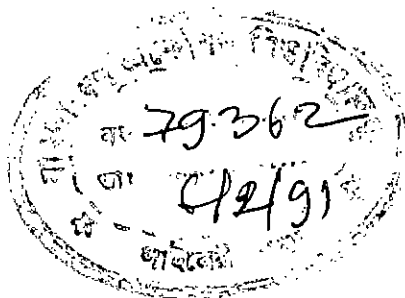# FINITE ELEMENT STUDY OF COMPOSITE BEHAVIOUR OF WALL-BEAM STRUCTURES

BY

HABIBUR RAHMAN KAMAL

Submitted to the Department of Civil Engineering of Bangladesh University of Engineering and Technology, Dhaka in Partial fulfillment of the requirements

for the degree of
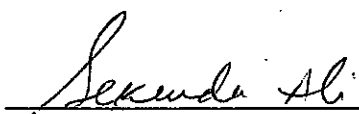
MASTER OF SCIENCE IN CIVIL ENGINEERING

October,1990.

FINITE ELEMENT STUDY OF COMPOSITE BEHAVIOUR OF
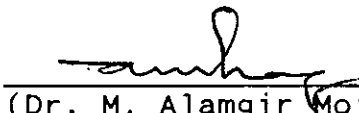
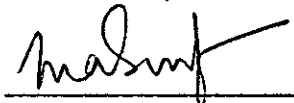# WALL-BEAM STRUCTURES

A   Thesis

By

HABIBUR RAHMAN KAMAL

Approved as to the style by:

(Dr.Sk. Sekender Ali)   Chairman
Associate Professor,
Department of Civil Engineering,
B.U.E.T., Dhaka
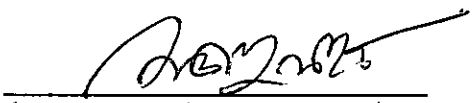
(Dr. M. Alamgir Mojibul Hoque)   Member
Professor and Head,
Department of Civil Engineering,
B.U.E.T., Dhaka.

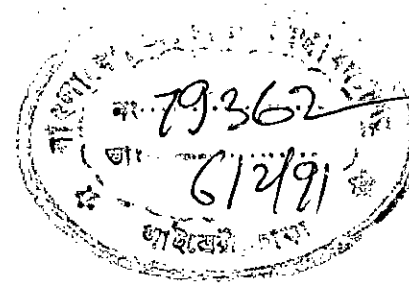(Dr. Abdur Rouf)   Member
Professor,
Department of Civil Engineering,
B.U.E.T., Dhaka.

(Dr. M. Mizanul Huq)   Member(external)
House No. 59, Road No.3/A
Dhanmondi,Dhaka.

October, 1990.

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Finally, the author acknowledges endless indebtedness to his brother Md. Lutfur Rahman,S.E.and his friend S.Kusum, whose inspirations and encouragements made this work possible.

ABSTRACT

It is a common practice in Bangladesh to design buildings in reinforced concrete frame ignoring the composite actions of wall-beam. Due to the ignorance of this composite action of wall-beam a considerable waste of materials result in building through under-utilazation. Although the concept is used extensively in many countries of the world, its optimum utilazation has been restrained to some extent due to the lack of proper investigations in the area. The previous investigations were mainly confined in the laboratory and very limited theoretical investigation based on finite element method was performed considering the brickwork as homogeneous material. This type of macro level model is suitable for the macro study but incapable of modeling local behaviour at the region of beam and wall ends where the stress gradients are very high and the fracture process is complex.

A project therefore has been undertaken in the Dept. of Civil Engineering , which involves both experimental and theoretical investigatios of the problem. The present study is a part of this on-going study which involves the development of linear elestic finite element model to study the composite action of wall-beam structures. Isoparametric

elements have been used to model the bricks, mortar joints, supporting beams, and the interface elements in between the wall and the beam. The brickwork has been modeled both as a homogeneous material or nonhomogeneous material (Bricks and joints as different materials).The model is very useful in predicting the local behaviour of the regions where the stress gradients are very high.

A series of analyses of a number of wall-beam structures with different height to span ratios, sizes of the beam, different stiffness parameters and different modular ratios have been made in this study. Particular emphasis has been given to the variation of shear stress, vertical stress, and the bending moment in the beam of the wall-beam structures. From this parametric study it has been found that the maximum moment in the beam occurs at about 1/15th of the span from the supports rather than at midspan. Whereas the tension attains its maximum value at or near the midspan. It was also found that the shear stress along the wall-beam interface is parabolic for lower values of relative stiffness parameter and the spread of the shear stress along the length of the beam is twice that of the vertical stress.

LIST OF FIGURES

LIST OF TABLES

NOTATIONS

A   Cross-sectional area of the Beam.

a   Ratio of the Contact Length to the Span

a1   Moment Distribution Factor.

a2   Moment Arm Factor.

B   A Constant.

b   Length of Bearing on the Support.

[B]   Strain-displacement Matrix

C   Stress Concentration Factor

[D]   Elasticity Matrix

{d}   Nodal Degree of Freedom.

d'   Equivalent Diameter.

E   Modulus of Elasticity of Concrete.

Ew   Modulus of Elasticity of Brick-work.

Es   Analogous Modulus of Subgrade Reaction for the Wall.

F   Stress Factor

fb   Bottom Fibre Stress in the Beam.

ft   Top Fibre Stress in the Beam.

fk   Characteristic Strength

G   Shear Modulus.

H   Height of the Wall in the Wall-Beam Structure.

H1   Calculated Height of Moment Arm.

H2   Calculated Height of Cross-section.

h   Depth of Beam.

I   Moment of Inertia.

[J]   Jacobian Matrix.

K   Relative Stifness Parameter (Smith and Riddington)

K'   Material Constant.

k   Bulk Modulus.

K_s Interface Stifness in the Tangential Direction.

K_n Interface Stifness in the Normal Direction.

k_1 Moment Coefficient.

L   Span of the Wall-Beam Structure.

L'   Specimen Width.

l_c Length of Contact.

M   Moment in the Beam.

N   Shape Function.

p   External Load on the Beam(Load of the Beam support).

P   Apllied Load.

q   Vertical Compressive Force in the Masonry per Unit Length.

r_m Partial Factor of Safety.

R_f Relative Stifness Parameter (Ahmed and Davies).

S   Shear Distribution Factor.

T   Tensile Force in the Beam.

t   Thickness of the Wall-Beam Structure.

xii

U   Strain Energy.

u   Displacement in the X-direction

v   Displacement in the Y-direction.

W   Total Load on the Wall-Beam Structure.

w   Load per unit length.

δ_x Longitudinal Stress

δ_y Vertical Stress.

$\epsilon_x, \epsilon_y, \gamma_{xy}$ Strains.

τ_{xy} Shear Stress along the Interface of the Wall and Beam.

λ_1 Stifness Parameter (Green)

β_1 Reduction Factor.

β_2 A Parameter (Green).

δ_1 Coeffient of Axial Tension.

Υ Influence Factor for Opening.

v   Poisson's Ratio.

μ, λ Lami's Constant.

α, γ, β Constants for Determining Stresse (Ahmed and Davies).

CONVERSION FROM ENGLISH TO SI UNIT

Length

1 Ft	=0.3048 m	=304.8 mm
1 In	=25.40 mm	

Area

1 Sq-ft	=0.092303 sq-m	=92903 sq-mm
1 sq-in	=16387 cu-mm	

Moment of Inertia

1 Ft	=0.008631 m	=86316000 mm
1 In	=416231 mm	

Mass

1 lb	=0.4536 kg

Weight

1 k	=4.44822 kN
1 lb	=4.44822 kN

Moment

1 k-ft	=1.35582 kN-m
1 lb-ft	=1.35582 kN-m
1 k-in	=112.985 N-m
1 lb-in	=0.112985 N-m

Stress

1 Psf	=47.88 Pa
1 ksi	=6.89476 Mpa
1 Psi	=6.89476 kPa

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The brick-wall on beam is one of the most frequently used structural system, yet its behaviour is often misunderstood. Out of numerous applications of wall-beam type structure, foundation beams, strip footings, lintels supporting the brick-walls with or without openings , grade beams etc are common examples. Whether a lintel over a window or a massive transfer beam at the base of a tall building, the composite interaction of a beam with the wall it supports is complex.

Masonry walls constructed above beams spanning between two supports acts compositely with the beams and distribute the major loads to the supports due to their high in plane rigidities as a result of arching action. The beams are thus required to tie the arch and hence, axial force are more predominant than the flexural forces that are generally expected in beams. Consideration of the composite behaviour will not only lead to a rational design of beams and walls, but also ensure satisfactory performance with respect to cracking. Design of beams for the composite behaviour will also result in a significant saving in concrete and steel.

Prior to 1952, it was common practice to assume the brick wall as a dead load on the beam and act as a filler and as a result their inherent strength was ignored. Later the beam was designed to support the load of an equilateral triangular area of brick wall, the span of the beam being the base of the triangle.

In the pervious years, the research work in this field was confined mainly in the laboratory and from these research works, many design recommendations emerged out which varies from country to country. Due to the inherent complexity involved in the composite action of the problem, theoretical work on the above field is almost nonexistent.

A program has been therefore undertaken in the Dept. of Civil Engg, which involves both theoretical and experimental works. The present study has been limited to inplane loading, both concentrated and distributed. The effects of loading history and time dependent deformation and other effects of similar type have not been considered. Two different finite elements, one four noded element (using linear displacement function) and the other one is eight noded element (using parabolic displacement function) are used to develop the program but the present study is based on the analysis using only four noded finite element.

Different techniques have been applied to make the program very efficient and versatile so that a real size wall-beam panel can be analyzed and it can be used for the investigations of composite behaviour of similar other types of structures with little or no modification of the program. The wall-beam structures are modeled as two different materials with an interface element in between. The wall is treated both as homogeneous material (considering the average properties of the brickwork) and a nonhomogeneous material (considering the brick and mortar as two different materials). The properties of the material needed to define this model have been obtained from results and tests, performed at different universities, and institutions. The finite element model has then been used to carry out a parametric study of the composite actions of wall-beam of standard sizes. The study illustrates the potential of the model both as a subsequent tool and as a means of preparing design procedures for practical purposes.

1.2 OUTLINE OF THESIS

The structure of this thesis can be summarized as follows:

i. A review of the state of art of the composite behaviour of the wall-beam structures with particular

emphasis on the areas significant to this study.

ii.   To   present   the   analytical   background   for
establishing the basis of calculation required for the
study of composite behaviour of wall-beam with the
introduction of interface element in between the
brickwork and the supporting beam.

iii. Development and description of   two dimensional
finite element model and verification of proposed
finite element model with the results of test on brick
prisms under concentrated and distributed load on the
wall-beam structure.

iv.   Description of different aspects of wall-beam
interaction and different assumptions   made in the
analysis.
v.   Finally, the application of the finite element
model to a parametric study of the behaviour of the
wall-beam structure subjected to both concentrated and
distributed load is presented.

CHAPTER 2

REVIEW OF LITERATURE

2.1 INTRODUCTION

If a wall and the beam on which it is supported act as a composite unit then the proportion of the load carried by the supporting beam must be determined. Prior to 1952, it was common practice to assume that the supported load is due to a triangular area of brickwork in which the span of the beam represented the base of the equilateral triangle. Since then experimental and theoretical studies have resulted in a better understanding of the problem.

The action of the load on the wall produces horizontal forces in the beam which partially restrain the supports so that arching action results in the panel. The degree of the arching action depends on the relative stiffness of wall and beam and in general, the stiffer the beam, the greater the beam bending moment since a large proportion of the load will be transmitted to the beam.

The above concept of composite action of wall and supporting beams has been used in many countries of the world. A number of studies have been carried out all over the world but most of them are experimental providing

different results and recommendations which vary from country to country. A few numerical models have been developed in the past with limited use for research purposes.

This chapter reviews the literature which have been published on various aspects of this problem in different countries of the world.

2.2. BRICK MASONRY WALLS IN COMPOSITE ACTION

2.2.1 Wall-Beams

Beams are provided to carry masonry walls over opening such as doors, windows and as grade beams. Such structures comprising beams and masonry above are called wall-beams. The behaviour of wall-beam structures are more complex than the designer expects in general. The high in plane rigidity of the wall makes it to act as an arch or deep beam, spanning across the opening. Additional vertical loadings on the wall are not transmitted vertically down to the beam below but are carried towards the stiff ends of the beam. The composite action can be described to be a tied arch in which the wall serves as the arch and the beam as the tie which prevents the arch from spreading. The uniformly distributed vertical load applied on the top of the wall

will be redistributed by the arching action so that high
vertical compressive stresses and horizontal shear stresses
are induced in the bottom corners of the wall. The
distribution of vertical stresses on the beam causes bending
moment in the beam to be substantially less than if the load
would have been applied with uniform distribution on the
beam itself.

2.2.2 Structural Action of wall-beams

It has long been recognized that structural interaction
takes place between a masonry wall and a supporting steel or
concrete beam. In simplest term this has been represented by
assuming that the beam supports only part of the brickwork
represented by a triangular load intensity diagram with zero
ordinates at the supports and maximum loading at midspan.
The loading from the remainder of the brickwork was assumed
to be transmitted to the support points by arching
action(22) as shown in figure 2.1.

A number of experimental studies (5,25,26,30,31)and a
very limited theoretical studies (7,8,13,14,17,27,32) of the
problem have shown that the vertical and shear stresses at
wall-beam interface are concentrated towards the supports as
shown in figure 2.2.
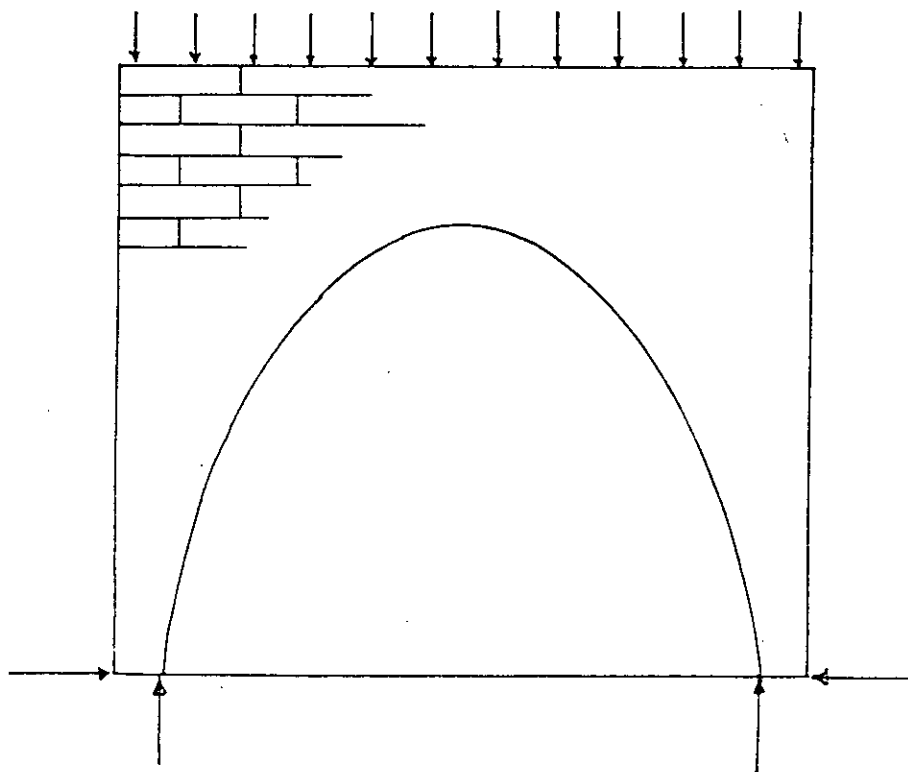
FIG 2.1 ARCHING FORCES IN WALL (HENDRY)
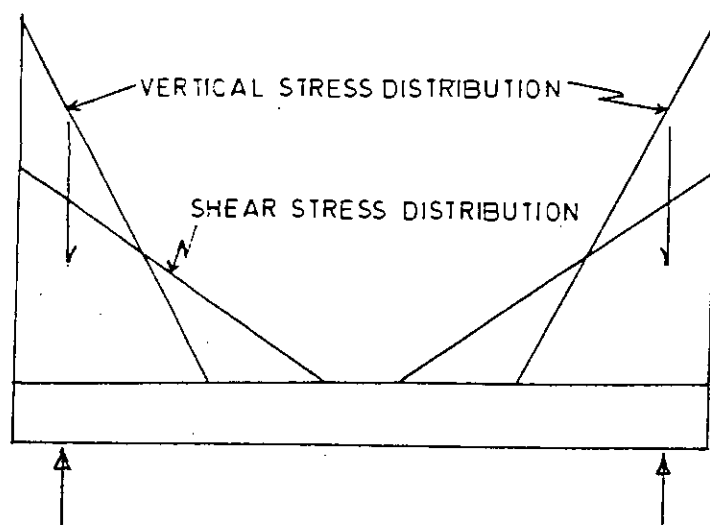
FIG. 2.2 VERTICAL AND SHEAR STRESSES IN THE BEAM (HENDRY)

Both the shear stresses and vertical stresses can be approximately represented by triangular diagram and the more flexible the beam, the more concentrated these stresses are towards the support. The shear force tries to counteract the downward deflection of the beam. Still, there is a tendency of this element to deflect downward i.e. away from the wall; with the possible development of cracks between the top of the beam and the bottom of the wall. The shear forces also induce tensile force in the beam.

Typical illustration of the vertical stresses along the wall-beam interface and horizontal stresses along the vertical centre line are as shown in figure 2.3. The distribution shows that

i.   Maximum vertical stresses occur at the support.

ii.   The maximum horizontal stress in the beam is at midspan, may be tensile, throughout the depth of the beam, i.e. it acts as a tie member.

Experiment also shows that composite action can not take place unless there is sufficient bond at the wall-beam interface to support the required shear stresses. The large compressive stresses near the supports result in large frictional forces along the interface and it has been shown that if the depth/span (H/L) ratio of the wall is greater
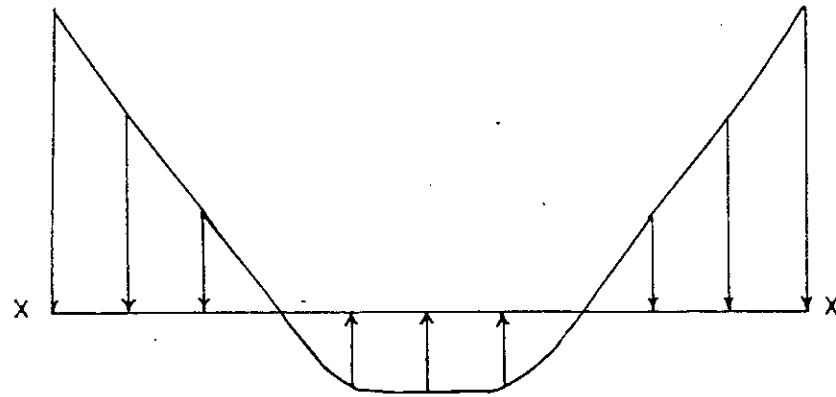
(a)   VERTICAL STRESS
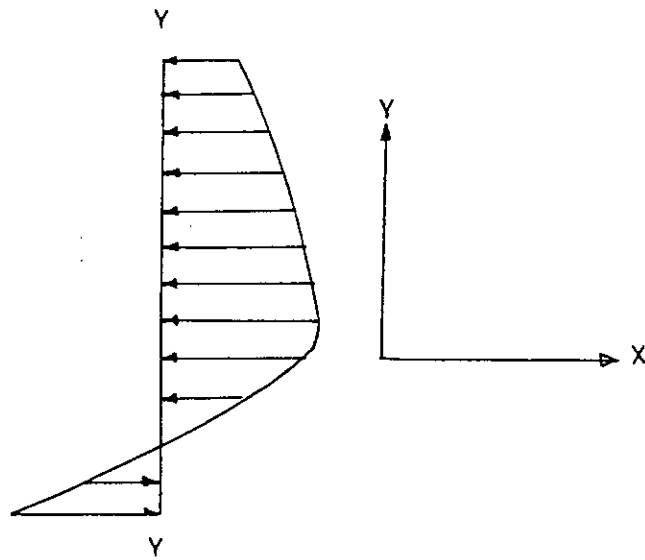
(b) HORIZONTAL TRESS

FIG. 2.3 DISTRIBUTION OF VERTICAL STRESSES ALONG THE
INTERFACE AND HORIZONTAL STRESSES ALONG THE VERTICAL
CENTRE LINE OF WALL BEAM STRUCTURE. (HENDRY)

than 0.6 then the friction forces developed are sufficient to supply the required shear capacity(31).

From different experimental investigations it was concluded that the degree of arching action depends mainly on the following parameters:

i.   The relative stiffness of the wall and beam.

ii. Type of loading.

iii. Span of the wall-beam.

iv. Wall height and thickness.

v.   The modular ratio of wall and beam.

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ii.   Type of loading.

iii. Span of the wall-beam.

iv.   Wall height and thickness.

v.   The modular ratio of wall and beam.

2.3. EXPERIMENTAL STUDY

2.3.1. General

Due to the complexity of the problem and nonavailability of the proper computing facilities in the past, the experimental studies were preferred. Those who are pioneer in this study are Wood, Simms, Smith, Riddington, Burhouse etc. In this section these studies will be reviewed and examined carefully.

2.3.2 Wood and Simms

In 1969, Wood and Simms (30) proposed a method which was based on the assumption that the vertical stresses in the vicinity of the supports form a rectangular stress block which extends at a distance of 'X' in to the span from the ends of the beam as shown in figure 2.4 instead of triangular distribution of load as described in the previous chapter. The bending moment coefficent, k1 was introduced.

$$M = WL/k1 \text{ and}$$

$$C = L/2X \qquad\qquad - - - - -(2.1)$$

where,

C=Stress concentration factor; that is C is the ratio of maximum to average stress.

W/UNIT LENGHT

TYPICAL DISTRIBUTION IN

ELASTIC PHASE

x

x

W/2

W/2

WL
2

L

WL
2
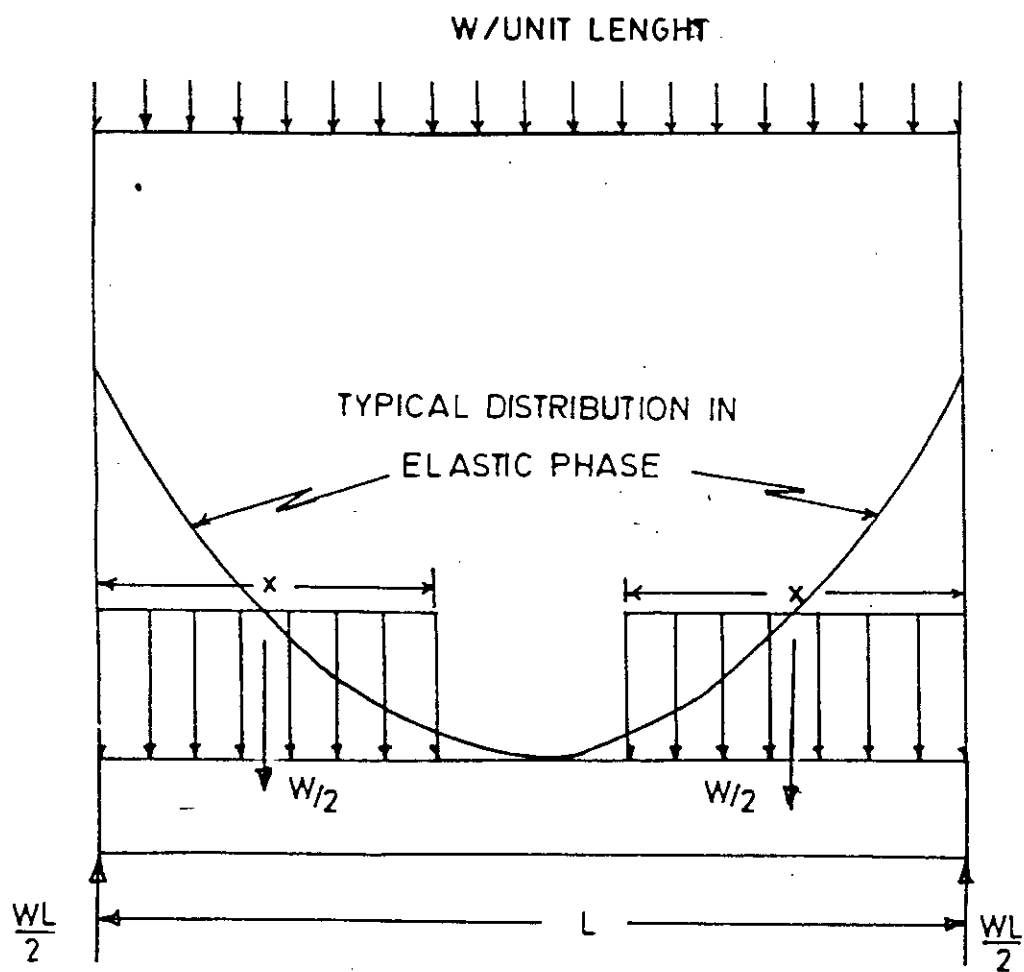
FIG. 2.4 ASSUMED EQUIVALENT BEAM LOADINGS (WOOD & SIMMS)

From figure 2.4,

M=WX/4.

WX/4 = WL/k1 or X/L = 4/k1 and

C=k1/8, i.e. bending moment coefficient,

k1=8C.   ....(2.2)

where,

W=Total distributed load on wall,

L=Span of the beam.

This means that the bending moment coefficient, the
stress block and the stress conentration factor have typical
values (31).

Their investigations reveal that for composite action
to be possible, the average compressive stress in the wall
must be relatively small. On the basis of these values of C,
the values of k1 may be determined.

Let the design strength per unit area of a wall

$= \beta_1 fk/rm$

where,

β_1 = Reduction factor

fk = Characteristic strength

rm = Partial factor of safety.

Now if the average stress in the wall is less than the
design stress by a factor of F, and if the design strength
may be increased by 50% in the region of concentrated stress

at the support, then, $C*F*\beta_1*fk/rm \not> 1.5 \ fk/rm$ but since $C=k1/8$. This equation leads to the bending moment factor for the beam, $k1 \not> 12(F*\beta_1)$, when $M=WL/k1$. This simple analysis has been elaborated by Wood and Simms to allow for the axial tension in the beam on the assumption that a parabolic line of thrust is developed in the wall.

Wood (31) based on the test results gave the following theoretical formulae to calculate the bending moment and the moment arm; the later being based on the elastic analysis of homogeneous deep walls. He recommended the depth of the beam to vary from 1/15 to 1/20 of the span; and bending moments, he recommended for door and window openings near the supports of walls to be WL/30 and WL/100 for plain walls with door or window openings at the centre. For freely supported deep walls a moment arm of 0.67 x depth of the wall with a limit of 0.70 times the span may be assumed. But in case of continuous beams the limiting moment arm at the centre is about 0.47 times the span and at the supports 0.34 times the span.

2.3.3. P. Burhouse

Burhouse (5) showed that for the majority of the cases of failure take place by crushing at the lower corners of

the panels, followed by failure of the supporting beam and similar results were reported by Smith and Riddington et.al.(27,28) for structures having light to medium supporting beam. With a very heavy support beam, local damage of the brickwork in the vicinity of the support was much less severe and at the ultimate load, failure was initiated from the support. In most cases the critical condition for failure will be concentrated vertical stress distrubition around the support. If the beam is exceptionally heavy, the stress concentration will be greatly reduced and in such cases overall compressive failure may be critical. Burhouse (5) presented a study of different parameters for the investigations of wall-beam in comparison with the investigations made by others. Method proposed by Davies and Ahmed (13, 14) appeared to give the most consistent agreement with the experimental results while the formula given by Smith and Riddington (27) gave a very high results for these beams. It is to be noted that the for the above study reinforced concrete beams were considered.

From a number of experiments Burhouse concluded that when load is applied on the top of a load bearing brickwork built on a beam, which spans between supports composite action between the walls and the beam significantly affects

the distribution of load transmitted through the wall to the beam. The composite action approximates in certain cases to that of a tied arch, the arch forming in the wall with the beam acting as a tie. From the tests carried out by Burhouse the following points was observed:

i.   In all tests, except one, primary failure occurred as a result of crushing of the brickwork at lower corner of the panel and followed by failure of the beam.

ii.   Assuming that the resultant tensile force in the beam acts at the centre of the reinforcement, the internal moment arm, the value of which varied with the applied load. Values of internal moment arm are expressed as a proportion of span and also compared calculated values of tensile force in the reinforcement and that measured in the test.

iii. In the majority of the tests the ratio of calculated and the measured tensile force is greater than one. This may be partly due to both concrete and brickwork carrying tensile stresses at the section at which the steel stresses is measured and hence giving a lower value.

iv. Assuming a limiting relationship between stresses and strains, it is possible to calculate the ratio of maximum strsses to the average applied strsses. The ratio varied with the applied load and the range together with an average values

2.3.4 Saky Rosenhaupt et.al.

Rosenhaupt et.al. (25,26) in his experimental study of masonry walls on beam tested a number of masonry walls under uniformly distributed load. The results were compared with a proposed theory. In his experimental study, he found the beam to behave like a tie taking all the tension and the compressive force being distributed to wall along the height. The ratio of the interior moment arm to height is approximately 0.6.

Twelve experiments were performed. A typical test specimen and the loading arrangement is shown in figure 2.5. The walls were simply supported and subjected to uniformly distributed load on the upper edge.

From the tests it was observed that the vertical load is uniformly distributed along the upper edge of the wall and then transmitted through the masonry to the beam supports. Vertical stresses in the bottom layer of the
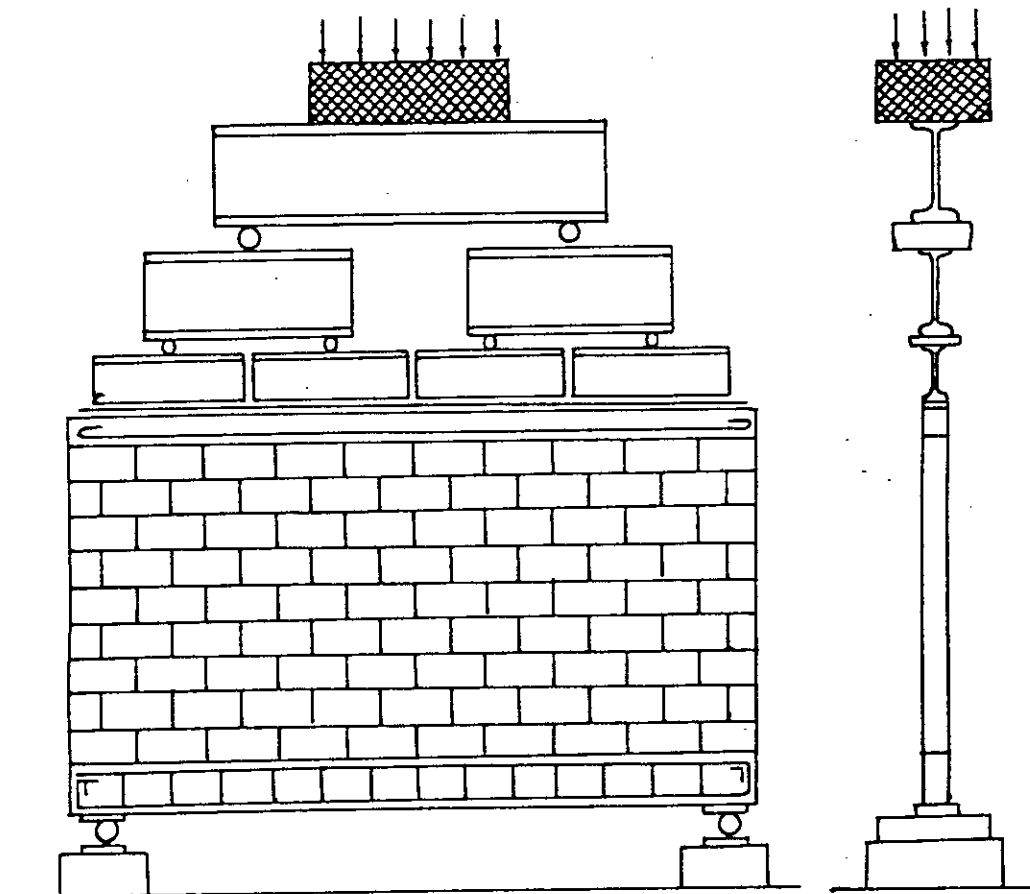
FIG. 2·5 LOADING ARRANGEMENT (ROSEN HAUPT)

masonry wall reaches a value approximately 3 to 4 times higher than the external load. When this stress exceeds the crushing strength of masonry, failure sets in through crushing of the building block near the edge. Another cause of failure is the vertical shear near the supports. Shear cracks appear in the vertical joints or between the vertical column (if any) and the masonry. It depends on the strength of the vertical joints and strength of the masonry, the height of the wall, inclusion or non inclusion of vertical ties etc.

The different factors influencing the behaviour of wall beams were also examined among which the H/L ratio and masonry materials are important. The height of the wall determines the moment of inertia and therefore the magnitude of the deflection as shown in figure 2.6. The first cracking load is also affected by height. In the composite cross section, the masonry constitutes the major part of the vertical dimension (about 90%). It is to be expected, therefore, that the masonry will have a major effect on the characteristics of the structures. Therefore it was concluded that these test program confirms the basic assumptions of the composite action that the beam acts as a tension tie and the wall as a compressive zone. The moment arm is approximately equal to one half the height.
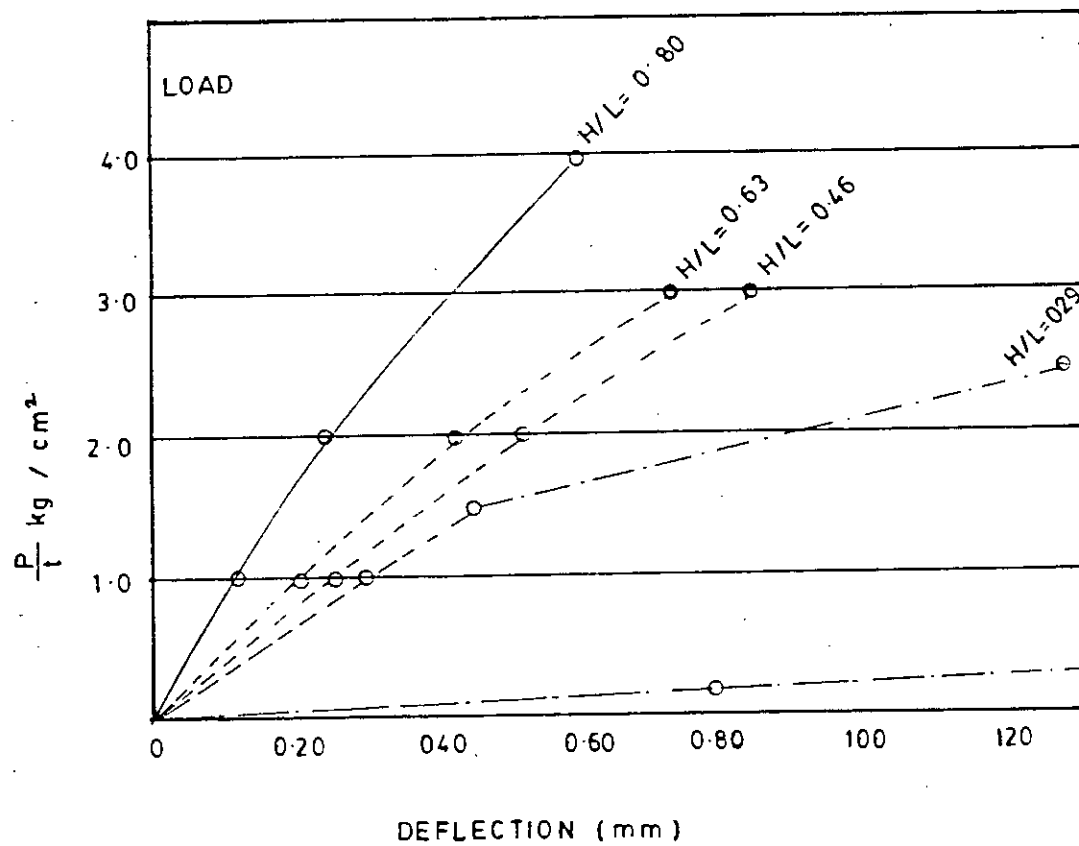
FIG. 2·6 LOAD DFFLECTION CURVE FOR DIFFERENT H/L RATIO (ROSEN HAUPT)

2.3.5 G. Annamalan, R. Jayaraman & A.G Madhaba Rao

Extensive experiments (1) were also carried out on the composite behaviour of reinforced brickwork in India. Their findings are described in the following paragraphs:

Ultimate load: Most of the tested specimens failed by crushing at the supports followed by final shear failure. In all the reinforced brickwork lintels, the cracks appeared along the major joints vertically between bricks in the beams and a horizontal separation was visible mostly along the second course of the wall. The compressive strength of the brickwork was found to have considerable influence on the strength of the composite structures. Reinforced brickwork thin lintels were found in general to have comparable ultimate strength with those of the reinforced concrete thin lintels considering the composite action with the same type of brick walls.

Vertical Strain Distribution: The effect of arch action is clearly brought out in the concentration of strain near the supports. The variation of compressive strains is found to be approximately triangular satisfying the theoretical assumptions. The theoretical

contact lengths on the average are about 20% less than that determined experimentally.

Maximum Masonry Compressive Stress: From the compressive strain the compressive stress near the supports were calculated. The theoretical and experimental values have a close correlation. The experimental stresses were found to be much lower than the allowable stresses (calculated from the crushing strength of brickwork prism with safety factor of 4) thus indicating the satisfactory working load behaviour.

Tensile force in Beams: The tensile force in the beam was determined by multiplying the average tensile strain by the axial rigidity of the beam. The experimental values have good correlation with that calculated from empirical formula, i.e. from the formula (30).

$$T=W/3.4. \qquad\qquad\qquad\dots\dots\dots\dots(2.3)$$

Where,

T= Maximum tension in the beam,

W= Total distributed load on the wall.

Bending Moment : The bending moment as found from these tests were found to vary from WL/30 to WL/50. The difference between the theoretical and experimental values of moment coefficients confirms the conservativeness of the assumptions made in the theoretical analysis and hence can be used for the safe design of lintels or other foundation beams.

Deflection: Deflection at service load was very insignificant. Actual midspan deflection at service loads are about span/1485 for specimen made with wire cut bricks and about span/2380 for special chamber brickworks. The load deflection behaviour indicates that the failure takes place by crushing of masonry walls rather than by flexure.

2.4 THEORETICAL INVESTIGATIONS

2.4.1. General

In comparison to the experimental studies, the theoretical studies in this area are very few and limited. Due to the complexity of the problem and non-availability of the proper computing facilities in those days, the theoretical studies especially finite element method of analysis could not advance properly. However, a few potential researchers took the courage to initiate the theoretical studies in this area. They developed numerical models to find the stresses, moments and displacements in the structure. In these section these studies will be reviewed and examined carefully.

2.4.2. S. Smith and Riddington.

Simth and Riddington (27) developed a four noded finite element program for the stress analysis. They used linearly varying displacement functions. The brickwork was modeled as a homogemeous material considering the average properties of brick and mortar , thus relatively coarser finite elements were choosen. The model although predicts the global behaviour of the structure quite satisfactorily,the analysis was handicapped due to the

selection of improper element sizes and improper representation of the material properties of the constituents. Thus the author considered the brickwork as a nonhomogeneous material, completely ignoring the orthotropy of the material.

From the analysis the compressive stress and horizontal shear stress distribution over the contact lengths were found to be approximated reasonably well to be triangular diagram. Their study covers a wide range of wall beam combinations and the results are summarized as in figure 2.7.

They perfomed a parametric study by choosing parameter like Poison's ratio, H/L ratio, modular ratio of brickwork and beam. From this parametric study they introduced a relative stiffness parameter, K.

The results of the parametric study also revealed that for wall height greater than 0.7L, the structural behaviour of a wall on beam is independent of height. Earlier experimental work of Wood and Simms (30) recommended this value to be 0.6L. Their investigation pointed out that the composite wall-beam is the same type of problem as beam on elastic foundation as shown in figure 2.10.

Also in these problems separation of the element is

FORCE = T
————   ———
LOAD W_w

AND

MOMENT
—————————
UNIFORMLY
DISTRIBUTED
MOMENT

$= \dfrac{M}{W_w L / 8}$

BENDING MOMENT

TIE FORCE

TIE FORCE $W_w / 3.4$

$E / E_w = 30$

$E / E_w = 4$

$E / E_w = 30$

MOMENT $W_w L / 4 K^{4/3}$

$E / E_w = 4$

RELATIVE STITIFNESS PARAMETER K
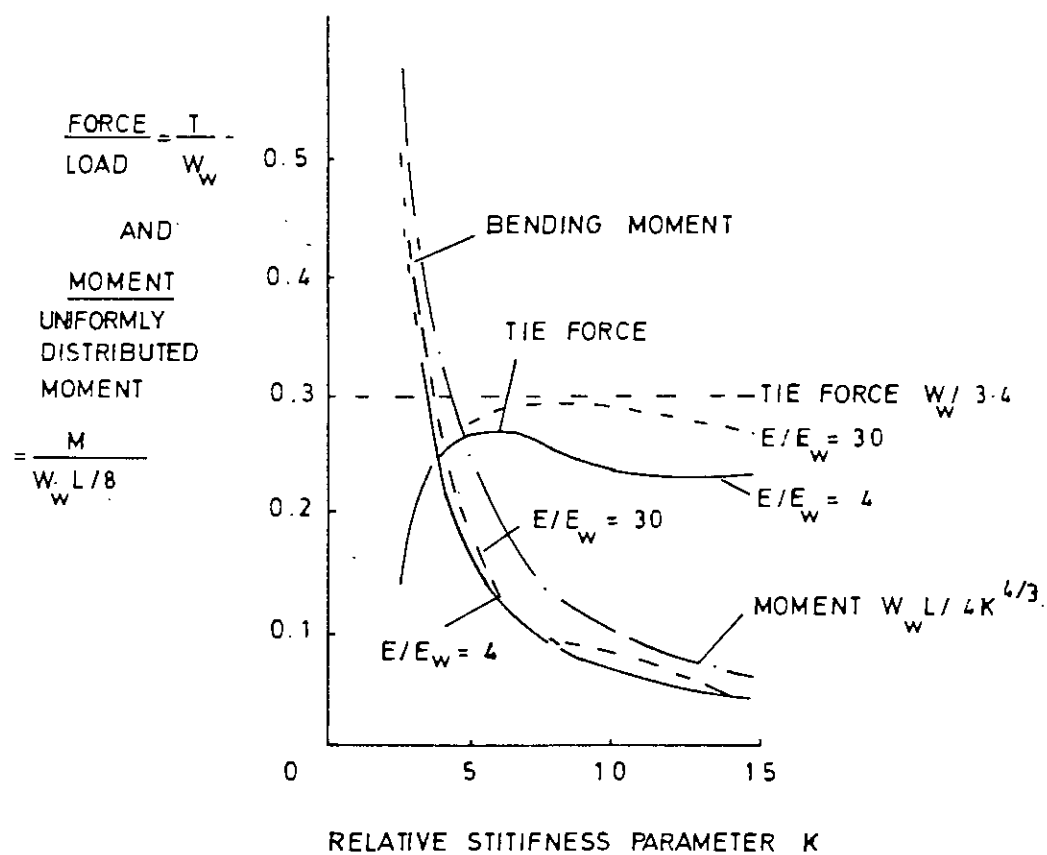
FIG. 2·7 MAXIMUM BENDING MOMENTS AND TIE FORCE IN BEAM (STAFFORD SMITH
AND RIDDINGTON)

possible. The length remaining in contact, after the separation has taken place due to the loading, bring a function of the relative stiffness. Let a L = Contact length, then

a $\propto$ ((E*I*L/(Ew*t))**0.25

where,

EI = Flexural rigidity of the beam

a = Ratio of the contact length to span

Ew = Elastic modulus of the wall material in compression.

t= Thickness of the wall and

L = Span ofthe beam or length of the wall.

Then,

a/L=(E*I/(Ew*t*L**3))**0.25

a/L=B/K

where K is the relative stiffness parameter and B is a constant, to be determined from experiment. The average value being unity and a = L/K. From this equation it is seen that the stiffer the beam relative to the wall, the longer will be the length of contact, in turn increasses the bending moment in the beam and thereby reduce the stress in the wall. From the theoretical investigation, they introduced ,

$$\delta y(max) = 1.63 \, W((Ew*t*L**3)/E*I))** \, 0.28/(L*t)$$

where δy is the maximum stress in the wall.

The effect of the axial stiffness of the beam was also considered and the effect of increased axial stiffness was to reduce the spread of the arch action. The principal effect of reducing the flexural stiffness of the beam by increasing K, is to increase the peak compressive stress at ends of the beam and to reduce the beam bending moment. For lower value of K (up to 5), reduction in the beam stiffness significantly increases the tie force when full composite action takes place. For values of K beyond 5, the reduction in the stiffness of the beam allows the arch action to spread slightly. They also concluded that the maximum beam moment occurs usually very close to the beam support and maximum tie force around the centre of the span. From the analyses Smith and Riddington proposed formulae for beam bending moment and tie force.

Maximum bending moment= $(W*L)/(4(Ew*t*L**3/(E*I)**(1/3)..$

$$............(2.4)$$

Maximum tie force in the beam = $W/3.4$. ....(2.5)

Here it may be noted that if H/L equals to 0.6 is put in the equation of tie force proposed by Wood (31)

i.e. T= 3WL/16H, then, T=W/3.2, which is 6.25 percent higher than the value proposed by Smith and Riddington.

2.4.3. Davies and Ahmed

In 1977, Ahmed (13,14) completed a linear elastic finite element study on the basis of which an approximate solution for the composite wall-beam problem was made by Ahmed and Davies (11,12,13).From the study they introduced a relative stiffness parameter.

$$Rf = ((Ew*t*H**3/(E*I))**0.25 \qquad \ldots\ldots\ldots(2.6)$$

where,

Rf = Relative stiffness parameter.

This parameter is similar to K introduced by Smith and Riddington (27). This paramter is more representative of the wall beam geometry than the parameter suggested by Smith & Riddington. In order to calculate the beam axial force, a relative axial stiffness parameter, Ra = EwtH/EA was introduced, Where, A is the area of the beam.

The basis of the method is that the vertical and shear stress distribution along the contact surface is mainly governed by flexural stiffness parameter, Rf in the

following manner

For Rf > 7 the stress distribution is triangular

For 5 < Rf < 7 the stress distribution is parabolic (quadratic) .

For Rf < 5 the stress distribution is parabolic (cubic) as shown in figure 2.8.

The axial force in the beam which varies from zero at the support and maximum at the centre, variation being linear as shown in figure 2.9.

From the finite element study they proposed few empirical formulae and these are

$$\delta y(max) = (W/Lt)*C$$
$$T = W(\alpha - \gamma Ra)$$
$$\tau(max) = W(\alpha - \gamma Ra)(1 + \beta Rf) \qquad \ldots \ldots (2.7)$$
$$C = 1 + \beta Rf$$

Where,

$\delta y(max)$ = maximum vertical stress in the brickwork,

T = maximum tensile force in the beam,

$\tau(max)$ = maximum shear stress along the interface,.

C = stress concentration factor.

C, α, γ, β may be determined from prescribed graph available at ref. (11,12).
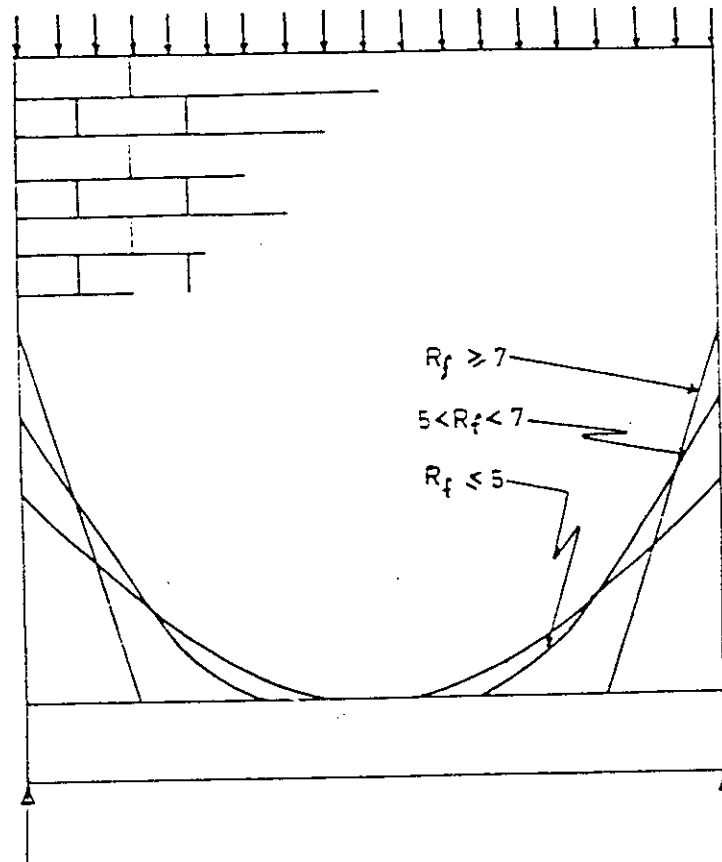
FIG 2.8 ASSUMED STRESS DISTRIBUTION (DAVIES AND AHMED)

$R_f \geqslant 7$

$5 < R_f < 7$

$R_f \leqslant 5$
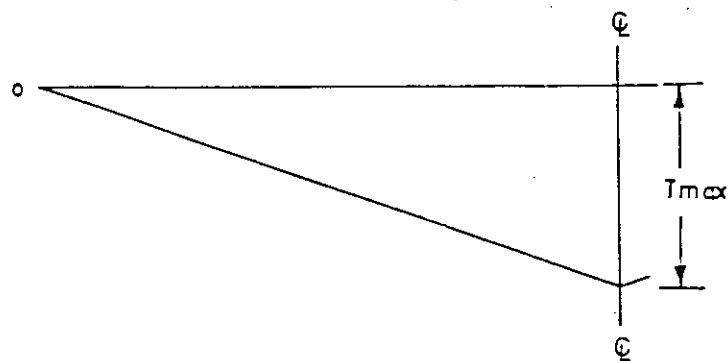
FIG. 2.9 ASSUMED TENSILE FORCE DISTRIBUTION (AHMED AND DAVIES)

2.4.4 **Green**

Green et. al. (17) developed equations for composite action of wall with height to width ratio greater than 1.5 and without openings. The equations were devoloped from a parametric study. The structural system consisting of masonry wall and its supporting beam was regarded, for the design purpose, as a beam on elastic foundation. The analysis of the wall-beam has been shown in figure 2.10. It is assumed that the beam is loaded through support and that the height of the wall is at least one third of the span. Then the load and the reaction may be expressed as

$$-EI \; d^4y \, /dx^4 = -(P-q) \qquad\qquad \ldots\ldots\ldots(2.8)$$

The prerequisite for analysis of the beam as an elastic foundation with the modulus of the subgrade reaction is that the settlement 'y' of any given point of a foundation does not depend on the settlement of other points and is in direct proportion to the pressure at that point i.e. $q = Ky$ and hence the general equation for the settlement, y becomes: $\qquad y = P \lambda_1 e^{-\lambda x} (\cos \lambda_1 x + \sin \lambda_1 x)/2Es$

From which

$q = P \lambda_1 e^{-\lambda x} (\cos \lambda_1 x + \sin \lambda_1 x)/2$

$\quad = (Es/(4*E*I))**0.25$

and from the boundary conditions:
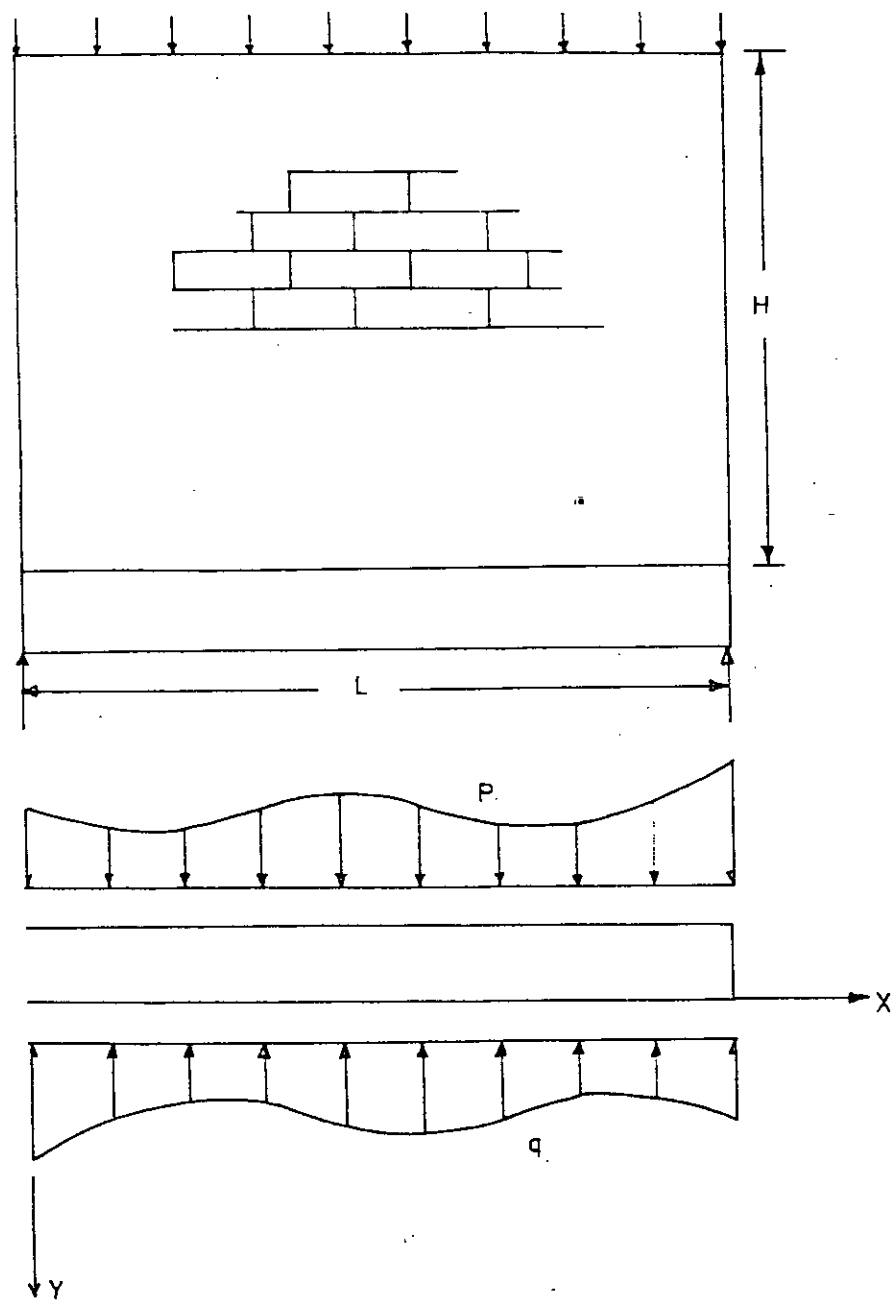
FIG. 2.10 WALL BEAM AS BEAM ON ELASTIC FOUNDATION (GREEN)

P= W/2 i.e. single load at support.

Subsequent simplification gives:

q (max) = 0.3W/2((Ew*t/(EI))** 0.33 and

Es = 0.5184 β_2**(1/3) where, ...(2.9)

β_2 = (Ew t)**4 /(E*I)

Contact length: The approximate force action in the wall beam may be assumed as shown in figure 2.11.

For a triangular distribution of stress, length of contact, l_c =4/λ_1

and a vertical force at support =Wλ_1/4   ...(2.10)

For determining the tie force in the supporting beam, it is assumed that centre of the compression is at mid height of the wall. Then taking moment about point 'O' of figure 2.11.

T= Wγ_1 where,

γ_1= L(1-16/3λ_1L) /4H   ....(2.11)

Horizontal Shear : For full composite action to develop between the wall and its supporting beam, the shear strength at beam and the interface should be adequate

H

L

t

THE RESULTANT COMPRESSION
IN THE ARCH

L_c = CONTACT LENGTH

SHEAR FORCE $\tau = \dfrac{4T}{L}$

$a = H/2$

l_c

$Wx/4$

o

T
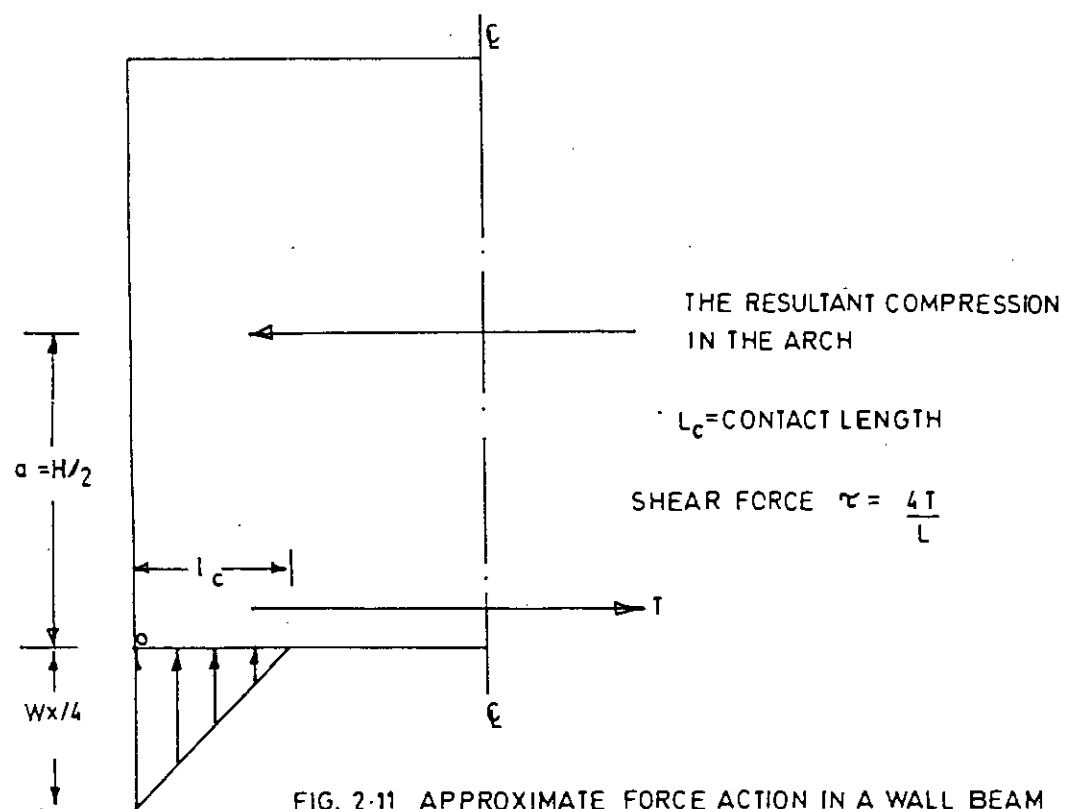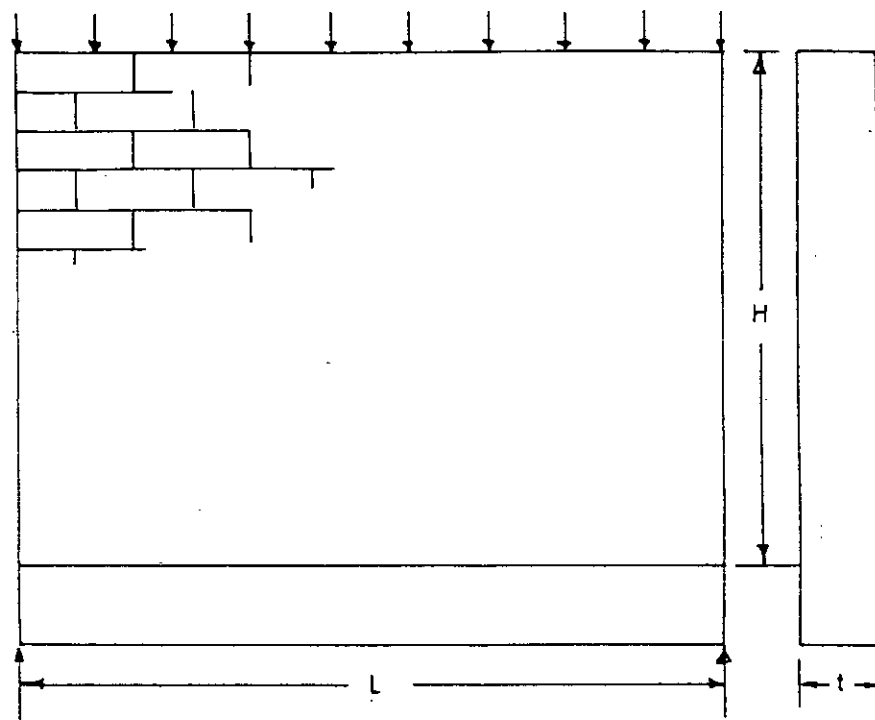
FIG. 2·11 APPROXIMATE FORCE ACTION IN A WALL BEAM
(GREEN)

to transfer the shear stress induced as a result of the arch action.

From this,

$$T = (\tau /2*)(L/2)$$

or .... (2.12)

$$\tau = 4T/L$$

Bending Moment : The assumed bending moment is calculated assuming the ends to be simply supported, though a certain amount of fixity is created at the supports due to the finite length of bearing in practical condition. Bending mement is calculated from the forces as shown in figure 2.11.

2.4.5 Coull, Colbourne

Coull (8) presented a simple variational analysis of the problem by representing the stress in wall with sufficient accuracy by a power series in the horizontal coordinates, the coeffecient of the series being the function of the height of the wall only. The magnitude of the wall stresses were found to be affected more by the wall height to the beam span ratio (H/L) and the relative wall to beam stiffness rather than by the beam depth to wall height ratio. Colbourne (7) has given theoretical solution for wall-beam system based on elastic analysis technique.

2.4.6. Gu Yisun

Gu Yisun et. al. (32) of China Central Engineering and Research Incorporation of Iron and Steel Industry and for Non-ferrous Metalurgy presented different formulae for design on the basis of data from computer calculation, based on engineering mechanics. Application of these for single storey mill buildings, since 1979, has resulted good technical and economic effects. After the first development in 1979, the formulae have been improved and they are discussed in the following paragraphs-

Determination of critical section for moment: In the case of walls without openings and openings symmetrical about the central line of beam span, the critical section is at mid span (figure 2.12.a and figure 2.12.b) and in the case of off-central openings, the section is at the vertical sides of the opening, which is nearer to the centre line of the beam span (figure 2.13 a, and figure 2.13 b).

Bending Moment : It is now a established fact that a wall- beam is a RCC member subjected to tension and bending. Computer analysis showed maximum tensile stresses in the bottom extreme fibres of the beam and small tensile stresses or compressive stress in the
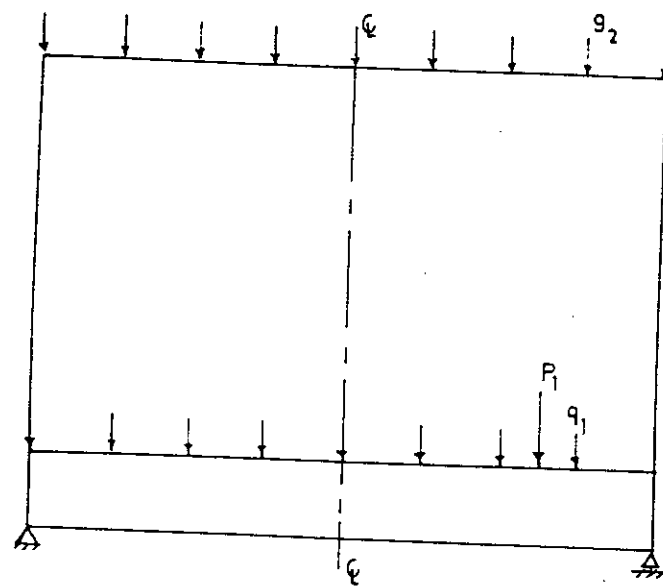
a. WALL WITHOUT OPENING
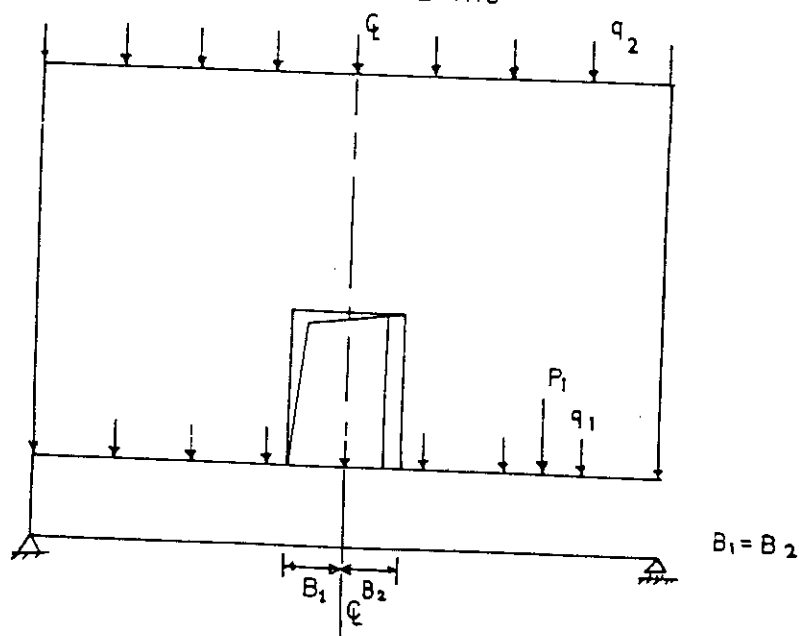
$B_1 = B_2$

b. WALL WITH CENTRAL OPENING

FIG. 2.12 WALLS WITHOUT OPENING AND WITH OPENING, CENTRALLY LOCATED (G.YISUN)

(a)
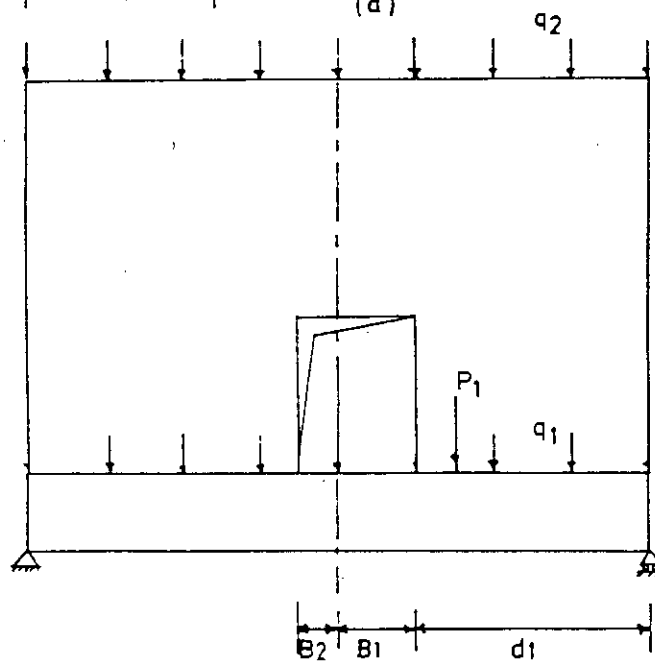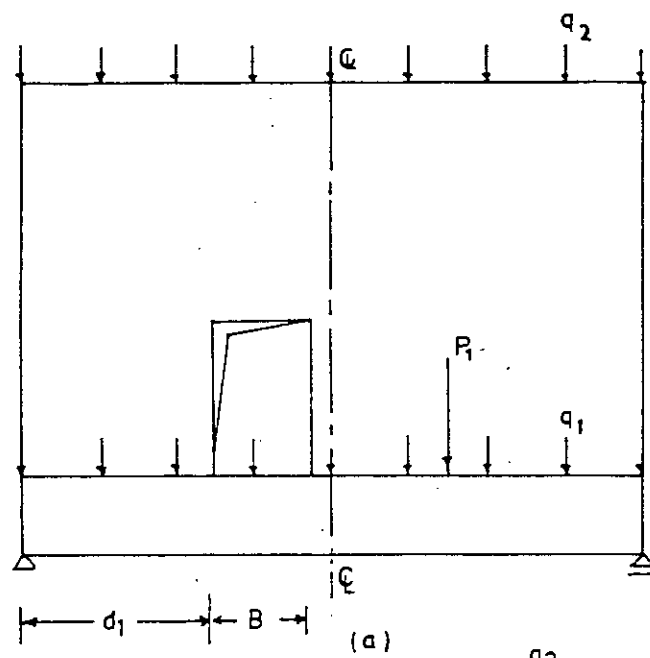
(b) B1 > B2 OR B1 < B2

FIG. 2.13 OFF CENTRAL OPENINGS.(G·YISUN)

top extreme fibre.In addition to the tension, T, there
is bending moment in the normal cross section of the
wall-beam (figure 2.14). Computer analysis also made
clear that the bending moment in a wall-beam under the
action of external load is a function of the bending
moment M2 of the composite beam at the section under
consideration. Then

M= a1*M2

where,

a1= Moment distribution factor. If M1 be the bending
moment in the case of multy storey building due to q1,
and P1, from the floor, then

$$M= M1 + a1*M2 \qquad \qquad \dots\dots\dots(2.13)$$

Axial Tension(T) in the wall beam: For brick wall
without openings, the axial tension,

$$T= M2(1-a1)/(a2*H1) \qquad \qquad \dots\dots(2.14)$$

where a2 is the moment arm factor. But for the wall
with opening

$$T= \Upsilon(1- a1)M2/(a2*H1) \qquad \qquad \dots\dots\dots(2.15)$$

where Υ is the influenc factor for opening. For wall
without opening or central opening, Υ = unity.

H1 = Calculated height of moment arm, H1 = H + 0.5h but
if H>L, H1 = L + 0.5h.
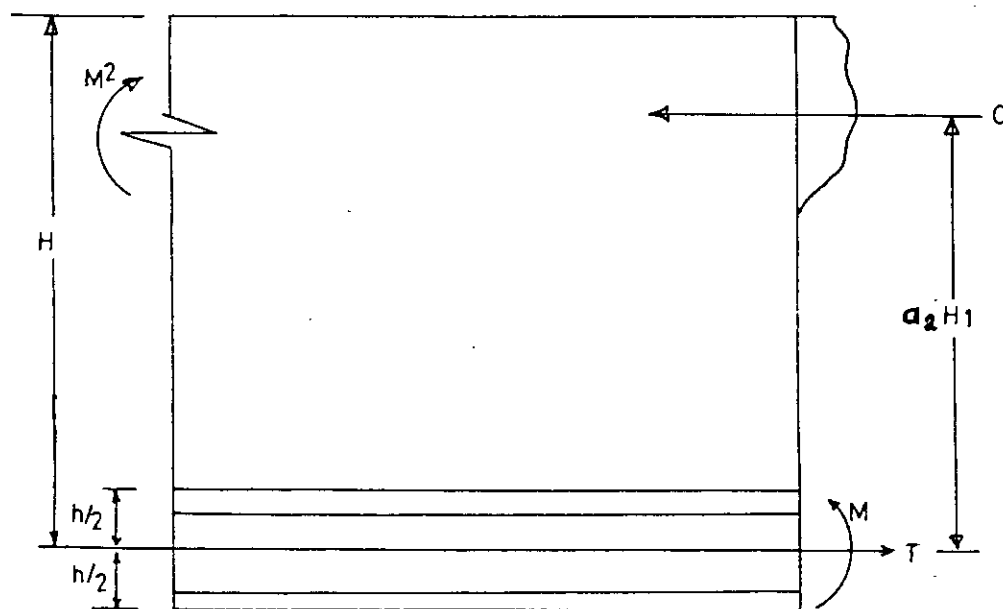
FIG. 2.14 FORCES IN A WALL BEAM STRUCTURE (G·YISUN)

Moment Distribution Factor, a1, for wall without opening: As a result of regression analysis, the following expression for X was obtained

$$a1 = h/H2 \ (0.82 + 0.3 \ E/Ew - 0.068L/h \qquad \ldots\ldots(2.16)$$

where,

L= span length of the beam, equal to the smaller value of 1.05 L1 and (L1 + b) where L1 is the clear span and b is the length of bearing on the support.

H2 = calculated height of cross section, to be used for determining the value of X, H2 = 0.75L + h when H> 0.75L and otherwise H2 = H + h.

a1 for Wall with Openings: a1 increases if the wall has an opening located off the centre of the beam span. The regression analysis gives the following relationship:

a1 = 0.33 + 2.8 R/L - 1.5 d1/L   if d1/L <0.25 and

a1 = 0.33 + 2.8 R/L - 0.78 > d1/L when d1/L > 0.25,

$$\ldots\ldots\ldots(2.17)$$

where d1 is the distance from the support to the vertical side of the opening nearer the support.

Moment Arm Factor,a2: From the regression analysis the following equation was obtained-

$$a2 = 1-0.54 \ H1/L \qquad \ldots\ldots\ldots(2.18)$$

Influence Factor for Opening : Based on the data of regression analysis the following relationship was obtained :

$$\gamma = 0.4 \ (1+(2d-B)/L) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2.17)$$

Shear Strength of Wall-Beam: On the basis of the results from Computer·analysis, the maximum shear,

$$\tau (max) = S*q2 + Q1 \quad where,$$

S = shear distribution factor.

Q2 = Shear in the cross section at support due to loading q2

Q1 = Shear in the cross section at support due to load from the floor supported by wall beam (q1 & P1)

Shear Distribution Factor, S:

Wall without opending: S varies from 0.36 to 0.48. But for practical purposes, recommended value is 0.42.

Wall with Openings: The value of S to be calculated from S= 0.4 [1.86 - 2.83B/d +3(B/d)**2]

2.5. **SUMMARY**

In this chapter a comprehensive literature survey (both experimental and theoretical)on composite behaviour of wall-beam structures have been made. From the survey it appears that most of the investigators confined their research activities in the laboratory. They performed several experiments with or without openings in the wall from which several design recommendations (empirical relations) have been emerged. The recommendations vary from country to country due to the nature and size of the specimens selected during their experimental investigations. The wide variations in the design recommendations in different countries reflect the importance of further investigations in this area. Some investigators proposed empirical formulae for axial force, shear force and bending moment considering the wall-beam as a beam on elastic foundation. Only Davis and Ahmed (11,12) and Smith and Riddington (27) adopted finite element method of analysis using linearly varying 4 noded element. They considered the brickwork as a homogeneous material thus relatively coarser finite elements were selected. With this model the local behaviour near the support can not be modeled accurately where the stress gradient is very high.

To model the high stress gradient and the fracture process in the beam and the brickwork near the support finer

finite element mesh is required. The main purpose of this study is to develop such a finite element model which can treat all the materials of structure separately. With this model the bricks and mortar joints can be modeled separately thus providing very fine mesh near the support. In the subsequent chapters the description of this model has been presented.

CHAPTER 3

NUMERICAL REPRESENTATION

3.1 INTRODUCTION

The discussion in the previous Chapter has thrown light on the present state of art of analysis of wall-beam structural analysis.

The study on this area starts in 1952, as for the first time proposed by Woods. Since then the problem has been examined in different ways in accordance with the analytical tools available at that time. Usually the magnitude of the computational task has restricted the detail stress analysis of the problem. Most of the formulations made in the past are empirical and at the same time the limitations of the technique have also precluded considerations of the physical separation which tends to occur between the wall and the beam, an important factor in the total behaviour.

But now with the availability of the electronic computational facilities and the finite element method of analysis, it is possible to make a close study of such a structure. However, the real problem is to develop a numerical model that closely resembles actual field condition and behaviour of the problem and at the same time

a balance must be made between mechanics and engineering simplicity. For the particular problem of wall- beam composite behaviour, the main task is to develop structural constitutive model for concrete, bricks, mortar and the interface elements between the wall and the bottom (or the sides) beam of the wall-beam structures. In this study an numerical model has been choosen for the wall-beam system incorporating all the relevant aspects to be investigated by using the finite element tool. The 2-D finite element model developed in this study is used to study the elastic response of the wall-beam structure. The model has the following essential features:

a) 2-D isoparametric element having 2 degrees of freedom (translational in X and Y directions) at each node has been used for bricks and joints of the brickwork. The material properties are kept constant for a particular study.

b) 2-D isoparametric element having 2 degrees of freedom at each node has been used to discretize the bottom beam of the structure.

c) 2-D isoparametric element with 2-degrees of freedom at each node has been used to represent the thin layer interface zone and their properties are kept

constant throughout the analysis.

d)   A mesh generator has been provided to generate the finite element mesh for brickwork and the supporting members automatically.

e)   Provision has been kept for selecting any type of boundary condition used in practice.

f)   Provision has been kept in the model for any type of geometry of the wall-beam structure.

g)   Provision has also been kept in the program to check the equilibrium of the structure at any stages of loading.

h)   The load on the structure can be applied either in the form of prescribed load (distributed or concentrated) or the prescribed displacements.

3.2 FINITE ELEMENT METHOD OF ANALYSIS

The finite element method is a general method of structural analysis by which the solution of a problem in a continuum mechanics may be approximated by analyzing a structure consisting of an assemblage of properly selected

finite elements interconnected at finite number of joints (nodal points).

For the purpose of structural analysis a structure (figure 3.1) can be idealized as a system of nodal points inter connected by discrete elements (figure 3.2).

The objectives of the analyses, is to find the resulting joint displacements and internal stresses in the structural elements given the joint loadings, the geometry of the structure (location of the joints) and the stiffness properties of the structural elements.

In this study two types of 2-D isoparamatric elements (figure 3.3) with two translational degrees of freedom at each node have been used. In the following section a brief description of the stiffness calculation of the element is represented.

3.2.1 Stiffness Calculation for Elements

Stiffness is defined as the ratio of the force to the displacements (9,24) and is represented by K. From elementary mechanics:

LOADING
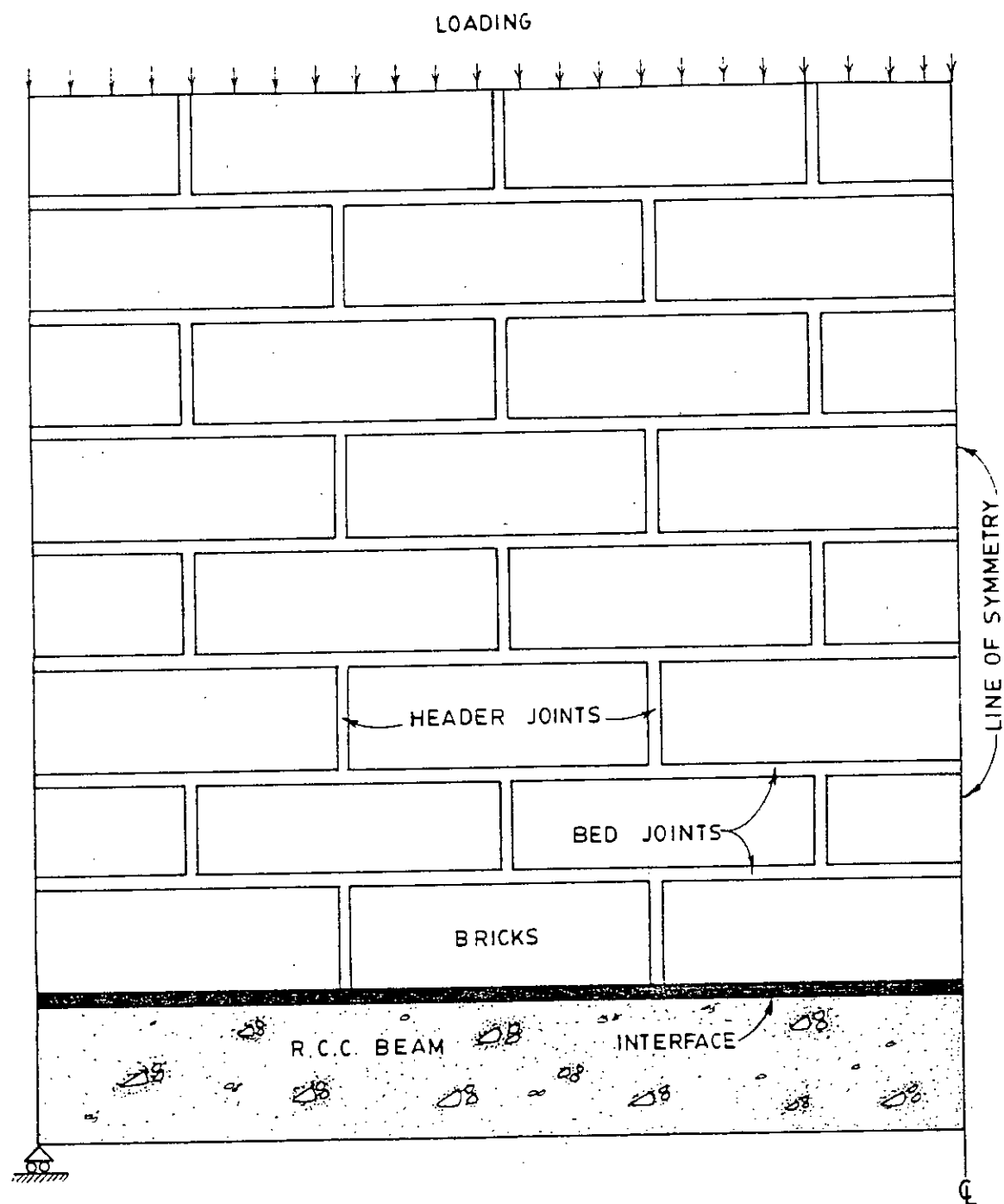
FIG. 3·1   WALL–BEAM STRUCTURE SYSTEM.
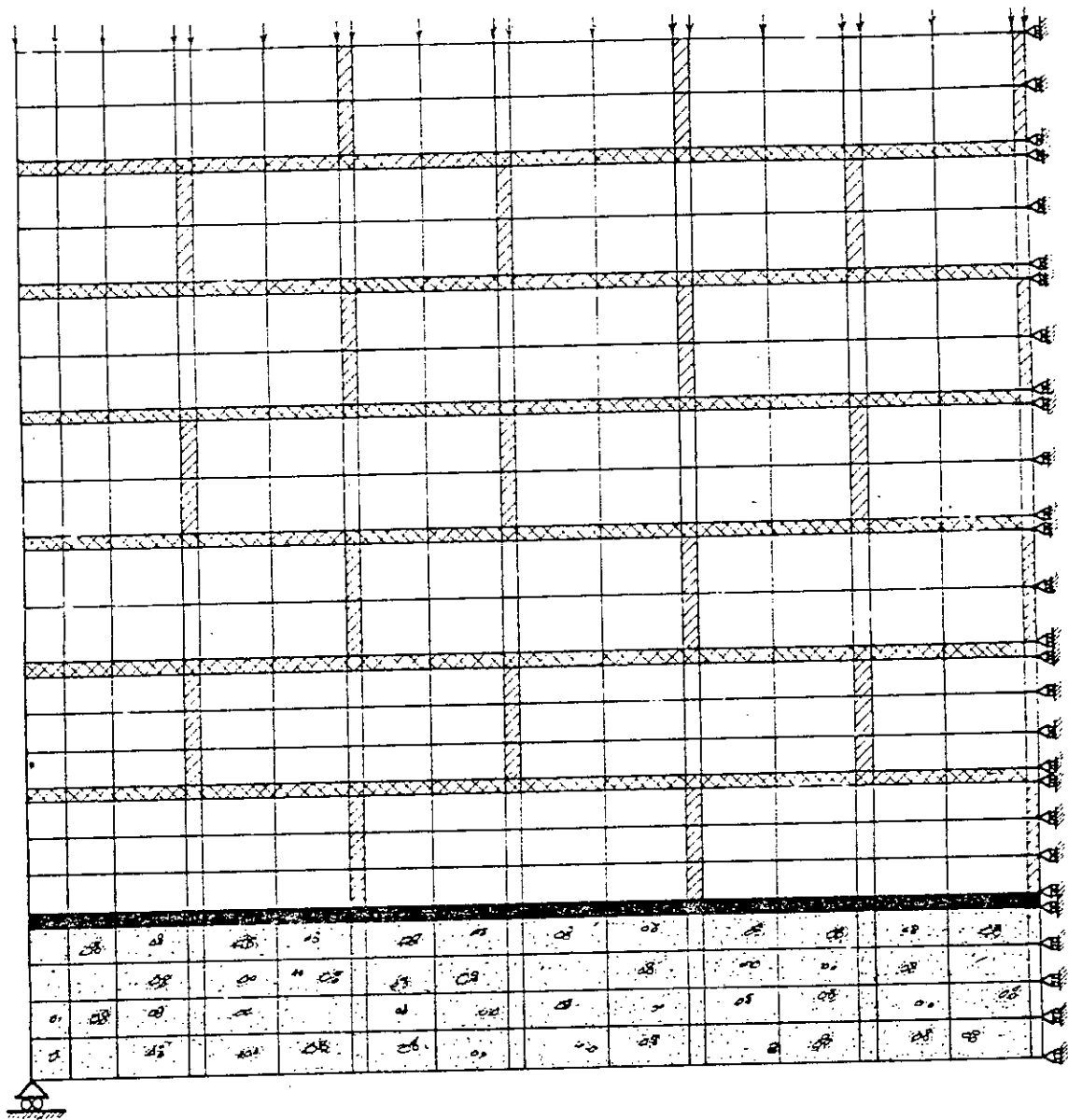
FIG. 3·2   FINITE ELEMENT DISCRETIZATION OF ALL-BEAM STRUCTURE,
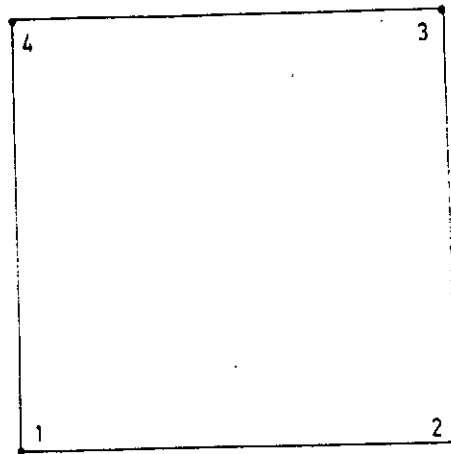
(a)   FOUR NODED ELEMENT
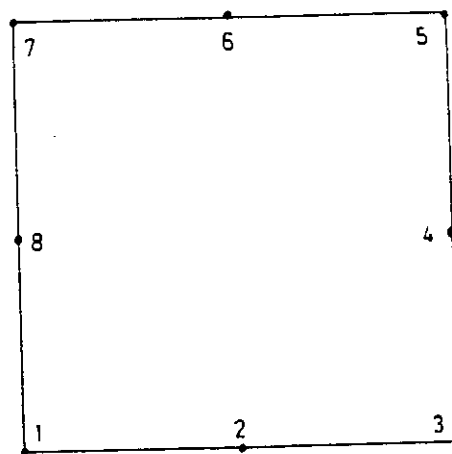
(b)   EIGHT NODED ELEMENT

FIG. 3.3   2-D ISOPARAMETRIC ELEMENTS.

e = FiLi/(AiEi) for a bar i. The other notations are as usual.

Ki=Fi/e

=AiEi/Li

or, [K] {D}={F}

where, [K] is the structure stiffness matrix, {D} is the nodal displacement vector (Global d.o.f), {F} is the total load (force) on structure nodes.

{R} = {P} +Σ{r} where {P} is the vector of externally applied loads on structure nodes and Σ{r} is the force applied by elements to nodes.

The jth column of [K] is the vector of nodal forces that must be applied to the nodes to maintain static equilibrium when the jth d.o.f has unit displacement and all other d.o.f have zero displacements.

For any continuum as shown in figure 3.2 the element stiffness matrix for the individual finite elements can be determined using an energy principle such as the principle of virtual work. The derivation of element stiffness matrix, [K] is a step by step procedure. The steps of the element stiffness matrix formulation may be briefly reviewed for an 'n' noded isoparametric elements as follows:-

The 2-D stress strain vectors may be represented as:

Strains, $E^T = \{\epsilon x \; \epsilon y \; \gamma xy\}$

$$\ldots\ldots(3.1)$$

Stress, $\sigma^T = \{\sigma x \; \sigma y \; \tau xy\}$. They are related by the elasticity matrix [D] as:

$\sigma^T = D E^T$ where [D] for isoparametric material is given as

$$D = \frac{E}{(1-v**2)} \begin{pmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{pmatrix} \qquad \ldots\ldots(3.2)$$

for plane stress condition and

$$D = \frac{E}{(1+v)(1-2v)} \begin{pmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & (1-2v)/2 \end{pmatrix} \ldots\ldots(3.3)$$

for plane strain condition.

E = Young's modulus

v = Poisson's ratio.

Here it is to be mentioned that temparature effects are not considered and also assumed that the elements (the structure as a whole) are free from initial stress and strain.

The global coordinates X,Y and the two translational degrees of freedom u,v along the coordinate directions are interpolated in terms of the nodal values using the relevent shape functions Ni(i= 1,2,3 . . - n) for the 'n' noded element as:

$$\left\{ \begin{matrix} x \\ y \end{matrix} \right\} = \sum_{i=1}^{n} Ni \left\{ \begin{matrix} xi' \\ yi' \end{matrix} \right\} \quad \text{and}$$

$$\ldots\ldots(3.4)$$

$$\left\{ \begin{matrix} u \\ v \end{matrix} \right\} = \sum_{i=1}^{n} Ni \left\{ \begin{matrix} ui' \\ vi' \end{matrix} \right\}$$

Where the dashed quantities are in the local coordinate system x', y'. Since the shape functions are expressed in terms of the natural coodinates ξ, η, the derivations are related to the x. y derivative through the jacobian J.

Let $\emptyset$ be some function of x and y. The chain rule yields (16):

$$\partial\emptyset/\partial\xi = \partial\emptyset/\partial x \ \partial x/\partial\xi + \partial\emptyset/\partial y \ \partial y/\partial\xi$$

$$\partial\emptyset/\partial\eta = \partial\emptyset/\partial x \ \partial x/\partial\eta + \partial\emptyset/\partial y \ \partial y/\partial\eta$$

or, $\left\{ \begin{array}{c} \partial\varphi/\partial\xi \\ \partial\varphi/\partial\eta \end{array} \right\} = [J] \left\{ \begin{array}{c} \partial\varphi/\partial x \\ \partial\varphi/\partial y \end{array} \right\}$

or, $\left\{ \begin{array}{c} \partial/\partial\xi \\ \partial/\partial\eta \end{array} \right\} = [J] \left\{ \begin{array}{c} \partial/\partial x \\ \partial/\partial y \end{array} \right\}$

Where   [J] is the Jacobian matrix which is   represented as:

$$[J] = \begin{bmatrix} \partial x/\partial\xi & \partial y/\partial\xi \\ \partial x/\partial\eta & \partial y/\partial\eta \end{bmatrix}$$

Since, the strains are expressed in the cartesian derivatives of the displacement, the inverse relation is used to express the strains in terms of the displacements as:

$$\{\epsilon\}^T = [B] \{\delta\}^T \qquad\qquad \ldots\ldots(3.7)$$

Where [B] = strain displacement matrix and

$\{\delta\}$ = nodal displacement vector for the   element

Now, from the principle of minimum potential energy of an   elastic   body we can relate the variations of the   work done by the loads with the strain energy. The strain   energy

density for a linear elastic body may be defined as:

$$dU = 1/2(strain)\ (stress)\ (total\ volume)$$

$$= 1/2\ \{\epsilon\}\ \{\sigma\}\ dV$$

$$= 1/2\ \{\epsilon\}\ [D]\ \{\epsilon\}\ dV$$

$$= 1/2\ \{B\}\ \{\delta\}\ [D]\ \{B\}\ \{\delta\}\ t\ dA$$

$$= 1/2\ \{B\}\ \{\delta\}\ [D]\ \{B\}\ \{\delta\}\ t\ dA$$

Where from we get the stiffness matrix K as

$$K = \int_A \{B\}\ [D]\ \{B\}\ t\ dA$$

$$= \int_{-1}^{+1}\int_{-1}^{+1} B\ D\ B\ t\ det\ J\ [J]\ d\xi\ d\eta \qquad \ldots\ldots(3.8)$$

Where, [D] = Elasticity matrix.

For the purpose of integration a 2x2 Gaussian quadrature is used to evaluate the integral and hence the stiffness matrix.

3.2.2 The Isoparametric Formulation.

Isoparametric elements were first introduced in 1966 (23). Isoparametric elements are useful in modeling structures with curved edges and in grading a mesh from coarse to fine. Element nodes define two things:

1. Nodal d.o.f {d} dictate displacements {u v w} of a point in the element. Symbolically,

$$\{u\ v\ w\} = [N]\ \{d\}. \qquad \ldots\ldots\ldots (3.9)$$

2. Nodal coordinates {c} define global coordinates {x y z} of a point in the element. Symbolically,

$$\{x\ y\ z\} = [\tilde{N}]\{c\}.$$

········(3.10)

Matrices [N] and [Ñ] are functions of ξ, η and ζ. An element is isoparametric if the node sets of item 1 and 2 are indentical and if [N] and [Ñ] are identical. More elaborately, the isoparametric elements are those group of elements, the geometry and displacements of the elements are described in terms of the same parameters and are of the same order. If the shape functions in natural coordinates fulfill the continuity of geometry and displacements both within the elements and between the adjacent elements, it can be shown that the compatibility requirements is satisfied in global coordinates also. For 2-D isoparametric elements, the stiffness matrix in global coordinates can be computed from the following relation:

$$[K] = \int_A [B]^T [D] [B]\ t\ dA$$
$$= [B]^T [D] [B]\ t\ A.$$

······(3.11)

Where, D matrices may be found from equation 3.2 and 3.3.

Now the expression for the displacement of isoparametric element can be represented as follows:

1.   For the Four Noded elements:

Definition   of   the   global   coordinates   and displacements are

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = [N]\ \{C\}\ \text{and}$$

$$.........(3.12)$$

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N]\ \{d\}$$

where,

$\{C\}$ $= \{x_1$ y_1 x_2 y_2 x_3 y_3 x_4 $y_4\}$,

$\{d\}$ $= \{u_1$ v_1 u_2 v_2 u_3 v_3 u_4 $v_4\}$ and

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix}$$

Here,

u = displacement of the nodes along x- direction

v = displacement of the nodes along y- direction

N = Shape function of the corresponding nodes that
means,

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \quad ......(3.13)$$

$$\{u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_4 \quad v_4\}$$

In case of four noded element the shape functions in natural coordinates system (ξ, η), as proposed by Irons and Ahmed (24) are as follows:

$$N1 = 1/4 \ (1-\xi) \ (1-\eta)$$

$$N2 = 1/4 \ (1+\xi) \ (1-\eta)$$

$$N3 = 1/4 \ (1+\xi) \ (1+\eta)$$

$$N4 = 1/4 \ (1-\xi) \ (1+\eta)$$

........(3.14)

2. For the eight noded elements, the formulation is the same as procedure (1) except:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N1 & 0 & N2 & 0 & N3 & 0 & N4 & 0 & N5 & 0 & N6 & 0 & N7 & 0 & N8 & 0 \\ 0 & N1 & 0 & N2 & 0 & N3 & 0 & N4 & 0 & N5 & 0 & N6 & 0 & N7 & 0 & N8 \end{bmatrix}$$

{u1 v2 u2 v2 u3 v3 u4 v4 u5 v5 u6 v6 u7 v7 u8 v8}

Where,

$$N1 = 1/4 \ (1-\xi) \ (1-\eta) - (N5+N8)/2$$

$$N2 = 1/4 \ (1+\xi) \ (1-\eta) - (N5+N6)/2$$

$$N3 = 1/4 \ (1+\xi) \ (1+\eta) - (N6+N7)/2$$

$$N4 = 1/4 \ (1-\xi) \ (1+\eta) - (N7+N8)/2$$

$$N5 = 1/2 \ (1-\xi^2) \ (1-\eta)$$

$$N6 = 1/2 \ (1+\xi) \ (1-\eta^2)$$

........(3.15)

$$N7 = 1/2 \ (1-\xi^2) \ (1+\eta)$$

$$N8 = 1/2 \ (1-\xi) \ (1-\eta^2)$$

Now combining strain displacement relationships:

$$\{ \epsilon \} = [B] \{ \delta \} \qquad \qquad \cdots\cdots\cdots(3.16)$$

Where,

$$[B] = \begin{bmatrix} \partial N1/\partial x & 0 & \partial N2/\partial x & 0 & \partial N3/\partial x & 0 & \partial N4/\partial x & 0 \\ \\ 0 & \partial N1/\partial y & 0 & \partial N2/\partial y & 0 & \partial N3/\partial y & 0 & \partial N4/\partial y \end{bmatrix}$$

$$\begin{bmatrix} \partial N5/\partial x & 0 & \partial N6/\partial x & 0 & \partial N7/\partial x & 0 & \partial N8/\partial x & 0 \\ \\ 0 & \partial N5/\partial y & 0 & \partial N6/\partial y & 0 & \partial N7/\partial y & 0 & \partial N8/\partial y \end{bmatrix}$$

$$\cdots\cdots(3.17)$$

in case of 8 noded element and

$$[B] = \begin{bmatrix} \partial N1/\partial x & 0 & \partial N2/\partial x & 0 & \partial N3/\partial x & 0 & \partial N4/\partial x & 0 \\ \\ 0 & \partial N1/\partial y & 0 & \partial N2/\partial y & 0 & \partial N3/\partial y & 0 & \partial N4/\partial y \end{bmatrix}$$

$$\cdots\cdots(3.18)$$

in case of 4 noded element.

But the terms in [B] matrix contains the derivatives in the global cartesian coordinates (x,y) and the shape functions are in terms of natural coordinate (ξ , η) system, so trasformation of the derivatives are necessasry.

Now, let $\emptyset$ be some function of x and y. Then the chain rule of differentiation in matrix form may be written as:

$$\left\{ \begin{array}{c} \partial\emptyset/\partial\xi \\ \partial\emptyset/\partial\eta \end{array} \right\} = \left[\begin{array}{cc} \partial x/\partial\xi & \partial y/\partial\xi \\ \partial x/\partial\eta & \partial y/\partial\eta \end{array} \right] \left\{ \begin{array}{c} \partial\emptyset/\partial x \\ \partial\emptyset/\partial y \end{array} \right\}$$

$$= [J] \left\{ \begin{array}{c} \partial\emptyset/\partial x \\ \partial\emptyset/\partial y \end{array} \right\} \quad\ldots\ldots\ldots\ldots(3.19)$$

Where [J] is the Jacobian matrix given by

$$[J] = \begin{pmatrix} \partial N1/\partial\xi & \partial N2/\partial\xi & \partial N3/\partial\xi & \partial N4/\partial\xi \\ \partial N1/\partial\eta & \partial N2/\partial\eta & \partial N3/\partial\eta & \partial N4/\partial\eta \end{pmatrix} \begin{pmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \\ x4 & y4 \end{pmatrix}$$

$$\ldots\ldots\ldots(3.20)$$

in case of 4 noded element and

$$[J] = \begin{pmatrix} \partial N1/\partial\xi & \partial N2/\partial\xi & \partial N3/\partial\xi & \partial N4/\partial\xi \\ \partial N1/\partial\eta & \partial N2/\partial\eta & \partial N3/\partial\eta & \partial N4/\partial\eta \\[6pt] \partial N5/\partial\xi & \partial N6/\partial\xi & \partial N7/\partial\xi & \partial N8/\partial\xi \\ \partial N5/\partial\eta & \partial N6/\partial\eta & \partial N7/\partial\eta & \partial N8/\partial\eta \end{pmatrix} \begin{pmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \\ x4 & y4 \\ x5 & y5 \\ x6 & y6 \\ x7 & y7 \\ x8 & y8 \end{pmatrix} \; .(3.21)$$

in case of 8 noded element.

[J] thus obtained is inverted numerically giving

$$\left\{\begin{matrix} \partial\varphi/\partial x \\ \partial\varphi/\partial y \end{matrix}\right\} = [J]^{-1} \left\{\begin{matrix} \partial\varphi/\partial\xi \\ \partial\varphi/\partial\eta \end{matrix}\right\}$$

and in particular the following array can be evaluated

$$\left|\begin{matrix} \partial N1/\partial x & \partial N2/\partial x \dots \dots \\ \partial N1/\partial y & \partial N2/\partial y \dots \dots \end{matrix}\right| \overset{=}{\underset{=}{[J]^{-1}}} \left[\begin{matrix} \partial N1/\partial\xi & \partial N2/\partial\xi \dots \\ \partial N1/\partial\eta & \partial N2/\partial\eta \dots \end{matrix}\right]$$

$$\dots \dots (3.22)$$

From knowing the Cartesian derivatives [B] matrix can easily be evaluated and [K] matrix can be obtained from the numerical integration of the expression for [K].

3.3. CHARACTERIZATION OF INTERFACE BEHAVIOUR

The need for an interactive analysis is appreciable but a few comprehensive methods are only available. Most of these simplify the behaviour of the brick wall or beam or both and gives insufficient or inaccurate results. The traditional concept attacks the problem as a two phase materials system, the brickwork is one and the bottom beam is the other. Attempts are then made to account for the interaction between these two phases by simplified approach. Either the brickwork is supported by a fictitious medium or

the beam is analyzed with the brickwork being represented by an artificial model.

In order to allow the interface behaviour (specially separation of beam from the brickwork), the finite element representation of the structure was asigned two separate nodes along the wall-beam interface: one set on the edge of the beam and the other set on the brickwork. A linking matrix was then introduced directly into the stiffness matrix, tying each pair of vertically adjacent nodes together. In effect the linking matrix represented a short, very stiff member connecting the two nodes, constraining them to have identical displacements. After each analysis extrapolation procedures were used perpendicular to and then along the wall-beam interface. The first was to improve the consistency of the stress results at the interface and the second was to obtain stresses at the intermediate points. Although the extrapolation procedure might appear questionable with regards to the validity of separation results alternative analysis of identical problems using different element grids have very consistent results (27).

Since the development of the concept of interaction of brickwork with their supporting beams and the need for inclusion of the structural rigidity in calculating the displacements and stress of the structures, this problem has

undergone extensive attention of research work.

Under actual conditions, relative movements at the wall-beam interface may occur causing slip and separation of the beam from the brick wall, specially under heavy loads. This takes place because of exceeding the limiting interface shear stress and excessive downward deflection of the bottom beam. In order to have a better simulation of the wall-beam interaction, it is desirable to incorporate techniques for accomodating this interface behaviour.

Zienkiewicz et. al.(34,35) advocated the use of continuous isoparametric elements with simple nonlinear material property for shear and normal stresses, assuming uniform strains in the thickness direction. In certain cases, ill conditions of the stiffness matrix takes place.

Goodman,Taylor and Brekke (19) developed an interface element to account for relative movements between rock joints. The element consists of two lines, each with two nodal points. The two lines occupy the same position before deformation. Each node has two degrees of freedom (horizontal and vertical). To simulate slippage across an interface, an arbitrary large normal stiffness and a very small tangent stiffness would be specified.

Desai et.al. (15) proposed a thin layer element. A special constitutive model is used for these elements. Various deformation modes such as stick, slip, debonding and rebonding (figure 3.4) can be handled with this element. It is capable of providing improved definition of normal and shear behaviour; hence, it can be computationaly more reliable than the zero thickness element. The formulation of this element is essentially the same as other elements. As such it is easier to program and implement. Inclusion of a finite thickness for the interface is realistic since there is thin layer of mortar joints which participiate in the interaction behaviour. The thin layer element can easily be introduced in an interface having the same configuration as the brickwork or the bottom beam.

In the finite element analysis of present work, the interface element should be such that it can represent the seats of the discontinuity and in absence of shear failure, debonding system maintains its character as a continuum and at the same time, when the limiting condition is reached, the element simulate the phenomenon adequately.

In view of the merits in the use of thin layer element, it is decided to use this element in the present study.
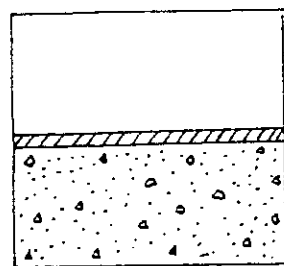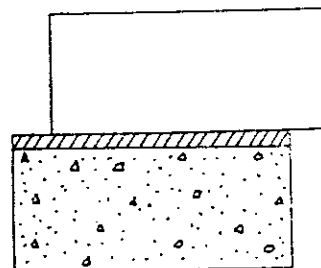
(a) STICK OR NO SLIP   (b)   SLIP

(c) DEBONDING   (d)   REBONDING

BRICKS

CONCRETE
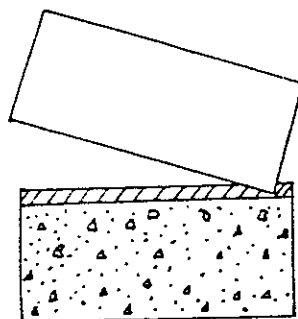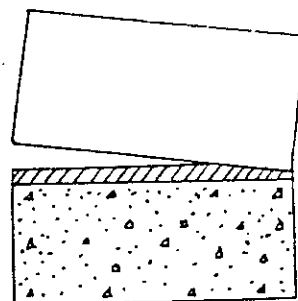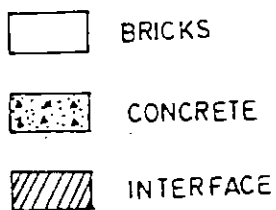
INTERFACE

FIG. 3.4 SCHEMATIC OF MODES OF DEFORMATION AT INTERFACE

3.3.1 Thin Layer Element for Interfaces

The thin layer interface, proposed by Desai et. al. (15) for the two dimesional idealization is shown figure 3.5 along with the other elements used in the study. The underlying idea of the thin layer element is based on the assumption that the behaviour near the interface involves a finite thin zone as shown in figure 3.6 rather than a zero thickness zone. The behaviour of this thin zone or layer or layer can be significantly different from the surrounding structural materials. However, the element may be treated like any other element by adopting appropriate constitutive laws.

The thin layer interface element can be formulated by assuming it to be linear elastic. The stiffness matrix of the interface element [K]i is written as

$$[K]i = \int [B]^T [D]i [B] \ t \ dA \qquad \ldots\ldots\ldots\ldots(3.23)$$

Where [D] is the constitutive matrix. Then the element equation can be written as

$$[K]i \ \{q\} = \{Q\} \qquad \ldots\ldots\ldots\ldots(3.24)$$

Where,

$\{q\}$ = vector of nodal displacement and

$\{Q\}$ = vector of nodal forces.
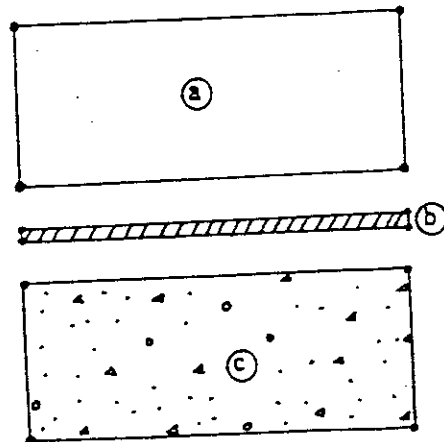
a   2-D ELEMENT FOR BRICKWORK

b   INTERFACE ELEMENT

c   RCC BEAM

FIG. 3.5 IDEALIZATION OF DIFFERENT ELEMENTS
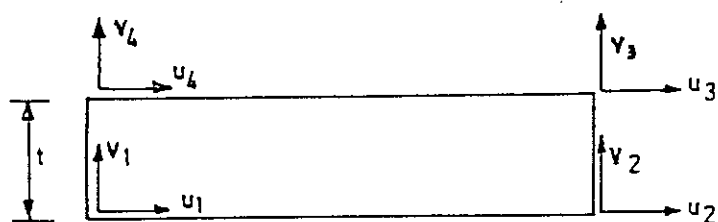
t = THICKNESS OF INTERFACE ELEMENT

FIG. 3.6 DEGREES OF FREEDOM FOR INTERFACE ELEMENT

For two dimensional idealization, the [D]i matrix is as given in equation 3.2 and eqnuation 3.3.

Hooke's law, for isotropic materials, is some times expressed in terms of Lamis' constant (18).

where,

$$\lambda = \frac{E}{(1+v) * (1-2v)}$$

$$\mu = \frac{E}{2 (1+v).}$$

The inverse relationship for E and v interms of λ and μ are

$$E = \frac{(3\lambda + 2\mu)}{(\lambda + \mu)} \quad \text{and}$$

$$v = \frac{\lambda}{2(\lambda + \mu)}$$

For the elastic behaviour of the materials the values of E, G and v can be computed from the laboratory tests. The shear modulus G represents the behaviour of a material under pure shearing stresses. To calculate the behaviour of material as a result of volumetric stresses, the bulk modulous K is given by(18):

$$K = \frac{E}{3(1-2v)}$$

$$G = \frac{E}{(1+v)*2}$$

$$\qquad\qquad\qquad\dots\dots\dots(3.27)$$

$$E = \frac{9K\ G}{(3K+G)} \quad \text{and}$$

$$v = \frac{(3K-2G)}{2*(3K+G)}$$

So, for interface element it is convenient to express the constitutive matrix in terms of the bulk modulus and shear modulus as :

$$[D]i = \begin{pmatrix} K+4G/3 & K-2G/3 & 0 \\ K-2G/3 & K+4G/3 & 0 \\ 0 & 0 & G \end{pmatrix} \qquad\dots\dots\dots(3.28)$$

Now since the behaviour of interface element is characterized by the relation between the relative displacements of the surfaces in contact and shear and normal stresses at the interface represented by Os and On respectively then the relation can be expressed as :

$$\left\{ \begin{array}{c} \sigma_s \\ \sigma_n \end{array} \right\} = \left[\begin{array}{cc} k_s & 0 \\ 0 & k_n \end{array} \right] \left\{ \begin{array}{c} \Delta U_s \\ \Delta U_n \end{array} \right\}$$

. . . . (3.29)

Where K_s and K_n are the interface stiffnesses, tangential and normal to the interface respectively and ΔU_s and ΔU_n are displacements in the tangential and normal to the interface respectively.

In this study X-direction is for the tangential stiffness and Y-direction is for the normal stiffness. So the constitutive matrix is modified accordingly to a new form. Since the interface element is capable of transferring only a normal stress and shear stress, the constitutive matrix takes up the form—

$$[D]i = \begin{pmatrix} 4G/3 & -2G/3 & 0 \\ -2G/3 & K+4G/3 & 0 \\ 0 & 0 & G \end{pmatrix} \qquad \ldots \ldots \ldots (3.30)$$

Where K = Bulk Modulus and

G = Shear Modulus of the interface element.

For the present study G is taken from the adjacent concrete materials of the bottom beam.

For the interface element of different wall-beam structures some fixed values are taken for Ks and Kn. According to Buragohain and Shah (6), for elastic cases Ks=Kn = 0 and Ks=Kn=10 for smooth and rough surfaces of contact repectively.

3.4 METHOD OF CALCULATION FOR BENDING MOMENT OF BEAM

As mentioned in chapter 2 that the RCC beam of a wall - beam structure acts as a tie beam. If the composite action of the wall-beam is not considered, the moment in the beam can be determined from statics provided the structure is statically determinate. But when the composite action is considered the moment in the beam can not be determined from statics. The normal stress distribution in this case differs quite significantly. Once the stresses are known from finite element analysis, the bending moment in the beam are calculated as below:

Referring to the figure 3.7.a and figure 3.7.b. force Fa and Fb balances each other. Only unbalanced force is Fc. Here fb and ft represent the bottom fibre stress and the top fibre stress respectively.
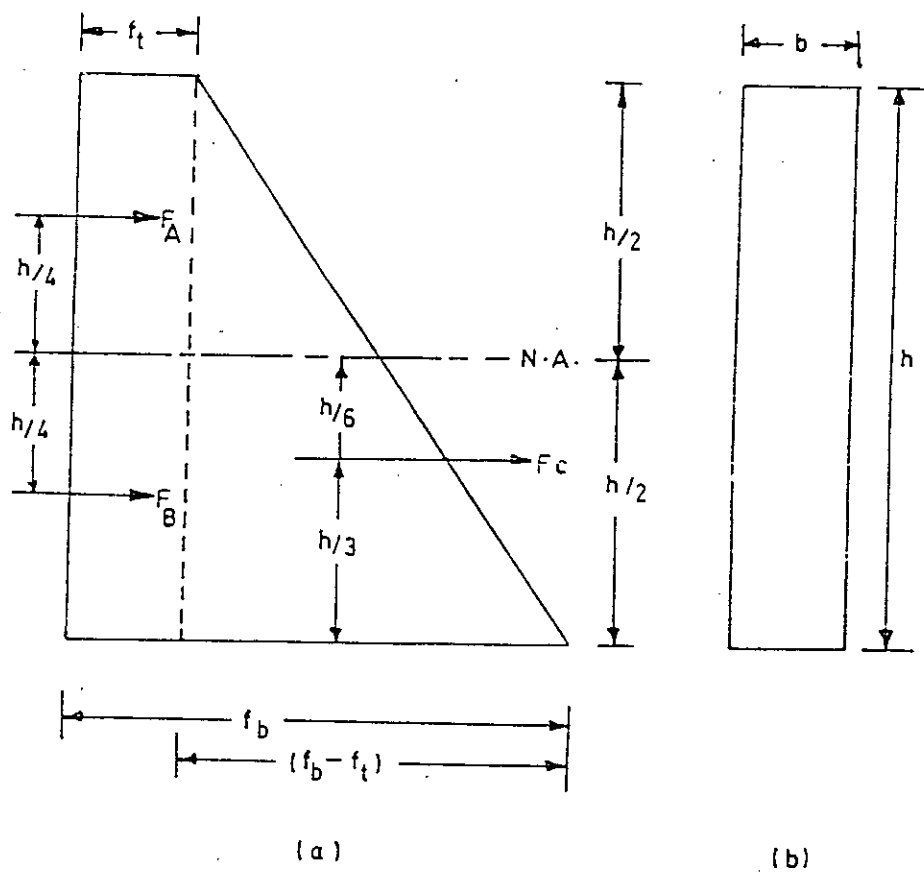
FIG. 3.7 DISTRIBUTION OF NORMAL STRESS IN THE BEAM

Fc = (fb-ft) h b/2

Now taking moment about the neutral axis

M = (fb-ft)bh**2/12   ....(3.31)

i.e. when the stresses at the top and bottom fibres (ft & fb) as well as the depth and width of the beam are known, the moment can be calculated from the equation (3.31)

From the present finite element analysis, the longitudinal stresses at the centre of each element and at the Gauss points are known. By extrapolation of these values the top and bottom fibre strsses can be found out but may be time consuming in some cases. So to reduce the effort in the calculation of moment, when stresses at any two points are known, the following simplification can be made.

Referring to figure 3.8, let the top and bottom stresses of any two points at a distance (y-dir) of hi be. ft1 and fb1 respectively. From the similar triangles:

(fb-ft)/h = (fb1-ft1)/hi

or   (fb-ft) = (fb1-ft1)* h/hi. From equation 3.4 (1)

M =((fb-ft)bh**2)/12. substituting the value of (fb-ft) in equation 3.31

M =((fb1-ft1) bh**2)*(h/hi)/12

or   M = I(fb1-ft1)/hi.   ....(3.32)
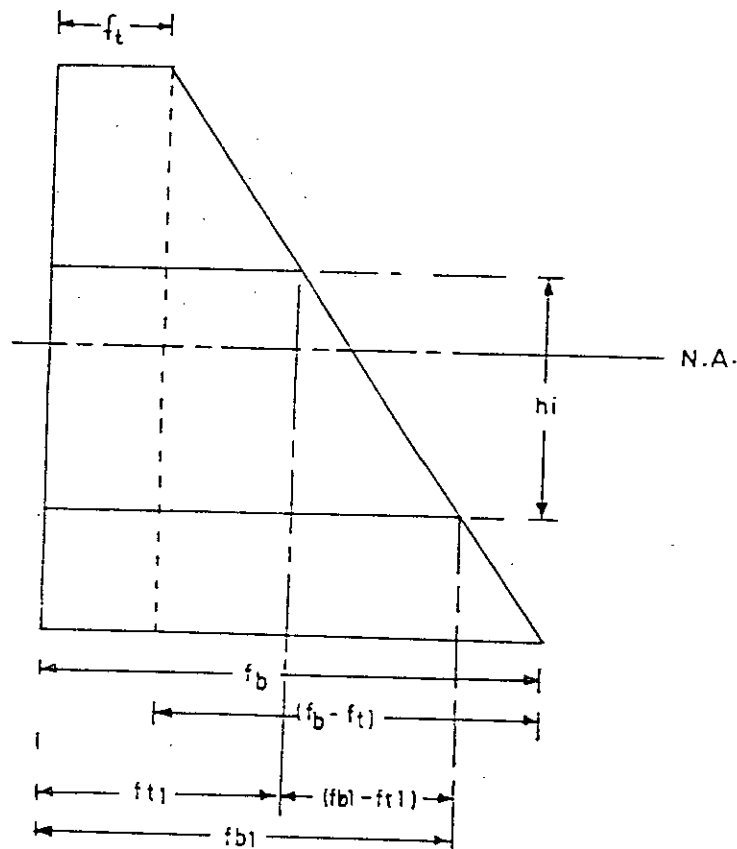
FIG. 3.8 STRESSES IN BEAM FOR SIMPLIFIED MOMENT CALCULATION METHOD

Therefore moment at any X-section of beam can be calculated from equation(3.32) when stresses (δx) at any two points and also their distance from each are known.

CHAPTER-4

FINITE ELEMENT PROGRAM

4.1 INTRODUCTION

In Chapter 3 the description of the tools used for the model,has been provided. In this chapter the finite element computer program using the above tools will be described. The development of this program is a part of on-going study of the composite behaviour of wall-beam structures, undertaken at the Dept. of Civil Engineering, of B.U.E.T.

The program capable of reproducing the linear behaviour of the materials at the present state .Allowances have been kept for the inclusion of the material nonlinearity and progressive fracture of the materials. The program is incremental in nature.Two types of finite elements have been used for the present study. More simpler four noded noded element has been used instead of more elaborate eight noded isoparametric element. The selection of this simpler element provided computing efficiency at the cost of little accuracy. The program can model the brickwork of the wall-beam structure either as homogeneous (considering the average properties of the brickwork) or nonhomogeneous (considering the brickwork as consist of two different materials, bricks and mortar joints) material. The program

has the option to check the stability of the solution at any
stage of loading. The following sections will provide more
about the model.

4.2 DESCRIPTION OF THE PROGRAM

A brief description of the the different subroutines of
the program and the format of data are presented in
Appendix.1. Figure 4.1 shows the flow chart of the program
used for the present study. In this section a flow chart and
main modules of the program are presented briefly.

CALLS SUBROUTINE

FIX THE MAXIMUM VALUES FOR THE

DIMENSION OF DIFFERENT ARRAYS USED   'DIMEN'

IN THE PROGRAM .

ACCEPTS THE DATA RELATED WITH THE

PROBLEM TYPE.   'PROBTP.'

DISCRETIZE THE TOTAL BLOCKS OF THE

STRUCTURE IN TO A NUMBER OF SMALL

PIECES OF UNIT WITH EACH POINT OF   'INPUT'

CONNECTION AS A NODE AND EACH

BOUNDED ZONE AS AN ELEMENT.

IDENTIFY THE DIFFERENT TYPES OF

MATERIALS OF THE WALL-BEAM   'INPUT','COMP'

STRUCTURE BY GIVING THEM INTEGER   'DIFMAT','MAT

NUMBERS FOR THE PROCESS OF   -IDN'

COMPUTATIONS.

CHOOSE A NUMBERING SCHEME FOR THE

NODES AS WELL AS THE ELEMENT AND   "   "

FOLLOW THE SEQUENCE FOR THE WHOLE

STRUCTURE.

PROVIDE THE CONNECTIVITY OF NODES
OF EACH ELEMENT AS WELL AS THE   'INPUT','COMP',
COORDINATES OF EACH NODE FOR THE   'MATIDN',
WHOLE MESH.   'DIFMAT'

FOR THE PARTICULAR TYPE OF THE
ELEMENT SELECT THE TYPE OF   'BMATPS'
DISPLACEMENT FUNCTION AT EACH NODAL
POINT.

FOR THE PARTICULAR DISPLACEMENT
FUNCTION SELECT THE CONSTITUTIVE   'DECSON'
RELATIONSHIP.

FROM THE SHAPE FUNCTION AND USING   'SFR2','BMATP
                                       -S',JACOB2',
ITS DERIVATIVES FORM DIFFERENT   'DMATPS'
MATRICES E.G. [B], [J], [D].

INTREGRATE NUMERICALLY TO OBTAIN THE STIFFNESS MATRICES OF THE ELEMENTS AND FORM STIFFNESS MATRIX FOR ALL ELEMENT OF THE MESH ONE BY ONE USING THE RESPECTIVE MATERIAL PROPERTIES.

'GAUSSQ','STIF 'BMATPS','DMA PS','DBE','JA-COB2'

FORM THE GLOBAL STIFFNESS MATRIX ACCORDING TO THE CONNECTVITY OF THE DIFFERENT ELEMENT OF THE MESH OF THE SYSTEM.

'STIFFP','SF R2','STIFP'

ON THE BASIS OF THE NODAL DISPLACEMENT VECTOR AND NODAL LOAD VECTOR, FORM A GLOBAL SET OF SIMULTANEOUS EQUATION OF TOTAL STIFFNESS MATRIX AND LOAD VECTOR AND NODAL DISPLACEMENT OF THE ENTIRE BODY.

'ALGOR','FRO NT','STIFFP', 'LOADPS'

CHOOSE A SOLUTION PROCESS FOR THE SET OF SIMULTANEOUS EQUATIONS FOR GETTING THE UNKNOWN VALUES ON THE BASIS OF BOUNDARY CONDITIONS.	'RESIDU','SFR -2','JACOB2', 'BMATPS','DMAT -PS','LINEAR', 'REDUCT','RED -JON','PRINC'

FIG(4.1)   FLOW CHART OF THE FINITE ELEMENT PROGR

The main modules of the program are:

a)   Discretization of the continuum on the basis of the type of elements and properties of elements (concrete, bricks, mortar joints and interface element).

b)   Selection of the displacement model that closely resembles the practical situation and at the same time maintains the compatibility with the type of element in the idealization.

c)   Derivations of the element stiffness matrix using the virtual 'work method that consists of the coefficients of the equilibrium equations derived from the material and geometric properties of the element obtained by the principle of minimum potential energy.

d)   Assembly of the algebraic equations for the overall discretized continuum on the basis that the nodal interconnections require the displacement at a node to be the same for all element adjacent to that node to form global set of simultaneous equation consisting of total stiffness matrix, total load vector and the nodal displacement vector for the entire body.

e)   Solution for the unknown displacements considering the boundary conditions and consequent appropriate modifications of the equations.

f) Computation of the element stresses(σ_x, σ_y, τ_{xy}) strains (ϵ_x, ϵ_y, γ_{xy}) principal stresses(σ_1, σ_2, θ), from the nodal displacements.

The present numerical procedure is based on the finite element method. The application of this method has been very common and many texts have been written on the subject (20, 21, 24). In general, finite element method can be called as a piece wise approximation and the solution of a problem in a continuum mechanics may be approximated by analyzing a structure consisting of any assemblage of properly selected finite elements interconnected at a finite number of joints (nodal points). For the purpose of our present study, a wall- beam structure has been idealized as a system of nodal points interconnected by discrete elements (Figure 3. 2). Two dimensional finite element analysis of the wall- beam subjected to distributed load (in the form of concentrated loads at the nodal points on the top surface/layer spaced closely) has been performed in this study. Two types of elastic finite element models have been used in the program. One assumes masonry to be a homogeneous continuum with an

average property of the brickwork, the other models the bricks and mortar separately. The concrete and the interface have been considered separately in both cases. For homogeneous case, an elastic modulus (E) of 7484 Mpa and a poison's ratio of 0.17 were assumed for the masonry. For the nonhomogeneous case, the finite elements corresponding to bricks and mortar were assigned different values of modulus of elasticity and poison's ratio (8000 Mpa and 0.16 for the bricks and 5000 Mpa and 0.20 for the mortar). In both the cases four noded elements with 2*2 Gaussian integration have been used.

There are mainly two types of solution methods available in finite element analysis e.g. band solution and frontal method of solution of which the later one is more efficient and adaptable for a computer with moderate storage capacity. Frontal method of solution of the equilibrium equations has been adopted from the finite element program of Owen and Hinton (20). In this program most of the data can automatically be generated. The nodal displacement, the reactions, the strains, and the stresses at the Gauss points of each element, as well as the average values at the centre of the elements are calculated.

4.3 IMPLEMENTATION OF THE PROGRAM

4.3.1. General

The program consists of 3124 lines (including comment statements). The finite element program of Ali (3) has been modified to the current form. The program has been modified at different stages. At every stage test runs are made and compared with the published results and modifications are done to obtain the desired results. Once the program was error free it was ready to run. Although the program at this stage is made for analysing wall-beam structures, but the provision has also been kept for analyses of infilled frames. . A simplied flow diagram of the program has been shown in figure 4.2.

4.3.2 Automatic Data Generation Scheme

Large amount of data is required to input for the analysis of wall beam structure, which is required which is cumbersome as well as error prone. To reduce the manual effort and also to reduce the error proneness an automatic data generation scheme has been introduced. The main program calls the subroutine 'INPUT' which reads most of the data
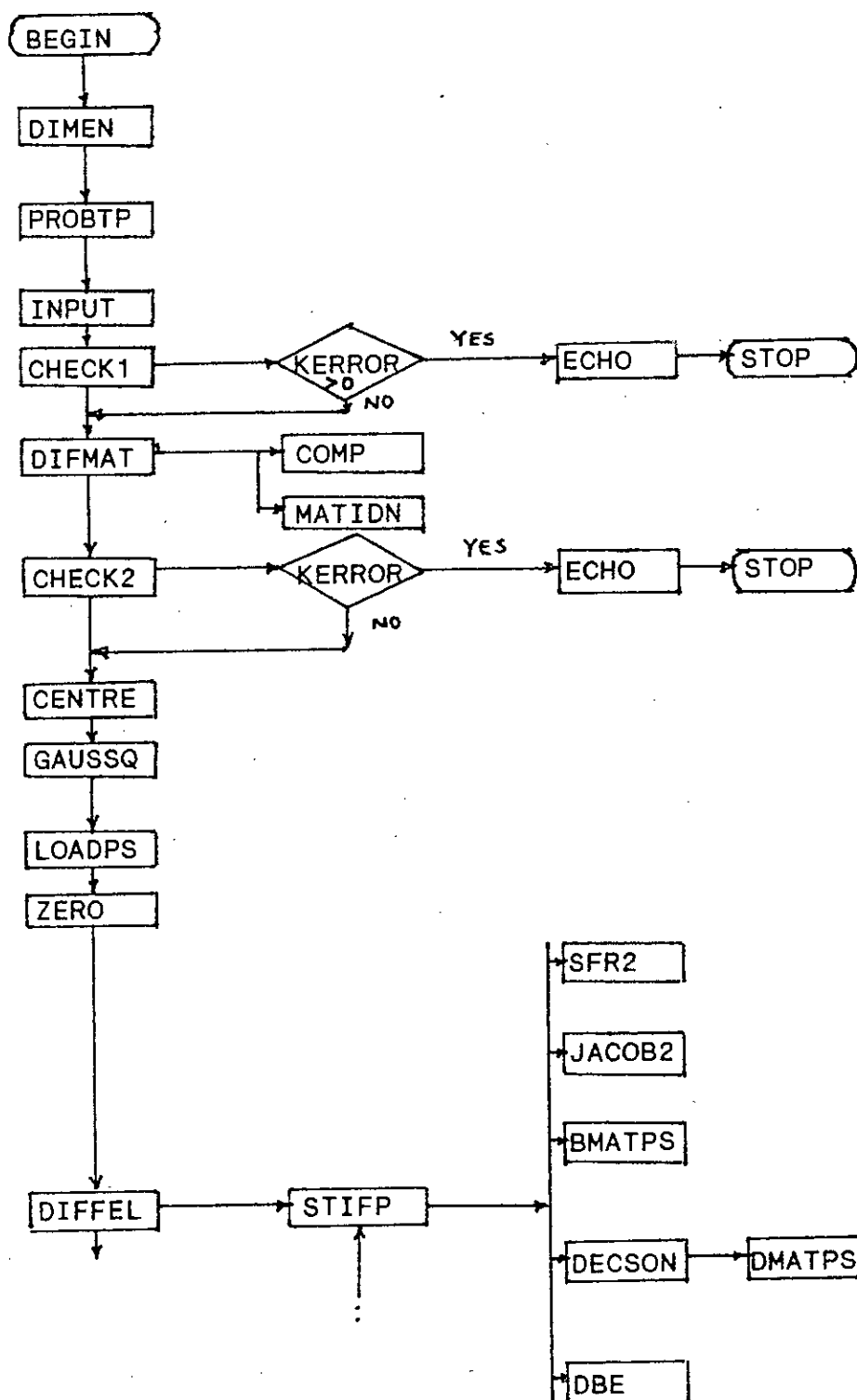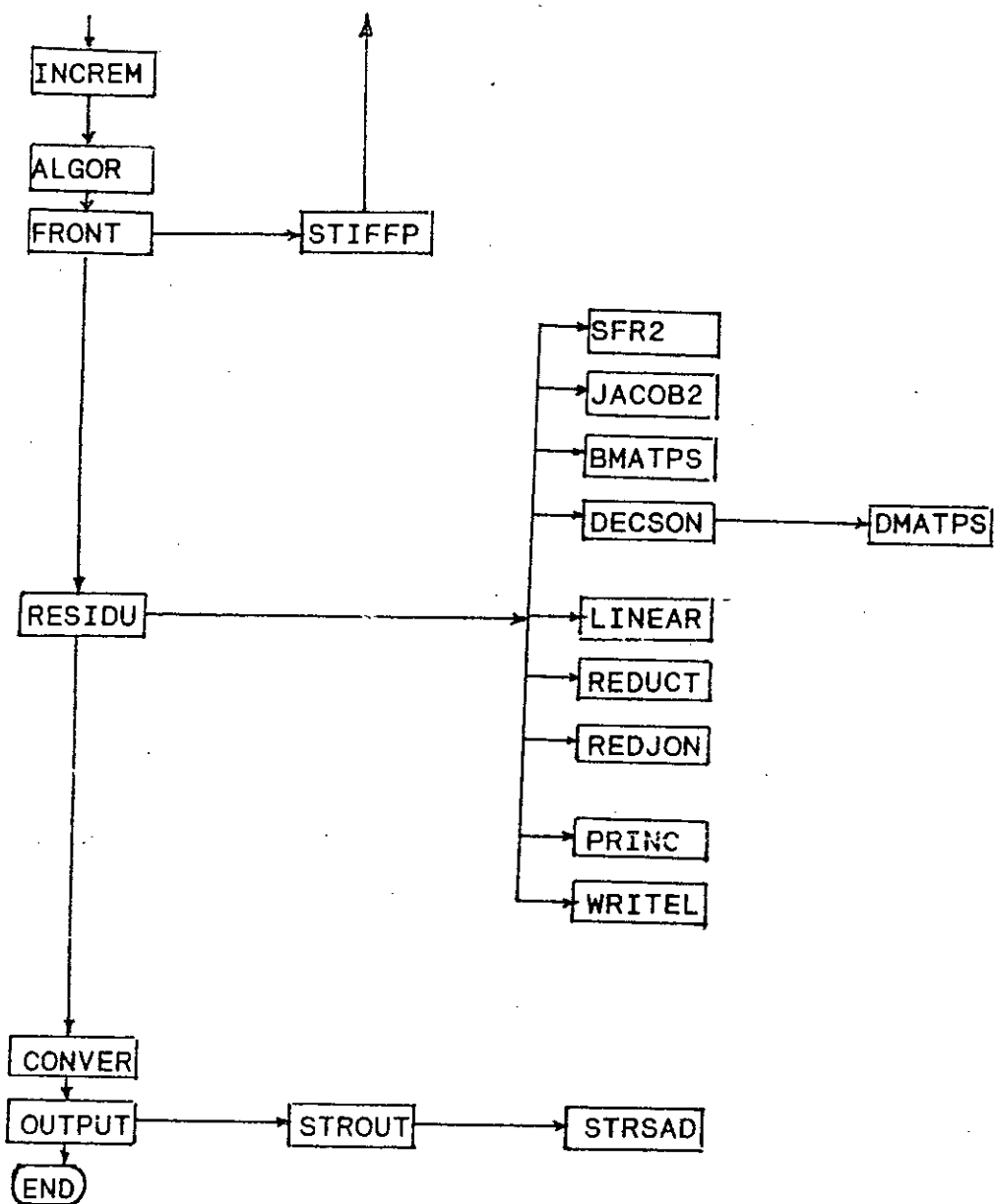
FIG.4.2 FLOW DIAGRAM OF THE PROGRAM.

and generates the finite element mesh, selects the nodal number, element number, nodal coordinates etc. If the number of material is more than two it again calls subroutine 'DIFMAT' to provide the proper position and orientation of the elements with different material properties. For these extensive and large operation only the number of nodes in the X and Y directions, the distance from the origin are required. For the structures having materials more than two, the location of different materials and the number of layers in concrete are required. The brickwork can be considered either as homogeneous or nonnomogeneous material through the counter 'IHOM' in the program. If 'IHOM' equals zero, the brickwork will be treated as homogeneous continuum and if IHOM is greater than 0 (zero) then the brickwork will be considered as nonhomogeneous material and the position of the interface and mortar joints will be selected as per the data provided in the subroutine 'DIFMAT'.

For fixing up the boundary conditions certain codes are used for nodes to be restrained against translation. The support informations are as follows :

    01===> Restrained along Y-direction.
    10===> Restrained along X-direction.
    11===> Restrained along both X and Y direction.

The program can tackle the case of symmetry along one or two axes of the problem which reduces the volume of calculation, requires less storage of the virtual storage block and consequently less computational time. For this no additional subroutines are required but need to modify the boundary conditions of the centre line.

The elements are numbered first along X-direction, then Y-direction is followed in the incerasing order. Same is the case with nodal numbers as shown in figure 4.3.a and 4.3.b. Connectivity is given in the anticlockwise order and the Gauss points are given first in the Y-direction, then in the X-direction. The process of mesh generation is shown in figure 4. 3(e) and 4. 3(f).

4.3.3 Checking of Input Data .

To check the correctness of data as provided in the subroutine 'INPUT', a subroutine called 'CHECK1' is provided. The purpose of this subroutine is to check for any error in the input of main control data. On meeting any irrational input data, it again calls a subroutine 'ECHO' to provide the diagnostic message. But if any data related with finite element mesh generation is encountered, it is checked by subroutine 'CHECK2' and corresponding diagnostic message
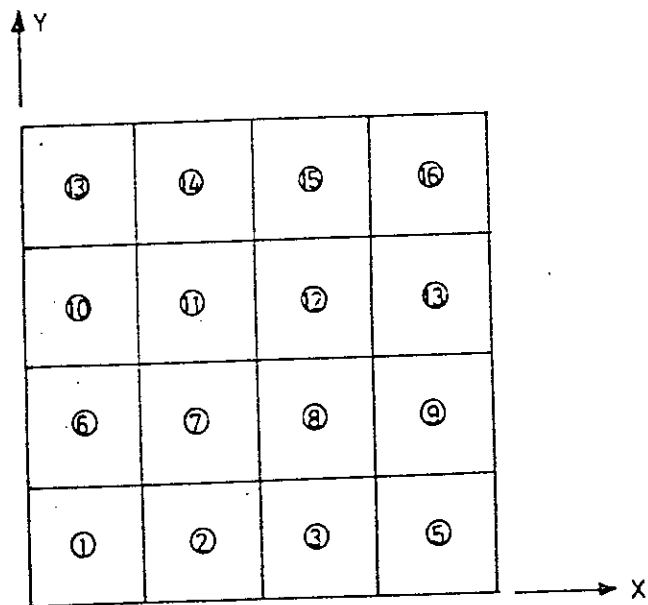
(a)   NUMBERING OF ELEMENTS
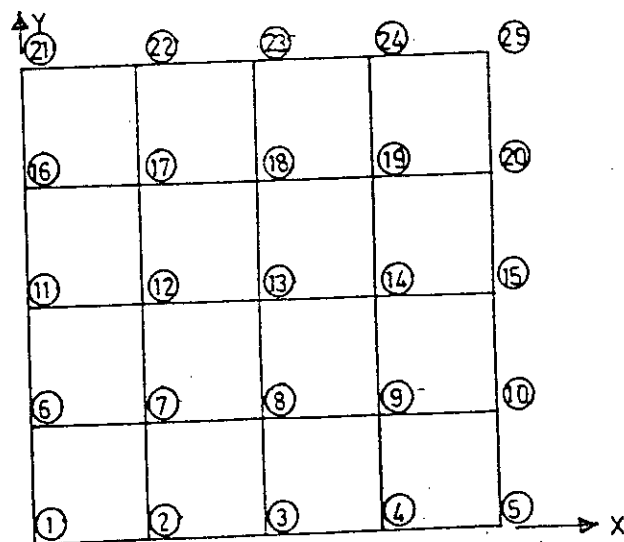
(b)   NUMBERING OF NODAL POINTS.
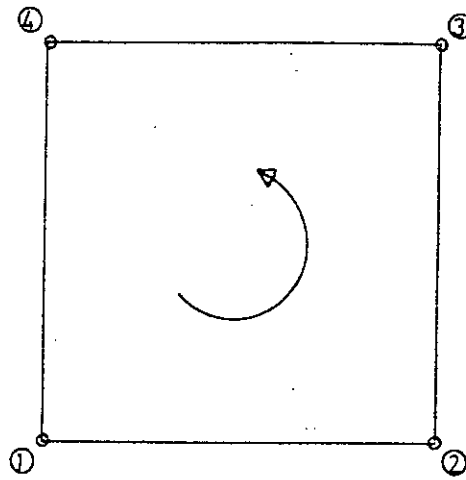
(c)   CONNECTIVITY DATA
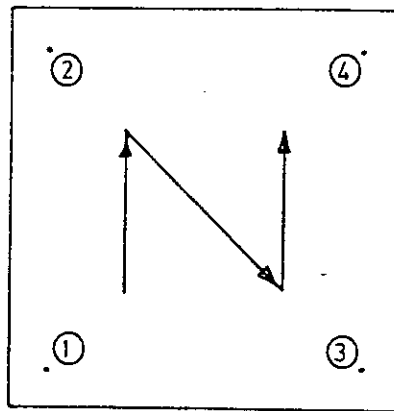
(d)   GAUSS POINTS

FIG. 4.3   MESH GENERATION

is given by subroutine 'ECHO'. The diagnostic messages are listed in Table 1 of Appendix A1.

4.3.4 Storage of Stiffness Matrices.

To take the advantages of the backing storage facilities, so that only a part of the stiffness is held in the core at a time, a program module is developed. This is attained by defining a specified buffer area and by opening a few working scratch files in the disk backing store. For a very economic and efficeient use of the storage, only half portion of the element stiffness matrix is computed and a minimum buffer area is used for a temporary storage. When the buffer is filled up with the part of the stiffness matrix, it is stored in sequential access file of the backing up storage for later use. Again the same area of the core space is used to store next part of the stiffness matrix . Therefore, when the formulation of the global stiffness matrix is in progress it deals mainly with the buffer area and in case of neccesity it reads values from those sequential access files. The stiffness matrix of the dirrerent isoparametric elements are formed and the global stiffness matrix is assembled by taking elements one by one.

4.3.5 The Solution Technique.

As described in sections 4.2 the frontal method of solution of the equilibrium equations is adopted from the finite element program of Owen and Hinton (20). Since the program can generate most of the data automatically, the nodal displacements, strains, and stresses at the Gauss points of each element and at the same time the average values of those are also calculated (i.e. at the centre of each element). For this a subroutine 'FRONT' is called. In its formation, provisions have been kept for choosing the correct path to call the correct routine which deals with the specified material type.

The operations in 'FRONT' has been done by specifying a number of scratch files where the temporary data are preserved. The stiffness matrix is divided in to a number of segments in such a way that each segment contains a number of complete rows. The sequential access backing storage file is also divided in to fixed length blocks in such a way that each complete segment wholly or partially fills the blocks.

4.3.6. Output of the Program.

The program calculates the stresses and strains at the Gaussian integration points as well as the average values of those at the centroid of each element. In addition to the stresses and strains it calculates the reaction at the restrained nodal points and also the displacements at the nodal points.

It calculates the stresses at the centroid of each element by calling the subroutine 'STRSAD'. For the principal stresses at the Gaussian intregration points, the subroutine 'PRINCP' is called. The strains are calculated at the subroutine 'LINEAR'. The displacements are provided by calling the subroutine 'OUTPUT'. Various options of output provided in the program is shown in the Table 2 of Appendix A1. In addition there is a provision for selection of different levels of the structures where the output is neccessary. 'NVSEC' and 'NHSEC' specifies the levels of the vertical and horizontal sections where outputs are neccessary.

4.4 VERIFICATION OF THE MODEL

To verify the accuracy of the finite element model test
runs are made and results are compared with published
results(2). The spliting test of brickwork specimen of
200mmx248mm size (figure 4.4) has been used for this
comparison. Since the stress gradients are very high near
the loading region and fairly uniform stress at the central
region of the specimen, the specimen was considered more
appropriate for the verification of the model. The indirect
tensile strength of a homogeneous material can be determined
from the following relationship (2).

$$6_T = K'.P/(d'.T) \qquad \ldots\ldots(4.1)$$

Where, P = applied load

t = specimen thickness

d' = equivalent diameter

  = (H.L'/0.7854)**0.50

H = specimen height

L' = specimen width

K = constant

6_T = Tensile bond strength
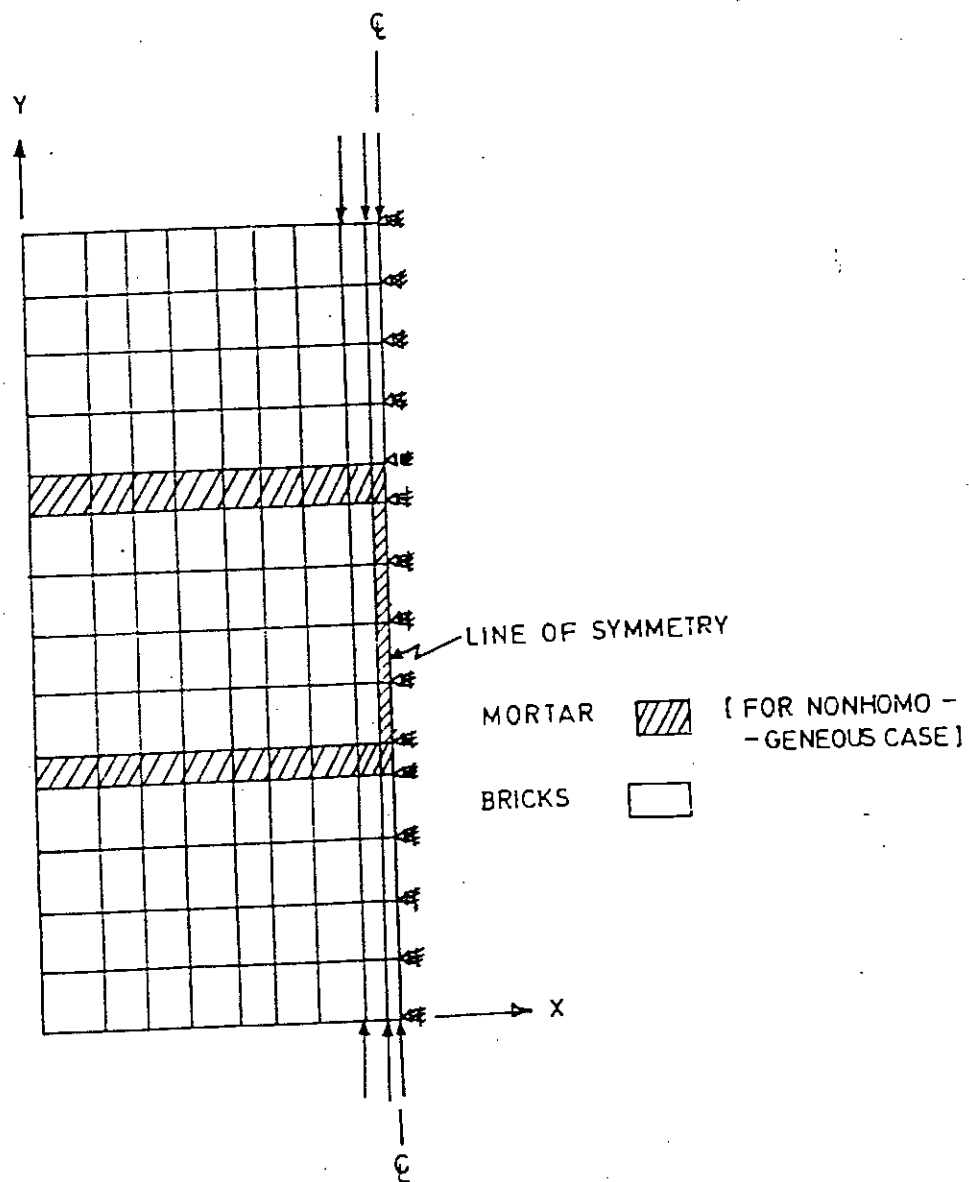
FIG. 4.4 THE FINITE ELEMENT DISCRETIZATION OF THE SPECIMEN SELECTED
FOR VERIFICATION OF THE MODEL.

The values of K is equal to 0.648 for a homogeneous material and for a brickwork varies with Eb/Em and found to be 0.65 to 0.71 as Eb/Em varied from 1 to 5 (2). To compare O$_T$ two specimens were considered, one modeling bricks and mortar joints as separate materials and the other treats brickwork as homogeneous material. The finite element discretization of the specimen is shown in figure 4.4.

From the equation 4.1 and from computer analysis the following results were obtained. The comparisom of results of the specimen is shown in table 4.1.

From table 4.1 it is observed that the results obtained from the present finite element analysis are in close agreement with those calculated from the equation 4.1. Once the program predicted the stress pattern of spliting test (2) accurately, the program was made ready for the study of composite action of wall-beam structures. The results of the present finite element study will be presented in Chapter 6 in comparison with the existing results of experimental as well as theoretical study made by many researchers of the world (11,12,13,27,28).

4.5 SELECTION OF WALL-BEAM FOR PARAMETRIC STUDY

To minimize the amount of computation both in terms of computing time and virtual storage requirements, it is

TABLE 4.1

COMPARISON OF PREDICTED RESULTS (δ_T,Mpa)WITH THE PUBLISHED RESULTS

TEST RUN	FINITE ELEMENT ANALYSIS	FROM EQN.4.1	VARIATION %
1.HOMOGENE -OUS	0.56065	0.5505	1.81
2.NONHOMOGE -NEOUS	0.6656	0.652	1.99

neccessary to find a relatively smaller size of wall-beam which will reproduce the behaviour of a complete wall. The wall-beam selected for the present study of composite behaviour is shown in figure 4.5. The half wall and the corresponding finite element mesh are shown in figure 3.2.

The analyses have been performed on half of the wall (taking the symmetry at the vertical centre line of the wall). A complete wall with one end hinged and the other end roller can be represented by half of the wall with proper boundary conditions at the vertical line of symmetry.

The length of the wall-beam was taken in such a way that two or three full bricks are taken on either sides so that a minimum of one or two vertical joints are there on either sides. The bricks size was taken to be 220 mm x 110 mm x 76 mm, whereas both the interface and mortar joints are taken to be 10 mm in depth and 110 mm in width.

4.6 SCOPE OF THE PROGRAM

The program is generalized for any geometry of the wall-beam structure. It can analyze the brickwork considering the brickwork as a homogeneous continuum as well as a nonhomogeneous material providing different properties to the elements. The application of load may be concentrated or distributed in the form of closely spaced

$H/L = 0.6$
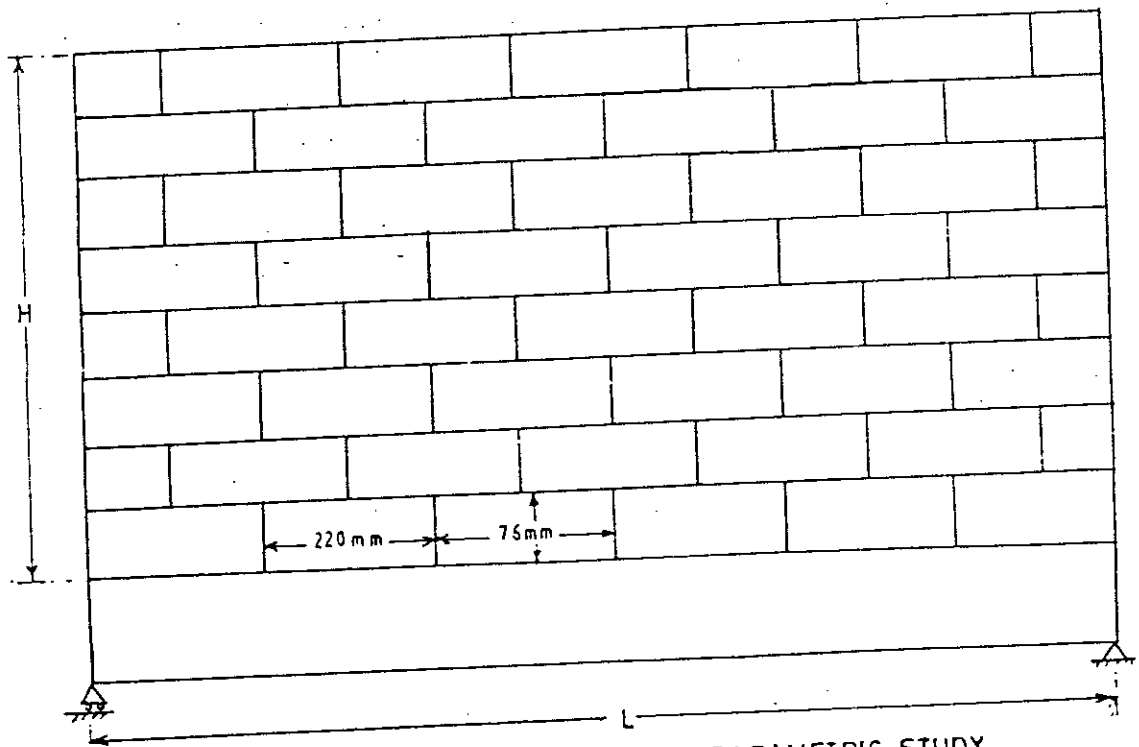
FIG. 4.5 WALL-BEAM AS SELECTED FOR PARAMETRIC STUDY

concentrated load . Consideration of gravity load and inclination of load can be imposed without any modification. The same is applicable for wall-beam as well as for infilled panel with any geometry. The program at this stage is designed for linear elastic behaviour but can be used for nonlinear analysis with certain modifications. The program is thus capable of modeling the behaviour of wall-beam, infilled frame and masonry walls subjected to inplane concentrated and distributed loads. As the bricks, mortar and concrete are modeled separately, the finite element model is suitable for any brick, mortar and concrete combination if the material parameters are known.

The program is written in FORTRAN 77 and it is mainly developed to run on the IBM 4331-K02 mainframe at B.U.E.T. computer centre. The program uses several ancillary subroutines for different tasks to be performed. Those subroutines were also written in FORTRAN 77 with the fortran compiler of the vax 11/780 computer with VMS operating system. To deal with the wall-beam structures and also that that the program can be used for infilled frames a few subroutines are introduced including making the provision of interface element in between the concrete beam and masonry walls. The program is also provided with the provision of automatic mesh generation.

4.7. WALL-BEAM AS TWO DIMENSIONAL STRUCTURE

In general ,the analysis of composite behaviour of wall beam structures subjected to concentrated or distributed load requires three dimensional effects to be considered. If the load is applied through a relatively small area of the wall then the distribution of stress through the wall thickness may be non-uniform. However ,from the result of three dimensional analysis(4), it may be concluded that a two dimensional analysis of the wall-beam structure subjected to concentrated or distributed load will be representative if the load is applied over 75% of its thickness. The findings of the two dimensional analyses described in the subsequent chapters will therefore be applied to this type of loading only. Therefore the problem may be idealized as two dimensional one. Linear elastic finite elment model has been used to study the behaviour of the structure regarding the deflection ,strain characteristics tensile stresses ,moment and stress pattern. The characteristics of the analytical tool (finite element) used for the present study has been described in the preeceding chapters.This chapter describes the application of this tool in the analyses of composite behaviour of wall-beam structures.

4.8 MATERIAL PROPERTIES

To develop a finite element model, for the composite action of wall-beam structures, which considers brickwork as an assemblage of bricks in the matrix of mortar and brickwork to be connected with the beam by interface elements, the properties of the bricks ,the mortar ,the concrete and the bond between the brickwork and the beam must be determined . For this study the values of these parameters are taken from available literature.In the present case the modulus of elasticity (E) and Poisson's ratio (V) of concrete are taken to be 14700 Mpa and 0.16 respectively. For mortar joints and the interface elements the values are 5000 Mpa and 0.2 respectively. For bricks the values are 8000 Mpa and 0.17 respectively. The shear and bulk modulous (G and K) for interface elements are calcuted from the values of E and V.

4.9. ASSUMPTIONS

The following assumptions have been made for the analysis of composite action of wall-beam structures:

a) Vertical and horizontal middle plane of the wall is continuous at every point before and after distortion and there are no holes or discontnuity in it.

b) Perfect bond exists at the joints of various components of the wall-beam structures.

c) Deformations are small in comparison to the wall thickness.

e) Concrete, bricks and mortar are homogeneous and isotropic linear elastic material.

4.10. FINITE ELEMENT DISCRETIZATION AND BOUNDARY CONDITIONS

The wall-beam structure considered in this theoretical study is simply supported at its ends (see figure 4.5). In practice this situation rarely exists . Usually the beam has some rotational restraint in its supports, due either to being built in to a wall or being connected , with some degree of rigidity, to supporting columns. In other cases, the wall is continuous over the supports . The number of factors affecting the rotational restraint and thereby controlling the behaviour of the structures under these conditions are therefore large. When the beam is built in, these factors include the extent of inbuilding and the height , physical properties and horizontal restraint of the wall below the support. When the beam is connected to the columns the influencing factors include the rigidity of

the fixing and the properties and the length of the columns. Other parameters affecting the structural behaviour include the length and height of the wall beyond the support.

A comprhensive study of these parameters is not only beyond the scope of this investigation but also would have produced too many results to allow usefull interpretation . Instead simple and idealized wall-beam structures are undertaken for analysis. The idealized wall on beam structure used for this study is shown in figure (3.2). The brickwork has been considered either as a homogeneous material or as an assemblage of bricks set in mortar matrix (nonhomogeneous). The beam has been considered to be simply supported. The length of wall is equal to the span of the beam. Figure 3.2. shows the idealization of the structure for the current analysis.

Although this basic wall on beam-structure represents a hypothetical structure , the result for stresses and deflection from this analysis should exceed those from practical cases. For example where the ends of the beam are built in should lead to a conservative design approach.

One of the supports of the wall-beam structure is a hinge and the other one is a roller. In case of roller it

is free to move in X-direction and restrained in Y-direction . But at the hinge both horizontal and vertical movements are restrained. Since the overall system is symmetric with respect to loading and geometry, only half of the structure has been discretized for the present study(figure 3.2). The appropriate boundary conditions have been provided for the nodes at the centre line of symmetry.

4.11. SUMMARY OF THE ANALYSIS SCHEME

For all the analyses performed in this study the overall dimensions of the wall-beam structure are taken which are compatible with the requirements of composite action of the structure. Different parameters have been considered to see their influence on the composite behaviour of wall-beam structure. The size of the bricks, the depth and thickness of the interface have been kept constant. The dimensions of the brick are taken to be 220 mm x 110 mm x 76 mm. The width of the wall is 110 mm. The thickness of interface element and morter joints are taken to be 10 mm. The stresses and the deflections are obtained at the Gauss points as well as at the centre of the elements and at the nodal points respectively. From the analyses of wall-beam

structure the following action in the R C C beam and wall have been determined.

    a)   Shear force / stress.

    b)   Tensile stress.

    c)   Moments and maximum moment.

    d)   Moment coefficient .

    e)   Tension coefficient .

    f)   Vertical stress and stress concentration

factor .

4.12 SUMMARY

The program is written in FORTRAN 77 mainly for the IBM 4331-K02 mainframe computer of the computer centre, B.U.E.T. The program used a few ancillary subroutines for the purpose of different fixed work to be performed. To make suitable for the present study a few new subroutines have been introduced and a number of old subroutines modified. The status of the program can be changed from linear elastic analysis to nonlinear fracture analysis by introducing a few additional subroutines. The program can also analyze infilled panels.

CHAPTER-5

RESULTS AND DISCUSSION

5.1   INTRODUCTION

A summary of the analyses scheme undertaken in this
study has been presented in Chapter 4. Results of these
analyses are presented and examined in this chapter. The
influence of various parameters on the distribution of
stresses, bending moment of the beam and tensile force of
the beam are systematically discussed.

In all the cases the upward displacements were taken to
be positive. Tensile stresses were taken to be positive and
moments causing bottom fibre tension is positive. All
negative stresse at sampling points were taken to be
compressive.

5.2. BRICKWORK AS HOMOGENEUOUS   AND   NONHOMOGENEUOUS MATERIAL.

5.2.1   General.

In the previous years almost all the finite element
models adopted to study the beheviour of wall-beam
structures considered the brickwork as a homogeneous
material. The modulus of elasticity of brickwork in these
cases were determined mainly from uniaxial compression test

of brick prisms. But for most of the cases , the results are not accurate due to the nonuniformity of load distribution on the top of the specimen . The complexity involved in the determination of correct modulus of elasticity of brickwork has been avoided in the present investigation. The following section provides the procedure adopted for the present study to determine the modulus of elasticity of homogeneous brickwork.

5.2.2 Determination of Combined Modulus of Elasticity of Brickwork

Normally the modulus of elasticity of brickwork is determined from either uniaxial or biaxial tests. As mentioned earlier, in most of the cases this type of test procedure produce erroneous results due to the artificial constraint imposed on the ends of the specimen. However, with the development of the present finite element model which can model brickwork as nonhomogeneous material,the ideal end conditions of the brickwork specimen could be simulated.

The combined modulus of elasticity of brickwork depends on the modulus of elasticity of bricks and mortar joints as well as the thickness of the wall and pattern of laying of bricks in the mortar matrix. In the current work ,

the size of bricks are taken to be 220mm x 110mm x 76mm.The thickness of mortar and interface element are considered to be 10mm. Whereas the width of the brickwork is taken to be 1.10mm.The modulus of elasticity of the brick and mortar is considered to be 8000 Mpa and 5000 Mpa respectively. Their poisson's ratio is taken to be 0.16 and 0.17.

For the determination of combined modulus of elasticity of brickwork, a brickwork panel of 680mm x 420mm has been considered.Total number of nodes and elements are 120 and 99 respectively. A distributed load of 0.01 KN / mm was applied on the top nodes of the structure. The details are shown in figure 5.1

From the analysis of above panel,

σ =6.8/(680 x 110) =9.090909 x 10E-05 KN/sq mm

ε =0.512020833 x 10E-02 / 420=0.000012147

Since the program is developed for the linear elastic analysis, the Hooke's Law is applicable and hence

E(comb)=σ / ε

= 9.090909 x 10 E-05 / 0.000012147

=7.48374 KN / sq mm.

FIG. 5·1 BRICK-WORK PANEL USED FOR THE DETERMINATION OF
COMBINED MODULUS OF ELASTICITY OF BRICK WORK.

5.2.3. Finite Element Study Considering Brickwork as Homogeneous and Nonhomogeneous Material

For the study of the behaviour of wall-beam structure subjected to both distributed and concentrated load, three finite element models were used; the first one assumed masonry to be a homogeneous continuum with coarse finite element mesh so that one element will encompass at least one header joint and one bed joint (fig.5.3), the second one treated bricks and joints separately (fig.5.2) , and the the third one considers brickwork as homogeneous material but used the same finite element mesh as the second one (fig.5.2).

To illustrate the differences among these three analyses, a wall-beam structure of the type shown in figure 5.2 and figure 5.3 have been considered. The brickwork has been considered both as a homogeneous (considering the average properties of the brickwork) and a nonhomogeneous (modeling bricks and mortar separately) material . In the last two cases the mesh has been generated giving due consideration to the location of different joints (figure 5.2). In the first case, the brickwork has been considered to be a homogeneous material but the element size has been choosen in such a way that one element will encompass at least one bed joint and a header joint (see figure 5.3).
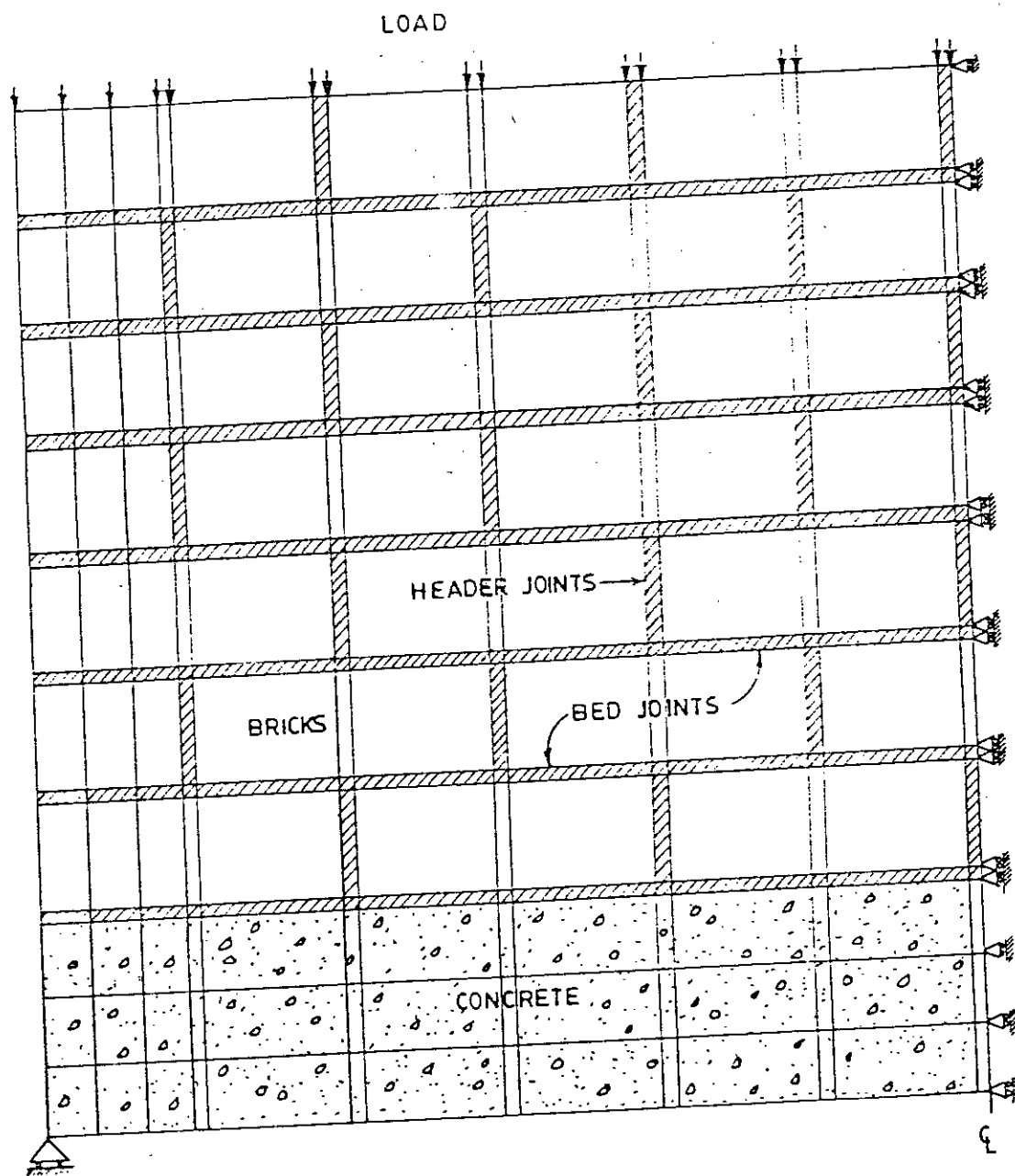
LOAD

FIG. 5.2   WALL- BEAM STRUCTURE USED TO COMPARE THE HOMOGENEOUS
AND NONHOMOGENEOUS CASES USING FINE MESH.
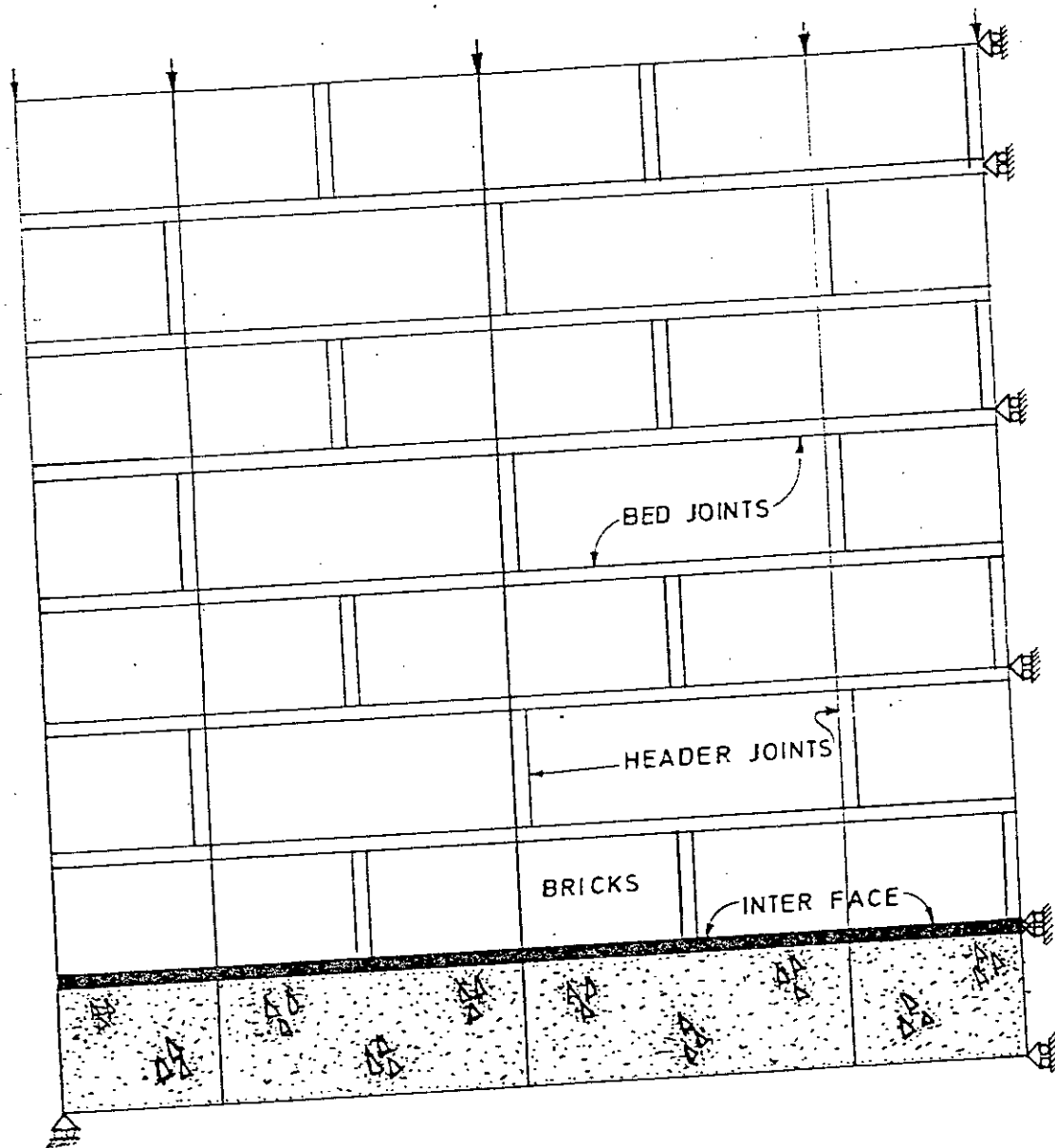
FIG 5·3   HOMOGENEOUS BRICK WORK WITH COARSE MESH

The load applied on the structure is shown in figure 5.2 and figure 5.3. For most of the analyses the wall-beam structure selected is 1370mm of span so that at least two vertical joints are there on either sides of the line of symmetry with a H/L ratio of 0.6. In other cases the span is so selected that at least one vertical joint is there on either sides of the vertical line of symmetry. When the brickwork is considered to be a homogeneous material the modulus of elasticity was determined as described in section 5.2.2. The maximum moment, maximum tension developed in the beam and maximum stresses developed for the last two cases are shown in table 5.1. The distribution of tension and moment along the length of the beam in these cases are shown in figure 5.4 and figure 5.5 respectively. figure 5.6 and figure 5.7 show the distribution of vertical stresses and shear stresses along the interface for all the three cases.

It can be seen from table 5.1 and figure 5.4, 5.5, 5.6, and 5.7 that the magnitude of stresses are very similar for the last two cases. Whereas for the first case the values differ quite significantly. In the wall-beam structures the shear stress at the interface of beam and brickwork plays a vital role in the composite action. Therefore , it appears from the analyses that consideration

TABLE 5.1

COMPARISON OF MAXIMUM STRESSES, MOMENTS, AND TENSION

SL.	ITEM	NH(#)	H(*)	% VARIATION
1	σ_x Mpa	4.960	4.963	0.06045
2.	σ_y Mpa	6.82	6.83	0.146
3.	τ_{xy} Mpa	1.78	1.796	0.891
4.	M N-mm	2051156	2050372	0.038
5.	T N	39881	39736	0.364
6.	k1	91.504	91.54	0.039
7.	k2	3.43	3.44	0.291

NH- NONHOMOGENEOUS CASE

* H- HOMOGENEOUS CASE WITH FINE MESH
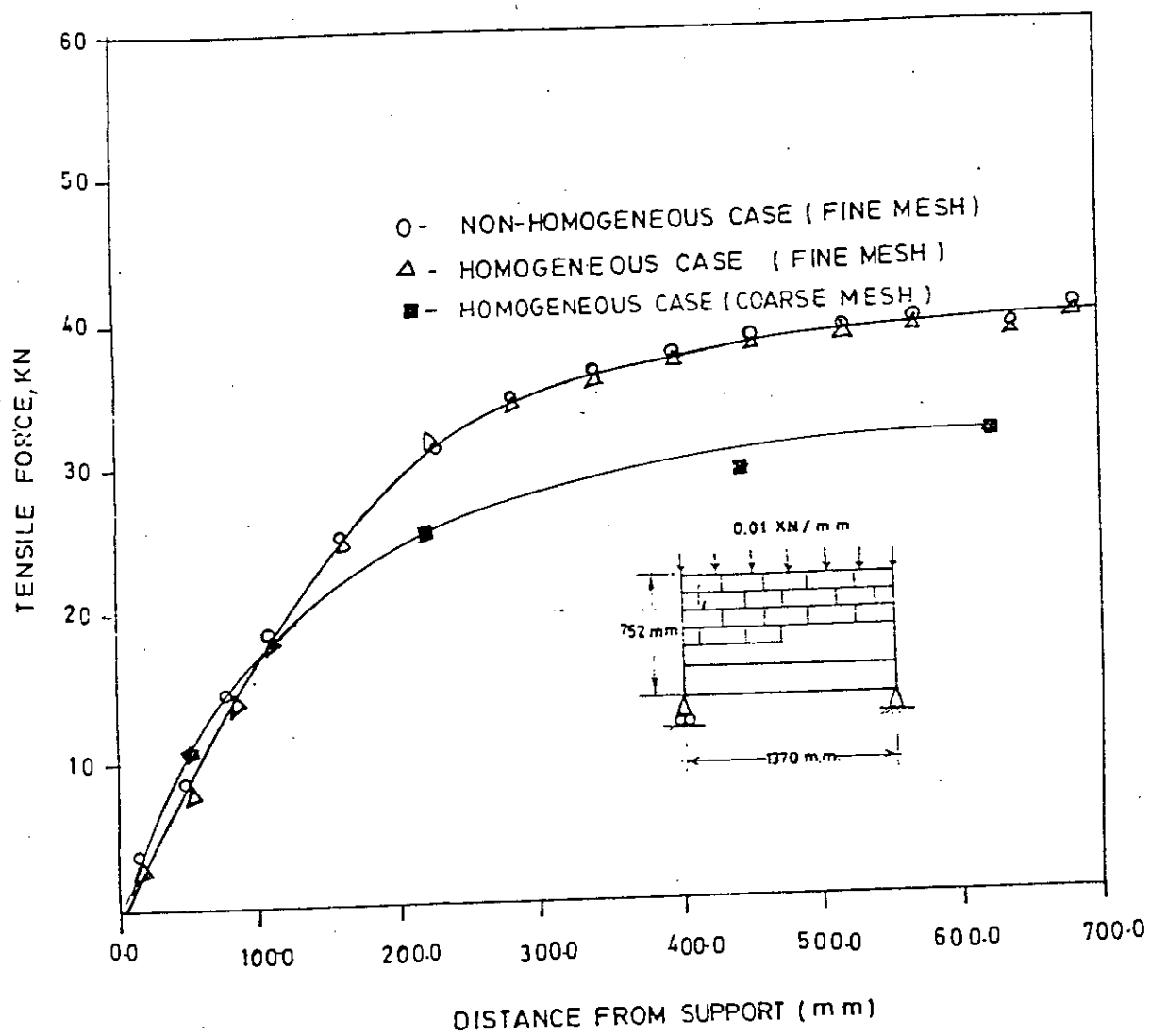
FIG 5.4 DISTRIBUTION AND MAGNITUDE OF TENSILE
FORCE ALONG THE LENGTH OF THE BEAM
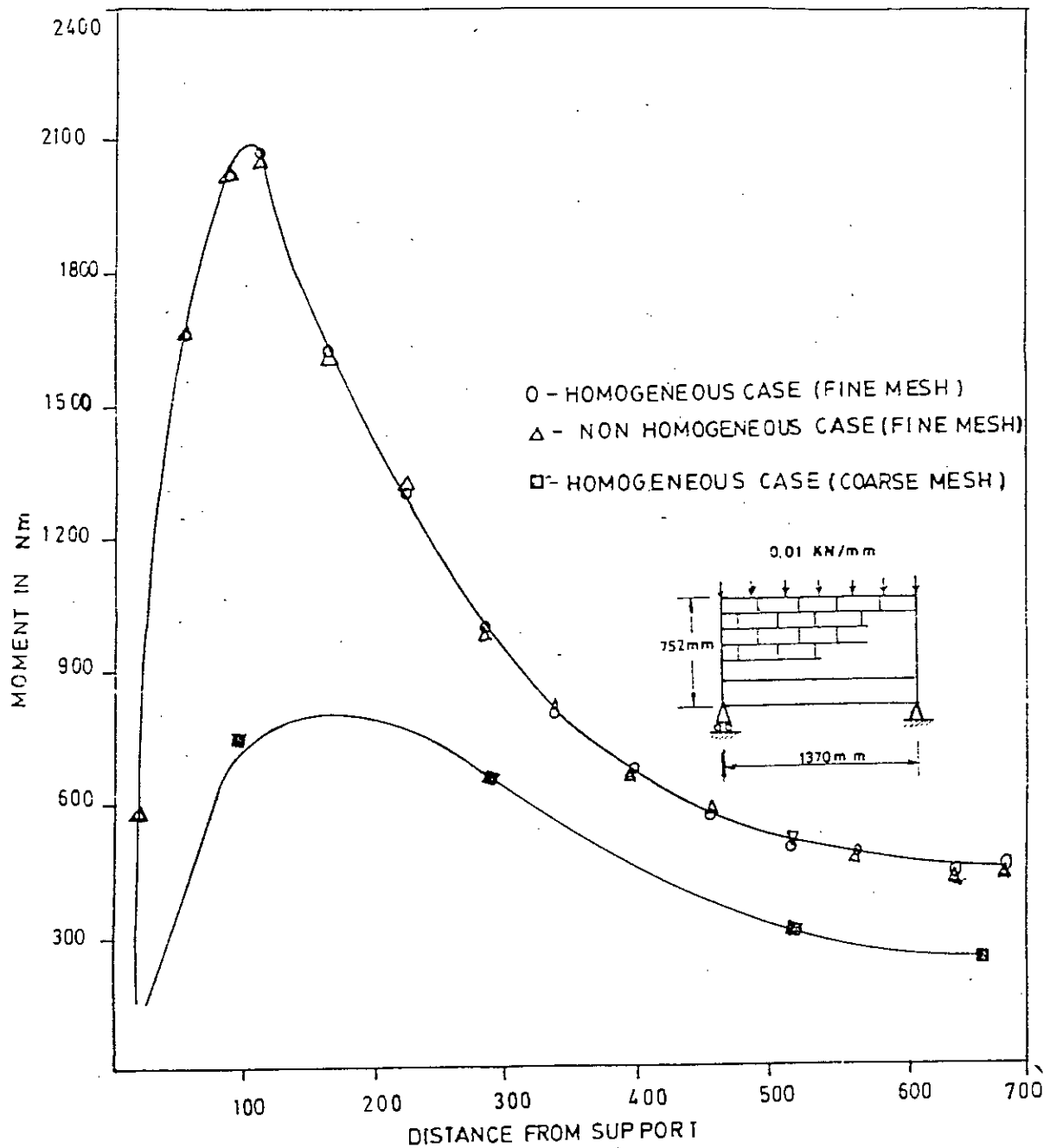
FIG 5-5 DISTRIBUTION OF MOMENT ALONG THE LENGTH OF THE BEAM
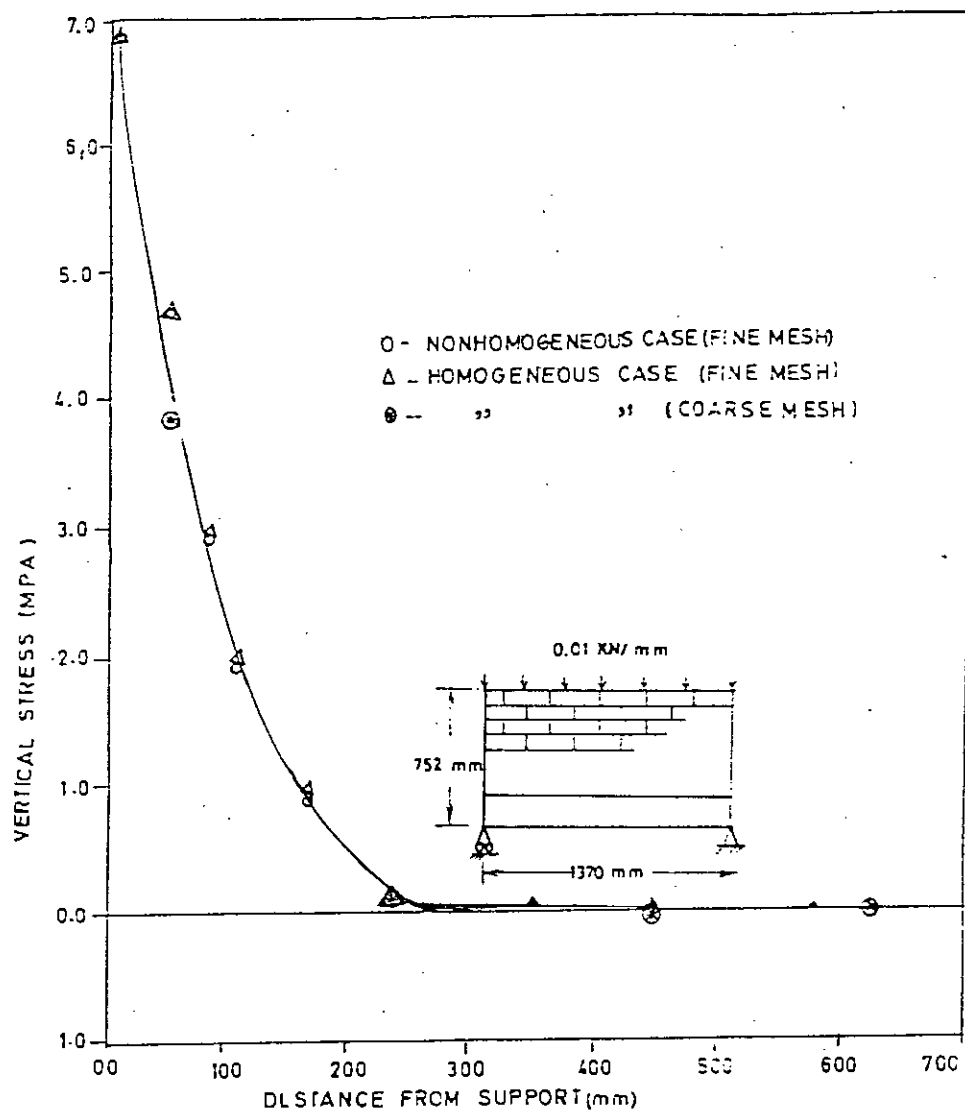
FIG 5·5 DISTRIBUTION OF VERTICAL STRESS ALONG THE INTERFACE
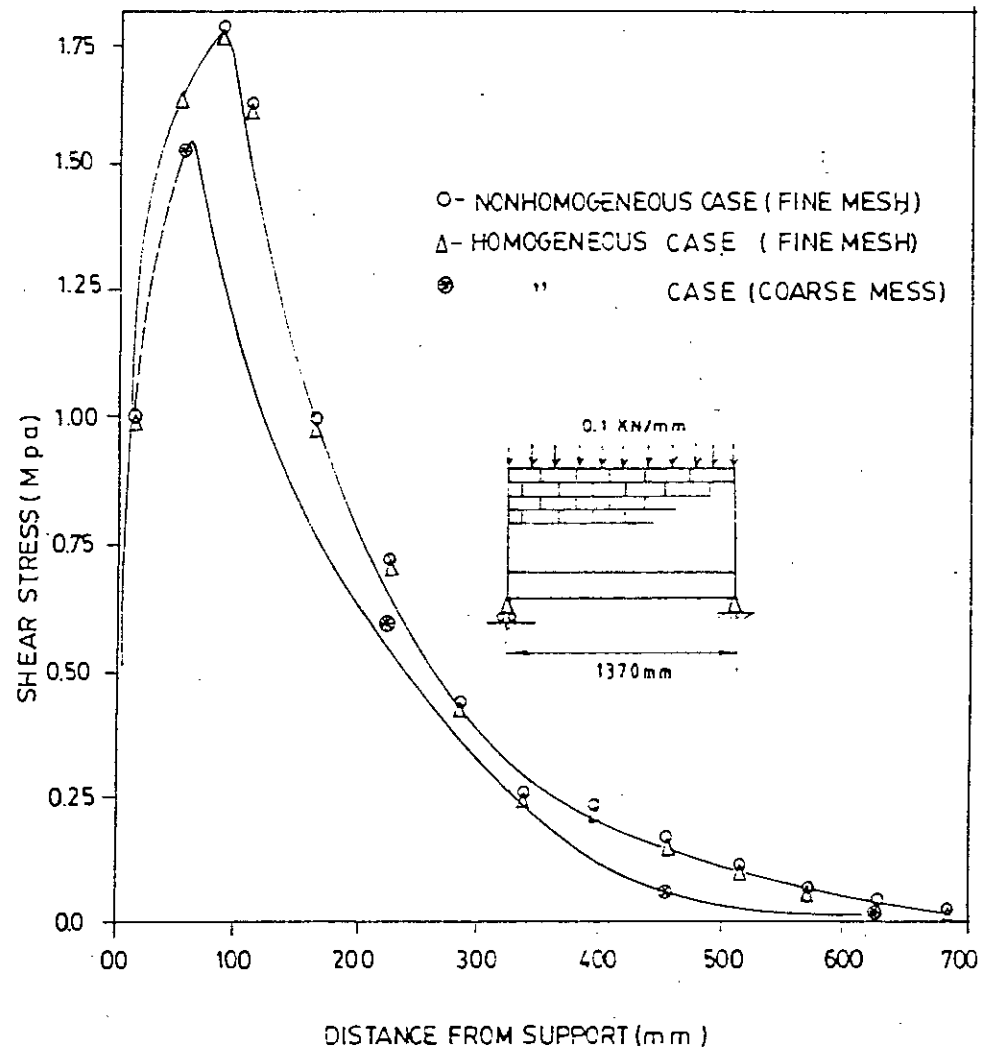
FIG. 5.7   DISTRIBUTION OF SHEAR STRESS ALONG THE INTERFACE

of brickwork as homogeneous material (encompassing bed joints and header joints) in the analysis of this type where local behaviour is to be predicted may not be correct.

From figures it is seen that for the elastic analysis the brickwork can be considered as a homogeneous material provided the size of the finite element matches with the size of the element used to calibrate material properties of the brickwork. It should be pointed out here that in the previous years the brickwork was considered as a homogeneous material using finite element mesh (size of the element) which actually could not model the material properties properly (10,11,12, 13,14,27,28,29). Therefore , for the subsequent investigations,the brickwork has been modeled as nonhomogeneous material (modeling bricks and mortar separately).

5.3 PARAMETRIC STUDY OF WALL-BEAM STRUCTURES

5.3.1 General

.＿ In the previous section the validity of the consideration of brickwork as nonhomogeneous material for the present study has been examined. In the subsequent sections a detailed study will be performed considering some of the important parameters of the wall-beam structures.

In all the cases the brickwork has been considered as nonhomogeneous materials (bricks and mortar as two different materials). In these cases the load has been applied either over the entire area of the wall or over the partial area of the wall (concentrated load).

5.3.2 Shear Stress

For full composite action to develop between the wall and its supporting beam, the shear strength at the wall-beam boundary should be adequate to transfer the horizontal shear stress along the interface as a result of the arching action. Therefore particular emphasis will be given on the shear stress distribution accross interface of wall and beam.

Distribution of Shear Stress along the Interface for Uniformly Distributed Load:

Figure 5.8 shows the distribution of shear stress along the interface for different values of Rf and K. Figure 5.9 shows that the shear stress at supports equals to zero and the value increases sharply and then decreases very slowly. The Figure also shows that for a value Rf = 7.0 or more the shear stress distribution is almost linear and more the value of Rf decreases the curve becomes more parabolic in
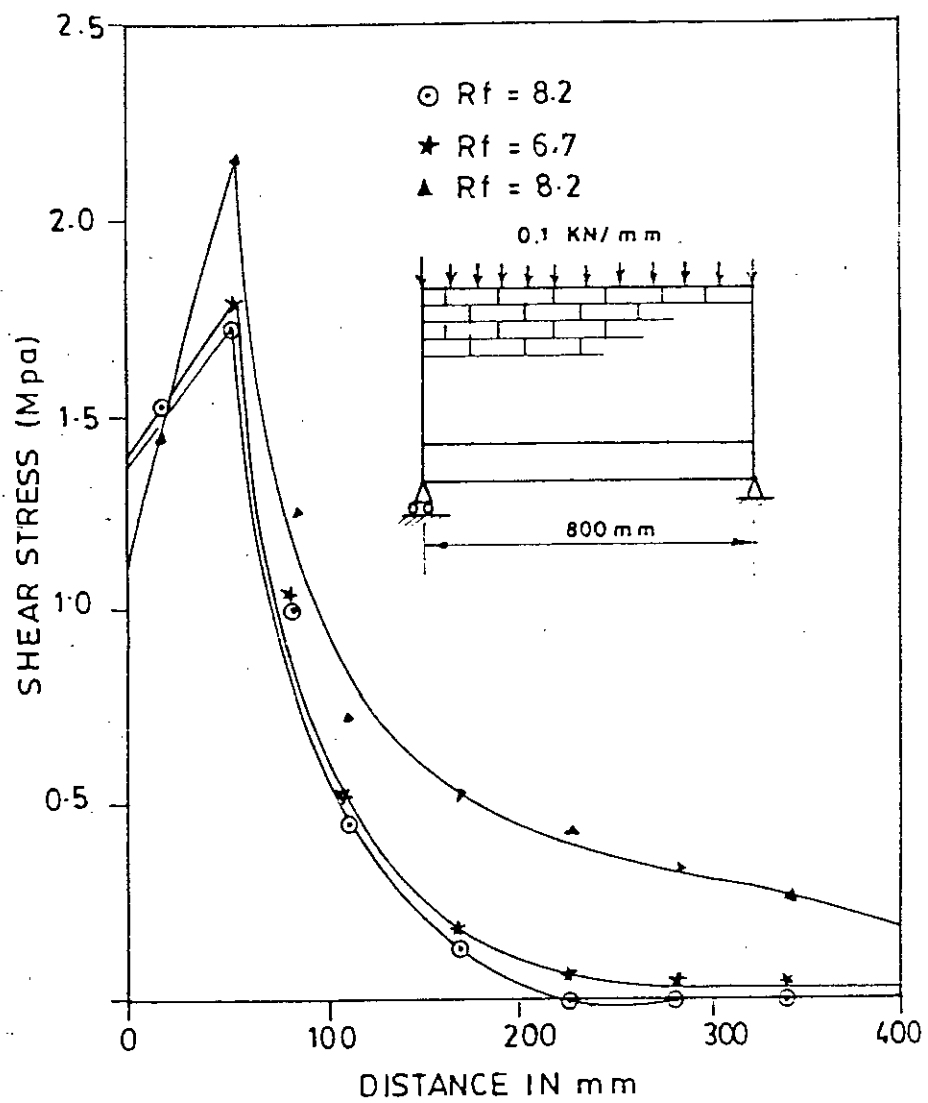
FIG. 5.8   SHEAR STRESS ALONG INTERFACE

nature. That is the stiffer the beam, the more is the distribution of shear stress along the interface.

Figure 5.9 shows the distribution of vertical stress and shear stress along the interface. From figure it is seen that the spread of shear stress along X-axis is almost twice the vertical stress .

Distribution of Shear Stress for Concentrated Load:

To observe the effects of type of loading on the shear stress distribution along the interface both concentrated load (load through a smaller area of the wall) and distributed load have been applied on the top of the wall-beam structures with four different values of Rf and K. In case of distributed load 80 KN of load are distributed on the top of the wall. In case of concentrated load the same load is applied at the centre, distributed over 8.00% area of the wall.

Figure 5.10 and figure 5.11 show the comparison of these two results. Figure 5.10 shows that the distribution of shear stress along the interface differs from each other. Figure 5.11 shows the magnitude of maximum shear stress for different values of K. From the figure it can be seen that the maximum shear stress for concentrated load
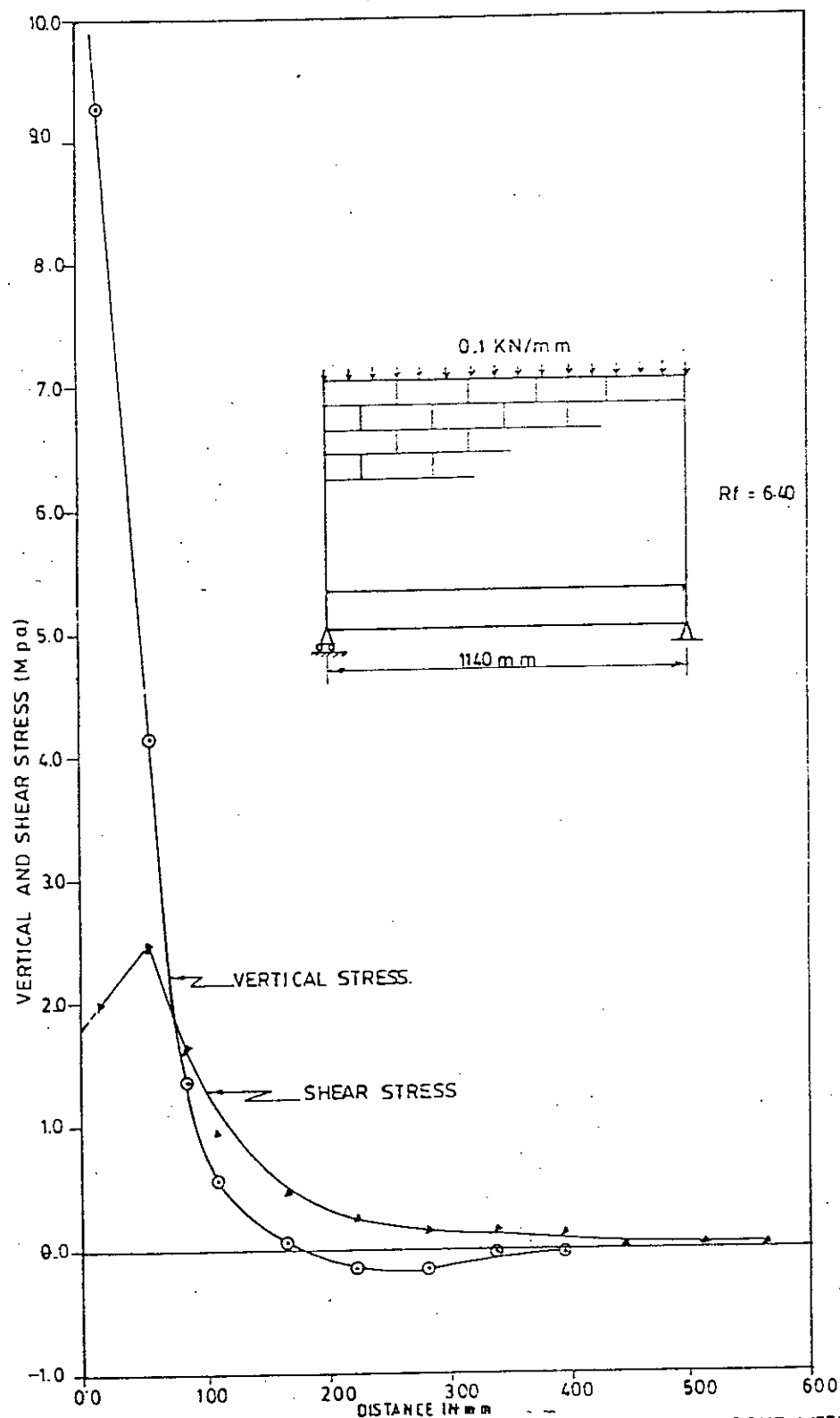
FIG. 5.9 DISTRIBUTION OF VERTICAL AND SHEAR STRESS ALONG THE INTERFACE.
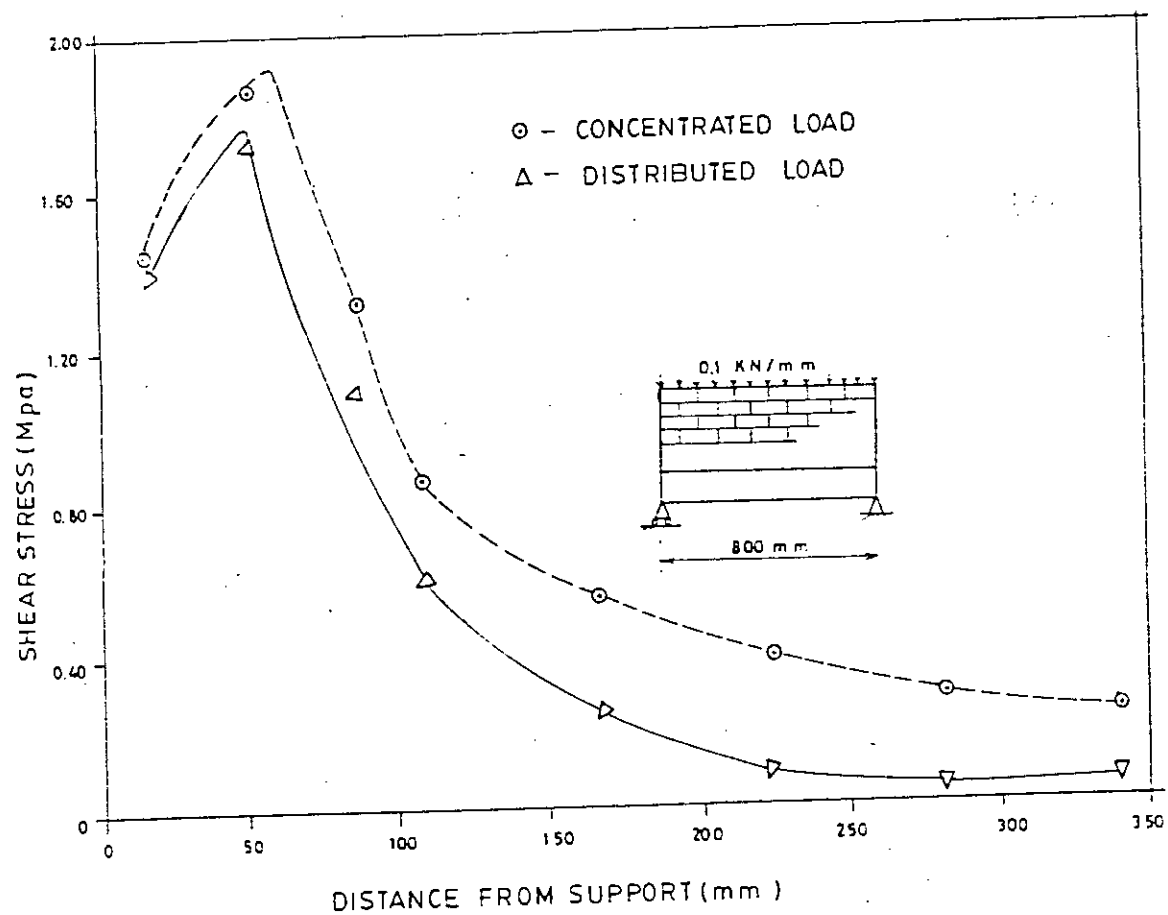
FIG 5·10 COMPARISON OF SHEAR STRESS FOR DIFFERENT TYPES OF LOADING
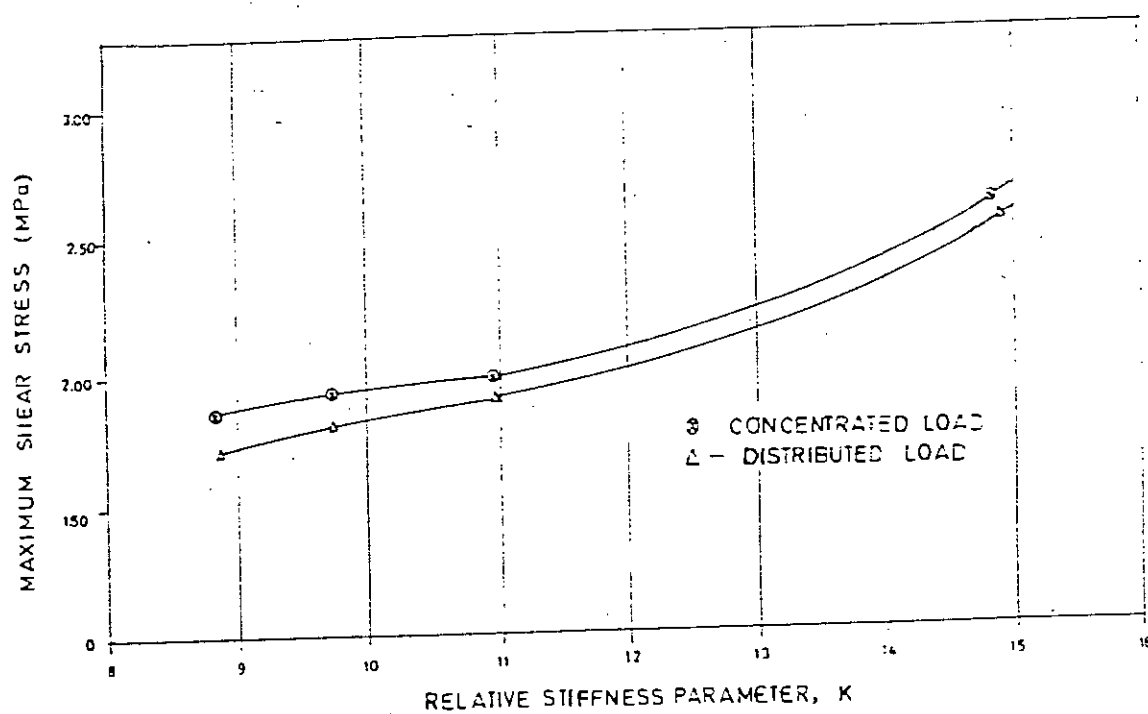
FIG 5.11   COMPARISON OF MAXIMUM SHEAR STRESS FOR DIFFERENT TYPE OF LOADING

is always higher than the corresponding distributed load.

In many practical situations the wall-beam structure receives partial loading from beam spanning perpendicular to the wall (fig 5.12). Till today no work (either experimental or theoretical) has been done on the wall-beam structure subjected to concentrated loading. This is possibly due to the complexity involved in the behaviour of the structure for this type of loading.

Though the present study is intended to develop an elastic finite element model for the wall-beam structure subjected to uniformly distributed load but its applicability for partially loaded wall-beam structure has also been explored in this thesis. In the subsequent sections the results of the present investigation have been compared with the findings of the previous investigators. In all the cases the uniformly distributed load has been considered.

Comparison of Maximum Shear Stress with the Previous Results:.

As mentioned in Chapter 2 different investigators have proposed different formulae from their experimental and theoretical investigations to determine the maximum shear
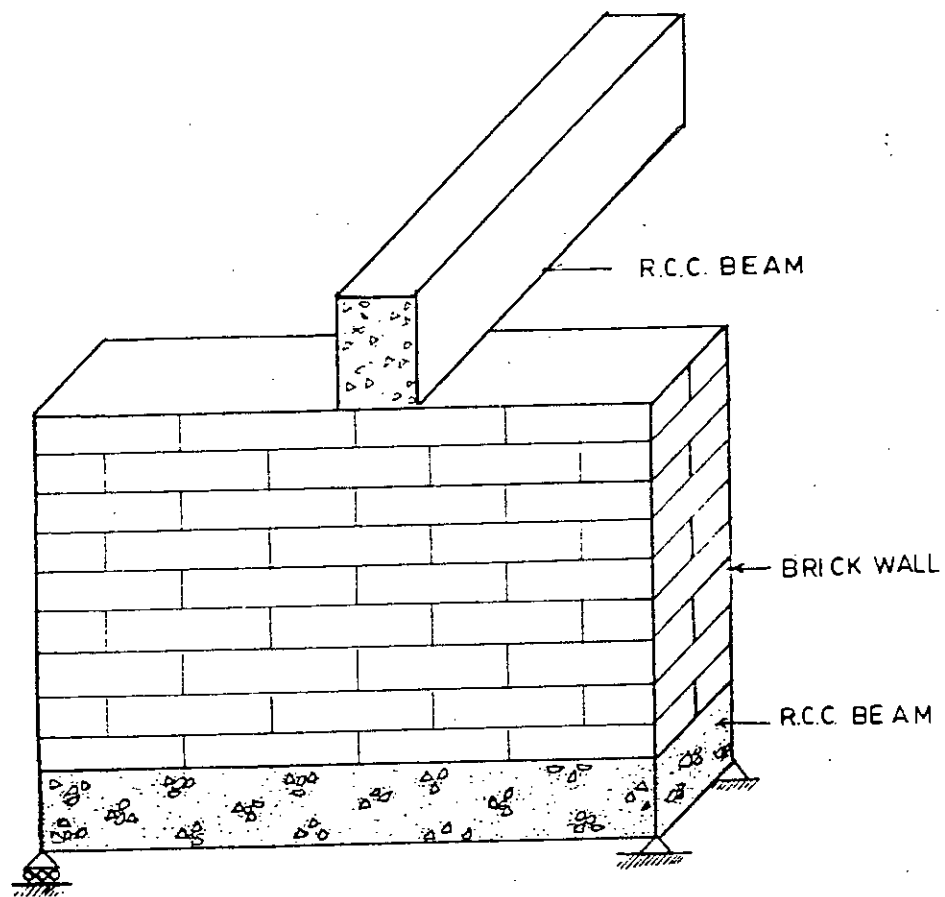
FIG 5·12 WALL BEAM STRUCTURE RECEIVING PARTIAL LOADING

force. Out of those, Davies and Ahmed (13,14) and Smith and Riddington (27) are popular. In this section the maximum shear stress determined from the present investigation has been compared with the previous results provided by those investigators . The comparisons are given in table 5.2 in a tabular form.

The table shows that the value of maximum shear stress (τ_{xy}) found from the present finite element study is in close agreement with that determined by Davies and Ahmed (13,14) whereas the stress determined by Smith and Riddington (27) is much higher.

5.3.3 Vertical Stress

Maximum Vertical Stress :

The vertical stress is maximum over the supports. Its effects on the beam is to produce a bending moment which is less than that would be obtained if the load was being carried directly by the beam.

Figure 5.13 shows the variation of magnitude and distribution of vertical stress (σ_y) with their Rf values along the interface. The figure indicates that the vertical stress is maximum over the supports and then the values

TABLE 5.2

COMPARISON OF MAXIMUM SHEAR STRESS (Mpa)

SL.	Rf	D.A.(*)	F.E.($)	S.S&R(@)
1.	5.57	2.6	4.9	6.82
2.	5.94	1.42	1.443	3.68
3.	6.4	1.274	1.862	4.01
4.	7.39	1.038	2.81	4.218
5.	10.13	3.024	3.025	6.75

* - D.A. -Davies and Ahmed (11,12)

$ - F.E. - Present Finite Element Method

@ - S.S.&R - Smith and Riddington (29)
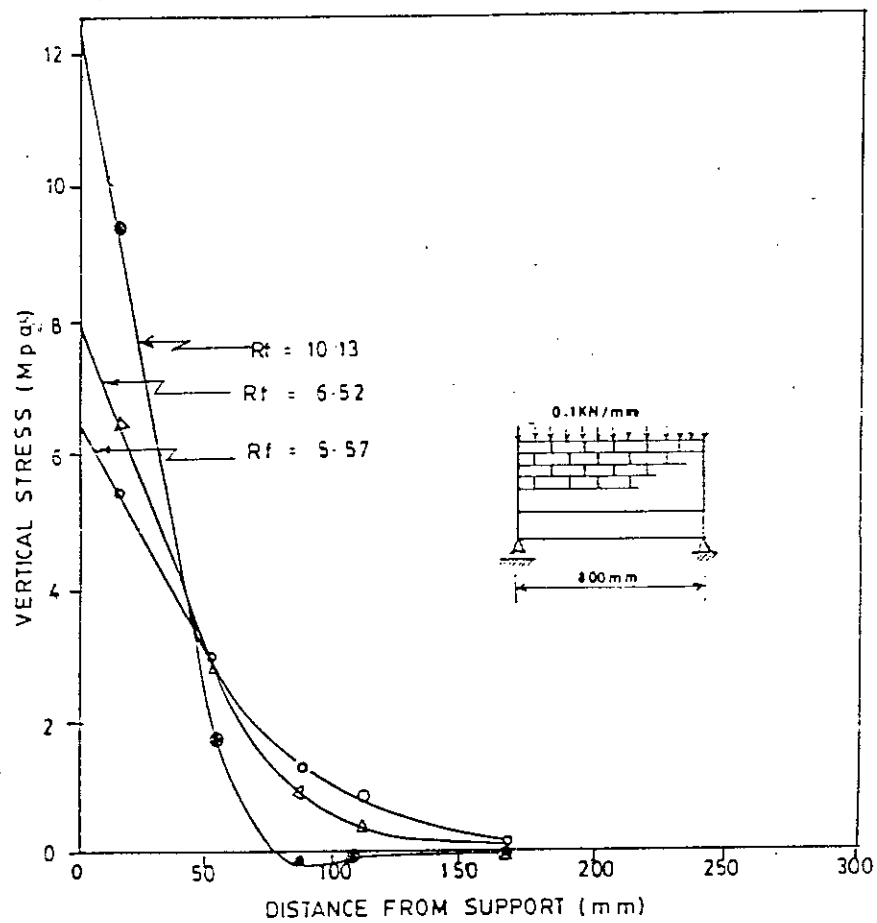
FIG. 5·13 DISTRIBUTION OF VERTICAL STRESS ALONG THE INTERFACE

decreases sharply for higher values of Rf and gradually for lower values of Rf along the length of the beam. The figure also indicates that the more is the value of Rf the more triangular is the distribution of vertical stress. In the present case the limiting value of Rf in between triangular and parabolic distribution may be estimated to be 6.5 as compared to this value of 7.00 as proposed by Davies and Ahmed (13,14).

Vertical Stress Concentration Factor.

The vertical stress concentration factor, C, may be defined as the ratio of the maximum vertical stress developed to the average vertical stress in the brickwork i.e.

$$C = \sigma_{max} / \sigma_{av}$$

where,

$$\sigma_{av} = \text{load/X-sectional area}$$

Figure 5.14 illustrates the variation of stress concentration factor along X-axis and figure 5.15 illustrates the variation of the same with Rf. From Figure 5.14 it is seen that the maximum vertical stress is developed over the supports and within a short distance the vertical stress changes to a value almost equal to σ_{av} i.e. $\sigma_{max} / \sigma_{av} \cong 1$. Figure 5.15 shows a linear correlation of C with Rf i.e. the more is the value of Rf, the more is
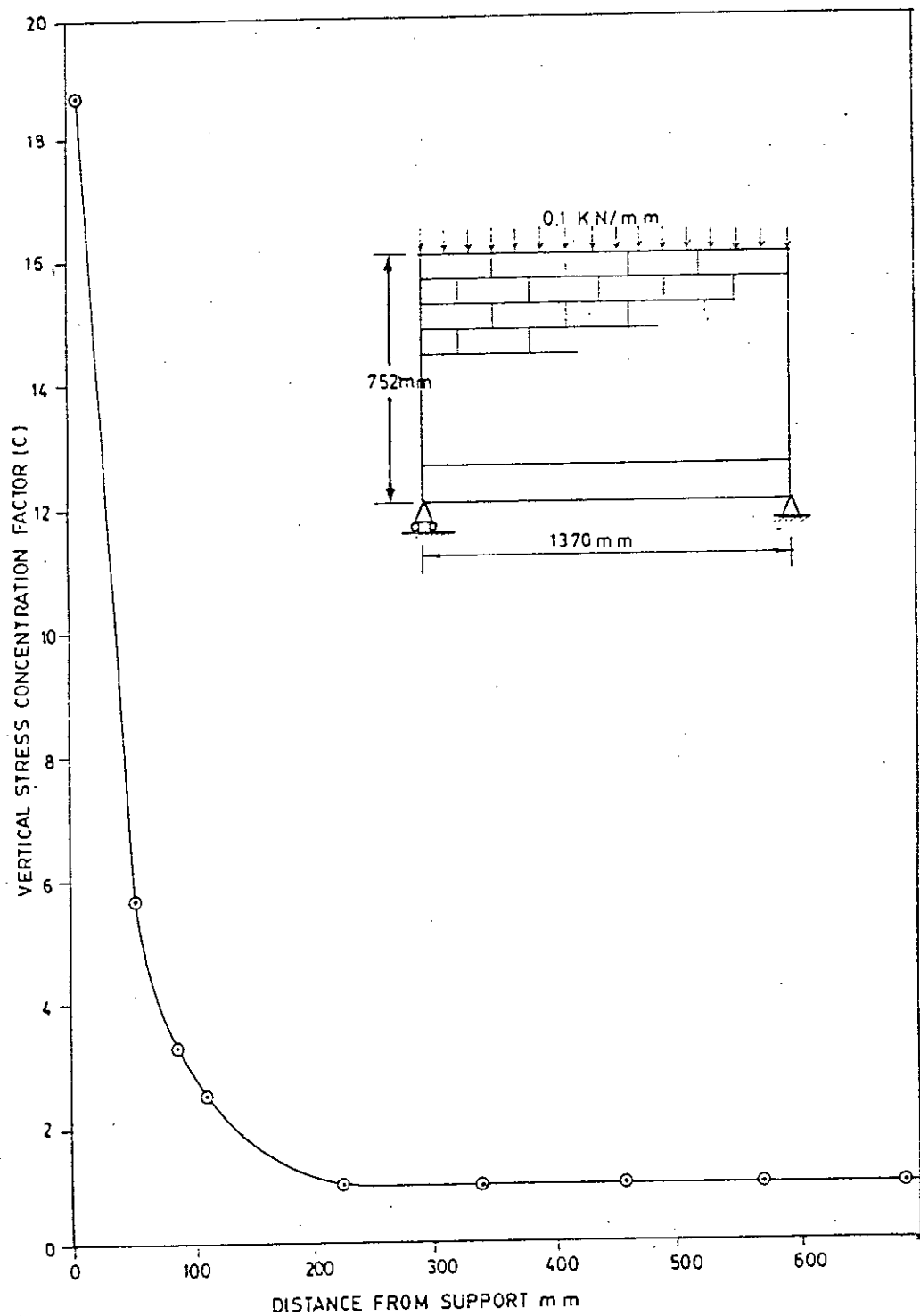
FIG. 5.14 VARIATION OF STRESS CONCENTRATION FACTOR ALONG THE LENGTH OF THE STRUCTURE.
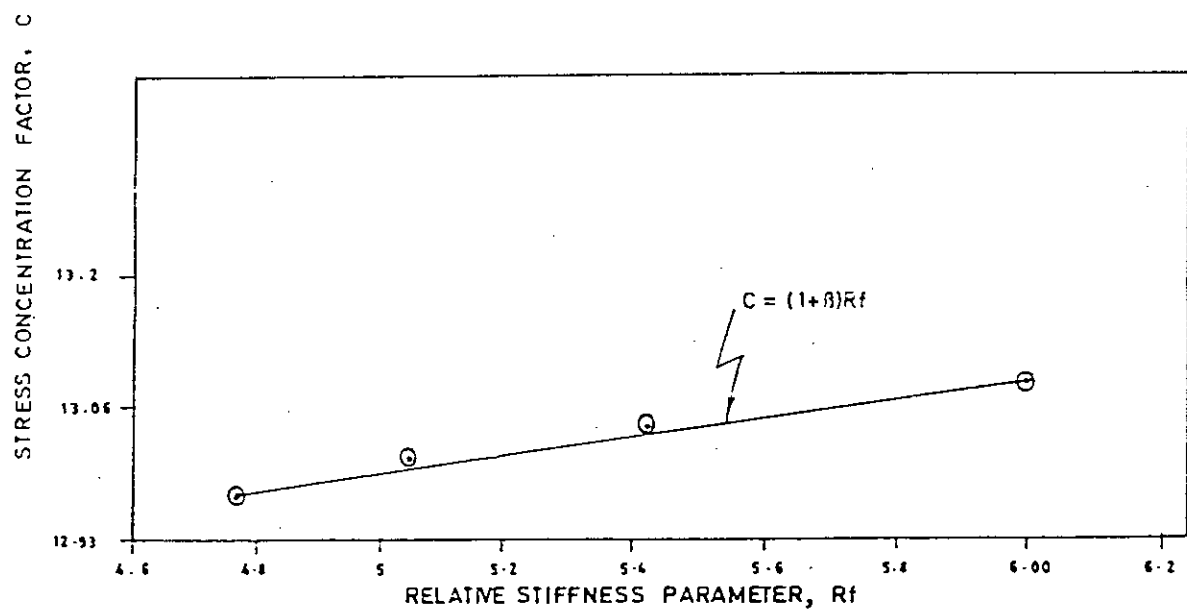
FIG 5·15 VARIATION OF STRESS CONCENTRATION FACTOR WITH Rℓ

the vertical stress concentration factor.

From above, it may be concluded that the vertical stress distribution along the contact surface is mainly governed by the stiffness parameter, Rf. For a very slender beam, that is with a higher values of Rf, the stress distribution is triangular with large vertical stress over the supports. In walls supported on relatively stiff beam with low values of Rf, the contact vertical stress spreads towards the centre of the span giving rise to smaller stress concentration over the supports and the distribution may be represented by a third degree parabola (13).

Comparison of Vertical Stress Concentration factor with the results of previous investigators:.

Table 5.3 shows the comparison the stress concentration factor as predicted by:

i)   Smith and Riddington (27).

ii)   Davies and Ahmed (13,14).

iii) The present finite element analysis.

Table 5.3 shows that the formulae proposed by Smith and Riddington (27) overestimates the values where as the values calculated by Davies and Ahmed (13,14) are on the lower side.

TABLE 5.3

COMPARISON OF VERTICAL STRESS CONCENTRATION FACTOR

SL	D.A.	F.E.	S.S.&R.
1.	16.7	29.96	33.5
2.	10.67	17.5	19.76
3.	11.535	13.72	13.079
4.	9.913	12.815	16.90
5.	9.353	12.55	15.52

D.A.- Davies and Ahmed (13,14)

F.E.- Present Finite Element Method

S.S.&R.- Smith and Riddington (27)

5.3.4 Bending Moment of the Supporting Beam

The bending moment at any section in the supporting beam of wall-beam structure results from vertical loading and the horizontal shear stress at the interface, which is eccentric to the axis (fig 5.16). In the present study the moment is determined from the longitudinal stress (σ_x) as described in section 3.4.
i.e.

M=I (fb1-ft1)/hi

From the calculated moment the coefficient of moment, k1, is calculated as ;

k1= W L/M

where,

W= Total load on the wall

L= Span of the beam

M= Maximum moment

In the following paragraphs details of the findings regarding moment and a few parameters affecting the moment are discussed.

Maximum Moment and Distribution of moment in the Beam.

To ascertain the values of maximum moment and distribution of moments along X-axis, moments are calculated
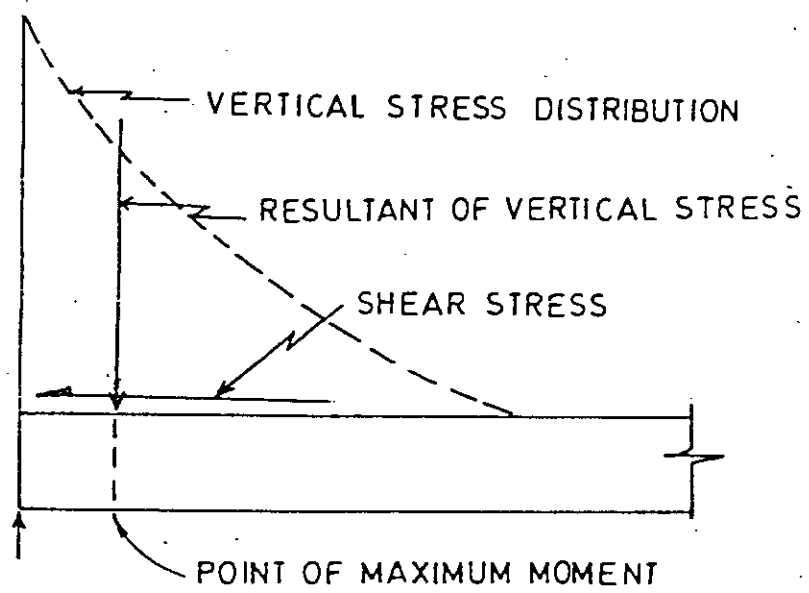
FIG. 5.16   STRESSES CONTRIBUTING MOMENT

at each sampling points along X-axis as well as at the centre of each element of the beam. Fig 5.17 illustrates that the maximum moment occurs at a distance of about 1/15th of span from the supports.

The supporting beam is subjected to the vertical forces and horizontal shear forces at the wall-beam interface. The horizontal shear force is thus eccentric with respect to the beam centroid. This has the effect of causing substantial reduction in the beam bending moment produced by the vertical force. Therefore the ultimate maximum moment is less than what is expected. The maximum moment always occurs near the supports. Distribution of beam moments are shown in figure 5.17.

Influence of Relative Stiffness Parameter on Maximum Moment in the Beam.

For the study of influence of relative stiffness parameter on maximum moment , analyses with different relative stiffness parameters have been performed. The results are compared with the maximum moments. The load applied was 0.1 KN/mm length of the wall, the modular ratio (Ec/Ew) was 2.0 and the details of the parameters used in the investigation are given in table 5.4. The parameters were so arranged that the values of K were variable.
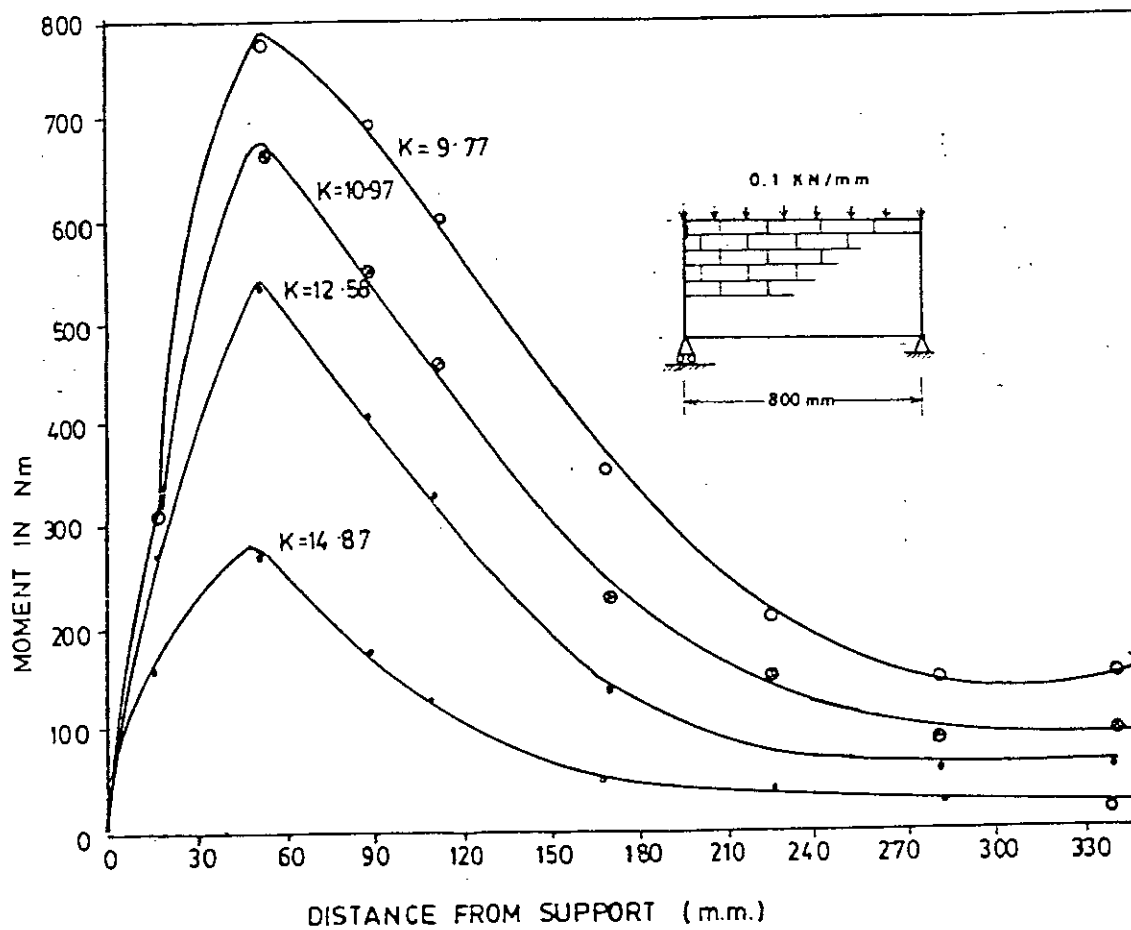
FIG 5·17 DISTRIBUTION OF MOMENT ALONG THE LENGTH OF BEAM.

TABLE 5.4

RELATIVE STIFFNESS VS COEFFICIENT OF MAXIMUM MOMENT

SL	H/L	K(*)	k1(#)
1.	0.74	9.279	118
2.	0.577	11.175	160
3.	0.514	12.18	178
4.	0.428	13.97	230
5.	0.40	15.61	236

(*)- K=((Ew*t*L**3/(E*I))**0.25

(#)- k1=(W*L/Mmax)

Figure 5.18 illustrates the relationship between the relative stiffness parameter, K and the coefficient of maximum moment, k1. The table 5.4 and the figure 5.18 show that the maximum moment is dependent on the value of K, The relationship of k1 and K is almost linear which is in good agreement with the findings of Smith and Riddington (27).

Again figure 5.18 shows that with the increase of values of K, the coefficient of moment increases, i.e. the moment decreases. Conversely it may be concluded that the stiffer the beam, the more is the bending moment developed in the beam. The lesser the stiffness of the beam, the more it behaves like a tension member.

Influence of Depth of Bottom Beam on Maximum Moment.

For studying the influence of depth of bottom beam on the maximum moment developed in the beam, five different sizes of the beam have been considered in this study. The H/L ratio was kept constant to 0.6, E/Ew was kept constant at 2.00, the width of the beam and width of the wall was the same. Table 5.5 shows the details of the analyses:

The variation of maximum moment and hence the coefficient of moment (WL/Mmax) with K and depth of the

TABLE 5.5

MAXIMUM MOMENTS FOR DIFFERENT BEAM SIZES

SL	BEAM SIZE mmxmm	K	Mmax N-mm	WL N-mm	k1
1.	40x110	14.87	210320	64000000	304
2.	50x110	12.58	309861	" "	206
3.	60x110	10.97	404464	" "	158
4.	70x110	9.77	498380	" "	128
5.	80x110	8.84	586081	" "	109
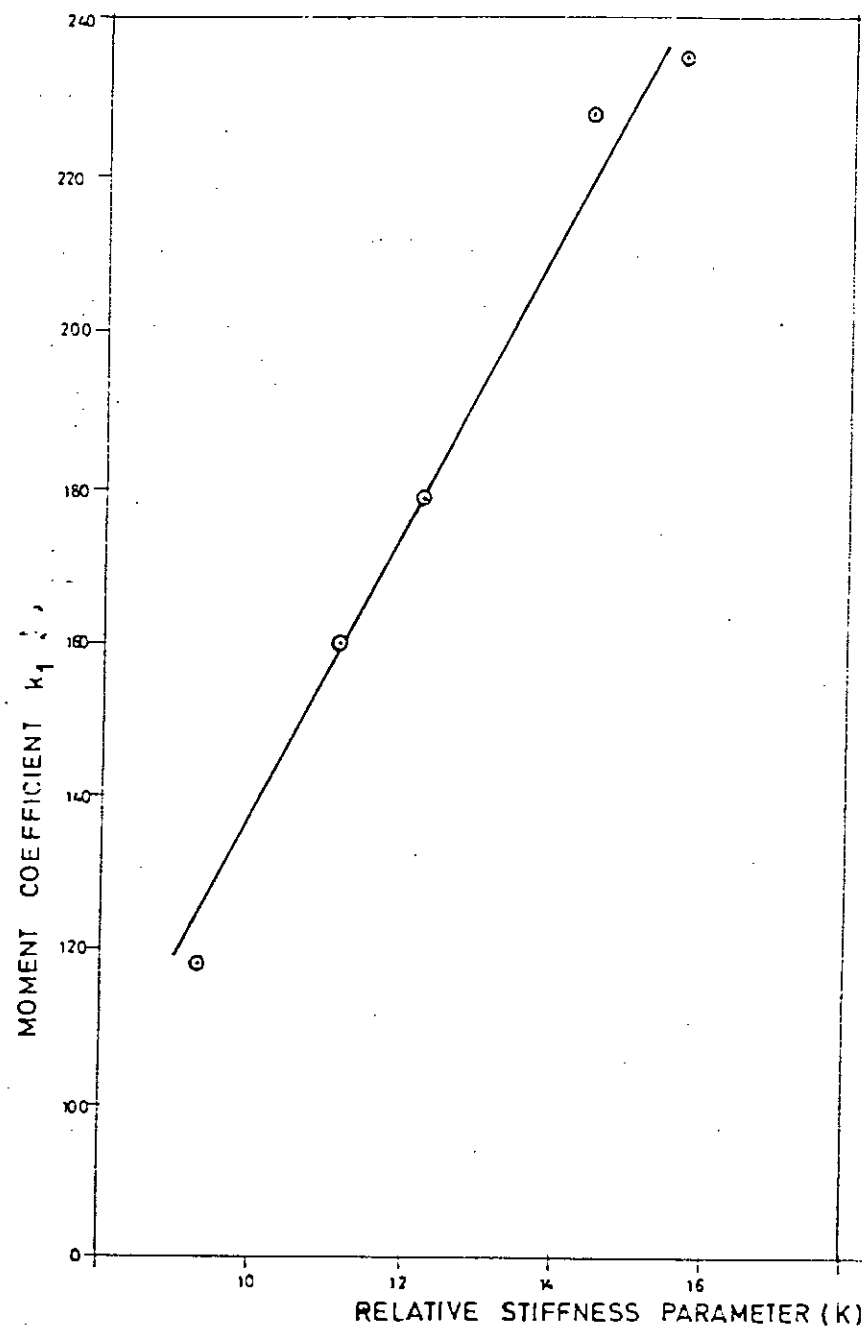
FIG 5·18   CORRELATION BETWEEN K AND k_1

TABLE 5.4

RELATIVE STIFFNESS VS COEFFICIENT OF MAXIMUM MOMENT

SL	H/L	K(*)	k1(#)
1.	0.74	9.279	118
2.	0.577	11.175	160
3.	0.514	12.18	178
4.	0.428	13.97	230
5.	0.40	15.61	236

(*)- $K=((Ew*t*L**3/(E*I))**0.25$

(#)- $k1=(W*L/Mmax)$

bottom beam are presented in figure 5.19 and 5.20. The above table and figures indicate that the maximum moment depends on the depth of the bottom beam provided other parameters are constant. Keeping other parameters constant, the more is the depth of the beam the more the beam behaves like a flexural member and the more is the magnitude of the maximum moment.

Influence of Concentrated Load on Maximum Moment.

For this analysis the same wall-beam structures as described in previous article are considered with concentrated load of same magnitude as in the case of uniformly distributed load. The maximum moment and the coefficient of moment are compared with those obtained in case of distributed load application. Table 5.6 shows the comparison. Fig 5.21 shows the magnitude and distribution of bending moments for beam along X-axis. From figure and the table it is seen that the maximum moment in the case of distributed load and concentrated load is in close agreement with each other. This is possibly due to the depth of the wall (H/L >0.6) which provides enough area for the dispersion of concentrated load through the wall. The distribution of load near the beam in this case resembles uniformly distributed load.Therefore, it may be said that the magnitude of maximum moment is not dependent on the area
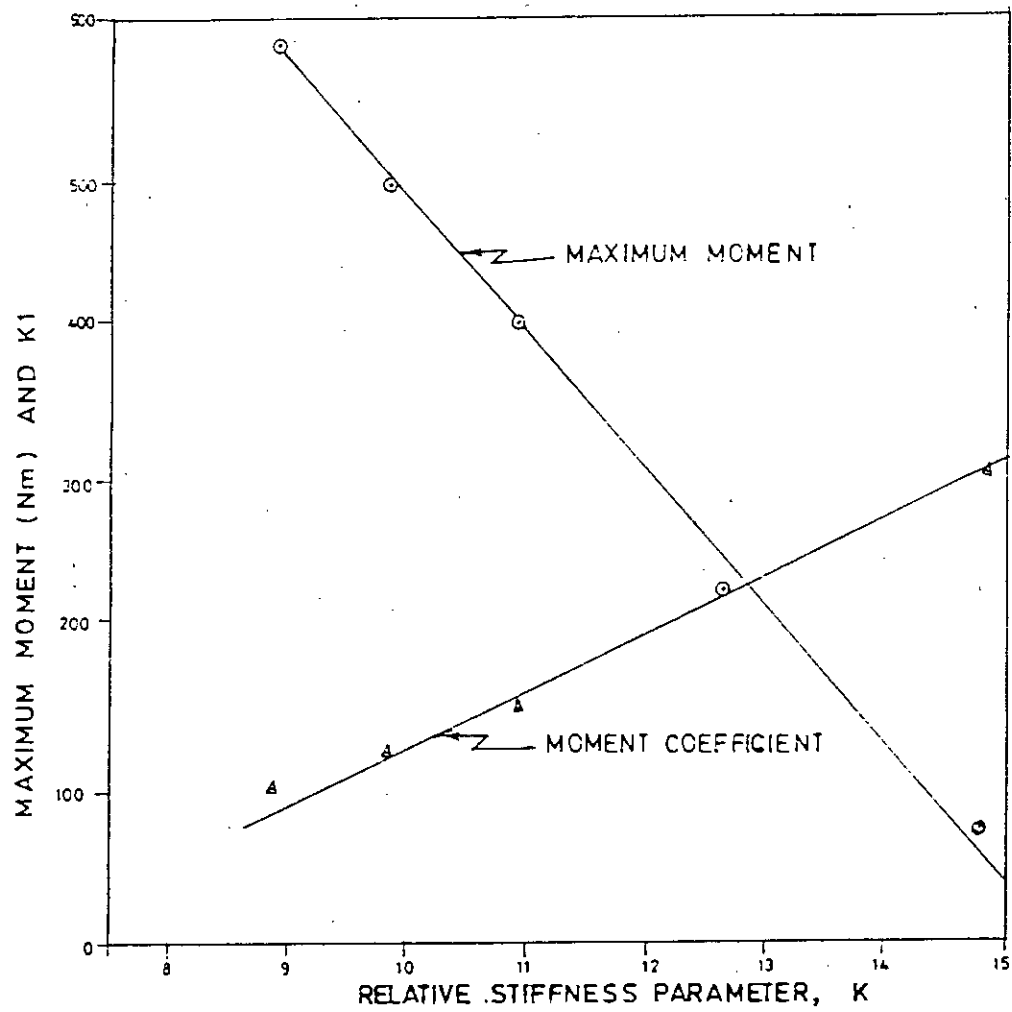
FIG 5·19   MAXIMUM MOMENT—RELATIVE STIFFNESS CURVE
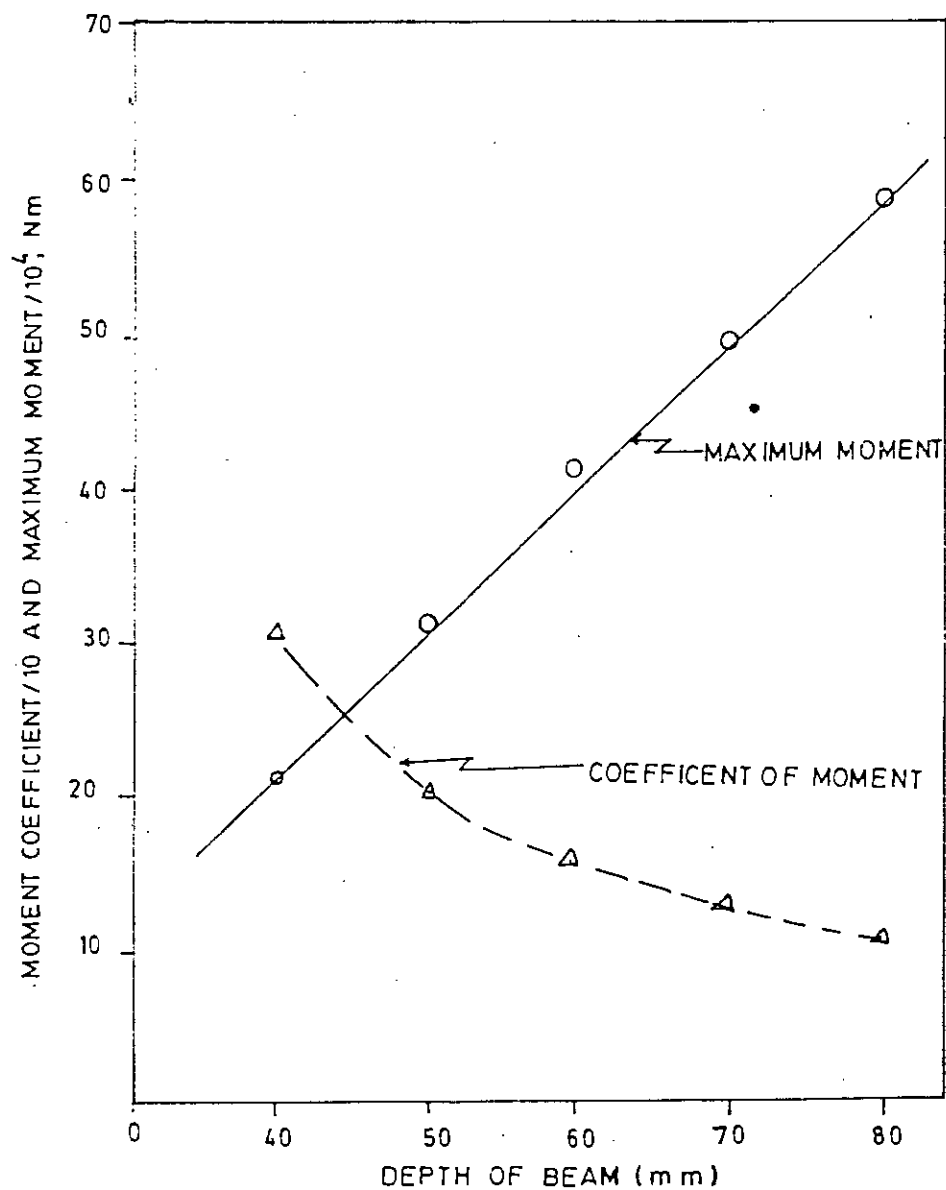
FIG 5·20 MIXIMUM MOMENT CURVE FOR DIFFERENT BEAM SIZES.

TABLE 5.6

INFLUENCE OF CONCENTRATED LOAD ON MOMENT IN THE BEAM

SL	BEAM SIZE mmxmm	MAX. MOMENT DISTR. LOAD N-mm	MAX. MOMENT CONC. LOAD N-mm	MOMENT COEFF. DIST N-mm	MOMENT COEFF. CONC N-mm
1.	40x110	203938	208032	304	307
2.	50x110	309868	307261	206	208
3.	60x110	404465	399316	158	160
4.	70x110	498373	491703	128	130
5.	80x110	586081	578072	109	110
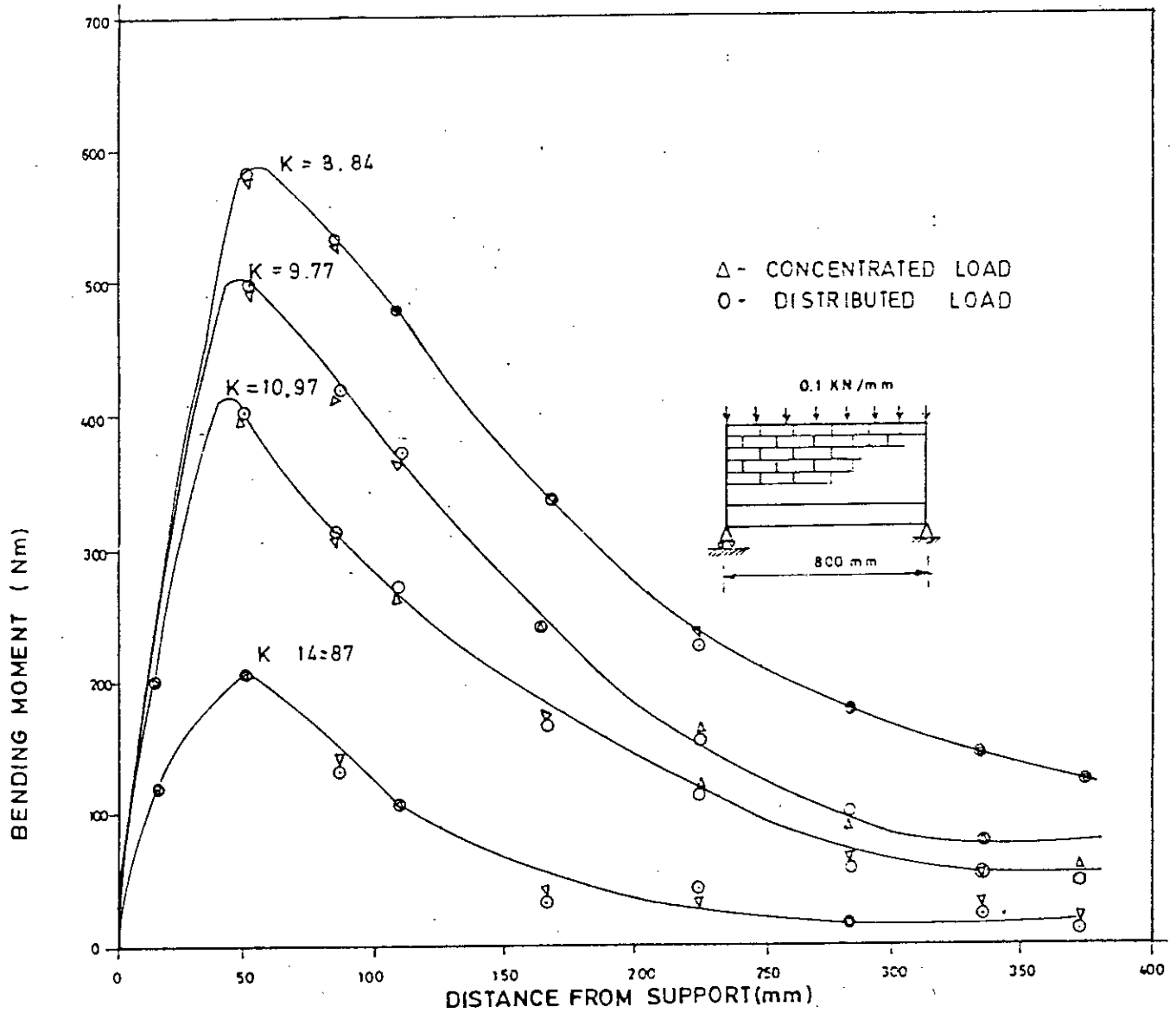
FIG 5.21 DISTRIBUTION OF BENDING MOMENT ALONG THE LENGTH OF THE BEAM FOR DIFFERENT LOADING TYPES.

through which the load has been applied but the magnitude of the applied load provided H/L is more than 0.6. Again the figure and table indicate that the magnitude of maximum moments in both the cases are very similar .

Here it should be mentioned that this particular investigation was carried out on the basis of wall aspect ratio of 0.6 or more. For shallow wall- beam structures (aspect ratio<0.6) the agreement may be different since the area through which the load dispersion of concentrated load takes place decreases and as a result the distribution of vertical stress on the beam will change.

Influence of Modular Ratio (E/Ew) on Maximum Moment.

For this study, five different wall-beams have been considered. In all the cases H/L was 0.6; and the loading was 0.1 KN/mm. The size of the beam was 80 x 110 mm . The ratio of modulus of elasticity of concrete and that of brickwork were varied from 2 to 10. The results of the analyses are given in table 5.7.

The variation of moment and hence k1 with K and E/Ew are shown figures 5.22 and 5.23 respectively.

Since the ratio of modulus of elasticity of brickwork and that of concrete contributes to the relative stiffness parameter, K, inversely it is evident that the ratio will influence the magnitude of maximum moment and coefficient of

TABLE 5.7

INFLUENCE OF MODULAR RATIO (E/Ew) OF WALL-BEAM

SL	E/Ew	K	Mmax N-m	k1
1.	2	8.80	585375	109
2.	3	7.95	632368	101
3.	4	7.40	672232	95.2
4.	5	7.00	706816	90.5
5.	10.286	5.84	839080	76.27
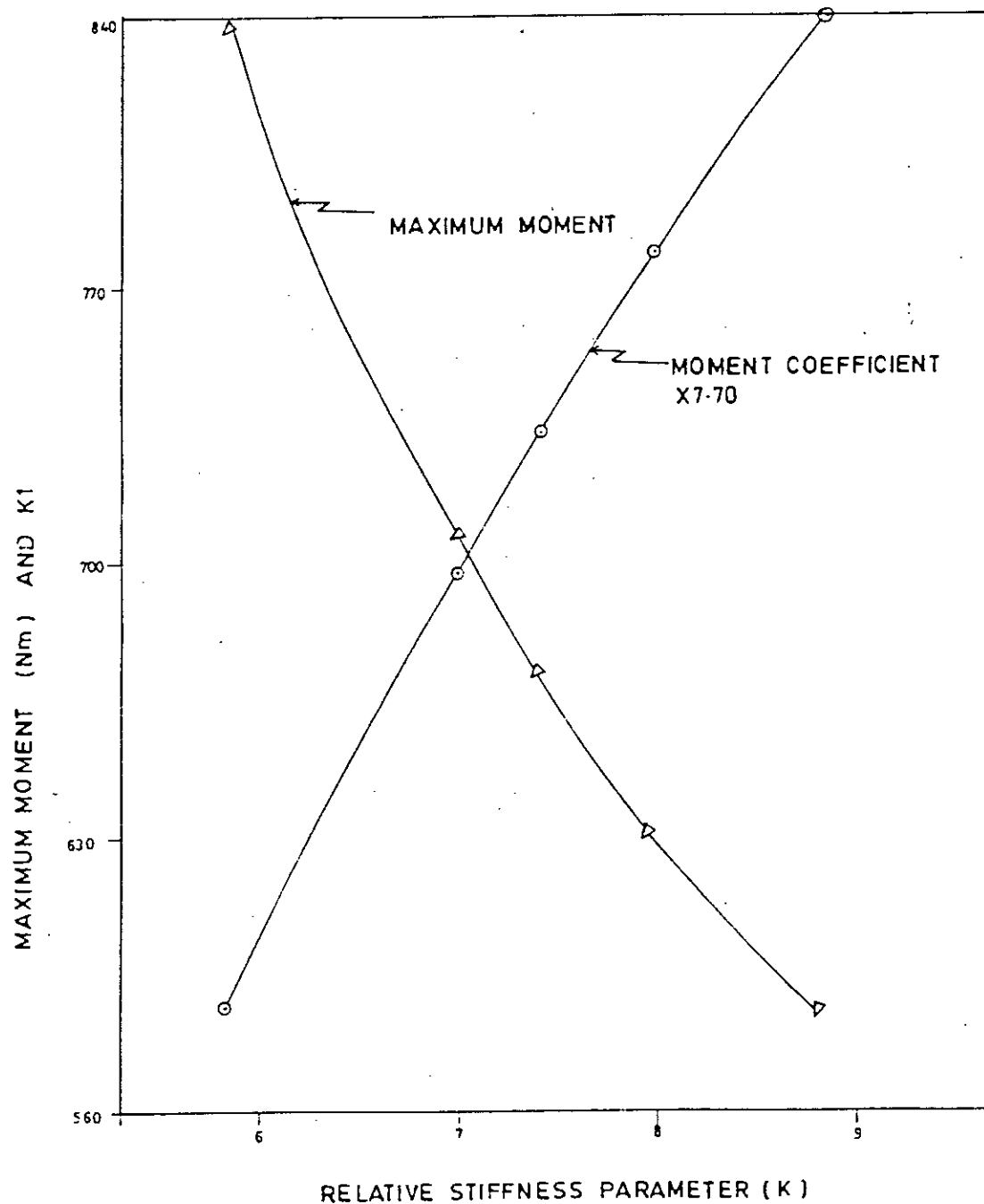
FIG 5.22 MAXIMUM MOMENT VS K CURVE FROM DIFFERENT MODULAR RATIO.
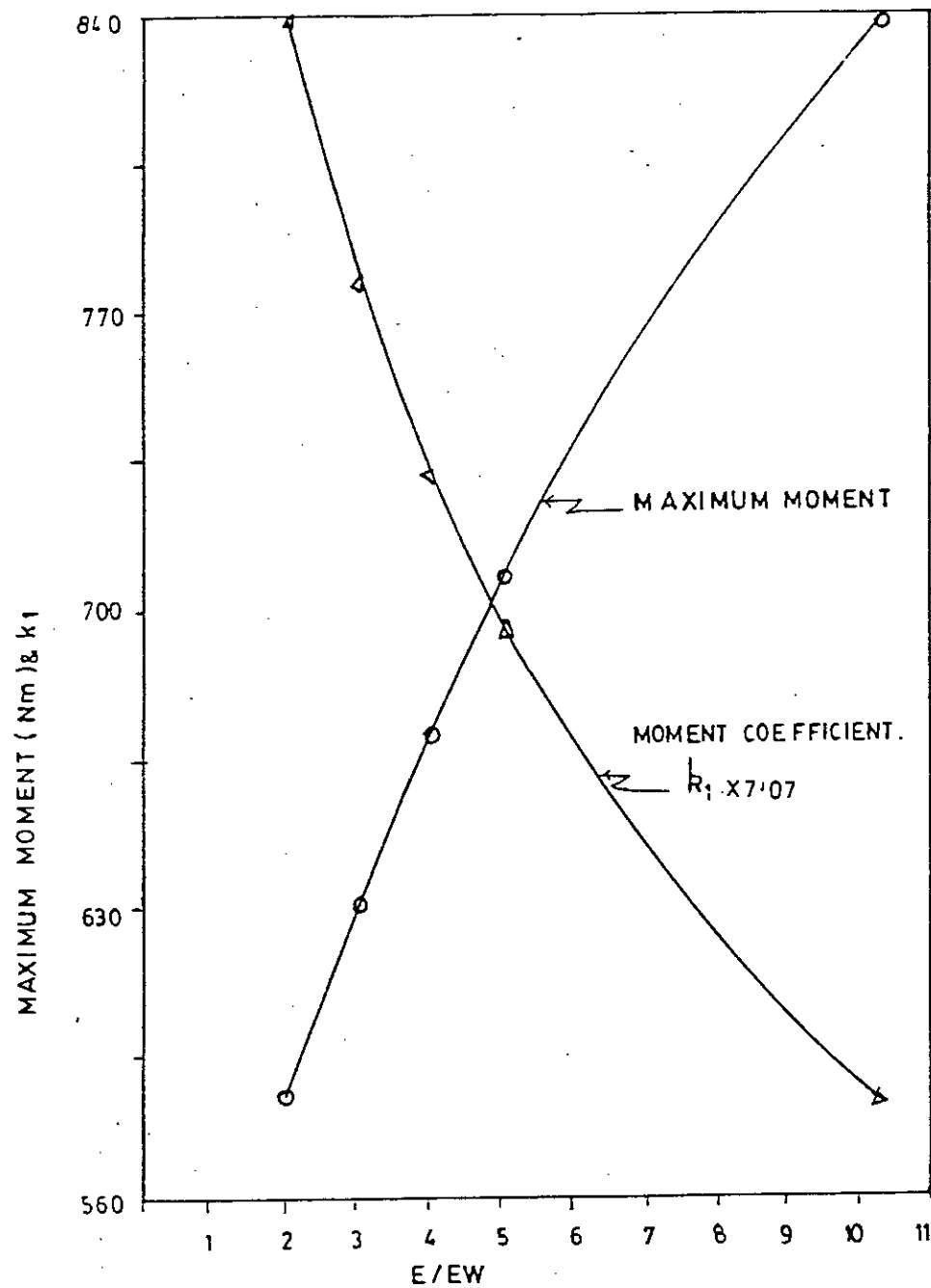
FIG 5·23 MAXIMUM MOMENT FOR DIFFERENT MODULAR RATIO CURVE

moment inversely as K does. Figure 5.23 indicates that with the increase of the modular ratio,the magnitude of maximum moment increases. That is the more is the relative stiffness the lesser is the magnitude of maximum moment. That is the stiffer is the beam relative to the brickwork, the more is the beam bending moment.

Comparison of Maximum Moment with the Results of other Investigators.

Comparison of maximum moments as determined by Smith and Riddington (27) and Davis and Ahmed (13,14) is shown in table 5.8 with the results of the present investigation.

The table indicated that in both cases, the magnitude of maximum moments are over estimated by a factor of 1.5 (approximately). From their results it may be mentioned that the formulae are conservative to some extent. Moreover it may so happen that, the magnitude of maximum moment is greater because they have considered triangular distribution of vertical stresses and shear stresses along the interface which is a mainly highly idealized case. In fact the distribution is parabolic.

Smith and Riddington (27) has proposed formula for calculating maximum moment as.:

TABLE 5.8

COMPARISON OF MAXIMUM MOMENTS

SL	H/L	K	MAXIMUM MOMENT N-mm			RATIO OF	
			(1)	(2)	(3)	1/3	2/3
1.	0.6	10.97	656420	806663	404465	1.623	1.9
2.	"	9.279	820525	890206	547017	1.50	1.60
3.	"	8.09	1269260	2132540	1334227	1.47	1.59
4.	"	8.84	875227	1010703	586080	1.49	1.72
5.	"	7.474	2188067	2174553	1480325	1.478	1.47

(1)- Smith &Riddington (27)

(2)- Davies and Ahmed (13,14)

(3)- Present Finite Element study

$$Mmax=(W*L)/(Fp*K**(4/3))$$

Where Fp=4.00. Ratio of maximum moment calculated from formula proposed by Smith and Riddington (27) and that calculated from the analysis is 1.5. Therefore, maximum moment may be calculated from the above formula with the modified value of Fp. The new proposed value of Fp may, therefore be considered as 6.0.

5.3.5. Tensile Force of the Beam.

General:

As mentioned earlier a wall on beam structure subjected to vertical loadings acts compositely in a way similar to a tied arch. The wall arches across the span and the beam serves as a tie to prevent the arch from spreading. The outward thrust of the wall is contained by the tying action of the beam, which is subjected, therefore, to additional axial tensile forces. In the following sections the influence of different parameters on tensile stresses are critically examined.

Maximum Tensile Force in the Supporting Beam:

For the determination of maximum tensile force, different beam sizes with different values of K and Rf are examined. The details of the investigations are shown in

figure 5.24. From figure it is seen that the tensile force in the beam is zero at the support and maximum at the centre of the beam. The force increases with the decrease of H/L ratio. Here the figure shows close resemblance with the distribution of tensile force as determined by lattice analogy of Yettram & Hirst (33).

Figure 5.24 also shows that the maximum tensile force occurs at or near the midspan of the beam irrespective of the H/L ratio, value of K or value of Rf. It is also revealed that upto a certain distance of about L/8 from the support, the magnitude is directly proportional to the distance from either support, then the curve flatens according to the value of H/L or Rf. The more is the value of Rf, the flatter is the curve and when the value of Rf exceeds 6.669 (=7) the distribution is almost horizontal i.e. for Rf = 7.0 the magnitude of tensile force remains unchange with the distance after a certain distance. The figure also indicates that the distribution of the tensile force along the length is not triangular as proposed by Ahmed and Davis (13,14) rather parabolic, the degree of parabola being dependent on the value of H/L.

The average value of maximum tensile force is 22.062 kK. Total applied load on the structure is 80.00 kN. Ratio
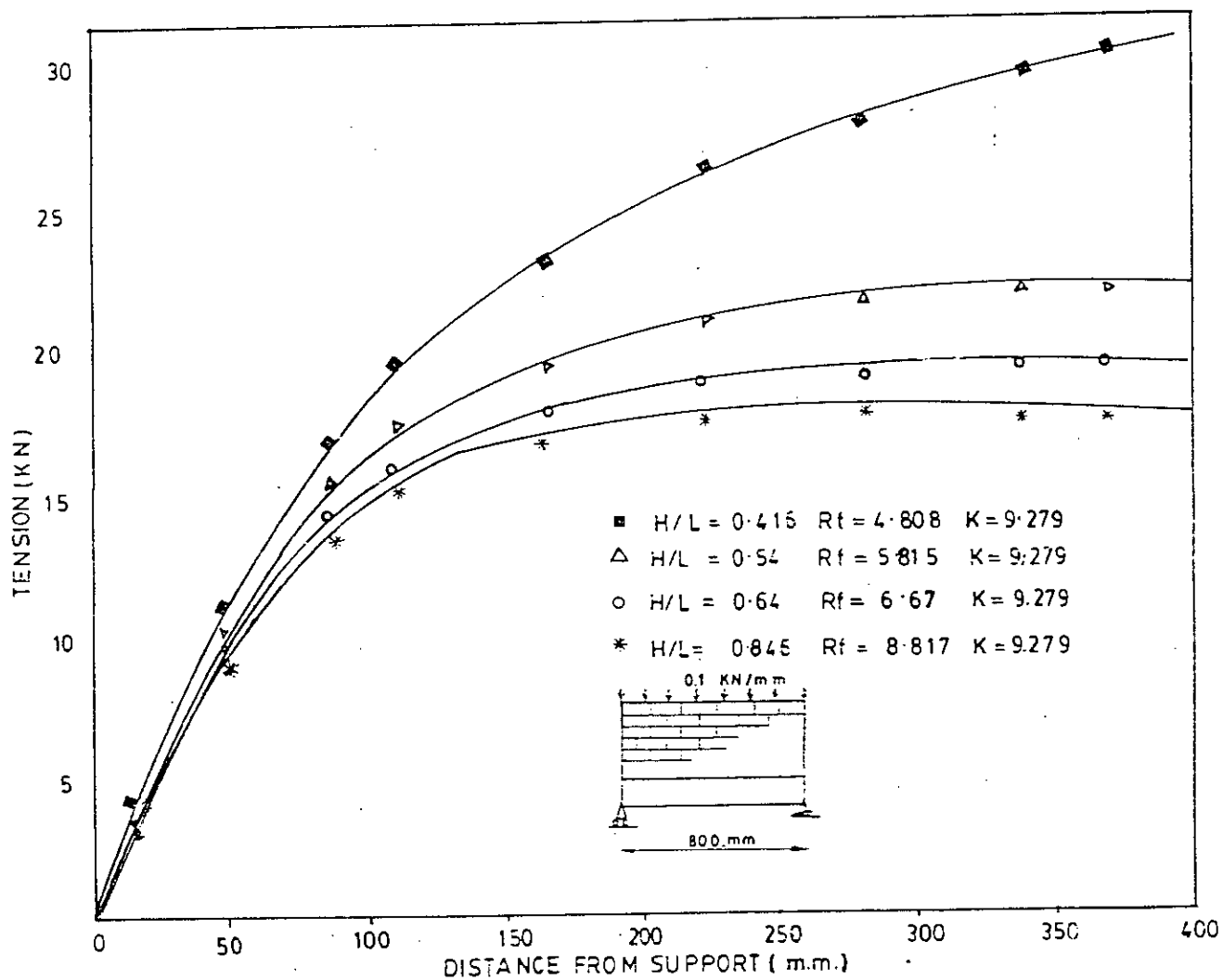
FIG. 5·24   TENSILE FORCE ALONG THE LENGTH OF THE BEAM FOR DIFFERENT H/L RATIO.

of the apllied load (W) and the maximum tensile force developed is approximately 3.70. In mathematical expresssion :

T=W/3.70.

Therefore, maximum tensile force may, therefore, be calculated from the above relationship.

CHAPTER-6

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY.

6.1 GENERAL

A linear elastic finite element model has been developed which can be used to analyze wall-beam structure subjected to inplane load (both distributed and concentrated). In this model concrete beam, interface element and brickwork have been modeled separately. The brickwork has been modeled separately giving due consideration to the bricks, mortars and their locations. In this study materials are assigned with their own properties. Provisions for both four and eight noded elements are kept in the program. A 2x2 integration scheme is adopted to evaluate element stiffness and load voctor. Automated nodal coordinates as well as element connectivity data generations are implemented in the program.

A parametric study of the behaviour of wall-beam structures has been carried out in chapter 6. From the results of this paramatric study the influence of different parameters on the composite behaviour of wall-beam structure has been studied. The results of the parametric study has been compared with the results of previous investigations and in most cases found to be conservative or nonconservative .

Although the model is efficient for the analysis of composite action of wall-beam structures, it has some limitations. The material model has not incorporated time dependent behaviour and the possibility of cyclic loading.The results are applicable to wall-beam structure with solid brick masonry subjected to inplane loadings which extends over the complete thickness of the wall. The model therefore can not predict the three dimensional effects when the load on the wall does not occupy the whole thickness.

Despite the limitations, the proposed finite element model appears to be more versatile, efficient and also more representative of the actual behaviour of the wall-beam structures. The present model can consider the brickwork both as homogeneous or nonhomogeneous material which is in contrast to the previous models which have considered brick masonry in the wall-beam structure as a continuum with average properties. The model can be used to prepare design recommendations for any inplane problems and can be used as a substitute for many experimental investigations. Since the material characteristics required for this finite element model can be determined from relatively simple tests, it can be readily adopted to any wall-beam structure built in any pattern.

From the present linear finite element study the

following conclusion can be drawn:

1) Finite element model of this type which treats the materials of the structure separately is more effective, since it reflects the influence of varying stiffness of its constituents.

2) The distribution of shear stress along the interface of the concrete beam and masonry is parabolic for a value of relative stiffness parameter (Rf) less than 7.00 but for a value of 7.00 or more the distribution of shear stress is linear.

3) Spread of the shear stress along the length of the beam is twice that of the vertical stress.

4) Vertical stress (compressive) is maximum over the supports and decreases gradually towards the centre of the beam to a value of average stress.

5) Vertical stress concentration factor varies linearly with the stiffness parameter (Rf). With the increase of this parameter the stress concentration factor increases.

6) Maximum moment occurs at a distance of about 1/15th of the span from either of the supports for a

simply supported beam.

7)   The magnitude of the maximum moment depends on the relative stiffness parameter (K). The lesser the stiffness of the beam, the more it behaves like a tie member.

8)   The maximum bending moment in the beam increases with the increase of the size of the supporting beam.

9)   Tensile force in a beam may be calculated from the formula:

T = W/3.7 in stead of W/3.4 as proposed by Smith and Riddington(27).

10)   Maximum moment may be calculated from

M=(W*L)/((Fp)*(K**(1/3)))

where Fp=6.0

6.2. RECOMMENDATION FOR FURTHER STUDY

The development of a linear elastic 2-D finite element program for the analysis of the behaviour of wall-beam structures could find considerable interest in various allied fields. To increase the applicability of the program developed for the present study, certain modifications can be done. The following recommendations can be made for further development of the present study:

1) The finite element computer program developed for the study can also be applied to the analysis of various other structural interaction problem with little or no modifications.

2) To increase the efficiency of the program the half band form of storing the stiffness matrix can be replaced by a variable band storage scheme which will provide considerable saving of core storage requirement during execution.

3) In this study the constituent materials are considered to be linearly elastic. But for a better understanding of the actual situation, material non-linearity and the progressive fracture of the materials should be incorporated.

4)   In ·this study the RCC beam is assigned with an
average property. But for a better representation of
stresses in different layers and moment of the concrete
beam, the reinforcement and the mass concrete may be
considered separately.

5)   The model may also be modified for the time
dependent deformation due to the sustained load and for
the cyclic loading.

6)   This model may be developed in the analysis and
design of reinforced masonry.

APPENDIX 1

TABLE-1

ERROR CODE FOR CHECK ON THE INPUT DATA

ERROR CODE   EXPLANATION

1   Total number of structure nodes less than
        or equal to zero

2.   The   possible maximum total number of nodal
        points   in   the   structure is less than   the
        specified number of structure nodes.

3.   The numbers of restrained nodes are less than
        two or greater than the number of structure
        nodes.

4.   The total number of load increament is less
        than 1

5.   The total number of nodes per element is less
        than 4 or greater than 8

6.   The numbers of degrees of freedom per node is
        less than two.

7.   The numbers of different materials is less
        than 1 or greater than the number of elements

8. The number of Gauss intregration points in each direction is less than 2 or greater than eight.

9. Two nodes have identical coordinates.

10. The material number of the element is less than 1 or greater than the number of different materials.

11. Nodal number of the element is zero.

12. Nodal number of the element is less than 1 or greater than the total number of nodal points.

13. Repetition of a node number with an element

14 & 15. Coordinates of the unused nodes have not been specified.

16. Unused node number is restrained node.

17. Required front width is greater than the front width available in the program.

18. Restrained node number is less than or equal to zero or greater than the total number of nodal points.

19. Restrained code is missing for restrained nodes.

20. Two identical restrained nodes.

TABLE-2

VARIOUS OPTION OF OUTPUT

OUTPUT CODE   INTERPRETATION

0.   No output neccesary

1.   Print only the displacement of nodes (both X
      & Y dir)

2.   Print the displacement of the nodes and
      reactions at support

3.   Print the displacement of nodes and reactions
      at   the   supports   and   stresses   at
      each sampling point of the element.

TABLE 3

BRIEF DESCRIPTION THE MAJOR SUBROUTINES

SUBROUTINE   FUNTION OF THE SUBROUTINE

PROBTP   Accept the data related with problem type

             NTYPE = 0 Represents Elastic solution without
             fracture

             NTYPE = 1 represents elastic fracture

             NTYPE = 2 represents non linear (for mortar
             only) fracture

             NTYPE = 3 represents nonlinear (brick and
             mortar) fracture

             NRELS = 0 represents brittle model

             NRELS = 1 represents inelastic collapse model

CENTRE   Finds the coordinate of the centre of the
             element

DIMEN   Assigns maximum values for the dimensions of
             arrays.

INPUT   Accepts most of the data for the problem.

DIFMAT, COMP, MATIDN Reassign the respective material
            number for the elements.

CHECK1   Checks the main control data

CHECK2   Checks the remainder of the input data(mesh
            generation)

ECHO   Echoes the error code and terminate the
            execution process.

LOADPS   Accepts the data on loading

DMATPS   Evaluates [D] matrix (elastic) for plane
            stress or plain strain elements.

INCREM   Increments the applied loading.

ZERO   Initiates the various arrays to zero.

PRINC   Evaluates the principal stress.

ALGOR   Sets the equation resolution index.

STIFP   Calculates the stiffness matrix.

DIFFEL   Stores the stiffness matrix of the representative elements in a 3-D array for further use.

DECSON   Sets the type of constitutive relation to be used for the materials.

STIFFP   Determines the status of the elements for stiffness calculation.

GAUSSQ   Sets the Gaussian Quadrature Rule.

BMATPS   Evaluates the [B] Matrix for the Plane Stress and Plane Strin Problem.

DMATPS   Evaluates the [D] Matrix for the Plane Stress and Plane Strain Problem.

JACOB2   Calculates the Jacobian Matrix.

SFR2   Determines the Shape Functions.

CONVER   Checks for the Convergence.

DBE　　　　Carries out the Multiplication of the [D] and [B] Matrices.

FRONT　　　Undertakes equation solution by frontal method.

RESIDU　　Evaluates the stresses according to the constitutive relations prevailing in the element.

REDJON　　Evaluate the stresses for the joint element on the basis of current deformation.

REDUCT　　Evaluate the stresses for the brick element on the basis of current deformation.

WRITEL　　Writes the failure code in addition to that it writes the stresses and principal stresses.

LINEAR　　Evaluate the stresses and strain from stress – strain relation and strain – displacement relationship.

OUTPUT　　Outputs the results.

STROUT　　Gives the output for the elements at particular level of the structure.

TABLE-4

SUB-ROUTINE	DATA	FORMAT
PROBTP	IBND,NTYPE,NRELS,NEQ,NENAL, ALPHA, REFAC,TOLNW.	5I5,3F10.4
	AC(I)	6F10.4
	XN	F10.4
INPUT	NPOIN,NPON,NPLEM,NVFIX,NNODE, NMATS,NGAUS,NALGO,NCRIT,NINCS, NX,NY,NIFLM,IPROB	16I5
	NPLTM,NXX,NYY,NFNOD,ISHER, NELCV,NPFIX	16I5
	NLYRC,NLYRB,NLYRI,NLYRM	16I5
	NVSEC,NHSEC	16I5
	LYEVL(I)	16I5
	LXEVL(I)	16I5
	XCRD	7F10.3
	YCRD	7F10.3
DIFMAT	IHOM	I5

COMP	INTR,ICOMP	2I5
DIFMAT	N1,JD(1)	15I5
	ILXO,NXD(I)	15I5
	ILYO,NYL(J)	15I5
	ILYE	15I5
	NYL(J)	15I5
	ILXE,NXD(I)	15I5
INPUT	NOFIX,IFPRE,PRESC(IVFIX,	I5,I10,
	IDOFN)	5F10.4
	NUMAT	I5
	PROPS(NUMAT,IPROP)	5F10.5
LOADPS	IPLOD,IGRAV,IEDGE	3I10
	LODPT,POINT(IDOFN)	I5,2F10.4
INCREM	FACTO,TOLFR,TOLDS,RESMX,	4F10.5,
	MITER,MITFR	2I5
	NOUTP(2),NOUTP(3)	2I5

REFERENCES

(1)   Annamalan,G.,Jayaraman,R.,Madhava Rao,"Experimental Investigation on the Composite Behaviour of Reinforced Brickwork thin Lintels",Indian Concrete Journal, June,1984, pp.154-61.

(2)   Ali,S.,and Page,A.W.,"A Failure Criterion for mortar joints in Brickwork Subjected to Combined Shear and Tension", Masornry International,No. 9, Dec 1986, pp.43-54.

(3)   Ali,S. ,"Concentrated Loads on Solid Masonry",Ph.D. Thesis, University of Newcastle,Feb 1987.

(4)   Ali,S.,"Concentrated Loads on Solid masonry", Ph.D. Thesis, University of Newcastle, Feb 1987, pp. 81-130.

(5)   Burhouse,P.,"Composite Action between Bricks Panel Wall and their Supporting Beams",Building Research Station, CP2/70

(6)   Buragohan,D.N.,and Shah,V.L.,"3-D Interactive Finite Element Analysis of Foundation Structures",International Conf. on Computer Application in Civil Engg, University of Roorkee, Oct. 23-25 ,1979.

(7)   Colbourne,J,R.,"Studies in Composite Construction-An Elastic Analysis of Wall-Beam Structures", CP15/69, Building Research Station,1962.

(8)   Coull,A., "Composite Action of Walls Supporting on Beams" , Bldg., Science , 1 (1966), pp. 259

(9) Cook, R.D., "Concept and Application of Finite Element Analysis" , 2nd Edn., John Wiley and Sons Inc, 1981.

(10) Davies ,S.R., "Composite Action ", University of Edinburg, Proc.of 2nd International Symp. on Masonry Structure, Kualampur, Malaysia.

(11) Davies, S.R., and Ahmed,A.E., "A Graphical Solution of Composite Wall-Beams", University of Edinburg.

(12) Davies,S.R., and Ahmed, A.E., "An Approximate Method for Analyzing Composite Wall-Beams", Proc. of Brick Ceram , Society , 27 1978, pp. 305-320.

(13) Ahmed, A.E.,"A Study of the Composite Action Between Masonry Panels and supporting Beams", Ph.D. thesis, University of Edinburg, 1977.

(14) Davies , S.R., and Ahmed, A.E., "Composite Action of Wall- Beams with Openings", Proc. of the Fourth Int. Brick Masonry Conf., Brugge, 1976, Paper 4.b.6.

(15)   Desai,   C.S. and Abel, J.F., "Introduction to the Finite Element Method - A Numerical Method for Engineering Analysis", International Journal of Numerical Method in Engineering, Vol-1, 1972.

(16) S. Moti, "The Engineering Companion ", John Wiley and Sons , Inc.,1966 p.60.

(17) Green, D.R., "The Interaction of the Solid Shear Walls and their Supporting Structures", Bldg. Sci., 7, 1972, pp. 239-248.

(18) Goodier, Timoshenko, "Theory of Elasticity"

(19) Goodman, R.E., Taylor, R.L. and Brekke, T.L., "A Method for the Mechanics of Jointed Rocks", Journal of the Soil Mechanics and Found. Dvn. , A S C E , vol. 94, No.SM3.

(20) Hinton,E. and Owen,D.R.J., "An Introduction to the Finite Element Computations", Prineridge Press Ltd.

(21)   Hinton, E. and Owen,D.R.J.,   "Finite   Element Programming", Academic Press.

(22) Hendry, A.W., " Structural Brickwork", John Wiley and Sons Inc., Newyork, 1981. pp.176-180.

(23) Irons, B.M., "Engineering Application of Numerical Intregation in Stiffness Method", AIAAJ, vol.-4, No.11, 1966 pp. 2035-2037.

(24) Irons, B.M. and Ahmed, S., "Techniques of Finite Elements", John Wiley and Sons Inc., 1980.

(25) Rosenhaupt,S. ,"Experimental Study of Masonry Walls on Beams",Proc. of the American Society of Civil Engg., June,1962, pp.137-66.

(26) Rosenhaupt, S.and Sokal,Y. ,"Masonry Walls on Continuous Beams",Journal of the Structural Dvn. Am. Soc.of Civil Engrs., 91, 1965, pp.175- 194.

(27) Stafford, S.B., and Riddington, J.R., "The Composite Behaviour of Elastic Wall-Beam System", Proc. of Instn.of Civil Engineers, Part-2, 1976 June, pp. 377-391.

(28) Stafford, S.B., Khan, M.A.H. and Wickens, H.G., " Tests on Wall-Beams Structures", Proc. of the Br. Ceram. Soc., 27, 1978, pp. 289-303.

(29) Smith, B.S. "The Composite Behaviour of Masonry wall on Steel Beam Structures", Proc. of Canadian Masonry Symposium, University of Calgary, Calgary, Alberta, 1976. pp.292-303.

(30) Woods, R.E. and Simms, L.G., "A Tentative Design Method for the Composite Action of Heavily Loaded Brickpanel Walls Supported on Reinforced Concrete Beams", Current Papers CP26/29, Bldg.,Research Station, Watford, Hertz, London, 1969.

(31) Woods, R.H., "Studies in Composite Action of Brickpanel Walls Supported on Reinforced Concrete Beams.", National Building Studies Research Paper 13, 1952.

(32) Yisun, g. et. al. ," Application of Finite Element Method to the Design of Wall-Beams", Proceedings of the 7th I C B M , 19-20 Feb., 1985, Austrailia. pp.501-508.

(33) Yettram, A.,and Hirst,M., "An Elastic Analysis for the Composite Action of Walls Supported on Simple Beams", Bldg. Sci. Vol.6, 1976, pp.151-59.

(34) Zienkiewicz, O.C., Valliappan,S. and King, I.P., "Elasto-plastic Solution of Engineering Problems 'Initial Stress' Finite Element Approach", International Journal of Numerical Method in Engineering, Vol.1.

(35) Zienkiewicz, O.C., " The Finite Element Methods", McGraw-Hills Inc. 3rd Edition.