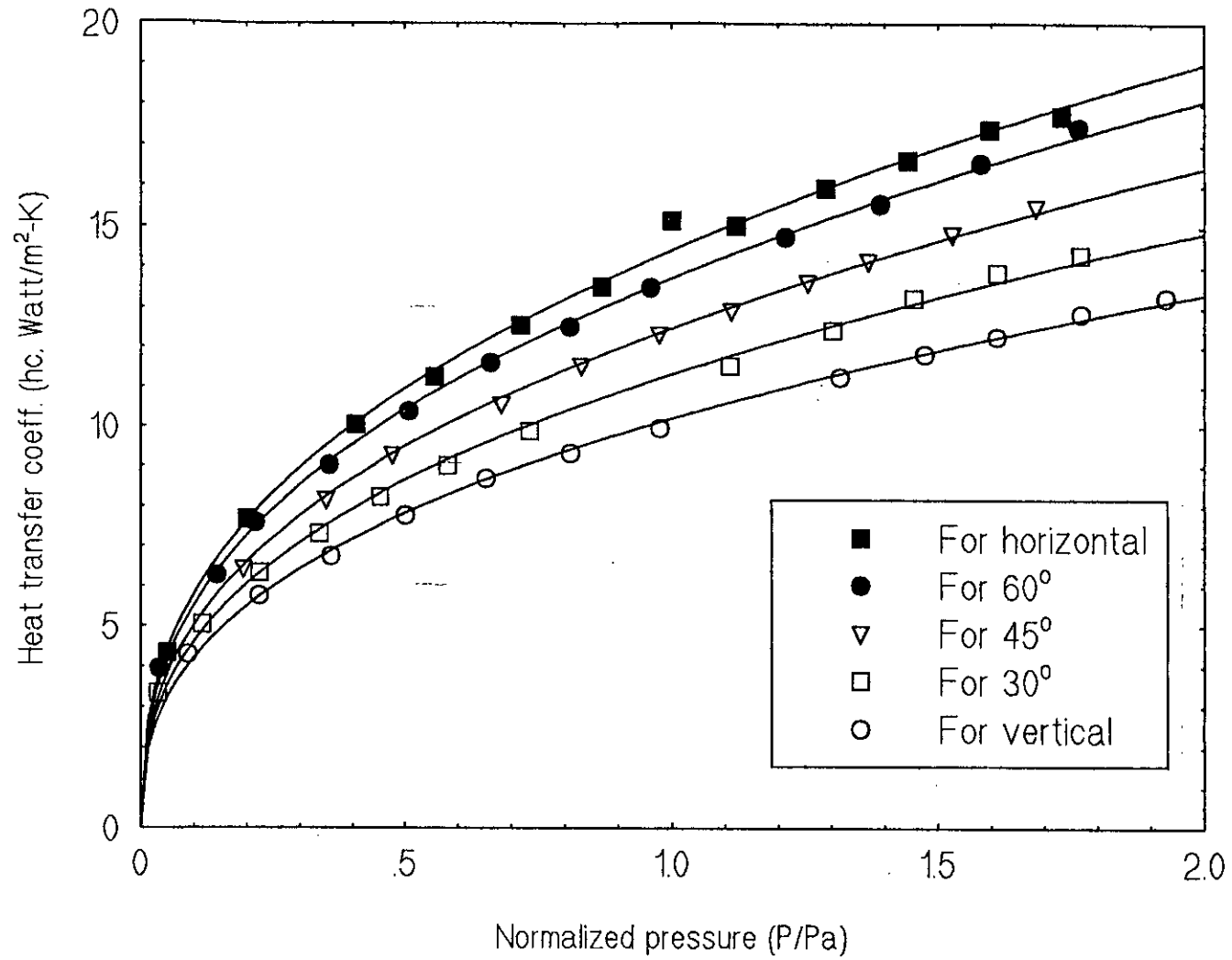


Fig. 2: Plot of heat transfer coefficient against normalised pressure for different inclinations of the cylinder in air.

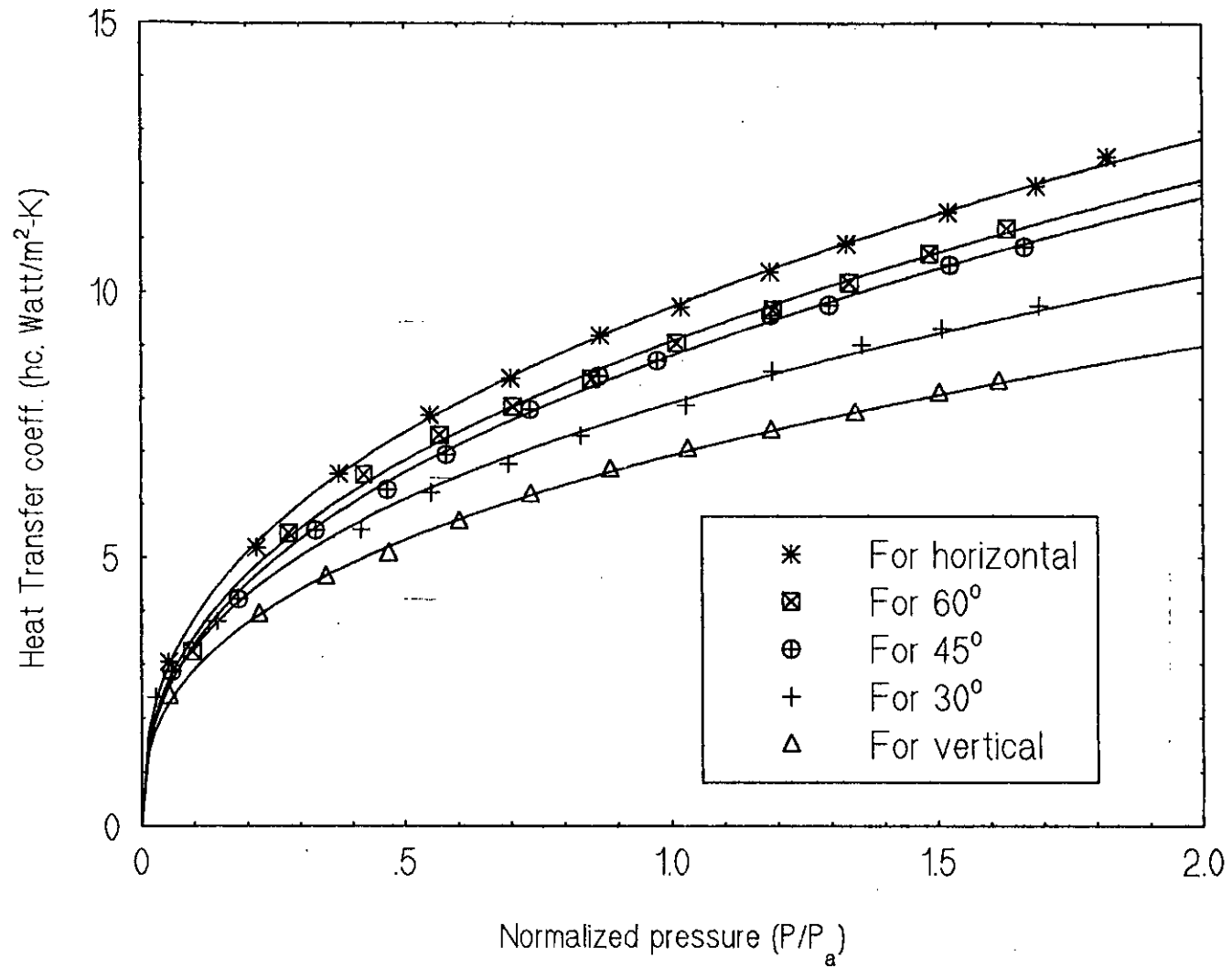


Fig. 3: Plot of heat transfer coefficient against normalised pressure for different inclinations of the cylinder in argon.

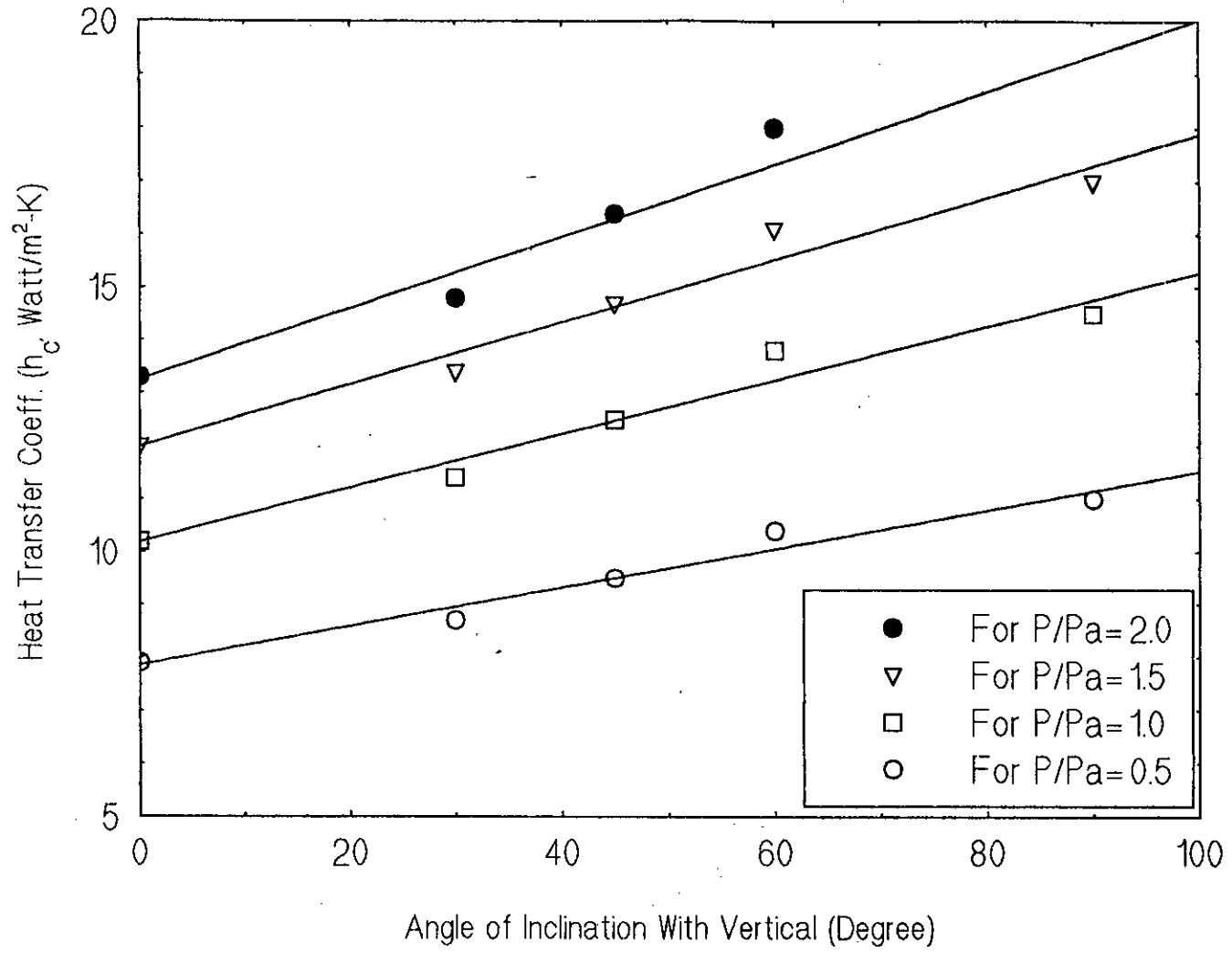


Fig. 4: Plot of heat transfer coefficient versus inclination of the cylinder in air.

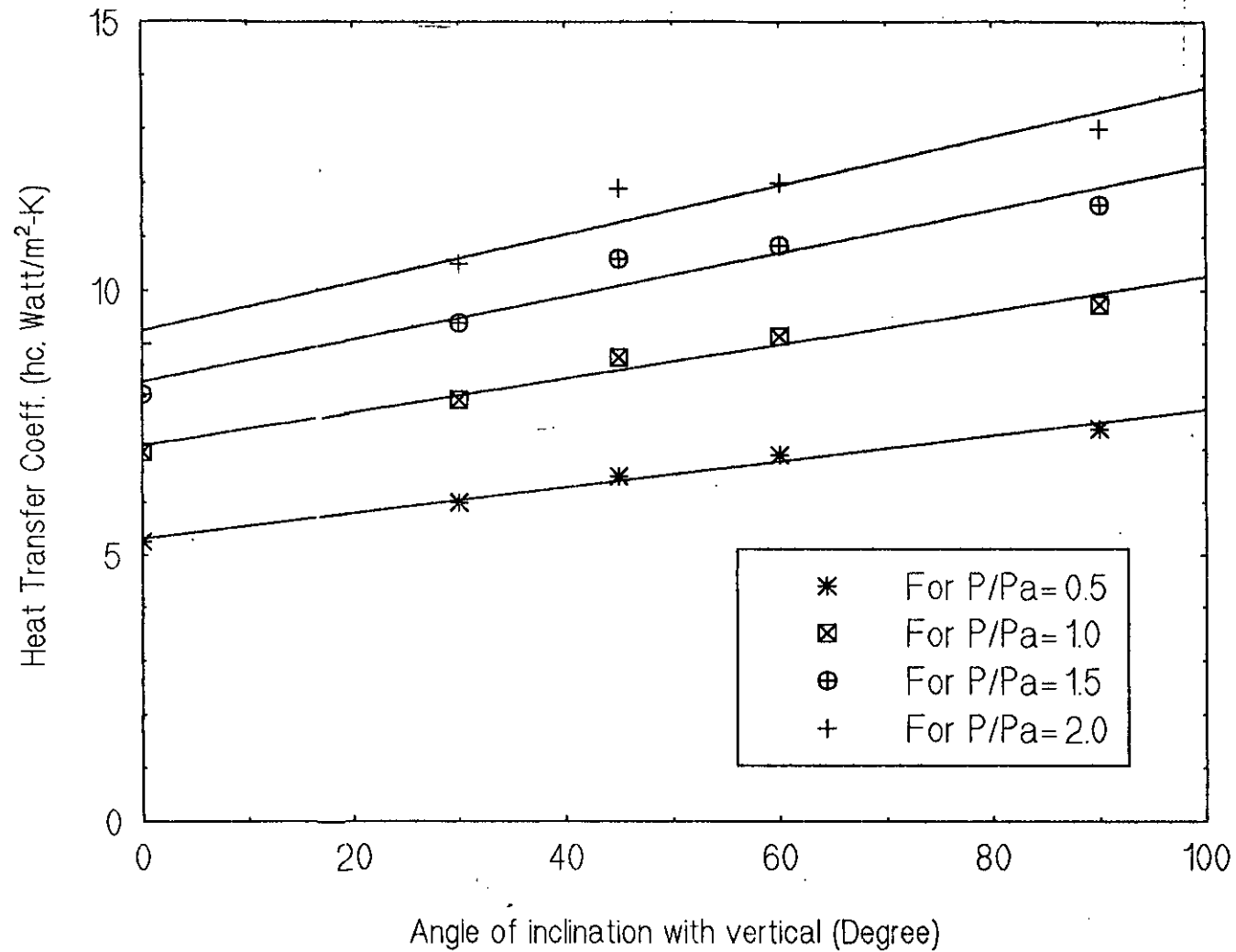


Fig. 5: Plot of heat transfer coefficient versus inclination of the cylinder in argon.

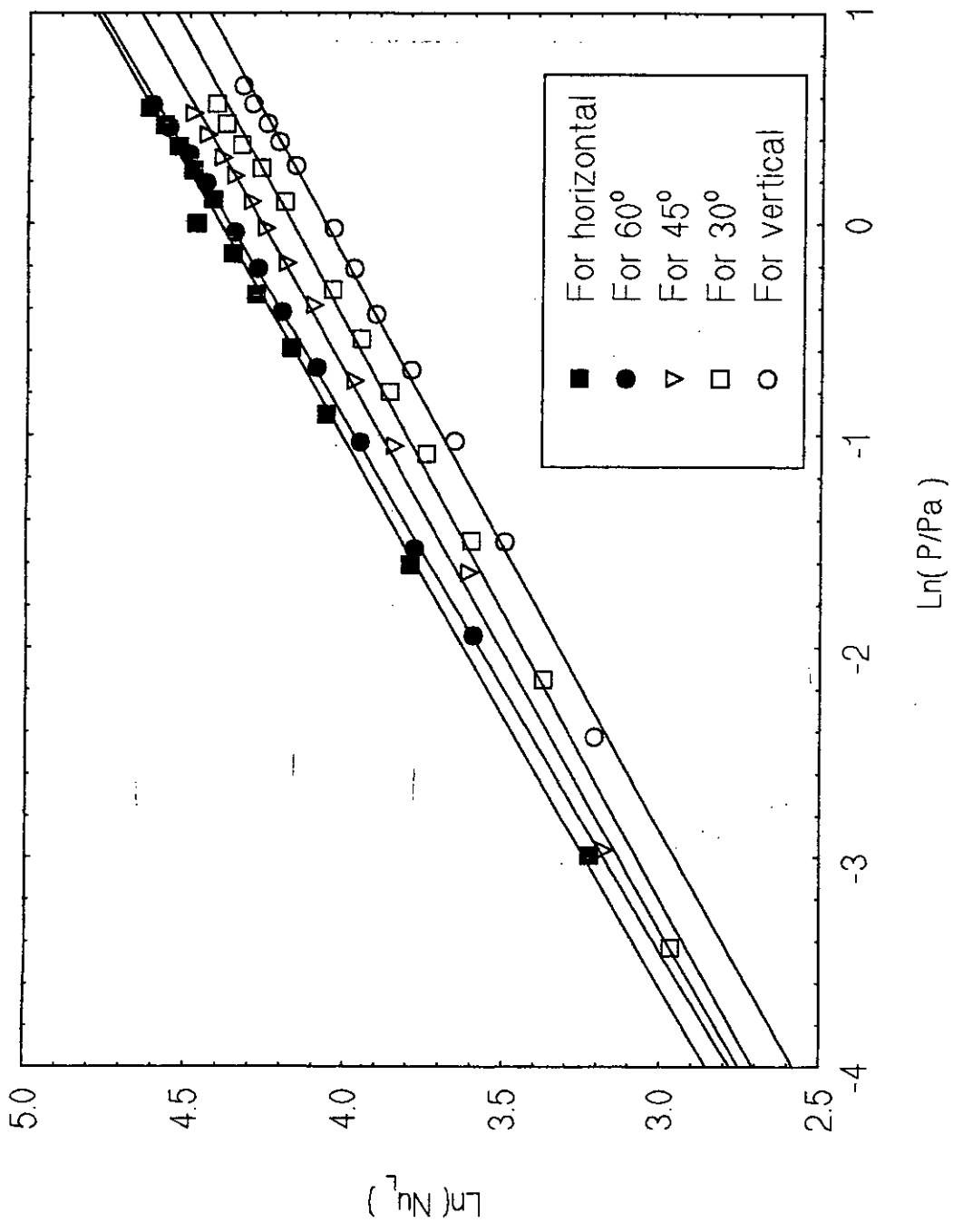


Fig. 6: Ln-Ln plot of Nusselt number versus Normalised pressure for air.

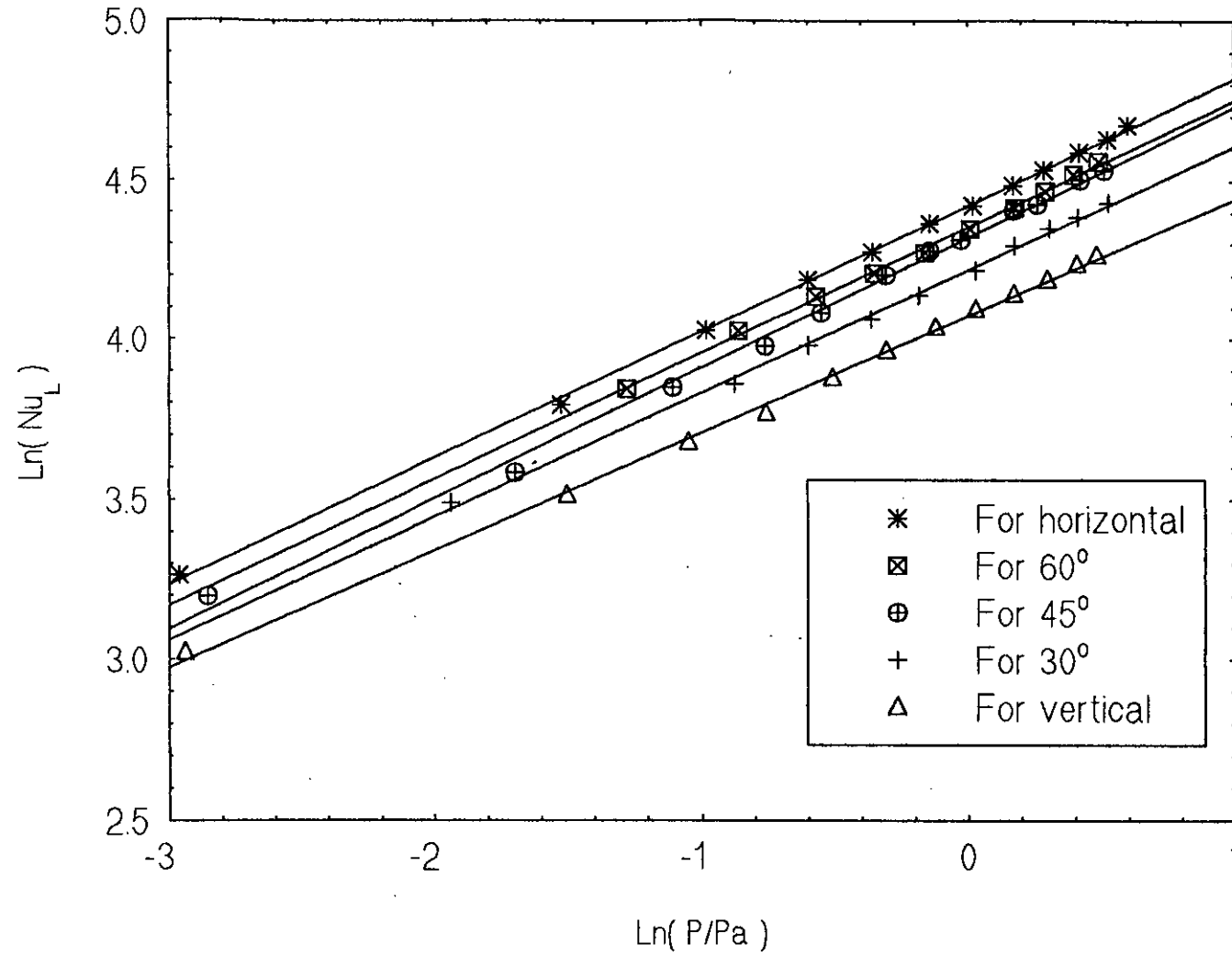


Fig. 7: Ln-Ln plot of Nusselt number versus normalised pressure for argon.

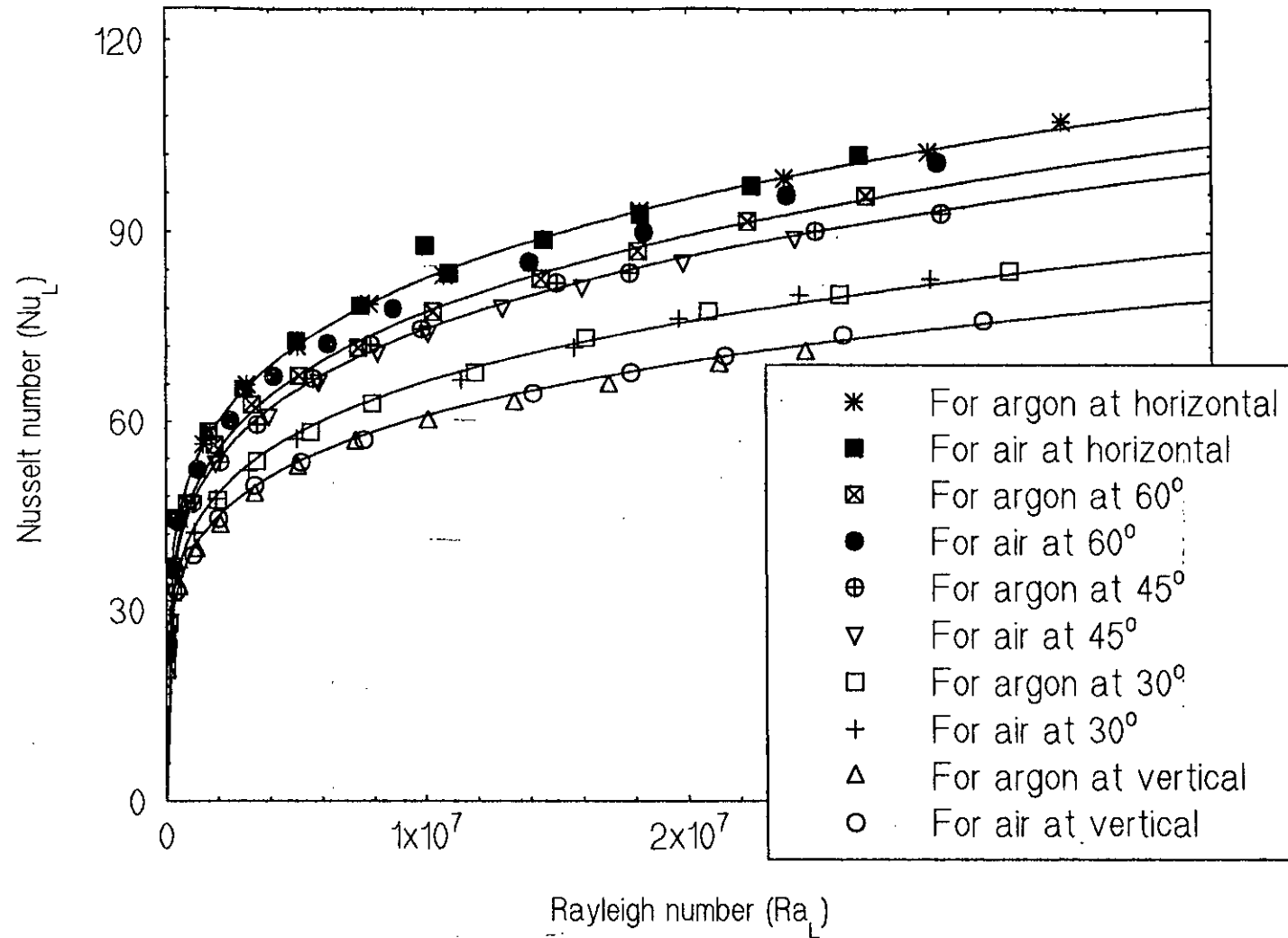


Fig. 8: Plot of Nusselt number against Rayleigh number for both air and argon.

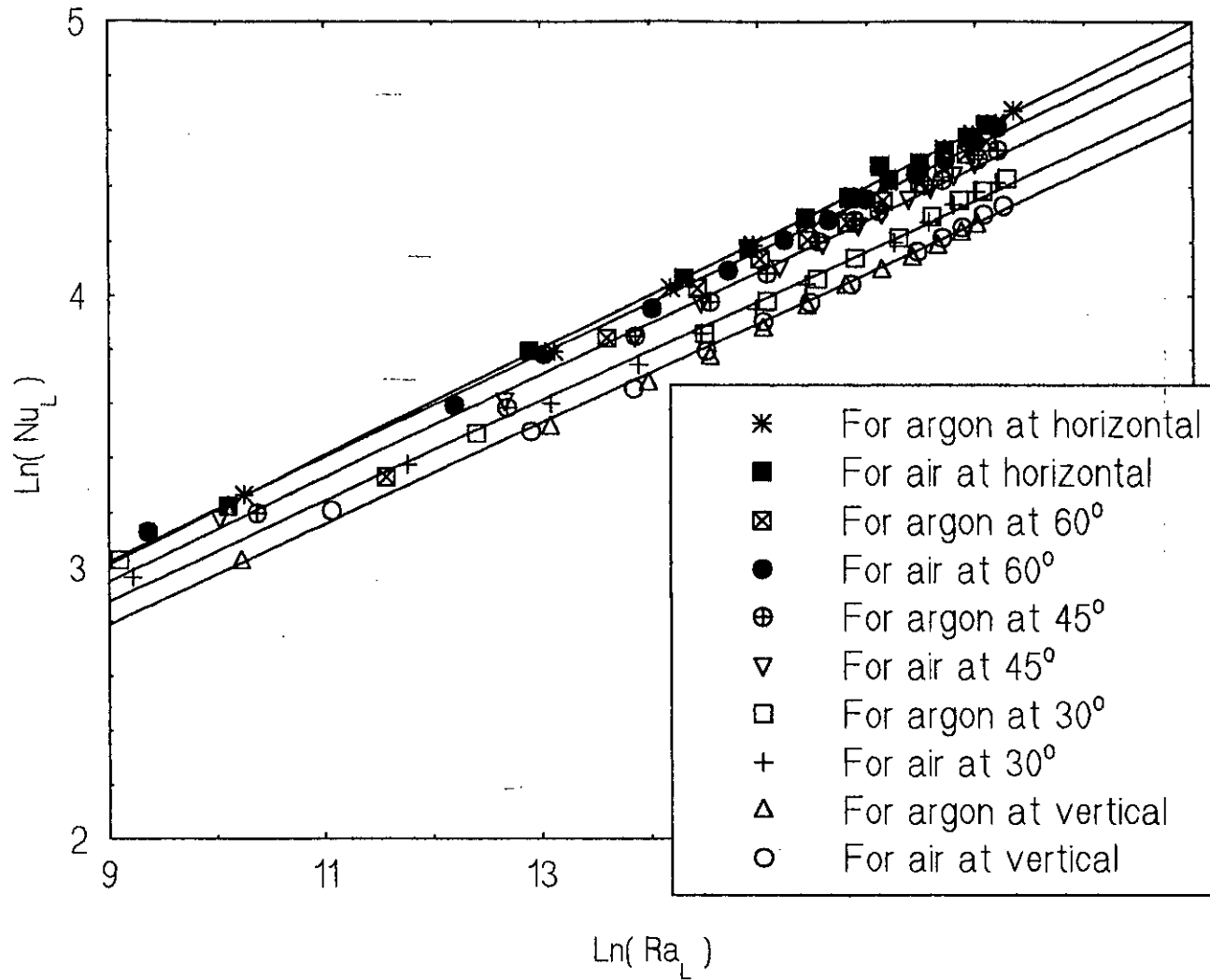


Fig. 9: Ln-Ln plot of Nusselt number versus Rayleigh number for both air and argon.

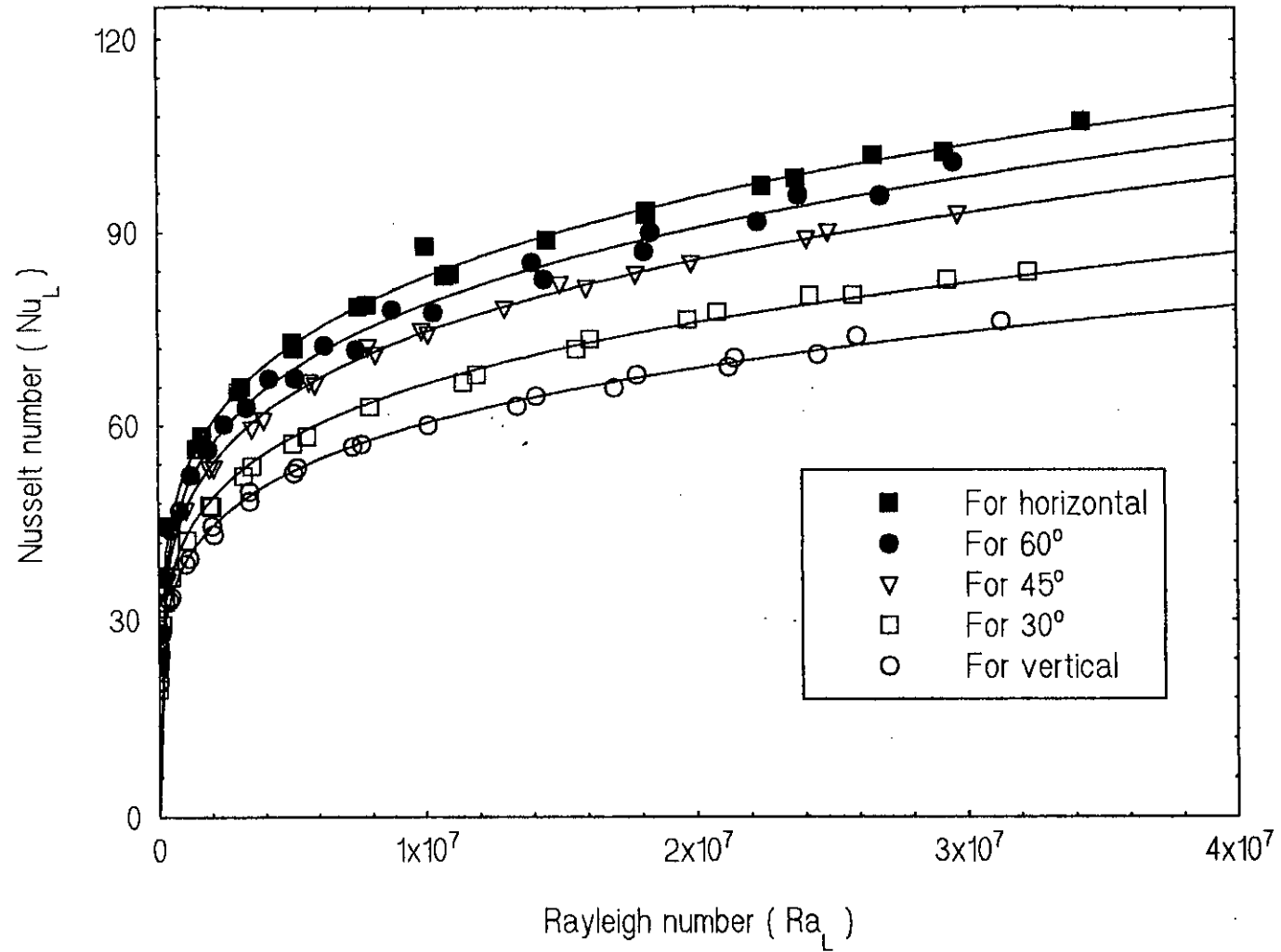


Fig. 10: Combined plot of Nusselt number against Rayleigh number for both air and argon.

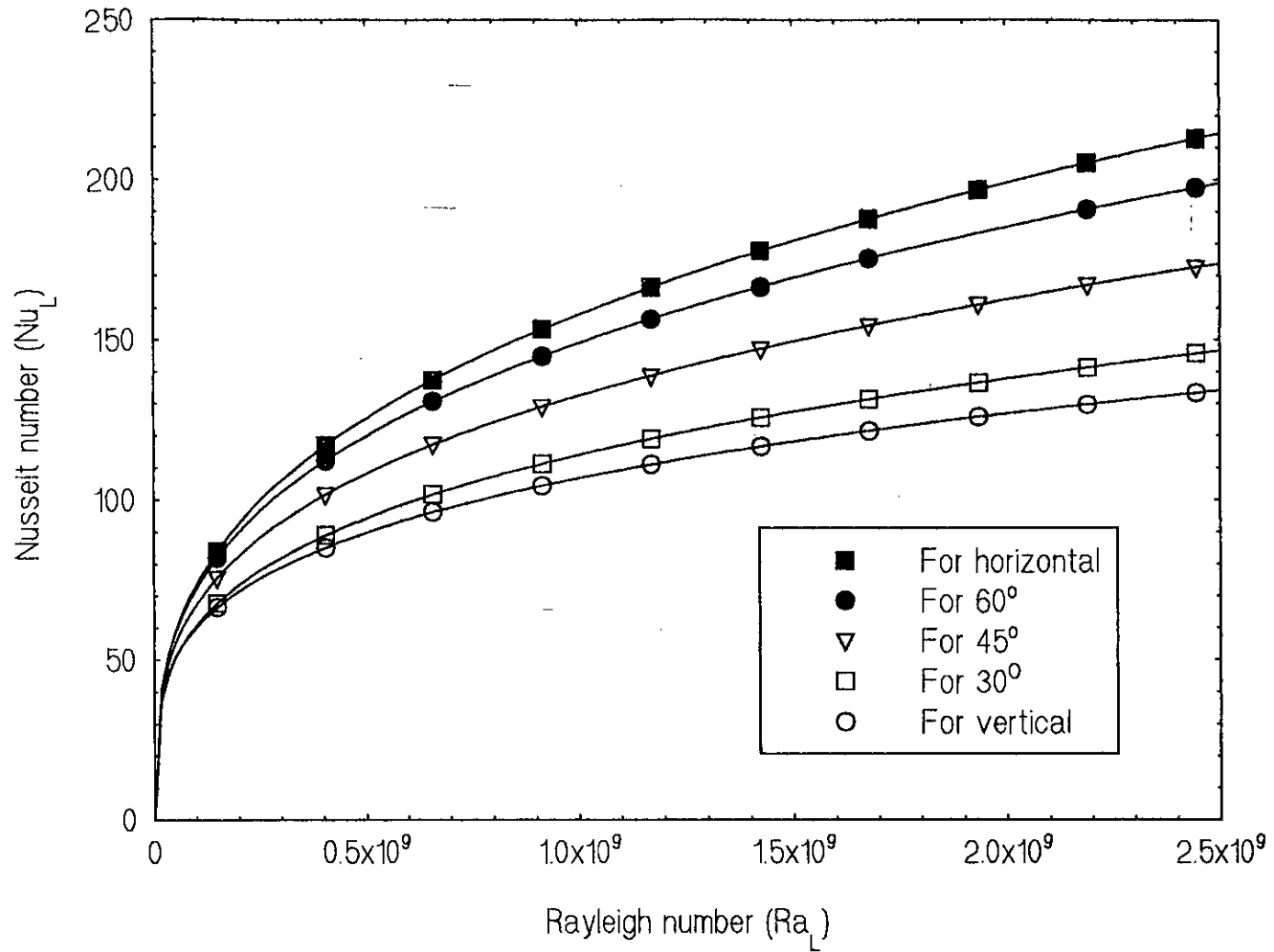


Fig. 11: Plot of nusselt number versus Rayleigh number at different inclinations with Arabi and Salman data.

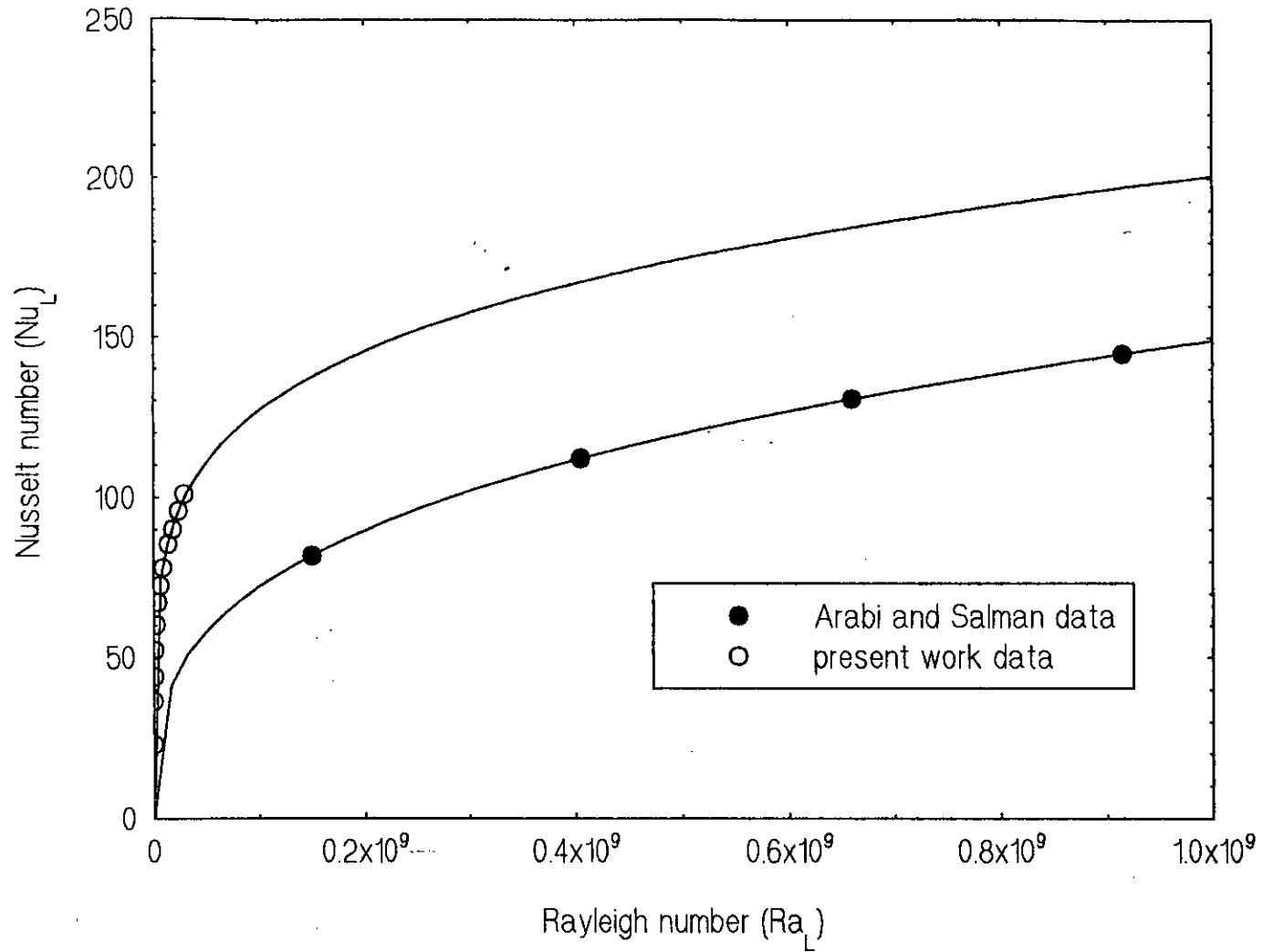


Fig. 12: Comparison of Nusselt number for Arabi and Salman data and present work data at 60° inclination.

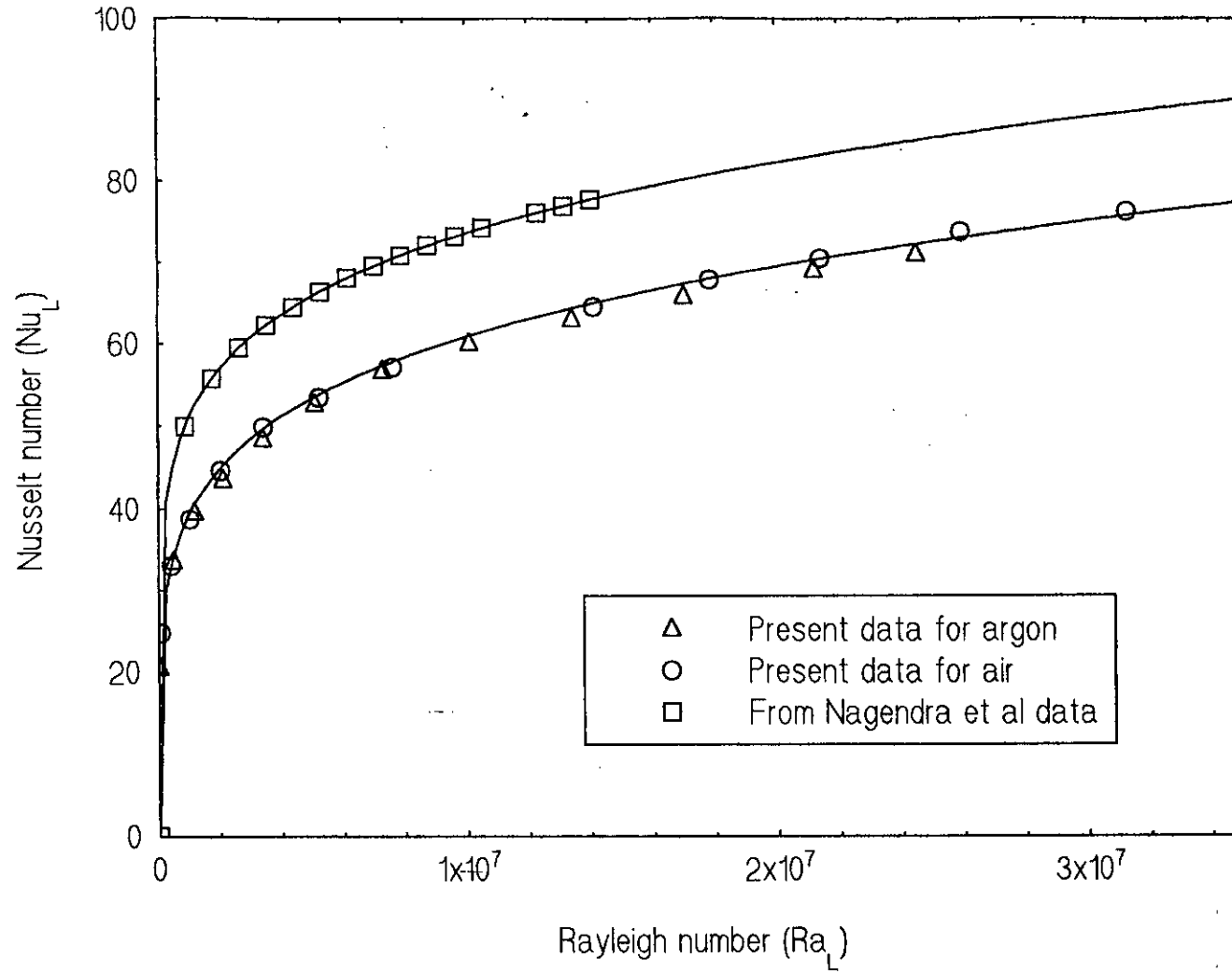


Fig. 13: Comparison of Nusselt number for present work data and Nagendra et al data for vertical cylinder.

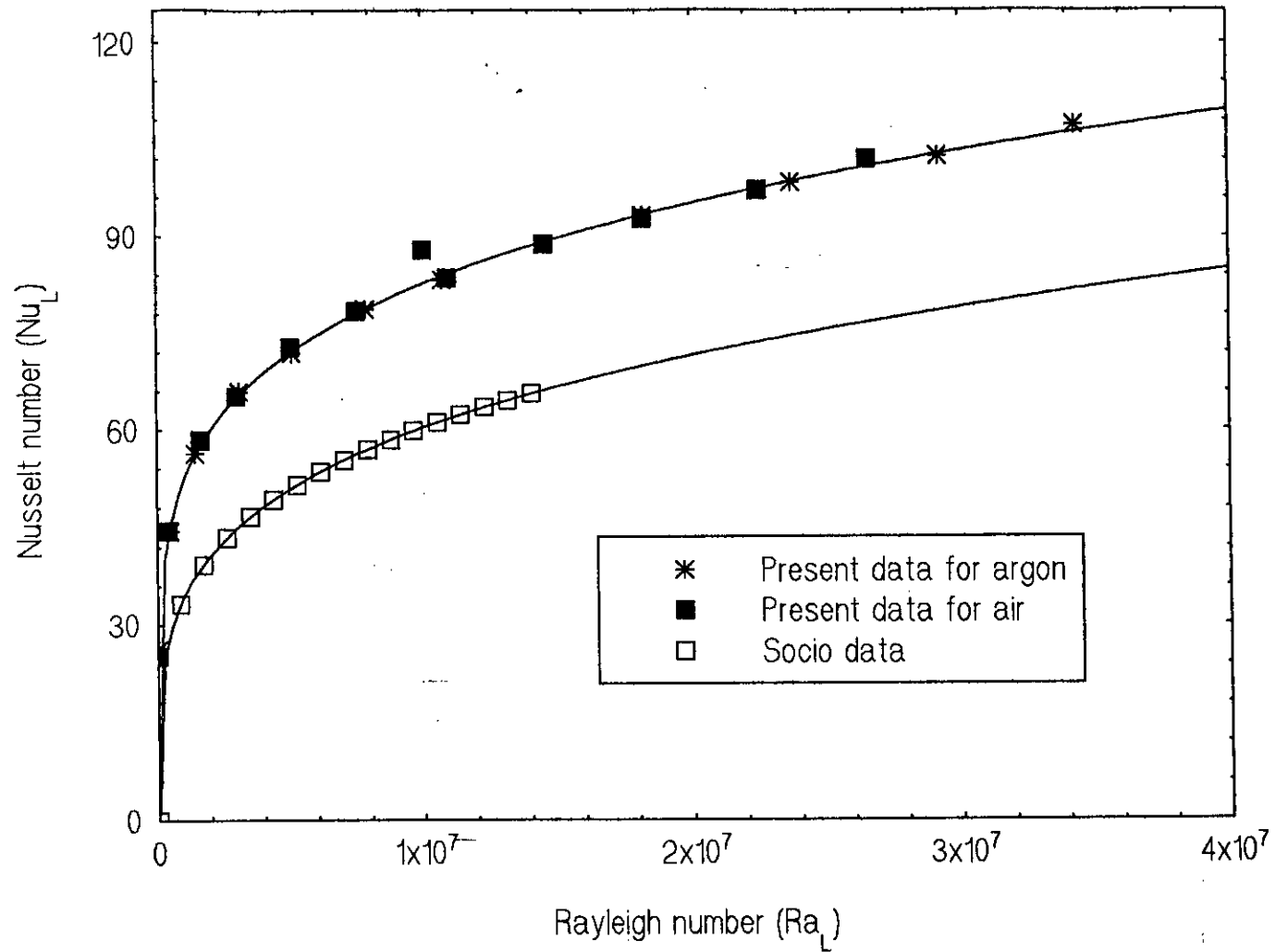


Fig. 14: Comparison of Nusselt number for Socio data and present work data for horizontal cylinder.

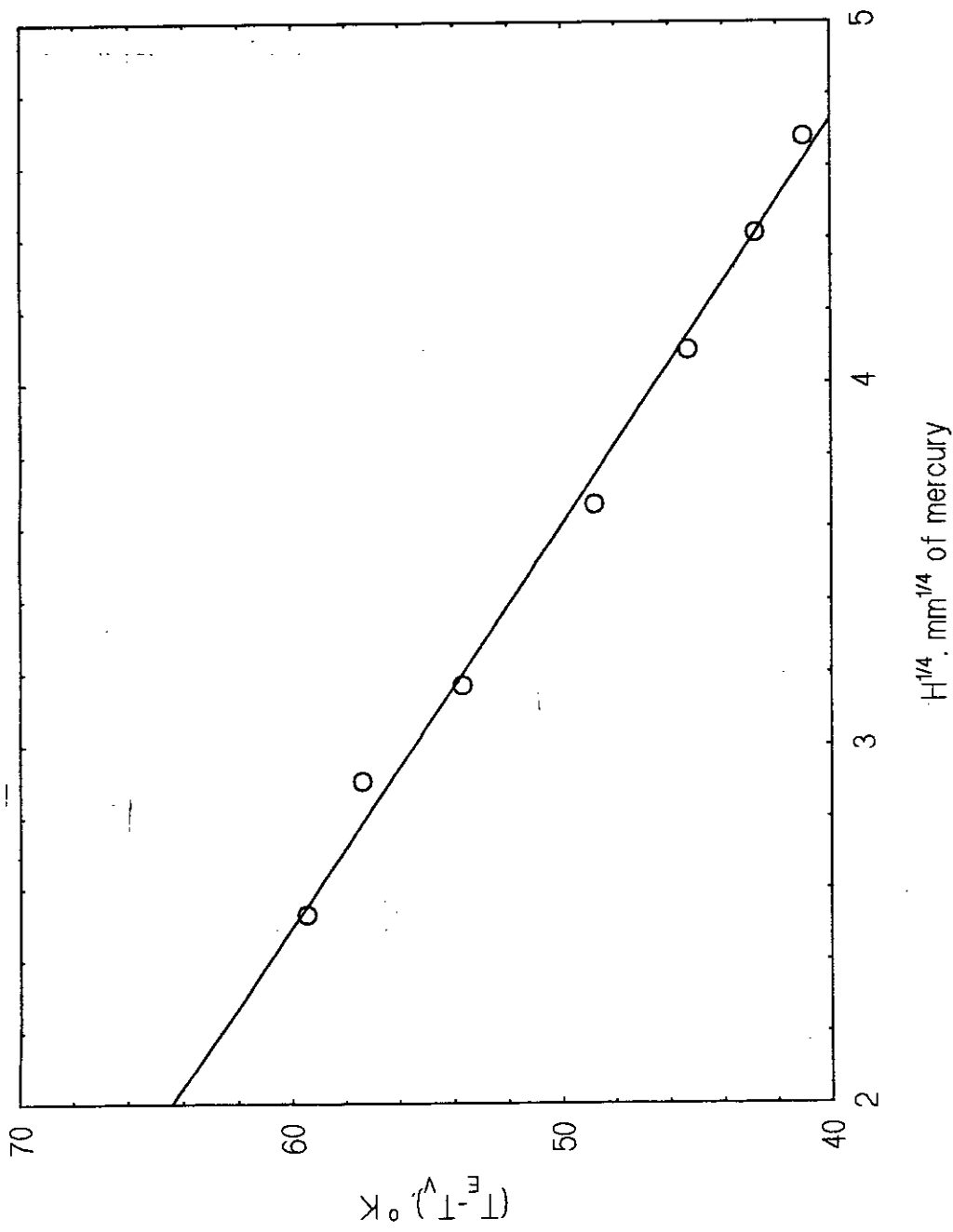


Fig. 15: Temperature difference versus $H^{1/4}$ curve.

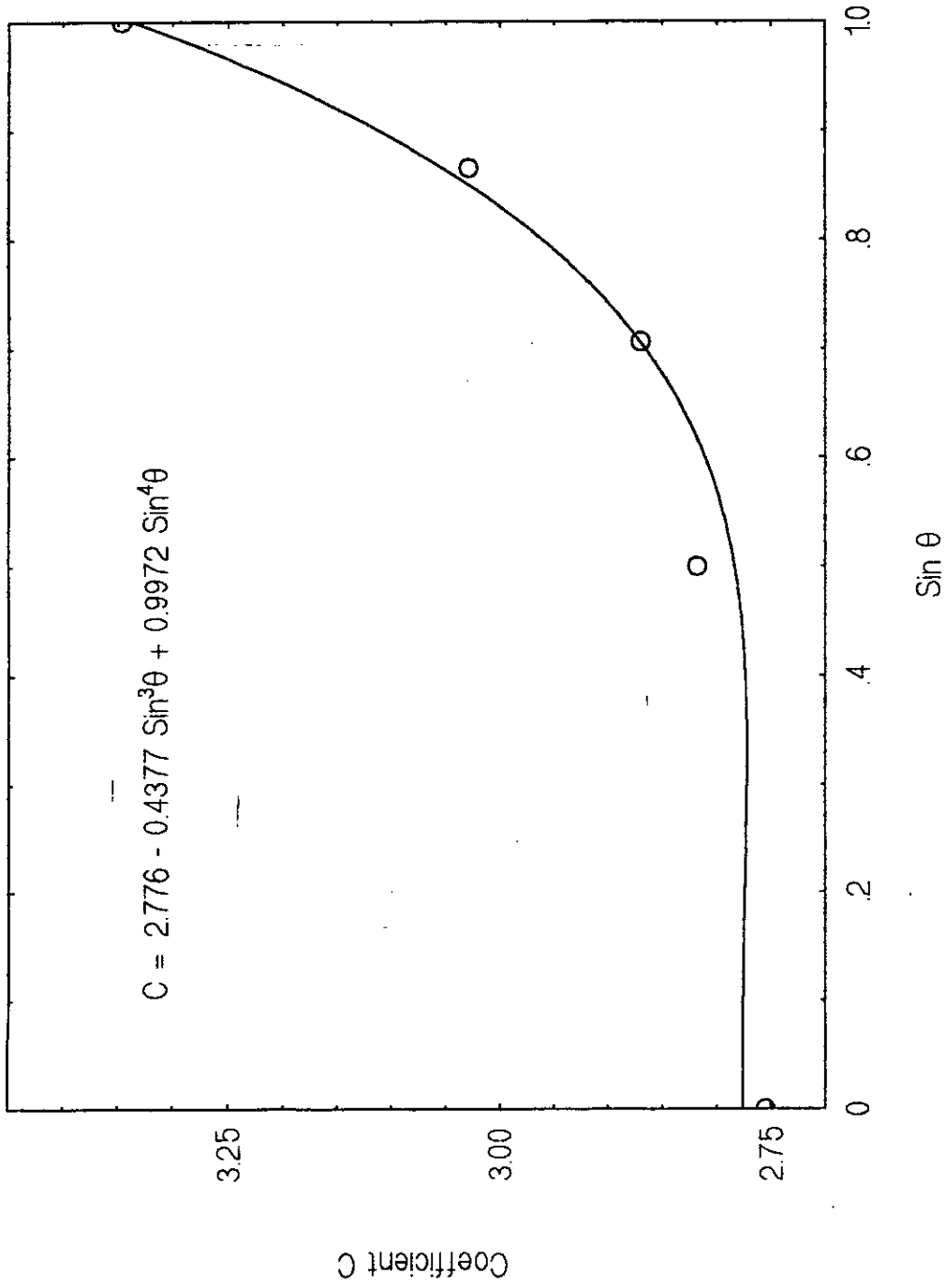


Fig. 16: Plot of coefficient C against Sin θ .

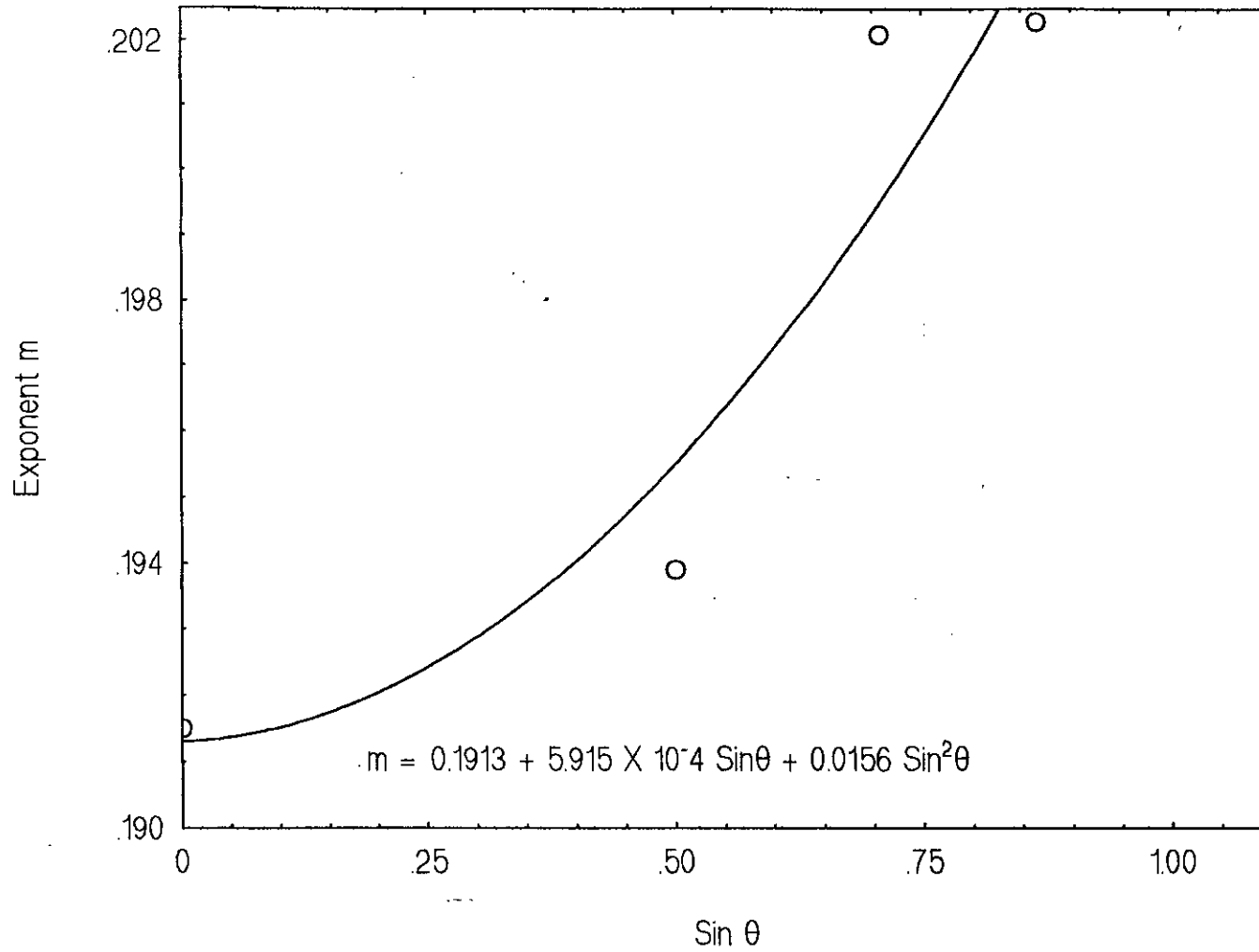
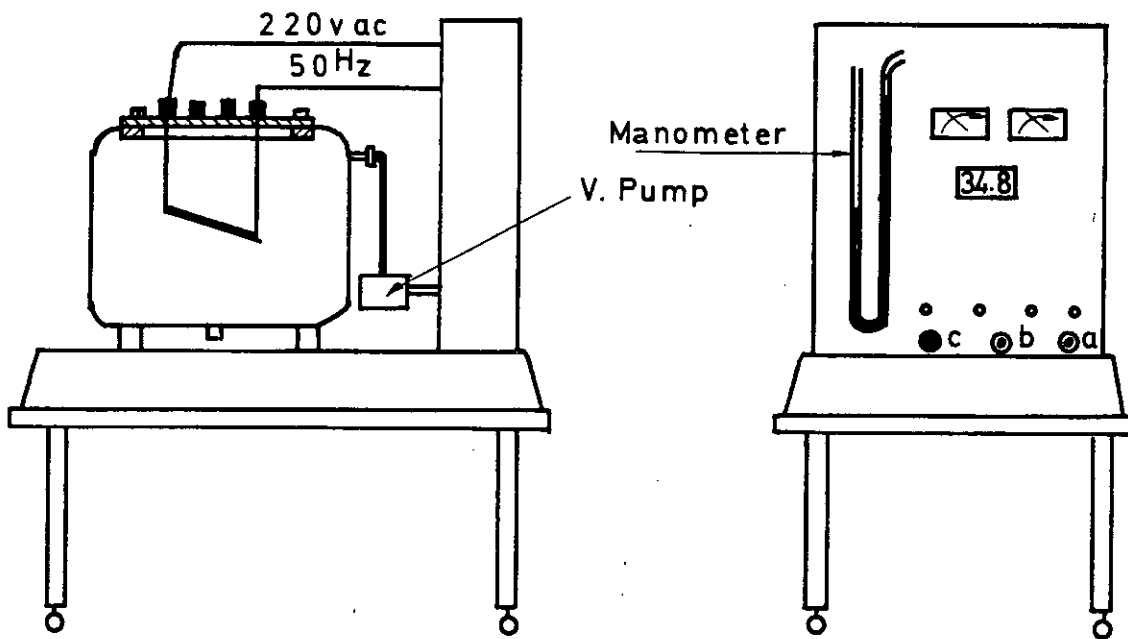
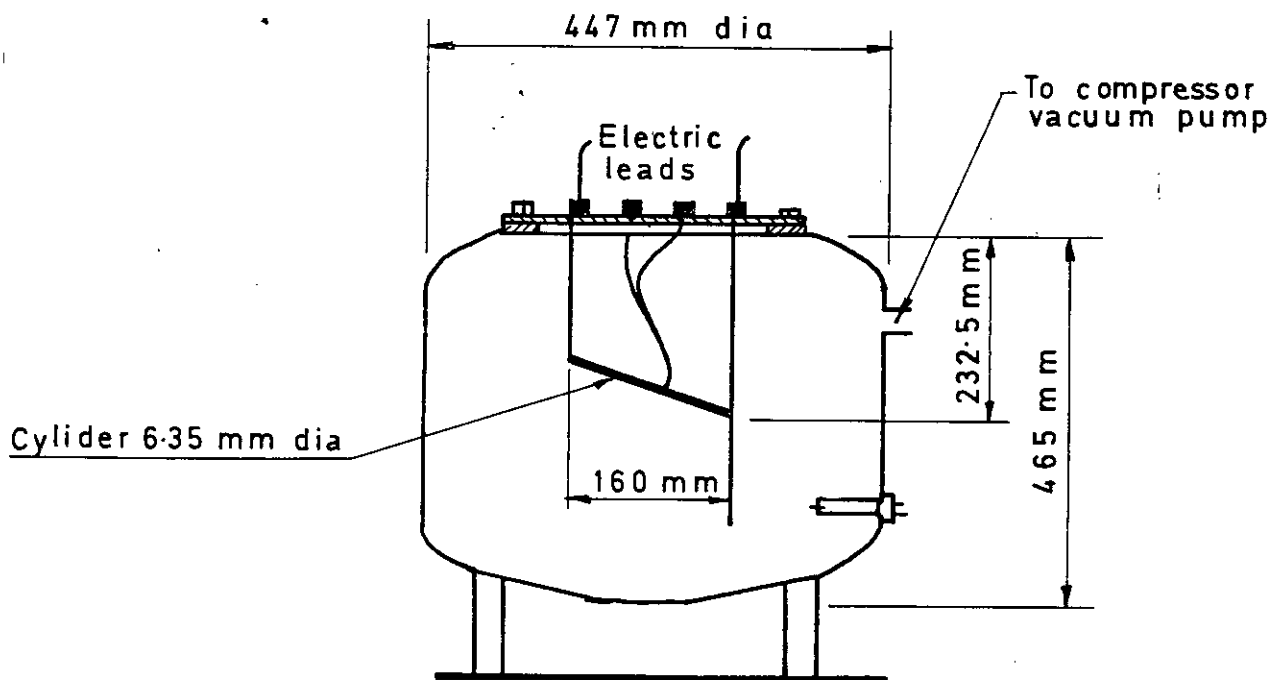


Fig. 17: Plot of exponent m against $\text{Sin } \theta$.



INSTRUMENT



VESSEL

FIG. 18

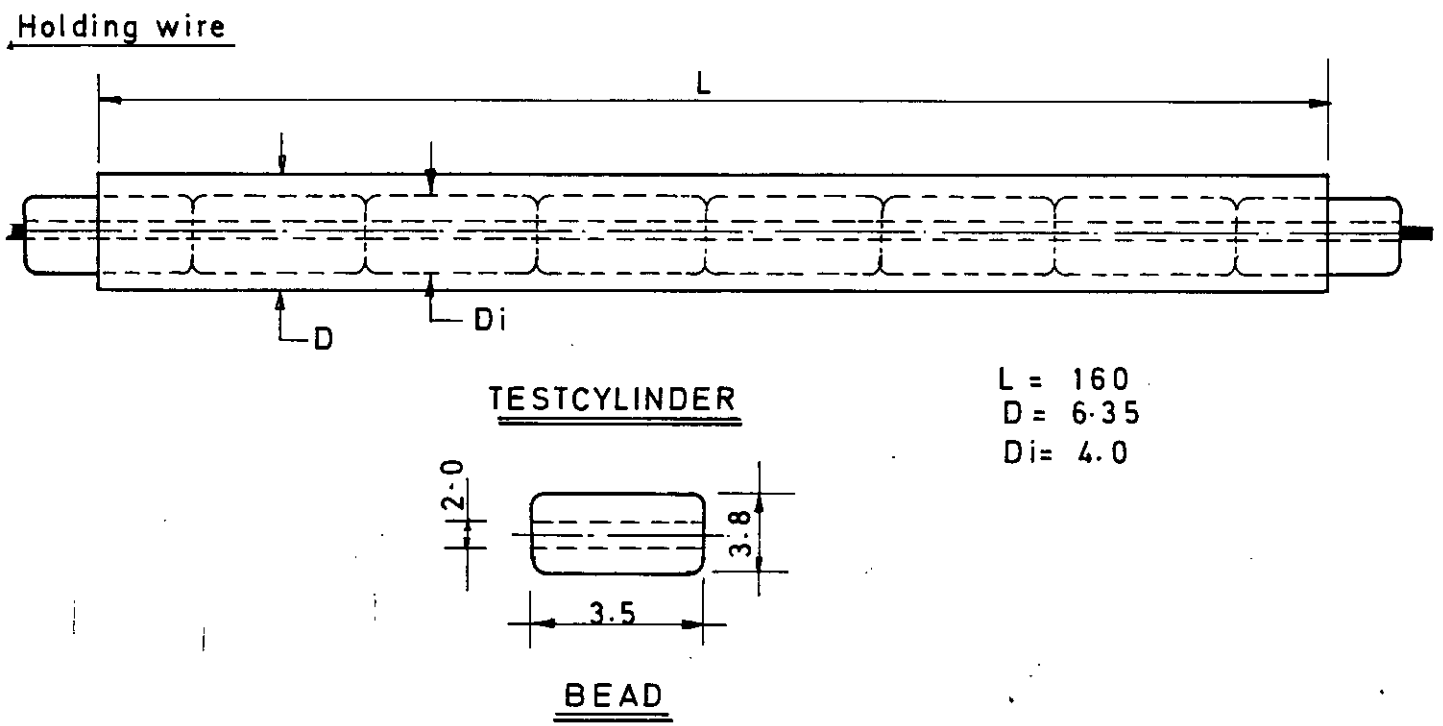


FIG. 19 TEST SPECIMEN

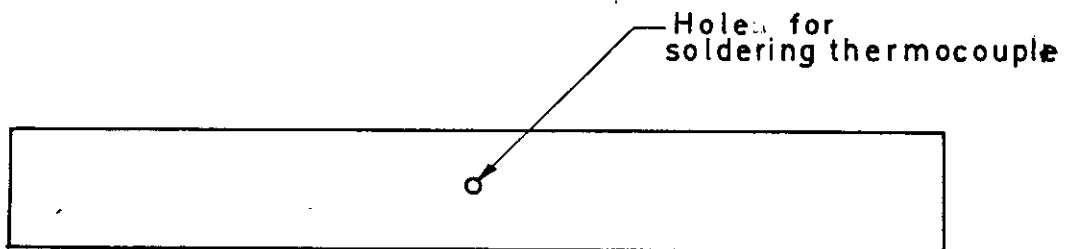


FIG. 20 CYLINDER WITH HOLE FOR THERMOCOUPLE

