MODEL INVESTIGATION OF CONOIDAL SHELL

By
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DECLARATION

I hereby declare that the project work reported herein has been performed by me and that this work has not been submitted for any other degree.

Firoz Montaser Ahmed
Signature of the student
This report presents a method for fabrication of a conoidal shell model. Moreover the experimental investigation on this model regarding deflection and stress distribution is presented.

The aim of the study was to investigate the stress pattern, deflection and overall behaviour of the conoidal shell model at service (design) load level and also at higher load levels. The study includes the comparison of the test results with those available in literature.

The test results show that bending moment developed throughout the shell body in addition to the membrane stress resultants. The location of maximum bending moment was near the interfaces of the shell and the edge members. Stress condition near the low end diaphragm was significant. The entire shell body remained in compression and no tension developed under loading conditions. The deflections were found to be small at the service load level. The measured compressive stresses were well below the allowable compressive strength of the constituent materials. Moreover, the measured stress resultants were comparable with those obtained from the membrane analysis.
From the model testing no crack was observed at service load level and only a few hair line cracks were observed at the edges of the shell at double the service load.

It was thus observed that in the conoidal shell model (which was designed on the basis of membrane analysis) bending moment of small magnitude developed under service load condition. Moreover, the membrane stresses were comparable to those obtained by membrane analysis.
Dedicated

to

My Parents
ACKNOWLEDGEMENTS

The author expresses his heartiest gratitude and profound indebtedness to Dr. Alamgir Habib, Professor and Head of Civil Engineering, BUET, Dhaka, under whose supervision this research project was carried out. Without his constant guidance and invaluable suggestions at every stage this work could not possibly materialize.

The support and services rendered by the staff of concrete laboratory, strength of materials laboratory and Workshop of BUET are also acknowledged with thanks.

Mr. A. Malek and Mr. Shahiduddin of Civil Engineering office, BUET, deserve credit for their assistance in typing the project report and drawing the sketches.
Strain causing elongation is indicated by +ve sign.

- **tensile force**

- **compressive force**

- **positive shear (upward on right face)**

- **moment producing compression at top**

- **moment producing tension at top**
NOTATIONS AND UNITS

\[ B \] = chord width  \\
\[ d \] = thickness  \\
\[ E_C \] = modulus of elasticity in psi  \\
\[ \varepsilon_{\text{bot}} \] = strain measured at bottom face of shell in inch/inch  \\
\[ \varepsilon_{\text{top}} \] = strain measured at top face of shell in inch/inch  \\
\[ f \] = rise at high end  \\
\[ f_C \] = allowable stress in concrete, psi  \\
\[ f_S \] = allowable stress in steel, psi  \\
\[ f_y \] = yield stress in steel, psi  \\
\[ g \] = total load on shell surface, psf  \\
\[ I \] = moment of inertia, \( \text{in}^4/\text{in} \)  \\
\[ M_x, M_y \] = bending moments, lb-in/in  \\
\[ n \] = modular ratio  \\
\[ N_x, N_y \] = direct force, lb/in  \\
\[ N_{xy} \] = shear force, lb/in  \\
\[ P_1, P_2 \] = principal normal stresses, lb/in  \\
\[ \theta \] = inclination of \( P_1 \) to x-axis, deg  \\
\[ F_x, F_y, F_z \] = real loads  \\
\[ X, Y, Z \] = fictitious loads
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1.1 GENERAL

Shell-like structures are familiar enough in nature but the use of such structures as containers, aircrafts fuselages, submarine hulls and roofing structures is only of recent origin. That the inherent strength of shells for structures has not been utilized much in the past is probably due to the difficulty in obtaining suitable material with which to construct them. Such difficulties no longer exist and shell structures in general are these days constructed of such varied materials as steel, light alloy, plastics, wood and reinforced concrete.

Thin shells are examples of strength through form as opposed to strength through mass. The effort in design is to make the shell as thin as practical requirements will permit so that the dead weight is reduced and the structure functions as a membrane free from large bending stresses. By this means, a minimum of materials is used to the maximum structural advantage. Shells of double curvature are among the most efficient of known structural forms. Conoids are such doubly curved shells which are widely used in civil engineering constructions.
1.2 STATEMENT OF THE PROBLEM

The present study involves testing of conoidal shell fabricated from micro-concrete and reinforced with mild steel wire. This study includes the review of available literature related to analysis and design of such shells. Extensive use of models as aids to structural design is necessary for shells with complex shapes and boundary conditions which are not easily amenable to treatment by analytical means. Membrane analysis considering only the in-plane forces is usually employed for the design of conoidal shells. The membrane stress resultants are insufficient to predict the true elastic behaviour of a shell element since of necessity the external load must give rise to flexure or bending of the shell and such bending may only be resisted by internal moments and forces induced in the shell. A limited number of previous studies revealed that bending is actually present in such shells together with the membrane forces. Extensive studies considering bending is, therefore, needed to understand the actual behaviour of conoidal shells properly. Attempts to investigate these shells analytically have been made but these are too tedious even with simplifying assumptions and approximations. Some recent efforts are, therefore, directed towards analysing such shells by model investigations.
1.3 OBJECTIVES OF THE PRESENT STUDY

The objectives of the present study are as follows:

1) To carry out test on the model shell with a view to observing the stress pattern and deflection of such shell at service load level. The behaviour of shell at twice the design loading level will also be observed.

2) To analyse the test results and to compare these with those available in literature. Particular attention will be given to observe the significance of the bending stress components.
2.1 GENERAL

Scale models are becoming increasingly more important in research on structural concrete and are also used quite widely in the design problems.

Around mid-thirties of this century, designers began to look for new forms of shells other than cylindrical ones. Investigations made on models of a structure are a useful tool in design procedure of these structures. These investigations represent an essential supplement to the methods of mathematical analysis of shell structures, which usually supply only general data about the stress and deformation state for a restricted number of fundamental shell types. The mutual action of analytical solutions and experiment provides concepts for creating new shell shapes for designing them safely and economically.

2.2 CONOID

A conoid is generated by a variable straight line which remains parallel to a given plane - the Director plane - and moves along two curves at the ends - known as the directrices. The more common cases of conoidal surfaces are obtained when one of the directrices is a straight line as in Fig. 2.1. It is assumed that both the straight line directrix and the
Fig. 2.1 Typical conoid.

Fig. 2.2 Truncated conoid with reference axes system.
plane, containing the curve directrix are at right angles to the director plane, the curve directrix being moreover symmetrical about its vertical axis. A part of a conoid, known as a truncated conoid is sometimes employed in preference to a full conoid (Fig. 2.2). Depending upon the curve used as the directrix, conoids are described as parabolic, circular, or catenary conoids. Of these the parabolic conoid is by far the most common. One of the greatest advantages in the use of conoidal shells is that a considerable amount of natural lighting is achieved at a minimum structural cost. Also the formwork for this ruled surface can be easily made from straight planks.

2.2.1 Geometry of a Conoid

Referring to Fig. 2.2 as mentioned earlier the director plane is perpendicular to the directrices. A complete conoidal surface may therefore be defined as \( z = f(y)x/\lambda \). If the directrix curve is a parabole, i.e. \( f(y)=f(B^2/4-y^2)/B^2/4 \), then the surface is termed a parabolic conoid and is defined by

\[
z = \frac{x}{\lambda} \frac{4f}{B^2} (B^2/4-y^2)
\]

(2.1)

The present study deals only with parabolic conoid.
2.3 MEMBRANE SOLUTIONS FOR CONOIDAL SHELLS

The derivatives of equation 2.1 are

\[ p = \frac{\partial z}{\partial x} = \frac{4f}{2B^2} (B^2/4 - y^2) \]

\[ q = \frac{\partial z}{\partial y} = \frac{8fxy}{2B^2} \]

\[ r = \frac{\partial^2 z}{\partial x^2} = 0 \]

\[ s = \frac{\partial^2 z}{\partial x \partial y} = \frac{8fy}{2B^2} \]

\[ t = \frac{\partial^2 z}{\partial y^2} = \frac{8fx}{2B^2} \]

It is assumed that the pseudo stress resultant \( \overline{N}_x \) is such that it exerts the same force in the \( x \) direction on the projected side as the real stress resultant does on the same side. Hence

\[ \overline{N}_x = N_x \sqrt{\frac{1 + p^2}{1 + q^2}}, \quad \overline{N}_y = N_y \sqrt{\frac{1 + q^2}{1 + p^2}} \]

\[ \overline{N}_{xy} = \overline{N}_{xy} \]

The equations of equilibrium in the \( x \), \( y \) and \( z \) directions may be formulated as follows by considering the projected element.
The above three equations of equilibrium can be reduced into a single differential equations by introducing a stress function $\phi$. The stress function $\phi$ is so defined that

$$
\tilde{N}_x = \frac{\partial^2 \phi}{\partial y^2} - \int_x X \, dx
$$

$$
\tilde{N}_y = \frac{\partial^2 \phi}{\partial x^2} - \int_y Y \, dy
$$

$$
\tilde{N}_{xy} = - \frac{\partial^2 \phi}{\partial x \partial y}
$$

For the condition of dead weight $(g)$ we have

$$
X = 0, \quad Y = 0, \quad Z = g \sqrt{1 + p^2 + q^2}
$$

Substituting the values from equation (2.5) in equation (2.4) and introducing a stress function $\phi$ and then solving the equation we get,

$$
\tilde{N}_x = \frac{\partial^2 \phi}{\partial y^2} = - ga \left[ \frac{3}{16} (\ell - x) y^2 - \frac{a^2}{32} (\ell - x) + \frac{1}{6} (\ell^3 - x^3) \right]
$$
\[ \bar{N}_y = \frac{\partial^2 \phi}{\partial x^2} = \frac{g a}{2} x y^2 \left( \frac{a}{2} \right) \left[ 1 + \frac{1}{2}(f/l)^2 \right] \]

\[ \bar{N}_{xy} = \frac{\partial^2 \phi}{\partial x \partial y} = -g a \left( \frac{y^3}{18} - \frac{y B^2}{32} + \frac{xy^2}{2} \right) \]

It is easily verified that the stress function chosen satisfies the boundary conditions. From symmetry, the first boundary condition to be imposed is

(i) \( \bar{N}_{xy} = 0 \) along, \( y = 0 \)

The second boundary condition is

(ii) \( \bar{N}_x = 0 \) at \( x = \lambda \)

This is because the traverse at the end \( x = \lambda \) is assumed incapable of receiving any forces at right angles to its plane.

2.4 MODEL TESTING

Model investigations of concrete shell roofs are usually concerned with the testing of model shells made from micro-concrete, reinforced with mild steel wire. Model testing may be classified into two types: one on which existing theoretical analysis can be tested by experimental observations of model shells, the other in which, because of either the lack or complexity of theoretical analysis, experimental
data from a model test can be directly used for designing the prototype.

Experimental research for shell structures is generally carried out with a view to substantiating analytical procedures or to obtaining values that cannot be exactly computed through analytical methods. Full scale experimental structures are by their nature, temporary but require many resources with much enclosed space and large loads for testing and therefore expensive in nature.

Most important use of models are those when results obtained from model studies are taken to be sufficiently correct for design purposes.

In order to simulate the applied force the distribution of the load should be strictly maintained. Response of the model under different loading conditions should be studied properly.

Among the numerous instrumentation systems available, a suitable one should be applied to a particular model. Special devices should be used in case of small scale models which can take care of gradual and small increment of loads. Light equipments are to be used in small scale models, so that no appreciable load or support is undesirably imposed.

2.5 DIMENSIONAL ANALYSIS APPLIED TO SHELLS

In a model experiment on a shell, it is the ultimate goal in arriving at the stresses and displacements in the
prototype based on the observed results on the model. For this purpose the relations connecting corresponding quantities - known as homologous quantities in dimensional theory - for the model and the prototype are required. The variables involved in the dimensional analysis of a physical problem are \( a, b, d, \sigma, E, \nu, y, \) and \( p \)

where \( a, b \) = linear dimensions of the shell

\( d \) = thickness of the shell

\( \sigma \) = stress in the shell

\( E \) = Young's modulus

\( \nu \) = Poisson's ratio

\( y \) = any displacement

\( p \) = a uniformly distributed load

During the simulation of the model with the prototype, certain non-dimensional numbers, known as \( \pi \) terms, have to be the same for the model as well as the prototype.

The \( \pi \) terms for the shell problem are as follows:

\[ \pi_1 = \nu, \quad \pi_2 = \frac{a}{b}, \quad \pi_3 = \frac{a}{d}, \quad \pi_4 = \frac{p}{E}, \]

\[ \pi_5 = \frac{\sigma}{E} \text{ and } \pi_6 = \frac{y}{a} \]

The \( \pi \) terms, \( \pi_1 \) to \( \pi_6 \) have to be the same for the model and the prototype. This means that

\[ \nu_m = \nu_p \quad \text{(i)} \]
\[
\frac{a_m}{b_m} = \frac{a_p}{b_p} \quad (ii)
\]
\[
\frac{a_m}{d_m} = \frac{a_p}{d_p} \quad (iii)
\]
\[
\frac{p_m}{E_m} = \frac{p_p}{E_p} \quad (iv)
\]
\[
\frac{\sigma_m}{E_m} = \frac{\sigma_p}{E_p} \quad (v)
\]

and \[\frac{y_m}{a_m} = \frac{y_p}{a_p}\]

Relation (i) implies that Poisson's ratio has to be the same for both the model and prototype.

Relation (ii) and (iii) means that all the geometrical dimensions of the prototype are scaled down in the same proportion in building the model. Let this scale be \(p\) so that

\[a_p = p\ a_m, \ b_p = p\ b_m, \text{ and } d_p = p\ d_m\]

The relation (v) enables to relate the stress in the model to that in the prototype.

The relation \(y_p = y_m \frac{a_p}{a_m} = p y_m\) is used to arrive at any deflection of the prototype from the corresponding deflection of the model. The relationship between prototype and
model loadings when the load is uniformly distributed may be arrived at from the equation (iv) as

$$p_m = p_p \frac{E_m}{E_p}$$

Thus if the material of the model and its linear scale are specified, $p$ and $E_p/E_m$ become fixed and the loading scale alone remains to be worked out.

2.6 THEORIES OF CONOIDAL SHELLS CONSIDERING BENDING

2.6.1 Need for Bending Theories

Membrane state of stresses can be interpreted as a statically determined system for simple structures in which the shell forces are in equilibrium with the support reaction at the boundaries. So membrane theory offers only an inadequate basis for the stress analysis of conoids. If a full conoid is considered, the surface loses all curvature close to the straight-line directrix. A part of the shell close to the straight-line directrix will function as a plate and the membrane theory will break down in this neighbourhood. The transfer of the load to the traverse at this edge will take place primarily through radial shear as opposed to in-plane shear. Moreover, disturbances emanating from the edge beams will penetrate far into the interior of the shell. The stresses resulting from the disturbance at the edges can only be found by applying a bending theory.
2.6.2 Theories Available in Literature

Early literature on conoids is chiefly in the Rumanian, French, Spanish, and Italian languages. The publication in German by Soare (3) is well known. Although the publication is rather lengthy, Soare's approach has served as a basis for the analysis of many authors. From the equations Soare (3) has presented, a comprehensive treatment of conoid with various directrices is possible.

Ramaswamy (4) presented a simplified approach for analysing the parabolic conoids. The method, which employs a polynomial stress function, leads to results which are in close agreement with Soare's theory if the shell considered is not too deep.

The analysis and calculation made by Banerjee (5) are attractive and very practical in their application. He adopted a third boundary condition, e.g., at \( Y = \pm \frac{y}{2} \), \( \bar{N}_y = 0 \) and then designed parabolic conoids. However, for the boundary line \( Y = \pm \frac{y}{2} \) it is not in agreement with the statical requirements that \( \bar{N}_y \) should be zero. Along with other imperfections in the derivation, the results cannot be taken as correct. Banerjee in his final remarks admitted the discrepancies stating that the resulting \( \bar{N}_y \) and \( \bar{N}_x \) values are too high. While comparing these values with those found by Ramaswamy using Soare's theory, one noted that at certain points, Ramaswamy's values were over twice those of Banerjee. However, it should be kept in mind that though this
is a large difference, the absolute values of the stresses remained low. This may also account for the fact that many conoid shells have been built by Banerjee with good results.

Even a calculation according to the membrane theory is rather complicated. For design and for tentative calculation a procedure\(^{(6)}\) has been developed which is sufficiently accurate and which has led to much simpler computations. This procedure uses the concept of the conoid as a curved thin plate subject to bending and axial force. By applying this approximation many conoids have been designed and constructed in Poland, in France, and some in the Netherland.

Vlasov\(^{(1)}\) derived a bending theory which deals with shallow doubly curved shells as a combination of a disc, plate, and doubly curved membrane. Although Vlasov's equations are strictly applicable only to shells of constant curvature such as translational surfaces of the second degree for which r, s, and t are constants, they may also be employed without appreciable error for the analysis of shallow shells of variable curvature. Thus, for instance, Keshava Rao and Sharma\(^{(7)}\) and Hadid\(^{(8)}\) have made use of these equations for the analysis of conoids.
CHAPTER 3
EXPERIMENTAL INVESTIGATION

3.1 GENERAL

A prototype parabolic truncated conoidal shell was designed and detailed in this study by using membrane theory. The dimensions of the prototype were 40'x30' the rise at the high and low ends 6' and 1'-6" respectively. The thickness of the shell was 3 inches. The details of the shell and its reinforcements are given in Appendix-A.

All linear dimensions including the diameter and spacing of reinforcing steel of the above prototype shell was scaled down by a ratio of 1:5 to obtain a model of the prototype. The model was constructed and experimental investigations were carried out with this model in order to predict the behaviour of the prototype.

The investigations were conducted in three phases. The first phase was concerned with the general tests to determine the physical properties of sand, mortar, concrete, steel etc. the samples of which were collected from the same mix of concrete and the same steel used for construction of the shell structure.

The second phase was the construction of shell model at the site.
The third phase was the testing of shell first, at working load level and then at twice the working load level.

### 3.2 PROPERTIES OF AGGREGATES

For model shell, the size of sand depends on the minimum thickness. For 0.60 inch thickness of shell it is recommended\(^9\) that the sand must pass through sieve No. 8 (Table 8.1, Appendix 8) and should have the gradation as given in Table 8.2, Appendix 8. Table 3.1 shows the gradation of the sand used in this study.

**Table 3.1 Gradation of sand**

<table>
<thead>
<tr>
<th>Sieve No.</th>
<th>% passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>16</td>
<td>87.78</td>
</tr>
<tr>
<td>30</td>
<td>52.82</td>
</tr>
<tr>
<td>50</td>
<td>12.08</td>
</tr>
<tr>
<td>100</td>
<td>3.70</td>
</tr>
</tbody>
</table>

The fineness modulus of the sand used in the test was calculated to be 2.54. The sand used for footings, columns, and beams was stored in a separate place after sieving through sieve no. 4.

Ordinary portland cement available in the concrete laboratory was used throughout the test. Maximum size of brick khoa used was 1" down graded in edge beams and columns.
In footings the aggregate was of 3/4" nominal size.

3.3 PROPERTIES OF MORTAR

The mortar used in shell element was prepared with water/cement ratio of 0.475 by weight. The proportion of cement to sand was 1:2.75. The compressive and tensile strengths confirmed by 2"x2" cube test and standard briquette test respectively are given in the Table 3.2.

Table 3.2 Strength of Mortar

<table>
<thead>
<tr>
<th>Type of test</th>
<th>Water/cement ratio</th>
<th>Age days</th>
<th>Strength psi</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive (2&quot; cube)</td>
<td>0.475</td>
<td>28</td>
<td>3917</td>
<td>average of three tests</td>
</tr>
<tr>
<td>Tensile (direct)</td>
<td>0.475</td>
<td>28</td>
<td>273</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

3.4 PROPERTIES OF CONCRETE

The concrete used in footings, columns and edge beams had a mix ratio of 1:1.5:3 with a W/C ratio of 0.475 in all cases. Cylinder (6"x12") compressive strengths for the concrete are given in Table 3.3.

Table 3.3 Strength of Concrete

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Water/cement ratio</th>
<th>Age days</th>
<th>Strength psi</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge beam</td>
<td>0.475</td>
<td>28</td>
<td>2746</td>
<td>average of three tests</td>
</tr>
<tr>
<td>Column and footing</td>
<td>0.475</td>
<td>28</td>
<td>2689</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
3.5 PROPERTIES OF STEEL

The reinforcement used in shell body was 1/13" non-galvanized steel wire, in diaphragm 1/8" dia steel wire and in edge beams and columns 3/8" diameter ordinary mild steel bar.

The vertical stirrups provided in edge beams and columns respectively were 1/4" dia M.S bars.

The tensile strengths of the reinforcing steel are given in Table 3.4.

Table 3.4 Tensile strength of reinforcing steel

<table>
<thead>
<tr>
<th>Size</th>
<th>No. of tests</th>
<th>Yield strength psi</th>
<th>Ultimate strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/13&quot; steel wire</td>
<td>3</td>
<td>Not found</td>
<td>1,338,18</td>
</tr>
<tr>
<td>1/8&quot; steel wire</td>
<td>3</td>
<td>Not found</td>
<td>1,177,60</td>
</tr>
<tr>
<td>1/4&quot; M.S rod</td>
<td>3</td>
<td>58,660</td>
<td>87,150</td>
</tr>
<tr>
<td>3/8&quot; M.S rod</td>
<td>3</td>
<td>51,478</td>
<td>67,533</td>
</tr>
</tbody>
</table>

3.6 THE MODEL

A shell model having 8'-0" x 6'-0" in plan was tested in this research project. The scale ratio of model to prototype was 1:5. The model was composed of two edge beams in the shorter side and two diaphragms in the longer side. The edge
Fig. 3.1 Cross-section for (a) edge beam & (b) column.

Fig. 3.2 Reinforcement details in footings & columns.
beams were 4"x4" in cross section and uniform throughout the length. Their cross sections along with steel reinforcement provided are shown in Fig. 3.1. The four corners of the shell were 4'-0" above the ground level and columns were 4"x4" in cross section. Details of column support and the footing are shown in Fig. 3.2. It may be noted that the columns and footings were overdesigned in order to ensure that failure did not occur in these before the shell fails.

The shell thickness was only 0.6 inch. No. 13 (dia. 1/13 inch) mild steel wires were placed at 2.5" on centres in both the directions parallel to the shell edges.

3.7 CONSTRUCTION PROCEDURE

The shell with the edge beams columns and footings was constructed at site.

3.7.1 Form Work

The formwork (Fig. 3.3) for the conoid shell was made in one unit by 1.5"x2"x5'-0" straight planks of Mango timber pinned side by side and supported on high end diaphragm in one side and on low end diaphragm on the other side. The formwork had a rise of 13.8" at the high end and 3" at the low end. For achieving straightness of the sloping planks, three intermediate supports were provided beneath the top surface of the formwork.
Fig. 3.3 Prepared formwork of shell and diaphragm.

Fig. 3.4 Finished formwork of shell, diaphragm, edge beam and column.
Fig. 3.5 Reinforcement details of footing and column.

Fig. 3.6 Edge beam reinforcement details.
The wooden forms for edge beams were attached to the formworks of shell diaphragm and column (Fig. 3.4).

The columns were cast inside rectangular form made of 1" thick timber planks.

3.8 REINFORCEMENT DETAILS

3.8.1 Footing Column and Edge Beam

Details of reinforcement in footing column and edge beam are shown in Figs. 3.1, 3.2, 3.5 and 3.6.

3.8.2 Shell Element

The mesh for the shell element was made up with 1/13" (No. 13) dia mild steel wires placed 2.5" on centres in both the directions. Figs. 3.7 to 3.10 show the reinforcement in the shell element. Development lengths of 5" on all the sides were kept for bonding with edge beams and diaphragms. Sample calculations are given in Appendix C.

3.9 CASTING

The model was cast-in-place. The columns reinforcement was kept extended above the columns which was bent later into the respective edge beams in a symmetrical sequence. The column reinforcement was also embedded into the footing. After proper placement of reinforcement meshes, steel in edge
Fig. 3.7 Wire mesh reinforcement of diaphragm.

Fig. 3.8 Wire mesh reinforcement over the forms of shell unit.
Fig. 3.9 Extra reinforcement details of shell with the edge beam and diaphragm.

Fig. 3.10 Completed reinforcement details of shell prior to casting.
Fig. 3.11  Shell after casting.

Fig. 3.12  Completed shell after 28-days.
beams and fitting of their shutterings, the shell element was cast with mortar. The 0.60 inch thickness was maintained accurately. The mortar had a cement/sand ratio of 1:2.75 with a water/cement ratio of 0.475. The edge beams were also cast on the same day with concrete using maximum size of khoa of 1/2 inch downgraded. The proportion of concrete mix was 1:1.5:3 and a water/cement ratio of 0.475 was used (Fig. 3.11).

3.10 CURING

The columns were cured with soaked canvas for two weeks following which the shuttering was struck off. The continuous curing of the shell element was carefully maintained for 28-days after which the shuttering was struck off (Fig. 3.12).

3.11 OUTLINE OF THE TEST

A brief outline of the tests performed in this research project is presented below:

a) Strains at the locations shown in Fig. 4.1 were measured and the stresses and moments were estimated.

b) Deflections at locations shown in Fig. 4.1 were determined.
3.12 INSTRUMENTATION

3.12.1 Strain Gage

Electrical resistance strain gages (Tokyo Sokki Kenkyujo Electronic Instruments Co. Ltd., Tokyo, Japan) having resistance of $120 \pm 0.3$ ohm were used for measuring strains. The gage length was 30 mm. Strain gage cement was used to fix the gage with the surface of the shell structure. Wooden box was used to cover the gage with a view to protect them when loading applies. Ordinary 14 thread wire of low resistance was used to connect the strain gage with the balancing box. To balance the temperature effect a dummy gage was fixed on a separate mortar block having the same property as that of the shell.

3.12.2 Dial Gage

To measure the deflections, dial gages with smallest division of $0.001$" were used (Fig. 3.13).

3.12.3 Strain Indicator

One 24 channel switch and balance unit (KYOWA Electronic Instruments Co. Ltd., Tokyo, Japan) was used with the indicator to record the strains directly. The minimum reading that could be taken with the strain recorder was $5 \times 10^{-6}$ in/in.
Fig. 3.13 Strain gage and dial gage arrangements, wires connected to the top and bottom strain gages.

Fig. 3.14 Sand bag and brick loads as viewed from shorter side on the shell element.
Fig. 3.15 Sand bag and brick loads as viewed from longer side on the shell element.
3.13 LOADING SYSTEM

In order to apply the load on the model, sand bags and bricks are generally used. Bricks of standard size and uniform weight are preferred.

3.13.1 Sand Bags

To apply uniformly distributed load over the curved surface of the shell, sand bags were chosen. The bags were made of thick cloths. Then the bags were manually filled-in with sand. The size of the bag was such that, no appreciable arching would result. The bags were so loosely filled that it would result in uniformly distributed load when the bags would be placed side by side. Sand bags placed on the shell element are shown in Fig. 3.14 and 3.15.

3.14 THE TEST PROGRAM

The test of model was performed in two stages.

a) The shell was tested at one third of the design load placed over the whole surface of the shell.

b) The shell was then loaded up to the design load subsequently.

3.15 TEST PROCEDURE

The following stages were performed in testing the shell.
1) Set of initial readings for strain gages at both top and bottom and dial gages were recorded when the shell was not loaded externally.

2) The sand bags were lifted over the shell and placed side by side starting from the centre of shell and ending at the shell edge. All the panels were being loaded simultaneously. This resulted in an intensity of 19.57 psf all over the surface.

3) Dial gage, top and bottom strain gage readings were taken when this loading was complete.

4) Another layer of sand bags were lifted over the shell in the same manner as before. This resulted in an intensity of 39.14 psf all over the surface and all readings were taken.

5) In order to arrive at the design load (60 psf) bricks were used for additional load in making 60 psf. Final readings were taken when this loading was complete.

6) The shell was then unloaded and all readings were taken under zero externally imposed load.

7) The shell was loaded first upto 60 psf and then to 120 psf. Final readings were taken when this loading was complete.
CHAPTER 4
TEST RESULTS

4.1 GENERAL

In the preceding chapter a detailed description of the model investigation performed by the author is presented. The test results of the shell model are presented in this chapter.

The loading level of the shell was kept low initially with a view to ensuring that no tension crack appears in the shell under loading. No tension crack was formed at the design load level of 60 psf. It may be mentioned here that stresses can be estimated fairly accurately from strain measurements when there is no cracking of the concrete over the zone of measured strains. Therefore, the stresses estimated from the measured strains are the elastic stresses. The locations of strain and dial gages on the shell model are shown in Fig. 4.1.

4.2 MEASURED STRAINS

The measured strains (\(\mu\) in \(\text{in}^{-1}\)) in the shell under 60 psf loading (i.e the design load) and 120 psf loading are shown in Figs. 4.2, 4.3, 4.4 and 4.5 respectively. Table 4.1 shows measured strains at different locations.

4.3 DEFLECTIONS

The values of vertical deflections (in inch) at different
Fig. 4.1 Locations of (a) strain gages & (b) dial gages on the shell model.
Fig. 4.2 Top surface strains in $\mu$ inch/inch at 60 psf load over the shell surface.

Fig. 4.3 Bottom surface strains in $\mu$ inch/inch at 60 psf load over the shell surface.
Fig. 4.4 Top surface strains μ inch/inch at 120 psf load over the shell surface.

Fig. 4.5 Bottom surface strains μ inch/inch at 120 psf load over the shell surface.
Fig. 4.6 Deflection (inch) under 60 psf load.

Fig. 4.7 Deflection (inch) under 120 psf load.
Fig. 4.8 Comparison of experimental results with the theoretical analysis.

Note: Enclosed values obtained from theoretical analysis.
Fig. 4.9 Moment stress resultant (lb-inch/inch) at 60 psf load.

Fig. 4.10 Moment stress resultant (lb-inch/inch) at 120 psf load.
locations are shown in Figs. 4.6 and 4.7 respectively. In the figure, positive values indicate downward deflections. Table 4.2 shows the measured deflection at different locations.

4.4 STRESS RESULTANTS

The stress resultants were calculated by using the observed strain readings as the basic data. The stress resultants thus calculated and compared with the analysis are shown in Fig. 4.8. The moment stress resultants for different load levels are shown in Figs. 4.9 and 4.10 respectively. Table 4.3 shows the stress resultants at different points. The theoretical analysis are presented in tabular forms in Appendix-A and the sample analytical calculations are given in Appendix-C.

4.5 BEHAVIOUR AT DIFFERENT LOAD LEVELS

The loading in the shell was performed in stages by applying loads using sand bags in the first and second layer and then bricks in the subsequent layers. The load was successively increased to 60 psf (design load) and then to 120 psf load (Fig. 4.11).

No significant crack was observed in the bottom surface of the shell. Only a few hair line cracks were observed at 120 psf load level along the edges of the shell. Maximum
deflection was found to be 0.108 inch at the midpoint of low end diaphragm (location C) at 120 psf load level. While the deflections at all other locations were much smaller under both the load levels.

After removal of loads no crack could be detected at the top surfaces.

Fig. 4.11 Shell element under 120 psf loading.
<table>
<thead>
<tr>
<th>Strain gauge location</th>
<th>Initial reading at 0 psf (a)</th>
<th>Final reading at 60 psf (b)</th>
<th>Unloading reading at 0 psf (c)</th>
<th>Final reading at 120 psf (d)</th>
<th>Strain at 60 psf 10^-6 in/in (b-a)</th>
<th>Strain at 120 psf 10^-6 in/in (d-c)</th>
<th>Unloading reading at 0 psf (e)</th>
<th>Residual strain for 60 psf (c-a)</th>
<th>Residual strain for 120 psf (e-c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(T)</td>
<td>-5110</td>
<td>-5145</td>
<td>-5110</td>
<td>-5175</td>
<td>-35</td>
<td>-65</td>
<td>-5130</td>
<td>0</td>
<td>-20</td>
</tr>
<tr>
<td>1(B)</td>
<td>-3050</td>
<td>-3105</td>
<td>-3050</td>
<td>-3135</td>
<td>-55</td>
<td>-95</td>
<td>-3075</td>
<td>0</td>
<td>-35</td>
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<td>2(T)</td>
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<td>-1310</td>
<td>-25</td>
<td>-50</td>
<td>-1295</td>
<td>-5</td>
<td>-25</td>
</tr>
<tr>
<td>2(B)</td>
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<td>-1175</td>
<td>-1150</td>
<td>-1220</td>
<td>-35</td>
<td>-70</td>
<td>-1175</td>
<td>-10</td>
<td>-25</td>
</tr>
<tr>
<td>3(T)</td>
<td>+2700</td>
<td>+2680</td>
<td>+2690</td>
<td>+2665</td>
<td>-20</td>
<td>-25</td>
<td>+2680</td>
<td>-5</td>
<td>-10</td>
</tr>
<tr>
<td>3(B)</td>
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<td>+510</td>
<td>+530</td>
<td>+495</td>
<td>-25</td>
<td>-35</td>
<td>+520</td>
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<td>-10</td>
</tr>
<tr>
<td>4(T)</td>
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<td>+130</td>
<td>+185</td>
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<td>-115</td>
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<td>-1365</td>
<td>-1280</td>
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<td>+85</td>
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<td>-5570</td>
<td>-50</td>
<td>-90</td>
<td>-5510</td>
<td>-20</td>
<td>-30</td>
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<td>5(B)</td>
<td>+335</td>
<td>+335</td>
<td>+335</td>
<td>+380</td>
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<td>-360</td>
<td>+10</td>
<td>-15</td>
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<td>6(T)</td>
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<td>+95</td>
<td>+75</td>
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<td>+25</td>
<td>+35</td>
<td>+60</td>
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<td>+4585</td>
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<td>-30</td>
<td>-45</td>
<td>+4575</td>
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<td>-10</td>
</tr>
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<td>7(T)</td>
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<td>-990</td>
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<td>+275</td>
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<td>-175</td>
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<td>-1950</td>
<td>-1785</td>
<td>-2110</td>
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<td>-325</td>
<td>-1801</td>
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<td>-1370</td>
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<td>+240</td>
<td>-1570</td>
<td>+20</td>
<td>+40</td>
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<td>9(T)</td>
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<td>-1640</td>
<td>-1840</td>
<td>-1495</td>
<td>225</td>
<td>+345</td>
<td>-1790</td>
<td>+25</td>
<td>+50</td>
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<td>-2290</td>
<td>-2010</td>
<td>-2485</td>
<td>-310</td>
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<td>-2070</td>
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T = Top  B = Bottom
### Table 4.2 Deflections for model

<table>
<thead>
<tr>
<th>Dial gauge location</th>
<th>Initial reading (1)</th>
<th>Final reading at 60 psf (2)-(1) inch</th>
<th>Deflection at 60 psf (2)-(1) inch</th>
<th>Unloading reading at 0 psf (3)</th>
<th>Final reading at 120 psf (4)-(3) inch</th>
<th>Deflection at 120 psf (4)-(3) inch</th>
<th>Unloading reading at 0 psf (5)</th>
<th>Residual reading at 60 psf (3)-(1)</th>
<th>Residual reading at 120 psf (5)-(3)</th>
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<tr>
<td>A</td>
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<td>11</td>
<td>0.011</td>
<td>6</td>
<td>39</td>
<td>0.033</td>
<td>25</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>0.0</td>
<td>37</td>
<td>0.037</td>
<td>15</td>
<td>109</td>
<td>0.094</td>
<td>49</td>
<td>15</td>
<td>34</td>
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<tr>
<td>C</td>
<td>0.0</td>
<td>43.5</td>
<td>0.0435</td>
<td>6</td>
<td>114</td>
<td>0.108</td>
<td>53</td>
<td>6</td>
<td>47</td>
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<tr>
<td>D</td>
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<td>5</td>
<td>37</td>
<td>0.032</td>
<td>20</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>E</td>
<td>0.0</td>
<td>10</td>
<td>0.010</td>
<td>5</td>
<td>36</td>
<td>0.029</td>
<td>18</td>
<td>5</td>
<td>13</td>
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*1 Div. = 0.001 inch*
Table 4.3 Stress resultants obtained from experimental investigation

<table>
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<tr>
<th>Location</th>
<th>Gauge position</th>
<th>60 psf</th>
<th>120 psf</th>
<th>60 psf</th>
<th>120 psf</th>
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<tr>
<td></td>
<td></td>
<td>$\varepsilon_t$</td>
<td>$\varepsilon_b$</td>
<td>Theoretical $N_y$ (lb/in)</td>
<td>Experimental $N_y$ (lb/in)</td>
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<tr>
<td>Shell body 7</td>
<td>1</td>
<td>-33</td>
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<td>-105.70</td>
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<td>2</td>
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<td>-53.70</td>
<td>-21.60</td>
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<td>5</td>
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<td>40</td>
<td>-5.90</td>
<td>-21.60</td>
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<td>6</td>
<td>+25</td>
<td>-30</td>
<td>-1.37</td>
<td>-5.40</td>
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<tr>
<td>Edge beam 8</td>
<td>8</td>
<td>175</td>
<td>145</td>
<td>+1029.2</td>
<td>-720.0</td>
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<tr>
<td></td>
<td>9</td>
<td>225</td>
<td>-310</td>
<td>-2854.2</td>
<td>-2040.0</td>
</tr>
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</table>
CHAPTER 5
ANALYSIS AND DISCUSSION OF TEST RESULTS

5.1 GENERAL

A detailed analysis and discussion of the test results are presented in this chapter. For the model, the deflection and stress values obtained experimentally at design load level of 60 psf have been compared with values obtained from the membrane solution.

5.2 DEFLECTION

For both the load levels the deflections observed (Figs. 4.6 and 4.7) at symmetrical location (D and E) were almost identical. Fig. 5.1 presents the variation of deflection along Sec. 3-3. The deflection at 120 psf loading was more than double of the corresponding deflection at the design load. The deflections at the edge beams were 0.012 inch and 0.032 inch at 60 psf and 120 psf respectively. The mid-point deflections of shell for 60 psf and 120 psf loadings were found to be 0.037 inch and 0.094 inch respectively.

Again the variation of deflection along the span (Sec. 1-1) is shown in Fig. 5.2. The maximum vertical deflection was observed at the location C at 120 psf load level and was of the order of 0.108 inch.
Fig. 5.1 Variation of deflection under different load level along Sec. 3-3.
Fig. 5.2 Variation of deflection under different load level along Sec. 1-1.
Fig. 5.3 Variation of $N_x$ (Sec. 2-2) at 60 psf load.
Fig. 5.4 Variation of $N_y$ (Sec. 1-1) at 60 psf load.
Fig. 5.5 Variation of $M_x$ (Sec. 2-2).
Fig. 5.6 Variation of $M_y$ (Sec. 1-1).
For conventional structure, the allowable deflection specified by ACI is $L/360$ which for the test model would be about 0.2 inch. The maximum experimentally estimated deflection (0.108 inch) is, therefore, well below this allowable limit.

### 5.3 Stress Resultants in the Working Load Level

The stress resultants were calculated directly from the surface strains (Fig. 4.1). These are presented in Table 4.3. Detailed calculations of these quantities in shell body are presented in Appendix-C and D.

Figures 5.3 and 5.4 show the variation of stress resultants ($N_x$ and $N_y$) at design load levels. The stress resultants in the shell body are compressive in nature as is the case with membrane analysis. The maximum value of $N_x$ and $N_y$ were found at locations 5 and 1 (Fig. 4.8).

Figures 5.5 and 5.6 show the bending moments in the shell body calculated directly from surface strains (Figs 4.9 and 4.10).

### 5.4 Comparison of the Actual Stresses with the Stresses Calculated by Membrane Theory

The results obtained from model testing agreed with results calculated using membrane theory. The actual stresses obtained from surface strains are the contribution of both
direct stresses and bending stresses. The edge beam was subjected to a much smaller moments than the designed moments (Art. 5.4.4) and due to the consequent reduction of stresses in the edge beam, the stresses in the shell body were expected to be of higher value than the values obtained from the membrane solution. All experimentally obtained values were far below the allowable mortar strength and these are compressive in nature. The discussion of the stress resultants have been made in the following articles.

5.4.1 $N_x$ - Stress Resultant

Figure 5.3 shows the comparison of the $N_x$ values between experimentally determined values and the values obtained from membrane solution. It can be mentioned that the stress resultants values obtained from membrane solution are much smaller than the experimental values.

5.4.2 $N_y$ - Stress Resultant

Figure 5.4 shows the comparison of the $N_y$ values between experimentally determined values and the values obtained from membrane solution. The nature of variation of stress resultants calculated by membrane solution was found to agree reasonably well with that of the stress resultants calculated from observed surface strains. Moreover the numerical values of stress resultants were also in fairly good agreement. The
experimental values were slightly greater than the theoretical ones. The theoretical values curve cut the experimental one near location 1 and at location 1 theoretical value is greater than the experimental one.

5.4.3 Bending Stress Resultants ($M_x$ and $M_y$)

One of the objective of this research project was to investigate the presence of bending in the conoidal shell. It may be noted here that membrane solution does not take into account the effect of bending. From the measured surface strain, it can be easily visualized that the bending moment is present within the body of the shell (Figs. 5.5 and 5.6). The surface stresses due to the bending moment are significant in comparison with the direct stresses in locations 4, 5, 6 and 7. While in other locations the values are much smaller than the direct stresses. The maximum bending moment under 60 psf load level found at locations 4 and 7 and the value was 10.8 lb-in/in. The variations of bending moments are also shown in the above mentioned figures. Maximum bending moment under 120 psf load level was found to be 72.36 lb-in/in at location 7.

5.4.4 Edge Beam

The moments estimated from test results in the midpoint of the edge beam (Locations 8 and 9 in Fig. 4.1) were
much smaller than the computed values, as shown by the Table 5.1.

Table 5.1 Comparison of computed value and measured value of edge beam moments

<table>
<thead>
<tr>
<th>Loading (psf)</th>
<th>Computed value (lb-ft)</th>
<th>Measured value (lb-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>1029.2</td>
<td>428</td>
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<tr>
<td></td>
<td>2855.7</td>
<td>715</td>
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</table>

This may be explained by the reasoning presented in Ref. 10 that the lower part of the shell acts like a bent horizontal beam transmitting the load directly to the longitudinal edges. In membrane solution it is assumed that edge beam provides only the necessary reactions needed by the shell body so that membrane state of stress can prevail in it. In afterwards edge beam is subjected to load opposite of the aforementioned reaction. However in actual case shell body and the edge beams act as a monolithic unit.

5.5 COMMENTS ON THE BEHAVIOUR OF A CONOIDAL SHELL

From the test results it can be seen that the shell and the edge beams act as a monolithic unit to carry the imposed load. Except the locations at the interfaces of shell
and edge members no significant bending moment occurs in the shell body. The edge zones and the regions near the columns were, in general, highly stressed. The major part of the shell was under compression and the nominal reinforcement provided was adequate even up to twice the design load. This was confirmed by the fact that no significant crack was observed at this double load. There was no tensile stresses found anywhere on the body of the shell. The actual edge beam moment was found to be much smaller than the computed moment from the membrane solution.
6.1 CONCLUSIONS

Within the scope and limitations of this study and the ambient conditions prevailing in the laboratory and at the testing site, the following conclusions may tentatively be made regarding the behaviour of conoidal shell.

1) The method adopted for the fabrication of the conoidal shell model can be used conveniently for the construction of a prototype.

2) Deflections in both the elastic range (design load) and double of the design load range are small in magnitude and well below the allowable deflection for such shells.

3) Transverse stress resultants \( (N_y) \) obtained from the test results agree fairly with the results obtained from the membrane solution.

4) Longitudinal stress resultants \( (N_x) \) are significantly larger than the analytical ones.

5) Bending moment is present throughout the shell body having significant magnitude at the interfaces of shell and edge members.

6) The ultimate load carrying capacity of conoidal shell designed in accordance with membrane solution is appreciable, being at least twice the service load.
6.2 RECOMMENDATIONS FOR FUTURE STUDY

From the analysis and discussion of test results of the present study the following further investigations on conoidal shells are recommended.

1) It is recommended to investigate edge beam experimentally with a view to suggest, to what extent the reduction of edge beam can be adopted.

2) It is suggested for any future study, the extensive investigation considering bending should be conducted upto its ultimate capacity with a view to suggest a simplified design method (considering bending) for conoidal shells.

3) Experiments on conoidal shell can be proposed to arrive at criteria for stability and safety against buckling which assumes greater importance and largely influences the selection of thickness and curvatures when the size of the shell is large.

4) It is also recommended to investigate the stress pattern, deflection beyond the service load level and to estimate the load carrying capacity and ultimate behaviour (including failure pattern) of conoidal shells.
APPENDIX-A
DIMENSIONS AND OTHER PROPERTIES IN PROTOTYPE AND MODEL

A prototype and model of conoidal shell was designed with the membrane theory. Its dimensions and other related properties are given below:

1) Geometry:

<table>
<thead>
<tr>
<th></th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span (l)</td>
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<td>8'</td>
</tr>
<tr>
<td>Chord width (B)</td>
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<td>Rise at low end</td>
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<td>Thickness (t)</td>
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</table>

2) Loads:

- Dead weight = 37.5 psf of shell surface
- Water proofing etc. = 7.5 " "
- Live load = 15.0 " "

Total load  g = 60 " "

3) Governing formula:

\[ a = \frac{8f}{\lambda B^2} \]
\[ p = -\frac{4f}{\lambda B^2} \left( \frac{B^2}{4} - y^2 \right), \quad q = \frac{8f}{\lambda B^2} xy \]
\[ \tilde{N}_x = - ga \left[ \frac{3}{16}(l-x)y^2 - \frac{B^2}{32}(l-x) + \frac{3}{16}x^3 \right] \]

\[ \tilde{N}_y = \frac{gaxy^2}{z} - \left[ 1 + \frac{1}{3}(r/l)^2 \right] \frac{8}{a} \]

\[ \tilde{N}_{xy} = (ga \frac{y^3}{16} - \frac{B^2}{32} + \frac{x^2y}{2}) \]

\[ N_x = \tilde{N}_x \sqrt{\frac{1+p^2}{1+q^2}}; \quad N_y = \tilde{N}_y \sqrt{\frac{1+q^2}{1+p^2}}; \quad N_{xy} = \tilde{N}_{xy} \]

\[ p_1 = \frac{N_x + N_y}{2} + \sqrt{\left( \frac{N_x - N_y}{2} \right)^2 + N_{xy}^2} \]

\[ p_2 = \frac{N_x + N_y}{2} - \sqrt{\left( \frac{N_x - N_y}{2} \right)^2 + N_{xy}^2} \]

\[ \tan 2\theta = \frac{2N_{xy}}{(N_x - N_y)} \]

The stresses \( N_x, N_y \) and \( N_{xy} \) were calculated by using the formulae given above and are presented in Table A-1 and A-2. The principal stress resultants for the model are shown in Fig. A.1.

4) Reinforcement in Shell Body

The stress in the model shell obtained from membrane analysis is well below the allowable stress of the mortar.
So in the body of the shell nominal reinforcement of the order of 0.3 percent of the gross micro concrete section was found generally satisfactory to cater for the effects of temperature and shrinkage i.e.

\[ A_s = 0.0216 \text{ in}^2/\text{ft} \]

Provided with No. 13 (dia. 1/13 inch) M.S wire @ 2.5" c/c in two perpendicular direction parallel to edges. At the edge zones, heavier reinforcements No. 13 @ 2.5" c/c were provided to cater for bending stresses. The diagonal reinforcements (using the same M.S wire) were provided for additional safety against shear (as shown in Figs. 3.8, 3.9 and 3.10).
### A.1 Stress Resultant for Prototype

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<th>Coordinates (x)</th>
<th>Projected stresses (lb/ft)</th>
<th>Real stresses (lb/ft)</th>
<th>Principal stresses (lb/ft)</th>
<th>Inclination of $p_1$ x-axis(degrees)</th>
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Table A.2  Stress Resultants for Model

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<th>Principal stresses (lb/ft)</th>
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Fig. A.1 Diagram of principal stress resultants (lb/ft).
APPENDIX-B

STANDARD TABLES

Table B.1 Maximum recommended aggregate size for minimum model dimensions

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Table B.2 Selected grading for micro concrete aggregate

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Table 8.3 ASTM C33-67 grading limits for finer aggregates

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APPENDIX-C
SAMPLE CALCULATIONS

Strains and Stress Resultants:

Sample calculations involving the strain transformations and stress-resultants will be presented in the following articles.

C.1 In the calculations below strain readings were taken in $10^{-6}$ in/in and $E$ in $10^6$ psi.

C.2 Calculations of Stress Resultants

Gage Position 1

\[
\begin{align*}
\varepsilon_{\text{top}} &= -35 \\
\varepsilon_{\text{bot}} &= -55 \\
N_y &= \frac{\varepsilon_{\text{bot}} + \varepsilon_{\text{top}}}{2} \times E \times t \\
M_y &= \frac{\varepsilon_{\text{bot}} - \varepsilon_{\text{top}}}{2 \times c} \times EI \quad \text{(where } c = t/2) \\
I &= \frac{1}{12} \times 0.6^3 = 0.018 \text{ in}^4/\text{in} \\
&= -2.16 \text{ lb-inch/inch}
\end{align*}
\]
Gage position 8 (on the edge beam)

\[ \varepsilon_{\text{top}} = -175 \quad \varepsilon_{\text{bot}} = +145; \quad I_{\text{beam}} = \frac{4 \times 4^3}{12} = 21.33 \text{ in}^4 \]

\[ N_x = \frac{+145 - 175}{2} \times 3 \times 4 \times 4 = -720 \text{ lb (total)} \]

\[ M_x = \frac{145 + 175}{2 \times 2} \times 3 \times 21.33 = 5120 \text{ lb-inch (total)} \]

\[ = 426.7 \text{ lb-ft} \]

The remaining calculations for stress resultants are exactly similar to those above in C.2.
APPENDIX-D
DESIGN OF EDGE BEAM, TIE AND DIAPHRAGM

EDGE BEAM

Refering to the results of model in Table A.2, in Appendix-A, the average bending moment was found by using the following analysis.

\[
\begin{align*}
\text{Moment at midspan due to } N_y &\text{ and } N_{xy} \text{ at diaphragm (Fig. D.1):} \\
&= 2855.7 \text{ lb-ft} \\
\text{Moment at midspan due to self weight of edge beam and the } &\text{components of } N_y \text{ and } N_{xy} \text{ at diaphragm (Fig. D.2):} \\
&= 1029.2 \text{ lb-ft}
\end{align*}
\]

Again from Fig. D.2 the moment at the midspan was found to be 1029.2 lb-ft due to the self weight of the edge beam and the components of \( N_y \) and \( N_{xy} \) at the diaphragm.
The edge beam was designed to satisfy the design requirements of the above analysis and as shown in Fig. 3.1(a).

![Diagram of edge beam and diaphragm](image)

**TIE AND DIAPHRAGM**

Refering to Fig.D.1, the tie was designed to counteract the horizontal thrust of 2854.2 lb and in the body of the diaphragm nominal reinforcement of the order of 0.25 percent of the gross micro concrete section was found generally satisfactory to cater the effects of temperature and shrinkage. These were overdesigned in order to ensure that failure did not occur in these before the shell fails as shown in Figs. D.3 & D.4.
REFERENCES


