STUDY OF THE STRENGTH CHARACTERISTICS OF JUTE-GLASS FIBRE REINFORCED COMPOSITE LAMINATES

BY

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CERTIFICATE OF RESEARCH

This is to certify that the work presented in this thesis is the outcome of the investigations carried out by the candidate under the supervision of Professor M.H. Khan, Vice Chancellor of Bangladesh University of Engineering and Technology, Dhaka.

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CANDIDATE'S DECLARATION

It is hereby declared that neither this thesis nor any part thereof has been submitted or is being concurrently submitted anywhere for the award of any degree or diploma or for publication.

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ABSTRACT

The tensile strength and stiffness of Jute-Glass fibre reinforced composite laminates have been determined experimentally at different volume fractions of jute and glass fibre. The "effective" fibre volume fraction has been defined to characterize a particular composition of the laminate in terms of a single unique fibre volume fraction. This "effective" fibre volume fraction has been found meaningful to interprete the strength and stiffness properties of the laminate. The strength and stiffness of Jute-Glass Reinforced Composite Laminate (JGRCL) have been found to vary linearly with the "effective" fibre volume fraction. The experimental data have been fed into the Bishop's model to determine the empirical strength and stiffness parameters so that predictions can be made using these parameters. The experimental values of stiffness of JGRCL have not been found to agree well with those predicted by the Lamination theory and the Law of Mixtures. The Flexural modulus of the JGRCL has been determined experimentally at different volume fractions of jute and glass fibre. The experimental values did not agree well with those calculated using the Lamination theory.

The experimental data for Jute Reinforced Plastics (JRP) which was performed by Kazim have been used to fit the Bishop's model. The empirical stiffness parameters have been determined so that they can be used for the prediction of stiffness of cross-ply JRP of any fibre volume fraction.

PREFACE

This work was intended since no experimental data were available in literature for Jute-Glass Reinforced Composites although they are in use in some form for the manufacture of chairs, tables, containers, baby cars, etc. in Bangladesh. This research was undertaken to find some experimental data for these composites in order to facilitate design.

Since the mechanical properties of fibre composites are dependent on the fibre volume fraction, fibre orientation and stacking sequence of the lamina, some method of prediction of these properties is to be practised for design purposes because it is impractical to determine these properties experimentally for almost infinite number of combinations of fibre volume fraction, fibre orientation and layer stacking sequence. Several models have been studied and one method has been suggested for this purpose.

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DEDICATION

TO MY PARENTS

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NOTATION

σ_x, σ_y	Normal stresses in X and Y directions respectively
σ_{xy}	Shear stress in X-Y plane
$\varepsilon_x, \varepsilon_y$	Normal strains in X and Y directions respectively
ε_{xy}	Shear strain in X-Y plane
$\sigma_c, \sigma_f, \sigma_m$	Normal Stresses in the composite, fibre and matrix respectively
$\varepsilon_c, \varepsilon_f, \varepsilon_m$	Normal Strains in the composite, fibre and matrix respectively
E_c, E_f, E_m	Modulus of elasticity of the composite, fibre and matrix respectively
V_f, V_m	Volume fractions of fibre and matrix respectively
ν_c, ν_f, ν_m	Poisson's ratio of the composite, fibre and matrix respectively
$\bar{\sigma}_k$	$\begin{vmatrix} \sigma_\alpha \\ \sigma_\beta \\ \sigma_{\alpha\beta} \end{vmatrix}$ Column matrix of normal and shear stresses in k-th layer referred to (α, β) coordinates
C_k	Matrix of k-th layer relating stress and strain in (α, β) coordinates
$\bar{\varepsilon}_k$	$\begin{vmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \varepsilon_{\alpha\beta} \end{vmatrix}$ Column matrix of normal and shear strains of layer k referred to (α, β) coordinates
E_α, E_β	Moduli of elasticity of a layer in α and β directions
$\nu_{\alpha\beta}$	Poisson's ratio for a layer resulting from a load applied in α direction
$\nu_{\beta\alpha}$	Poisson's ratio for a layer resulting from a load applied in β direction

$G_{\alpha\beta}$	Shear stiffness of a layer in (α,β) directions
T_k	Matrix relating strain components in (x,y) coordinates to strain components in (α,β) coordinates of k-th layer
σ_k	$\begin{vmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{vmatrix}$ Column matrix of normal and shear stresses in k-th layer referred to (x,y) coordinates
ε_k	$\begin{vmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{vmatrix}$ Column matrix of normal and shear strains in k-th layer referred to (x,y) coordinates
ϕ_k	Angle of principal axis of orthotropy, α, of k-th layer measured from plate reference axis, x.
N	$\begin{vmatrix}N_x\\ N_y\\ N_{xy}\end{vmatrix}$ Column matrix of normal and shear load intensities referred to (x,y) coordinates
M	$\begin{vmatrix}M_x\\ M_y\\ M_{xy}\end{vmatrix}$ Column matrix of bending moment intensities acting in xz and yz planes and twisting moment intensity acting about x and axes respectively
e	$\begin{vmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{vmatrix}$ Column matrix of midplate normal and shear strains referred to (x,y) coordinates
K	$\begin{vmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ \frac{\partial^2 w}{\partial x \partial y}\end{vmatrix}$ Column matrix of curvature components of the plate
I_k	Second moment of area per unit width of k-th layer
θ	Angle defining the orientation of fibre from x-axis
$f(\theta)$	Fibre distribution function
G_c	Shear modulus of the composite
E_l	Modulus of elasticity of lateral fibre

ε_l	Strain in most strained lateral fibre
$\varepsilon_{l_1}, \varepsilon_{l_2}$	Strain in first and second lateral fibres
μ	Empirical constant allowing for contiguity effects
ψ	Half the included angle between the lateral fibres
$\alpha_1, \alpha_2, \alpha_3$; $\beta_1, \beta_2, \beta_3$; $\gamma_1, \gamma_2, \gamma_3$	Coefficients in stress/strain equations
V_v	Volume fraction of voids in the composite
ϑ	Poisson's ratio of the resin matrix in Bishop's Model
λ	Empirical constant applied to shear strength of the composite
ϕ_s	Angle between s-th band of fibres and x-axis
p_s	Fraction of total fibres lying in band at angle ϕ_s to x-axis
V_{feff}	Effective fibre volume fraction of jute-glass laminate
W	Weight
ρ	Density
S_{ij}	Material compliance
σ_{ex}	Experimental failure strength of jute-glass laminate in tension
ε_{ex}	Experimental strain at failure in tension of jute-glass laminate
E_{ex}	Experimental Young's modulus of jute-glass laminate
	Standard error in fitting Bishop's model in least square method
R	$(3 + \text{Cos}4\psi) - S(1 - \text{Cos}4\psi)$
S	$\varepsilon_y/\varepsilon_x$

Subscripts

f	Fibre
m	Matrix
l	Lateral fibre
l_1	First lateral fibre
l_2	Second lateral fibre
s	Ordinal integer
T	Total
v	Voids
x	In X-direction
y	In Y-direction
xy	In X-Y plane
ex	Experimental
k	Layer identification number
j	Jute
g	Glass
eff	Effective
c	Composite

CHAPTER 1

INTRODUCTION

A composite material is defined as a material containing two or more distinct phases on a macroscopic scale. Composite materials are commonly classified into three types, namely 1. Fibrous composite, 2. Laminated composite, and 3. Particulate composite. Fibrous composites consist of fibres in a matrix. Laminated composites consist of layers of various materials. And particulate composites are composed of particles in a matrix.

Composite materials have a long history of usage. Their beginnings are unknown, but all recorded history contains references to some form of composite material like plywood, concrete, etc.. Fibre reinforced resin composites are the most recent addition to the composite family. These composites have high strength-to-weight and stiffness-to-weight ratios. These facts facilitated their use in light weight structures such as aircraft and space vehicles. The use of fibrous composites is expanding rapidly in other engineering applications because of superior physical and electrical properties over metals. These applications include pressure vessel, bearing, boat hull, electrical insulators, casings, etc..

In most engineering applications, fibrous composites are used in the form of laminates which consist of layers of various materials. These individual layers are constructed of unidirectional continuous or discontinuous fibres or woven cloths. In conventional design, engineers are supposed to choose a material according to the design requirement. Contrary to that they can now prescribe a particular composition of a composite which will give the desired strength and stiffness properties required by design. This has become possible because stiffness and strength of a fibre composite can be tailored by the use of multilayered laminates with appropriate fibre orientation of each layer.

1.1 FIBRES AND RESINS

Fibres

It is a well-known fact that long fibres in various forms are inherently much stronger and stiffer than the same material in bulk form. The paradox of a fibre having superior properties to the bulk form is due to the more perfect structure of a fibre. The crystals are aligned in the fibre along the fibre axis, Moreover, there are fewer internal defects in fibres than in bulk material.A fibre is characterized geometrically by its very high length-to-diameter ratio and its near crystal-sized diameter.

Glass fibre is a non-organic fibre made out of specific type of glasses. Three commercial type of glass fibres are used in practice. They are A-glass, E-glass, and S-glass fibres. A glass is made from soda-lime glass and is cheap. E-glass is made from calcium alumina borosilicate glass and has higher strength and better electrical insulating properties than A-glass. S-glass consists of silica, magnesia, and alumina combined in certain proportions. S-glass is about 40 percent stronger and more temperature resistant than E-glass.

Jute fibre is a naturally grown organic fibre. Various grades of jute fibres are produced in Bangladesh. Of them, Tosa jute has been found to show good mechanical properties. Jute fibres are not very long and its diameter is not constant.

Fibres are usually obtained in various forms such as woven cloth, non-woven mat, rovings, etc.. Woven cloth may again be classified according to the type of weave such as plain weave, satin weave, etc..

Resin

Resin acts as a binding material for the fibres in a fibre composite. A thermosetting resin has been found to be a suitable material. But until the advent of polyester type resin which can be cured at atmospheric pressure, fibre reinforced composites were not a reality. Presently two types of resin are in use 1. Polyester, 2. Epoxide resin. Polyester resin is more widely used than epoxide resin because it is cheap and has less curing problem than epoxide resin.

1.2 FIBRE REINFORCED LAMINATES

Fibre reinforced composites have certain superior physical, mechanical and electrical properties over metals which enhances the use of fibre composites in numerous engineering applications. Fibre reinforced laminates are usually tested and analysed for prediction of composite properties for their simpler geometry. The results obtained from the analytical and experimental study of laminates can then be used for the design of any other structural components. There are different methods of manufacturing fibre reinforced laminates. Usually fibres or woven fibre mats are wetted by liquid resin and then cured for several hours with or without pressure. During the curing period, the resin undergoes copolymerization resulting in crosslinking of polymer chains and thereby gets solidified. To make objects of any other shape, fibres are cut according to the shape of the mould and placed in the mould. Resin is applied. After curing the object is taken out of the mould.

1.3 MOULDING TECHNIQUES

Currently four different methods of manufacturing fibre reinforced laminates are in practice. A brief description of each method is given below.

a. HAND LAY-UP : The laminate is made by consolidating layers of brush or spray applied polyester resin and fibre reinforcement by hand in an open mould. The mould itself is made of the same composite. The process is particularly suitable for making large components and small number of mouldings.

b. COLD PRESS MOULDING : Laminates are made by consolidating resin and reinforcement in matching tools, usually made of concrete with a polyester of epoxide resin facing . Simple clamping rigs, combined with the weight of the moulds themselves, provide the required pressure. This is particularly suitable for making large mouldings required in moderate quantities.

c. HOT PRESS MOULDING : Mouldings are made from resin and fibre "preforms" in heated steel moulds mounted between the platens of a hydraulic press. The preforms are made by depositing the fibre on a male mould and coating them with a binder to retain their shape. The process is suitable for long run of small components. Mouldings made by this process have a higher fibre to resin content than mouldings made by hand lay up or by cold press moulding.

d. FILAMENT WINDING : This process consists of winding continuous filament or roving on a mandrel after they have been passed through a resin bath. This method is generally used for making cylindrical pipes and tanks.

1.4 OBJECTIVES OF PRESENT INVESTIGATION

The objectives of the present investigation are mentioned below.

1. Experimental determination of tensile strength and stiffness, and flexural modulus of Jute-Glass fibre reinforced composite laminates at different volume fractions of jute and glass fibre.

2. Determination of the strength and stiffness of the composite laminate using the Lamination theory and Law of Mixtures for different volume fractions of jute and glass fibre.

3. To compare the experimental values of tensile strength and stiffness, and flexural modulus with obtained from the aforementioned theories for different volume fractions of jute and glass fibre.

4. To compare the values of strength and stiffness of jute fibre, glass fibre and glass-jute fibre reinforced composite laminates.

5. To study of the effect of number of fibre layers on the strength of the composite laminate.

6. Determination of the empirical parameters in the Bishop's analytical/empirical model using the experimental data so that they can be used directly for the prediction of strength and stiffness properties for design purposes.

7. Determination of the stiffness parameters of Bishop's model using the experimental results [22] for jute reinforced composite so that they can be used for design purposes.

CHAPTER 2

LITERATURE REVIEW

A number of theoretical and analytical models have been suggested for the analysis of fibre composite materials. Experimental works are also reported for different fibre and resin compositions. Attempts have been made to evolve a suitable mathematical model to predict the strength and stiffness of the composite for almost any combination of fibre and matrix variables. The ultimate strength and elastic moduli of a fibre composite are basically dependent upon the stress-strain relationship of its components. Other factors also influence these properties such as fibre volume fraction, physical properties, temperature, etc..

2.1 COMPOSITE STIFFNESS

Cox [8] first developed the netting analysis for fibrous felts and papers in 1952. He only considered the fibres and ignored the effect of matrix. Although this model has certain drawbacks such as it predicts zero strength perpendicular to fibre direction for unidirectional composites, it was found useful for the analysis and design of a large class of spun or filament-wound pressure vessels [34].

The limitations of the netting analysis were recognized by Gordon [15] and Arridge [2] who proposed several modifications to account for the fibre-matrix interaction. They included the familiar Law of Mixtures in their models. Few authors, after Cox and Arridge, considered composite with a random array of fibres, and most of the later work has been concerned with the analysis of unidirectional fibre-reinforced composites. Shaffer [33] derived the elastic constants for a unidirectional composite based upon the Law of

Mixtures, but also including the effects of matrix yielding. In this analysis, the matrix was assumed to be ideally elastic-plastic. The results were strictly applicable only for elastic strains. But this analysis was inappropriate for brittle matrices.

Ekvall [10] suggested a mechanics of materials type model in 1961. This analysis attempted to take into account the actual cross-sectional shape of the reinforcing fibres. Specifically square, rectangular and round fibres were considered. This model attempted to find the material properties necessary for an elastic analysis of a two-dimensionally orthotropic composite structure. One drawback of this model is the assumption of regularity in geometry.

In 1965, Whitney and Riley [39] proposed an analysis based on the theory of elasticity. This analysis did not use minimum energy theorems to obtain the property bounds. This is also a unidirectionally fibre reinforced composite model. The theoretical predictions compared well both for the transverse and longitudinal moduli with experimental values. However, the predicted shear modulus was significantly different from experimental values.

In 1966, Tsai et al [35] presented a finite-difference model based on the theory of elasticity. The results were more acceptable as compared to Whitney-Riley model. But the complexity of this model was higher. This model allowed the longitudinal modulus and major poisson's ratio to be predicted from the law of mixture formula and concentrated on better prediction for the transverse and shear modulus.

Hashin, Dow and Rosen [9,17] proposed another model based on the variational techniques. It was found that the longitudinal stiffness and the major poisson's ratio were linear functions of fibre volume fractions, thus verifying the Law of Mixtures for these moduli.

In 1966, Bishop [4] proposed an improved method for predicting the mechanical properties of fibre composite materials. This model was primarily based on netting analysis, but the indirect contributions of the fibre had been allowed for mathematically by introducing two hypothetical "Lateral Fibres". The model has been used to analyse glass-fibre/epoxide-resin laminate, and to predict the behaviour of a glass-fibre pressure vessel under simultaneous bending and internal pressure. Data predicted by the model did not differ from observed data by more than 17 percent. The stress-strain behaviour of the pressure vessel was predicted satisfactorily.

The classical lamination theory for multilayer laminates was derived by Ekvall [11] from the classical work by Pister and Dong [28] and Reissner and Stavsky [29]. This theory embodies a collection of stress and deformation hypothesis. Laminate stiffness predicted by this theory was found to correlate well with those obtained from experiment by Tsai [36] and Azzi and Tsai [3]. A generalized thermoelastic analysis of multilayer composite laminates was presented by Tsai [36,37]. It was assumed that each constituent layer was homogeneous and anisotropic and is in a state of two-dimensional stress. Tsai presented some experimental data to verify the analytical results. The test specimen layers were made up of unidirectional glass fibres preimpregnated with an epoxy resin. The laminated specimen consisted of two or three layers. The test results were obtained by measuring the surface strains of the loaded specimens. The measured components of the [A], [B] and [D] matrices agreed reasonably well with the theoretically predicted values for both cross-ply and angle-ply laminates.

Recently, finite-element techniques have been gaining popularity for the determination of the elastic moduli. The first application of this technique to composite problems was apparently made by Foye [13] to find elastic moduli of unidirectional composites. The results obtained using this technique was found to agree well with experimental values.

For prediction of stiffnesses of composites reinforced with anisotropic filaments, Whitney [40] proposed to utilize the appropriate properties of the fibre when calculating the associated directional properties of the composite. The results of this "Whitney Correction Method" were compared with experimental data. It was found that fibre anisotropy characteristics have significant effect on the composite elastic moduli. The Whitney correction was quite approximate **because it** assumed that only transverse component properties would affect the transverse properties of overall composite. This assumption was eliminated by Chen and Cheng [7]. They utilized a procedure similar to the one for isotropic fibres for the analysis of anisotropic fibres. It was seen that the theory and experiment were in good agreement for both shear and transverse moduli.

2.2 COMPOSITE STRENGTH

Boue [6] established the effect of fibre to matrix volume ratio on the failure mode of fibre-reinforced composite in 1962. He found that, for specimens with low fibre volume fraction, the failure commences by transverse resin cracking followed by fibre fracture and fibre pullout from both sides of the resin crack. For the high fibre volume fraction specimens, random fibre failure occured below 50 percent of ultimate load. The failure of the composite occured by an accumulation of random fractures.

Jech, et al [20] considered a Law of Mixtures type model to predict the strength of fibre composite in which during loading, a uniform state of strain exists up to the moment of fracture. When this uniform strain reaches the failure strain of the fibres, failure of the composite occurs. But this model predicted too large a composite tensile strength as compared to experimental data.

Kelly and Davies [23] proposed the maximum stress theory of failure for unidirectionally reinforced fibre composites under biaxial loading. According to this theory, failure occurs if one of the three ultimate strengths ($\sigma_x, \sigma_y, \sigma_{xy}$) is reached. Tsai [38] proposed the maximum strain theory of failure. Here it was assumed that associated with three strain components ($\epsilon_x, \epsilon_y, \epsilon_{xy}$), there exist ultimate strains. Hill [18] proposed the maximum work theory based upon a yield criterion for anisotropic materials. This theory assumes that the yield stress and ultimate strength are identical. Tsai [38] compared the uniaxial strength predicted by maximum stress, maximum strain and maximum work theories with test data obtained from uniaxial tensile and compressive test on a unidirectional E-glass composite. It was found that the maximum work theory offers better agreement with experimental data than do the other theories.

Zweben [42] proposed a noncumulative fracture model for the tensile strength of fibre composites. According to this model, failure of composite occurs after the first fibre fails or at most a few isolated fibre fail. This model finds a correlation between theoretical strength of the weakest fibre and the observed composite failure loads. Fore some fibre composites, this model gives reasonable agreement between theory and experiment.

Gucer and Gurland [16] first proposed the so-called "Cumulative Weakening Model" to account for the failure mode of brittle composites. In this model, the composite is divided into a series of layers of unit thickness. Thus the composite becomes a chain of bundles. This model did not agree well with experimental results.

Fariborg, et al [12] investigated the tensile behaviour of intraply hybrid composites. They modified the basic chain of bundles probability model. The existence of the hybrid effect for strain was shown along with its sensitivity of volume ratio and dispersion of the type of fibres.

Kazim [22] performed experiments for the determination of different mechanical properties of Jute fibre reinforced composite laminates at varying fibre volume fractions in 1986. He used the netting analysis for prediction of laminate properties. But this analysis was not found effective for cross-ply for Jute reinforced plastics.

Present work has been undertaken by the author to study the strength characteristics of Jute and Glass fibre reinforced composite and to suggest a suitable model for the prediction of strength and stiffness of of both Jute and Glass-Jute composite for any combination of fibre volume fraction and fibre orientation.

CHAPTER 3

MATHEMATICAL MODEL ON THE PREDICTION OF STIFFNESS AND STRENGTH PROPERTIES OF FIBRE COMPOSITES

There are numerous mathematical models for the prediction of strength and elastic properties of fibre composite materials. Some of them have been analysed from micromechanics point of view and others have been analysed from macromechanics point of view.

The micromechanics approach begins with a study of individual constituent materials. The aim here is to find how the behaviour of the composite depends on the composition, size, volume fraction, distribution and orientation of the constituents. General relationships of this sort will then provide prediction of the behaviour of a composite resulting from an arbitrary combination of these constituents.

In macromechanics approach, the composite is viewed as if it were a single material. if the composite is approximately isotropic, then its average properties may be substituted into the usual design formulas. A more sophisticated approach is required for anisotropic materials. Empirical descriptions are necessary whenever there is insufficient data to sustain an analytical model.

Both these approaches aim at predicting "effective" material properties that may be utilized in conventional design formulas. In practice the two approaches complement each other.

A few existing models, which are to be studied, are discussed in the following sections.

3.1 LAW OF MIXTURES [15,2]

A composite made up of a cylindrical fibre with Young's modulus E_f surrounded by a tubular matrix with Young's modulus E_m may be considered. Assuming equal strain in the components under a uniform axial load, it may be written that

$$\varepsilon_f = \varepsilon_m = \varepsilon_c, \qquad \sigma_f V_f + \sigma_m (1-V_f) = \sigma_c \qquad 3.1.1$$

Combining this equation with Hooke's law, the following expression can be obtained.

$$E_c = V_f E_f + (1-V_f) E_m \qquad 3.1.2$$

This is the well known Law of Mixtures formula. This formula has been found to be useful for the estimation of properties in the direction of fibre of unidirectional fibre composites.

3.2 LAMINATION THEORY [11]

The general formulae given in this section apply to flat plates built up from a number of thin layers having arbitrary thickness and orientation of their axes of orthotropy. The properties of a plate are referred to a set of reference axes, o(x,y,z), and the properties of an individual layer k, to the principal axes of orthotropy of the layer, $o(\alpha,\beta)$. The angle between x and α is termed as ϕ_k (Fig. 3.1). In plates which behave orthotropically, the axes o(x,y) are chosen as the principal axes of orthotropy. It is convenient to take the central plane as the plane of reference because this minimizes the coupling terms between in-plane loading and bending. This is entirely appropriate for balanced arrangements where layers k and n-(k-1) are identical.

Stress-strain relationship for a single layer

The elastic properties of an orthotropic layer k, can be defined using matrix notations as shown below (Appendix B).

$$\bar{\sigma}_k = C_k \bar{\varepsilon}_k \qquad 3.2.1$$

where $C_k = \begin{vmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{vmatrix}_k$

in which, $C_{11}=(E_\alpha/\rho)_k$, $C_{22}=(E_\beta/\rho)_k$, $C_{66}=G_{\alpha\beta k}$

$C_{12}=(\nu_{\alpha\beta} E_\alpha/\rho)_k = C_{21}=(\nu_{\beta\alpha} E_\beta/\rho)_k$

The stresses and strains are referred to the layer co-ordinate axes $o(\alpha, \beta)$ and can be related to those for the plate reference axes $o(x,y)$ by the standard formulation for the transformation of stress and strain components.

Thus, $\bar{\epsilon}_k = T_k \epsilon_k$ and $\sigma_k = T_k' \bar{\sigma}_k$ 3.2.2

where $T_k = \begin{vmatrix} \cos^2\phi_k & \sin^2\phi_k & 1/2\sin 2\phi_k \\ \sin^2\phi_k & \cos^2\phi_k & -1/2\sin 2\phi_k \\ -\sin 2\phi_k & \sin 2\phi_k & \cos 2\phi_k \end{vmatrix}$

For the reference axes the stress-strain relationship of layer k may be deduced from equation 3.2.1 and 3.2.2 as follows:

$\sigma_k = T_k' \bar{\sigma}_k = T_k' C_k \bar{\epsilon}_k = T_k' C_k T_k \epsilon_k$ 3.2.3

Thus $\sigma_k = b_k \epsilon_k$ where $b_k = T_k' C_k T_k$

Stiffnesses of an assembly of layers:

In a single plate consisting of an assembly of n layers, the in-plane and flexural load deformation relationship for the plate can be written as follows [29] -

$$\begin{vmatrix} N \\ M \end{vmatrix} = \begin{vmatrix} A & B \\ B & D \end{vmatrix} \begin{vmatrix} e \\ k \end{vmatrix} \qquad 3.2.4$$

The plate stiffnesses A,B and D are given by-

$$A = \sum_{k=1}^{k=n} t_k b_k \qquad B = \sum_{k=1}^{k=n} t_k z_k b_k$$

and $$D = \sum_{k=1}^{k=n} (t_k z_k^2 + I_k) b_k \qquad 3.2.5$$

The thicknesses of the individual layers of practical laminates are usually such that I_k is negligibly small in relation to $t_k z_k^2$ and D therefore usually reduces to -

$$D = \sum_{k=1}^{k=n} t_k z_k^2 b_k \qquad 3.2.6$$

From equation 3.2.5 , it can be seen that for balanced laminates B is zero and equation 3.2.4 reduces to two uncoupled equations -

$$N = Ae \quad \text{and} \quad M = Dk \qquad 3.2.7$$

Estimation of layer elastic properties :

To estimate elastic properties of a lamina , the following basic assumptions are made.

1. The lamina is assumed to be macroscopically homogeneous, linearly elastic, generally orthotropic, or transversely orthotropic and initially stress-free.
2. The fibres are assumed to be linearly elastic, homogeneous, regularly spaced and perfectly aligned.
3. The matrix is assumed to be isotropic, linearly elastic and homogeneous.
4. The fibre and matrix are assumed to be free of voids and the bonding between the fibres and matrix is assumed to be perfect.

The fibre volume fraction V_f, and the matrix volume fraction V_m, in a lamina without voids, satisfy the relationship $V_f + V_m = 1$.

The layer modulus of elasticity in the α-direction is given by-

$$E_k = E_f V_f + E_m V_m$$

where E_f and E_m are the corresponding fibre and matrix elastic moduli respectively.

The layer Poisson's ratio resulting from a load applied in the α-direction is given by -

$$\nu_{\alpha\beta} = \nu_f V_f + \nu_m V_m$$

where ν_f and ν_m are the corresponding fibre and matrix Poisson's ratios respectively.

The Poisson's ratio for a lamina resulting from a load in the β-direction is given by -

$$\nu_{\beta\alpha} = \nu_{\alpha\beta}(E_\beta / E_\alpha)$$

3.3 NETTING ANALYSIS [8]

The mathematical analysis for this model is based upon the following assumptions :

1. The effect of the binder phase (matrix) on the stiffness of the laminate is neglected.
2. Fibres are long, thin and straight.
3. Load applied only at the fibre ends (no interfacial forces), and.
4. No bending stiffness for the fibres.

A planar mat of fibres is considered which is subjected to tensile strains ε_x and ε_y in two directions at right angles to each other and to a shear strain ε_{xy} between these directions. The strain of a fibre inclined at an arbitrary angle θ to the direction x (Fig. 3.2) is given by [25] -

$$\varepsilon_x \mathrm{Cos}^2\theta + \varepsilon_y \mathrm{Sin}^2\theta + \varepsilon_{xy}\mathrm{Sin}\theta\mathrm{Cos}\theta \qquad 3.3.1$$

The stress in the fibre is assumed to be proportional to this strain. If the load in the fibre is L, then the contribution of the fibre to the loads in the directions x and y will be LCosθ and LSinθ, respectively..

Let f(θ) be the distribution function, i.e. the fraction of fibres inclined at angle θ to the direction x in the unit width of transverse to their direction such that

$$\int_0^\pi f(\theta)\, d\theta = 1 \qquad 3.3.2$$

The fractions of the fibres intersecting lines of unit width perpendicular to the directions x and y are then $f(\theta)\cos\theta$ and $f(\theta)\sin\theta$, respectively. Therefore, the loads per unit width of edge, i.e. σ_x (along x, on the edge perpendicular to x), σ_y (along y, on the edge perpendicular to y), and σ_{xy} (along x, on the edge perpendicular to y or vice versa) are-

$$\sigma_x = E_f V_f \int_0^{\pi} (\varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \varepsilon_{xy} \cos\theta\sin\theta)\cos^2\theta f(\theta)d\theta$$

$$\sigma_y = E_f V_f \int_0^{\pi} (\varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \varepsilon_{xy} \cos\theta\sin\theta)\sin^2\theta f(\theta)d\theta \qquad 3.3.3$$

$$\sigma_{xy} = E_f V_f \int_0^{\pi} (\varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \varepsilon_{xy} \cos\theta\sin\theta)\sin\theta\cos\theta f(\theta)d\theta$$

where E_f is the fibre modulus and V_f is the fibre volume fraction. Alternately, the above equations can be rewritten as -

$$\sigma_x = C_{11}\varepsilon_x + C_{12}\varepsilon_y + C_{16}\varepsilon_{xy}$$

$$\sigma_y = C_{12}\varepsilon_x + C_{22}\varepsilon_y + C_{26}\varepsilon_{xy} \qquad 3.3.4$$

$$\sigma_{xy} = C_{16}\varepsilon_x + C_{26}\varepsilon_y + C_{66}\varepsilon_{xy}$$

where,

$$C_{11} = E_f V_f \int_0^{\pi} \cos^4\theta f(\theta)d\theta$$

$$C_{16} = E_f V_f \int_0^{\pi} \cos^3\theta\sin\theta f(\theta)d\theta$$

$$C_{22} = E_f V_f \int_0^{\pi} \sin^4\theta f(\theta)d\theta \qquad 3.3.5$$

$$C_{26} = E_f V_f \int_0^{\pi} \sin^3\theta\cos\theta f(\theta)d\theta$$

$$C_{12} = C_{66} = E_f V_f \int_0^{\pi} \cos^2\theta\sin^2\theta f(\theta)d\theta$$

For the isotropic, two-dimensional case with a random distribution of fibres, the distribution function $f(0)$ used is -

$$f(\theta) = 1/\pi \,, \qquad 0 \leq \theta \leq \pi$$

Thus the elastic constants are found to be -

$C_{11} = C_{22} = 3/8 E_f V_f$, $\quad C_{12} = 1/8 E_f V_f$ and $C_{16} = C_{26} = 0$

These yield for the composite, -

Young's Modulus, $E_c = C_{11} - (C_{12}/C_{22})^2$

or $E_c = 1/3\, E_f V_f$

Shear Modulus, $G_c = C_{12} = 1/8 E_f V_f$ 3.3.6

and Poisson's ratio, $\nu_c = (E_c/2G_c) - 1 = 1/3$.

3.4 BISHOP'S MODEL FOR THE PREDICTION OF MECHANICAL PROPERTIES OF FIBRE COMPOSITE MATERIALS

The structural behaviour of fibre reinforced materials is analysed either by the netting analysis [8] which assumes that only fibres bear the load, or by the Continuum analysis [30,36] which bases prediction on the measured properties of a single unidirectional ply or reinforcement. The continuum analysis is more complicated. A modified form of Netting analysis is more convenient for the straight-forward correlation of properties. The following modifications are made to standard netting analysis.

1. Associated with each real fibre, there are two hypothetical fibres lying in the plane of the laminate at angles $(\pi/2 + \psi)$ and $(\pi/2 - \psi)$ to the real fibre. These hypothetical fibres have modulus E_l and are called "lateral fibres".

2. For a laminate of unit thickness and area, volume of the lateral fibres is $2V_f(1 + \mu V_f)$, where μ is a constant.

3. If the strain in one lateral fibre is ε_{l_1} and strain in the other is ε_{l_2}, the modulus E_l of both fibres is a function of the strain ε_l where $\varepsilon_l = \varepsilon_{l_1}$ or ε_{l_2}, whichever is greater.

The lateral fibre concept :

The concept of lateral fibre takes into account the indirect but substantial contributions of the real fibres to transverse and shear stiffnesses. These contributions are ignored by the netting analysis and whilst there are more sophisticated methods of allowing for them, the lateral fibre concept is simple and convenient for computation.

The two lateral fibres, with an included angle of 2ψ, take account of indirect fibre contributions to both transverse and shear stiffnesses whereas to assume only one lateral fibre at right angles to the real fibre would allow no mathematical contribution to shear stiffness.

The volume and therefore, the cross-sectional area of the lateral fibres are assumed to contain the factor $(1+\mu V_f)$ to allow for "Contiguity effects", that is extra indirect contribution to stiffness due to the extra fibres coming into sideways contact as fibre content increases; the factor 2 in the expression giving the volume of the lateral fibres arises from consideration of the two lateral fibres for each real fibre.

The Model :

The final form of the model can be stated as follows. Derivation of the model is provided in Appendix B.

$$\begin{aligned}\sigma_x &= (\alpha_1)_T\varepsilon_x + (\beta_1)_T\varepsilon_y + (\gamma_1)_T\varepsilon_{xy}\\ \sigma_y &= (\alpha_2)_T\varepsilon_x + (\beta_2)_T\varepsilon_y + (\gamma_2)_T\varepsilon_{xy}\\ \sigma_{xy} &= (\alpha_3)_T\varepsilon_x + (\beta_3)_T\varepsilon_y + (\gamma_3)_T\varepsilon_{xy}\end{aligned} \qquad 3.4.1$$

where-

$$\begin{aligned}(\alpha_1)_T &= 1/8V_f\{3C_o + 4C_2 + C_4\} + (1-V_f-V_v)E_m/(1-\nu^2)\\ (\beta_1)_T &= (\alpha_2)_T = 1/8V_f\{C_o-C_4\} + \nu(1-V_f-V_v)E_m/(1-\nu^2)\\ (\beta_2)_T &= 1/8V_f\{3C_o-4C_2+C_4\} + (1-V_f-V_v)E_m/(1-\nu^2)\\ (\gamma_1)_T &= (\alpha_3)_T = 1/8V_f\{2S_2 + S_4\}\\ (\gamma_2)_T &= (\beta_3)_T = 1/8V_f\{2S_2-S_4\}\\ (\gamma_3)_T &= 1/8V_f\{C_o-C_4\} + \lambda/2(1-V_f-V_v)E_m/(1+\nu)\end{aligned} \qquad 3.4.2$$

and

$$C_o = \sum_{s=1}^{s=n} p_s\{E_{fs} + 2E_{ls}(1+\mu V_f)\}$$

$$C_2 = \sum_{s=1}^{s=n} \{p_s \cos 2\phi_s\}\{E_{fs} - 2E_{ls}(1+\mu V_f)\cos 2\psi\}$$

$$C_4 = \sum_{s=1}^{s=n} \{p_s \mathrm{Cos}4\phi_s\}\{E_{fs} + 2E_{ls}(1+\mu V_f)\mathrm{Cos}4\Psi\} \qquad 3.4.3$$

$$S_2 = \sum_{s=1}^{s=n} \{p_s \mathrm{Sin}2\phi_s\}\{E_{fs} - 2E_{ls}(1+\mu V_f)\mathrm{Sin}2\Psi\}$$

$$S_4 = \sum_{s=1}^{s=n} \{p_s \mathrm{Sin}4\phi_s\}\{E_{fs} + 2E_{ls}(1+\mu V_f)\mathrm{Sin}4\Psi\}$$

CHAPTER 4

PREPARATION OF SPECIMEN AND TESTING PROCEDURE

4.1 TENSILE TEST SPECIMENS

Woven cross-ply Jute and Glass fibre mats were cut into sizes 254x508 mm. The jute fibre mat was made from BTA grade jute. The woven glass fibre mat was made from E-glass fibres. Weight of each fibre mat was recorded before using. Polyester resin was mixed with catalyst and accelerator in certain proportions so that the copolymerization reaction occurs at the desired rate.

Using Hand Lay-up method, several Composite laminates were made. Two non-reacting polythene sheet was used to cover the layup area. One was placed on a plain surface on which the glass fibre or jute fibre mats were kept for resin laying. The prepared polyester resin was then poured on them uniformly so that no air gap remained within the laminate. For multilayer laminates, more fibre mats were added one after another to the first layer of fibres and then again resin was poured on it consecutively. Finally the other polythene sheet was placed on the laminate and pressure was given using a flat plate of smooth surface so that any remaining air entrapped in the laminate goes out.

Using the above procedure, different composite laminates were made with various combination of jute and glass fibre mats. The orientation of fibres of each layer was kept same that means fibres of all the layers of a laminate were aligned in 0 and 90 directions along the length of the specimens.

The laminates were cured slowly at room temperature and under little pressure. After solidification of the resin, each laminate was weighed. The weight of the resin can be obtained by subtracting the weight of fibre mats from total weight of the laminate. The fibre volume fractions of jute and glass fibre were found using the procedure described in Appendix A.

Ten specimens each of dimension 254 mm in length and 25.4 mm in width were cut from each type of laminate. They were numbered simultaneously for identification. Each specimen was then polished using a fine grinding wheel in order to avoid any stress concentration effect at the edges. Tabs of size 38.1x25.4 mm were also made using the above procedure. Tabs were made approximately 1.5 times thicker than the specimen. Tabs were then attached to the ends on both sides of each specimen using very strong adhesive (Araldite). The dimensions of the test specimen were according to the standard specified by ASTM D 3039-76 and are shown in Figure 4.1.

After all the specimens were prepared, different dimensions of each specimen (gage length, width, minimum thickness, etc.) were measured. Tensile test of each specimen was then conducted using a universal testing machine at BEEB. Although the straining rate was not possible to keep constant in this machine it was kept approximately constant during tast. Test was performed at room temperature and without any initial stress. Load was applied in the direction of fibre alignment. The breaking load was recorded from the display and the load-elongation curve was recorded on a chart. The elongation at failure was measured and recorded from the chart. The load and elongations at different points upto the elastic limit on the load-elongation curve were also measured to find the Young's Modulus of Elasticity for each specimen. Ten specimens were tested for each type of laminate. Experimental data of the tensile test for jute-glass reinforced composite laminate specimens are given in Tables 4.2 to 4.6.

4.2 FLEXURE TEST SPECIMENS

Flexure test specimens were prepared in the same way as described in section 4.1. Here woven cross-ply glass and jute fibre mats were sized 15 cm in length and 4 cm in width. Multilayered laminates were made by hand layup method using various combination jute and glass fibre layers with thicknesses ranging from 1.3 mm to 2.2 mm. The laminates were cured at room temperature and under little pressure for two days so that the laminate develops full strength upon the completion of copolymerization reaction of the resin. Ten specimens each of dimension 40 mm in length and 14 mm in width were cut from each type of laminate having a particular combination of jute and glass volume fractions. These specimens were then polished using a fine grinding wheel to avoid stress concentration effects.

The prepared specimens were then tested in a "Flexural Modulus Measuring Apparatus" at BITAC. Before starting the experiment, the mean thickness (d) of each specimen over its full width at the mid section was measured and recorded. In order to ensure that the deflection of each specimen upon loading does not exceed the elastic limit of the material, the deflection should not cause a strain more than 0.2 percent. Therefore, the desired deflection (maximum) for each specimen was calculated from the following equation-

$$D = 0.21505/d$$

where D = deflection of the specimen at its midpoint,mm

d = thickness of the specimen,mm

The specimens were tested on the apparatus using three-point test method (Fig. 4.2). Each specimen was placed centrally on the supports and then the load beam was placed on the middle of the specimen. The dial gauge adjusting screw was turned so that the proximity switch functions. The bezel locking screw was loosened and the dial gauge bezel was turned so that "zero" coincides with the position of the pointer. Loose weights were applied at the centre of the beam successively until the dial gauge reading becomes approximately 2D. At this position, the red light should be balanced with green light by turning the gauge adjusting screw to the right position. The applied load "W" was recorded. The applied load W should be placed on the beam as quickly as possible. Exactly one minute after the completion of loading, the resultant deflection D is measured to the nearest 0.002 mm and recorded. The remaining specimens were tested in the same way one after another.

The flexural modulus of each specimen was calculated using the beam deflection theory from the following equation.

$$E = L^3W/4d^3Db$$

where

b = specimen width, mm

L = specimen span length, mm

W = Load, Newton

d = specimen thickness,mm

D = deflection of the specimen at midpoint, mm

The experimental data for flexural modulus of jute-glass fibre reinforced composite laminates are summarized in Tables 4.7 to 4.11.

CHAPTER 5
RESULTS AND DISCUSSIONS

5.1 EFFECTIVE FIBRE VOLUME FRACTION

It has been observed by Boue [6] that the mechanical properties of a fibre composite are a function of the fibre volume fraction of the composite. Thus knowledge of fibre volume fraction of a fibre composite is very important to predict its properties. In the present study, two types of fibre have been used for reinforcement of the composite. As a result, there are two fibre volume fractions, one for jute and the other for glass. Since the strength properties of the composite will depend on each of the fibre volume fractions, some hypothetical effective fibre volume fraction has to be developed which should interprete the properties in a right way. Simple summation of the two fibre volume fractions has no significance on the elastic and plastic properties of the composite, because this total fibre volume fraction can be kept same by increasing the volume fraction of one fibre by an amount and by decreasing the volume fraction of the other fibre by the same amount. This would result in a significant change in the elastic and strength properties although the volume fraction remained same as before. That means different mechanical properties can be obtained at same total fibre volume fraction. To find a unique effective fibre volume fraction to characterize a particular composition of the composite, the following relation has been defined by the author.

$$V_{feff}=2(V_{fj}E_j+V_{fg}E_g)/(E_j+E_g)$$

The above expression for fibre volume fraction has been found effective in interpreting the composite properties because of the following features:

I. It takes the weighted average of the two fibre volume fractions based on their elastic moduli multiplied by 2 which means that a small increase in a fibre volume fraction having higher modulus will increase the V_{feff} significantly and vice versa. Physically it is expected that the strength and stiffness of the composite will increase significantly if the volume fraction of the fibre having higher modulus is increased. On the other hand, if the volume fraction of the fibre having lower modulus is increased in the same amount which would result in a small increase in V_{feff}, the mechanical properties of the composite are not expected to increase in the same way as in the previous case. Thus the above expression of effective fibre volume fraction shows a unique relationship between fibre volume fraction and mechanical properties of a composite consisting of two or more types of fibre.

2. If the two types of fibre have the same elastic moduli, then the above expression for V_{feff} becomes simple summation of the two individual fibre volume fractions which also valididates the applicability of the effective fibre volume fraction.

However, the expression for the effective fibre volume fraction should not be considered something absolute. It may not be effective in interpreting other mechanical properties such as fatigue strength, etc.. Also V_{feff} may not be a useful expression if the volume fraction of either of the fibres becomes extremely small. The expression for V_{feff} must be justified before using in any other work.

5.2 TENSILE AND FLEXURE TEST OF JUTE-GLASS FIBRE REINFORCED COMPOSITE LAMINATES

Tensile test of Jute-Glass Reinforced Composite Laminate (JGRCL) has been performed for five different fibre volume fractions. Throughout this and the following sections, volume fraction will be used to mean the effective fibre volume fraction of JGRCL as described in the previous section. Test results are summarized in Table 5.1 along with predicted properties.

The experimental and predicted Young's Modulus of the JGRCL for different volume fractions have been plotted in Fig. 5.1. The experimental modulus increases linearly with volume fraction. The maximum deviation between experimental values of stiffness and those predicted by Bishop's Model was about 5 percent. This deviation may be due to the scatter of experimental values and some unavoidable experimental inaccuracies. The maximum deviation of the predicted stiffness of Lamination theory and Law of Mixture from experimental stiffness within the range of fibre volume fractions covered in the experiment is about 51 percent.

The breaking strength of the JGRCL is found to increase linearly with volume fraction as shown in Fig. 5.2. The maximum deviation between the breaking strength predicted by Bishop's model and the experimental values was about 7 percent within the range covered. The fitted curves both for experimental and predicted strength are plotted in Fig. 5.2. A higher degree of scatter in experimental results has caused a little more deviation in the predicted strength in this case. The divergence of the stiffnesses and strengths predicted by Bishop's model from experimental values at higher volume fractions may be decreased using more experimental data in fitting the model and thereby reducing the scatter. Voids in the laminate have been assumed to be absent while fitting the Bishop's model. This assumption might have caused some discrepancies in the predicted properties, because voids are invariably present in every laminate made by hand lay-up method. Usually number of voids in the composite increases with volume fraction.

The experimental stiffness and strength of JGRCL in comparison those of only Jute fibre RP [22] and only Glass fibre RP [42] at different volume fraction is shown in Table 5.2.

In Fig. 5.3, a better picture of the variation of stiffness with volume fraction of these three composites is shown. Fig. 5.4 shows the variation of breaking strength with volume fraction for JGRCL and JRP. It is seen that addition of glass fibres to JRP although boosted the stiffness of it, but the strength did not increase in that way. This may be due to the fact that as glass fibre was added to the laminate, the elongation of the specimen within elastic limit became very small. As a result the elastic modulus increased a lot. But as ultimate strength of the laminate is a plastic property, it has no relation with elastic strain. The strength increased proportionately with the increase in glass fibre volume in the laminate.

In Fig. 5.5, breaking strain has been plotted against fibre volume fraction for JGRCL. It is found that strain at failure decreases logarithmically with increase in the effective fibre volume fraction of the laminate. This means that the laminate becomes more and more brittle with the increase in effective volume fraction, i.e. increase in glass fibre volume fraction. This is because glass fibre contributes more to the effective volume fraction and thus raises the brittleness.

Fig. 5.6 shows the effect of increasing number of jute lamina on the strength of JGRCL while keeping the number of glass lamina constant. It shows that increase in the number of jute lamina decreases the strength of the laminate and the change in strength occurs nonlinearly. This is because with increase in the number of jute lamina keeping the number of glass lamina constant, the effective volume fraction of the laminate decreases and as a result the strength also decreases as illustrated in Fig. 5.4.

The test results from flexure testing of the JGRCL are summarized in Table 5.3 along with the predicted results. The table shows the experimental and predicted flexural modulus of the laminate at different fibre volume fractions.

The experimental and theoretical results are plotted in Fig. 5.7. It can be seen from the figure that both theoretical and experimental flexural modulus increase with increase in volume fraction in a nonlinear way and the rate of increase in flexural modulus with volume fraction is less at higher values of fibre volume fractions. Also the deviation of theoretically calculated modulus from experimentally determined modulus increases rapidly with increase in volume fraction. At a volume fraction of 0.386, the predicted modulus becomes about twice more than the experimental value.

The Bishop's model has been fitted using the experimental stiffness data of JRP [22] to find the stiffness parameters so that they can be used for further prediction of stiffness of JRP. The fitting was almost perfect and the the difference between predicted stiffness and experimental values was very small. The maximum deviation of predicted stiffness from experimental value within the range of fibre volume fraction of 0.208 is found to be only 0.25 percent which is negligible. The experimental and predicted stiffness at different volume fractions are summarized in Table 5.4 and plotted in Fig. 5.8.

The stiffness and strength parameters obtained from fitted Bishop's model for JGRCL for cross-ply reinforcement are given below :

	For Stiffness	For Strength
$E_f =$	9.5E+06	1.1E+07
$E_l =$	1.1E+04	9.3E+03
$E_m =$	-9.7E+02	-8.7E+02
$\mu =$	899.223	898.23
$\nu =$	0.30	0.000001
$\psi =$	35°	35°

The above values can now be used for the prediction of stiffness and strength of cross-ply jute-glass fibre reinforced composite laminates with any other fibre volume fractions of jute and glass fibre.

The stiffness parameters obtained from fitted Bishop's model for JRP for cross-ply laminates are given below :.

$E_f =$ 8.7E+05

$E_l =$ -2.8E+02

$E_m =$ -6.0

$\mu =$ 53.742

$\nu =$ 0.30

$\psi =$ 35°

The above values are now ready for the prediction of stiffness of crossply Jute reinforced plastics using the Bishop's model.

Table 5.5 shows the breaking strains of JGRCL and JRP at different volume fractions. It is seen that the breaking strain is drastically reduced in case of JGRCL as compared to JRP due to the addition of glass fibres.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The present research work was oriented towards the experimental study of some strength properties of Jute-Glass fibre reinforced composite laminates as well as comparison of the same with different analytic and empirical model to predict the strength properties. The following conclusions can be made as a result of the present research work :

1. As the composite under study consisted of two types of fibres, namely Jute and Glass fibres, the "effective" fibre volume fraction is found to be meaningful and effective in interpreting the strength properties of the laminate.
2. Jute fibre mat was found to be 3 to 4 times thicker than glass fibre mat. As a result as the number of jute mat increased by decreasing the number of glass mat in a particular laminate, thickness of the laminate increased.
3. As the number of jute layer increased in the laminate keeping the number of glass layer constant, the "effective" volume fraction decreased. And as a result the magnitude of strength and stiffness decreased linearly.
4. The brittleness of the laminate increases with the increase in "effective" fibre volume fraction.
5. Bishop's analytical/empirical model has been found to be most effective in predicting the strength and stiffness of the present laminate. The maximum deviation between predicted and experimental values within the range of fibre volume fraction covered in the present work was 7 percent for strength and 5 percent for stiffness which is quite acceptable from engineering point of view.
6. The experimental values of stiffness of the laminate did not match well with those predicted by both the lamination theory and the law of mixtures.

7. The increase in flexural modulus of the laminate with "effective" fibre volume fraction was smaller for experimental values than for values obtained from lamination theory.

8. Bishop's Model has been found to predict the Young's Modulus of cross-ply JRP [22] at different physical fibre volume fraction with very high accuracy.

9. Bishop's model is suggested for the prediction of strength and elastic properties of fibre composite laminates where some experimental data are available so that the parameters can be determined in order to predict the stiffness and strength of the same fibre composite with desired combination of fibre volume fraction and fibre orientation.

10. The stiffness and strength parameters established here can be used for prediction of stiffness and strength of only cross-ply Jute-Glass laminates.

6.2 RECOMMENDATIONS

The following suggestions may be recommended as an extension of present research work:

1. Experimental study of shear and compressive strength and transverse and shear modulus of oriented jute-glass composites and fitting the data to different models for prediction.

2. Experimental study of angle-ply laminates at various orientations of fibre in tension, compression, shear and impact and fitting the data into the Bishop's model in order to predict these properties at various fibre volume fractions and fibre orientations.

3. Experimental study of creep and viscoelastic behaviour of the jute-glass laminate at various fibre volume fractions.

4. Experimental study of longitudinal, transverse and shear modulus and strength and Poisson's ratio of unidirectional jute-glass composite and comparison with theoretical models.

FIGURES

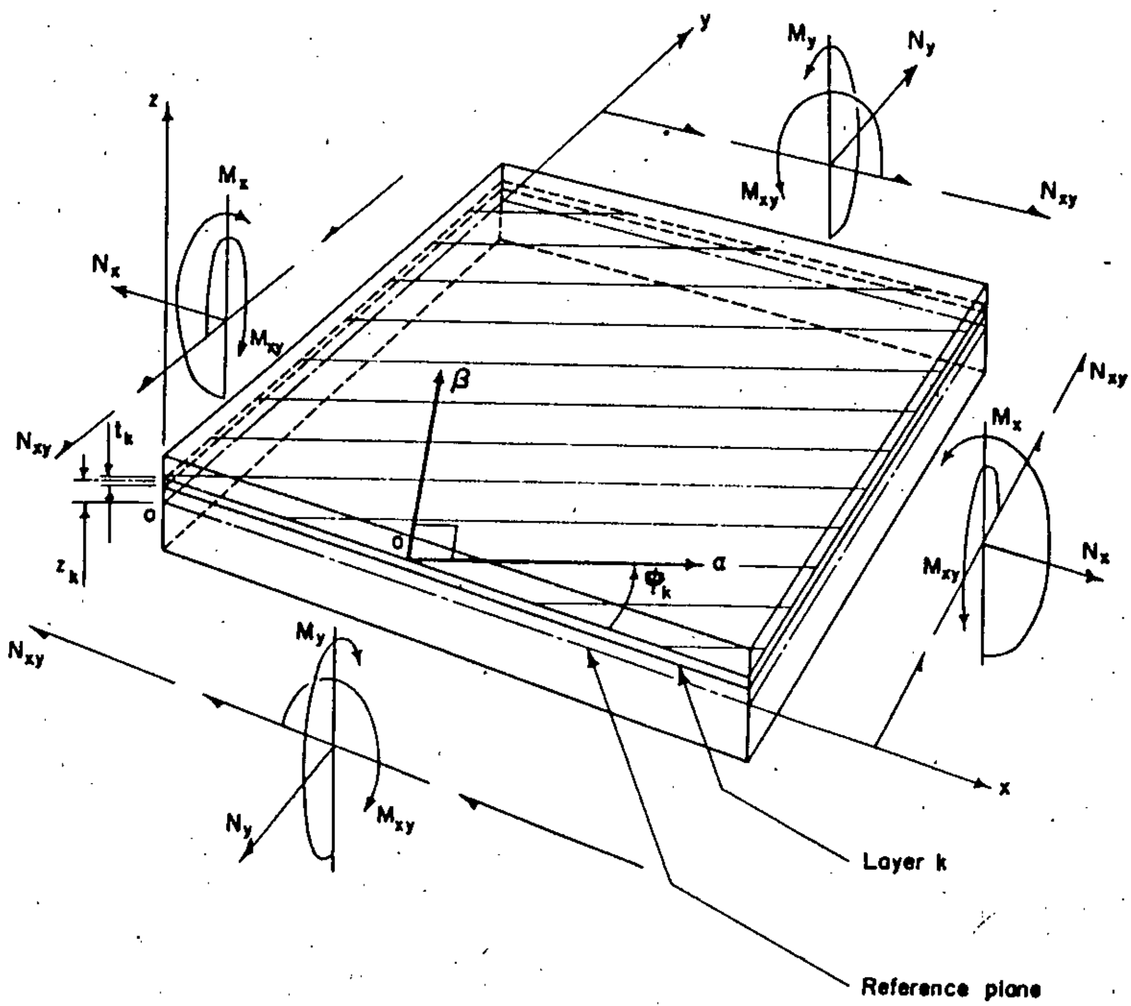


Figure 3.1 Lamination analysis of orthotropic multilayer fibre composite laminate

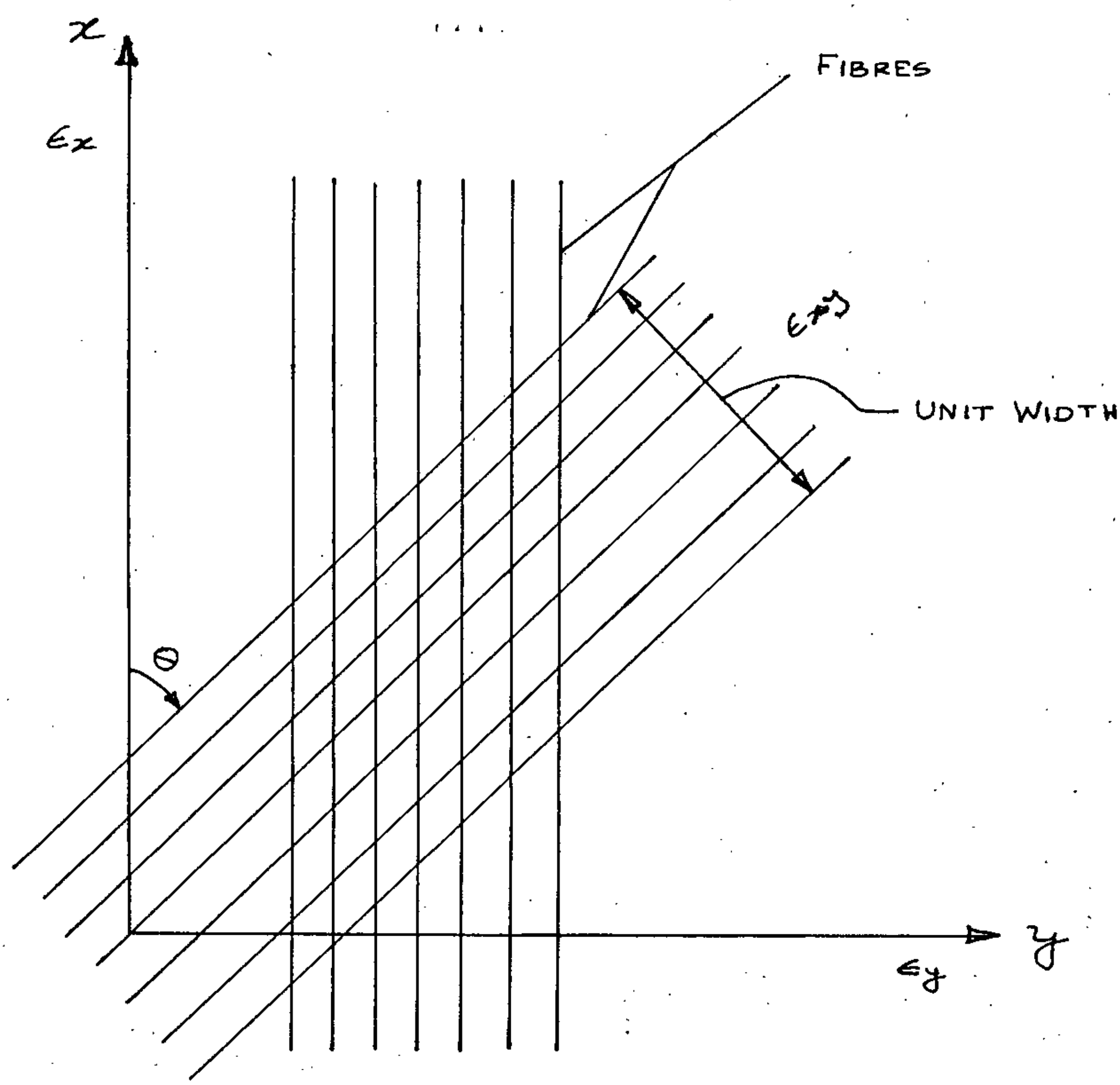


Figure 3.2 Planar mat of fibres

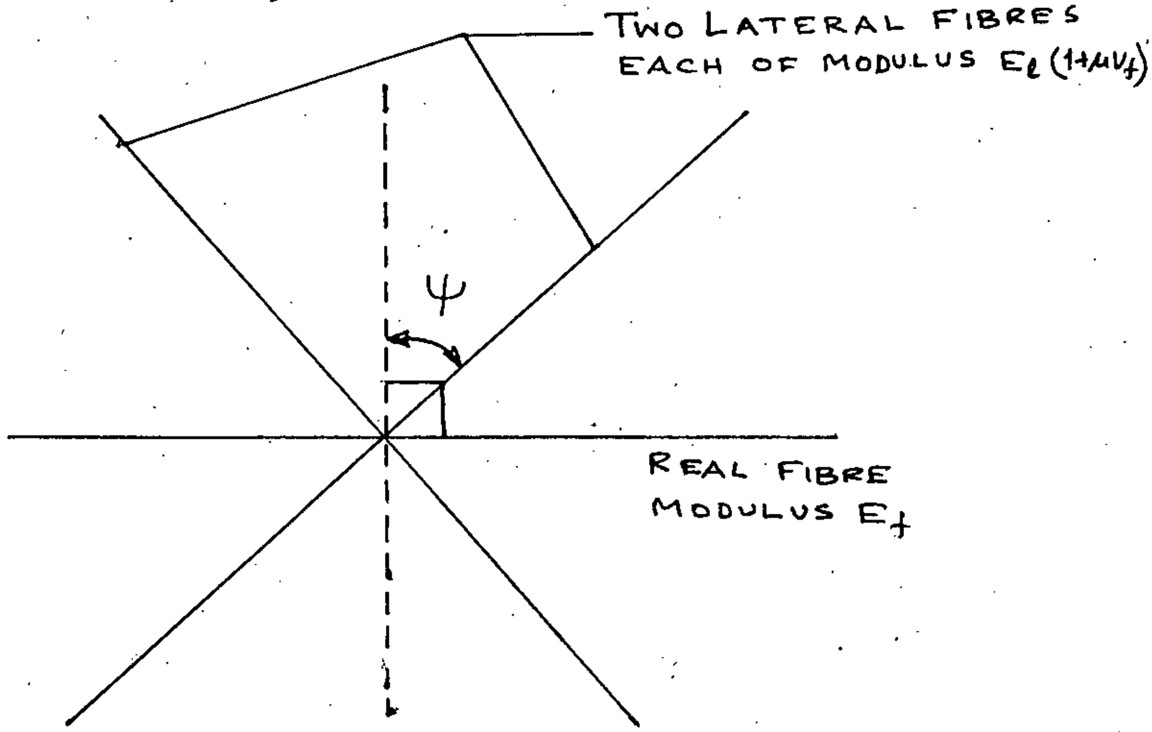


Figure 3.3 Fibre notation (Bishop's Model)

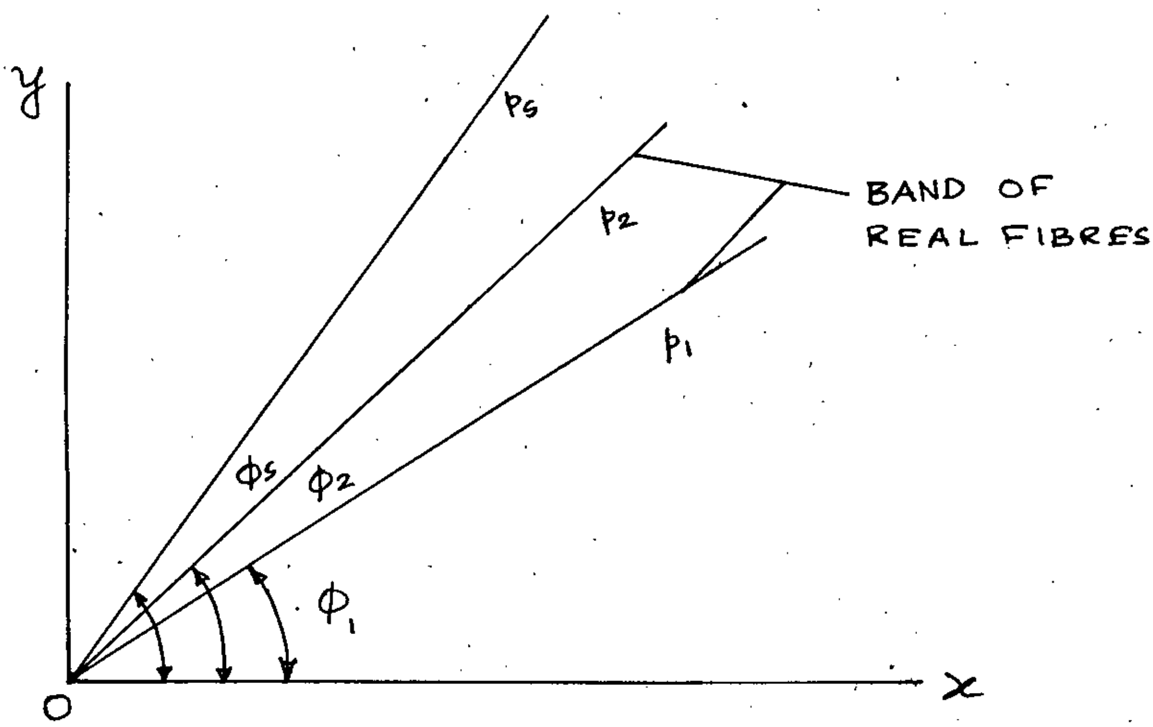
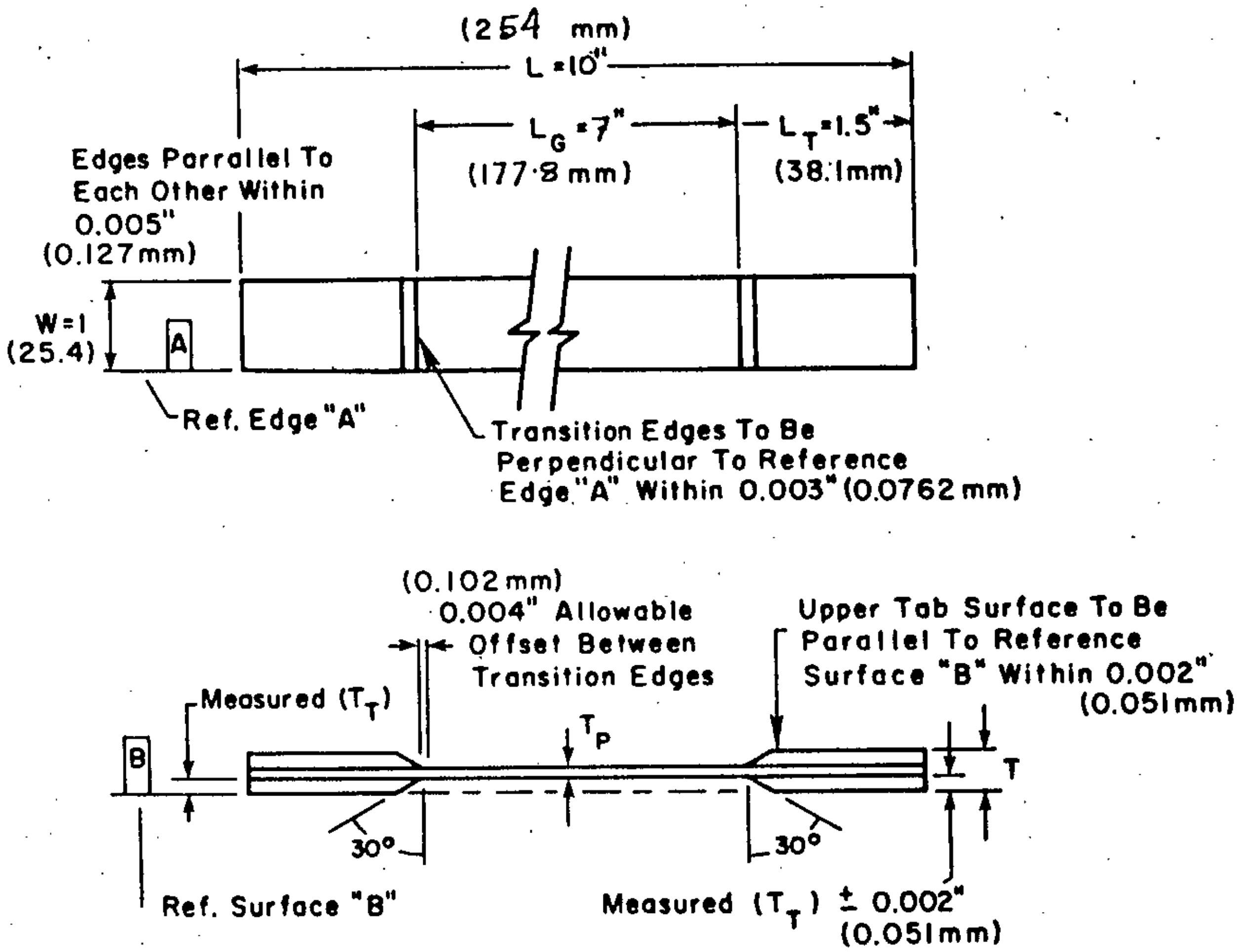


Figure 3.4 Orientation notation (Bishop's Model)



Tolerance Limits for different dimensions :

W	L	L_G	L_T	T	T_T	T_P	
±0.0156	±0.1250	±0.0625	±0.0625	±0.0150	±0.0050	—	inches
0.396	3.175	1.5875	1.5875	0.381	0.127		mm

Figure 4.1 Dimension of the tensile test specimen of fibre composite according to ASTM standard D 3039-76.

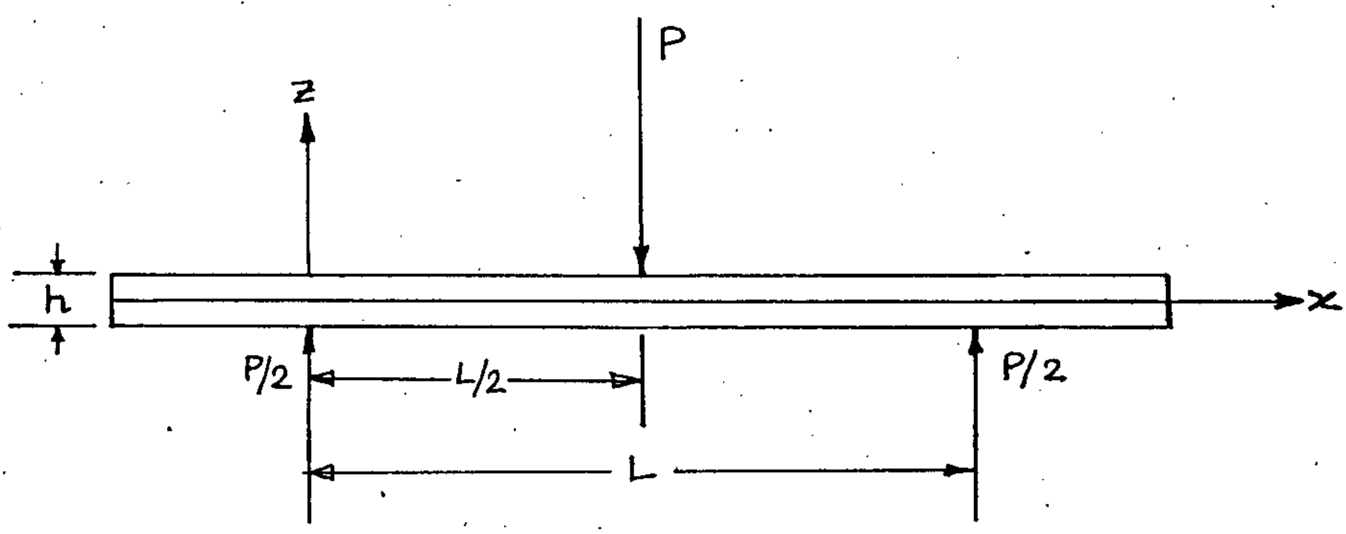


Figure 4.2 Three point flexure test

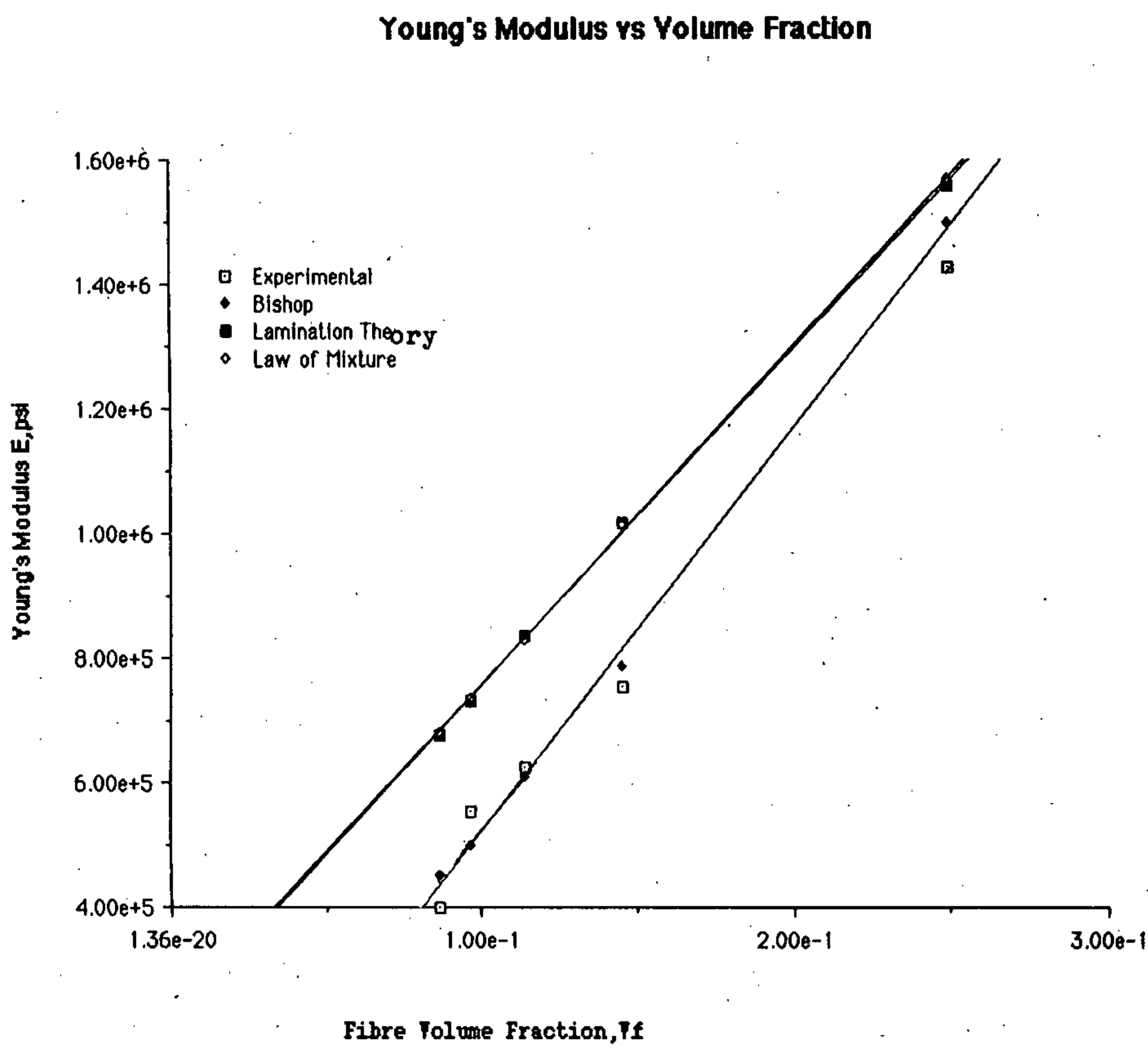


Figure 5.1 Variation of Young's Modulus with "Effective" fibre volume fraction

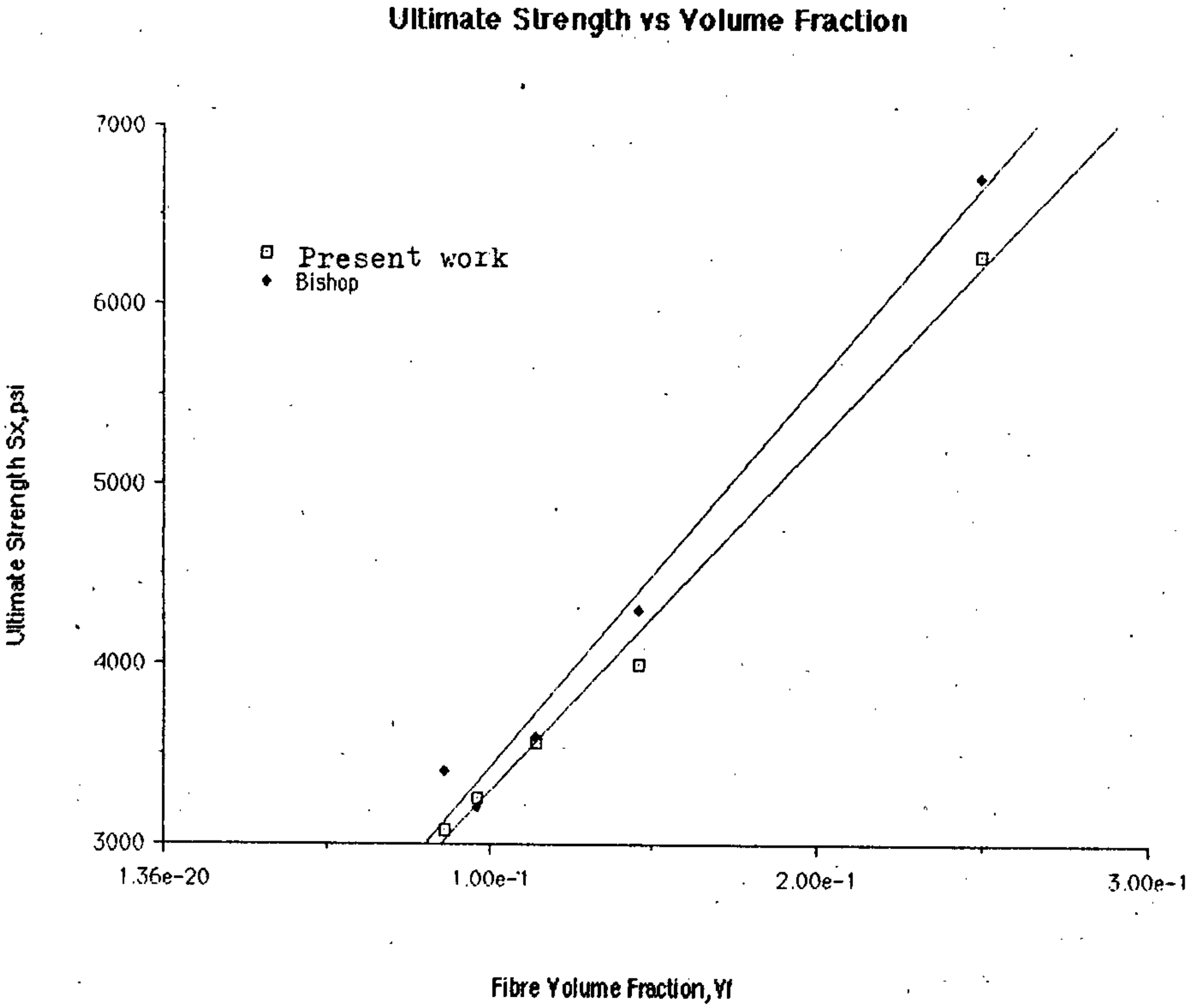


Figure 5.2 Variation of Ultimate strength with "Effective" fibre volume fraction

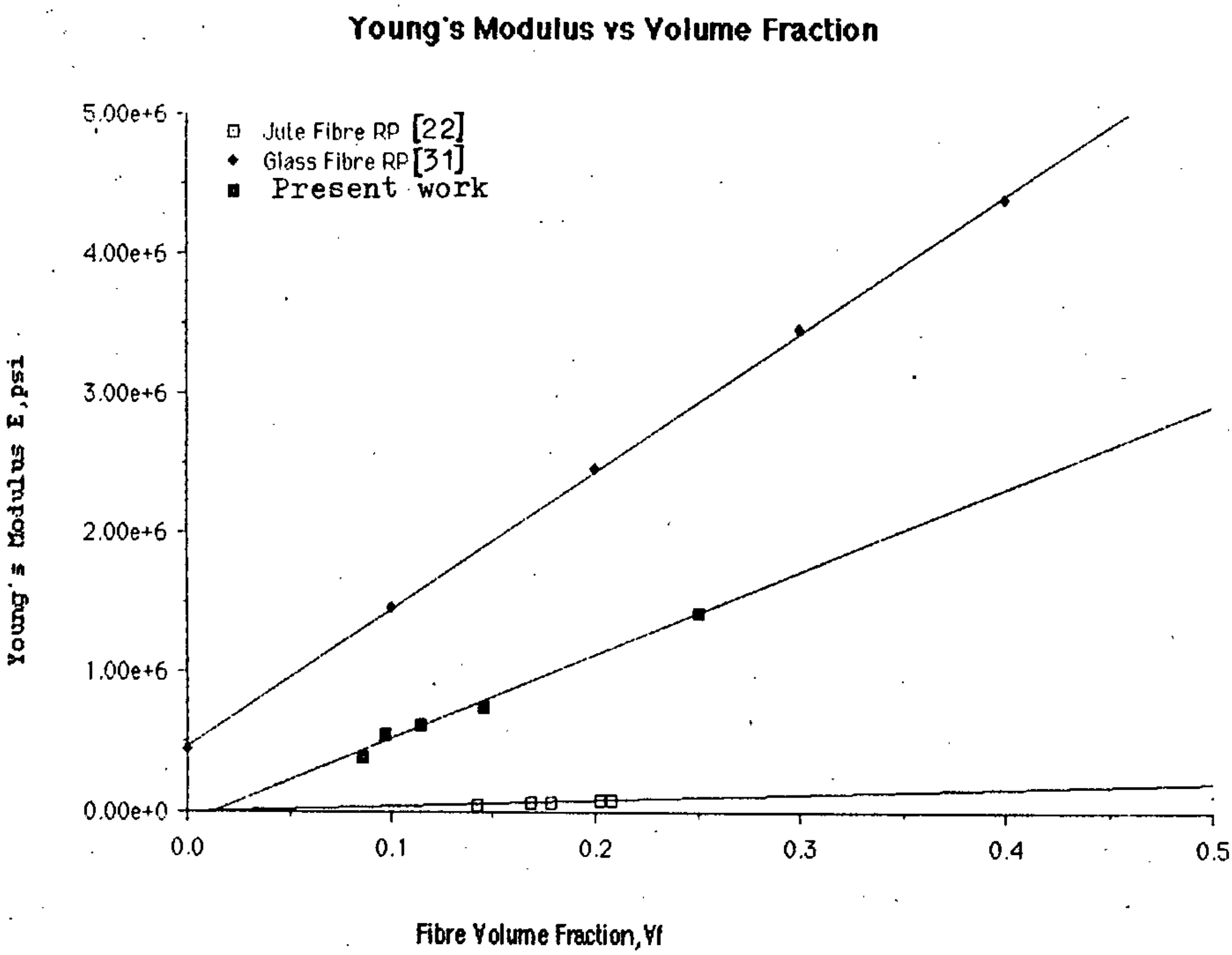


Figure 5.3 Comparison of Young's modulus at different fibre volume fractions between three types of composites

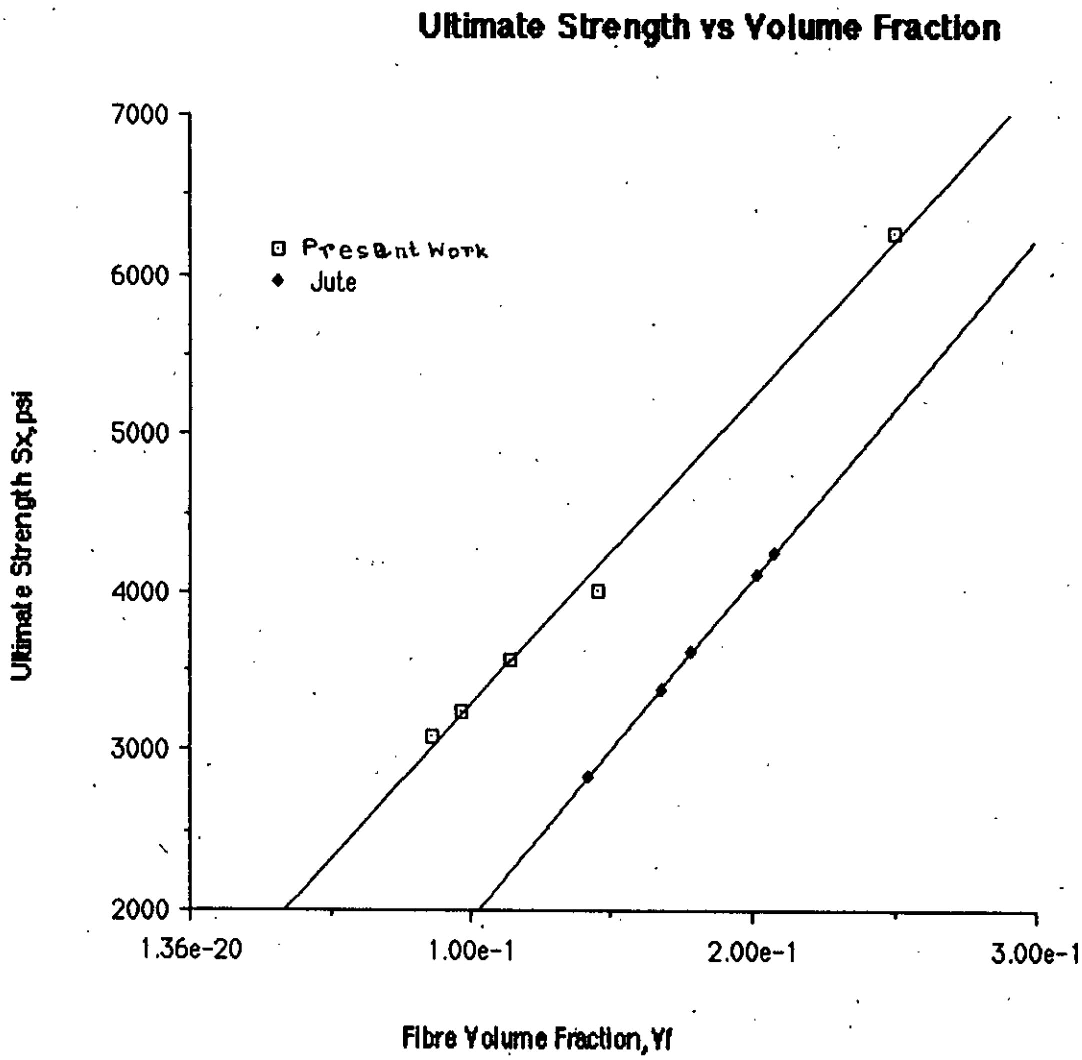


Figure 5.4 Variation of Ultimate Strength with fibre volume fraction for Jute-Glass and Jute RP

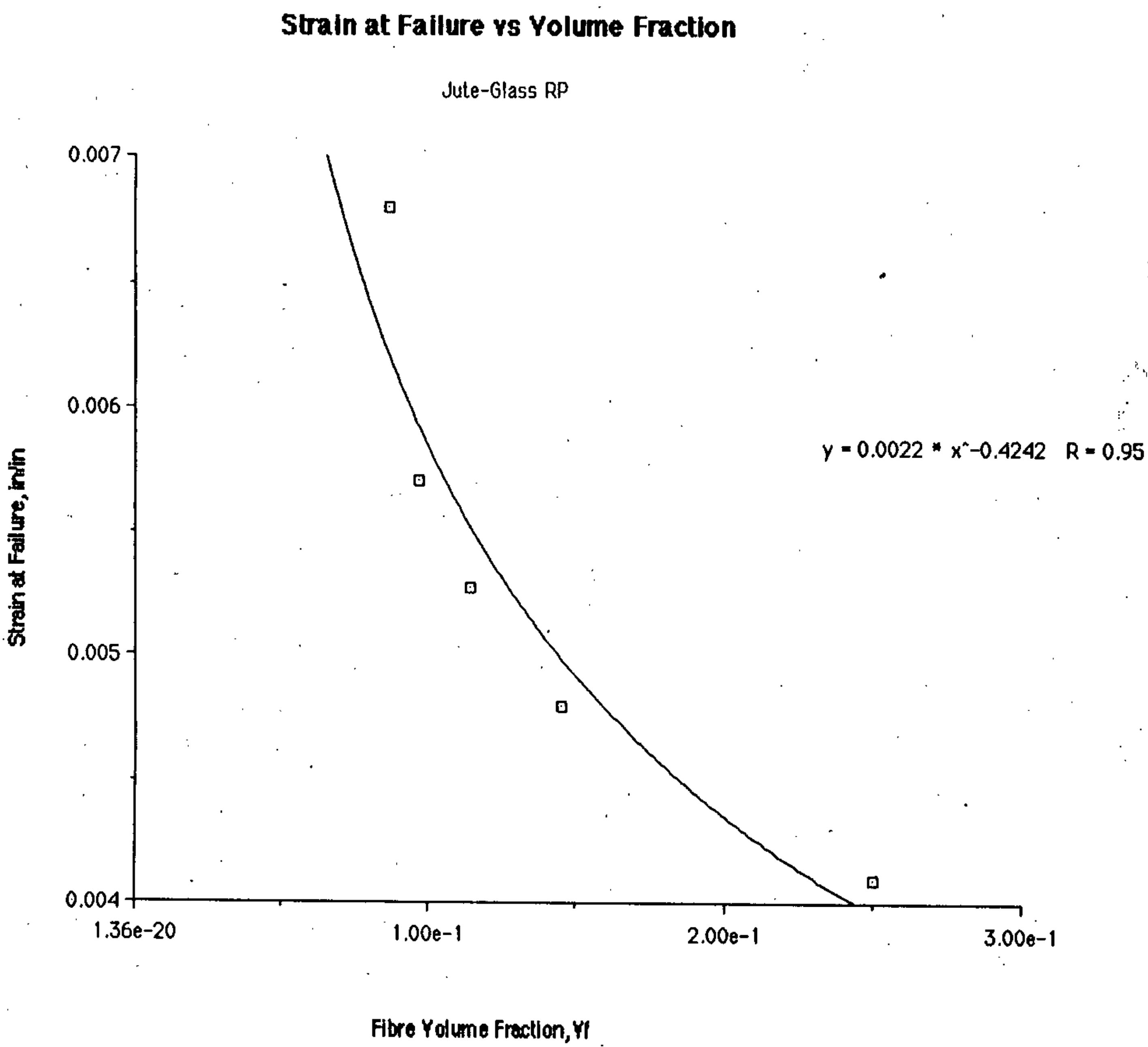


Figure 5.5 Variation of Failure strain with "Effective" fibre volume fraction

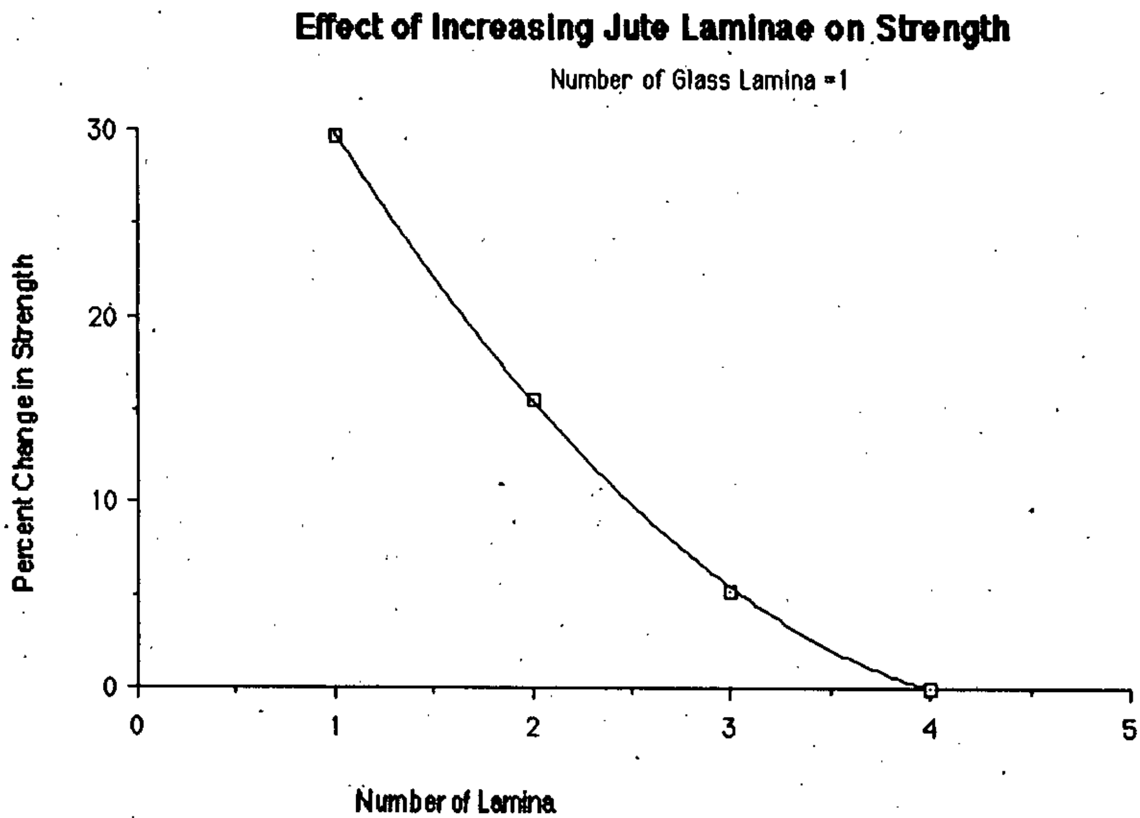


Figure 5.6 Effect of increasing jute lamina on the ultimate strength of Jute-Glass RP

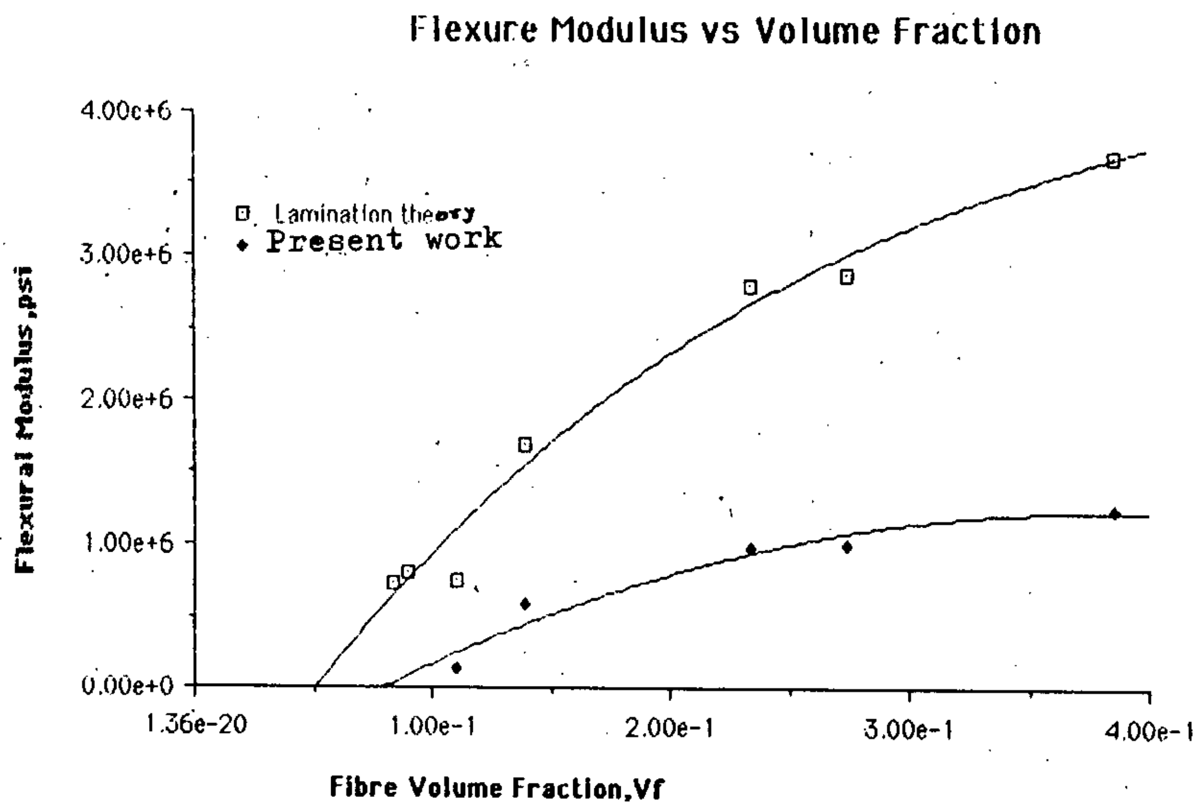


Figure 5.7 Variation of Flexural modulus with "Effective" fibre volume fraction

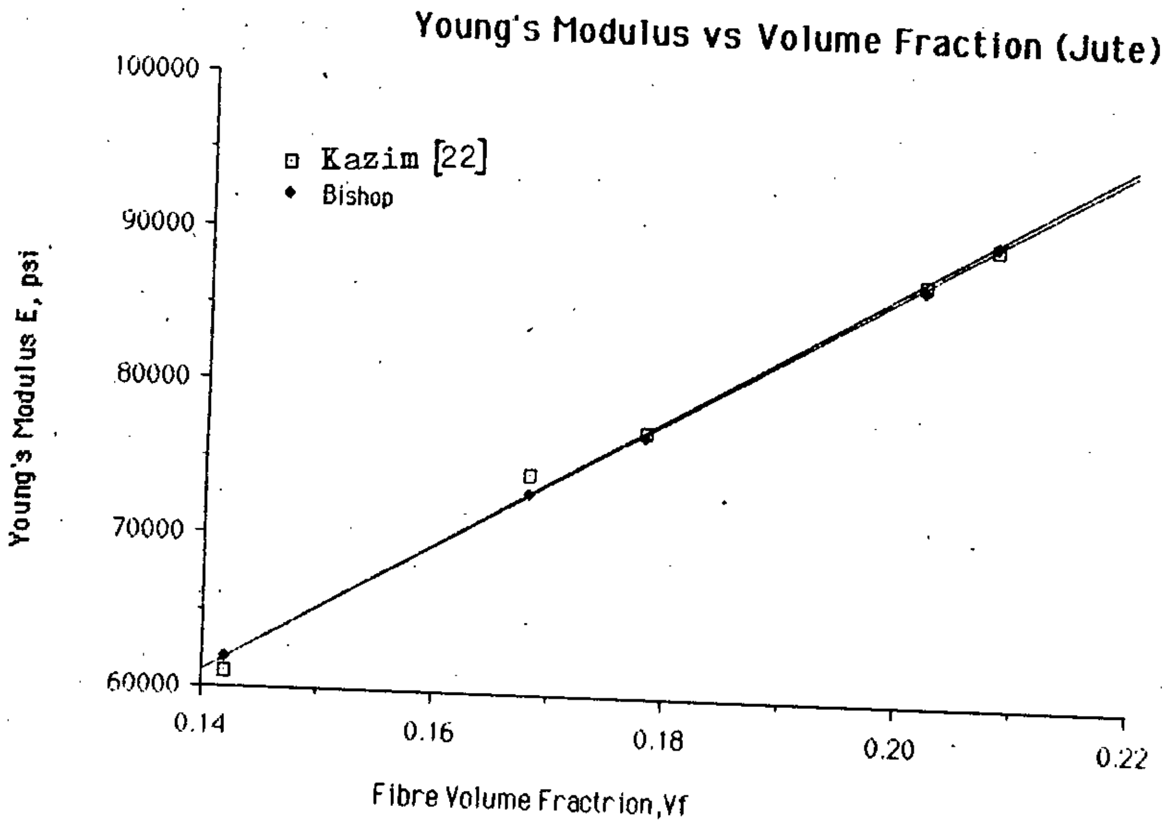


Figure 5.8 Young's moduli vs. Volume fraction for Jute RP showing the accuracy of fitting the Bishop's model

APPENDICES

APPENDIX A
DETERMINATION OF VOLUME FRACTIONS

In the prediction of mechanical properties of fibre composites, volume fractions are used. These quantities can not usually be measured directly and are estimated from weight fractions.

The fibre volume fraction V_f, and the matrix volume fraction, V_m in a laminate without voids, satisfy the relationship ,

$$V_f + V_m = 1 \qquad \text{A.1}$$

In the present study, two types of fibre have been used for reinforcement which have to be considered separately. So the following relation should be written instead of equation A.1.

$$V_{fj} + V_{fg} + V_m = 1 \qquad \text{A.2}$$

The composition of each laminate is known in terms of weight of fibres and matrix W_{fj}, W_{fg}, and W_m respectively. Therefore the volume of each type of fibre and matrix in a laminate can be written as

$$Vol_{fj} = W_{fj}/\rho_j$$

$$Vol_{fg} = W_{fg}/\rho_g$$

$$Vol_m = W_m/\rho_m$$

The total volume of these three constituents should be equal to the volume of the laminate if no voids inside are assumed.

$$Vol_T = W_{fj}/\rho_j + W_{fg}/\rho_g + W_m/\rho_m \qquad \text{A.4}$$

Therefore, volume fractions of fibres and matrix are given by,

$$V_{fj} = W_{fj}/\rho_j \;/(W_{fj}/\rho_j + W_{fg}/\rho_g + W_m/\rho_m)$$

$$V_{fg} = W_{fg}/\rho_g \;/(W_{fj}/\rho_j + W_{fg}/\rho_g + W_m/\rho_m) \qquad \text{A.5}$$

$$V_m = W_m/\rho_m \;/\; (W_{fj}/\rho_j + W_{fg}/\rho_g + W_m/\rho_m)$$

APPENDIX B

B.1 LAMINATION THEORY : Derivation of C Matrix from General Matrix for Homogeneous Solids

The complete three-dimensional system of relationships for a homogeneous Solid is as follows.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & \text{(Symmetric)} & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{vmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{Bmatrix} \qquad \text{B.1.1}$$

where S_{ij} are the material compliances.

In the standard theory of plates, it is assumed that the stress $_z$ in the z-direction (through the plate's thickness) are zero. Thus the terms of the third column of the matrix of compliances make no contribution to the strains. Additionally, there will be no coupling between the in-plane strains ε_x, ε_y and ε_{xy} and out-of-Plane direct strains ε_z and out-of plane shear stresses σ_{xz} and σ_{yz}. Also ε_z is not coupled to σ_{xy}. Consequently equation B.1.1 reduces to-

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{22} & S_{23} & 0 & 0 & S_{26} \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & S_{45} & 0 \\ & \text{(Sym.)} & & & S_{55} & 0 \\ & & & & & S_{66} \end{vmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ 0 \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{Bmatrix} \qquad \text{B.1.2}$$

Equation B.1.2 is most conveniently written as the following two smaller matrices-

$$\begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{66} \end{vmatrix} \begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{vmatrix} \qquad \text{B.1.3}$$

and

$$\begin{vmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{vmatrix} = \begin{vmatrix} S_{44} & S_{45} \\ & S_{55} \end{vmatrix} \begin{vmatrix} \sigma_{xz} \\ \sigma_{yz} \end{vmatrix}$$

together with an equation for $_z$ which is not required. In the principal axes of orthotropy, $o(\alpha, \beta)$ of a single layer, the compliances S_{16} ($=S_{61}$) and S_{26} ($=S_{62}$) become zero since , in the directions of the axes of orthotropy, coupling between in-plane shear and direct stresses is eliminated. Thus,-

$$\begin{vmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \varepsilon_{\alpha\beta} \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} & 0 \\ & S_{22} & 0 \\ & & S_{66} \end{vmatrix} \begin{vmatrix} \sigma_\alpha \\ \sigma_\beta \\ \sigma_{\alpha\beta} \end{vmatrix} \qquad \text{B.1.4}$$

This gives the identifications-

$E_\alpha = 1/S_{11}$, $E_\beta = 1/S_{22}$, $\nu_{\alpha\beta} = -S_{12}/S_{11} = -S_{12}E_\alpha$

and $G_{\alpha\beta} = 1/S_{66}$

In order to calculate layer stresses from layer strains the compliance matrix of equation B.1.4 must be inverted to become the C matrix giving-

$$\begin{vmatrix} \sigma_\alpha \\ \sigma_\beta \\ \sigma_{\alpha\beta} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{vmatrix} \begin{vmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \varepsilon_{\alpha\beta} \end{vmatrix} \qquad \text{B.1.5}$$

in which $C_{11} = S_{22}/(S_{11}S_{22} - S_{12}^2) = E_\alpha/(1 - \nu_{\alpha\beta}\nu_{\beta\alpha})$

$C_{22} = S_{11}/(S_{11}S_{22} - S_{12}^2) = E_\beta/(1 - \nu_{\alpha\beta}\nu_{\beta\alpha})$

$C_{12} = C_{21} = -S_{12}/(S_{11}S_{22} - S_{12}^2) = \nu_{\alpha\beta}E_\beta/(1 - \nu_{\alpha\beta}\nu_{\beta\alpha}) = \nu_{\beta\alpha}E_\alpha(1 - \nu_{\alpha\beta}\nu_{\beta\alpha})$

and $C_{66} = 1/S_{66} = G_{\alpha\beta}$

B.2 MATHEMATICAL DERIVATION OF BISHOP'S MODEL TO CORRELATE STIFFNESS AND STRENGTH WITH ORIENTATION

The theory of this appendix is based on the analysis by H.L. Cox [8]. A mat of straight fibres consolidated to a volume fraction V_f, of unit thickness and area, and arbitrary orientation is considered. It is assumed that the orientation can be defined by a fibre distribution function $f(\theta)$ such that the volume of fibre that lies between angles θ and $\theta + d\theta$ to the ox axis is proportional to $f(\theta)d\theta$. $f(\theta)$ will obviously be a periodic function of period .Let the factor of proportionality be K; since the total volume of fibre is V_f, then-

$$K \int_0^{\pi} f(\theta)d\theta = V_f \qquad \text{B.2.1}$$

If the fibres lying between θ and $(\theta + d\theta)$ are intercepted by a line at right angles to them, the cross-sectional area of the fibres cut by unit length of this line will be $Kf(\theta)d\theta$. The cross-sectional area of fibre, viewed lenghtwise, lying between θ and $(\theta + d\theta)$ and intercepted by a line of unit length perpendicular to OX, is $Kf(\theta)\text{Cos}\theta\, d\theta$. Cross-sectional area for fibres intercepted by a line of unit length parallel to OX is $Kf(\theta)\text{Sin}\theta d\theta$.

If the mat of fibre is subjected to tensile strain ε_x parallel to OX, ε_y parallel to OY and shear strain ε_{xy} between OX and OY, the strain ε_f in a fibre inclined at an angle θ to OX can be shown to be [8]

$$\varepsilon_f = \varepsilon_x \text{Cos}^2\theta + \varepsilon_y \text{Sin}^2\theta + \varepsilon_{xy} \text{Sin}\theta\text{Cos}\theta \qquad \text{B.2.2}$$

If the load is proportional to strain, contributions of the fibre in the directions OX and OY respectively are the product of load in the fibre with $\text{Cos}\theta$ and $\text{Sin}\theta$ respectively. Combining these data and noting that modulus of the fibre is E_f, the following relations are obtained :

$$\sigma_x = KE_f \int_0^\pi (\epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \epsilon_{xy} \sin\theta\cos\theta)\cos^2\theta \, f(\theta) \, d\theta$$

$$\sigma_y = KE_f \int_0^\pi (\epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \epsilon_{xy} \sin\theta \cos\theta)\sin^2\theta f(\theta) d\theta \qquad B.2.3$$

$$\sigma_{xy} = KE_f \int_0^\pi (\epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \epsilon_{xy} \sin\theta\cos\theta)\sin\theta\cos\theta f(\theta) d\theta$$

where σ_x, σ_y are tensile stresses and σ_{xy} is the shear stress. Because f(θ) is periodic, it may be taken as -

$$f(\theta) = a_0 + a_1 \cos 2\theta + a_2 \cos 4\theta + \text{------} + b_1 \sin 2\theta + b_2 \sin 4\theta + \text{------} \qquad B.2.4$$

The double angles ensure that f(θ) is periodic over an angular interval of π . Integrals are evaluated from 0 to π so that a fibre contributes only once to an integral and does not make a further contribution when viewed lengthwise in the opposite direction. Therefore,

$$\int_0^\pi f(\theta) d\theta = \int_0^\pi (a_0 + a_1 \cos 2\theta + \text{--------} + b_1 \sin\theta + \text{-----}) d\theta = a_0 \pi \qquad B.2.5$$

and from equation B.2.1

$$K = V_f / a_0 \qquad B.2.6$$

Equations for σ_x, σ_y and σ_{xy} may be written as

$$\sigma_x = \alpha_1 \epsilon_x + \beta_1 \epsilon_y + \gamma_1 \epsilon_{xy}$$

$$\sigma_y = \alpha_2 \epsilon_x + \beta_2 \epsilon_y + \gamma_2 \epsilon_{xy} \qquad B.2.7$$

$$\sigma_{xy} = \alpha_3 \epsilon_x + \beta_3 \epsilon_y + \gamma_3 \epsilon_{xy}$$

where the coefficients α, β ,etc. are evaluated from the definite integrals of which the following is typical-

$$\alpha_1 = KE_f \int_0^\pi \cos^4\theta (a_0 + a_1 \cos 2\theta + \text{------} + b_1 \sin 2\theta + \text{------}) d\theta$$

$$= a_0 KE_f \int_0^\pi \cos^4\theta (1 + a_1/a_0 \cos 2\theta + \text{-----}) d\theta \qquad B.2.8$$

Terms higher than suffix 2 vanish during integration, and the following expression is obtained.

$$\alpha_1 = a_0 \; KE_f (6 + 4a_1/a_0 + a_2/a_0)/16$$

$$\beta_1 = \alpha_2 = \gamma_3 = a_0 \; KE_f (2 - a_2/a_0)/16$$

$$\beta_2 = a_0 \; KE_f (6 - 4a_1/a_0 + a_2/a_0)/16 \qquad B.2.9$$

$$\gamma_1 = \alpha_3 = a_0 \; KE_f (2b_1/a_0 + b_2/a_0)/16$$

$$\gamma_2 = \beta_3 = a_0 \; KE_f (2b_1/a_0 - b_2/a_0)/16$$

To represent a system of parallel fibres parallel to the direction ϕ_1, Cox takes for the distribution function $f(\theta)$ a Dirac Delta function [24] -

$$f(\theta)=\{1+2\cos 2(\theta-\phi_1)+2\cos 4(\theta-\phi_1)+----\}/\pi \qquad B.2.10$$

which is a step function having values

$\theta=\phi_1$, $f(\theta)=\infty$

$\theta\neq\phi_1$, $f(\theta)=0$

Although the function of the above equation contains a discontinuity at $\theta=\phi_1$, it is legitimate to use it as a factor in an integrand .

Equation above may be written as

$$f(\theta)=\{1+2\cos 2\phi_1\cos 2\theta+2\cos 4\phi_1\cos 4\theta+----+2\sin 2\phi_1\sin 2\theta+2\sin 4\phi_1\sin 4\theta+---\}/\pi$$

By comparison with equation ,

$$\begin{aligned} a_0 &= 1/\pi \\ a_1 &= 2/\pi\cos 2\phi_1 \\ a_2 &= 2/\pi\cos 4\phi_1 \\ b_1 &= 2/\pi\sin 2\phi_1 \\ b_2 &= 2/\pi\sin 4\phi_1 \end{aligned} \qquad B.2.11$$

Substituting these values and K from equation , into equation , the coefficients become

$$\begin{aligned} \alpha &= V_f E_f/8(3+4\cos 2\phi_1+\cos 4\phi_1) \\ \beta_1 &= \alpha_2 = \gamma_3 = V_f E_f/8(1-\cos 4\phi_1) \\ \beta_2 &= V_f E_f/8(3-4\cos 2\phi_1+\cos 4\phi_1) \\ \gamma_1 &= \alpha_3 = V_f E_f/8(2\sin 2\phi_1+\sin 4\phi_1) \\ \gamma_2 &= \beta_3 = V_f E_f/8(2\sin 2\phi_1-\sin 4\phi_1) \end{aligned} \qquad B.2.12$$

If there are n similar bands of parallel fibres lying at n different angles to OX, the contribution to stiffness of the sth band of fibre, modulus E_{fs}, fraction p_s, angle ϕ_s, is given by coefficients $(\alpha_1)_{fs}$ to $(\gamma_3)_{fs}$ where

$$(\alpha_1)_{fs} = 1/8 p_s V_f E_{fs} (3+4\cos 2\phi_s + \cos 4\phi_s)$$
$$(\beta_1)_{fs} = (\alpha_2)_{fs} = (\gamma_3)_{fs} = 1/8 p_s V_f E_{fs} (1-\cos 4\phi_s)$$
$$(\beta_2)_{fs} = 1/8 p_s V_f E_{fs} (3-4\cos 2\phi_s + \cos 4\phi_s) \qquad \text{B.2.13}$$
$$(\gamma_1)_{fs} = (\alpha_3)_{fs} = 1/8 p_s V_f E_{fs} (2\sin 2\phi_s + \sin 4\phi_s)$$
$$(\gamma_2)_{fs} = (\beta_3)_{fs} = 1/8 p_s V_f E_{fs} (2\sin 2\phi_s - \sin 4\phi_s)$$

The contribution of the lateral fibres are given by coefficients $(\alpha_1)_{ls}$ to $(\gamma_3)_{ls}$ where

$$(\alpha_1)_{ls} = 1/8 p_s V_f (1+\mu V_f) E_{ls} [\{3+4\cos 2(\phi_s+\pi/2-\psi)\} + \cos 4(\phi_s+\pi/2-\psi) + 3+4\cos 2(\phi_s+\pi/2+\psi) + \cos 4(\phi_s+\pi/2+\psi)\}] \qquad \text{B.2.14}$$

Similarly for

$(\beta_1)_{ls} = (\alpha_2)_{ls} = (\gamma_3)_{ls} =$ etc.

Stiffness contributions of the resin matrix are given by classical elasticity theory for isotropic materials. Noting that the matrix volume fraction is $(1-V_f-V_v)$, the contributions are

$$(\alpha_1)_m = (\beta_2)_m = (1-V_f-V_v) E_m/(1-\nu^2)$$
$$(\alpha_2)_m = (\beta_1)_m = \nu(1-V_f-V_v) E_m/(1-\nu^2) \qquad \text{B.2.15}$$
$$(\gamma_1)_m = (\alpha_3)_m = (\gamma_2)_m = (\beta_3)_m = 0$$
$$(\gamma_3)_m = \lambda(1-V_f-V_v) E_m/2(1+\nu)$$

The coefficients for the complete laminate, comprising real fibres, lateral fibres and resin matrix are obtained by summing the individual coefficients i.e.

$$(\alpha_1)_T = \sum_{s=1}^{s=n} (\alpha_1)_{fs} + \sum_{s=1}^{s=n} (\alpha_1)_{ls} + (\alpha_1)_m \qquad \text{B.2.16}$$

etc.

where the subscript T signifies total coefficient summing up the coefficients and applying trigonometrical simplifications, the final model which describes the relation between stress and strain is obtained as stated in chapter 3.

APPENDIX C

SIMPLIFICATION OF BISHOP'S MODEL AND PROGRAMMING FEATURES

C.1 SIMPLIFICATION OF THE MODEL

Most theories of fibre composite materials attempt to predict laminate properties from known properties of the separate fibre and resin. It is not possible, however, to take account of all the factors that affect laminate properties and properties finally predicted by the theories are no more than a guide to what may be obtained in practice. A reverse procedure is used in this model. It is proposed to adjust certain parameters (e.g. fibre modulus) so that the theoretical model is tailored to fit the observed data. The fitted function may then be used to smooth experimental data and to make predictions of an interpolative type. If the basic observed data are low, because of for example inferior resin , the predicted properties will be appropriately reduced . The designer should be able to place almost as much reliance on the values predicted by the fitted model as on directly observed data.

Simplification of the model

In our present study, we had the following conditions

$n=2$

$\phi_{s=1}=0$ $\quad p_{s1}=0.5$ C.1

$\phi_{s=2}=90$ $\quad p_{s2}=0.5$

Substituting these values in the final model,(equation), the following final expressions are obtained

$C_0 = E_f + 2E_l(1 + \mu V_f)$

$C_2=0$

$C_4 = E_f + 2E_l(1 + \mu V_f)\mathrm{Cos}4\psi$ C.2

$S_2=0$

$S_4=0$

In the present study, load was applied parallel to fibres in one direction which means that the shear stress and stress in the direction perpendicular to load are zero.

$\sigma_y = 0$ and $\sigma_{xy} = 0$.

Therefore, the equation of stress are further simplifies as

$$\sigma_x = (\alpha_1)_T \varepsilon_x + (\beta_1)_T \varepsilon_y \qquad C.3$$

$$0 = (\alpha_2)_T \varepsilon_x + (\beta_2)_T \varepsilon_y$$

Assuming V_v to be zero, the coefficients come out to be,

$$(\alpha_1)_T = (\beta_2)_T = V_f/8\{4E_f + 2E_1(1+\mu V_f(3+\text{Cos}4\psi) + (1-V_f)E_m/(1-\tilde{\nu})$$

$$(\beta_1)_T = (\alpha_2)_T = V_f/8\{2E_1(1+\mu V_f)(1-\text{Cos}4\psi)\} + \nu(1-V_f)E_m/(1-\tilde{\nu}) \qquad C.4$$

$$(\alpha_3)_T = (\beta_3)_T = (\gamma_3)_T = (\gamma_1)_T = (\gamma_2)_T = 0$$

Thus the simplified model becomes

$$\sigma_x = (\alpha_1)_T \varepsilon_x + (\beta_1)_T \varepsilon_y$$

or, $\sigma_x = \{V_f/8[4E_f + 2E_1(1+\mu V_f)(3+\text{Cos}4\psi)] + (1-V_f)E_m/(1-\tilde{\nu})\}\, \varepsilon_x + \{V_f/8[2E_1(1+\mu V_f)(1-\text{Cos}4\psi)] + \nu(1-V_f)E_m/(1-\tilde{\nu})\}\, \varepsilon_y$ C.5

and $0 = \{V_f/8[2E_1(1+\mu V_f)(1-\text{Cos}4\psi)] + \nu(1-V_f)E_m/(1-\tilde{\nu})\}\, \varepsilon_x + \{V_f/8[4E_f + 2E_1(1+\mu V_f)(3+\text{Cos}4\psi)] + (1-V_f)E_m/(1-\tilde{\nu})\}\, \varepsilon_y$

Method of Fitting the Model with Observed Data using Least Square Method

We can write

$$\sigma_x = f(E_f, E_1, E_m, \psi, \mu, \nu, V_f, \varepsilon_x, \varepsilon_y) \qquad C.6$$

The empirical constants E_f, E_1, E_m, μ, ν and ψ are to be found by fitting the observed data to the model using least square method.

Let σ_x be the predicted strength and σ_{ex} be the experimental strength. Then the standard error γ can be written as

$$\gamma = \sum_{V_f} (\sigma_x/\sigma_{ex} - 1)^2 \qquad C.7$$

Now should be minimized in order to obtain the best fit. Therefore, the following equations are to be satisfied.

$$\frac{\partial Y}{\partial E_f} = 0$$

$$\frac{\partial Y}{\partial E_l} = 0$$

$$\frac{\partial Y}{\partial E_m} = 0 \qquad \text{C.8}$$

$$\frac{\partial Y}{\partial \mu} = 0$$

$$\frac{\partial Y}{\partial \psi} = 0$$

$$\frac{\partial Y}{\partial \nu} = 0$$

or,

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial E_f} = 0$$

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial E_l} = 0$$

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial E_m} = 0 \qquad \text{C.9}$$

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial \mu} = 0$$

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial \psi} = 0$$

$$\sum_{v_f} \frac{2}{\sigma_{ex}} \left(\frac{\sigma_x}{\sigma_{ex}} - 1 \right) \frac{\partial \sigma_x}{\partial \nu} = 0$$

Solution of the above equations will give the values for $E_f, E_l, E_m, \mu, \psi, \nu$. Once these empirical constants are known for a particular type of fibre composite, then the designer can predict the strength and stiffness of that particular fibre composite with different combination of fibre volume fraction, orientation, etc. using these constants.

Substitution the expression for σ_x in the equations C.9 will make very lengthy equations. So only the final form which has been obtained after a great deal of mathematical manipulations, is given below.

$$\{E_f/2\}C+\{E_1R/2\}(C+\mu D)-A=0$$

$$\{E_f/2\}(C+\mu D)+\{E_m(1-\varsigma\nu)/(1-\nu^2)\}(F+\mu G)+\{E_1R/4\}(C+2\mu D+\mu^2E)-(A+\mu B)=0$$

$$(E_f/2)F+\{E_1R/4\}(F+\mu G)+\{E_m(1-\varsigma\nu)/(1-\nu^2)\}H-I=0$$

$$\{E_f/2\}D+\{E_1R/4(D+\mu E)+\{E_m(1-\varsigma\nu)/(1-\nu^2)\}G-B=0 \qquad \text{C.10}$$

$$\{E_f/2\}(C+\mu D)+\{E_1R/4\}(C+\mu D+\mu^2E)+\{E_m(1-\varsigma\nu)/(1-\nu^2)\}(F+\mu G)-(A+\mu B)=0$$

$$\{E_f/2\}F+\{E_1R/4\}(F+\mu G)+\{E_m(1-\varsigma\nu)/(1-\nu^2)\}H-I=0$$

where $R=(3+\text{Cos}4\psi)-\varsigma(1-\text{Cos}4\psi)$ and $\varsigma=E_f/E_m$

The expressions for A,B,C,D,E,F,G,H,and I will depend on the stiffness or strength which property is to predicted. For the prediction of stiffness of the composite laminate—

$$A=\sum V_f/E_{ex}$$

$$B=\sum V_f^2/E_{ex}$$

$$C=\sum V_f^2/E_{ex}^2$$

$$D=\sum V_f^3/E_{ex}^2$$

$$E=\sum V_f^4/E_{ex}^2 \qquad \text{C.11}$$

$$F=\sum (1-V_f)V_f/E_{ex}^2$$

$$G=\sum (1-V_f)V_f^2/E_{ex}^2$$

$$H=\sum (1-V_f)^2/E_{ex}^2$$

$$I=\sum (1-V_f)/E_{ex}$$

For the prediction of strength of the laminate—

$$A=\sum \frac{V_f\varepsilon_x}{\sigma_{ex}}$$

$$B=\sum \frac{V_f^2\varepsilon_x}{\sigma_{ex}}$$

$$C=\sum \frac{V_f^2\varepsilon_x^2}{\sigma_{ex}^2}$$

$$D=\sum \frac{V_f^3\varepsilon_x^2}{\sigma_{ex}^2} \qquad \text{C.12}$$

$$E=\sum \frac{V_f^4\varepsilon_x^2}{\sigma_{ex}^2}$$

$$F=\sum \frac{(1-V_f)V_f\varepsilon_x^2}{\sigma_{ex}^2}$$

$$G=\sum \frac{(1-V_f)V_f^2\varepsilon_x^2}{\sigma_{ex}^2}$$

$$H=\sum \frac{(1-V_f)^2\varepsilon_x^2}{\sigma_{ex}^2}$$

$$I=\sum \frac{(1-V_f)\varepsilon_x}{\sigma_{ex}}$$

V_f, E_{ex}, σ_{ex} and ε_{ex} are experimental data. A computer program is required to solve the equations mentioned above.

C.2 PROGRAMMING FEATURES

Determination of Stiffness parameters

The experimental data for V_f, E_{ex}, σ_{ex}, and ε_{ex} for all laminates are appended to the computer program. The program first calculates the values of A,B,C,D,E,F,G,H, and I for stiffness correlation. Then the equations are solved using the method of iteration. The initial value of the parameters are assumed on the basis of actual properties. If the program does not converge, these values have to be changed so that the iteration process converges. After having the solution, these stiffness parameters are fed back into the model using a subroutine in order to calculate the predicted stiffness properties of the laminate with different volume fractions.

Determination of Strength Parameters

Strength parameters are also found in the same way as for stiffness parameters with a slight change in the program. Here, the values A through I are calculated for strength correlation. Strength parameters are then fed into a subroutine to predict the strength of the laminate having different volume fractions.

APPENDIX D

PROGRAM LISTING

D.1 Listing of the Program for the Prediction of Stiffness Properties of Fibre Composites using Lamination Theory

This program has been developed by ESDU International Ltd, 251-259 Regent Street, London W1R 7AD, UK. It is written in BASIC. The program listed below.

Listing of Program in BASIC

```
100 REM PROGRAM A83035
110 REM ==============
120 INIT
130 SET DEGREES
140 P1=41
150 PRINT "INPUT NUMBER OF LAYERS IN PLATE AND SYMMETRY CODE"
160 INPUT N1,C1
170 DIM Q(N1,8)
180 Q=0
190 IF C1=3 THEN 340
200 IF C1=2 THEN 240
210 A$="     SYMMETRIC LAMINATE"
220 K1=1
230 GO TO 260
240 A$="ANTI-SYMMETRIC LAMINATE"
250 K1=-1
260 K3=INT(N1/2)
270 IF N1/2-K3>0 THEN 310
280 K2=1
290 I1=K3
300 GO TO 370
310 K2=2
320 I1=K3+1
330 GO TO 370
340 A$="    ASYMMETRIC LAMINATE"
350 I1=N1
360 K2=0
370 PRINT @P1: USING 380:A$,N1
380 IMAGE /,4X,23A,4D," LAYERS",/,/
390 PRINT "ARE ANY LAYER PROPERTIES TO BE ESTIMATED ?"
400 PRINT "ANSWER YES OR NO"
410 INPUT Y$
420 IF Y$="YES" THEN 440
430 GO TO 450
```

```
440 GOSUB 1180
450 H1=0
460 FOR I=1 TO I1
470 PRINT "LAYER ";I;"   INPUT CODE,PSI(degrees)"
480 INPUT X,Q(I,1)
490 IF X<0 THEN 540
500 IF X>0 THEN 620
510 PRINT "INPUT THE 5 LAYER PROPERTIES"
520 INPUT Q(I,2),Q(I,3),Q(I,4),Q(I,5),Q(I,6)
530 GO TO 600
540 PRINT "INPUT LAYER THICKNESS,LAMINA NUMBER"
550 INPUT Q(I,2),N3
560 Q(I,3)=Q2(N3,1)
570 Q(I,4)=Q2(N3,2)
580 Q(I,5)=Q2(N3,3)
590 Q(I,6)=Q2(N3,4)
600 Q(I,8)=1-Q(I,6)*Q(I,6)*Q(I,4)/Q(I,3)
610 GO TO 650
620 FOR J=2 TO 8
630 Q(I,J)=Q(X,J)
640 NEXT J
650 H1=H1+Q(I,2)
660 IF C1=3 THEN 730
670 IF I=I1 AND K2=2 THEN 730
680 Q(N1-I+1,1)=Q(I,1)*K1
690 FOR J=2 TO 8
700 Q(N1-I+1,J)=Q(I,J)
710 NEXT J
720 H1=H1+Q(I,2)
730 NEXT I
740 X=0
750 FOR I=1 TO N1
760 Q(I,7)=H1/2-(X+Q(I,2)/2)
770 X=X+Q(I,2)
780 NEXT I
790 PRINT @P1:"    PLATE CONSTRUCTION"
800 PRINT @P1: USING 810:
810 IMAGE /,4X,"LAYER LAYER   LAYER",6X,"DISTANCE FROM  E(ALPHA)",5X,S
820 PRINT @P1: USING 830:
830 IMAGE "E(BETA)",6X,"G(ALPHA-   POISSONS RATIO"
840 PRINT @P1: USING 850:
850 IMAGE 4X,"NO.   ANGLE   THICKNESS  REF. PLANE",S
860 PRINT @P1: USING 870:
870 IMAGE 34X,"BETA)    (ALPHA-BETA)",/
880 FOR I=1 TO N1
890 PRI @P1: USI 910:I,Q(I,1),Q(I,2),Q(I,7),Q(I,3),Q(I,4),Q(I,5),Q(I,6)
900 NEXT I
910 IMAGE 6D,7D.1D,5(2X,3E),4D.3D
920 GOSUB 1550
930 GOSUB 2250
940 PRINT @P1: USING 950:
950 IMAGE /,/,4X,"IN-PLANE STIFFNESS SUB-MATRIX A",/
960 V=A
970 GOSUB 2170
980 PRINT @P1: USING 990:
990 IMAGE /,/,4X,"COUPLED IN-PLANE AND FLEXURAL STIFFNESS",S
1000 PRINT @P1: USING 1010:
1010 IMAGE " SUB-MATRIX B",/
1020 V=B
1030 GOSUB 2170
1040 PRINT @P1: USING 1050:
1050 IMAGE /,/,4X,"FLEXURAL STIFFNESS SUB-MATRIX D",/
1060 V=D
1070 GOSUB 2170
1080 B$="THE PLATE STIFFNESS MATRICES"
```

```
1090 C$=" SATISFY THE CONDITIONS FOR SPECIAL ORTHOTROPY"
1100 IF X9>0 THEN 1140
1110 PRINT @P1: USING 1120:B$,C$
1120 IMAGE /,/,4X,28A,46A,/,/
1130 GO TO 1160
1140 PRINT @P1: USING 1150:B$,C$
1150 IMAGE /,/,4X,28A," DO NOT",46A,/,/
1160 GOSUB 2640
1170 END


1180 REM SUBROUTINE B83035
1190 REM ================
1200 REM CALCULATE LAMINA ELASTIC PROPERTIES
1210 PRINT "INPUT NUMBER OF LAMINA FOR WHICH ELASTIC"
1220 PRINT "PROPERTIES ARE TO BE CALCULATED"
1230 INPUT N2
1240 DIM Q1(2,5),Q2(N2,4)
1250 FOR I=1 TO N2
1260 PRINT "LAMINA ";I;"   INPUT THE 5 FIBRE PROPERTIES"
1270 INPUT Q1(1,1),Q1(1,2),Q1(1,3),Q1(1,4),Q1(1,5)
1280 PRINT "LAMINA ";I;"   INPUT THE 4 MATRIX PROPERTIES"
1290 INPUT Q1(2,1),Q1(2,2),Q1(2,3),Q1(2,4)
1300 Q1(2,5)=1-Q1(1,5)
1310 Q2(I,1)=Q1(1,1)*Q1(1,5)+Q1(2,1)*Q1(2,5)
1320 X1=Q1(1,2)/Q1(2,2)
1330 X2=(X1-1)/(X1+2)
1340 X3=X2*Q1(1,5)
1350 Q2(I,2)=Q1(2,2)*(1+2*X3)/(1-X3)
1360 X1=Q1(1,3)/Q1(2,3)
1370 X2=X1*(1+Q1(1,5))+Q1(2,5)
1380 X3=X1*Q1(2,5)+1+Q1(1,5)
1390 Q2(I,3)=Q1(2,3)*X2/X3
1400 Q2(I,4)=Q1(1,4)*Q1(1,5)+Q1(2,4)*Q1(2,5)
1410 PRINT @P1: USING 1420:I
1420 IMAGE "    LAMINA",2D," PROPERTIES",/
1430 PRINT @P1: USING 1440:
1440 IMAGE 13X,"E(ALPHA)      E(BETA)      G(ALPHA-    POISSONS RATIO",S
1450 PRINT @P1: USING 1460:
1460 IMAGE "  VOLUME",/,42X,"BETA)   (ALPHA-BETA)   FRACTION",/
1470 PRINT @P1: USING 1480:Q1(1,1),Q1(1,2),Q1(1,3),Q1(1,4),Q1(1,5)
1480 IMAGE 4X,"FIBRE ",3(2X,3E),4D.3D,10D.3D
1490 PRINT @P1: USING 1500:Q1(2,1),Q1(2,2),Q1(2,3),Q1(2,4),Q1(2,5)
1500 IMAGE 4X,"MATRIX",3(2X,3E),4D.3D,10D.3D
1510 PRINT @P1: USING 1520:Q2(I,1),Q2(I,2),Q2(I,3),Q2(I,4)
1520 IMAGE 4X,"LAMINA",3(2X,3E),4D.3D,11X,"-",/,/
1530 NEXT I
1540 RETURN


1550 REM SUBROUTINE C83035
1560 REM ================
1570 REM CALCULATE STIFFNESS MATRICES
1580 DIM A(3,3),B(3,3),C(3,3),D(3,3),T(3,3)
1590 DIM T1(3,3),U(3,3),V(3,3),W(3,3)
1600 A=0
1610 B=0
1620 C=0
1630 D=0
1640 FOR I=1 TO N1
1650 C(1,1)=Q(I,3)/Q(I,8)
1660 C(1,2)=Q(I,6)*Q(I,4)/Q(I,8)
1670 C(2,1)=C(1,2)
1680 C(2,2)=Q(I,4)/Q(I,8)
1690 C(3,3)=Q(I,5)
```

```
1700 X1=SIN(Q(I,1))
1710 X2=COS(Q(I,1))
1720 X3=SIN(2*Q(I,1))
1730 X4=COS(2*Q(I,1))
1740 T(1,1)=X2*X2
1750 T(1,2)=X1*X1
1760 T(1,3)=0.5*X3
1770 T(2,1)=T(1,2)
1780 T(2,2)=T(1,1)
1790 T(2,3)=-T(1,3)
1800 T(3,1)=-X3
1810 T(3,2)=X3
1820 T(3,3)=X4
1830 FOR J=1 TO 3
1840 FOR J1=1 TO 3
1850 T1(J,J1)=T(J1,J)
1860 NEXT J1
1870 NEXT J
1880 V=T1
1890 W=C
1900 GOSUB 2050
1910 V=U
1920 W=T
1930 GOSUB 2050
1940 X=Q(I,2)
1950 V=X*U
1960 A=A+V
1970 X=X*Q(I,7)
1980 V=X*U
1990 B=B+V
2000 X=X*Q(I,7)+Q(I,2) 3/12
2010 V=X*U
2020 D=D+V
2030 NEXT I
2040 RETURN


2050 REM SUBROUTINE D83035
2060 REM ================
2070 REM MATRIX MULTIPLY,V*W
2080 FOR J1=1 TO 3
2090 FOR J2=1 TO 3
2100 U(J1,J2)=0
2110 FOR J3=1 TO 3
2120 U(J1,J2)=U(J1,J2)+V(J1,J3)*W(J3,J2)
2130 NEXT J3
2140 NEXT J2
2150 NEXT J1
2160 RETURN


2170 REM SUBROUTINE E83035
2180 REM ================
2190 REM MATRIX OUTPUT
2200 FOR J1=1 TO 3
2210 PRINT @P1: USING 2230:V(J1,1),V(J1,2),V(J1,3)
2220 NEXT J1
2230 IMAGE 2X,3(2X,4E)
2240 RETURN


2250 REM SUBROUTINE F83035
2260 REM ================
2270 REM CHECK FOR SPECIAL ORTHOTROPY
2280 A1=0
```

```
2290 D1=0
2300 FOR J1=1 TO 3
2310 FOR J2=1 TO 3
2320 IF ABS(A(J1,J2))>A1 THEN 2350
2330 IF ABS(D(J1,J2))>D1 THEN 2370
2340 GO TO 2380
2350 A1=ABS(A(J1,J2))
2360 GO TO 2330
2370 D1=ABS(D(J1,J2))
2380 NEXT J2
2390 NEXT J1
2400 B1=SQR(A1*D1)
2410 A2=0
2420 B2=0
2430 D2=0
2440 FOR J1=1 TO 3
2450 FOR J2=1 TO 3
2460 IF ABS(B(J1,J2))/B1>1.0E-6 THEN 2550
2470 B(J1,J2)=0
2480 IF J1=J2 THEN 2600
2490 IF J1+J2=3 THEN 2600
2500 IF ABS(A(J1,J2))/A1>1.0E-6 THEN 2570
2510 A(J1,J2)=0
2520 IF ABS(D(J1,J2))/D1>1.0E-6 THEN 2590
2530 D(J1,J2)=0
2540 GO TO 2600
2550 B2=B2+ABS(B(J1,J2))
2560 GO TO 2480
2570 A2=A2+ABS(A(J1,J2))
2580 GO TO 2520
2590 D2=D2+ABS(D(J1,J2))
2600 NEXT J2
2610 NEXT J1
2620 X9=A2+B2+D2
2630 RETURN

2640 REM SUBROUTINE G83035
2650 REM ================
2660 REM CALCULATE PLATE APPARENT ELASTIC PROPERTIES
2670 A=A/H1
2680 Q(1,6)=A(2,1)/A(2,2)
2690 Q(1,7)=A(1,2)/A(1,1)
2700 Q(1,8)=1-Q(1,6)*Q(1,7)
2710 Q(1,3)=A(1,1)*Q(1,8)
2720 Q(1,4)=A(2,2)*Q(1,8)
2730 Q(1,5)=A(3,3)
2740 PRINT @P1:"    APPARENT ELASTIC PROPERTIES"
2750 PRINT @P1: USING 2760:Q(1,3),Q(1,4),Q(1,5)
2760 IMAGE /,4X,"E(ALPHA)=",3E,"  E(BETA)=",3E,"  G(ALPHA-BETA)=",3E
2770 PRINT @P1: USING 2780:Q(1,6)
2780 IMAGE 4X,"POISSONS RATIO(ALPHA-BETA)=",1D.4D
2790 PRINT @P1: USING 2800:Q(1,7)
2800 IMAGE 4X,"POISSONS RATIO(BETA-ALPHA)=",1D.4D
2810 PRINT @P1: USING 2820:H1
2820 IMAGE 4X,"PLATE THICKNESS=",3E
2830 RETURN
```

D.2 Listing of the Program for the Prediction of Strength and Stiffness Properties of Fibre Composites using Bishop's Model

This program has been written in FORTRAN language by the author. The program is listed below.

```
1       DIMENSION ERROR(6),VF(6),ELAS(6),SX(6),EX(6)
2       OPEN(UNIT=8,FILE='IN',STATUS='OLD')
3       OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
        WRITE(9,1001)
1001    FORMAT('EXPERIMENTAL RESULTS FOR JUTE-GLASS COMPOSITE',//,
       +5X,'VF',10X,'ELAS',10X,'STRENGTH',10X,'STRAIN',//)
        DO 201 I=1,5
5       READ(8,10) VF(I),ELAS(I),SX(I),EX(I)
10      FORMAT(F5.3,E8.2,E8.2,F7.5)
        WRITE(9,9) VF(I),ELAS(I),SX(I),EX(I)
9       FORMAT(1X,F6.4,8X,E8.2,8X,E8.2,8X,F7.5)
201     CONTINUE
        CALL BISHOP(VF,ELAS,SX,EX,A,B,C,D,E,F,G,H,AI)
        WRITE(9,101) A,B,C,D,E,F,G,H,AI
101     FORMAT(//,'A= ',E8.2,/,5X,'B= ',E8.2,/,5X,'C= ',E8.2,/,5X,
       +'D= ',E8.2,/,5X,'E= ',E8.2,/,5X,'F= ',E8.2,/,5X,'G= ',E8.2,
       +/,5X,'H= ',E8.2,/,5X,'AI= ',E8.2,//)
11      EF=8.7E+06
12      EL=3.2E+03
13      EM=5.7E+02
14      AM=4.86
15      AN=1.00
16      R=1.65
        WRITE(9,951) EF,EL,EM,AM,AN,R
951     FORMAT('INITIAL VALUES',//,1X,'EF=',E9.3,5X,'EL=',E9.3,5X,'EM=',
       +E9.3,/,1X,'AM=',F8.2,5X,'AN=',F8.2,5X,'R=',F8.2,//)
17      EFF=EF
18      ELL=EL
19      EMM=EM
20      AMM=AM
21      ANN=AN
22      RR=R
        WRITE(*,*) EF,EL,EM,AM,AN,R
30      EF=(4.*A-EL*R*(C+AM*D))/(2.*C)
31      EL=(4.*(A+AM*B)-2.*EF*(C+AM*D)-4.*EM*AN*(F+AM*G))/(R*(C+2.*AM*D+
       +E*AM**2))
32      R=(4.*(A+AM*B)-2.*EF*(C+AM*D)-4.*EM*AN*(F+AM*G))/(EL*(C+2.*AM*D+
       +E*AM**2))
33      EM=(4.*AI-2.*EF*F-EL*R*(F+AM*G))/(4.*AN*H)
34      AN=(4.*AI-2.*EF*F-EL*R*(F+AM*G))/(4.*EM*H)
35      AM=(4.*B-2.*EF*D-EL*R*D-4.*EM*AN*G)/(EL*R*E)
40      ERROR(1)=ABS(EFF-EF)
41      ERROR(2)=ABS(ELL-EL)
42      ERROR(3)=ABS(EMM-EM)
43      ERROR(4)=ABS(AMM-AM)
44      ERROR(5)=ABS(ANN-AN)
45      ERROR(6)=ABS(RR-R)
46      T=ERROR(1)
```

```
51    IF(ERROR(I+1).LE.T)GOTO 53
52    T=ERROR(I+1)
53    CONTINUE
      WRITE(*,*) T
60    IF (T.GE.1.5) GOTO 17
      WRITE(9,324)
324   FORMAT(5X,'FINAL VALUES',//)
61    WRITE (9,65) EF,EL,EM,AM,AN,R,T
65    FORMAT(5X,'EF= ',E8.2,5X,'EL= ',E8.2,5X,'EM= ',E8.2,/,5X,'AM= '
     +,F8.3,5X,'AN= ',F8.5,5X,'R= ',F8.5,/,'MAXM ERROR=',F8.3,//)
      ANU=0.30
      COSF=(R-2.67)/1.33
      WRITE(9,205)
205   FORMAT('PREDICTION OF STIFFNESS AND STRENGTH OF JUTE-GLASS',/,
     +'FIBRE REINFORCED COMPOSITE LAMINATES USING BISHOPS MODEL',//)
      CALL PREDIC (EF,EL,EM,AM,ANU,COSF,VF,EX,ELAS,SX)
      WRITE(9,501)
501   FORMAT(5X,'VOLUME FRACTION',10X,'YOUNGS MODULUS',10X,
     +'TENSILE STRENGTH',10X,'STRAIN',//)
      DO 108 I=1,5
      WRITE (9,105) VF(I),ELAS(I),SX(I),EX(I)
105   FORMAT(10X,F6.4,15X,E8.2,20X,E8.2,15X,F8.6)
108   CONTINUE
      STOP
      END
      SUBROUTINE BISHOP(VF,ELAS,SX,EX,A,B,C,D,E,F,G,H,AI)
C     PROGRAM TO CALCULATE THE EMPIRICAL CONSTANTS A,B,C,D,E,F,G,H,AI
C     IN BISHOP'S MODEL
      DIMENSION VF(6),ELAS(6),SX(6),EX(6)
      AT=0.0
      BT=0.0
      CT=0.0
      DT=0.0
      ET=0.0
      FT=0.0
      GT=0.0
      HT=0.0
      AIT=0.0
      DO 90 I=1,5
      AT=AT+VF(I)/ELAS(I)
      BT=BT+(VF(I)**2)/ELAS(I)
      CT=CT+(VF(I)**2)/(ELAS(I)**2)
      DT=DT+(VF(I)**3)/(ELAS(I)**2)
      ET=ET+(VF(I)**4)/(ELAS(I)**2)
      FT=FT+(1.-VF(I))*VF(I)/(ELAS(I)**2)
      GT=GT+(1.-VF(I))*(VF(I)**2)/(ELAS(I)**2)
      HT=HT+((1.-VF(I))**2)/(ELAS(I)**2)
      AIT=AIT+(1.-VF(I))/ELAS(I)
90    CONTINUE
      A=AT
      B=BT
      C=CT
      D=DT
```

```
      E=ET
      F=FT
      G=GT
      H=HT
      AI=AIT
      RETURN
      END
      SUBROUTINE PREDIC(EF,EL,EM,AM,ANU,COSF,VF,EX,ELAS,SX)
C**********************************************************************
C     SUBROUTINE TO FIND PREDICTED STIFFNESS AND STRENGTH OF JUTE-GLASS-
C     FIBRE REINFORCED COMPOSITE LAMINATES USING BISHOPS MODEL
C**********************************************************************
      DIMENSION VF(5),CO(5),CF(5),ALOT(5),ALTT(5),BOT(5),BTT(5),GTH(5),
     +ELAS(5),SX(5),EX(5)
      DO 1000 I=1,5
      CO(I)=EF+2.*EL*(1.+AM*VF(I))
      CF(I)=EF+2.*EL*(1.+AM*VF(I))*COSF
      ALOT(I)=VF(I)*(3.*CO(I)+CF(I))/8.+(1.-VF(I))*EM/(1.-ANU**2)
      BOT(I)=VF(I)*(CO(I)-CF(I))/8.+ANU*(1.-VF(I))*EM/(1.-ANU**2)
      BTT(I)=ALOT(I)
      ALTT(I)=BOT(I)
      GTH(I)=VF(I)*(CO(I)-CF(I))/8.+(1.-VF(I))*EM/(2.*(1.+ANU))
      ELAS(I)=ALOT(I)-(BOT(I)*ALTT(I))/BTT(I)
      SX(I)=ELAS(I)*EX(I)
 1000 CONTINUE
      RETURN
      END
```

```
C      PROGRAM TO FIND VALUES FOR EF,EL,EM,M,N & COSF FOR PREDICTION OF
C      STIFFNESS AND STRENGTH USING BISHOP'S MODEL BY ITERATION
  1    DIMENSION ERROR(6),VF(6),ELAS(6),SX(6),EX(6)
  2    OPEN(UNIT=8,FILE='IN',STATUS='OLD')
  3    OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
       WRITE(9,1001)
 1001  FORMAT('EXPERIMENTAL RESULTS FOR JUTE-GLASS COMPOSITE',//,
      +5X,'VF',10X,'ELAS',10X,'STRENGTH',10X,'STRAIN',//)
       DO 201 I=1,5
  5    READ(8,10) VF(I),ELAS(I),SX(I),EX(I)
  10   FORMAT(F5.3,E8.2,E8.2,F7.5)
       WRITE(9,9) VF(I),ELAS(I),SX(I),EX(I)
  9    FORMAT(1X,F6.4,8X,E8.2,8X,E8.2,8X,F7.5)
  201  CONTINUE
       CALL BISHOP(VF,ELAS,SX,EX,A,B,C,D,E,F,G,H,AI)
       WRITE(9,101) A,B,C,D,E,F,G,H,AI
  101  FORMAT(//,'A= ',E8.2,/,5X,'B= ',E8.2,/,5X,'C= ',E8.2,/,5X,
      +'D= ',E8.2,/,5X,'E= ',E8.2,/,5X,'F= ',E8.2,/,5X,'G= ',E8.2,
      +/,5X,'H= ',E8.2,/,5X,'AI= ',E8.2,//)
  11   EF=8.7E+06
  12   EL=8.2E+03
  13   EM=5.7E+02
  14   AM=4.86
  15   AN=1.00
  16   R=1.65
       WRITE(9,951) EF,EL,EM,AM,AN,R
  951  FORMAT('INITIAL VALUES',//,1X,'EF=',E9.3,5X,'EL=',E9.3,5X,'EM=',
      +E9.3,/,1X,'AM=',F8.2,5X,'AN=',F8.2,5X,'R=',F8.2,//)
  17   EFF=EF
  18   ELL=EL
  19   EMM=EM
  20   AMM=AM
  21   ANN=AN
  22   RR=R
       WRITE(*,*) EF,EL,EM,AM,AN,R
  30   EF=(4.*A-EL*R*(C+AM*D))/(2.*C)
  31   EL=(4.*(A+AM*B)-2.*EF*(C+AM*D)-4.*EM*AN*(F+AM*G))/(R*(C+2.*AM*D+
      +E*AM**2))
  32   R=(4.*(A+AM*B)-2.*EF*(C+AM*D)-4.*EM*AN*(F+AM*G))/(EL*(C+2.*AM*D+
      +E*AM**2))
  33   EM=(4.*AI-2.*EF*F-EL*R*(F+AM*G))/(4.*AN*H)
  34   AN=(4.*AI-2.*EF*F-EL*R*(F+AM*G))/(4.*EM*H)
  35   AM=(4.*B-2.*EF*D-EL*R*D-4.*EM*AN*G)/(EL*R*E)
  40   ERROR(1)=ABS(EFF-EF)
  41   ERROR(2)=ABS(ELL-EL)
  42   ERROR(3)=ABS(EMM-EM)
  43   ERROR(4)=ABS(AMM-AM)
  44   ERROR(5)=ABS(ANN-AN)
  45   ERROR(6)=ABS(RR-R)
  46   T=ERROR(1)
  50   DO 53 I=1,6
```

```
51    IF(ERROR(I+1).LE.T)GOTO 53
52    T=ERROR(I+1)
53    CONTINUE
      WRITE(M,*) T
60    IF (T.GE.1.5) GOTO 17
      WRITE(9,324)
324   FORMAT(5X,'FINAL VALUES',//)
61    WRITE (9,65) EF,EL,EM,AM,AN,R,T
65    FORMAT(5X,'EF= ',E8.2,5X,'EL= ',E8.2,5X,'EM= ',E8.2,/,5X,'AM= '
     +,F8.3,5X,'AN= ',F8.5,5X,'R= ',F8.5,/,'MAXM ERROR=',F8.3,//)
      ANU=0.30
      COSF=(R-2.67)/1.33
      WRITE(9,205)
205   FORMAT('PREDICTION OF STIFFNESS AND STRENGTH OF JUTE-GLASS',/,
     +'FIBRE REINFORCED COMPOSITE LAMINATES USING BISHOPS MODEL',//)
      CALL PREDIC (EF,EL,EM,AM,ANU,COSF,VF,EX,ELAS,SX)
      WRITE(9,501)
501   FORMAT(5X,'VOLUME FRACTION',10X,'YOUNGS MODULUS',10X,
     +'TENSILE STRENGTH',10X,'STRAIN',//)
      DO 108 I=1,5
      WRITE (9,105) VF(I),ELAS(I),SX(I),EX(I)
105   FORMAT(10X,F6.4,15X,E8.2,20X,E8.2,15X,F8.6)
108   CONTINUE
      STOP
      END
      SUBROUTINE BISHOP(VF,ELAS,SX,EX,A,B,C,D,E,F,G,H,AI)
C**********************************************************************
C     PROGRAM TO CALCULATE THE EMPIRICAL CONSTANTS A,B,C,D,E,F,G,H,AI
C     IN BISHOP'S MODEL
C**********************************************************************
      DIMENSION VF(6),ELAS(6),SX(6),EX(6)
      AT=0.0
      BT=0.0
      CT=0.0
      DT=0.0
      ET=0.0
      FT=0.0
      GT=0.0
      HT=0.0
      AIT=0.0
      DO 90 I=1,5
      AT=AT+VF(I)*EX(I)/SX(I)
      BT=BT+(VF(I)**2)*EX(I)/SX(I)
      CT=CT+(VF(I)**2)*(EX(I)**2)/(SX(I)**2)
      DT=DT+(VF(I)**3)*(EX(I)**2)/(SX(I)**2)
      ET=ET+(VF(I)**4)*(EX(I)**2)/(SX(I)**2)
      FT=FT+(1.-VF(I))*VF(I)*(EX(I)**2)/(SX(I)**2)
      GT=GT+(1.-VF(I))*(VF(I)**2)*(EX(I)**2)/(SX(I)**2)
      HT=HT+((1.-VF(I))**2)*(EX(I)**2)/(SX(I)**2)
      AIT=AIT+(1.-VF(I))*EX(I)/SX(I)
90    CONTINUE
      A=AT
      B=BT
      C=CT
      D=DT
```

```
      E=ET
      F=FT
      G=GT
      H=HT
      AI=AIT
      RETURN
      END
      SUBROUTINE PREDIC(EF,EL,EM,AM,ANU,COSF,VF,EX,ELAS,SX)
C***********************************************************************
C     SUBROUTINE TO FIND PREDICTED STIFFNESS AND STRENGTH OF JUTE-GLASS
C     FIBRE REINFORCED COMPOSITE LAMINATES USING BISHOPS MODEL
C***********************************************************************
      DIMENSION VF(5),CO(5),CF(5),ALOT(5),ALTT(5),BOT(5),BTT(5),GTH(5),
     +ELAS(5),SX(5),EX(5)
      DO 1000 I=1,5
      CO(I)=EF+2.*EL*(1.+AM*VF(I))
      CF(I)=EF+2.*EL*(1.+AM*VF(I))*COSF
      ALOT(I)=VF(I)*(3.*CO(I)+CF(I))/8.+(1.-VF(I))*EM/(1.-ANU**2)
      BOT(I)=VF(I)*(CO(I)-CF(I))/8.+ANU*(1.-VF(I))*EM/(1.-ANU**2)
      BTT(I)=ALOT(I)
      ALTT(I)=BOT(I)
      GTH(I)=VF(I)*(CO(I)-CF(I))/8.+(1.-VF(I))*EM/(2.*(1.+ANU))
      ELAS(I)=ALOT(I)-(BOT(I)*ALTT(I))/BTT(I)
      SX(I)=ELAS(I)*EX(I)
 1000 CONTINUE
      RETURN
      END
```

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TABLES

TABLE 1

Mechanical Properties of the Constituents of the Laminate

Jute Fibre (BTA) *			Glass Fibre (E) *			Resin Matrix+(Polyester)		
Density gm/cc #	Young's Moduli psi	Poisson's Ratio	Density gm/cc	Young's Moduli psi	Poisson's Ratio	Density gm/cc(s)	Young's Moduli psi	Poisson' Ratio
1.12245	8.67E+5	0.35	2.55	1.E+7	0.2	1.26	3.E+5	0.3

*- Taken from literature

#- Experimentally found by the author

+- Obtained from Manufacturer's Catalog

TABLE 2

Specifications of Unsaturated Polyester Resin :

Used in Present Work			Used by Kazim		
Specification	Stiffness psi	Strength psi	Specification	Stiffness psi	Strength psi
EPOLAC G-153ALX	3.0E+05	5000	EPOLAC G-774 TSY	2.5E+04	2000

TENSILE TEST DATA

TABLE 4.2

LAMINATE I

No. of Jute Layers = 1 No. of Glsss Layers = 1

Volume Fraction of Jute = 0.189 Volume Fraction of Glass = 0.0626

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.1454

Stacking Sequence : Jute + Glass

Specimen No.	Plate Thick-ness,mm	Width of Plate mm	Gage length of Specimen mm	Load at Failure Kg	Strain at Failure mm/mm	Tensile Strength Kg/mm**2	Young's Modulus Kg/mm**2
1	1.1	27.7	178.0	86.0	0.00421	2.825	535.5
2	1.2	27.2	175.0	81.0	0.0052	2.4816	525.6
3	1.2	26.9	176.5	84.0	0.0056	2.6022	527.0
4	1.15	26.8	177.0	76.0	0.0054	2.4659	524.9
5	1.1	26.9	178.0	88.0	0.0051	2.974	537.5
6	1.15	27.6	179.0	90.0	0.0042	2.824	531.9
7	1.1	26.85	176.5	93.5	0.0041	3.16	543.9
8	1.1	26.8	175.5	87.25	0.0046	2.96	537.5
9	1.1	27.55	174.5	92.5	0.0046	3.05	539.7
10	1.15	27.25	177.5	90.5	0.0049	2.886	536.9

TABLE 4.3

LAMINATE 2

No. of Jute Layers = 2 No. of Glass Layers = 1

Volume Fraction of Jute = 0.257 Volume Fraction of Glass = 0.0397

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.1141

Stacking Sequence : Jute + Glass + Jute

Specimen No.	Plate Thick-ness, mm	Width of Plate mm	Gage Length of Specimen mm	Load at Failure Kg	Strain at Failure mm/mm	Tensile Strength Kg/mm**2	Young's Modulus Kg/mm**2
11	1.8	28.15	177.0	118.0	0.0042	2.3288	428.4
12	1.9	28.1	178.0	131.0	0.0048	2.4536	434.0
13	2.0	27.9	179.5	139.0	0.0046	2.4885	431.9
14	2.05	27.6	180.0	143.0	0.0054	2.5269	444.6
15	2.1	27.65	180.5	146.0	0.0055	2.5144	436.1
16	2.2	27.4	181.5	155.0	0.0061	2.576	438.2
17	2.15	27.55	180.0	155.0	0.0062	2.612	453.0
18	2.1	27.65	182.5	150.5	0.0058	2.59	448.8
19	2.05	27.75	182.0	142.0	0.0049	2.496	442.4
20	1.95	27.9	184.0	140.0	0.0054	2.568	446.0

TABLE 4.4

LAMINATE 3

No. of Jute Layers = 3 No. of Glass Layers = 1

Volume Fraction of Jute = 0.257 Volume Fraction of Glass = 0.03

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.09622

Stacking Sequence = Jute + Jute + Glass + Jute

Specimen No.	Plate Thick-ness, mm	Width of Plate mm	Gage Length of Specimen mm	Load at Failure Kg	Strain at Failure mm/mm	Tensile Strength Kg/mm**2	Young's Modulus Kg/mm**2
21	2.45	27.9	180.0	158.0	0.0048	2.3115	395.9
22	2.45	27.65	179.5	147.0	0.00496	2.17	386.1
23	2.5	27.4	181.0	147.0	0.0063	2.146	384.0
24	2.6	27.55	179.5	162.0	0.00615	2.2616	392.4
25	2.65	27.35	180.5	166.0	0.0054	2.2904	393.5
26	1.95	26.95	181.0	122.0	0.0059	2.324	397.7
27	2.6	28.05	178.5	177.0	0.00576	2.432	386.9
28	2.55	28.1	178.0	163.0	0.00578	2.278	388.2
29	2.55	27.45	178.0	167.0	0.00624	2.386	396.7
30	2.3	28.2	180.5	150.0	0.00582	2.312	393.8

TABLE 4.5

LAMINATE 4

No. of Jute Layers = 1

Volume Fraction of Jute = 0.173

Fibre Orientation = 0 and 90

No. of Glass Layers = 2

Volume Fraction of Glass = 0.121

Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.25

Stacking Sequence = Glass + Jute + Glass

Specimen No.	Plate Thick-ness, mm	Width of Plate mm	Gage Length of Specimen mm	Load at Failure Kg	Strain at Failure mm/mm	Tensile Strength Kg/mm**2	Young's Modulus Kg/mm**2
31	1.2	28.2	178.5	154.0	0.00386	4.561	1013.1
32	1.2	27.95	179.0	148.0	.00432	4.4126	1011.7
33	1.25	27.75	179.0	143.5	0.00413	4.136	1004.0
34	1.3	27.8	178.5	153.0	0.00532	4.24	1004.7
35	1.3	27.15	179.0	146.0	0.00419	4.142	1003.3
36	1.25	27.55	181.0	152.0	0.0041	4.4138	1012.4
37	1.3	28.0	178.5	165.0	0.00371	4.532	1013.8
38	1.25	27.15	179.0	157.0	0.00454	4.628	1015.2
39	1.2	27.35	178.0	151.0	0.00326	4.589	1011.7
40	1.15	28.2	178.0	147.5	0.00379	4.546	1008.9

TABLE 4.6

LAMINATE 5

No. of Jute Layers = 4 No. of Glass Layers = 1

Volume Fraction of Jute = o.2687 Volume Fraction of Glass = 0.0235

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.0861

Stacking Sequence = Jute + Jute + Glass + Jute + Jute

Specimen No.	Plate Thick-ness, mm	Width of Plate mm	Gage Length of Specimen mm	Load at Failure Kg	Strain at Failure mm/mm	Tensile Strength Kf/mm**2	Young's Modulus Kg/mm**2
41	2.8	28.25	177.0	172.0	0.0069	2.1745	290.3
42	2.9	27.75	178.5	174.5	0.0065	2.168	283.9
43	3.0	27.85	180.0	182.0	0.0071	2.1783	287.4
44	3.15	27.65	179.5	190.5	0.0067	2.187	286.8
45	3.15	27.45	177.0	191.0	0.0074	2.208	279.0
46	3.15	27.45	179.0	188.0	0.0061	2.176	277.6
47	3.25	28.0	178.0	196.5	0.0069	2.159	279.7
48	3.0	27.55	179.5	180.0	0.0066	2.182	274.1
49	3.15	27.9	179.0	189.0	0.0071-	2.153	278.3
50	3.1	27.7	180.0	186.0	0.0068	2.164	288.2

FLEXURE TEST DATA

TABLE 4.7

LAMINATE 1

No. of Jute Layers = 2

No. of Glass Layers = 1

Volume Fraction of Jute = 0.254

Volume Fraction of Glass = 0.038

Fibre Orientation = 0 and 90

Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.1104

Stacking Sequence = Jute + Glass + Jute

Specimen No.	Plate Thickness mm	Width of Plate mm	Span Length mm	Load Newton	Deflection mm	Flexural Modulus Newton/mm**2
1	2.1	14.7	25.4	4.55	0.104	1316.57
2	2.1	14.5	25.4	2.55	0.0845	920.67
3	2.15	14.5	25.4	1.55	0.061	722.37
4	2.125	14.75	25.4	2.55	0.086	858.25
5	2.125	14.25	25.4	3.5	0.11	953.3
6	2.15	14.25	25.4	3.5	0.127	797.2
7	2.1	14.2	25.4	5.0	0.154	1011.45
8	2.125	14.3	25.4	3.0	0.10	895.67
9	2.125	14.05	25.4	3.0	0.09	1012.9
10	2.15	14.45	25.4	3.0	0.11	778.0

TABLE 4.8

LAMINATE 2

No. of Jute Layers = 1 No. of Glass Layers = 1

Volume Fraction of Jute = 0.205 Volume Fraction of Glass = 0.058

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.1394

Stacking Sequence = Jute + Glass

Specimen No.	Plate Thickness mm	Width of Plate mm	Span Length mm	Load Newton	Deflection mm	Flexural Modulus Newton/mm**2
11	1.375	14.65	25.4	5.0	0.1302	4137.35
12	1.375	14.75	25.4	5.0	0.137	3885.16
13	1.4	14.625	25.4	6.0	0.145	4224.2
14	1.425	14.75	25.4	6.0	0.136	4234.6
15	1.425	14.08	25.4	5.0	0.125	4006.1
16	1.45	14.625	25.4	5.0	0.115	3995.0
17	1.425	14.25	25.4	6.0	0.16	3725.74
18	1.40	14.5	25.4	5.0	0.1145	4496.28
19	1.425	14.95	25.4	5.0	0.111	4265.82
20	1.40	14.575	25.4	5.0	0.128	4001.37

TABLE 4.9

LAMINATE 3

No. of Jute Layers = 1

Volume Fraction of Jute = 0.195

Fibre Orientation = 0 and 90

No. of Glass Layers = 2

Volume Fraction of Glass = 0.11

Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction =0.234

Stacking Sequence : Glass + Jute + Glass

Specimen No.	Plate Thickness mm	Width of Plate mm	Span Length mm	Load Newton	Deflection mm	Flexural Modulus Newton/mm**2
21	1.275	14.45	25.4	5.0	0.1065	6421.9
22	1.30	14.2	25.4	5.0	0.0945	6948.0
23	1.35	14.45	25.4	5.0	0.0795	7247.3
24	1.325	14.325	25.4	5.0	0.098	6272.5
25	1.3	14.4	25.4	5.0	0.100	6474.7
26	1.425	14.3	25.4	7.0	0.107	6477.0
27	1.35	14.55	25.4	5.0	0.0795	7197.4
28	1.40	14.475	25.4	5.0	0.083	6213.4
29	1.45	14.5	25.4	6.0	0.076	7316.5
30	1.40	14.525	25.4	7.0	0.106	6787.8

TABLE 4.10

LAMINATE 4

No. of Jute Layers = 1 No. of Glass Layers = 3

Volume Fraction of Jute = 0.156 Volume Fraction of Glass = 0.1357

Fibre Orientation = 0 and 90 Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.274

Stacking Sequence : Glass + Glass + Jute + Glass

Specimen No.	Plate Thickness mm	Width of Plate mm	Span Length mm	Load Newton	Deflection mm	Flexural Modulus Newton/mm**2
31	1.78	12.0	25.4	7.0	0.0582	7272.1
32	1.67	11.175	25.4	7.0	0.0837	6582.0
33	1.67	11.425	25.4	6.0	0.0639	7230.4
34	1.67	12.175	25.4	5.0	0.0565	6391.9
35	1.62	13.025	25.4	5.0	0.055	6708.3
36	1.725	11.55	25.4	4.0	0.0385	7177.1
37	1.625	11.85	25.4	4.0	0.0435	7423.1
38	1.70	11.60	25.4	4.0	0.043	6662.4
39	1.7	11.075	25.4	4.0	0.0465	6456.2
40	1.65	11.775	4.0	4.0	0.0465	6659.3

TABLE 4.11

LAMINATE 5

No. of Jute Layers = 1

Volume Fraction of Jute = 0.1424

Fibre Orientation + 0 and 90

No. of Glass Layers = 4

Volume Fraction of Glass = 0.1976

Fibre Orientation = 0 and 90

Effective Fibre Volume Fraction = 0.386

Stacking Sequence = Glass + Glass + Jute + Glass + Glass

Specimen No.	Plate Thickness mm	Width of Plate mm	Span Length mm	Load Newton	Deflection mm	Flexural Modulus Newton/mm**2
41	1.475	12.1	25.4	7.0	0.0805	9176.3
42	1.475	11.575	25.4	7.0	0.0925	8324.4
43	1.5	12.1	25.4	7.0	0.083	8430.5
44	1.525	11.125	25.4	7.0	0.082	8845.5
45	1.5	11.425	25.4	7.0	0.088	8445.1
46	1.525	11.725	25.4	7.0	0.0805	8555.9
47	1.5	11.925	25.4	7.0	0.085	8398.0
48	1.5	12.25	25.4	7.0	0.08	8681.2
49	1.525	11.975	25.4	7.0	0.0785	8615.2
50	1.475	12.8	25.4	7.0	0.078	8922.9

TABLE 5.1

$V_{f\,eff}$	No. of Lamina		Young's Modulus psi				Ultimate Strength psi		Strain at Failure
	Jute	Glass	Experimental	Bishop	Lamination Th	Law of Mix.	Experimental	Bishop	
0.086	4	1	4.01E+5	4.5E+5	6.78E+5	6.8E+5	3086.81	3400.0	0.0068
0.096	3	1	5.56E+5	5.0E+5	7.33E+5	7.4E+5	3250.88	3200.0	0.00571
0.114	2	1	6.25E+5	6.1E+5	8.37E+5	8.3E+5	3568.82	3600.0	0.00527
0.145	1	1	7.56E+5	7.9E+5	1.02E+6	1.0E+6	4005.98	4300.0	0.0048
0.25	1	2	1.43E+6	1.5E+6	1.56E+6	1.6E+6	6263.6	6700.0	0.0041

TABLE 5.2

V_{feff}	Young's Modulus psi			Ultimate Strength psi	
	JRP	JGRCL	GRP	JRP	JGRCL
0.00			4.5E+05		
0.0861		401190.43			3086.81
0.00962		555779.3			3250.88
0.10			1.46E+06		
0.1141		624971.54			3568.82
0.142	6.108E+04			2836.86	
0.1454		757712.0			4005.98
0.168	7.428E+04			3385.01	
0.178	7.72E+04			3614.66	
0.20			2.46E+06		
0.202	8.74E+04			4118.53	
0.208	8.978E+04			4247.98	
0.2501		1429665.91			6263.6
0.30			3.46E+06		
0.40			4.385E+06		

TABLE 5.3

V_{feff}	No. of Lamina		Flexural Modulus psi	
	Jute	Glass	Experimental	Lamination Theory
0.083	4	1		7.45E+05
0.0902	3	1		8.16E+05
0.1104	2	1	1.34E+05	7.6E+05
0.1394	1	1	5.9E+05	1.7E+06
0.234	1	2	9.74E+05	2.8E+06
0.274	1	3	9.92E+05	2.87E+06
0.386	1	4	1.25E+06	3.68E+06

TABLE 5.4

V_{fj}	Young's Modulus psi	
	Experimental	Bishop
0.142	6.108E+04	6.2E+04
0.168	7.428E+04	7.3E+04
0.178	7.72E+04	7.7E+04
0.202	8.74E+04	8.7E+04
0.208	8.978E+04	9.0E+04

TABLE 5.5

JRP		JGRCL	
V_{fj}	Breaking Strain	V_{feff}	Breaking Strain
0.142	0.05295	0.0861	0.0068
0.168	0.0417	0.0962	0.00571
0.178	0.0344	0.114	0.00527
0.202	0.026	0.1454	0.0048
0.208	0.0246	0.2501	0.0041