OPTIMIZATION OF DUCTING SYSTEM FOR AIRCONDITIONING

By

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It is hereby declared that neither this thesis nor any part thereof has been submitted or is being concurrently submitted elsewhere for the award of any degree or diploma.

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ABSTRACT

In this study, an attempt has been made to determine the optimum duct sizes for minimum owning and operating cost (life cycle cost) of an airconditioning ducting system. In this regard, a mathematical model has been developed considering multipath ducting system. A computer program has also been developed to find the optimum solution of the model. The model has been applied to a specified ducting system. The result of the proposed method has been compared with those of other conventional duct sizing methods for the specified ducting system. Comparison shows that the minimum life cycle cost obtained by the proposed method is 9.23 percent and 4.12 percent lower than those obtained by Static regain and Equal friction methods respectively.

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LIST OF SYMBOLS

A	Cross sectional area of a duct (m^2)
$a_1, a_2 \cdot\cdot a_R$	Number of duct sections in the 1st, 2nd ...Rth path
$a_{t_o n}$	Exponent of the n-th independent variable of the t-th term in the objective function
a_{tn}	Exponent of the n-th independent variable of the t-th term in the constraint equation
C	Local loss coefficient for different fittings
CF	Cost of blower and unit casing (Tk)
C_k	Local loss coefficient in the k-th fitting
CM	Cost of motor, controls, switches, starters and accessories (Tk)
CP	Cost of motor for each KW (Tk/KW)
C_t	Coefficient of each term in the constraint equation
C_{t_o}	Coefficient of each term in the objective function
D_i	Duct diameter in the i-th duct section (m)
E	Present worth owning and operating cost (life cycle cost) (Tk)
E_c	Unit energy cost (Tk/Kwh)
E_d	Energy demand cost (Tk/KW)
EF	Blower cost including motor, controls, switches, starter, accessories etc. (Tk)
$(EO)_i$	Total cost of equipment (Coils, registers, diffusers, dampers, silencers) etc. in the i-th section (Tk)

E_p	First year energy cost (Tk)
E_s	Initial (present) cost of the duct material, insulation, blower etc. (Tk)
F	Minimum cost of a motor (Tk)
FP	Power of the motor (Assumed as equal to theoretical power consumption) (KW)
f	Friction factor
h_d	Total head downstream to a fitting (Pa)
h_o	Loss in pressure due to friction or fittings(Pa)
h_u	Total head upstream to a fitting (Pa)
h_v	Loss in pressure due to velocity (Pa)
I(i)	Dummy function
$(L_R)_i$	Actual duct length in the i-th section (m)
l	Number of fittings in a duct section
m	Total number of duct sections in a duct system
n	Number of duct sections in a path
n_e	Motor drive efficiency (decimal)
n_f	Overall efficiency of the blower (decimal)
P	Total pressure developed by the blower
PWF	Present worth factor (dimensionless)
Q_i	Volume flow rate of air through the i-th section (m^3/sec)
Q_f	Air flow rate of the blower (m^3/sec)
R	Number of total paths in a duct system
S	Unit ductwork cost including material and labour cost (Tk/kg)
T	System operating time (hr/yr)

t_i	Duct material thickness in the i-th section (m)
T_o	Total number of terms in the objective function
T_c	Total number of terms in the constraint equation
U_i	Quantity of ductwork in the i-th duct section (kg)
V_i	Velocity of air in the i-th duct section (m/s)
V_d	Downstream velocity of air across a transition(m/s)
V_u	Upstream velocity of air across a transition (m/s)
W	Duct material density (kg/m^3)
ΔP_f	Pressure loss in fittings (Pa)
ΔP_s	Pressure loss in a straight duct section due to friction (Pa)
ΔP_j	Total loss in pressure in the j-th section (Pa)
ρ	Density of air (kg/m^3)

CHAPTER I

INTRODUCTION

1.0 GENERAL

With the development of technology and subsequent increase in standard of living airconditioning has become a part of modern life either for creating comfortable environment or necessary environment for storage and production.

A comfortable and healthy environment is now considered a necessity rather than luxury, and many modern processes and products would not have existed without precise control of environmental conditions. Application of airconditioning has made all these things possible. Maintenance of warm space in the cold months and cool space in the warm months provides comfort and increase productivity through the efficient use of human resources.

During the last few decades the prices of natural resources have increased a lot and with it the prices of energy, services, commodities and other things have increased. Scientists and engineers in different fields are giving their highest efforts to introduce new products or modified versions of the existing products/systems to perform the same task at lower cost but in a better way and thus to cope up with the price hike. In line with the rise in cost of materials, manufacturing and energy, the cost of airconditioning has also increased tremendously. Researches are being carried out to find out possible avenues for cost reduction in airconditioning. Any optimization technique to reduce cost of airconditioning will bring blessings to its potential users.

Airconditioning as a whole is a composite system comprising of some subsystems. Ducting system of central airconditioning, also known as the air distribution system, is one of its subsystems that involves substantial amount of owning and operating costs. Optimization of the complete airconditioning system may be somewhat difficult but optimization of any subsystem may not be so difficult. Since ducting system is one of the most important subsystems of a central airconditioning system, where saving in the owning and operating costs may give substantial benefit to its owner, it is worthwhile to optimize airconditioning ducting system.

1.1 STATEMENT OF THE PROBLEM

In airconditioning ducting system, blower capacities are very much dependent on the duct sizes and vice-versa. The purpose of the blower is to impart energy to air so that it can flow from one part to another part overcoming the frictional losses in different duct sections and duct fittings.

Losses in pressure due to friction in straight ducts or due to fittings are directly proportional to the square of the velocity of air in the duct. Again duct cross-sectional area is inversely proportional to the velocity of air. From the above relationship it can be stated that duct area will reduce with increase in air velocity in a duct. But the energy consumption by the blower will increase with increase in air velocity. The relationship of ducting cost and present worth of energy cost with velocity of air is shown in fig.1.1. From this figure (Fig. 1.1) it is evident that the minimum of combined owning cost of ducting and present worth of the energy cost occurs at a specific air velocity.

The problem in this research work is to develop a mathematical model, along with solution process, to findout the optimum duct sizes and corresponding blower capacity to minimize the combined owning and energy costs of a ducting layout with specified amount of air flow.

An arbitrary duct layout (Fig. 1.2) is considered here to explain the problem. In the arbitrary ducting system shown in Fig. 1.2, there are n number of straight duct sections and one blower to deliver Q_1, Q_2 ...... Q_k m^3/Sec through the terminals m, m + 1, ... n respectively. It is now required to find out the sizes of the n duct sections and the size of the blower for which life cycle owning cost of ducting and present worth of the energy cost of the blower is minimum.

1.2 OBJECTIVES OF THE STUDY

The objectives of the present research work are:

i) To develop a mathematical model to design an airconditioning ducting system which will give an optimized combined owning and operating cost of the air distribution system.

ii) To develop a computer program for solution of the above mathematical model for optimum duct design.

iii) Application of the above model in a specific airconditioning ducting system (case study) and to compare the optimum cost of the system with those found by the conventional design methods.

1.3 LIMITATIONS OF THE STUDY

To avoid complicated mathematical manipulation and to understand the contribution of duct size and capacity of motor to the total owning and operating cost of ducting system, the following limitations have been considered.

i) The model considers circular ducts only.

ii) Factors affecting noise generation have not been considered.

iii) Materials for attenuating noise have not been considered.

CHAPTER II

LITERATURE REVIEW

2.0 GENERAL

Because of increase in cost of energy and materials, production cost as well as owning and operating costs of different systems are increasing day by day. Scientists and Engineers in different fields are working ceaselessly to reduce cost of production, owning and system operation costs. One group is working to improve the quality of a product and/or efficiency of a system technically so that ultimately the effective cost is reduced. Another group is working to find optimized selection and application of different variable parameters so that a product may be produced economically keeping quality within satisfactory level or a system may be owned and operated with a minimum cost but keeping the performance within satisfactory level.

In line with the above endeavours, engineers and scientists are also working in the field of Airconditioning and Refrigeration to find ways so as to reduce the cost of production and/or owning and the operating costs. Both scientific approach and optimization techniques are applied in this field to achieve the goal. The following paragraphs describe briefly some of the works done in this field.

2.1 SCIENTIFIC APPROACH

In 1982, LORNE W. NELSON and THOMAS BECKEY[1] conducted a field experiment with varying thermostat setup at different time of the day to varify cooling energy savings. They used hybrid computer to analyse weather data to predict thermostat setup temperature schedule for each day of a week. They concluded from their observations that with a reasonably sized air conditioning unit, people can expect to save energy and get comfort using thermostat setup in the cold season. Setting the thermostat up at night and during the day provides significantly more energy savings compared to a simple setup period. They also observed that computer simulation results using thermostat setup schedules agree with results from actual family residences.

In 1985, NASENBENY, ROBERT J.[2] showed that variable-air-volume (VAV) airconditioning can cut operating costs by up to 50% over conventional constant volume (CV) ventilating systems.

The basic economizer control system, is a proven method of reducing airconditioning costs. This system brings in cool outdoor air (when available) to reduce or eliminate the need for mechanical cooling. Cost savings of up to 30 percent can easily be realized with this type of system when compared to systems without economizers, even in systems with simultaneous heating and cooling. However, during hot weather the outdoor air will increase cooling costs and the amount of air brought in must be kept to a minimum. Thus, the "high limit cut-off" must determine if the outdoor air can provide free cooling, and disconnect the mixed air controller from the dampers if it cannot. The performance of this device is critical to the economizer system. Several different types of high limit controls are in use today. DALE K. DICKSON and STEVEN T. TOM[3] in 1986 described the advantages and disadvantages of these controls and proposed a new control system which provides a good compromise between performance and maintainability.

An ideal high limit controller compares the enthalpies of the outdoor and return air and disconnect the economizer whenever the enthalpy of the outdoor air is higher. In contrast to the ideal high limit controller Dale K. Dickson and Steven T. Tom[3] proposed a high limit cut-off switch, which is adjusted to minimize error caused by the difference in outdoor and room humidities, known as optimized dry bulb economizer. They showed that economizer control system with optimized dry bulb operation is more efficient than that of enthalpy comparison. A computer simulation of a dual duct system in Indiana, showed that this type of control would reduce annual heating and cooling costs by 21 percent compared to operation on minimum outdoor air.

GIL AVERY[4] one of the ASHRAE members, studied the design aspects of an outside air economizer cycle in 1986.

He showed that to ensure the proper control of the dampers, it should generally be sized smaller than the return duct or outside air intake hood or louver. He also showed that dampers with combination of opposed and parallel blades would be the best choice to achieve the optimum performance. He further concluded that an approximate savings of over 4 percent of the total system cooling operating costs can be achieved, when

i) the return fan requires 5 percent of the total power required for the mechanical cooling system.

ii) the system is geographically located where the economizer cycle can be used for half of the operating time.

iii) an average yearly mix of 50 percent return air and 50 percent outside air satisfies the mixed air thermostat.

iv) Fan power varies directly with flow.

In 1986 R.J. TSAL and H.F. BEHLS[5] provided a mathematical model for duct designing with cost optimization. This model considered only a single path ducting system with circular ducts. The model does not consider the effects of high velocities of air in the ducts, influence of high velocity air on local loss coefficients, acoustics and other practical constraints associated with multipath ducting system. As such a wide scope still exists for further optimization.

Besides these works some other scientists worked on different softwares to reduce cost of designing airconditioning system. DD4M Air duct design(6) is a software program to aid in a wide range of air duct design applications. This has been updated by MC^2 Engineering Software, Miami, Florida. The new program incorporates automated entry of 100-plus standard ASHRAE fitting, seperate calculations for exhaust and distribution applications and pop-up windows for display of fittings and their variation. It accepts unlimited entry of other devices and fittings by equivalent length, direct drop or C-factors.

Another such software is "E20-II piping and duct design software" prepared by Carrier Corporation. None of these softwares consider cost optimization of airconditioning ducting system.

FRANK KOSTYUN and DOUGLAS A. AMES(7) installed a eutectic storage unit in Arizona. The eutectic composition is a mixture of organic salts, water, nucleating and stabilizing materials. The primary salt ingredient is sodium sulfate, a common chemical found in detergents and other household products. Being inorganic, eutectics are nontoxic, flammable or biodegradable. They do not expand or contract while changing between solid and liquid states. The eutectic salt approach combines the attractive aspects of both chilled water and ice storage system. Eutectic provide the same storage capacity as water with one-third the tank volume and one-ninth the water. With eutectic storage system two third of the chiller plant operation time is reduced and thus two important goals of energy management that is reduced equipment time and reduced energy consumption are achieved.

2.2 OPTIMIZATION APPROACH

There had been a number of researches in the field of optimization, wherein mathematical solutions are sought to maximize or minimize an objective, known as objective function, under a number of certain constraints. In present day world, widely used optimization techniques are linear Programming, Dynamic Programming, Geometric Programming, Quadratic Programming, Calculus of Variation, Queueing etc. Researches are also being done to find optimized solutions in case of multicriteria objectives viz. Goal Programming, Minimum Deviation Approach[12], Step-Method (STEM) etc. Researches are also being carried out to find new approaches towards optimization of different systems. Studies are being carried out to apply optimization techniques in Airconditioning systems, especially to determine optimum ducting system with minimum life cycle cost. Descriptions of a few optimization techniques are given below.

2.2.1 QUADRATIC PROGRAMMING

A special case of the general nonlinear programming problem occurs when the objective function is quadratic, and all the constraints are linear. This quadratic program may be written as

$$\text{Minimize} \quad y = \sum_{n=1}^{N} C_n x_n + \frac{1}{2} \sum_{n=1}^{N} \sum_{p=1}^{N} x_n q_{np} x_p$$

Subject to

$$\sum_{n=1}^{N} a_{kn} x_n \geqslant b_k ; \quad k = 1,2,\ldots, M$$

$$x_j \geqslant 0 ; \quad j = 1,2, \ldots, N$$

2.2.2 DYNAMIC PROGRAMMING

Dynamic programming is a mathematical optimization technique used for making a series of interrelated decisions. Usually, a multi-stage decision process is transformed into a series of single-stage decision processes. Dynamic programming starts with small portion of the problem and finds the optimal solution for this smaller problem. It then gradually enlarges the problem, finding the current optimal solution from the previous one, until the original problem is solved in its entirety.

In contrast to other mathematical programming techniques, there does not exist a standard mathematical formulation of the dynamic programming problem. Dynamic programming is a general strategy for optimization rather than a specific set of rules.

2.2.3 GEOMETRIC PROGRAMMING

Latter on in chapter IV it will be seen that the mathematical model for optimization of ducting system for airconditioning system conforms to the geometric programming form as such geometric programming problem has been given special consideration and presented below.

Geometric programming first emerged in 1961 when Clarence Zener, then Director of Science at Westinghouse Corporation, observed that many engineering design problems consisting of a sum of component costs could sometimes be minimized almost by inspection under suitable condition.

Wild and Beighter describe the method as follows:

Instead of seeking the optimum values of the independent variables first, geometric programming finds the optimal way to distribute the total cost among the various terms of the objective functions. Once these optimal allocations are obtained, often by inspection of simple linear equations, the optimal cost can be found by routine calculation.

The generalized geometric programming problem appears as follows:

$$\text{Minimize} \quad y_o = \sum_{t=1}^{T_o} \sigma_{ot} \, C_{ot} \prod_{n=1}^{N} x_n^{a_{otn}}$$

Subject to constraints of geometric form

$$\sum_{t=1}^{T_m} \sigma_{mt} \, C_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \leqslant \sigma_m$$

for $m = 1, 2 \ldots\ldots M$

Where

σ_{ot} and $\sigma_{mt} = \pm 1$	(Sign of each term in the objective function and m-th constraint respectively).
C_{ot} and $C_{mt} \geqslant 0$	(The coefficient of each term in the objective function and m-th constraint respectively).
$x_n \geqslant 0$	(The independent variables).
$\sigma_m = \pm 1$	(The constant bound of the constraint).
a_{otn} and a_{mtn}	are the exponents of the n-th independent variable of the t-th term of the objective function and m-th constraint respectively.
$M =$	Number of constraints
$T_o =$	Number of terms in the objective function.
$T_1, T_2 \ldots\ldots T_M$	are the number of terms in each constraint, 1 to M, respectively.

CHAPTER III

THEORY OF DUCT DESIGNING

3.0 GENERAL

The purpose of the airconditioning ducting system is to deliver a specified amount of air to each diffuser in the conditioned space, so as to ensure proper amount of cooling effect and the proper air motion within the space. The method used for layout and sizing of the duct system must result in a reasonably quiet operation of the air-conditioning system and must not require unusual adjustments to achieve the proper distribution of air to each space.

3.1 DUCT DESIGNING FACTORS

3.1.1 SOUND

A low noise level is achieved by limiting the air velocity inside the duct by using sound-absorbing duct materials or linings, and avoiding drastic restriction in the duct such as nearly closed dampers. The use of fibrous glass duct material has gained wide acceptance in recent times because they are very effective for noise control. These ducts are also atractive from the fabrication point of view, because the duct, insulation, and vapor barrier are all the same piece of material. Metal ducts are usually lined inside with fibrous glass material in the vicinity of the air distribution equipment and upto a reasonable distance away from the equipment. The remainder of the metal duct is then wrapped or covered outside with insulation and a vapor barrier. Insulation on the outside of the duct also act as an agent for attenuating noise and vibration.

3.1.2 LAYOUT

The layout of the duct system is very important to the final design of the system. Generally the location of the air diffusers and air moving equipment is first selected with some attention given to how a duct system may be installed. The ducts are then laid out with attention given to space and ease of construction.

3.1.3 PRESSURE

The total pressure requirement of a duct system is an important consideration. From the stand point of first cost, the ducts should be small; however, small ducts tend to give high air velocities, high noise levels and large losses in total pressure. Therefore, a reasonable compromise between first cost, operating cost and practice must be reached. The total pressure requirements of a duct system are determined in two main ways. For residential and light commercial applications all of the heating, cooling and air moving equipment is determined by the heating and/or cooling load. Therefore, the fan characteristics are known before the duct design is begun. Furthermore, the pressure losses in all other elements of the system except the supply and return ducts are known. The total pressure available for the ducts is then the difference between the total pressure characteristic of the fan and sum of the total pressure losses of all of the other elements in the system excluding ducts. Large commercial and industrial duct systems are usually designed using velocity as a limiting criterion and the fan requirements are determined after the design is complete.

When the above factors are taken into consideration the next step is to size the ducts.

3.2 DUCT DESIGNING METHODS

At present duct designing are done with the help of any one of the following methods:-

i) Equal friction method

ii) Static regain method

iii) Balanced capacity method

iv) Constant velocity method

v) Velocity reduction method.

A brief description of the above duct design methods are given below:

3.2.1 EQUAL FRICTION METHOD

The principle of this method is to make the pressure loss per unit length of all duct sections same for the entire system. The usual procedure is to select the velocity in the main duct adjacent to the fan to keep the noise level within certain limit for the particular application or to keep frictional loss within limits. Usually an acceptable velocity is chosen for the main duct section adjacent to the blower and the duct size alongwith frictional loss per unit length are determined for that duct section from the Friction Chart. This established frictional loss per unit length is then kept constant for all sections of the said ducting system. Any specific duct section in that ducting system can be sized for the flow rate of that specific section and frictional loss per unit length determined earlier.

3.2.2 STATIC REGAIN METHOD

This method reduces the air velocity in the direction of flow in such a way that the increase i.e. regain in static pressure in each transition just balances the pressure losses in the following section.

The procedure for the use of the static regain method is to first select a velocity for the duct adjacent to the blower. With the flow capacity, this velocity establishes the size of the main duct. The run of the duct that appears to have the largest flow resistance is then designed first. A velocity is assumed for the next section in this run and the static pressure regain is compared to the lost pressure for that section. Usually two or more velocities are needed to be assumed to find a reasonable balance between the static pressure regain and the losses of a section.

3.2.3 BALANCED CAPACITY METHOD

The basic principle of this method of design is to make the total pressure loss same in each duct run, from blower to the outlet.

The design procedure for the balanced capacity method is the same as the equal friction method for the run of the largest flow resistance. In finding the duct sizes for the largest frictional loss, the main duct sections are already designed. To find the sizes of branch ducts of other duct runs different frictional loss (es) per unit length is (are) chosen and ducts are designed by equal friction method to make the total pressure drop equal in all duct runs.

3.2.4 EQUAL VELOCITY METHOD

The procedure for this method is to select the velocity in the main duct adjacent to the blower to provide a satisfactory noise level. This velocity is then maintained throughout the duct system. From known flow rates and this velocity, duct sizes are determined from charts for duct design.

3.2.5 VELOCITY REDUCTION METHOD

The design procedure of this method is similar to the equal velocity method. In this method the velocity from the main run is reduced with a constant ratio. Velocity in the main duct and this constant ratio determines the velocities in the next sections. These velocities and known flow rate then determine the sizes of different duct sections.

CHAPTER IV
MATHEMATICAL MODELING

4.1 MODEL FORMULATION

4.1.1 OBJECTIVE FUNCTION

The goal of the present research is to find out the optimum ducting system under different practical constraints. The total life cycle cost of the airconditioning ducting system will be the owning cost of the ducts along with its insulation cost, blower cost and the present worth value of the yearly operating cost. Then the objective will be to minimize the life cycle cost and the mathematical model for objective function can be written as per following:

$$\text{Minimize,} \quad E = E_s + E_p\,(PWF) \qquad (1)$$

Where,

E = Present worth owning and operating cost (life cycle cost) (Tk)

E_s = Initial (present) cost of the duct material, insulation, blower etc. (Tk)

E_p = First year energy cost (Tk)

PWF = Present worth factor (dimensionless)

Initial cost E_s can be calculated from the following relation:

$$E_s = \sum_{i=1}^{m} \left[SU_i + (EO)_i \right] + EF \qquad (2)$$

Where,

i = Counter of duct sections and varies from 1 to m

m = Total number of sections in a duct system.

U_i = Quantity of duct work in the ith duct section (kg)

S = Unit ductwork cost including material and labour cost (Tk/kg)

$(EO)_i$ = Total cost of equipment (coils, registers, diffusers, dampers, silencers) etc. in the i-th section (Tk)

EF = Blower cost including motor, controls, switches, starter, accessories etc. (Tk)

In designing airconditioning ducting system it is the usual practice to design with circular ducts first and subsequently to convert circular sections into equivalent rectangular sections considering equal pressure loss and equal flow rate. Thus considering circular duct, quantity of duct work U_i can be expressed as follows:

$$U_i = \pi D_i (L_R)_i \; t_i \; W \qquad (3)$$

Where,

D_i = Duct diameter in the i-th section (m)

$(L_R)_i$ = Actual duct length in the i-th section (m)

t_i = Duct thickness in the i-th section (m)

W = Duct material density (kg/m^3)

When the value of U_i from equation no. (3) is put in equation no. (2) it gives:

$$E_s = \sum_{i=1}^{m} \left[\pi S D_i \, (L_R)_i \, t_i \, W + (EO)_i \right] + EF \qquad (4)$$

Initial cost for blower and motor EF may again be expressed as follows:

$$EF = CF + CM \qquad (5)$$

Where,

CF = Cost of blower and unit casing (Tk)

CM = Cost of motor, controls, switches, starters and accessories etc. (Tk)

Cost of blower for the same delivery capacity under different pressure head may be assumed constant, which is logical and cost of motor may be expressed as a function of the motor power required to drive the blower. Thus CM becomes:

$$CM = (FP) \times (CP) + F \qquad (6)$$

Where,

FP = Power of the motor (Assumed as equal to theoretical power consumption) (KW)

CP = Cost of motor for each KW (Tk/KW)

F = Minimum cost of a motor (Tk)

Then combining equation (5) and (6) gives

$$EF = CF + (FP) \times (CP) + F \qquad (7)$$

ENERGY CONSUMPTION COST

Energy cost E_p in equation (1) can be written as:

$$E_p = (FP)(E_d + E_c T) \qquad (8)$$

Where,

FP = Power consumed by the motor (KW)

E_d = Energy demand cost (Tk/KW)

E_c = Unit energy cost (Tk/Kwh)

T = System operating time (h/yr)

Theoretical power FP for the motor may be calculated from the following relationship.

$$FP = \frac{Q_f P}{1000\ n_f n_e} \qquad (9)$$

Where,

Q_f = Air flow rate of the blower (m^3/sec)

P = Total pressure developed by the blower (Pa)

n_f = Overall efficiency of the blower (decimal)

n_e = Motor drive efficiency (decimal)

Then

$$E_p = \frac{Q_f\ P}{1000\ n_f n_e} (E_d + E_c T) \qquad (10)$$

Combining equation no. (4) and (7) it is found that:

$$E_s = \sum_{i=1}^{m} \left[\pi LSD_i (L_R)_i \; t_i \; W + (EO)_i\right] + CF$$

$$+ (FP)(CP) + F \qquad (11)$$

Putting the experessions for E_p and E_s from equations (8) and (11) respectively in equation no. (1) it is found:

$$E = \sum_{i=1}^{m} \left[\pi LSD_i (L_R)_i \; t_i \; W + (EO)_i\right] + CF + (FP)x(CP)$$

$$+ F + FP(E_d + E_c T)(PWF)$$

$$= \sum_{i=1}^{m} \left[\pi LSD_i (L_R)_i \; t_i \; W + (EO)_i\right] + \left[CP+(E_d+E_c T)x\right.$$

$$\left.(PWF)\right] \; x \; FP + CF + F \qquad (12)$$

Now when the expression for FP from equation (9) is put in the above equation (Eqn. no. (12) it gives the objective function in the following form:

$$E = \sum_{i=1}^{m} \left[\pi LSD_i (L_R)_i t_i W+(EO)_i\right] + \left[CP+(E_d+E_c T)(PWF)\right]x$$

$$\frac{Q_f P}{1000 \; n_f n_e} + CF + F \qquad (13)$$

In the above objective function all variables excepting D and P are known for any specific application. Both D and P are the design variables that needed to be determined for the minimum value of the objective function i.e minimization of life cycle cost. But both D and P are functions of duct velocities. As such these design variables are expressed in terms of duct velocities in the following steps.

Duct diameter can be expressed in terms of duct velocity from the following equation:

From continuity equation it is known that

$$Q = AV \tag{14}$$

Where,

Q = Volume flow rate of air through a duct section (m^3/s)

A = Cross sectional area of a duct section (m^2)

V = Average velocity of air in a duct section (m/s)

For circular ducts

$$A = \frac{\pi D^2}{4} \tag{15}$$

Thus equation (14) and (15) gives

$$Q = \frac{\pi D^2 V}{4} \tag{16}$$

which again gives

$$D = 1.128\ Q^{0.5}\ V^{-0.5} \tag{17}$$

Now, assuming an absolute roughness of 0.1 mm[8] for the duct material friction factor f can be expressed by equation (18). Equation (18) is valied for air velocities ranging from 1.8 m/s to 25 m/s.

$$f = 0.0185(Q\rho)^{-0.1} \qquad (18)$$

Where,

f = friction factor (dimensionless)

ρ = density of air (kg/m^3)

Pressure loss in duct section of length L_R and diameter D as given by the Darcy-Weisbach equation is as follows:

$$\Delta P_s = \frac{f(L_R)}{D} \; \frac{\rho V^2}{2} \qquad (19)$$

Where

ΔP_s = Pressure loss in a straight duct section due to friction (Pa).

Combining equation no. (17), (18) and (19) it is found that

$$\Delta P_s = 0.0082\, \rho^{0.9}\, Q^{-0.6}\, V^{2.5}\, (L_R) \qquad (20)$$

Loss of pressure in different types of fittings in a section is given by the equation:

$$\Delta P_f = (\sum_{k=1}^{l} C_k) \frac{\rho V^2}{2} \qquad (21)$$

Where,

ΔP_f = Pressure loss in fittings in a section (Pa)

C_k = Local loss coefficient in the k-th fitting (dimensionless)

l = Number of fittings in a section.

The total loss of pressure in a duct section is equal to the sum of frictional loss in straight ducts and local losses due to different type of fittings.

$$\text{Thus } \Delta P = \Delta P_s + \Delta P_f \qquad (22)$$

Where,

ΔP = total loss of pressure in a section (Pa)

Combining equation no. (20), (21) and (22) gives

$$\Delta P = 0.0082 \rho^{0.9} Q^{-0.6} V^{2.5} (L_R) + (\sum_{k=1}^{l} C_k) \frac{\rho V^2}{2} \qquad (23)$$

The values of C_k are available in the ASHRAE handbook[9]. Most of the local loss coefficients are functions of upstream and downstream velocities across different fittings, and a few are independent of velocity. Those coefficients which are functions of velocities may be corelated by using least square curve fitting[10] method.

The total pressure P required by the blower is simply the summation of P along the index run, i.e. the path of largest flow resistance. If duct sections are reckoned along the index run starting from the section next to the blower,

$$\text{Then} \quad P = \text{max. of } \left(\sum_{j=1}^{n} \Delta P_j \right)_{ip} \qquad (24)$$

Where,

ip = 1st, 2nd.... R-th

R = Number of total paths in the system.

n = Number of duct sections in a path.

Density of air for an airconditioning ducting system may be considered constant. As such equations (23) and (24) provide the following relationship.

$$P = \text{max. of } \Big[0.0082\, \rho^{0.9} \sum_{j=1}^{n} Q_j^{-0.6} V_j^{2.5} (L_R)_j + \frac{\rho}{2} \sum_{j=1}^{n} \Big(\sum_{k=1}^{l} C_k\Big)_j V_j^2 \Big]_{ip} \qquad (25)$$

Putting the expression of D and P in terms of duct velocity V from equations (17) and (25) respectively in equation (13) gives the following final form of the objective function :

$$E = \sum_{i=1}^{m} \left[1.128\, \pi WS(L_R)_i t_i\, Q_i^{0.5}\, V_i^{-0.5} + (EO)_i \right]$$

$$+ \left[(E_d + E_c T)(PWF) + CP \right] \frac{Q_f}{1000\, n_f n_e}$$

$$\times \left[\text{max. of } \left\{ 0.0082\, f^{0.9} \sum_{j=1}^{n} Q_j^{-0.6}\, V_j^{2.5}\, (L_R)_j \right. \right.$$

$$\left. \left. + \frac{f}{2} \sum_{j=1}^{n} \left(\sum_{k=1}^{l} C_k \right)_j V_j^2 \right\} \right]_{ip} + CF + F \qquad (26)$$

Where,

$i = 1, 2 \ldots . m$

$j = 1, 2 \ldots . n$

$k = 1, 2 \ldots . l$

$ip = 1, 2 \ldots . R$

m = Total number of sections in a duct system

n = Number of duct sections in a path

l = Number of fittings in a duct section

R = Number of total paths in the system

4.1.2 CONSTRAINT EQUATIONS

It is necessary for all duct system that the pressure loss along every path should be equal. But in most cases designers do not size duct systems in that way, rather they use balancing devices to balance the pressure losses along different paths. In this present case pressure losses along different paths are considered equal.

Let there be R paths in a ducting system starting from the blower, where there are a_1, a_2.... a_R sections in the 1st path, 2nd path ... and R-th path respectively. Since pressure loss is equal along each path, the following constraint equation can be written.

$$\left(\sum_{j=1}^{a_R} \Delta P_j \right)_{ip} = \text{constant} \qquad (27)$$

It is evident that thus (R-1) equality constraints develop from the above equation.

For a straight-through transition, no data for local loss coefficient is available for velocity ratio (downstream to upstream velocity) greater than one. It may be considered in designing that for straight-through transition the down stream velocity will always be equal or lower than the upstream velocity. If, therefore there are b number of such transitions in a duct system then b number of following constraints equation may be considered.

$$\left(\frac{V_d}{V_u} \right)_{it} \leq 1 \qquad (28)$$

Where,

it = 1,2,.... b

b = Number of straight-through transitions

V_d = Downstream velocity of air across a straight-through transition.

V_u = Upstream velocity of air across a straight-through transition.

4.1.3 MATHEMATICAL MODEL

From the previous sections the final mathematical model for airconditioning ducting system is

$$\text{minimize} \quad E = \sum_{i=1}^{m} \left[1.128\pi\, WS(L_R)_i t_i \, Q_i^{0.5} \, V_i^{-0.5} + (EO)_i \right]$$

$$+ \left[(E_d + E_c T)(PWF) + CP \right] \times \frac{Q_f}{1000\, n_f n_e}$$

$$\times \left[\text{max. of} \left\{ 0.0082\, \rho^{0.9} \sum_{j=1}^{n} Q_j^{-0.6} \, V_j^{2.5} \, (L_R)_j \right. \right.$$

$$\left. \left. + \frac{\rho}{2} \sum_{j=1}^{n} \left(\sum_{k=1}^{l} C_k \right)_j V_j^2 \right\} \right]_{ip} + (CP + F) \qquad (26)$$

Subject to

$$\left(\sum_{j=1}^{a_R} \Delta P_j \right)_{ip} = \text{Const}$$

$$\Delta P_j = 0.0082\, \rho^{0.9} \, Q_j^{-0.6} \, V_j^{2.5} \, (L_R)_j$$

$$+ \left(\sum_{k=1}^{l} C_k \right)_j \frac{\rho V_j^2}{2}$$

$$\left(\frac{V_d}{V_u} \right)_{it} \leqslant 1$$

4.2 SOLUTION OF THE MODEL

From the previous section the objective function in terms of duct velocities is as follows:

$$E = \sum_{i=1}^{m} \left[1.128\,\pi W S (L_R)_i t_i Q_i^{0.5} \bar{V}_i^{-0.5} + (EO)_i\right]$$

$$+ \left[(E_d + E_c T)(PWF) + CP\right] \times \frac{Q_f}{1000\, n_f n_e}$$

$$\times \left[\max.\text{ of } \left\{0.0082\, \rho^{0.9} \sum_{j=1}^{n} Q_j^{-0.6} V_j^{2.5} (L_R)_j\right.\right.$$

$$\left.\left. + \frac{\rho}{2} \sum_{j=1}^{n} \left(\sum_{k=1}^{l} C_k\right)_j V_j^2 \right\}\right]_{ip} + (CF + F) \qquad (26)$$

Where,

$i = 1, 2, \ldots m$

$j = 1, 2, \ldots n$

$k = 1, 2, \ldots l$

ip = 1st, 2nd ... R-th

m = Total number of duct sections in a duct system.

n = Number of duct sections in a path

l = Number of fitting in a duct section

R = Number of total paths in the system.

Let $\left[(E_d + E_c T)(PWF) + CP\right] \dfrac{Q_f}{1000\ n_f n_e} = B$

and $ip = ip_{index}$ corresponding to the path of maximum pressure loss. Then expanding the above expression for objective function gives.

$$E = \Big[1.128\pi WS(L_R)_1 t_1 Q_1^{0.5} V_1^{-0.5} + 1.128\pi WS(L_R)_2 t_2 Q_2^{0.5} V_2^{-0.5} + \ldots + 1.128\pi WS(L_R)_m t_m Q_m^{0.5} V_m^{-0.5} + (EO)_1 + (EO)_2 + \ldots + (EO)_m\Big]$$

$$+ \Big[0.0082B\rho^{0.9} Q_1^{-0.6}(L_R)_1 V_1^{2.5} + 0.0082B\rho^{0.9} Q_2^{-0.6}(L_R)_2 V_2^{2.5} + \ldots\ldots + 0.0082B\rho^{0.9} Q_n^{-0.6}(L_R)_n V_n^{2.5}$$

$$+ B\frac{\rho}{2}(C_1 + C_2 + \ldots + C_1)_1 V_1^2 + B\frac{\rho}{2}(C_1 + C_2 + \ldots + C_1)_2 V_2^2 + \ldots + B\frac{\rho}{2}(C_1 + C_2 + \ldots + C_1)_n V_n^2\Big]_{ip_{index}} + (CF+F)$$

This may again can be written in the following form:

$$E = \Big[1.128\pi WS(L_R)_i t_1 Q_1^{0.5} V_1^{-0.5} V_2^o V_3^o \ldots V_m^o + 1.128\pi WS(L_R)_2 Q_2^{0.5} V_1^0 \times t_2 \times V_2^{-0.5} V_3^o \ldots V_m^o + \ldots\ldots + 1.128\pi WS(L_R)_m t_m Q_m^{0.5} V_1^o V_2^o \ldots V_m^{-0.5}$$

$$+ \left\{(EO)_1 + (EO)_2 + \ldots + (EO)_m\right\} V_1^o V_2^o \ldots V_m^o\Big]$$

$$+ \left[0.0082B \, \rho^{0.9} \, Q_1^{-0.6}(L_R)_1 V_1^{2.5} \, V_2^{o} \cdots V_m^{o} + 0.0082B \, \rho^{0.9} Q_2^{-0.6} \right.$$

$$x(L_R)_2 V_1^{o} \, V_2^{2.5} \, V_3^{o} \cdots V_m^{o} + \cdots + 0.0082B \, \rho^{0.9} Q_n^{-0.6} (L_R)_n V_1^{o} V_2^{o} \cdots$$

$$\cdots V_n^{2.5} \, V_{n+1}^{o} \, V_m^{o} + B \frac{\rho}{2}(C_1 + C_2 + \cdots C_l)_1 V_1^{2} V_2^{o} \cdots V_m^{o}$$

$$+ B \frac{\rho}{2}(C_1 + C_2 + \cdots + C_l)_2 V_1^{o} \, V_2^{2} \cdots V_m^{o} + \cdots$$

$$\left. + B \frac{\rho}{2}(C_1 + C_2 + \cdots + C_l)_n V_1^{o} \, V_2^{o} \cdots V_n^{2} \, V_{n+1}^{o} \cdots V_m^{o} \right] ip_{index}$$

$$+ (CF + F) \, V_1^{o} V_2^{o} \cdots V_m^{o}$$

Now this can be expressed in the following generalized form:

$$E = \sum_{t_o=1}^{T_o} \sigma t_o \, C_{t_o} \prod_{n=1}^{m} V_n^{a_{t_o n}} \qquad (29)$$

Where

$\sigma t_o = \pm 1$ (sign of each term in the objective function)

$C_{t_o} = 1.128 \pi WS(L_R)_1 t_1 Q_1^{0.5}$, $\left[(EO)_1 + (EO)_2 + \cdots + (EO)_m\right]$,

$0.0082 \, B \rho^{0.9} \, Q_1^{-0.6}(L_R)_1$ etc. i.e the coefficient of each term in the objective function.

m = Total number of variables in a system.

T_o = Total number of terms in objective function

$a_{t_o n}$ = Exponent of the n-th independent variable of t-th term.

Again the constraint equations in section 3.1.2 considering equal pressure loss along each path is

$$\left[\sum_{j=1}^{a_1} \Delta P_j\right]_1 = \left[\sum_{j=1}^{a_2} \Delta P_j\right]_2 = \ldots\ldots = \left[\sum_{j=1}^{a_{R-1}} \Delta P_j\right]_{R-1}$$

$$= \left[\sum_{j=1}^{a_R} \Delta P_j\right]_R = \text{Constant.}$$

From the above equation for R different paths (R-1) no. of independent equality constraints of the following form can be developed.

$$\left[\sum_{j=1}^{a_{R-1}} \Delta P_j\right]_{R-1} = \left[\sum_{j=1}^{a_R} \Delta P_j\right]_R \qquad (30)$$

To adopt the terms both in the objective function and in the constraint equations let the following arbitrary duct layout be considered, where there are m duct sections in total and

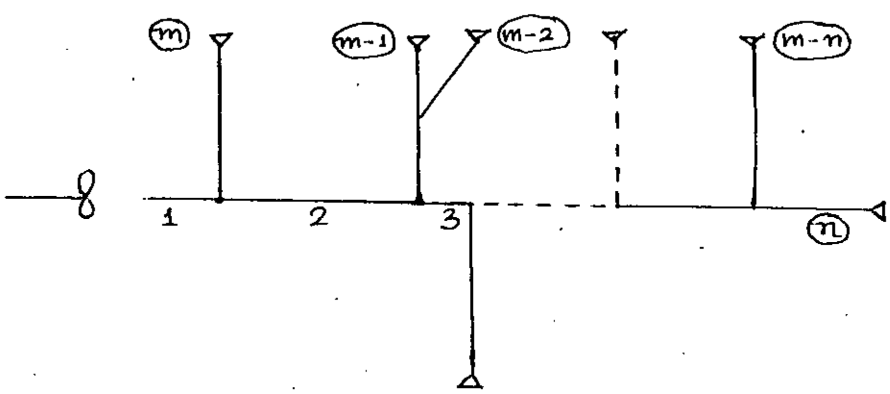


Figure 4.1

R duct paths each starting from section 1 and terminating at section no; m, m-1 .... m-n etc. respectively (encircled numbers indicate terminal sections of different paths). It is quite evident from the Fig. 4.1 that consecutive duct section serial numbers may not appear along all the paths. As such, to generalize the expression for pressure losses along any path a dummy function I(i) may be introduced in the above experession where i indicates the counter of duct sections. If any duct section does not lie in a particular path then I(i) = 0, otherwise I(i) = 1. Then the generalized form of equation number 30 is

$$\left[\sum_{i=1}^{m} I(i)_{R-1}\,\Delta P_i\right] = \left[\sum_{i=1}^{m} I(i)_R\,\Delta P_i\right] \qquad (31)$$

Now when the expression for ΔP in terms of V from eqn.(23) is put into the equation no. (31) it becomes:

$$\sum_{i=1}^{m} I(i)_{R-1}\left[0.0082\,\rho^{0.9} Q_i^{-0.6}\, V_i^{2.5}(L_R)_i + \left(\sum_{k=1}^{l} C_k\right)_i \frac{\rho V_i^2}{2}\right]$$

$$= \sum_{i=1}^{m} I(i)_R\left[0.0082\,\rho^{0.9}\, Q_i^{-0.6}\, V_i^{2.5}(L_R)_i + \left(\sum_{k=1}^{l} C_k\right)_i \frac{\rho V_i^2}{2}\right.$$

Expanding both sides, it is found

$$I(1)_{R-1} 0.0082\rho^{0.9}\, Q_1^{-0.6}\, V_1^{2.5}(L_R)_1 + \ldots + I(m)_{R-1} 0.0082\rho^{0.9} Q_m^{-0.6}$$

$$\times\, V_m^{2.5}(L_R)_m$$

$$+ I(1)_{R-1}(C_1+C_2+\dots+C_l)_1 \frac{\rho V_1^2}{2} +\dots+ I(m)_{R-1}(C_1+C_2+\dots+C_l)_m \frac{\rho V_m^2}{2}$$

$$= I(1)_R 0.0082\rho^{0.9} Q_1^{-0.6} V_1^{2.5}(L_R)_1 +\dots+ I(m)_R 0.0082\rho^{0.9} Q_m^{-0.6}$$

$$\times V_m^{2.5}(L_R)_m$$

$$+ I(1)_R(C_1C_2+\dots+C_l)_1 \frac{\rho V_1^2}{2} + \dots\dots\dots +$$

$$I(m)_R(C_1+C_2+\dots+C_l)_m \frac{\rho V_m^2}{2}$$

which when rearranged keeping any one of the non-zero terms on the right hand side, gives

$$I(1)_{R-1} 0.0082\rho^{0.9} Q_1^{-0.6} V_1^{2.5}(L_R)_1 +\dots+ I(m)_{R-1} 0.0082\rho^{0.9}$$

$$\times Q_m^{-0.6} V_m^{2.5}(L_R)_m$$

$$+ I(1)_{R-1}(C_1+C_2+\dots+C_l)_1 \frac{\rho V_1^2}{2} +\dots+ I(m)_{R-1}(C_1+\dots+C_l)_m \frac{\rho V_m^2}{2}$$

$$- I(1)_R 0.0082\rho^{0.9} Q_1^{-0.6} V_1^{2.5} (L_R)_1 +\dots+ I(m-1)_R 0.0082\rho^{0.9}$$

$$\times Q_{m-1}^{-0.6} V_{m-1}^{2.5}$$

$$\times (L_R)_{m-1} - I(1)_R(C_1+\dots C_l)_1 \frac{\rho V_1^2}{2} - \dots - I(m)_R(C_1+\dots+C_l)_m \frac{\rho V_m^2}{2}$$

$$= I(m)_R 0.0082\rho^{0.9} Q_m^{-0.6} V_m^{2.5}(L_R)_m$$

Now dividing both sides by the right hand side term, it is found

$$\frac{I(1)_{R-1}\ Q_1^{-0.6}(L_R)_1}{I(m)_R\ Q_m^{-0.6}(L_R)_m}\ V_1^{2.5}\ V_m^{-2.5} + \ldots\ldots + \frac{I(m)_{R-1}(C_1+\ldots C_l)m}{I(m)_R 0.0082\ Q_m^{-0.6}(L_R)_m}$$

$$\times\ \frac{\rho^{0.1}}{2}\ V_m^{0.5} = 1. \qquad (32)$$

Since Q_1, ... Q_m, $(L_R)_1$....$(L_R)_m$ are constants, the left hand side of the above equation may again be expressed in a general form like that of the objective function. Thus the above equation can be written in the following generalized form:

$$\sum_{t=1}^{T^c} \delta_t C_t \prod_{n=1}^{m} V_n^{a_{tn}} = 1 \qquad (33)$$

Equation no. (28) is

$$\frac{V_d}{V_u} \leqslant 1$$

i.e $\qquad V_d\ V_u^{-1} \leqslant 1$

If V_d and V_u are replaced by the duct section velocities across the transition then the above equation may also be expressed in the following generalized form

$$\sum_{t=1}^{T_c} \delta_t C_t \prod_{n=1}^{m} V_n^{a_{tn}} \leq 1 \qquad (34)$$

Where,

δ_t = $\pm$ 1 (Sign of each term in the constraint equation)

C_t= Coefficient of each term in the constraint eqn.

m = Total number of variables in a system

T_c= Total number of terms in the constraint equation

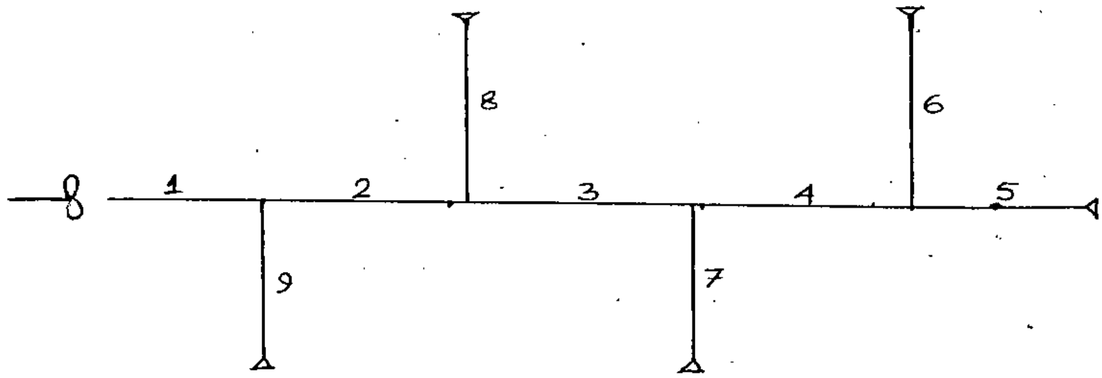
a_{tn} = Exponent of the n-th independent variable of the t-th term.

Thus it is found that the objective function and the constraint functions can be expressed in the geometric[11] form. As such geometric programming technique is suitable for the solution of the present problem.

Logic diagram for geometric programming algorithm is presented in appendix- VII.

CHAPTER V
CASE STUDY

The following ducting system has been taken into account to compare different duct design methods and to find out the optimum duct design criteria.



In this oase study, following factors have been considered for ease of comparison among different duct design methods.

1) Same duct layout
2) Duct sections equal in length
3) Equal discharge through each terminal
4) Same duct material throughout the system
5) Duct material thickness constant throughout the system.
6) Constant thermal insulation thickness throughout the system.
7) No acoustically lined ducts.

Data for the above mentioned system has been assumed and is presented in Appendix-I.

Three different duct design methods have been applied to the given system with the assumed data to determine the dimension of each duct section, air velocity in each duct section, power required to run the blower to deliver the required amount of air and the life cycle cost of the duct system. Global optimum life cycle cost are then determined for each duct design method by changing the initial velocity at the blower outlet, where the change in initial velocity determines the velocity of air and other parameters in the downstream duct sections as per the duct design methods. The global optimum life cycle costs are then compared to select the best duct design method.

The three duct design methods applied to the present duct layout are

1) Equal friction method
2) Static regain method

and 3) The optimized duct design method which considers equal pressure loss along each path of the duct system.

Sample calculation for the above three duct design methods are presented in Appendices II, III and IV.

CHAPTER VI

RESULTS AND DISCUSSION

Results obtained by putting numerical values in the mathematical model for the specific ducting system, shown in Figs. 6.1, 6.2 and 6.3.

Fig. 6.1 presents curves of life cycle costs versus air velocities in the main duct next to blower for the ducting system, designed seperately by the three design methods viz. static regain method, equal friction method and optimized design method. Two sets of curves are presented each for different local loss coefficients. From these curves the following statements can be made.

i) The life cycle cost reduces with increase in air velocity in the main duct to an optimum value from where the cost increases with further increase in air velocity seperately for ducting system designed by each method.

ii) The life cycle cost for a ducting system designed by optimized method is always less than the costs of the system when it is designed by the other two method for a wide range of practical air velocities.

iii) The global optimum life cycle cost for the ducting system designed by the optimized method occurs at lower initial velocities with respect to the other two methods. This indicates that the ducting system designed by this method will cause less noise problem compared to a system designed by other two methods.

iv) For higher local loss coefficient at the entry section minimum life cycle costs for the three different design methods occur at air velocities in the main duct very close to each other, whereas for lower local loss coefficient at the entry section minimum life cycle cost for the optimized method occurs at relatively lower velocity of air in the main duct than those of the other two methods.

v) Comparison among the minimum life cycle costs found by the three different duct design methods shows that the minimum life cycle cost obtained by the optimized method is 9.23 percent and 4.12 percent lower than those obtained by the static regain and the equal friction methods respectively.

In Fig. 6.2 life cycle cost versus different amount of total air flow curves are drawn for static regain, equal friction and optimized duct design methods. Curves numbering from 1 to 3 refer to the static regain, equal friction and optimized duct design method respectively for the same velocity of air in the main ducts. Curve number 4 is optimum life cycle cost versus total airflow rate for the optimized duct design method. For the present case these curves show that for the same velocity of air in the main ducts, life cycle cost is minimum for the optimized duct design method irrespective of the total air flow rate and the optimum life cycle cost found by the optimized design method is the lowest among the three design methods mentioned above.

In Fig. 6.3 a curve is presented, which shows a relationship between the initial velocities at optimum condition for the optimized method and total air flow rate for the present duct layout. This curve shows that as the total flow rate increases air velocity at optimum condition gradually decreases.

Velocities at different duct sections for the three different design methods are given in Table 6.1. It is evident from the numerical value of the velocities, that for the optimized method, velocity in the straight-through sections remain same, whereas for the other two methods velocities in the straight-through sections gradually decrease. Again both in case of optimized method and the static regain method velocities in the branches gradually decrease starting from the branch next to the main duct, but in case of optimized method the decrease rate of velocity in the branches is lower than that of the static regain method. In case of equal friction method all the branch velocities are same and it is only due to the assumption that flow rate of air through the branches are same, otherwise this case may not happen.

CHAPTER VII

CONCLUSION AND RECOMMENDATION

7.1 Based on the study made in this research, the following conclusions can be drawn:

i) A mathematical model can be formulated for determination of different design variables at optimum life cycle cost of airconditioning ducting system considering all practical constraints of the system.

ii) Geometric programming may be used to solve the mathematical model to find different design variables at optimized condition.

iii) Computer programming for the solution of the mathematical model makes it possible to get the optimized solution in case of complex ducting layout, which otherwise would have been tedious and highly time consuming if done manually.

iv) Results of the study show that the life cycle cost of a ducting system designed by Optimized method is lower by 9.23 percent (approx.) and 4.12 percent (approx.) from that designed by static regain method and Equal friction method respectively.

v) It also appears from the study that

a) Equal pressure loss along each duct run of the airconditioning ducting system provides optimum design of the system.

b) In case of optimum ducting system design, velocity of air across straight-through transition should remain same.

c) For an optimized ducting system, the optimum velocity in the main duct next to the blower reduces with increase in total air volume flow rate.

It is recommended to study further on the following:

1) To study on optimization of ducting system considering acoustic problems and related costs.

2) To study on optimization of ducting system with different duct materials and to find the influence of material properties and relevent material costs.

3) To study on optimization of overall airconditioning system consisting of refrigeration equipment, air-conditioning apparatus/equipment and other ancillary equipment.

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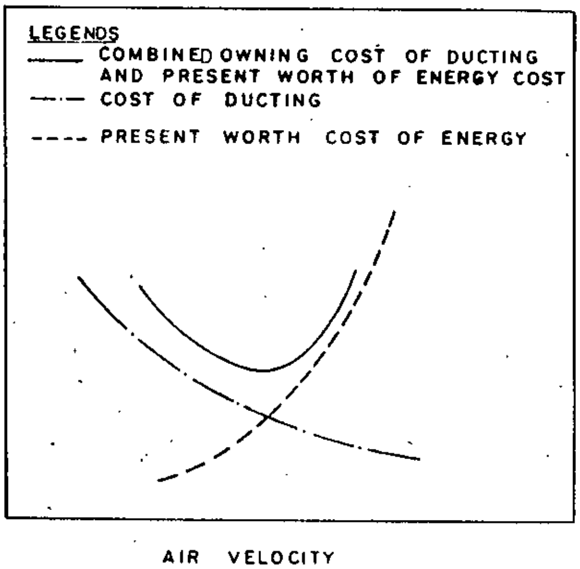


FIG. 1.1 TYPICAL RELATIONSHIP OF AIR VELOCITY AND COST OF AIR DISTRIBUTION SYSTEM

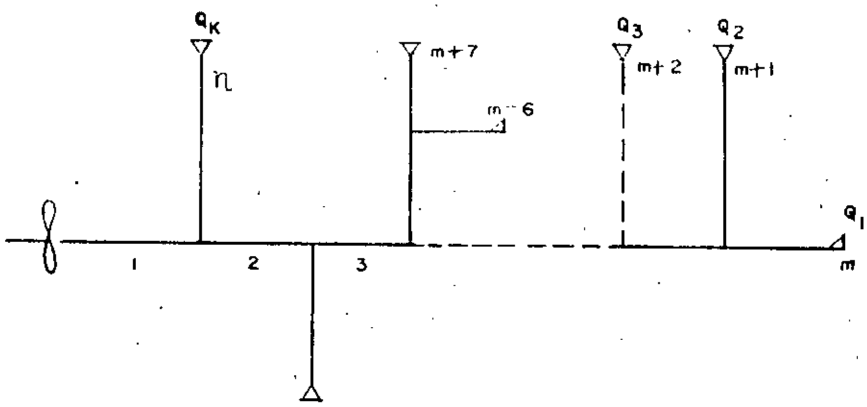


FIG. 1.2 TYPICAL DUCT LAYOUT

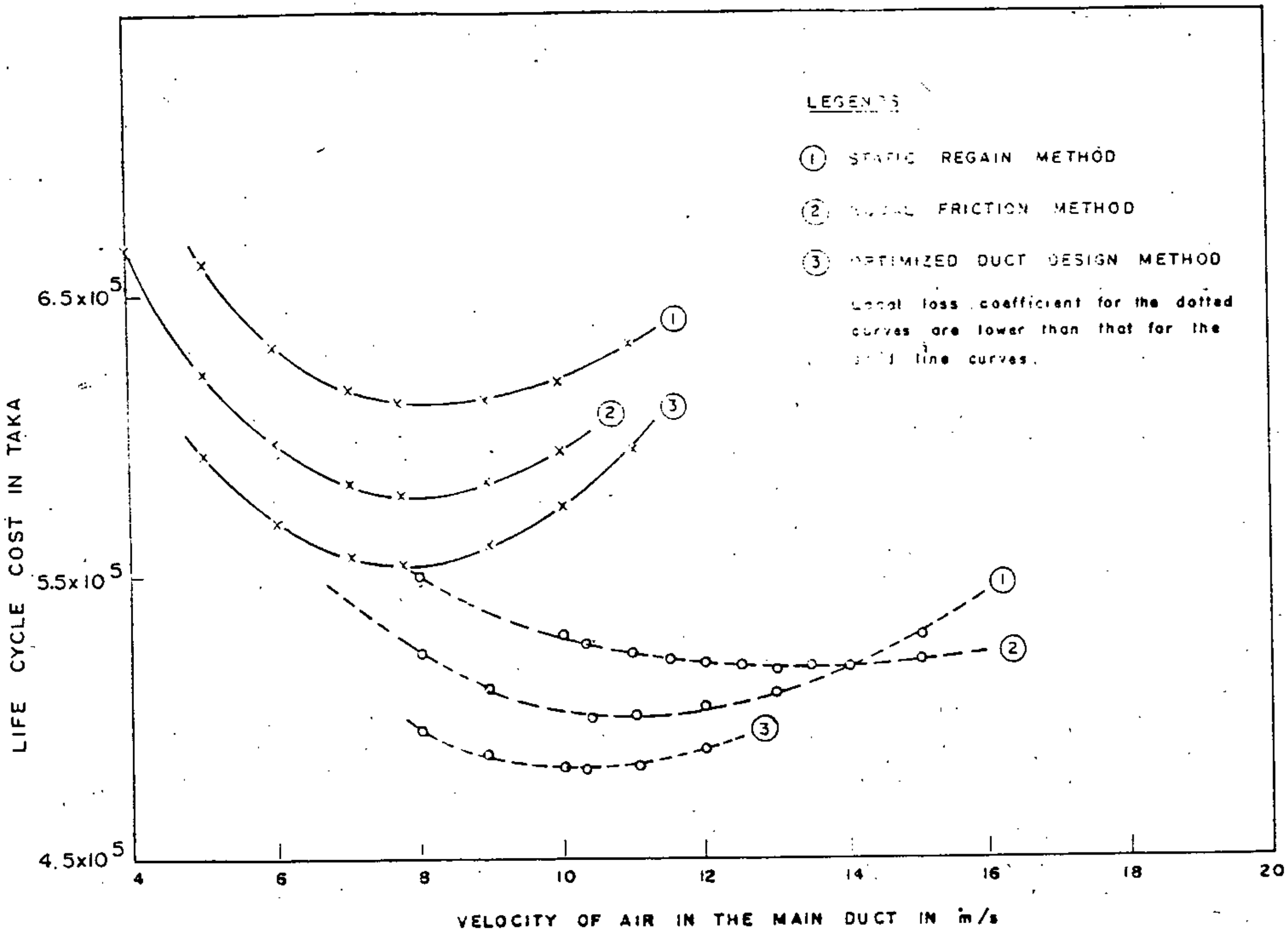


Fig. 6.1 Life cycle cost with respect to velocity of air in the main duct for static regain, equal friction and optimized duct design methods.

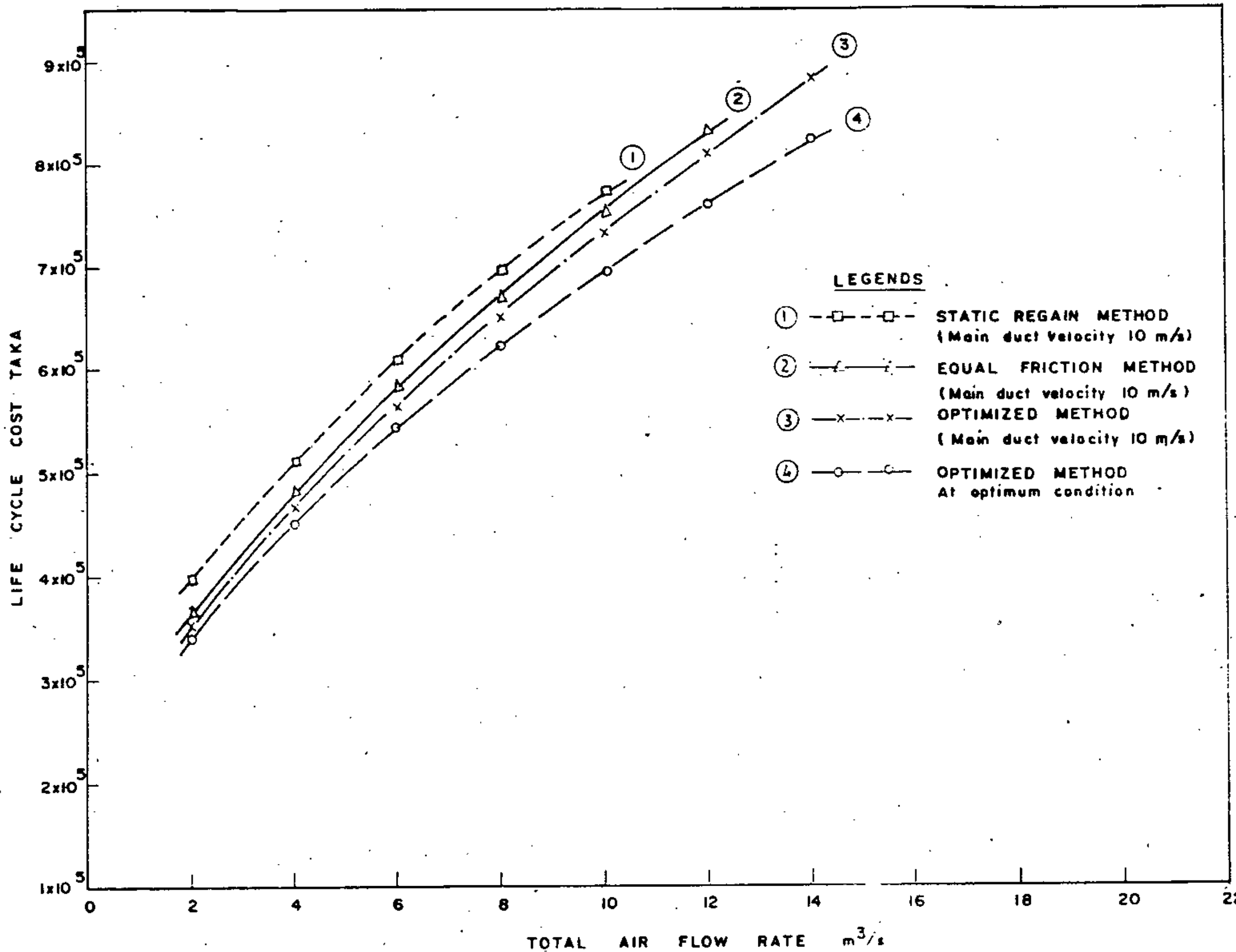


Fig. 6.2 Variation of life cycle cost with increase in total volume flow rate of air.

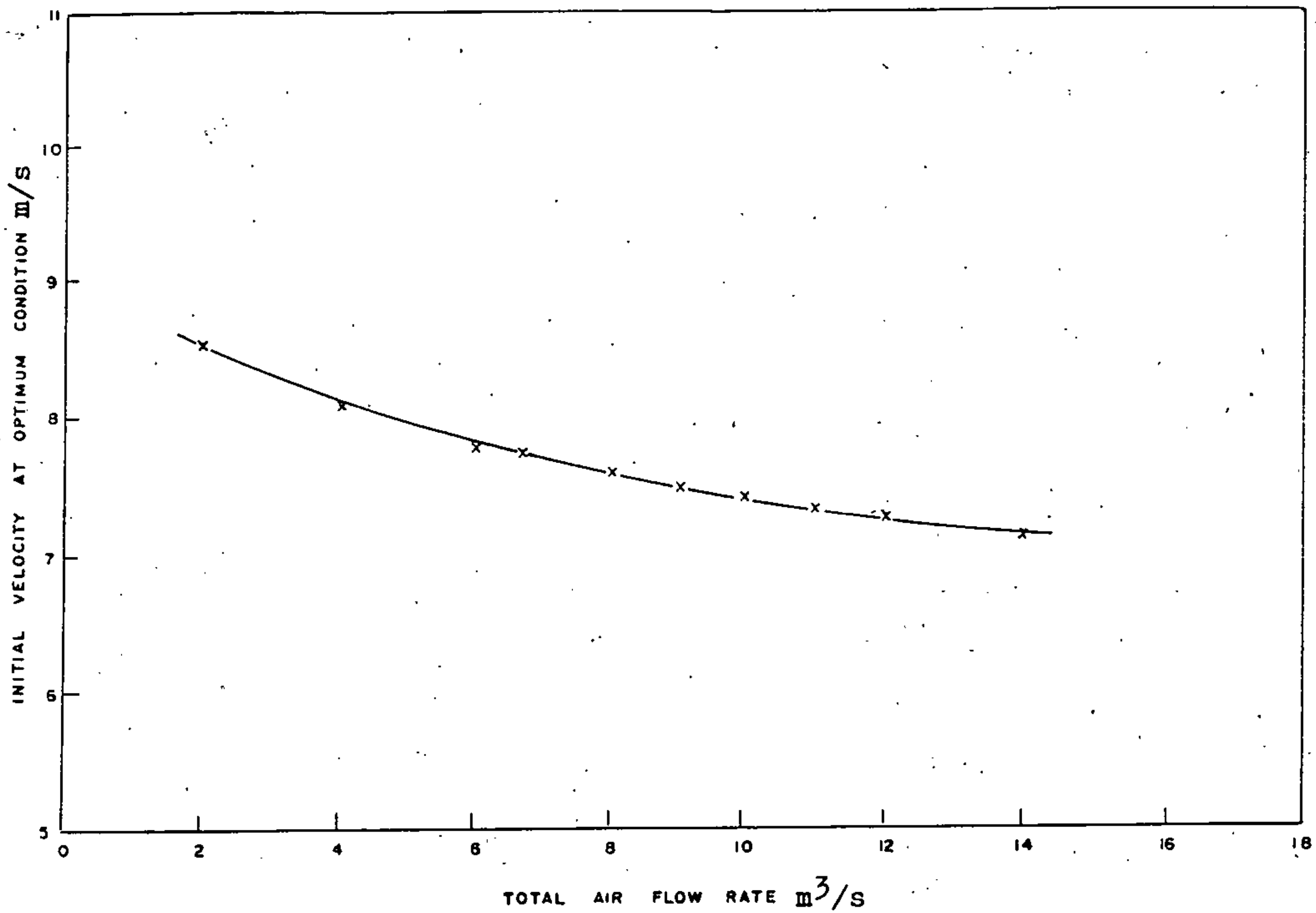


Fig. 6.3 Relationship between initial velocity of air at optimum condition and total air flow rate.

Table 6.1: Velocity of air in different duct sections for Equal friction Static regain and Optimized method.

Initial velocity	Duct section	Velocity for equal friction method	Velocity for static regain method	Velocity for optimized method
7.00	1	7.00	7.00	7.00
	2	6.635	6.144	7.00
	3	6.192	5.328	7.00
	4	5.618	4.521	7.00
	5	4.757	3.653	7.00
	6	4.757	2.802	4.492
	7	4.757	3.280	5.575
	8	4.757	3.759	6.243
	9	4.757	4.257	6.729
7.718	1	7.718	7.718	7.718
	2	7.316	6.738	7.718
	3	6.828	5.810	7.718
	4	6.194	4.902	7.718
	5	5.245	3.935	7.718
	6	5.245	3.028	5.021
	7	5.245	3.563	6.229
	8	5.245	4.105	6.974
	9	5.245	6.671	7.515

Appendix-I: Data used in the case study

Discharge through each terminal = 1.344 m^3/s

Length of each duct section = 20 m

E_d = 10.00 Tk/KW

E_c = 2.30 Tk/KWh

T = 2200 hr/Hr

n_f = 0.85

n_e = 0.70

= 1.204 kg/m^3

m = 9 sections in the system

R = 5 paths in the system

PWF = 4.8577 for an amortization period of 9 years at the annual interest rate of 14.5% on the basis of equal payment series.

CP = Tk. 2200.00

F = Tk. 3300.00

CF = Tk. 110880.00

S_i = Tk. 968.75/m^2 (Including cost of material, cost of insulation and cost of fabrication).

Appendix-II: Sample calculation for equal friction method.

Section	Item	D(m)	Q m^3/sec	V m/sec	$h_v = (\frac{V}{1.29})^2$ (Pa)	L(m)	h'_o or c	h_o (Pa)	$\sum h_o$ (Pa)
1	Duct	1.07	6.72	7.718	35.86	20	0.5 6.03	10.0 216.24	226.24
2	Duct	0.975	5.376	7.315	32.22	20	0.5 0.0052	10.0 0.168	10.168
3	Duct	0.87	4.032	6.83	28.0	20	0.5 0.0066	10.0 0.185	10.185
4	Duct	0.74	2.688	6.194	23.096	20	0.5 0.0093	10.0 0.215	10.215
5	Duct	0.57	1.344	5.25	16.59	20	0.5 0.015	10.0 0.249	10.249
6(4)	Duct (branch)	0.57	1.344	5.25	16.59	20	0.5 1.3	10.0 21.569	31.567
7(3)	Duct (branch)	0.57	1.344	5.25	16.59	20	0.5 1.284	10.0 21.3	31.3
8(2)	Duct (branch)	0.57	1.344	5.25	16.59	20	0.5 1.259	10.0 20.887	30.877
9(1)	Duct (branch)	0.57	1.344	5.25	16.59	20	0.5 1.24	10.0 20.57	30.57

Total Duct Area = $\sum_{i=1}^{9} \pi D_i L_i$ = 408.72 m^2

Pressure loss along path	1-2-3-4-5	= 267.057 Pa
" " " "	1-2-3-4-6	= 288.375
" " " "	1-2-3-7	= 277.893
" " " "	1-2-8	= 267.295
" " " "	1-9	= 256.81

Highest pressure loss takes place along the path 1-2-3-4-6, and its value is 288.375 Pa.

$\therefore$ Required power = $\frac{6.72 \times 288.375}{1000 \times 0.7 \times 0.85}$ = 3.257 KW

And life cycle cost = 3.257(10+2.3x2200) 4.8577
+ 408.72x968.75
+ 3.257x2200+114180.00
= 597508.05 Taka

Appendix-III: Sample calculation for static regain method.

Section	Item	D(m)	Q m^3/sec	V m/sec	$h_v = (\frac{V}{1.29})^2$ (Pa)	L(m)	h'_o or c	h_o(pa)	$\sum h_o$(Pa)	$(h_u-h_d)a$ (Pa)
1	Duct	1.07	6.72	7.718	35.795	20	0.5	10.0		
	Fitt.						6.03	215.85	225.85	
2	Duct	1.02	5.376	6.738	27.33	20	0.4	8.0		
	Fitt.						0.0127	0.347	8.347	-0.1186
3	Duct	0.94	4.032	5.81	20.32	20	0.35	7.0		
	Fitt.						0.014	0.28	7.28	0.27
4	Duct	0.82	2.688	4.90	14.454	20	0.28	5.6		
	Fitt.						0.0157	0.227	5.827	0.039
5	Duct	0.66	1.344	3.935	9.32	20	0.24	4.8		
	Fitt.						0.02	0.186	4.986	0.148
6(4)	Duct	0.75	1.344	3.03	5.527	20	0.13	2.6		
	Fitt.						1.209	6.682	9.282	0.355
7(3)	Duct	0.68	1.344	3.563	7.64	20	0.19	3.8		
	Fitt.						1.207	9.22	13.02	0.34
8(2)	Duct	0.65	1.344	4.105	10.144	20	0.26	5.2		
	Fitt.						1.2046	12.219	17.419	0.233
9(1)	Duct	0.60	1.344	4.671	13.137	20	0.38	7.7		
	Fitt.						1.203	15.80	23.40	0.74

Total Duct Area $\sum_{i=1}^{9} \pi D_i L_i = 451.76\ m^2$

Pressure	loss	along	path	1-2-3-4-5	= 252.29
"	"	"	"	1-2-3-4-6	= 256.586
"	"	"	"	1-2-3-7	= 254.497
"	"	"	"	1-2-8	= 251.616
"	"	"	"	1-9	= 249.25

Highest pressure loss takes place along the path 1-2-3-4-6, and its value is 256.586 Pa

$\therefore$ Required power $= \frac{6.72 x 256.586}{1000 x 0.7 x 0.85} = 2.8979$ KW

And life cycle cost = 2.8979x(10+2.3x2200)x4.8577

+ 451.76x968.75+2.8979x2200

+ 114180.00

= 656568.92 Taka

Appendix-IV: Sample calculation for optimized method.

Section	Item	D(m)	Q m^3/sec	V m/sec	$h_v = (\frac{V}{1.29})$ (Pa)	L(m)	h'_o or c	h_o (pa)	$\sum h_o$ (Pa)
1	Duct	10.07	6.72	7.718	35.86	20	0.5 6.03	10.0 216.24	226.24
2	Duct	0.94	5.376	7.718	35.86	20	0.60 0.0	12.0 0.0	12.0
3	Duct	0.82	4.032	7.718	35.86	20	0.7 0.0	14.0 0.0	14.0
4	Duct	0.64	2.688	7.718	35.86	20	0.95 0.0	19 0.0	19.0
5	Duct	0.46	1.344	7.718	35.86	20	1.5 0.0	30 0.0	30.0
6(4)	Duct	0.575	1.344	5.02	15.17	20	0.5 1.225	10.0 18.58	28.58
7(3)	Duct	0.5	1.344	6.229	23.358	20	0.85 1.3	17.0 30.37	47.37
8(2)	Duct	0.48	1.344	6.97	29.246	20	1.1 1.3	22.0 38.02	60.02
9(1)	Duct	0.46	1.344	7.515	33.998	20	1.4 1.3	28.0 44.197	72.197

Total Duct Area = $\sum_{i=1}^{9} \pi . D_i L_i = 373.54 \text{ m}^2$

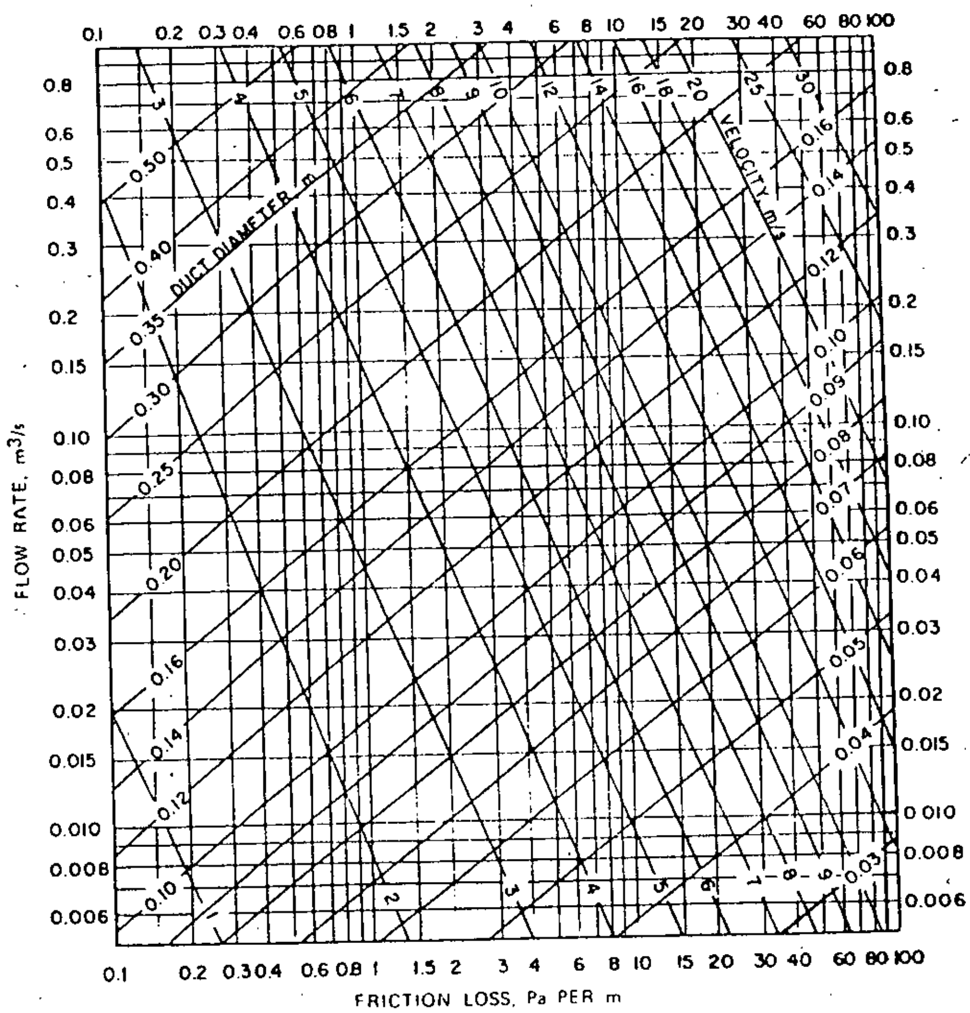
Pressure	loss	along	path	1-2-3-4-5	= 301.24
"	"	"	"	1-2-3-4-6	= 299.82
"	"	"	"	1-2-3-7	= 299.61
"	"	"	"	1-2-8	= 298.26
"	"	"	"	1-9	= 298.437

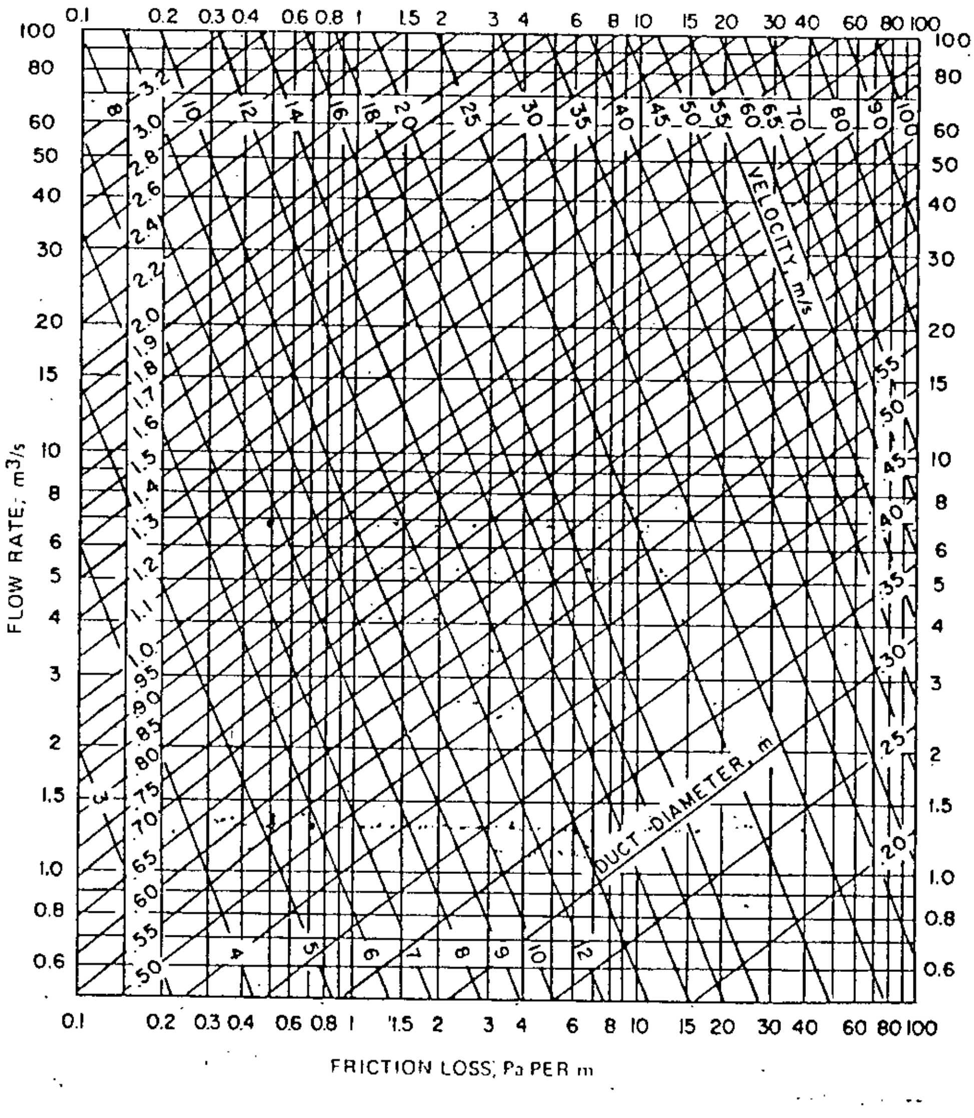
Highest pressure loss takes place along the path 1-2-3-4-5 and its value is 301.24 Pa

$\therefore$ Required power = $\frac{6.72\text{x}301.24}{1000\text{x}0.7\text{x}0.85}$ = 3.402 KW

and life cycle cost = 3.402x(10+2.3x2200)4.8577+373.54x968.75
+ 3.402x2200+114180.00
= 567317.56 Taka

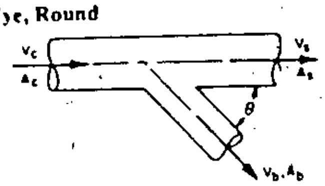
Appendix-V: Friction chart for galvanized iron sheet duct material




FLOW RATE, m³/s
FRICTION LOSS, Pa PER m
VELOCITY, m/s
DUCT DIAMETER, m

Appendix - VI: Local loss coefficients for duct fittings.

6-5 Diverging Wye, Round



$\theta = 15° - 90°$

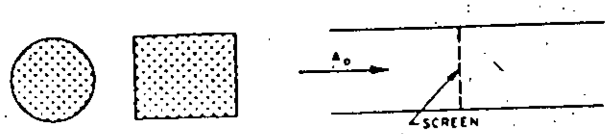
$A_c = A_s$

Branch, $C_{c,b}$					
V_b/V_c	θ, degrees				
	15	30	45	60	90
0	1.0	1.0	1.0	1.0	1.0
0.1	0.92	0.94	0.97	1.0	1.0
0.2	0.65	0.70	0.75	0.84	1.0
0.4	0.38	0.46	0.60	0.76	1.1
0.6	0.20	0.31	0.50	0.65	1.2
0.8	0.09	0.25	0.41	0.80	1.3
1.0	0.10	0.25	0.52	0.90	1.3
1.2	0.11	0.32	0.67	1.1	1.4
1.4	0.22	0.63	0.88	1.4	1.6
1.6	0.41	0.72	1.2	1.8	1.8
2.0	0.99	1.4	1.9	2.7	2.2
2.6	2.5	2.9	2.7	4.6	—
3.0	6.5	6.7	7.0	7.3	—

Main

V_s/V_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0
$C_{c,s}$	0.40	0.32	0.26	0.20	0.15	0.10	0.06	0.02	0

7-7 Obstruction, Screen in Duct, Round, and Rectangular



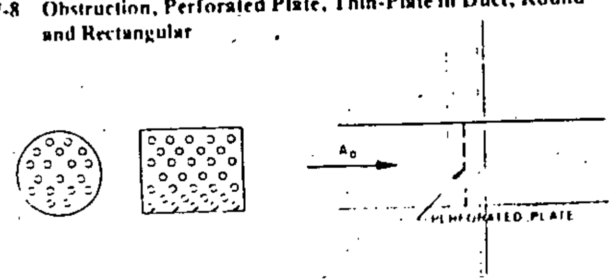
$n = A_{or}/A_o$

where

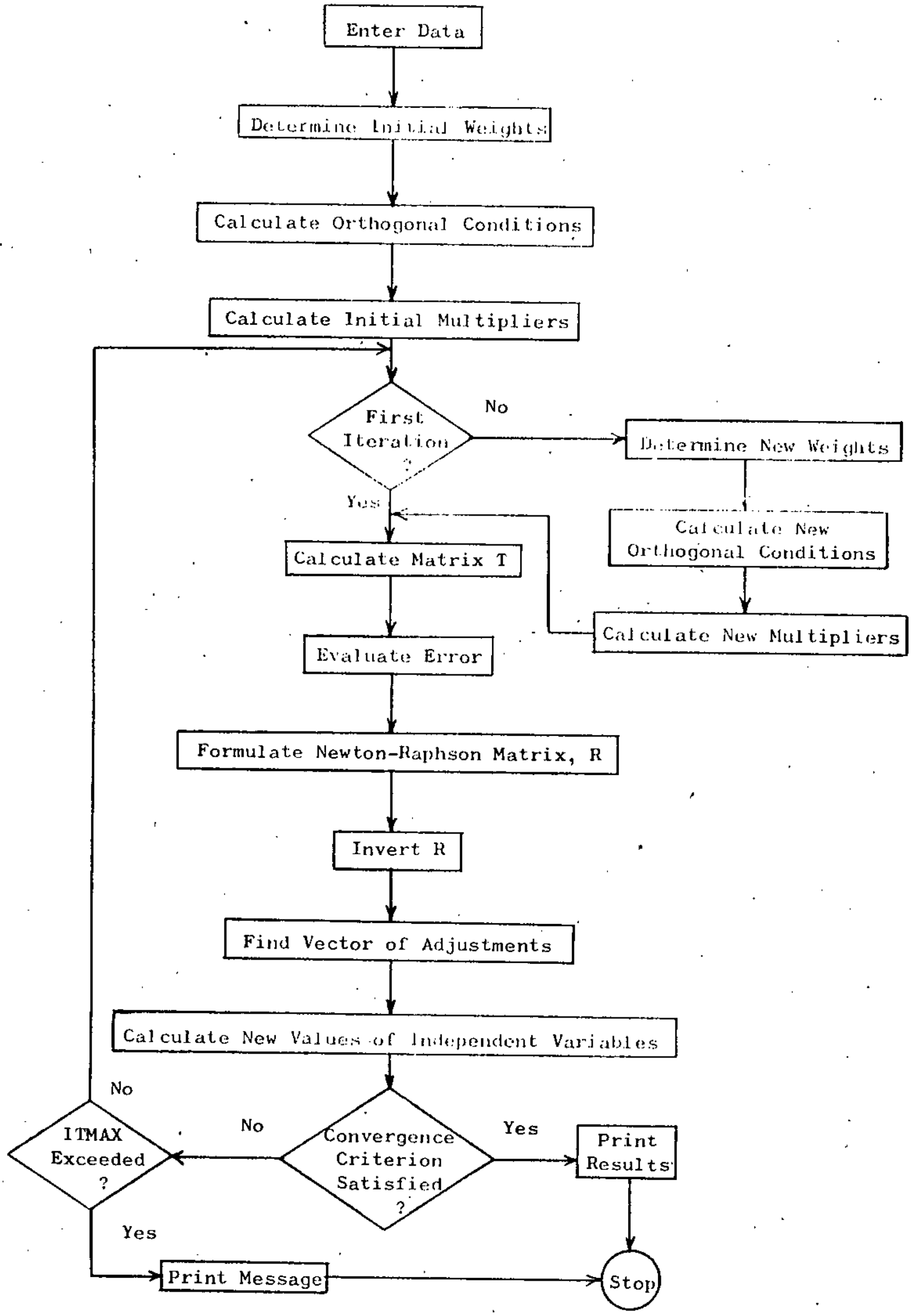
n = free area ratio of screen
A_{or} = total flow area of screen
A_o = area of duct

n	0.30	0.40	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.90	1.0
C_o	6.2	3.0	1.7	1.3	0.97	0.75	0.58	0.44	0.32	0.14	0

7-8 Obstruction, Perforated Plate, Thin-Plate in Duct, Round and Rectangular



Appendix-VII:Logic diagram for Geometric Programming Algorithm



Appendix-VIII: Computer program for Geometric Programming Algorithm.

```
FILE: GPROG    FORTRAN  A1  BUET COMPUTER CENTRE, DHAKA           VM/SP (4331-L02)

C    GEOMETRIC PROGRAMMING ALGORITHM FOR THE SOLUTION
C     OF NONLINEAR OPTIMIZATION PROBLEMS
C      DESCRIPTION OF PARAMETERS USED IN THIS PROGRAM
C   ARE GIVEN BELOW:
C       N         TOTAL NUMBER OF VARIABLES
C       M         NUMBER OF CONSTRAINTS
C       SIG       ASSUMED SIGN OF THE VALUE OF THE OBJECTIVE
C                 FUNCTION
C       KT        VECTOR GIVING NUMBER OF TERMS PER POLYNOMIAL
C       C         VECTOR OF COEFFICIENTS
C       A         ARRAY OF EXPONENTS
C       X         VECTOR OF INITIAL ESTIMATES OF OPTIMAL SOLUTION
C       W         VECTOR OF WEIGHTS (DUAL VARIABLES)
C       SUM       OBJECTIVE FUNCTION
C       V         DUAL FUNCTION
C       KTT       TOTAL NUMBER OF TERMS IN THE PROBLEM
C       M1        TOTAL NUMBER OF POLYNOMIALS
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
      COMMON N,M,SIG,M1,KTT,KT0,MR,N1,NR,MM1,V,SUM,DP
      COMMON C(80)
      COMMON X(13),KT(30),W(80),A(80,13),R(30,80),PIVOT(30),ADD(3),E(80)
      COMMON MM
      DIMENSION IPVOT(30),INDEX(80,2)
      OPEN (UNIT=10,FILE='IN',STATUS='OLD')
      OPEN (UNIT=11,FILE='OUT',STATUS='NEW')
C
      WRITE(11,399)
C **********INPUT TO THE PROBLEM************************
      WRITE(11,315)
      READ(10,100)N,M,SIG
      KTT=0
      M1=M+1
C*******NUMBER OF TERMS IN EACH POLYNOMIAL**********
      DO 200 L=1,M1
      READ(10,101)KT(L)
200   KTT=KTT+KT(L)
      KSIG=SIG
      IF(M-1)263,263,264
263   WRITE(11,107)N,M,KTT,KSIG
      GO TO 500
264   WRITE(11,109)N,M,KTT,KSIG
500   CONTINUE
      WRITE(11,301)
      WRITE(11,305)
C******COEFFICIENTS AND EXPONENTS*************
      I2=0
      DO 202 L=1,M1
      I1=I2+1
      I2=I1+KT(L)-1
      DO 202 I=I1,I2
      READ(10,109)C(I)
      READ(10,102)A(I,1),A(I,2),A(I,3),A(I,4),A(I,5),A(I,6),A(I,7),
     +A(I,8),A(I,9),A(I,10),A(I,11),A(I,12),A(I,13)
202   WRITE(11,107)L,I,C(I),A(I,1),A(I,2),A(I,3),A(I,4),A(I,5),A(I,6),
     1A(I,7),A(I,8),A(I,9),A(I,10),A(I,11),A(I,12),A(I,13)

FILE: GPROG    FORTRAN  A1  BUET COMPUTER CENTRE, DHAKA           VM/SP (4331-L02)

C*****INITIAL SOLUTION******************************
      DO 203 J=1,N
      READ(10,103)X(J)
203   WRITE(11,103)X(J)
      MR=M+N+1
      N1=N+1
      NR=N+2
```

```
      MR1=MR+1
      KTO=KT(1)
      IT=1
      MRX=56
      DO 213 IR=1,MRX
      DO 213 JR=1,MRX
213   R(IR,JR)=0
C*******************************
C***** EVALUATION OF THE INITIAL WEIGHTS******************
C      THESE WEIGHTS WILL CHANGE AT EACH ITERATION *******
      CALL GP3
C*** EVALUATION OF THE OBJECTIVE FUNCTION ********
      SUM=0.
      DO 206 I=1,KTO
206   SUM=SUM+W(I)*(C(I)/ABS(C(I)))
C     INITIAL VALUE OF V
      V=ABS(SUM)
      DO 207 I=1,KTO
207   W(I)=W(I)/V
100   FORMAT(I4,I4,F5.1)
109   FORMAT(1H0,9X,I3,10H VARIABLES,7X,I3,14H RESTRICTIONS ,7X,I3,
     19H TERMS    ,7X,14HOBJECTIVE SIGN,I3)
197   FORMAT(1H0,9X,I3,10H VARIABLES,7X,I3,14H RESTRICTION  ,7X,I3,
     19H TERMS    ,7X,14HOBJECTIVE SIGN,I3)
301   FORMAT(1H0,37X,26HCOEFFICIENTS AND EXPONENTS)
305   FORMAT(1H0,1X,4HTERM,2X,11HCOEFFICIENT,2X,4HX(1),2X,4HX(2),2X,
     14HX(3),2X,4HX(4),2X,4HX(5),2X,4HX(6),2X,4HX(7),2X,4HX(8),2X,
     14HX(9),2X,5HX(10),2X,5HX(11),2X,5HX(12),2X,5HX(13))
102   FORMAT(13F5.2)
107   FORMAT(/,1H0,I2,1H-,I3,E13.6,13F6.3)
199   FORMAT(E15.8)
101   FORMAT(I4)
103   FORMAT(6X,F12.7)
315   FORMAT(10X,'**     GEOMETRIC PROGRAMING PROBLEM      **')
399   FORMAT(1H1)
C     ***********************************************
      CALL GP22
C     PRODUCT OF THE TRANSPOSE MATRIX OF ORTHOGONAL CONDITIONS ... T
C     CONSTRAINTS BY THE SAME MATRIX WITHOUT TRANSPOSING
      DO 215 JR=NR,MR
      DO 215 IR=NR,MR
      DO 215 KR=1,N
215   R(IR,JR)=R(IR,JR)+R(KR,IR)*R(KR,JR)
      MM=NR
C**INVERSION OF THE MATRIX FOUND ABOVE********
      CALL GP10
C****
      DO 224 JR=NR,MR
```

FILE: GPROG   FORTRAN A1 BUET COMPUTER CENTRE, DHAKA   VM/SP (4331-L02)

```
      DO 224 IR=1,N
224   R(N1,JR)=R(N1,JR)+R(IR,JR)*R(IR,N1)
C ***  INITIAL MULTIPLIERS***
      DO 225 IR=NR,MR
      L=IR-N1
      AMD(L)=0.
      DO 225 JR=NR,MR
225   AMD(L)=AMD(L)+R(N1,JR)*R(IR,JR)
      DO 273 KKK=1,20
      IF(IT-1)232,226,232
C**THE FIRST ITERATION IS NOT OVER ****
226   CONTINUE
C*** MATRIX T ****
      DO 231 J=1,N
      DO 231 JJ=1,N
      SU=0.
      DO 228 I=1,KTO
228   SU=SU+C(I)/ABS(C(I))*A(I,J)*A(I,JJ)*W(I)
      I2=KTO
      SUD=0.
      DO 230 L=2,M1
      I1=I2+1
      I2=I1+KT(L)-1
      SUS=0.
      DO 229 I=I1,I2
229   SUS=SUS+C(I)/ABS(C(I))*A(I,J)*A(I,JJ)*W(I)
      SUL=SUS*AMD(L-1)
230   SUD=SUD+SUL
231   R(J,JJ)=SUD-SU
      GO TO 235
```

```
232    CONTINUE
C** NEW WEIGHT GENERATED BY THE ALGORITHM ******
       CALL GP3
       SUM=0.
       DO 243 I=1,KTO
       SUM=SUM+W(I)*(C(I)/ABS(C(I)))
       IF(I.EQ.13)GO TO 321
       GO TO 243
321    WRITE(11,979)SUM
979    FORMAT(//,5X,'SUM OF FIRST 1  T','/,F15.5)
243    CONTINUE
[illegible]
243    CONTINUE
       DO 233 I=1,KTO
233    W(I)=W(I)/V
C*** NEW VECTOR AND MATRIX OF ORTHOGONAL CONDITIONS *********
       CALL GP22
       DO 235 L=1,M
       IR=L+N1
235    AMD(L)=AMD(L)+PIVOT(IR)
       GO TO 226
236    CONTINUE
C **** EVALUATION OF THE ERROR ***************
       SUMV=0.
       DO 286 I=1,KTO
286    SUMV=SUMV+W(I)*(C(I)/ABS(C(I)))

       DO 254 IR=1,N
9      E(IR)=0.
       DO 253 JR=NR,MR
       J=JR-N1
253    E(IR)=E(IR)+R(IR,JR)*AMD(J)
254    E(IR)=R(IR,N1)-E(IR)
       E(N1)=SIG-SUMV
       I2=KTO
       DO 256 L=2,M1
       I1=I2+1
       I2=I1+KT(L)-1
       SG=0.
       DO 255 I=I1,I2
255    SG=SG+W(I)*(C(I)/ABS(C(I)))
       IR=L+N1-1
256    E(IR)=1.-SG
C ************************************
       WRITE(11,314) IT
       DO 396 J=1,N
396    WRITE(11,108) J,X(J)
       WRITE(11,308)
       VV=V*SIG
       WRITE(11,104) SUM
       WRITE(11,105) VV
C **************************
104    FORMAT(24X,18HOBJECTIVE FUNCTION,1H=,F25.8)
105    FORMAT(24X,13HDUAL FUNCTION      ,1H=,F25.8)
108    FORMAT(1H0,42X,2HX(,I2,5H ) = ,F15.8)
314    FORMAT(1H1,//,1X,'*************************************',
      11X,9HITERATION,I3,1X,'*************************************')
308    FORMAT(1X)
C ***FINAL EXPRESSION OF THE NEWTON-RAPSON MATRIX **************
       DO 250 IR=N1,MR
       DO 250 JR=1,N
250    R(IR,JR)=R(JR,IR)
       DO 251 IR=N1,MR
       DO 251 JR=N1,MR
251    R(IR,JR)=0.
       R(N1,N1)=-SIG
C***** INVERSION OF THE MATRIX R *********************
       MM=1
       CALL GP10
C*** THE INVERSE OF THE MATRIX R IS MULTIPLIED BY THE VECTOR E IN ORDER*
C*** TO FIND THE VECTOR OF ADJUSTMENT **********************************
       DO 257 IR=1,MR
       PIVOT(IR)=0.
       DO 257 JR=1,MR
257    PIVOT(IR)=PIVOT(IR)+R(IR,JR)*E(JR)
C ** THE NEW ITERATION STARTS HERE ***********
       IT=IT+1
C** THE PRIMITIVE SOLUTION IS MODIFIED AND A NEW VALUE OF V IS FOUND****
       DO 258 J=1,N
258    X(J)=X(J)*EXP(PIVOT(J))
       V=V*EXP(PIVOT(N1))
       DO 298 IR=1,N
```

```
      DO 298 JR=M1,MR
298   R(IR,JR)=0.
C TEST OF THE SOLUTION ******************************
      IF(ABS(V-SIG*SUM)/V-0.000003)274,274,273
C *THE SOLUTION IS SATISFACTORY *************************
274   GO TO 299
C*THE SOLUTION IS NOT SATISFACTORY *********************
273   CONTINUE
      WRITE(11,778)
      GO TO 897
299   WRITE(11,388)
897   CONTINUE
      DO 395 J=1,N
395   WRITE(11,108)J,X(J)
      WRITE(11,308)
      WRITE(11,104)SUM
      VV=V*SIG
      WRITE(11,105)VV
C***************************************************
388   FORMAT(1H0,45X,15HOPTIMAL RESULTS)
778   FORMAT(30X,39HNOT ENOUGH CONVERGENCE IN 20 ITERATIONS)
C**
      STOP
      END
      SUBROUTINE GP10
C** THE SUBROUTINE GP10 IS USED TO INVERT THE NEWTON AND RAPSON MATRIX,
C** OR ONE SUBMATRIX OF IT      ****************************************
      COMMON N,M,SIG,M1,KTT,KTO,MA,N1,I1,MM1,V,SUM,NP
      COMMON X(80)
      COMMON R(13),KT(30),W(80),A( ,13), ( , ),PIVOT(  ),AND( ),( 30)
      COMMON MM
      DIMENSION IPVOT(80),INDEX(80,2)
      DET=1.
      DO 216 JR=MM,MR
216   IPVOT(JR)=0
      DO 241 IR=MM,MR
C** INVESTIGATION OF THE PIVOT ************************
      T=0.
      DO 220 JR=MM,MR
      IF(IPVOT(JR)-1)217,220,217
217   DO 219 K=MM,MR
      IF(IPVOT(K)-1)218,219,239
218   IF(ABS(T)-ABS(R(JR,K)))242,219,219
242   IROW=JR
      ICOL=K
      T=R(JR,K)
219   CONTINUE
220   CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
C* THE PIVOT IS LOCATED ON THE DIAGONAL ****************
      IF(IROW-ICOL)221,223,221
221   DET=-DET
      DO 222 LL=MM,MR
      T=R(IROW,LL)
      R(IROW,LL)=R(ICOL,LL)
```



```
222   R(ICOL,LL)=T
223   INDEX(IR,1)=IROW
      INDEX(IR,2)=ICOL
      PIVOT(IR)=R(ICOL,ICOL)
      DET=DET*PIVOT(IR)
C**THE PIVOT ROW IS DIVIDED BY THE PIVOT ********
      R(ICOL,ICOL)=1.
      DO 284 LL=MM,MR
284   R(ICOL,LL)=R(ICOL,LL)/PIVOT(IR)
C*** ROWS WITHOUT PIVOT ARE REDUCED **************
      DO 241 KL=MM,MR
      IF(KL-ICOL)245,241,245
245   T=R(KL,ICOL)
      R(KL,ICOL)=0.
      DO 240 LL=MM,MR
240   R(KL,LL)=R(KL,LL)-R(ICOL,LL)*T
241   CONTINUE
C*********COLUMNS ARE EXCHANGED *********************
      DO 238 IR=MM,MR
      LL=MR-IR+MM
      IF(INDEX(LL,1)-INDEX(LL,2))227,238,227
227   JROW=INDEX(LL,1)
```

```
      JCOL=INDEX(LL,2)
      DO 237 K=MM,MR
      T=R(K,JROW)
      R(K,JROW)=R(K,JCOL)
      R(K,JCOL)=T
237   CONTINUE
238   CONTINUE
239   RETURN
      END
      SUBROUTINE GP3
C     THE SUBROUTINE GP3 CALCULATES THE ABSOLUTE VALUE OF ALL THE TERMS
C     OF THE MODEL      ***************************************************
      COMMON N,M,SIG,M1,KTT,KTO,MR,N1,IT,MR1,V,SUM,NR
      COMMON C(80)
      COMMON X(13),KT(30),W(80),A(80,13),R(80,80),PIVOT(80),AMD(3),E(80)
      COMMON MA
      I2=0
      DO 205 L=1,M1
      I1=I2+1
      I2=I1+KT(L)-1
      DO 205 I=I1,I2
      W(I)=1.
      DO 204 J=1,N
204   W(I)=W(I)*X(J)**A(I,J)
205   W(I)=W(I)*ABS(C(I))
      RETURN
      END
      SUBROUTINE GP22
C     THE SUBROUTINE GP22 IS USED TO [illegible]
C     CONDITIONS FOR THE OBJECTIVE [illegible]
C     FOR THE CONSTRAINTS      [illegible]
      COMMON N,M,SIG,M1,KTT,KTO,[illegible],N1,IT,[illegible],NR
      COMMON C(80)
```

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```
      COMMON X(13),KT(30),W(80),A(80,13),R(80,80),PIVOT(80),AMD(3),E(80)
      COMMON MM
      DO 212 IR=1,N
      I2=0
      DO 212 JR=N1,MR
      L=JR-N1+1
      I1=I2+1
      I2=I1+KT(L)-1
      J=IR
      DO 212 I=I1,I2
212   R(IR,JR)=R(IR,JR)+C(I)/ABS(C(I))*A(I,J)*W(I)
      RETURN
      END
```