THEORETICAL INVESTIGATION

OF

AERODYNAMICS AND DESIGN ANALYSIS OF A HAWT WITH CIRCULAR ARC BLADE SECTION.

BY
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The thesis titled "THEORETICAL INVESTIGATION OF AERODYNAMICS AND DESIGN ANALYSIS OF A HAWT WITH CIRCULAR ARC BLADE SECTION" submitted by Most. Hosney Ara Begum, Roll No. 901420F, Registration No. 88573 has been accepted as satisfactory in partial fulfilment of the degree of Master of Science in Mechanical Engineering on December 23, 1998.

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This is to certify that the work presented in this dissertation is outcome of the investigation carried out by the candidate under the supervision of Dr. Amalesh Chandra Mandal, Professor, Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh.

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ABSTRACT

A theoretical investigation is performed to determine performance characteristics of a Horizontal Axis Wind Turbine (HAWT) with circular arc blade section. The results are compared with those of HAWT having airfoil blade section. It is observed that the results of turbine with circular arc profile are comparable to those with airfoil blade section. With a view to diminishing cost of production and eliminating manufacturing complexity with airfoil blade section, the study is taken into account.

The design of HAWT with circular arc blade profile is also carried out. The usual design uses the minimum value of the ratio of drag co-efficient to lift co-efficient, but in this design maximum value of lift co-efficient (0.9 of maximum lift co-efficient) is considered to obtain optimum values of power coefficient and other parameters.This condition rather improves the power coefficient a little bit.

The maximum value of power coefficient obtained by using circular arc blade profile is 0.38 while using conventional blade profile of NACA or Wortman, this value becomes around 0.40.

The investigation is carried out by using only Momentum Theory and Blade Element Theory. Hence it may not be useful to apply for the multibladed wind turbine.

CONTENTS

LIST OF SYMBOLS

a	axial interference factor
a′	tangential interference factor
A	turbine disc cross-sectional area $=\pi R^2$
A_1	incoming wind cross-sectional area
A_2	wake cross-sectional area
B	number of blades in the rotor
C	chord of the blade
Cc	constant chord
C_D	blade drag coefficient$=\frac{dD}{1/2\,\rho\, C\, W^2\, dr}$
C_L	blade lift coefficient $= \frac{dL}{1/2\,\rho\, C\, W^2\, dr}$
C_{LD}	design lift coefficient
C_{Lmax}	maximum lift coefficient
C_P	power coefficient $= \frac{P}{1/2\,\rho\, A\, V_\infty^3}$
C_Q	torque coefficient $= \frac{Q}{1/2\,\rho\, A\, V_\infty^2\, R}$
C_t	tapered chord
C_T	thrust coefficient $= \frac{T}{1/2\,\rho\, A\, V_\infty^2}$
D	drag force
f	maximum camber of circular arc profile
KE	kinetic energy
L	lift force
$\dot{m}$	mass flow rate of the air
P	turbine power
P_{max}	maximum power
P^+	pressure immediately in front of the rotor
P^-	pressure immediately behind the rotor
P_∞	free stream pressure
Q	torque
r	local blade radius
R	rotor radius
T	thrust force

V wind speed through the turbine
V_2 wind velocity
V_d design wind velocity
V_t wake tangential velocity
V_∞ undisturbed wind velocity
W relative wind velocity

GREEK ALPHABETS

α angle of attack
α_d design angle of attack
β blade twist angle
β_{linear} blade twist angle (linear)
β_{opt} blade twist angle (optimum)
Δ change (of energy)
η efficiency of windmill
η_{max} maximum efficiency of windmill
λ tip speed ratio
λ_d design tip speed ratio
λ_r local tip speed ratio
ρ air density
σ solidity ratio=$BC/2\pi R$
σ_r local solidity ratio=$BC/2\pi r$
ϕ angle of relative wind velocity
ψ ratio of radius=r/R
ω wake rotational velocity
Ω angular velocity of rotor

CHAPTER ONE : INTRODUCTION

1.1. General

With the rise of population and industrial development, the use of energy has been increased by manifold times. Most of the energy is being produced by using conventional energy sources, as a result there has been an alarming situation in concern to environmental pollution all over the country. Now-a-days people are rethinking about the use of conventional energy sources. This has resulted a search for alternative energy sources without environmental pollution.

In view of this point in mind, people are trying to use various renewable energy sources like wind, solar, biomass etc. But still today no useful and economic energy extraction process from wind or solar has not yet been developed. However, researchers have been giving much effort in this field especially during the last two decades. In this respect, Bangladesh should not sit idle. As such, its researchers are also trying to find suitable ways to extract energy either from wind or solar in an effective way.

In the field of wind for exploitation of energy, there have been reasonable developments. Still people are doing research and development in this sector to find efficient and cost effective machinery. Now-a-days both horizontal and vertical wind machines of different kinds have been developed in different part of the World. Our attention has been paid on horizontal axis wind machine only. Horizontal axis wind machine with conventional blade like NACA or Wortman have been practiced so far in many places. But they are not cost effective and economic rather complicated to manufacture.

Due to this problem the simplified type of wind machines has been taken into consideration which can be manufactured locally; still it would give reasonable power at moderate wind speed. As such, a horizontal axis wind machine with circular arc blade section has been chosen for theoretical investigation primarily. This machine can be manufactured with semi-skilled technician and on the contrary cost of production is not high.

1.2. Wind as a source of energy

The conventional energy sources such as fossil fuel and uranium are mostly used now-a-days. But the main problem of using this sources is that they cause severe environmental pollution. As a result both environmentalists and concerned scientists are paying attention for alternate energy sources which do not create health hazard.

On the other hand, with the rising demand of energy and for many other reasons, prices of these fuels are increasing day by day. So people are trying to find the alternate sources of energy to exploit them at the cheapest rate.

As a consequence of the facts as mentioned above, wind energy (and other non-conventional energy sources such as solar energy) are being strongly promoted. It is wellknown that wind energy is a kind of energy source which will never be finished and is available almost all the times round the year in all the places and in large quantities all over the World. Wind Energy is a kind of energy produced by the earth's rotation as well as the irregular solar heating over the earth's surface. It is a clean and renewable source of energy which has been serving mankind for several centuries. For the past few centuries, people are extracting energy from the wind in various ways. One conventional way for converting wind energy to a more useful form is through the use of windmills.

Total reserve of fossil and uranium fuels will be finished one day and are not available in all countries and besides these, people are interested to be self-sufficient ; they do not want to depend on others and spend huge amount of foreign currency for import of fuel.

As a result, in different parts of the world, people are more interested to develop efficient devices to collect energy from the wind, although, power generation cost through wind machine is higher in comparison to that by conventional energy sources. Now-a-days the utilization of wind energy is increasing highly in many developed as well as under developed countries.

1.3. Aim of the thesis

During the past several years researchers have given much effort to find reasonable and reliable analytical prediction methods for horizontal axis wind turbines. Also a lot of extention works are performed in many parts of the world in order to improve the accuracy of the calculated results. From the very morning of the research works, a number of works are being carried out for the development of this kind of turbines. Because of the lack of appropriate and reliable performance prediction method, the researchers are to depend mostly on the experimental results. The researchers are doing both the model and the prototype tests, but the experimental work solely cannot give the proper idea about the good design.

Most of the research work are performed for horizontal axis wind turbine with airfoil blade profile, whose construction are complicated. The prime object of this work is to use circular arc blade profile instead of conventional complex airfoil blade profiles whose construction is easier in comparison to the complex one. In the present work much attention is paid on the performance study of wind turbine with circular arc blade profile.

The analysis is based on the use of the combination of the Momentum theory and the Blade Element theory which is known as the "Strip theory". For multibladed wind turbines which operate at very low tip speed ratios, it is found that the strip theory approach is suitable for the performance analysis of a horizontal axis wind turbine.

The study will be confined to the theoretical investigation only. This investigation will incorporate to determine performance characteristics of a horizontal axis wind turbine with circular arc blade profile including a number of variables. The study will also include the design of a wind machine to extract small amount of power only which may be suitable to produce power for generation of electricity at the remote areas applicable at a low wind velocity that prevails in some areas of Bangladesh.

1.4. Scope of the thesis

Chapter 1 presents the general introduction in regard to the works concerning performance prediction and design of a horizontal axis wind machine. Some general idea with respect to the present study is also mentioned in this Chapter.

Chapter 2 describes the literature survey containing historical background and review of the existing literature. In historical background, it gives a short description of the history of using wind power from the ancient period upto modern age by mankind and related papers which have been published so far by the different authors in the different places recently.

Chapter 3 describes the existing theories: the momentum theory and the blade element theory. Only the theories which are related to the present work have been described neatly and in a simple manner.

The design procedure of a horizontal axis wind turbine using the circular arc blade section has been described in the Chapter 4. The discussion in regard to the blade characteristics have also been incorporated in this Chapter.

Chapter 5 gives the elaborate discussion concerning the performance prediction of the horizontal axis wind turbine using circular arc blade section. Various related and important variables have been considered for discussion. Every related matters have been discussed in a simple manner.

Finally, in the Chapter 6, the conclusions with regard to the findings of the theoretical investigation of the horizontal axis wind turbine using circular arc blade section are drawn. Some recommendations for future work have also been incorporated into this Chapter.

CHAPTER TWO : LITERATURE SURVEY

2.1. General

Wind has been used to propel ships for a long time. In the past, many countries supported their prosperity on their ability to navigate. America being discovered by such ships. Roughly speaking, wind was the only power source to move ships until Watt developed the steam engine in the 19th century.

Windmills and windpumps have been used by mankind for many centuries. In deed before the coming of the steam engine, they were the main sources of motive power (together with watermills, animal and human power). Most of these windmills have been replaced by modern engines such as steam or electric motors over the last two centuries.

Over the last decade new sophisticated and reliable wind turbines have been developed internationally to compete with conventional power plants. These wind turbines are commercially available at a competitive price compared to other electricity production units. Of course, the price of electricity production by these turbines depends on the local wind conditions.

Researchers in different parts of the world have contributed greatly to the knowledge of analytical prediction methods of wind turbines, which are discussed in a nutshell in the following sections.

2.2. Historical background

The development of wind power has been characterized greatly by periods of progress and longer periods of neglect, but nevertheless the employment of wind power has never been absent from the time of its inception. From the ancient period people have been working mainly on various classes of wind machinery to extract energy from the wind. It is difficult to assess the time from which this kind of machine was first used by man. However, it could be said that the origin of wind turbines was located in the meridional part of the Mediterranean Sea as well as in China where only the vertical axis machines were known. It is assumed that works on windmills have probably been started from 2000 B.C. In this regard, the period from the ancient time upto the end of the 19th century may be categorized as the "Ancient Development Period" while that from the end of 19th century upto date may be termed as the "Modern Development Period".

A number of windmills have been designed and constructed to extract power from the wind. Hero from Alexandria, who lived during the 3rd century B.C., described a simple turbine with a horizontal axis and four sails, which was used to produce the power for an organ working with compressed air.

The Persian people utilized wind turbines extensively by the middle of the 7th century B.C. The machine was mounted on a horizontal axis and used axially disposed sails described by Golding (1929). This type of early windmills which was used to grind grain, consisted of several sails.

After this, horizontal axis windmills were constructed of upto ten wooden booms, rigged with jib sails, were developed. This type of windmills was at field upto about 12th century, when at a time in France and England Dutch type of windmills were made whose purpose were to grind grain and pump water. These were horizontal axis types windmills. Upto the end of 19th century people had been working with these types of windmills only. Such primitive types of windmills are still found in use today in many Mediterranean regions (Figure 2.1).

In the 11th century, windmills were frequently used in the Middle East and were introduced to Europe in the 13th century by returning crusaders. At that time in Europe, many horizontal axis windmills had been fabricated for grinding grains and moving water.

In the 14th century, the Dutches posses the lead in improving the design of windmills and used extensively for draining the marshes and lakes (Figure 2.2).

In 1582, the first oil mill was constructed in Holland and in 1586 the first paper mill was fabricated to apply it to serve the purpose of the huge demand for paper that caused from the invention of the printing press. In the last portion of the 16th century, sawmills were invented for processing timber. In this century , the primitive jib sails on wooden booms were converted to sails, which were typically made of sailcloth. The sails were flat planes and inclined at a constant angle to the direction of rotation. Modern designs replaced the cloth sails for sheet metal and introduced different types of the rotor to face heavy weather. In Holland the use of wind power was decreased during industrial revolution after the invention of steam engine. In the Netherlands about 2500 windmills were still in operation in the end of 20th century and in 1960, fewer than 1000 were still in working condition.

The Dutch introduced many improvements in the design of windmills and, in particular, the rotors. By the 16th century, the primitive jib sails on wooden booms had given way to sails supported by wooden bars on both sides of the stock. Later, the bars were moved to the trailing edge of the rotor to improve the aerodynamic design (Figure 2.3). More modern designs substituted sheet metal for the cloth sails, used steel stocks, and introduced various types of shutters and flaps to control the speed of the rotor in heavy weather.

In the 17th century, Hammurabi, emperor of the Babylonic Empire, used wind turbines for irrigation was described by Golding (1976).

In the mid of 19th century, about six million multibladed windmills have been constructed and used in the United States for pumping water, electricity generation and other purposes. Wind mills are used for pumping water in different parts of the United States for farms, rural households, watering livestock on ranges in remote areas. These types of machines have consisted of metal fan-blades, 12 to 16 feet in diameter, mounted on a horizontal shaft, with a tail-vane to keep the rotor facing into the wind (Figure 2.4). In that type of machine the shaft is usually connected to a set of gears and a cam that move a connecting rod and this rod operates a pump at the bottom of the tower. A 3.7 m (12 feet) diameter rotor of this type develops about 1.5 to 2.2 kW (2 to 3 hp) in a 6.6 m/sec (15 mph) wind and can pump about 10 gallons of water per minute to a height of about 30.5 m (100 feet).

In the mid of 19th century they also used windmills for producing electricity and these windmills have two or three propeller-type blades which are incorporated by a shaft and gear to a d.c. generator (Figure 2.5). They usually incorporate some type of energy storage system, often consisting of a bank of batteries.

In the early part of 20th century another interesting windmill was built named "Smith - Putnam". Palmer Putnam had concluded that a large machine was required to minimize the cost of electricity, generated by the wind. Smith-Putnam wind turbine is shown in Figure 2.6.

At the end of 19th century, in Denmark there were about 2500 industrial windmills in operation, supplying about 25% of the total power required at that time which was about 30 MW described by Eldridge (1980). Besides this, about 4600 windmills were used at about 2% of the Danish farms for various applications, such as thrashing, milling of grains and water pumping. In 1930 the number of industrial windmills was decreased to about 1000, but the number of farm units was increased to about 16000. During World war II, they developed and operated a number of new types of large windmills for producing electricity. The number of this machines increased from 16 to 88 from the summer of 1940 to the beginning of 1944. The number of that type of machines started to decrease after World war II and dropped to 57 at the end of 1947. This dropping continued up to 1950; by the end of this year the production of electricity using windmills was being conducted only on an experimental basis. The 200 kW Gedser mill (Figure 2.7), which was the latest in this series, was operated until 1968, when it was shut down because it was found that by that time the cost of electricity supplied by this wind-powered unit was about twice the equivalent fuel cost of the steam-powered electric utility plants which were being operated in Denmark.

The Russians in 1931, built an advanced wind turbine near Yalta on the Black Sea of 100 kW capacity (Figure 2.8). About 280,000 kWh per year was the annual output of this machine. At that time the Russians worked extensively for the development of windmills and in 1930 they considered building of a large system of 5 MW rated capacity as mentioned in Eldridge (1980).

Remarkable work was done on wind powered electrical generation plants from the late 1940 to 1950 in England. The North Scotland Hydroelectric Board commissioned the John Brown Company to fabricate an experimental wind turbine in 1950. The design capacity of this unit was 100 kW in 15.8 m/sec (35 mph) wind speed. In 1955 it was operated coupled to a diesel-powered electric utility network, but was shutdown for mechanical causes. A Frenchman named Andreau, designed a wind powered generator in 1950, shown in the Figure 2.9. This machine was first operated at St. Albans, England and then at Algeria. The design capacity of this machine was 100 kW of a.c. power in 13.2 m/sec (30 mph) wind speed. In this machine the propeller blades were hollow and as they rotated, acted as centrifugal pumps. Although the efficiency of this unit was low compared to conventional horizontal axis wind powered rotors; the main advantage of this machine is that power generating equipment was not supported at high altitude.

In 1958 to 1966, the French constructed and operated many large wind powered electric generators, these included three horizontal axis units, each of which with three propeller type blades. From 1958 to 1963, this kind of unit was operated intermittently near Paris, a unit of this kind is shown in Figure 2.10. The design capacity of this unit was 800 kW in 16.7 m/sec (37 mph) wind speed. In south of France, two other units were constructed among which the smaller one had 21.4 m (70 feet) diameter rotor and the larger one was designed for 1000 kW in 16.7 m/sec (37 mph) wind speed, weighed 96 tons except tower. During this period, the French also built and operated many experimental vertical axis windmills.

It has been estimated, the number of windmills in use in Germany was about 18000 in 1895, 17000 in 1907, 11400 in 1914 and between 4000 to 5000 in 1933. They improved a number of parameters including light-weight constant speed rotors which were controlled by variable pitch propeller blades. They also took the credit of using composite material for these machines. The capacity of largest unit was 100 kW in 8.1 m/sec (18 mph) wind speed, designed by the German.

Figure 2.11(a) shows a three-bladed rotor and Figure 2.11(b) shows a single circular arc blade with cross section which are considered in this study.

Now a days, a number of countries around the world, are interested about windmills and as a result large wind energy conversion systems (WECS) have been built and tested. But only one problem lies herewith, which is the installation cost per kilowatt. It has been too high in comparison to the cost of other conventional electric power. Besides this, for wind variability some other form of energy storage must be considered. In the present days, high cost of fuel coupled with fuel scarcities together with the environmental pollution has make the human being to think wind energy as a prime source of power in future.

2.3. Review of existing prediction methods

In case of a propeller, it adds energy into the air from another source of energy, where as the wind turbine extracts energy by driving air and converts it into a mechanical power. Considering the similarity of the wind turbine and the propeller, the same theoretical development can be used for the performance analysis of wind machine. The concerned theory was based on two different independent approaches; one is the Momentum Theory and the another is the Blade Element Theory.

The starting point in wind turbine analysis lies in the Control Volume approach contained in the Actuator Disk concept originated by Rankine (1865) and improved later for marine propellers. It was modified for wind turbines and has received further treatment in recent years from others. The main theme of the theory is the determination of the forces acting on the rotor which produce the motion of the fluid. In the actuator disk theory the induced velocity, i.e. the rotor axial flow velocity is assumed to be constant through the "disk" and is obtained by equating the streamwise drag with the change in axial momentum. Also the actuator disc is considered as the surface of the imaginary body of revolution. This theory predicts the ideal efficiency of the rotor.

The most frequently used theory for performance analysis of a horizontal axis wind turbine is the modified blade element theory and the strip theory. The assumption in this theory is local two dimensional flow at each radial rotor station to determine forces and the torque with the help of known airfoil sectional aerodynamics, chord and pitch angles.

The application of the momentum theory is limited where reverse flow begins to occur at the downstream of the rotor but in reality no such phenomenon happens. When a wind turbine is in operation, the state of operation can be categorized theoretically into four categories, which occur simultaneously at different positions of a blade as shown in the Figure 2.3.1 as described by Wilson (1974) -

- the propeller state
- the windmill state
- the turbulent wake state and
- the vortex ring state.

In the analysis of helicopter, many experimental studies were conducted on the vortex ring state and on the turbulent wake state. Lock (1923) used two model rotors for wind tunnel tests and Glauert (1926) used the data of Lock to define a characteristic curve. On the other hand Gessow (1952) obtained a similar characteristic curve by a flight test of a helicopter and Johnson (1980) give its approximate formula.

To calculate the aerodynamic performance of a horizontal axis wind turbine at tip speed ratios it is observed that aerodynamic stall can occur as the blade experiences high angles of attack.

A method has been developed to determine the blade shapes for maximum power by Walker (1976). Here the blade chord and twist are continuously varied at each radial station until the elemental power coefficient has been maximized. This is obtained when every radial element of the blade is operating at the airfoils maximum lift to drag ratio.

A comparison between near-optimum and optimum blade shapes has been done by Anderson (1980) for turbines which are operating at both constant tip speed ratio and constant rotational speed. A simple method for design and performance analysis of a horizontal axis wind turbine has been suggested by Shepherd (1984) which eliminate iteration process and is based on the use of ideal and optimized analysis to determine the blade geometry. It uses fixed values of the axial induction factors and corresponding optimized rotational induction factors.

Work of Jansen (1976) on the theories make the basis for calculation of the design parameters and the behavior of a windmill and he took a conclusion that using simple materials high power coefficient can be achieved.

For design of the large horizontal axis wind turbines, it must be kept in mind that structural reliability and efficiency should be high and maintenance and weight should be minimum in order to produce energy at a competitive cost. The major parts which contribute for structural weight are the vibratory loads act on the rotor and the tower. These loads are unsteady and may be aerodynamic, gravitational or inertial in origin. Vibratory stresses caused by dynamic loads will be the main design consideration for large wind turbines. An analysis made by Sepra (1975) on the vibratory loads and stresses in the hingeless and teetered rotor for the NSF - NASA MOD-0 wind turbine and took conclusion that the teetered rotor is advantageous then hingeless one with respect to shank, stresses, fatigue life and tower loading. The hingeless rotor is not structurally stable when overloaded and on the other hand a teetering rotor will achieve a long service life in a wind turbine which need minimum attention.

A method has been given to determine the overall design, performance and structural analysis of a horizontal axis wind turbine by Islam (1986).

Rotor performance was analysed regarding blade loadings, stresses and size limits of horizontal axis wind turbines. Simple method has been developed and used to illustrate the process of rotor design in the view of identifying the variables which influence performance and blade loadings. The tip speed ratio and the number of blades are shown to have a great influence on the stress levels and the analysis predicts why a performance for two blades has emerged for large machines.

CHAPTER THREE : EXISTING THEORIES

3.1. General

Probably the most simple type of wind machine is the device that moves in a straight line under the action of the wind. The power available in the wind is taken as the flux of kinetic energy through the active cross-sectional area intercepting the apparatus which is utilizing this energy for mechanical or electrical output as the purpose required. For calculation of different parameters of the wind turbines, it is assumed that the flow is steady and there is no turbulence.

The simple linear momentum theory together with the blade element theory, is known as the "Modified blade element theory" or the "Strip theory" and this theory is adequate for wind machines performance analysis. The strip theory is presented in this chapter with the effects of wake rotation.

3.2. Axial momentum theory

For a conventional wind turbine operating under steady state condition in an initially free flowing windstream of constant initial velocity, the velocity of a unit volume of air will decrease monotonically as it approaches the turbine, passes through it and starts to recede from it. As the air recedes farther from the turbine, it will receive kinetic energy from the surrounding winds and its velocity will increase until it again reaches its initial velocity. The density cannot be controlled except within the limits of altitude string, and usually other considerations prevail. Far from the turbine , the air is developed and used to illustrate the process of rotor design in the view of identifying the variables which influence performance and blade loadings.

By the use of axial momentum theory alone it is possible to determine the forces acting on a rotor which is responsible for the motion of the fluid and thereby the power output from the windmill.

3.2.1. Non-rotating wake :

The starting point in wind turbine analysis lies in the "Control volume" approach contained in the "Actuator disc" concept originated by Rankine (1965) and extended it for marine propellers. This theory has been useful in the derivation of ideal efficiency of a rotor and also applied for wind turbines.

The assumptions required for the application of the axial momentum theory for the maximum possible output of a wind turbine are :

a. The fluid is incompressible and inviscid
b. The blades number is infinite
c. The thrust loading is uniform
d. The static pressure far ahead and far behind the rotor are equal to the undisturbed ambient static pressure
f. The flow is homogeneous
g. No frictional drag

Let us assume the control volume as shown in the Figure 3.2.1.

The notations used are :
V_∞ = undisturbed wind velocity far from the upstream, m/s
V = wind velocity when it flows through the rotor, m/s
V_2 = wind velocity far behind the rotor, m/s
A = turbine disc area, m^2
A_1 = cross sectional area of incoming wind, m^2
A_2 = wake cross sectional area, m^2
and
ρ = air density, kg/m^3
The conservation of mass gives,

$$\rho A_1 V_\infty = \rho A V = \rho A_2 V_2 \qquad (3.2.1)$$

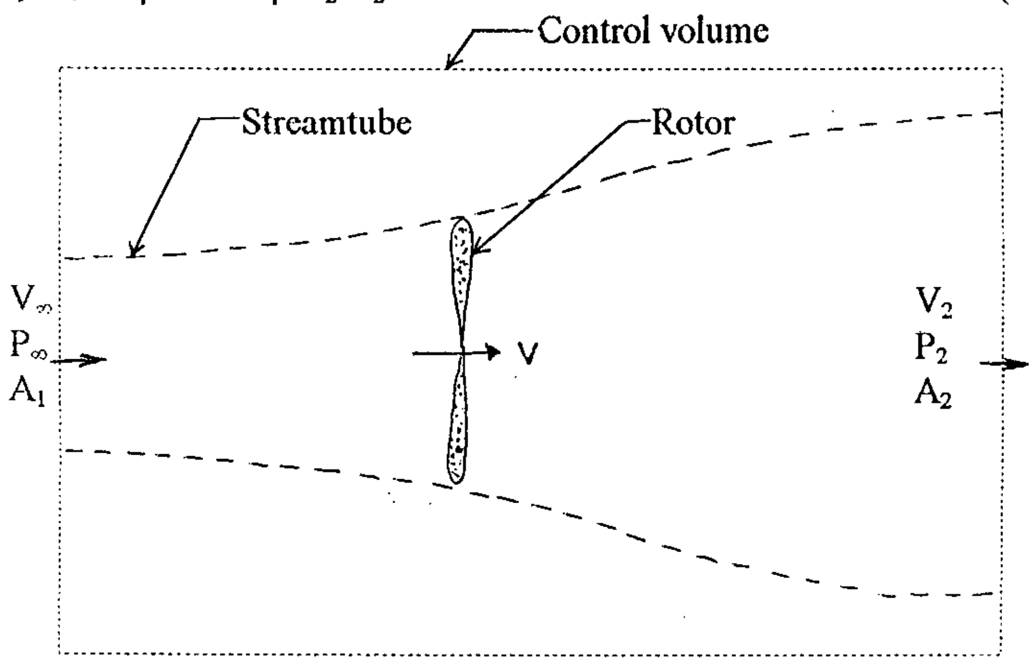

Figure 3.2.1 : Control Volume of a Wind Turbine

The change of momentum of the flow gives the thrust force T and is expressed as,

$$T = \dot{m}\,(V_\infty - V_2) = \rho\, A_1\, V_\infty^2 - \rho\, A_2\, V_2^2 \qquad (3.2.2)$$

Inserting equation (3.2.1) into equation (3.2.2) one obtains,

$$T = \rho\, A\, V\, (V_\infty - V_2) \qquad (3.2.3)$$

On the other hand, the thrust on the rotor can also be expressed from the pressure difference over the rotor area as,

$$T = A\,(P^+ - P^-) \qquad (3.2.4)$$

Where
P^+ = Pressure immediately in front of the rotor
P^- = Pressure immediately behind the rotor

Introducing Bernoulli's equation,

For upstream of the rotor, $P_\infty + \frac{1}{2}\rho\, V_\infty^2 = P^+ + \frac{1}{2}\rho\, V^2$ .... (3.2.5)

For downstream of the rotor, $P_\infty + \frac{1}{2}\rho\, V_2^2 = P^- + \frac{1}{2}\rho\, V^2$ .... (3.2.6)

One finds by substracting equation (3.2.6) from equation (3.2.5),

$$P^+ - P^- = \frac{1}{2}\rho\,(V_\infty^2 - V_2^2) \qquad (3.2.7)$$

From equations (3.2.4) and (3.2.7) the expression for the thrust can be written as,

$$T = \frac{1}{2}\rho\, A\,(V_\infty^2 - V_2^2) \qquad (3.2.8)$$

Equating the expressions of thrust from equation (3.2.8) and (3.2.3),

$$\frac{1}{2} A\, \rho\,(V_\infty^2 - V_2^2) = \rho\, A\, V\,(V_\infty - V_2)$$

Simplifying one obtains, $V = \frac{V\infty + V_2}{2}$ .... (3.2.9)

The rotor velocity V is often defined in terms of an axial interference, induction, or perturbation factor "a" as,

$V = V_\infty (1 - a)$ .... (3.2.10)

Now from equations (3.2.9) and (3.2.10), the wake velocity V_2 can be expressed as ,

$V_2 = V_\infty (1 - 2a)$ .... (3.2.11)

From the momentum theory, the change in kinetic energy of the mass flowing through the rotor area is the power absorbed by the rotor and is given by,

$P = \dot{m}\, \Delta KE = ½ \rho\, A V (V_\infty^2 - V_2^2)$ .... (3.2.12)

Introducing equations (3.2.10) and (3.2.11) in equation (3.2.12), the expression for power becomes,

$P = 2\rho A V_\infty^3 a(1 - a)$ .... (3.2.13)

This is a cubical equation of velocity and from the mathematical point of view, the maximum power occurs when first derivative of P with respect to a becomes zero,

i.e. $\frac{dp}{da} = 0$

From equation (3.2.13), one obtains the derivative,

$\frac{dp}{da} = 2\rho\, A V_\infty^2 (1 - 4a + 3a^2) = 0$

or, $a = \frac{1}{3}$

This is the value of optimum interference factor and substituting this value in equation (3.2.13), the expression for maximum power becomes,

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho A V_\infty^3\right) \qquad (3.2.14)$$

Here the factor 16/27 is called the "Betz-coefficient", Golding E.W.(1976) and it represents the maximum fraction of power which an ideal rotor can extract from the flow. This fraction 16/27 is related to the power of an undisturbed flow which is arriving at an area A, whereas in reality the mass flow rate through A is not ρAV_∞ but is equal to ρAV. So the maximum efficiency η_{max} for the maximum power output can be expressed as,

$$\eta_{max} = \frac{P_{max}}{1/2\,\rho\, A\, V\, V_\infty^2}$$

$$= \frac{16}{27} \cdot \frac{3}{2}$$

$$= \frac{8}{9} \qquad (3.2.15)$$

The theory described above is called the axial momentum theory for non - rotating wake as it does not take into account of the additional effects of wake rotation. The incoming stream is not rotational and interaction of the stream with the rotating windmill cause the wake to rotate in opposite direction. If the wake contained kinetic energy together with the translational kinetic energy, then from the thermodynamic point of view, it is expected to have lower power extraction than in the case of the wake having only translational kinetic energy.

3.2.2. Effect of wake rotation

In this section the wake rotation is taken into account in the axial momentum theory. Considering the effect of wake rotation, the following assumptions are made,

(a) At the upstream of the rotor the flow is entirely axial

(b) At the down stream of the flow it rotates with an angular velocity ω but remains irrotational.

(c) The angular velocity of the downstream flow ω is assumed to be small in comparison to the angular velocity Ω of the wind turbine itself.

These assumptions mentioned above maintains the approximation of axial momentum theory that the pressure in the wake region is equal to that of the free stream .

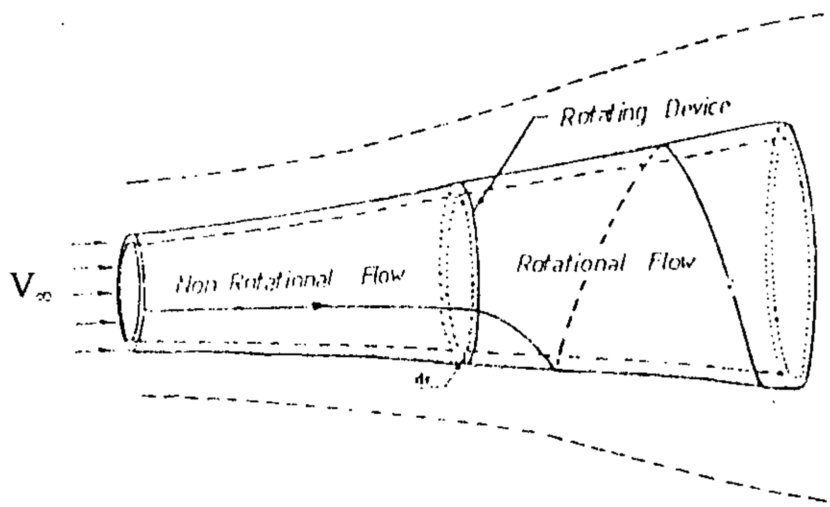

Figure 3.2.2. : Streamtube Model Showing the Rotation of the Wake

The wake rotates in opposite direction of the rotor rotation and causes an additional loss of kinetic energy for the rotor. The product of the torque Q acting on the rotor and the angular velocity Ω of the rotor gives the power. To get maximum power it is essential to have a high angular velocity and low torque because high torque will causes in large wake rotational energy. The angular interference factor "a′" relates the angular velocity ω of the wake and the angular velocity Ω of the rotor in the following way,

$$a' = \frac{\text{Angular velocity of the wake}}{\text{Twice the angular velocity of the rotor}}$$

$$= \frac{\omega}{2\Omega} \qquad \text{(3.2.16)}$$

The annular ring through which a blade element will pass is described in the following Figure 3.2.3,

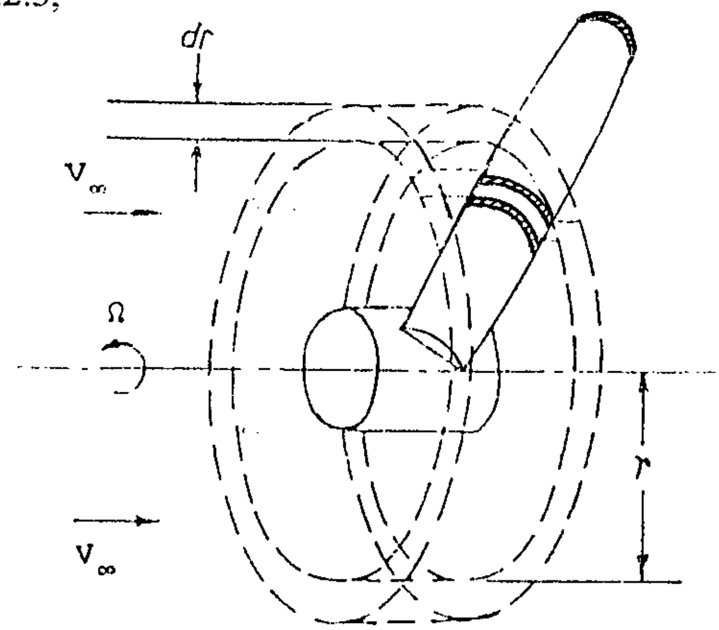

Figure 3.2.3 : Blade Element Annular Ring.

The axial thrust force dT can be expressed as the product of elemental mass flow rate dm and the change of velocity in the following way,

$$dT = d\dot{m}\,(V_\infty - V_2)$$

$$= \rho\, dA\, V\,(V_\infty - V_2) \qquad \text{(3.2.17)}$$

Substituting the values of V and V_2 from equations(3.2.10) and (3.2.11) into equation (3.2.17) and expressing the area of the annular ring dA as,

$$dA = 2\pi\, r\, dr \qquad \text{(3.2.18)}$$

The thrust can be written as,

$$dT = \rho\, 2\pi\, r\, V_\infty\,(1 - a)\,[\,V_\infty - V_\infty(1 - 2a)]\, dr$$

$$= 4\,\pi\, r\, \rho\, V_\infty^2\, a\,(1 - a)\, dr \qquad \text{(3.2.19)}$$

On the other hand, by the use of Bernoulli's equation, the thrust force can also be calculated from the pressure difference over the blades. Bernoulli's equation yields, as the relative angular velocity changes from Ω to $(\Omega + \omega)$ and the axial components of the velocity remain as before,

$$P^+ - P^- = \frac{1}{2}\rho\ (\Omega + \omega)^2\ r^2 - \frac{1}{2}\rho\,\Omega^2\ r^2$$

$$= \rho\,(\Omega + \frac{1}{2}\omega)\,\omega\, r^2$$

The thrust on the annular element is given by the product of pressure difference and the elemental area,

$$dT = (P^+ - P^-)\, dA$$

$$= \rho\,(\Omega + \frac{1}{2}\omega)\,\omega\, r^2\ 2\,\pi\, r\, dr$$

Replacing ω by $2\,\Omega\, a'$ as obtained from the equation (3.2.16),

$$dT = 4\, a'\,(1 - a')\,\frac{1}{2}\rho\ \Omega^2\ r^2\ 2\ \pi\ r\,dr \quad \text{.......................................} \quad (3.2.20)$$

Equating right hand sides of equations (3.2.19) and (3.2.20), one finds,

$$\frac{a\,(1 - a)}{a'\,(1 + a')} = \frac{\Omega^2\, r^2}{V_\infty^{\ 2}} = \lambda_T^{\ 2} \quad \text{..} \quad (3.2.21)$$

Where, "λ_T" is the local tip speed ratio and is expressed as,

$$\lambda_T = \frac{\Omega r}{V_\infty} \quad \text{..} \quad (3.2.22)$$

Let us now derive an expression for the torque Q acting on the rotor for which the change in angular momentum flux dQ through the annular ring is considered. The elemental torque is given by,

$$dQ = d\dot{m}\ V_t\ r$$

$$= \rho\,\omega\, r\, V\, r\, dA$$

Where V_t is the wake tangential velocity.

The expression for the torque acting on the annular ring is obtained from the equations (3.2.10), (3.2.16) and (3.2.18) as,

$$dQ = 4\,\pi\, r^3\ \rho\, V_\infty\, (1 - a)\ a'\, \Omega\, dr \qquad (3.2.23)$$

The power generated through the annular ring is equal to

$$dP = \Omega\, dQ$$

So the total power can be given by the integration of dP over the radius 0 to R and the expression becomes,

$$P = \int_0^R \Omega\, dQ \qquad (3.2.24)$$

The tip speed ratio is defined as,

$$\lambda = \frac{R\,\Omega}{V_\infty} \qquad (3.2.25)$$

Substituting the value of dQ from equation (3.2.23) into equation (3.2.24), the expression for total power becomes,

$$P = \int_0^R 4\,\pi\, r^3\ \rho\, V_\infty\, (1 - a)\, a'\, \Omega^2\, dr$$

$$= \frac{1}{2}\rho\, A\, V_\infty^3\, \frac{8}{\lambda^2} \int_0^\lambda a'\,(1 - a)\, \lambda_r^3\ d\lambda_r \qquad (3.2.26)$$

In the above expression, "A" is the turbine swept area which is given by,

$$A = \pi\, R^2$$

Power coefficient is defined as,

$$C_p = \frac{P}{\frac{1}{2}\rho\, A\, V_\infty^3}$$

Inserting the value of power P from equation (3.2.26), the expression of power coefficient can be written as,

$$C_p = \frac{8}{\lambda^2} \int_0^\lambda a'\,(1 - a)\, \lambda_r^3\, d\lambda_r \qquad (3.2.27)$$

From equation (3.2.21) the tangential interference factor a′ in terms of axial interference factor a becomes,

$$a' = -\frac{1}{2} + \frac{1}{2}\sqrt{\{1 + \frac{4}{\lambda_T^2} a(1-a)\}} \quad \text{............} \quad (3.2.28)$$

Inserting value of a′ from equation (3.2.28) into equation (3.2.27) and taking derivative of Cp equal to zero for maximum power coefficient, the expression of λ_T in terms of a becomes,

$$\lambda_T = \frac{(1-a)(4a-1)^2}{1-3a} \quad \text{............} \quad (3.2.29)$$

From equations (3.2.21) and (3.2.29), the relation between a and a′ can be obtained as,

$$a' = \frac{1-3a}{4a-1} \quad \text{............} \quad (3.2.30)$$

3.3. Blade element theory

The forces acting on a differential element of the blade can be calculated with the help of "Blade element theory". In this theory, the performance of the entire rotor is determined by integrating the concern characteristic over the length of the blade.

The blade element theory is based on the following assumptions -

(a) Along each blade there is no interference between the adjacent blade elements.

(b) The forces acting on the blade element are determined from only the lift and drag characteristics of the sectional profile of the element.

(c) The pressure in the far wake is equal to that of the free stream.

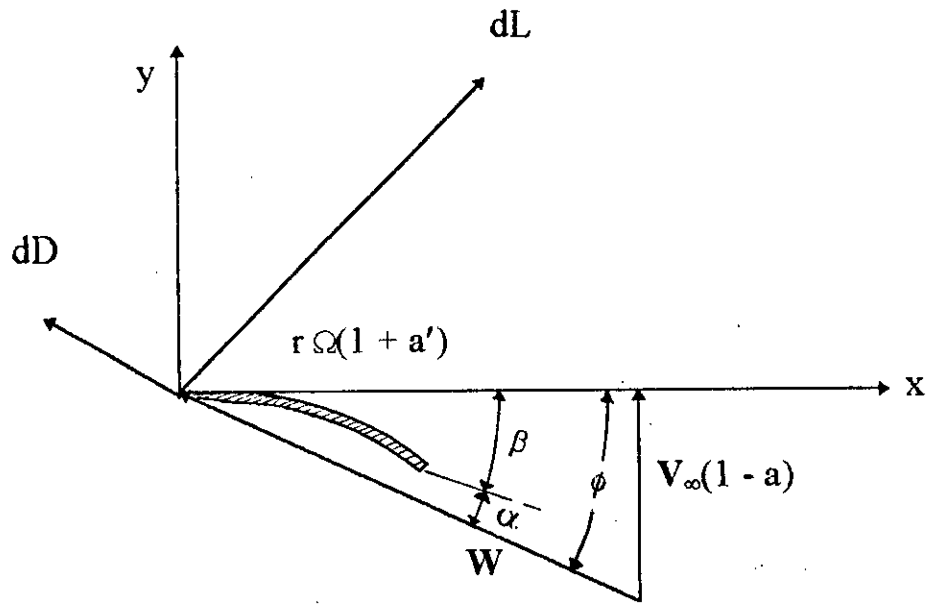

Figure 3.3.1. : Velocity Diagram of a Blade Element.

The aerodynamic force components acting on the blade element are shown in the above Figure 3.3.1 where x - y coordinate system is used. Among the forces the lift force component dL acts perpendicular to the resulting velocity vector **W** and the drag force component dD acts in the same direction of the resulting velocity vector. The sectional lift and drag forces may be defined as,

$$dL = C_L \frac{1}{2} \rho W^2 C\, dr \quad \text{.......... (3.3.1)}$$

$$dD = C_D \frac{1}{2} \rho W^2 C\, dr \quad \text{.......... (3.3.2)}$$

From the geometry of the figure the thrust and torque experienced by the blade element are given by the following expressions,

$$dT = dL \cos\phi + dD \sin\phi \quad \text{.......... (3.3.3)}$$

$$dQ = (dL \sin\phi - dD \cos\phi)\, r \quad \text{.......... (3.3.4)}$$

Assuming that "B" is the number of blades of the rotor, the expressions for the thrust and torque can be written as,

$$dT = B\, C \frac{1}{2} \rho W^2 (C_L \cos\phi + C_D \sin\phi)\, dr$$

$$= B\, C \frac{1}{2} \rho W^2 C_L \cos\phi \left(1 + \frac{C_D}{C_L} \tan\phi\right) dr \quad \text{.......... (3.3.5)}$$

and

$$dQ = B\, C \frac{1}{2} \rho W^2 (C_L \sin\phi - C_D \cos\phi)\, r\, dr$$

$$= B\, C \frac{1}{2} \rho W^2 C_L \sin\phi \left(1 - \frac{C_D}{C_L} \frac{1}{\tan\phi}\right) r\, dr \quad \text{.......... (3.3.6)}$$

From Figure 3.3.1., the relative velocity can be expressesd as,

$$W = \frac{(1 - a) V_\infty}{\sin\phi}$$

$$= \frac{(1 + a') \Omega r}{\cos\phi} \quad \text{.......... (3.3.7)}$$

From the geometry of the Figure 3.3.1, the following trigonometric relations can be obtained,

$$\tan\phi = \frac{(1 - a) V_\infty}{(1 + a') \Omega r}$$

$$= \frac{1-a}{1+a'}\frac{1}{\lambda_r} \quad \text{......} \quad (3.3.8)$$

Also,

$$\beta = \phi - \alpha \quad \text{......} \quad (3.3.9)$$

On the other hand, the local solidity ratio σ_r can be expressed as,

$$\sigma_r = \frac{B\,C}{2\pi r} \quad \text{......} \quad (3.2.10)$$

Finally, the equations of blade element theory from equations (3.3.5) and (3.3.6) in terms of local solidity ratio σ_r become,

$$dT = (1-a)^2 \frac{\sigma_r C_L \cos\phi}{\sin^2\phi}\left(1 + \frac{C_D}{C_L}\tan\phi\right) \frac{1}{2}\rho V_\infty^2\, 2\pi r\, dr \quad \text{......} \quad (3.3.11)$$

$$dQ = (1+a')^2 \frac{\sigma_r C_L \sin\phi}{\cos^2\phi}\left(1 - \frac{C_D}{C_L}\frac{1}{\tan\phi}\right) \frac{1}{2}\rho\, \Omega^2 r^3\, 2\pi r\, dr \text{......} \quad (3.3.12)$$

3.4. Strip theory

The performance of a wind turbine can be determined from the axial momentum theory and blade element theory by developing a couple of relationships. In order to do this, the thrust obtained from the momentum theory, equation (3.2.19) is equated to the thrust obtained from the blade element theory for an annular element at radius r equation (3.3.11),

$$dT_{\text{momentum}} = dT_{\text{blade element}}$$

$$\text{or,} \quad \frac{a}{1-a} = \frac{\sigma_r C_L \cos\phi}{4\sin^2\phi}\left(1 + \frac{C_D}{C_L}\tan\phi\right) \qquad (3.4.1)$$

This is an important relation which relates axial interference factor “a” with the local solidity ratio σ_r together with the lift coefficient C_L and the drag coefficient C_D.

On the other hand, equating the expression of angular momentum obtained from the momentum theory, the equation (3.2.23) with the expression of the same obtained from the blade element theory, the equation (3.3.12) and substituting equation (3.3.8), one finds

$$dQ_{\text{momentum}} = dQ_{\text{blade element}}$$

$$\text{or,} \quad \frac{a'}{1+a'} = \frac{\sigma_r C_L}{4\cos\phi}\left(1 - \frac{C_D}{C_L}\frac{1}{\tan\phi}\right) \qquad (3.4.2)$$

The equation (3.4.2) determines the angular interference factor which contains the lift coefficient and the local solidity ratio.

The drag coefficient C_D i.e. the drag term should be omitted in calculations of axial interference factor “a” and the angular interference factor “a′ “ because the retarded air due to drag is confined to thin helical sheets in the wake and have a little effects on the induced flow as described by Wilson R.E. (1974). Thus omitting the drag coefficient C_D the expressions for the induction factors a and a′ can be obtained as,

$$\frac{a}{1-a} = \frac{\sigma_r C_L \cos\phi}{4\sin^2\phi} \qquad (3.4.3)$$

$$\frac{a'}{1+a'} = \frac{\sigma_r C_L}{4\cos\phi} \quad \text{.......} \quad (3.4.4)$$

Inserting equation (3.4.3) into equation (3.3.11), the expression for elemental thrust can be determined as,

$$dT = 4\,a\,(1-a)\,(1+\frac{C_D}{C_L}\tan\phi)\,\frac{1}{2}\,\rho\,V_\infty^2\,2\,\pi\,r\,dr \quad \text{.......} \quad (3.4.5)$$

Also inserting equations (3.4.4) and (3.3.8) into equation (3.3.12), the expression for elemental torque can be obtained as,

$$dQ = 4\,a'\,(1-a)\,(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\,\frac{1}{2}\,\rho\,V_\infty\,\Omega\,2\,\pi\,r^3\,dr \quad \text{.......} \quad (3.4.6)$$

The expression for elemental power is given by,

$$dP = dQ\,\Omega$$

Now substituting the expression of dQ from the equation (3.4.6) into the above equation, we find,

$$dP = 4\,a'\,(1-a)\,(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\,\frac{1}{2}\,\rho\,V_\infty\,\Omega^2\,2\,\pi\,r^3\,dr \quad \text{.......} \quad (3.4.7)$$

Introducing the local tip speed ratio $\lambda_r = \Omega r / V_\infty$, from the equation (3.2.22), the expressions of total thrust, total torque and total power can be obtained by integrating equations (3.4.5), (3.4.6) and (3.4.7) respectively over the total tip speed ratio ranging from 0 to λ and the expressions are found as,

$$T = \frac{1}{2}\,\rho\,A\,V_\infty^2\,\frac{8}{\lambda^2}\int_0^\lambda a\,(1-a)\,(1+\frac{C_D}{C_L}\tan\phi)\,\lambda_r\,d\lambda_r \quad \text{.......} \quad (3.4.8)$$

$$Q = \frac{1}{2}\,\rho\,A\,V_\infty^2\,R\,\frac{8}{\lambda^3}\int_0^\lambda a'\,(1-a)\,(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\,\lambda_r^3\,d\lambda_r \quad \text{.......} \quad (3.4.9)$$

and

$$P = \frac{1}{2}\,\rho\,A\,V_\infty^3\,\frac{8}{\lambda^2}\int_0^\lambda a'\,(1-a)\,(1-\frac{C_D}{C_L}\frac{1}{\tan\phi})\,\lambda_r^3\,d\lambda_r \quad \text{.......} \quad (3.4.10)$$

From definition the coefficient of thrust C_T, the coefficient of torque C_Q and the coefficient of power C_P can be given by,

$$C_T = \frac{T}{1/2\rho A V_\infty^2} \quad \text{..........} \quad (3.4.11)$$

$$C_Q = \frac{Q}{1/2\rho A V_\infty^2 R} \quad \text{..........} \quad (3.4.12)$$

$$C_P = \frac{P}{1/2\rho A V_\infty^3} \quad \text{..........} \quad (3.4.13)$$

Substituting the values of T, Q and P from equations (3.4.8), (3.4.9) and (3.4.10) into equations (3.4.11), (3.4.12) and (3.4.13) respectively and simplifying the thrust coefficient, torque coefficient and the power coefficient can be expressed as,

$$C_T = \frac{8}{\lambda^2}\int_0^\lambda a(1-a)\left(1+\frac{C_D}{C_L}\tan\phi\right)\lambda_r\, d\lambda_r \quad \text{..........} \quad (3.4.14)$$

$$C_Q = \frac{8}{\lambda^3}\int_0^\lambda a'(1-a)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\lambda_r^3\, d\lambda_r \quad \text{..........} \quad (3.4.15)$$

$$C_P = \frac{8}{\lambda^2}\int_0^\lambda a'(1-a)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\lambda_r^3\, d\lambda_r \quad \text{..........} \quad (3.4.16)$$

The equations (3.4.14), (3.4.15) and (3.4.16) can be written as in the following forms by replacing local tip speed ratio with the equation (3.2.22) and considering $\psi=r/R$,

$$C_T = 8\int_0^1 a(1-a)\left(1+\frac{C_D}{C_L}\tan\phi\right)\psi\, d\psi \quad \text{..........} \quad (3.4.17)$$

$$C_Q = 8\lambda\int_0^1 a'(1-a)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\psi^3\, d\psi \quad \text{..........} \quad (3.4.18)$$

$$C_P = 8\lambda^2\int_0^1 a'(1-a)\left(1-\frac{C_D}{C_L}\frac{1}{\tan\phi}\right)\psi^3\, d\psi \quad \text{..........} \quad (3.4.19)$$

3.5. Expressions for maximum power

We have the expressions for axial interference factor a , angular or tangential interference factor a′ and the relation between a and a′ as obtained from the equations (3.4.3), (3.4.4) and (3.2.30) respectively,

$$\frac{a}{1-a} = \frac{\sigma_r C_L \cos\phi}{4 \sin^2\phi} \quad \text{......} (3.4.3)$$

$$\frac{a'}{1+a'} = \frac{\sigma_r C_L}{4 \cos\phi} \quad \text{......} (3.4.4)$$

$$a' = \frac{1-3a}{4a-1} \quad \text{......} (3.2.30)$$

Now eliminating a and a′ from the above three equations one obtains,

$$\sigma_r C_L = 4\,(1 - \cos\phi) \quad \text{......} (3.5.1)$$

Inserting the value of local solidity ratio $\sigma_r = \frac{BC}{2\pi r}$ from equation (3.3.10), the above expression becomes,

$$C = \frac{8\pi r}{B C_L}\,(1 - \cos\phi) \quad \text{......} (3.5.2)$$

From the equations (3.2.22) and (3.3.8), one finds,

$$\lambda_r = \frac{\Omega r}{V_\infty} \quad \text{......} (3.2.22)$$

$$\tan\phi = \frac{1-a}{1+a'}\,\frac{1}{\lambda_r} \quad \text{......} (3.3.8)$$

Now replacing the value of (1 - a) from (3.4.3) and (1 + a′) from (3.4.4), one obtains,

$$\tan\phi = \frac{4a\sin^2\phi}{\sigma_r C_L \cos\phi}\;\frac{\sigma_r C_L}{4a\cos\phi}\;\frac{1}{\lambda_r}$$

Inserting the value of $\sigma_r C_L$ from equation (3.5.1), eliminating a and a′ the expression of λ_r becomes after simplification,

$$\lambda_T = \frac{\sin\phi\,(2\cos\phi - 1)}{(1 - \cos\phi)\,(2\cos\phi + 1)} \quad \text{.........} \quad (3.5.3)$$

The expression of ϕ can be obtained from the above equation as,

$$\phi = \frac{2}{3}\tan^{-1}\frac{1}{\lambda_T} \quad \text{.........} \quad (3.5.4)$$

From Figure 3.3.1, the equation of blade twist angle β can be written as,

$$\beta = \phi - \alpha \quad \text{.........} \quad (3.3.9)$$

The above expressions are very important for the calculation of blade configuration.

CHAPTER FOUR : DESIGN OF WIND MACHINE

4.1. General

The appropriate design of wind turbine for a particular region is obviously an important task. As, it is fact that the air density is low, so to obtain reasonable amount of power extraction, the wind velocity has to be very reasonable preferably around 10 m/s. Wind power varies with cube of wind velocity, as such, the wind velocity has significant role in production of power. However, uptil now the wind data, which has been published so far for the various regions of Bangladesh show mostly unfortunate values. Only in some areas, the values of wind velocity are moderate, even are not available throughout the year. As such, the design of wind turbine for such regions is very difficult, if reasonable power has to be extracted. On the contrary, in respect of technology, Bangladesh is not that much developed. As a result to construct a wind turbine with sophisticated blade is again a problem. Considering all these difficulties, design approach has been taken into consideration with simple type of blade section and for production of small amount of power at the available mean wind speed in Bangladesh for some regions only.

The conventional design uses the minimum value of lift to drag ratio for the design. But in this particular design, the lift at 0.9 of maximum lift coefficient has been chosen. It has been already checked by calculation that, this rather improves the situation a little bit in consideration of power extraction for the blade section with circular arc profile.

4.2. Selection of number of blades, the design tip speed ratio and twist angle.

Fixed pitch wind turbines have been considered only which operate at maximum efficiency in a limited range of wind speed conditions. The first problem is to choose the appropriate number of blades of the turbine. As the number of blades increases, the power production should increase, but it does not have linear effect. In multi-bladed turbine, the turbine speed reduces, efficiency reduces; on the contrary the cost of production rises. In this regard we should not neglect the fact that, a finite number of blades instead of the ideal infinite blade number, which causes an extra reduction in power, particularly at low tip speed ratio. This is caused by the pressure leakage around the tip of the blade. To design a rotor with a given tip speed ratio one can choose between many blades with a small chord width or lower number of blades with a large chord. In this point of view, it is clear that for a given tip speed ratio a rotor with lower number of blades will have larger tip losses. Since the chord become smaller for high tip speed ratios, this effect is smaller for higher tip speed ratios.

The selection of design tip speed ratio and the number of blades are more or less related. If the value of the design tip speed ratio is low, a higher number of blades should be considered. The objective of doing this is that, the influence of number of blades, B, on the power coefficient, C_P is larger at lower tip speed ratio.

On the other hand, for higher value of design tip speed ratio lower number of blades should be chosen. This is done because higher number of blades for a high design tip speed ratio will lead to very small and thin blades which causes huge cost involvement.

The choice of the design tip speed ratio also depend on, for what purpose the turbine will be used. If it is used for a piston pump or some other slow running equipment, which need a high starting torque, then the design speed of the rotor will usually be chosen low. If the turbine is used for fast running equipment such as generator or a centrifugal pump, a high design tip speed will be considered.

The table given in the next page should be considered as the guidelines for the choice of the design tip speed ratio and the corresponding number of blades as per Lysen (1983).

λ_d	B
1	6 - 20
2	4 - 12
3	3 - 6
4	2 - 4
5 - 8	2 - 3
8 - 15	1 - 2

Table 4.2.1 : Table showing the Design tip speed ratio and Number of blades.

In the calculation procedure of a wind turbine, the blade is divided into a number of radial stations in order to obtain the optimum configuration of the turbine. The formulas which are used to describe the information about the twist angle β are given bellow -

Local design tip speed ratio : $\lambda_r = \lambda_d \frac{r}{R}$ .... (3.2.22)

Angle of relative wind velocity : $\phi = \tan^{-1} [\frac{1 - a}{1 + a'} \frac{1}{\lambda_r}]$ .... (3.3.8)

and

Twist angle : $\beta = \phi - \alpha$ .... (3.3.9)

In this thesis, the turbine is considered for small power output specifically for electricity generation at remote area for that reason, the design tip speed ratio is selected as $\lambda_d = 6$ and the number of blades are B = 3 from the above table.

The twist angle may be zero twist, linear twist or the optimum twist. In this particular case, the linear twist angle is chosen. To construct a blade with optimum twist is very expensive and complicated while the performance of the linear twist blade does not deviate much in comparison to that with optimum twist. There is no justification to use zero twist angle, however, only in case of constructional simplicity it may be used in that case power production has to be sacrificed.

4.3. Selection of airfoil section

For an airfoil used in wind energy conversion system, the lift to drag ratio is of great importance for the efficiency of the rotor. All types of airfoils have a high lift to drag ratio (C_L/C_D) over a certain range of angle of attack α . In view of the fluctuations that occur in the wind speed and therefore, in the angle of attack, it is important to choose airfoil sections that possess a high lift to drag ratio for a relatively large range of α values. From this point of view, though the circular arc blade section is not perfect, its simplicity in consideration of manufacture and cost has encouraged to select this as blade section.

Pandey et al (1988) conducted wind tunnel tests on several steel plates of circular arc to determine their lift and drag characteristics. The test were conducted at a Reynolds Number $2.23X10^5$ and the angle of attack was varied from -20° to 90°. Their test results are shown in the Figures 4.1 to 4.6.

The driving factors for the power coefficient C_P , of a wind turbine are the lift coefficient C_L and the drag coefficient C_D values of the airfoil sections.

For the design and performance calculations of the wind turbines two-dimensional airfoil data are used in terms of lift coefficient C_L and drag coefficient C_D.

The airfoil data taken for the calculation are mentioned as follows,

The concavity of airfoil i.e. the camber of the airfoil to the chord length ratio (f/C) is chosen from 0.00 to 0.14 from Pandey et al (1988) and seven curves are shown for f/C equal to 0.00, 0.04, 0.06, 0.08, 0.10, 0.12 and 0.14 with respect to angle of attack ranging from -20° to +90° in the Figures 4.1 to 4.6.

From C_L versus α graph (Figure 4.2), it is found that, the value of C_L is maximum for f/C = 0.08.

The design lift coefficient value of C_{Ld} is chosen as $C_{Ld} = 0.9C_{Lmax}$ where C_{Lmax} is the maximum lift coefficient value in C_L vs. α curve.

The design value of α_d is obtained corresponding to the value of C_{Ld}.

The value of Reynolds Number is kept fixed at $2.23X10^5$ in this investigation. Since lift-drag characteristics at variable Reynolds Number are not available at the moment.

4.4. Calculation scheme of performance

In this investigation some performance characteristics have been studied. For the calculation of the performance characteristics, the following parameters have been chosen beforehand,

Design wind velocity	: V_d
Chord of the blade	: C
Rotor radius	: R
Number of blades	: B
Design tip speed ratio	: λ_d

The steps which are needed for the calculation of performance characteristics of the wind turbine are as follows,

1) The value of local solidity ratio σ_r is determined from the equation (3.3.10).

2) A certain number of radial stations are assumed for which the blade twist angles are found.

3) The value of λ_r is obtained for each radial station using the equation (3.2.22).

4) From the C_L versus α graph it is seen that the maximum value of C_L is occurred for f/C = 0.08 and the design value of C_L is chosen by $C_{Ld} = 0.9\ C_{Lmax}$.

5) Optimum value of β is calculated next.

a. At the first step, the initial values of λ, β and a are assumed as 2, 4° and 0.9 respectively.

b. λ_r is calculated.

c. The value of a′ from the equation (3.2.28) is calculated.

d. ϕ from equation (3.3.8) is calculated.

e. From the equation (3.3.9), α is obtained.

f. Corresponding to the above α, the value of C_L and C_D is determined by linear interpolation.

g. Finally a from the equation (3.4.1) is found by iteration process.

h. Using the value of a, β is determined

i. The above calculation procedure for each radial station is repeated.

6) Calculation of C_p.

a. Using the above iteration process from the steps (a) to (f) the value of a and a′ for each radial station, are found.

b. The value of C_P is obtained by numerical integration from the equation (3.4.19) by using Simpson's rule.

7) Calculation of C_Q.

a. Using the iteration process as mentioned above in section 5), from steps (a) to (f), the value of a and a′ are calculated.

b. The value of C_Q by numerical integration from the equation (3.4.18) is obtained by using Simpson's rule.

8) Calculation of C_T.

a. Using the iteration process as mentioned above in section 5), from steps (a) to (f), the value of a and a′ are calculated.

b. The value of C_P are found by numerical integration from the equation (3.4.17).

CHAPTER FIVE : RESULTS AND DISCUSSION

5.1. General

The performance characteristics of a horizontal axis wind turbine with circular arc blade profile are discussed in this section elaborately. In addition the design of the horizontal axis wind turbine with the same type of blade profile is also mentioned. The conventional blade sections are NACA or Wortman. Obviously they are suitable for use in horizontal axis wind machine. But they incur high cost of production and render complicated manufacturing technique. As such, simple type of blade has been used to observe their nature of performance. However, the result with circular arc blade profile is definitely comparable to those with conventional NACA or Wortman blade profile. At the moment few experimental data of circular arc blade profile are available for different values of f/c ratios. Among them the ones which give higher values have been considered only for calculating performance characteristics.

The manufacture of circular arc blade profiles are easy, simple, take smaller effort, need moderate skilled persons and can be made from easily available plain galvanized iron sheets or aluminium alloy sheets.

5.2. Discussion on calculated result

The Figure 5.1 shows the power coefficient (Cp) vs. tip speed ratio curves for different values of camber ratios (f/C). It can be observed from this figure that the value of power coefficients are higher at the value of f/C = 0.10 and lower at f/C = 0.0. It is further seen that the values of power coefficients at f/C = 0.80 and f/C = 0.10 are very near. So, it can be pointed out that the turbine blades may be manufactured either with f/C=0.80 or 0.10. However, preferably f/C = 0.80 will be suitable. It can be observed from this figure that the peak power coefficient is around 0.39 and the peak values are not sharp rather a little bit flattened. This nature of curve obviously very promising for application as the wind turbine blade.

The variation of optimum twist angle vs. ratio of radius at various tip speed ratios is shown in the Figure 5.2. Three tip speed ratios have been considered. It is observed that the nature of the curves are similar and there is little difference in magnitude of twist angle among the three. Near the hub the values of twist angle are positive while near the tip the values are negative. The values are obtained for any particular wind machine having, number of blades = 3, chord = 200 mm and radius = 750 mm.

In the Figure 5.3, the variation of the twist angle with ratio of radius has been shown. This figure shows both the optimum twist angle, linear twist angle and zero twist angle at the design tip speed ratio of 6. The values are considered for a 50 Watt wind machine at mean wind speed of 5 m/s.

The comparison of power coefficient with tip speed ratio at different types of twist angles is seen from the Figure 5.4. It is obvious from this figure that the power coefficient at optimum tip speed ratio is higher whereas at zero twist angle it is lower. At linear twist angle, the predicted values remain more or less at the middle. Comparing among the values of three types, significant difference can be found. However, in each case the peak values are occurred around design tip speed ratio of 6 and near the peak the nature of curve shows flat trend which is important from the operational point of view.

The Figure 5.5 shows the comparison of power coefficients with tip speed ratio at α_{opt} and minimum C_D/C_L ratio. α_{Opt} corresponds to the lift characteristics at $0.9C_{Lmax}$. It is vivid from this figure that if the lift-drag characteristics corresponding to the α_{opt} are considered in the calculation of power coefficients than the conventional values of minimum C_D/C_L ratio, there occurs little improvements in the power coefficient values. As such, in the present design, lift-drag characteristics corresponding to α_{opt} has been chosen.

The comparison of power coefficients with tip speed ratio for constant and tapered chord at various twist angle is shown in the Figure 5.6. It can be observed from this figure that, the effect of chord taper has insignificant effect on the power coefficient. However, the values with constant chord rather improves a little bit.

In the Figure 5.7, the comparison of torque coefficients with tip speed ratio at various twist angle is shown. It is seen from this figure that torque coefficient at optimum twist angle is higher than those at zero twist angle and the values are almost at the middle for linear twist angle. The peak values are seen at around the design tip speed ratio of 6. This figure has similar trend as is found in the Figure 5.4.

Comparison of torque coefficients with tip speed ratio for constant and tapered chord at various twist angle is shown in the Figure 5.8. It is observed from this figure that the value for constant chord are a little bit higher than those with tapered chord. Hence from manufacturing point of view application of constant chord blade will be suitable. It is further seen that the values with optimum twist angle is higher while those with zero twist is lower and those at linear twist nearly at the middle of the two.

The Figure 5.9 shows the comparison of thrust coefficient with tip speed ratio at various twist angle. The thrust coefficient values at optimum twist angle is higher than those at zero twist angle and nearly at the middle of the two at linear twist angle at the higher tip speed ratio range, as this figure reveals. However, at the lower tip speed ratio range, they are almost identical at any twist angle.

Comparison of power with tip speed ratio at various twist angle is shown in the Figure 5.10. Corresponding to optimum twist angle, the power is higher while it is lower at zero twist angle and at linear twist angle, it is nearly at the middle of the two. The peak values are observed at around design tip speed ratio. It is further observed that with the inclusion of optimum twist, near the peak the values are flattened over a wide range of tip speed ratio, as the figure reveals.

The Figure 5.11 shows the comparison of power coefficient with tip speed ratio with NACA 4418 blade and circular arc blade section. Optimum value of twist angle is used for the calculation. It can be seen from the figure that, using conventional NACA 4418 profile, the power coefficient values are higher than those with circular arc blade profile. However, the values are definitely comparable. At the lower tip speed ratio side using circular arc blade section, some improvements in values are observed while at higher tip speed ratio values, they are lower with circular arc blade section.

CHAPTER SIX : CONCLUSION AND RECOMMENDATION

6.1. Conclusions

The following conclusions are drawn in regard to the application of circular arc blade profile as the blade of a horizontal axis wind turbine and its design.

(1) The application of circular arc blade profile as the blade element of a horizontal axis wind turbine in place of conventional blade section such as NACA or Wortman, reduces the cost of production and manufacturing complexity reasonably.

(2) Using circular arc blade profile, the values of power coefficients increases at the lower tip speed ratio in comparison to those with conventional NACA blade and at higher tip speed ratio, there is reverse effect.

(3) Employing the optimum twist angle, the power values becomes flattened over a wide range of tip speed ratio around design tip speed ratio .

(4) The power coefficient values of a horizontal axis wind machine with the circular arc blade profile are comparable to those with the conventional blade section .

(5) The blade of circular arc profile may be chosen with camber ratio (f/C) of either 0.08 or 0.10, preferably 0.08.

(6) In the design of a horizontal axis wind machine with circular arc blade profile the design lift value may be chosen as $C_{Ld}= 0.9C_{Lmax}$ in place of conventional practice of using minimum value of drag to lift ratio, as such rather power coefficient improves a little bit.

(7) Circular arc blade section can be manufactured easily by semiskilled worker, as a result it may be manufactured locally.

(8) Since the wind velocity in various parts of Bangladesh is not very high, so, design of low powered wind machine will be suitable and as such this has been taken care of.

(9) For the lack of experimental results of power coefficients, the theoretical characteristic curves could not be verified. However, the nature of the curve obviously represent that, they will give reasonable correlation with experimental results.

(10) The prediction of performance characteristics could have been more realistic if the lift-drag characteristics variation on Reynolds Number could have been incorporated into the analysis.

(11) Since the variation of power production for constant chord blade section and tapered blade section is insignificant; rather for constant chord type, power improves a little bit, as such it is recommended to use constant chord blade section.

6.2. Recommendations

From the analysis of performance characteristics and design of a horizontal axis wind machine with circular arc blade profile, the following recommendations are suggested.

(1) The further theoretical investigation with large number of blades can be done.

(2) The design of a horizontal axis wind machine for high wind velocity can be made.

(3) The experimental data can be obtained for both the lower and higher number of blades at different wind speeds.

(4) The stress analysis with both the galvanized iron sheet and aluminium alloy sheets may be done.

(5) The natural frequency of the wind machine with circular arc blade profile can be considered for study.

(6) Wind shear may be considered for obtaining the performance of wind machines.

(7) Effects of tip loss, coning and tilting may be taken into consideration for study.

(8) Starting torque characteristics may be studied by changing the number of blades.

(9) The study can be conducted by employing blade pitching both positive and negative blade pitching.

(10) The experimental results of lift-drag characteristics of circular arc blade section at various Reynolds Number can be determined.

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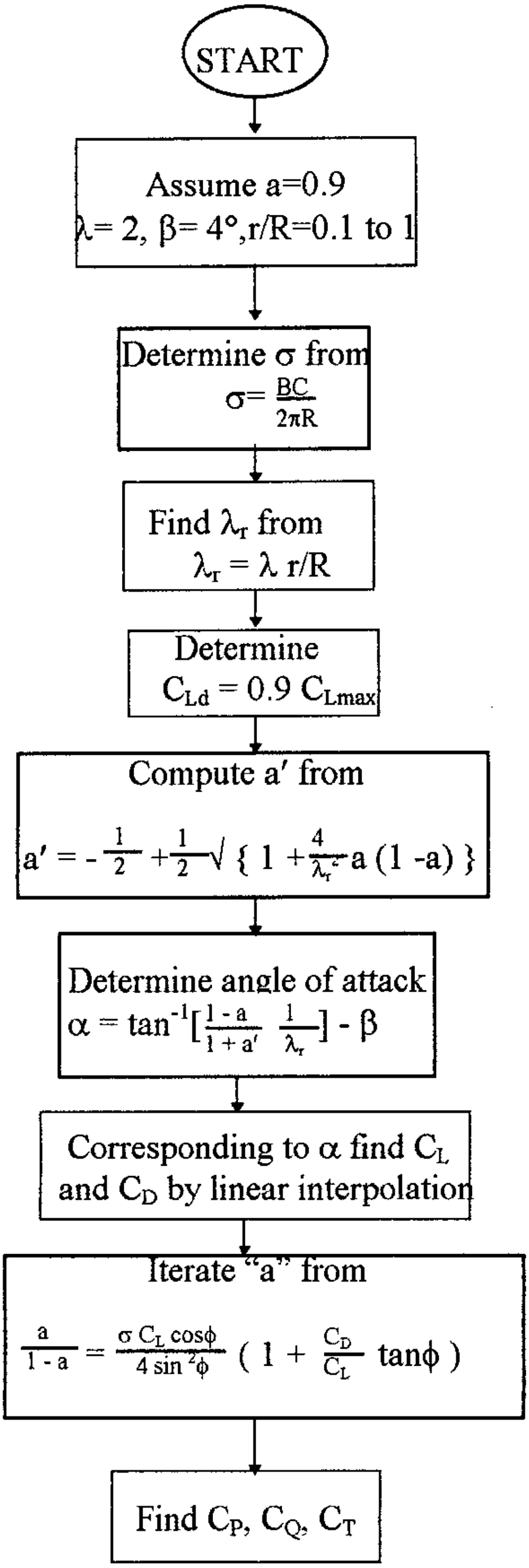
APPENDIX-A : FLOW DIAGRAM



APPENDIX - B : FIGURES

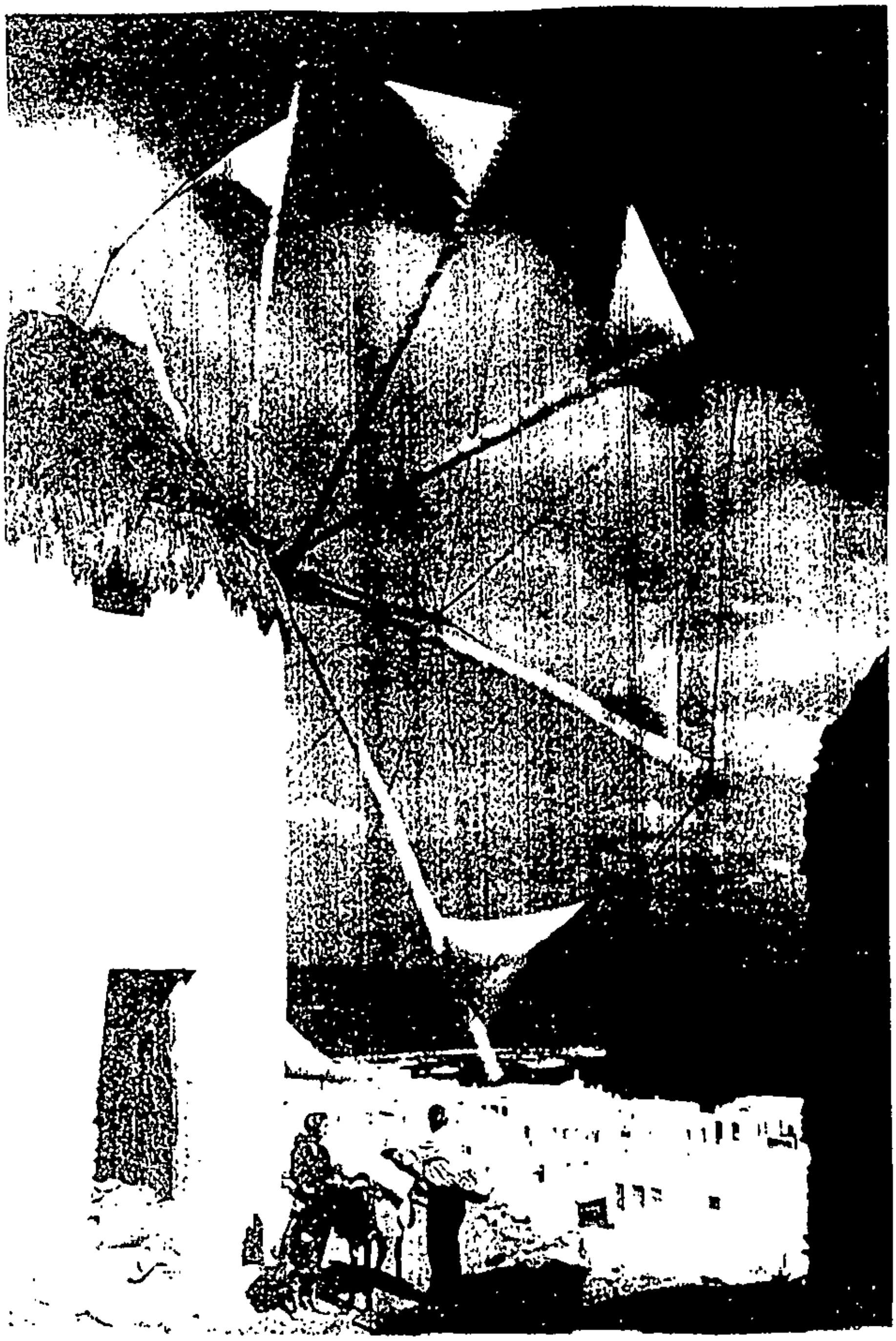


Figure 2.1 : Primitive type of horizontal axis windmill with jib sails wrapped on wooden booms.

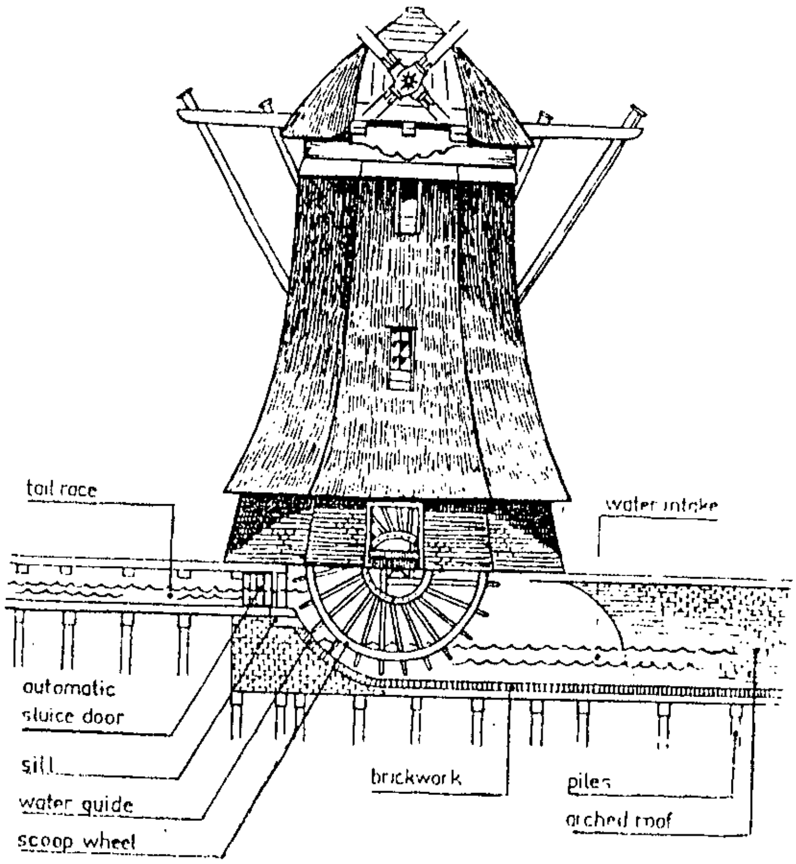


Figure 2.2A : Large octagonal drainage mill used for draining the marshes and lakes.

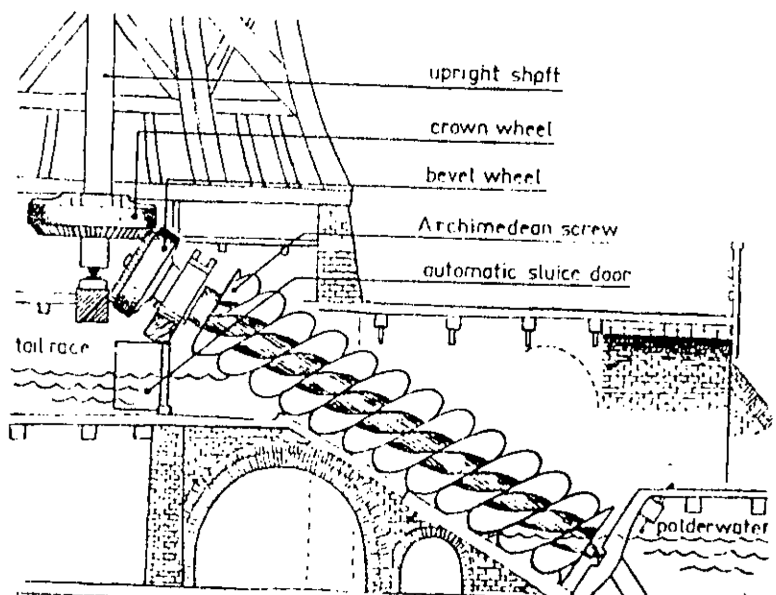


Figure 2.2B : Arrangement showing method of driving the Archimedean screw of a drainage mill.

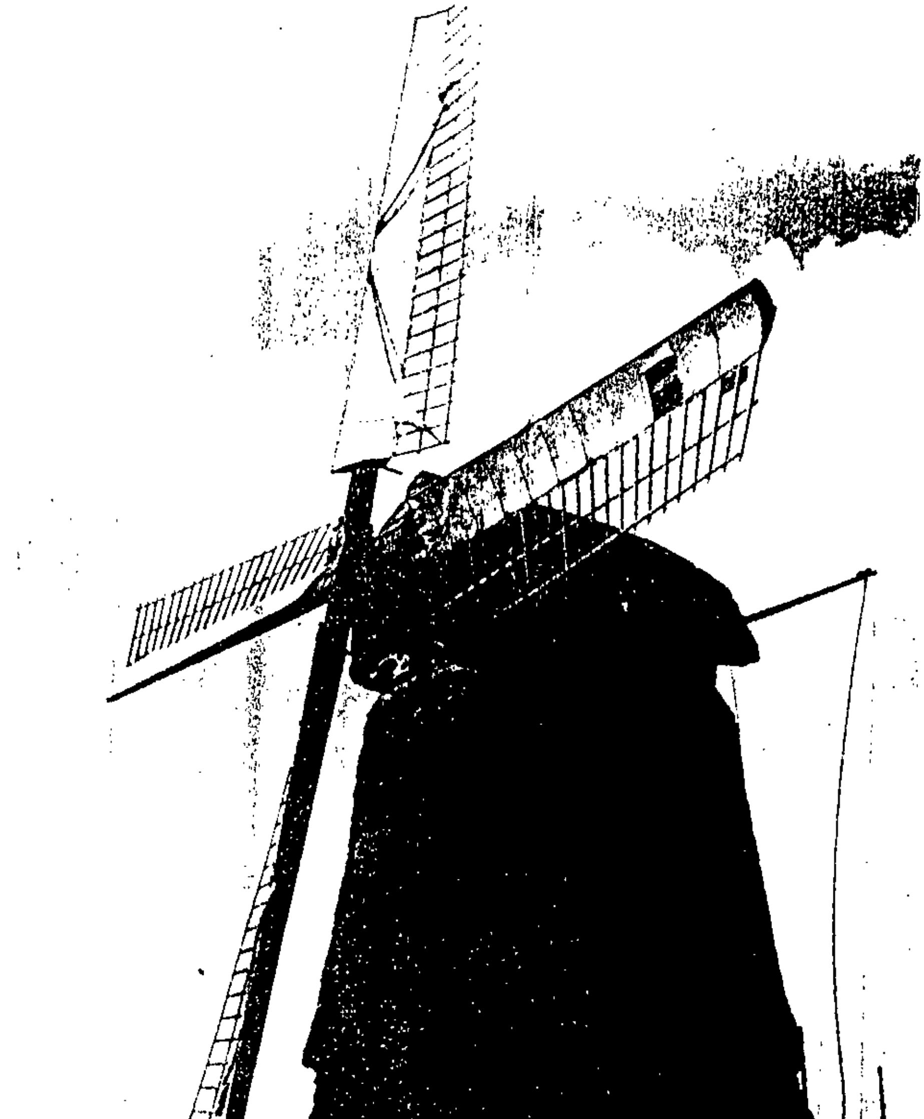


Figure 2.3 : Drainage mill, North-Holland type, built in 1761.

Figure 2.4 : Multibladed windmill for pumping water.

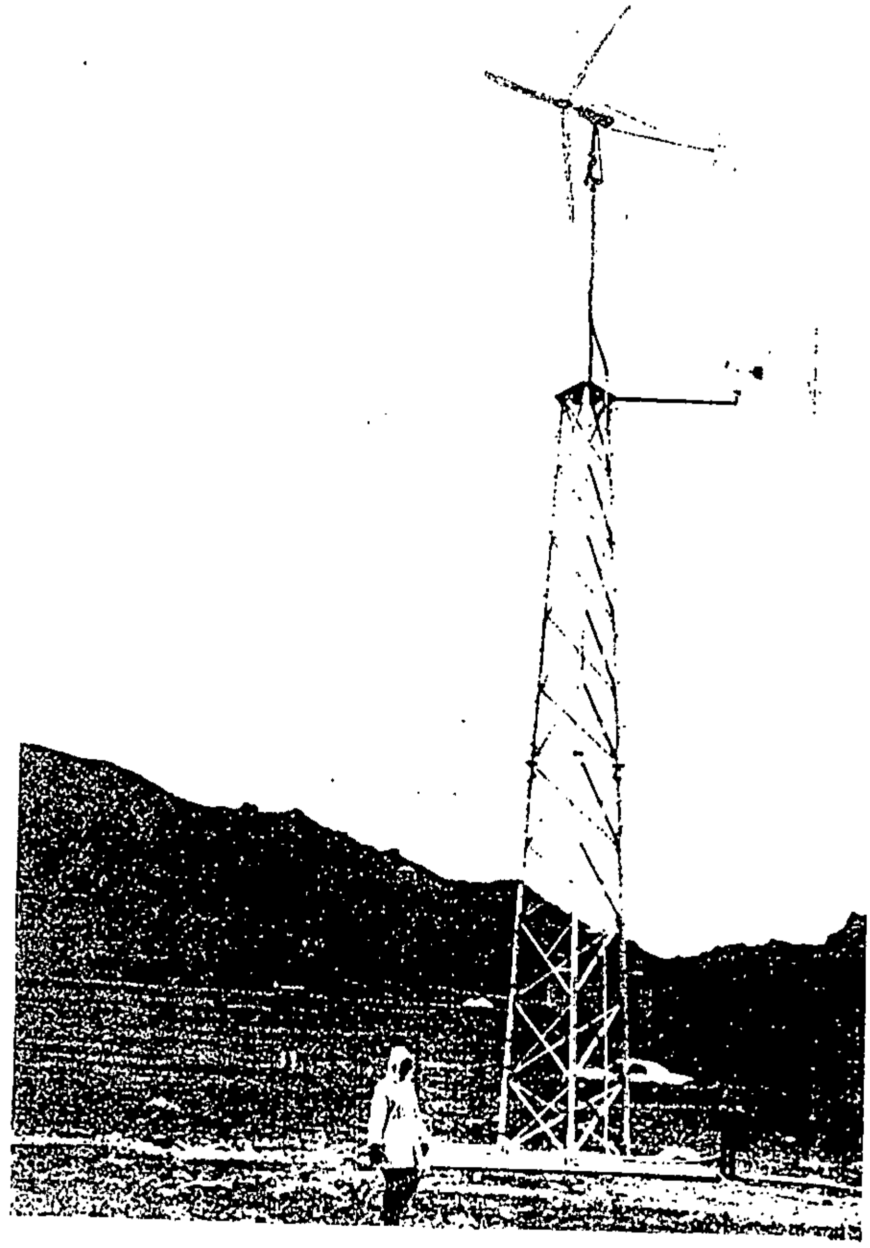


Figure 2.5 : Windmill for generating electricity.

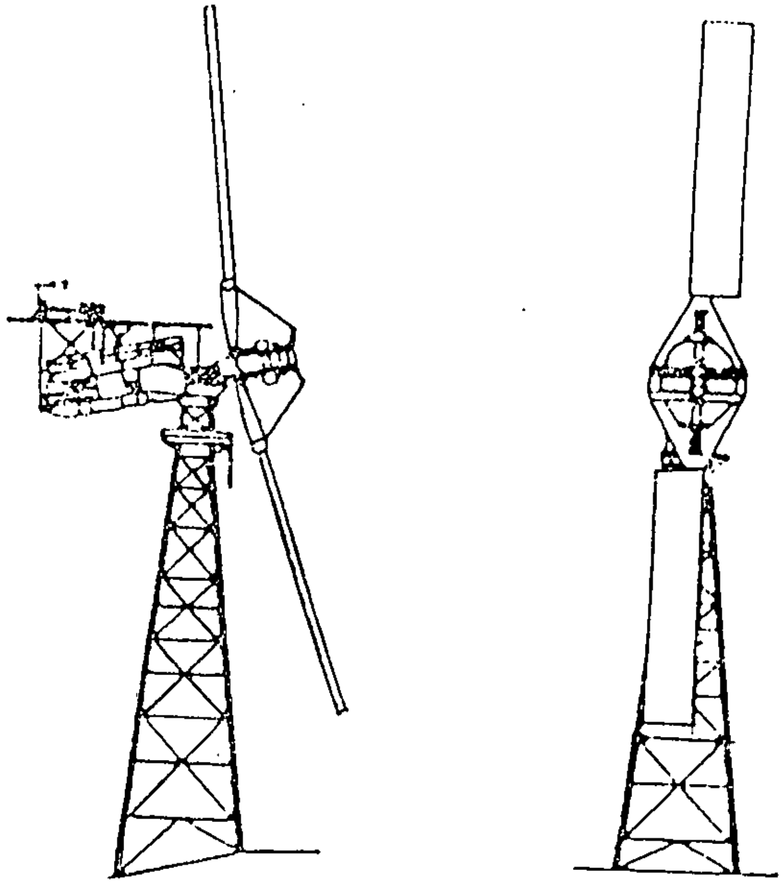


Figure 2.6 : Smith-Putnam wind turbine.

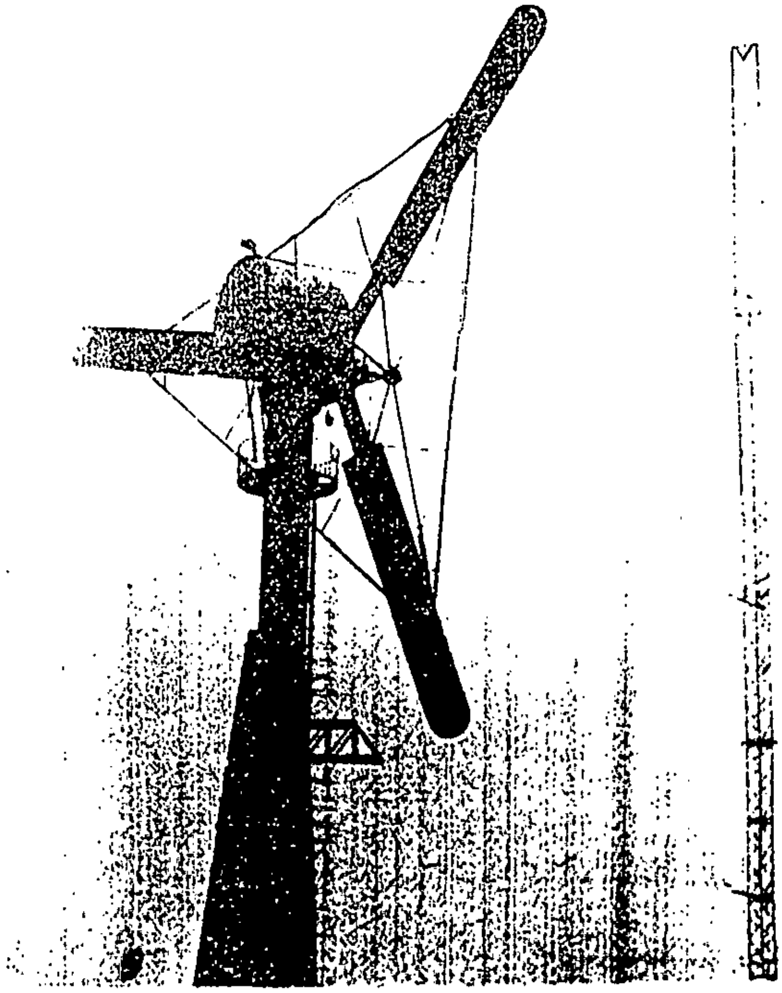


Figure 2.7 : Restored Danish Gedser wind-turbine.

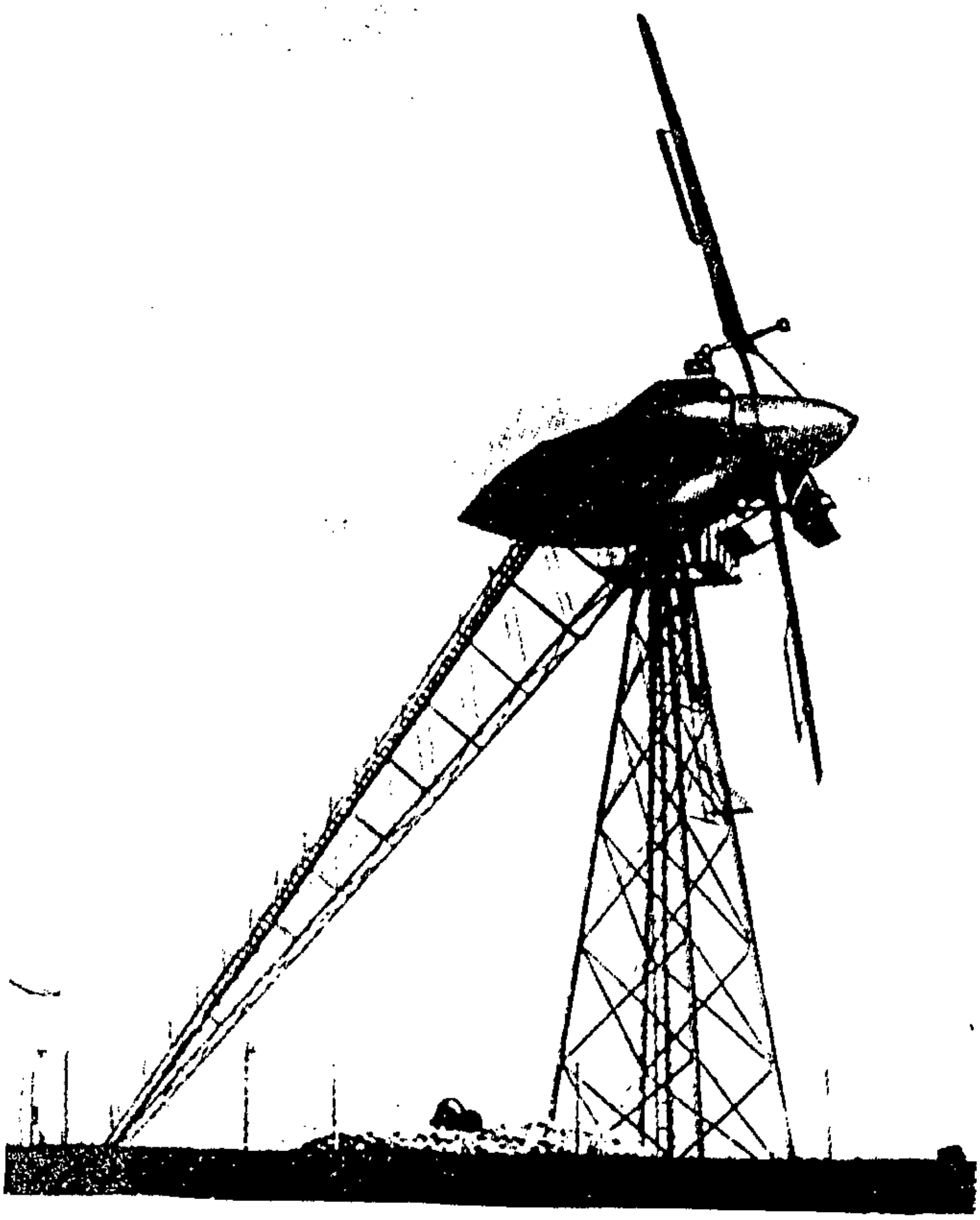


Figure 2.8 : Russian wind turbine of 100 KW output.

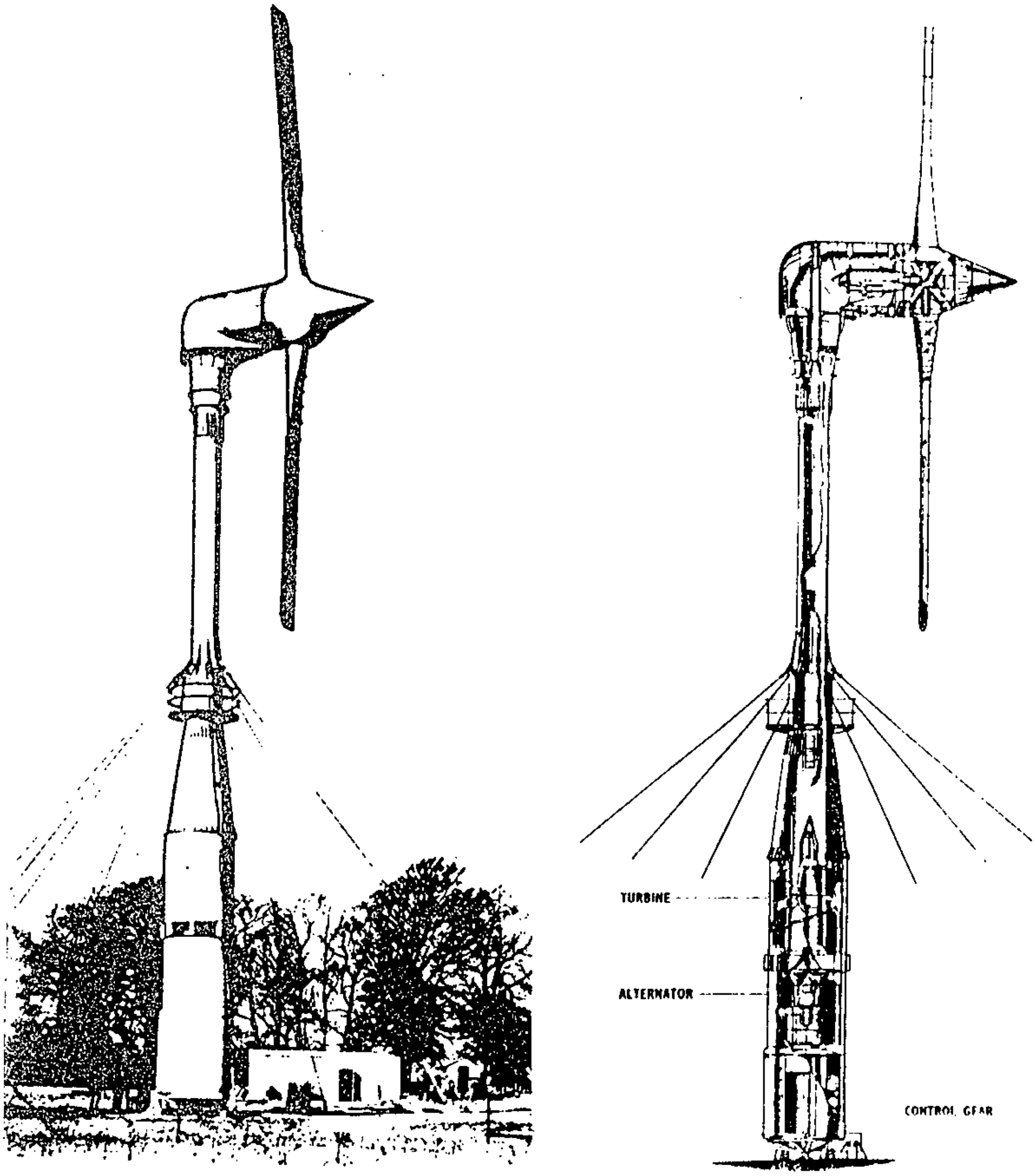


Figure 2.9 : Enfield-Andreau wind turbine of 100 kW output.

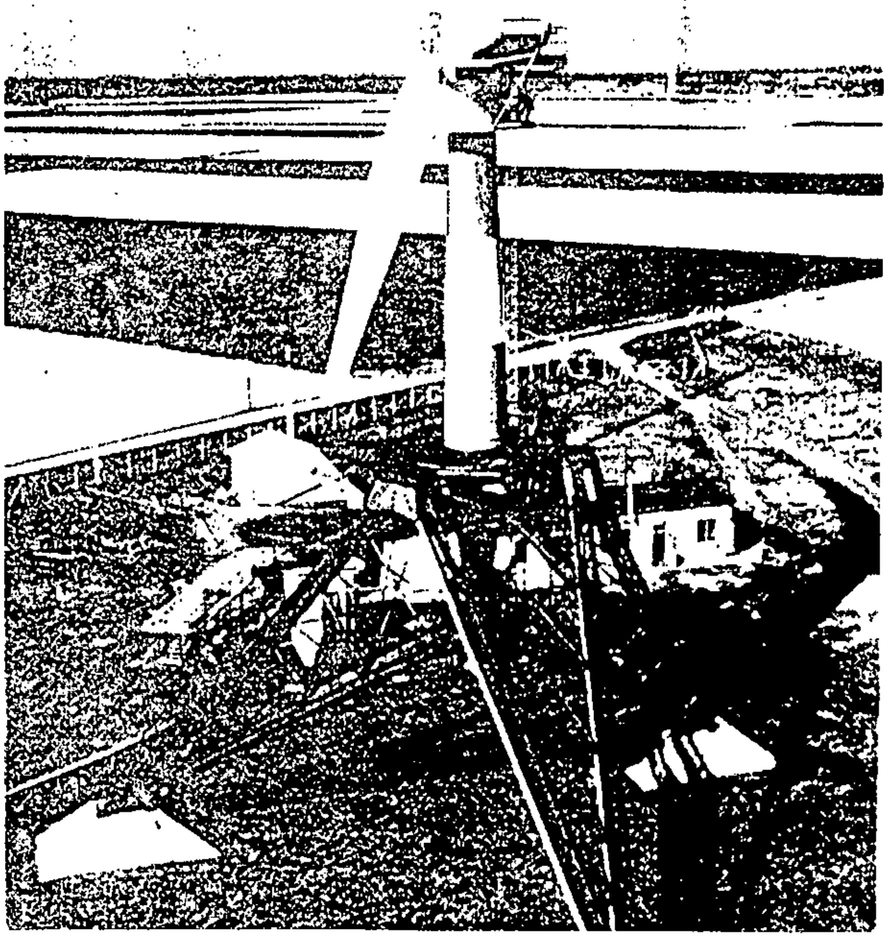


Figure 2.10 : French wind turbine located near Paris.

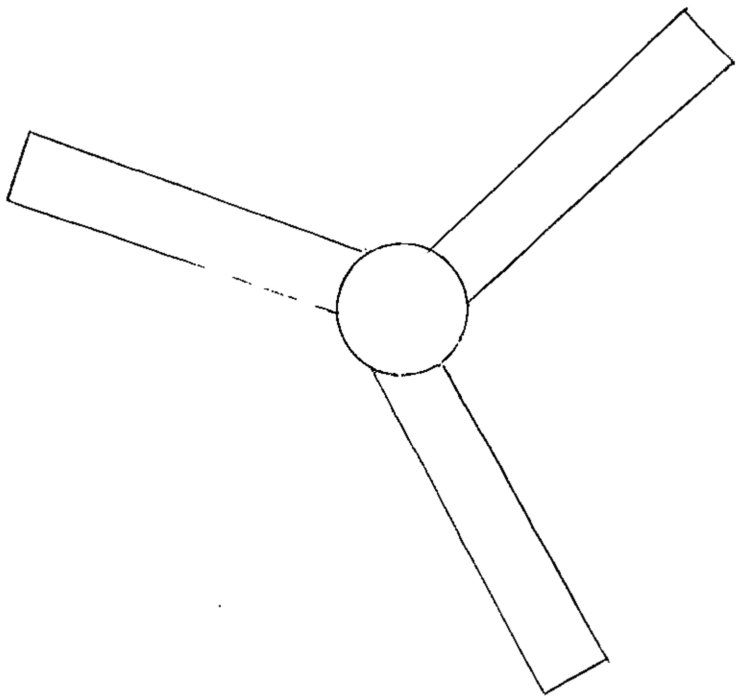


Figure 2.11(a) : Three-bladed rotor.

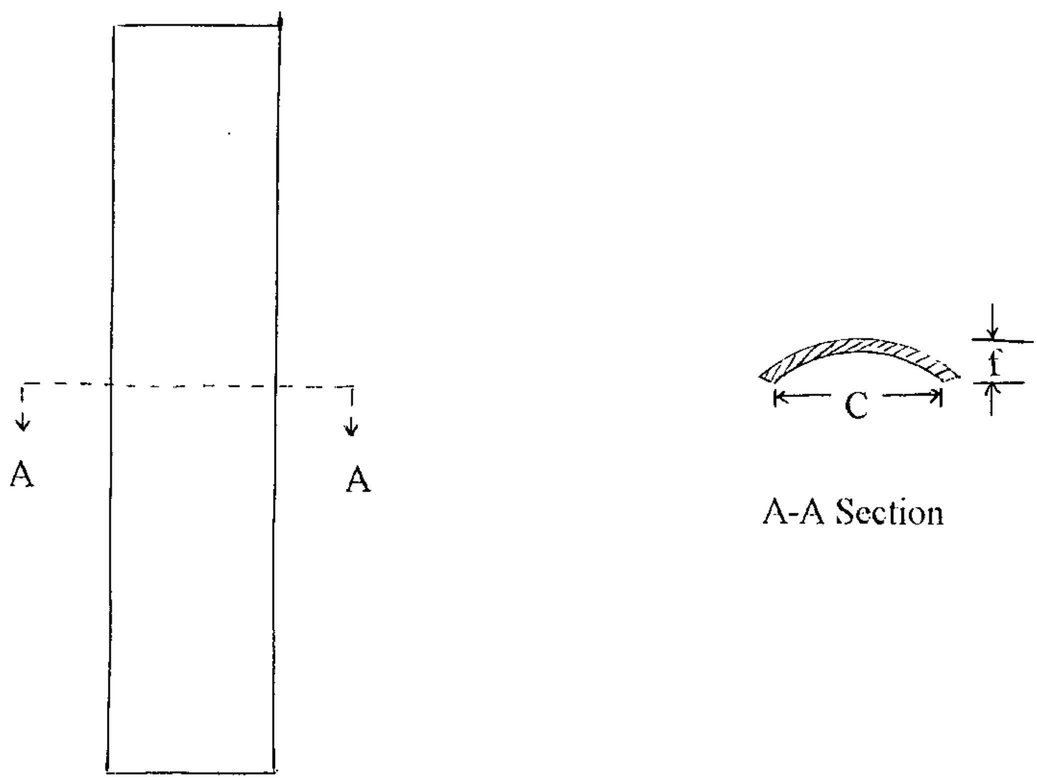


Figure 2.11(b) : Circular arc blade.

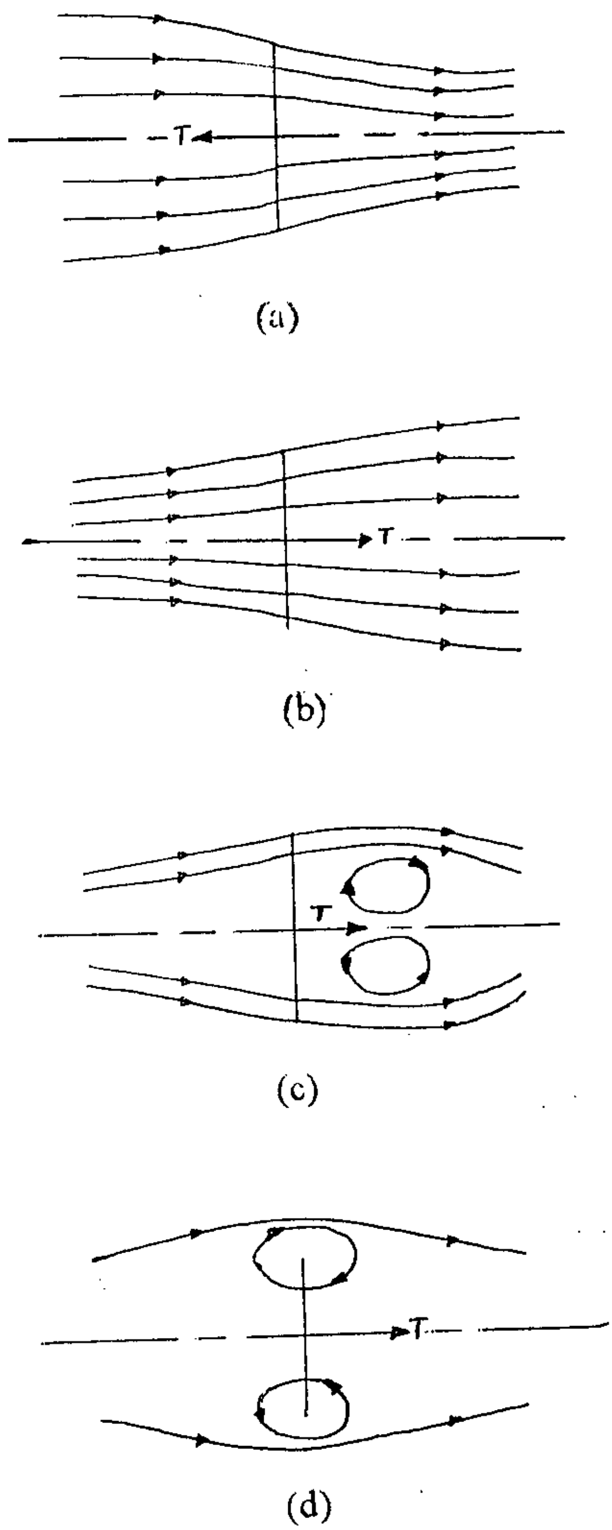


Figure 2.3 : Working states of a rotor. (a) Propeller state (b) Windmill state (c) Turbulent wake state (d) Vortex ring state

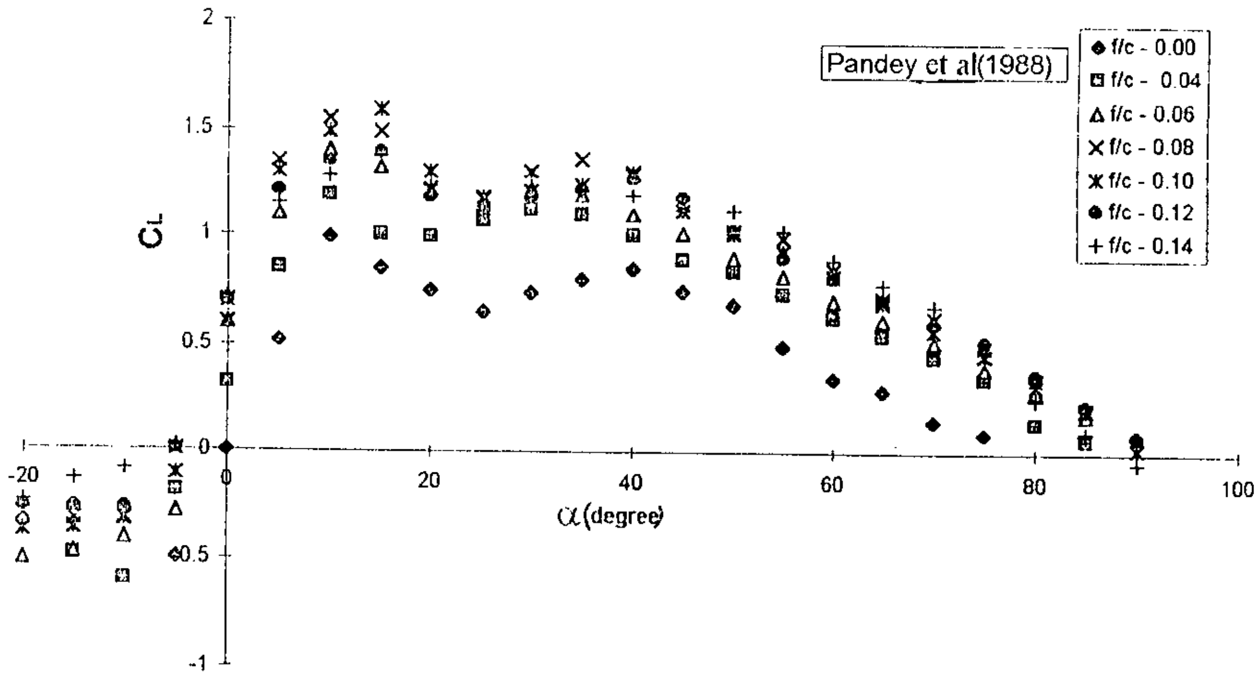


Figure 4.1 : Lift coefficient vs angle of attack at various camberness ratio from -20° to 100°.

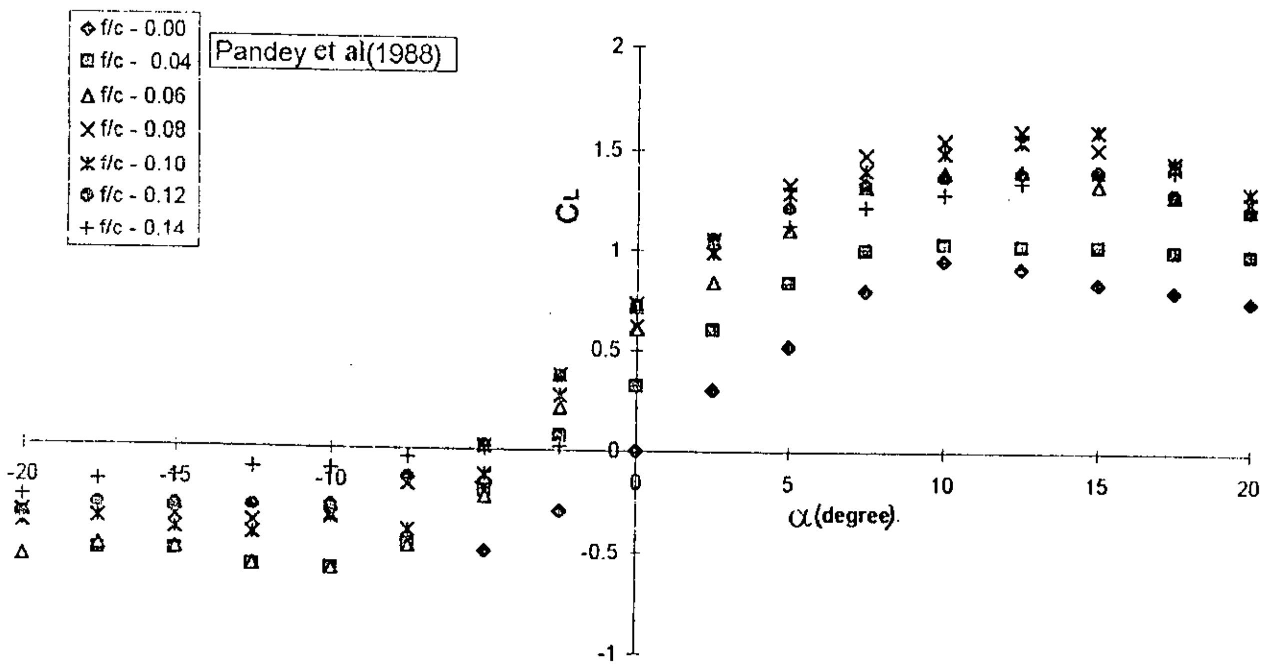


Figure 4.2 : Lift coefficient vs angle of attack at various camberness ratio from -20° to 20°.

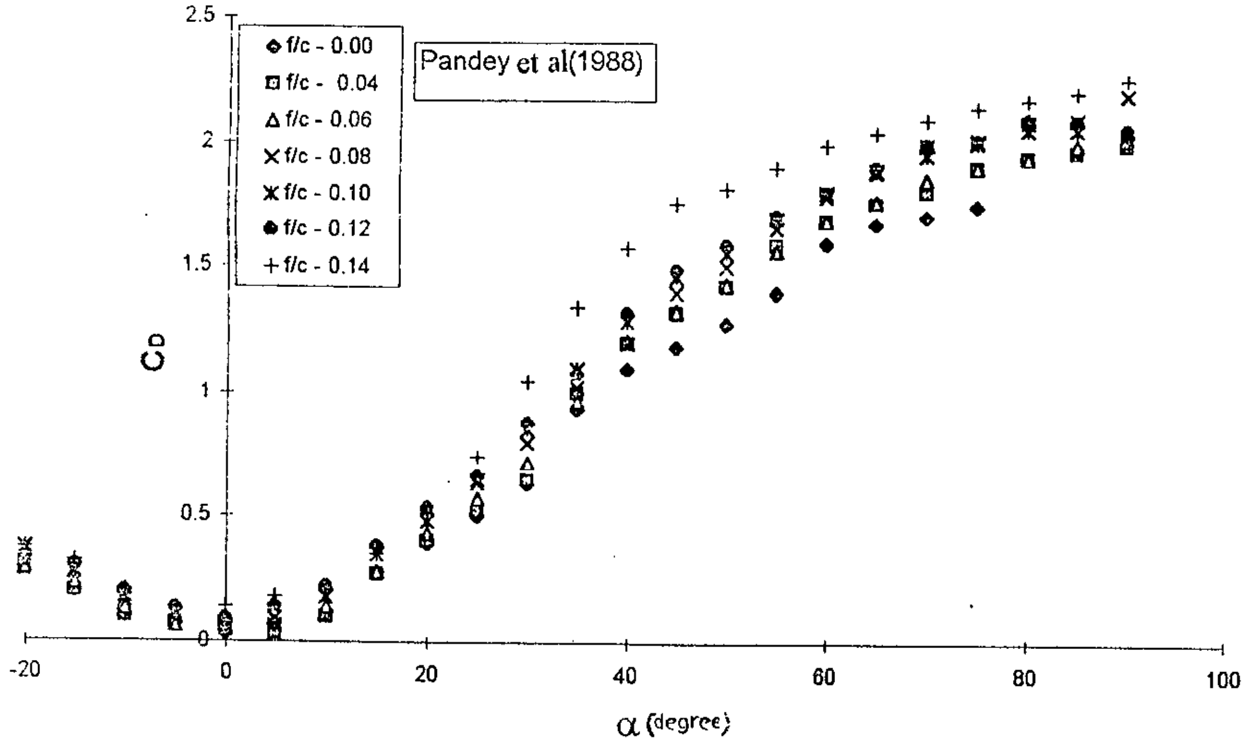


Figure 4.3 : Drag coefficient vs angle of attack at various camberness ratio from -20° to 100°.

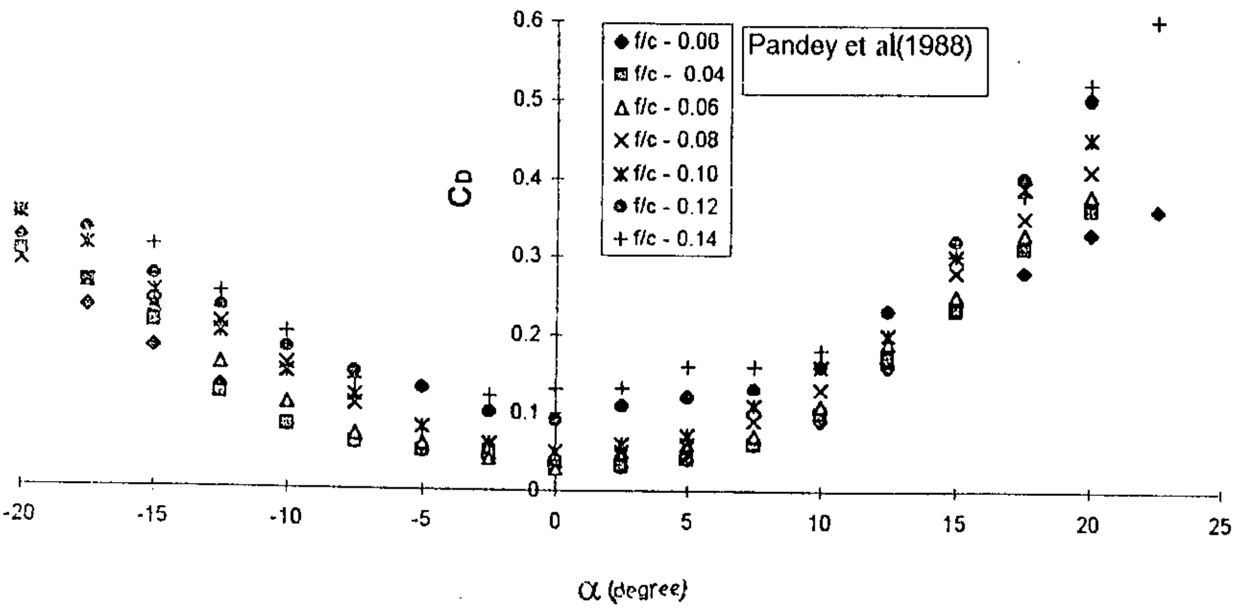


Figure 4.4 : Drag coefficient vs angle of attack at various camberness ratio from -20° to 20°.

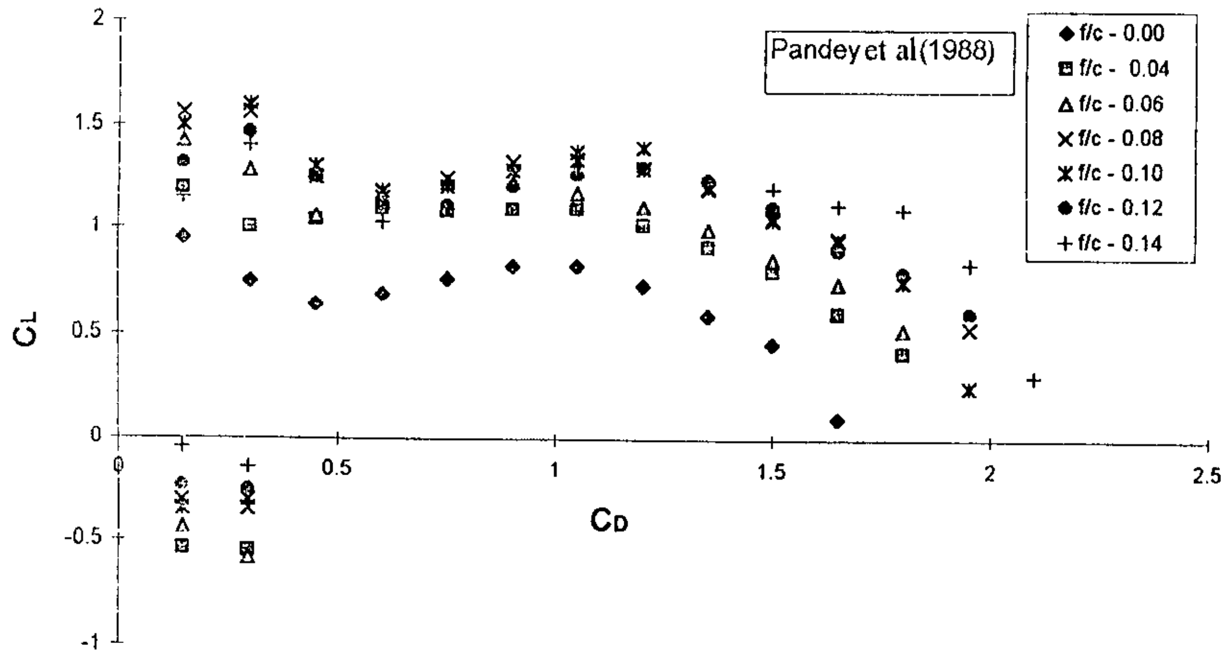


Figure 4.5 : Lift coefficient vs drag coefficient at various camberness ratios.

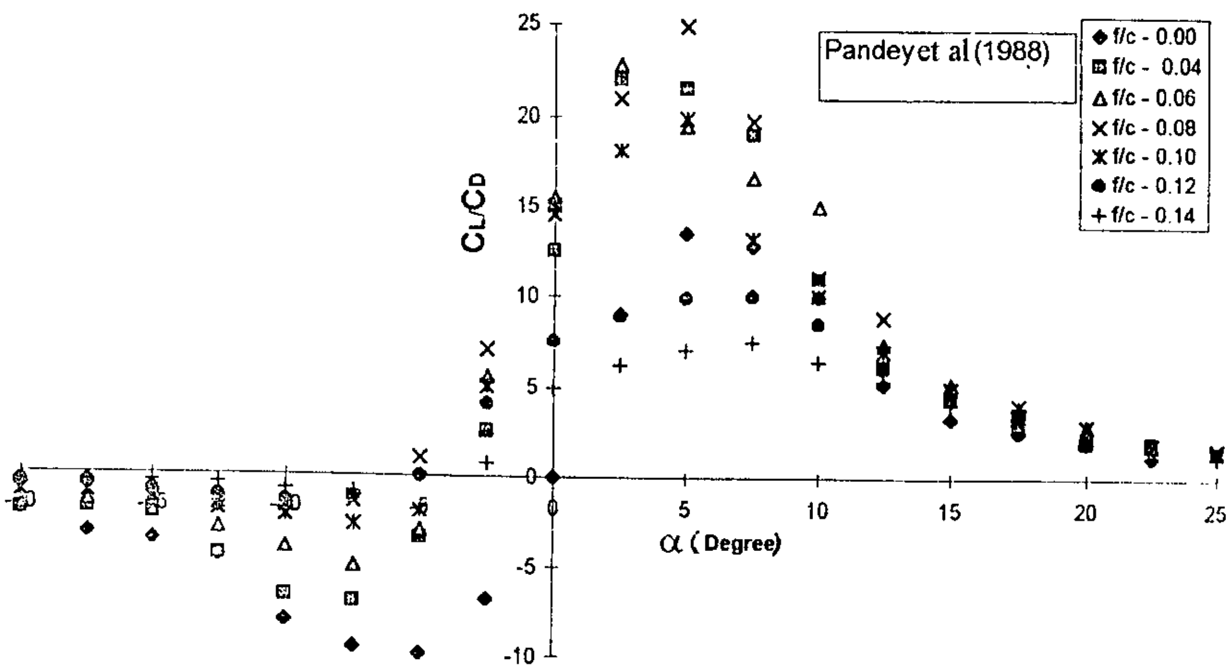


Figure 4.6 : Ratio of lift coefficient to drag coefficient vs angle of attack at various camberness ratio.

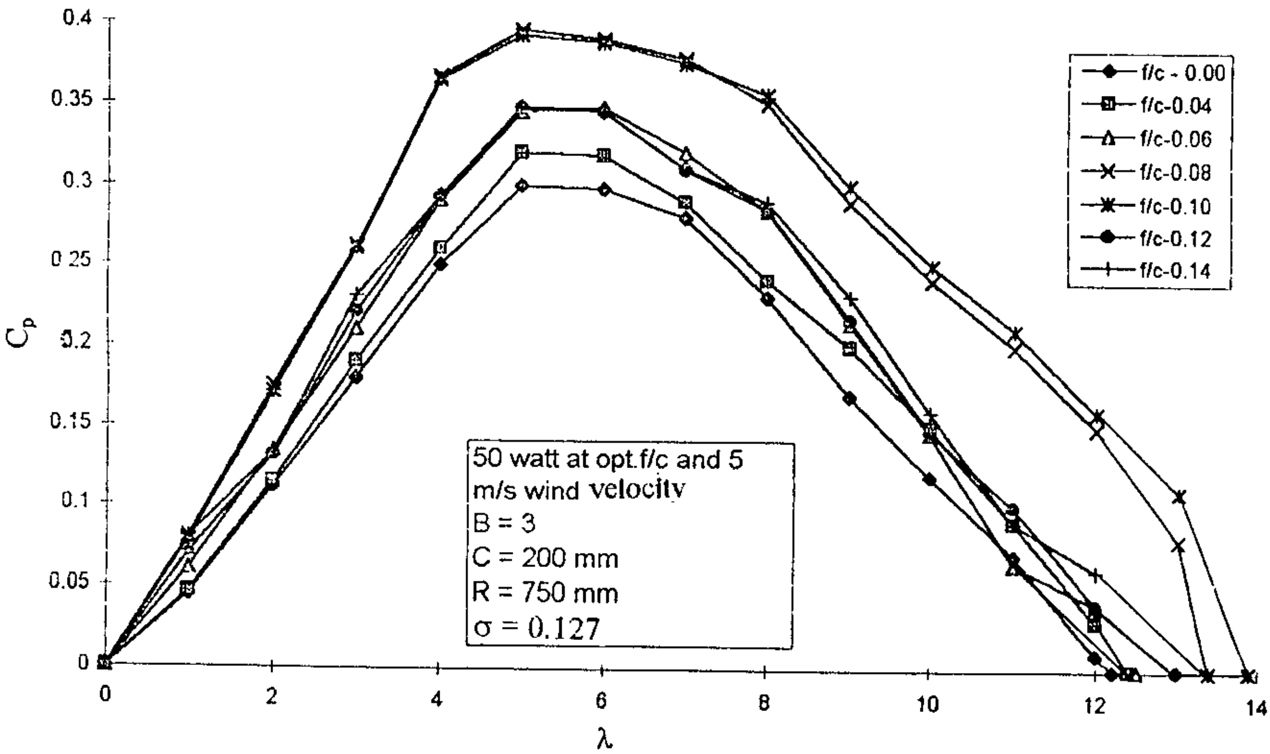


Figure 5.1 : Power coefficient vs tip speed ratio at various camberness ratio with optimum twist angle.

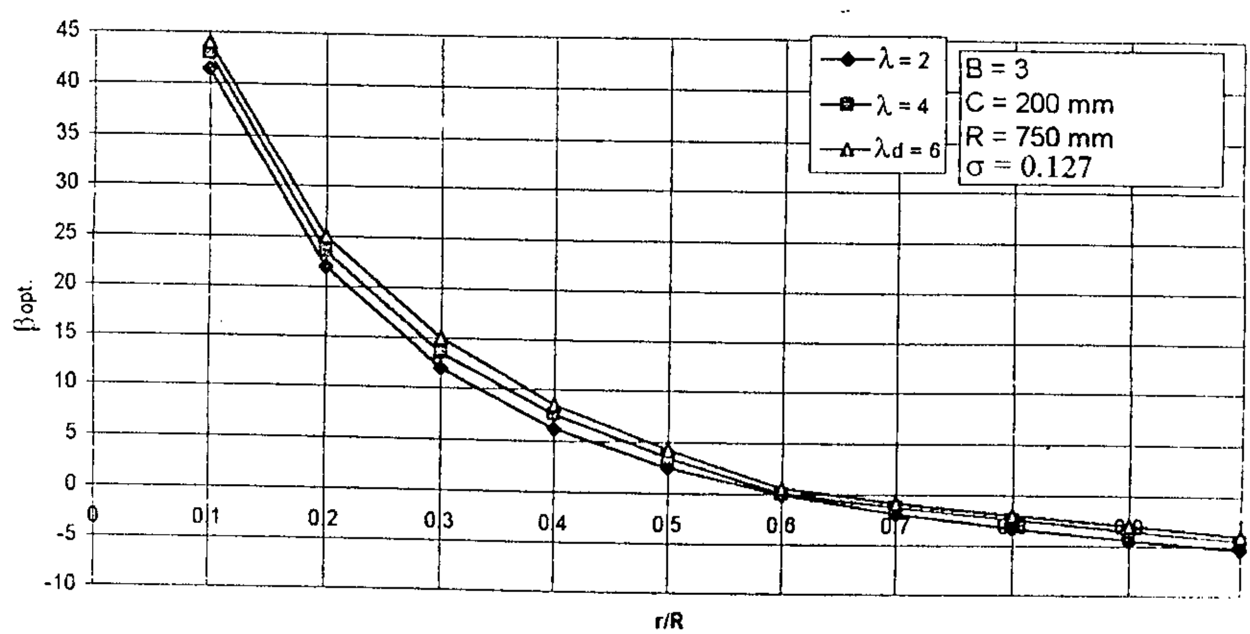


Figure 5.2 : Optimum twist angle vs ratio of radius at various tip speed ratios.

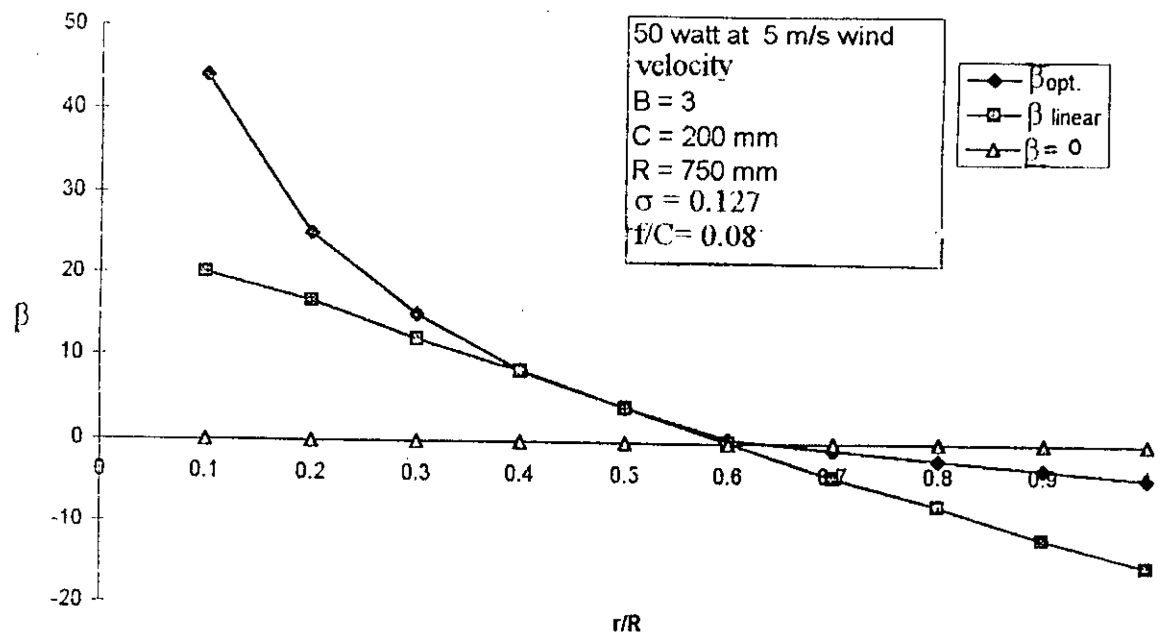


Figure 5.3 : Twist angle vs.ratio of radius at design tip speed ratio.

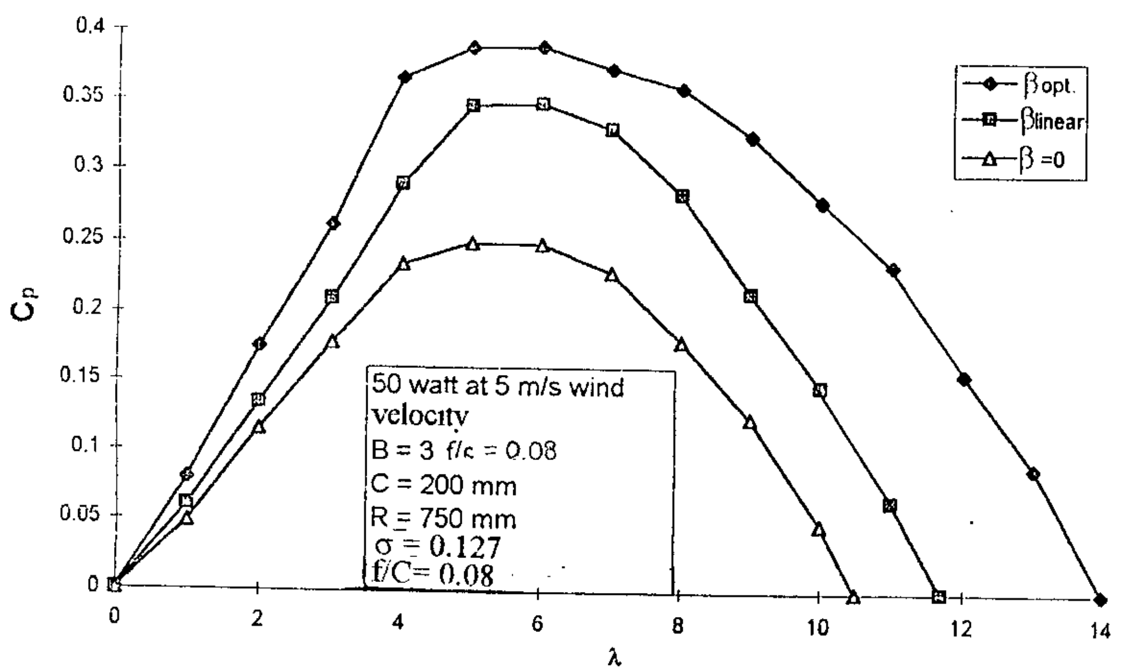


Figure 5.4 : Power coefficient vs.tip speed ratio at various twist angles.

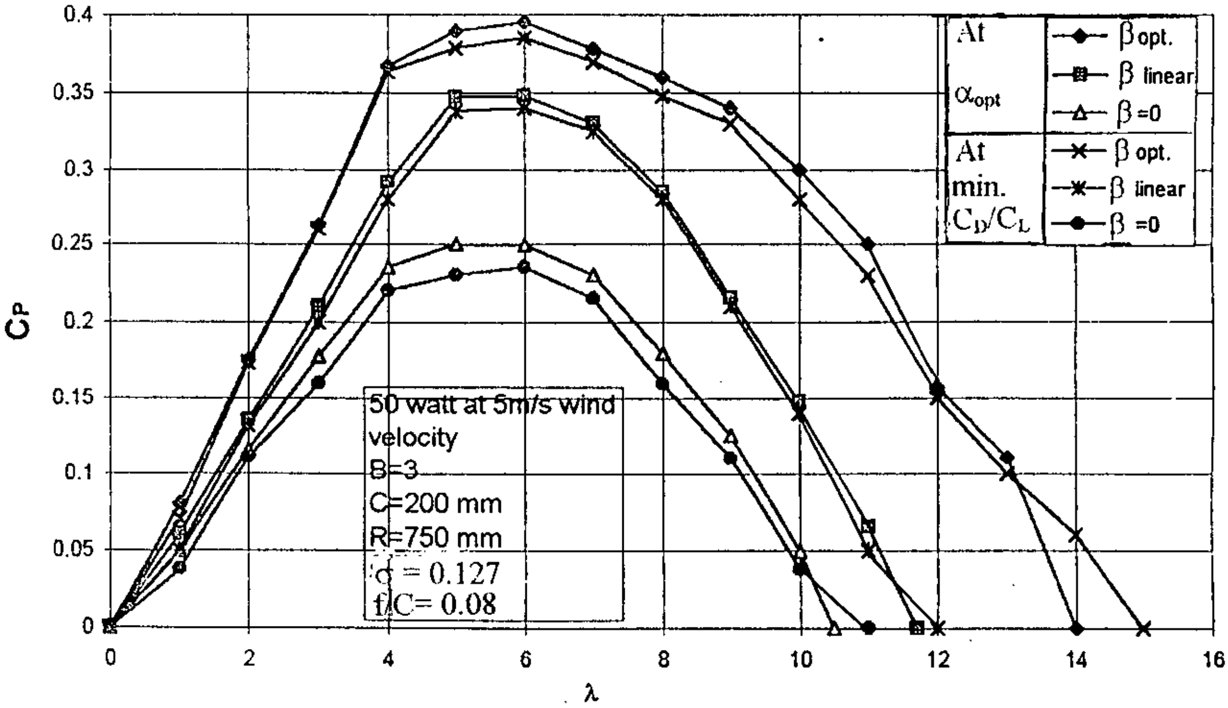


Figure 5.5 : Comparison of power coefficient with tip speed ratio at optimum angle of attack and minimum ratio of drag coefficient to lift coefficient.

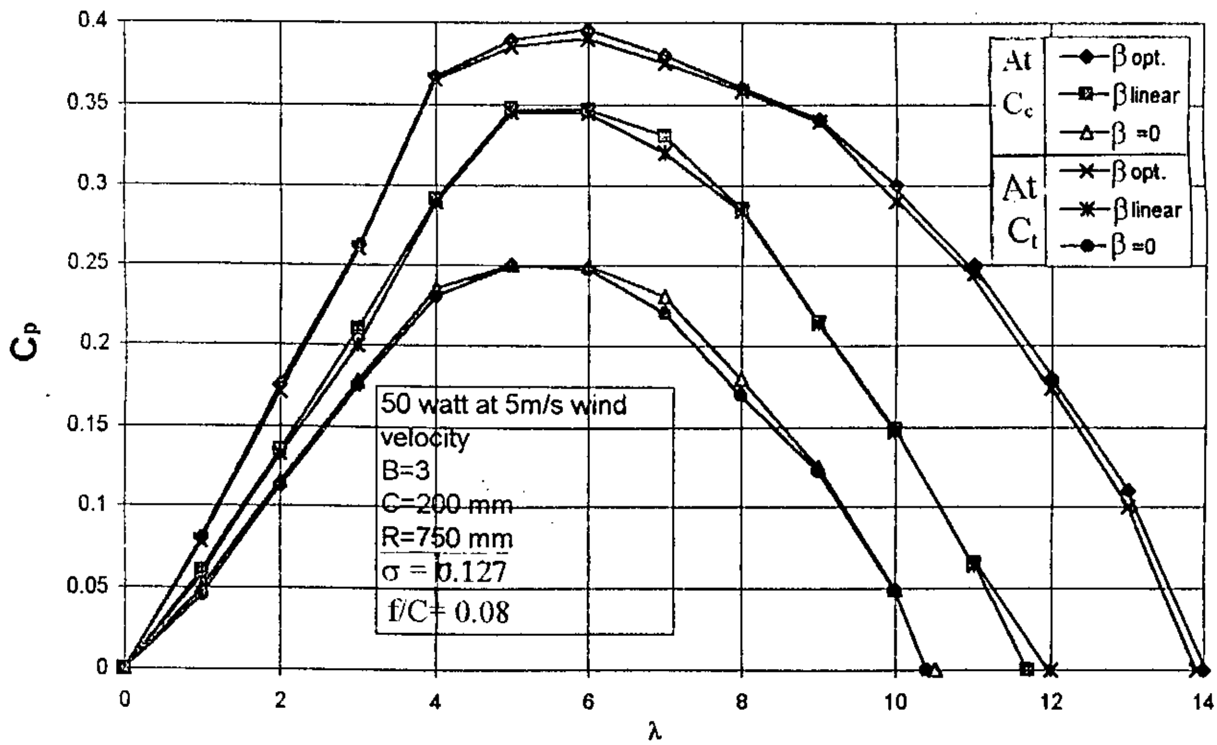


Figure 5.6 : Power coefficient vs tip speed ratio for constant and tapered chord at various twist angles.

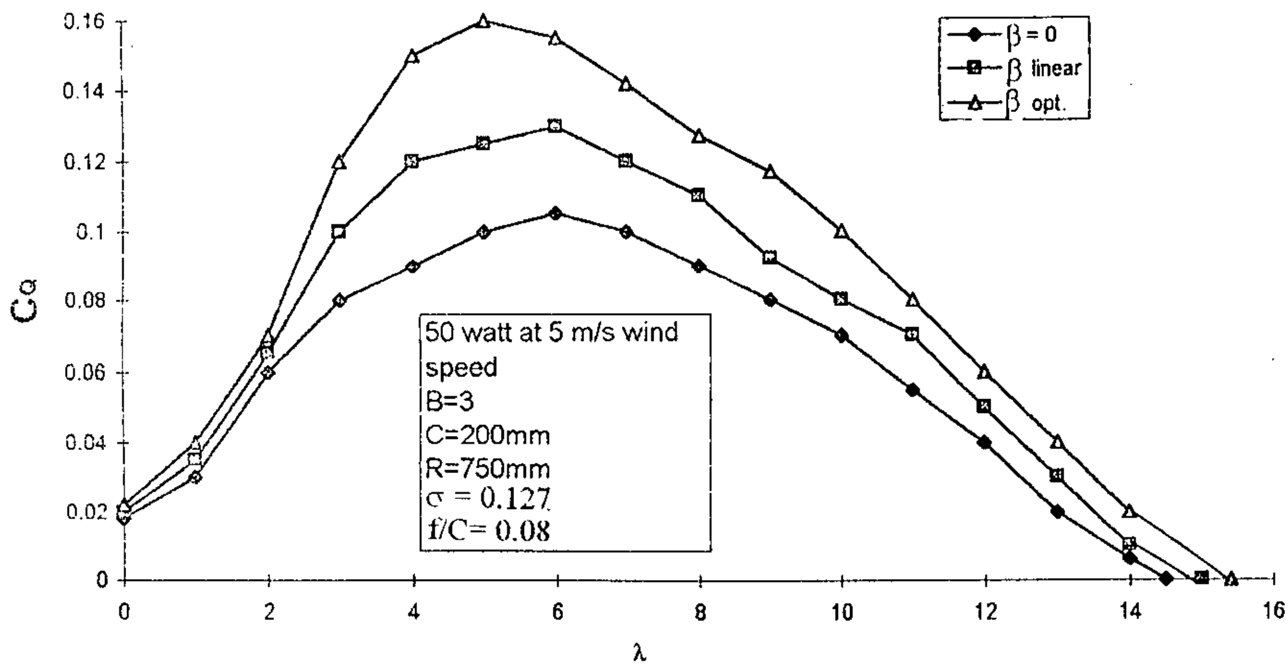


Figure 5.7 : Torque coefficient vs tip speed ratio at various twist angle.

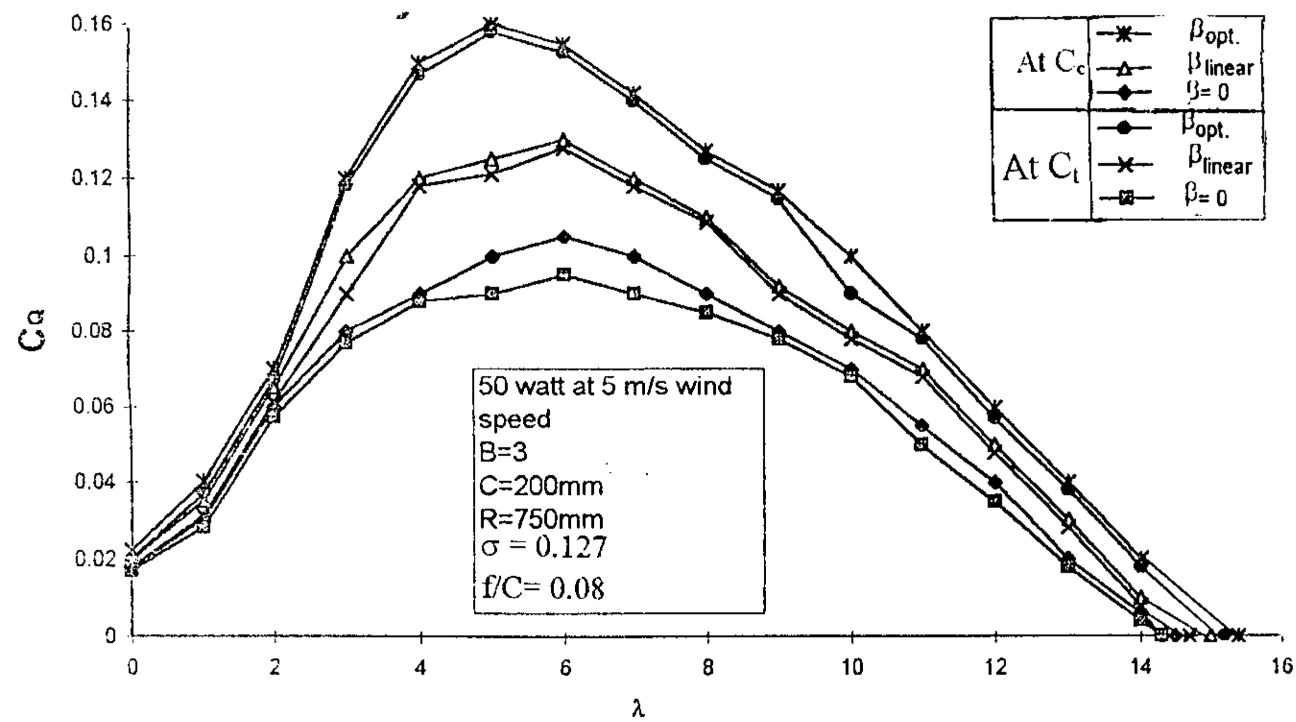


Figure 5.8 : Torque coefficient vs tip speed ratio for constant chord (Cc) and tapered chord (Ct) at various twist angle.

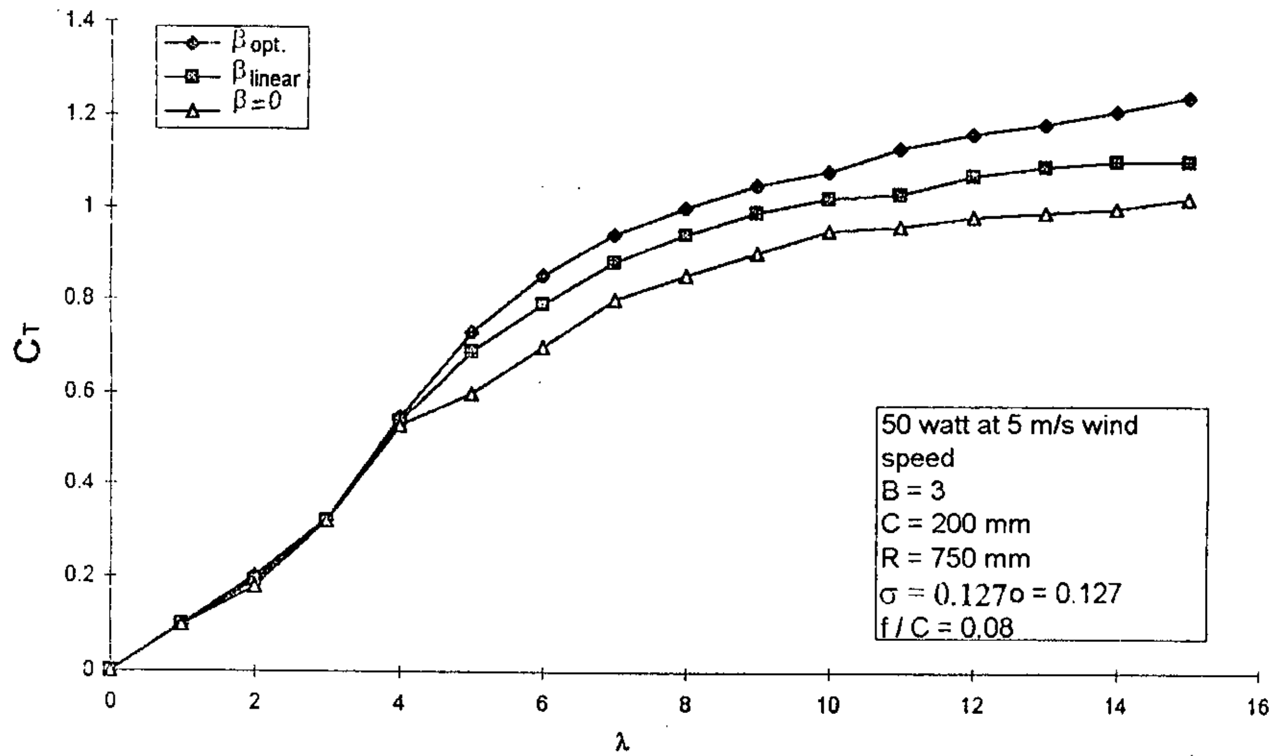


Figure 5.9 : Thrust coefficient vs tip speed ratio at various twist angles.

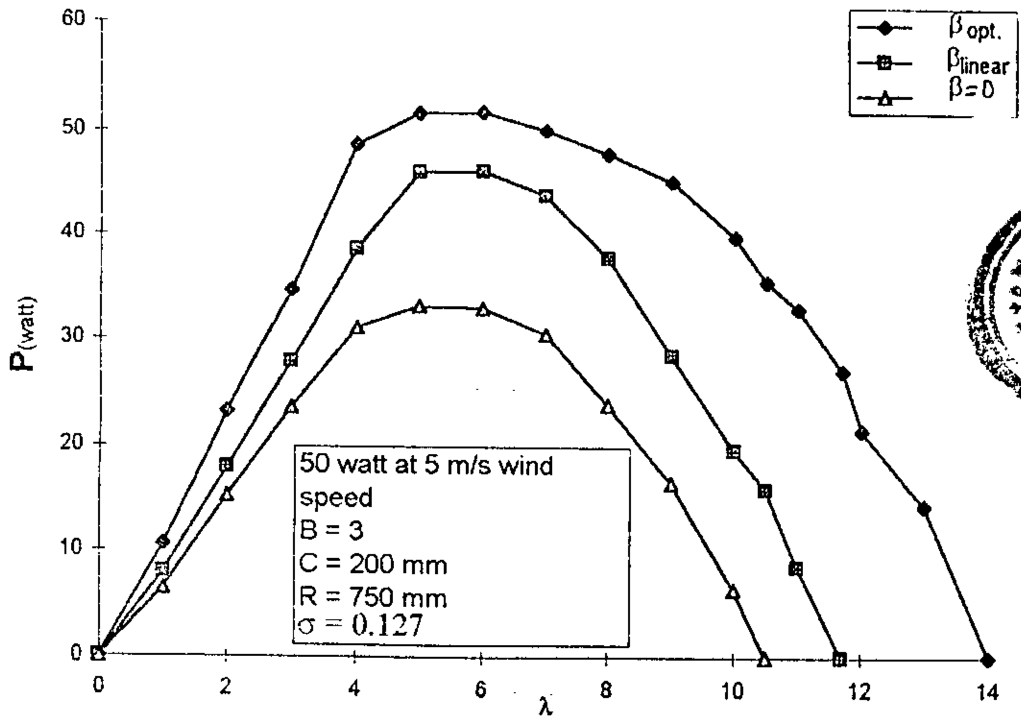


Figure 5.10 : Power vs tip speed ratio at various twist angle.

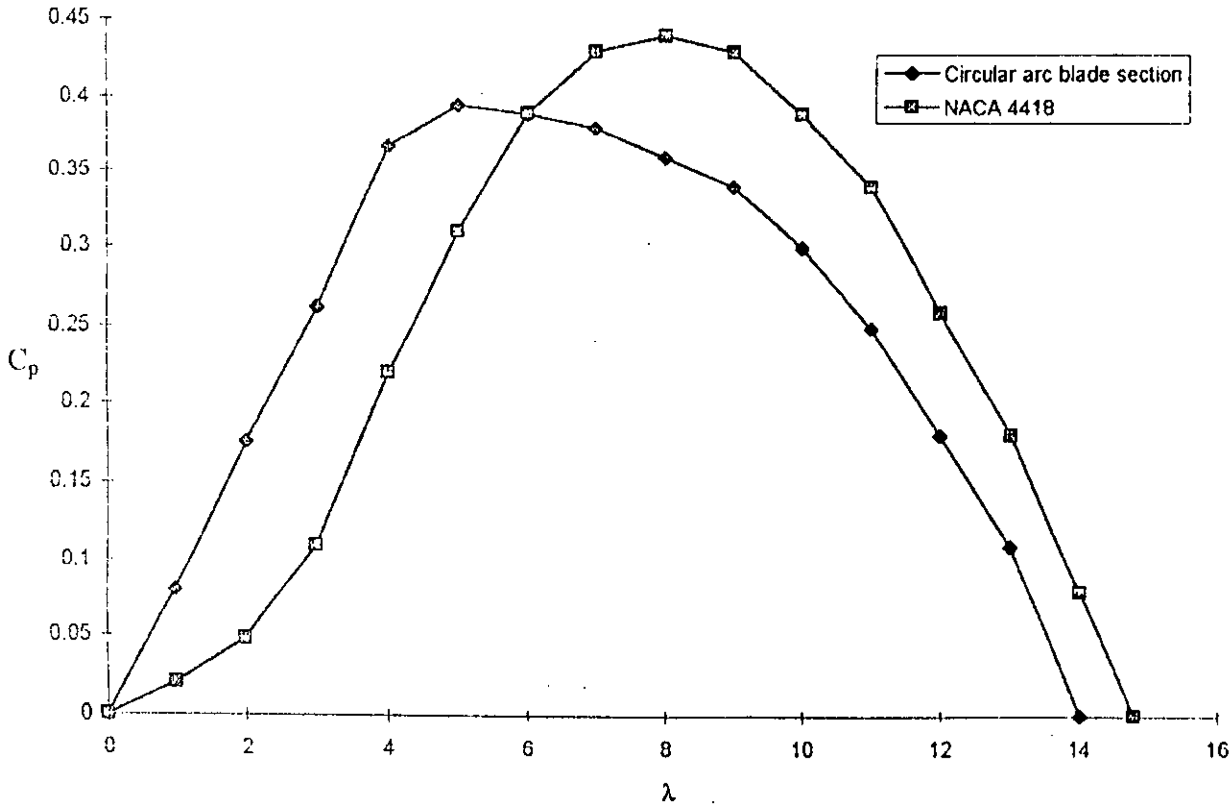


Figure 5.11 : Power coefficient vs. tip speed ratio for NACA 4418 blade and circular arc blade section at optimum twist angle.