A DESIGN RATIONALE FOR STAIR SLABS BASED ON FINITE ELEMENT ANALYSIS

A THESIS

BY

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DECLARATION

I do hereby declare that the work embodied in this thesis is the result of investigation carried out by me and this has not been submitted nor is being concurrently submitted in candidature for any degree at any other university.

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CONTENTS

ABSTRACT

The results of a study of stair slabs by using finite element technique have been presented in this thesis. The behavior of both Dog-legged and Open-well type of stair have been analyzed. Both thick and thin shell finite elements have been used for this purpose and the results of the analyses by both of these elements compare well. Sensitivity study of the critical parameters of the stair slab has also been carried out to make the findings more generalized.

The stair slabs, usually being supported on walls or beams at landing levels, derive significant rigidity from such supports. This reduces the magnitude of the moments that would have otherwise resulted.

The leading codes of practice do not provide proper appreciation to this distinctive feature of stair slabs arising out of its supporting arrangements. The design moments for the stair slabs, under study, have been found to be considerably smaller than those commonly suggested by the Codes of Practice.

Based on the findings of the present study, a new design rationale has been suggested, for both Dog-legged and Open-well type of stairs.

CHAPTER 1
INTRODUCTION

1.1 General Introduction

Stairs, though an essential element in every building, have hardly received adequate attention from the designers. Lack of well defined procedures for both analysis and design of stairs have resulted in over-designed stairs in almost every case. Considerable differences prevail among the practicing engineers in their opinion regarding the design of stair slabs.

The general tendency of designing stair slab as a simple slab, without recognizing the distinctive features of a stair slab and a simple slab, is clearly wasteful. As a stair case of typical dimension covers some 10 percent area of a common residential building and since the structural cost involvement is about 50 percent higher in stair case than the average floor area cost, it is clear that any improvement in the design of stair would certainly contribute to the overall economy of construction of buildings.

Recent research on stair slab, as reported by Saquib et al[1] is indicative of the possibility of rationalizing the design characteristics of stair slabs. Later investigations concerning the actual behavior of the stair slab by Ahmad[2] and Zahedi[3] upheld the fact that the stair slabs do not respond in the same manner as do the simple slabs. Their findings are undoubtedly fascinating so far structural suitability as well as the economy is concerned. Their findings and suggestions are, however, limited by the scope of their research, largely because of the capacity of the computer available at that time. Now with the availability of larger computer and suitable finite element packages, a comprehensive study into the behavior of stair slab is warranted.

1.2 Background of the Research

The inherent folded shape of stair slab prevents itself from behaving as a simple slab under transverse loading. The landing slab, commonly being supported by walls on three sides, would have stiffening effect on the stair slab, providing an efficient way of transferring load than it would otherwise be in the case of a stair slab supported at the end walls only. Moreover, a stair slab of cranked shape and usual support condition is subjected to both membrane and flexural stresses[2,3]. The leading codes of practice do not seem to provide guidelines to ascertain such effects .

Regarding the design of stairs the American Concrete Institute (ACI) Code do not provide any appreciation to the restraining effect on stair slab owing to its shape and support conditions. As a result the followers of this code usually design a stair slab as a simple one way slab with span equal to the horizontal distance between the supports[4]. The British Code[5], on the other hand, provides some reduction in the effective span of a stair slab. Reduction in the effective span as suggested by the British Code, obviously results some saving in the design.

A limited experimental study by Saquib et-al[1] on a full-scale single flight stair suggested that the stair slabs do not respond in the same manner as do the simple slabs. The study further revealed that a typical stair slab of same thickness designed by British Code[5] requires nearly half the reinforcement required by the conventional American design practice[4]. A single flight stair designed by British Code was constructed and tested. It was found that the stair sustained 133 percent of the load corresponding to the ultimate moment capacity without any sign of failure. This fact indicated that even though the British Code of Practice regarding design of stairs is liberal than American design practice, there was scope for further savings.

The above findings have encouraged later researchers like Zahedi[3] and Ahmad[2] to pursue further investigations on stair slabs. Ahmad carried out both experimental and numerical studies in an attempt to establish the actual behavior of stair slab. He came out with conclusive evidence regarding the restraining effect of the landing slabs on the waist slabs. Zahedi, studied the problem using thick shell finite element program. He reported that axial force of appreciable magnitude exists near the junction (Kink zone) of landing and waist slab. He suggested to provide reinforcement to tackle this axial force.

The studies carried out by Zahedi and Ahmad were of limited nature because of the storage limitation of the computer available at that time. Due to this limitation they analyzed a single flight of a two flight stair assuming simplifying boundary conditions . But the anti-symmetrical geometry of the stair near landing presents a rather complicated behavior near the kink than the usual simple condition of symmetry. The situation warrants a more rigorous analysis of the stair slab .

In the present study, attempts have been made to investigate the behavior of stair with two and three flights to eliminate or minimize the boundary effects on the stair slab . This would generate more appropriate response of the slab under loading. It is expected that this study would be able to suggest definite guidelines for ascertaining the restraining effect of landing slab and would provide guideline for a specific design procedure for stair slabs.

1.3 Scope and Methodology of the Research

In an attempt to investigate the behavior of stair slab and to come out with a recommendation for a rational design procedure, a survey of related literatures, codes of practices have been made. A brief comparison of the conventional design approaches is presented in order to delineate the superfluity of those design approaches; thus enabling a relative comparison with the findings of the present study.

Comprehensive analyses of a dog-legged and an open-well type stairs have been made using both thick shell[6,7] and thin shell[8] finite elements. In analyzing the dog-legged type stair first the analysis was made on a two flight stair by applying symmetry conditions at either floor level. Later a three flight analysis with boundary conditions applied at the extreme landings of first and third flight was carried out. The second flight of a three flight analysis was free from undue boundary restraints and stresses of this flight was considered in interpreting the behavior of stair slab. Results of both thick and thin shell analyses have been found to be in close agreement. The open-well type stair was also analyzed using both thick shell and thin shell elements for a wide range of well dimensions.

The sensitivity analyses for the parameters of stair slab have been carried out and the results of this parametric study are helpful to envisage the structural response of the stair slab and to provide indication for a future study in this regard for developing design charts.

On the basis of the findings of the present study suggestions have been made regarding the design of stair slabs.

The thesis is organized in the following order:

1. A review of the leading codes of practice for the design of stair slabs is presented in Chapter 2.

2. Chapter 3 summarizes the results obtained, together with the methodology followed for analysis.

3. The findings and results are discussed, to enable interpretation of the over-all behavior of stair slab, in Chapter 4.

4. Proposals for a rational design of stair slabs are formulated on the basis of conclusions of the present study in Chapter 5.

5. Chapter 6 summarizes the conclusions of the present study.

1.4 Objective of the Research

The objectives of the research were as follows:

1. To investigate the actual behavior of stair slabs by Finite Element Method using different shell element programs.

2. To investigate the effects of various parameters on the behavior of the stair slabs.

3. To develop guidelines for a rational design procedure for stair slabs.

CHAPTER 2
LITERATURE REVIEW

2.1 General

The successful functioning of a multistory building requires easy and safe circulation of traffic in normal situation as well as in case of emergencies. In the design of buildings due care should be given to the type of vertical circulation. The means of communication between various floors is offered by various structures such as stairs, lifts, ramps, ladders etc. The differences in elevation of two areas in a building is commonly overcome by providing stairs.

A stair may be defined as a series of suitably arranged steps for the purpose of connecting different floors of a building. It may also be defined as an assemblage of treads, risers, stringers, newel posts, hand rails and balusters so designed and constructed as to provide an easy and quick access to the different floors rendering comfort and safety to the users. The enclosure containing the complete stair way is termed as stair case. A glossary of the technical terms used for stair case and a brief description of the different type of stairs are given in Appendix-I.

In the following articles a review of the leading codes of practice is made.

2.2 A Review of the Codes and Practices

In the absence of definite guidelines for designing stair slabs, engineers may come with considerably different designs for a particular problem. This will be delineated here by presenting case studies in which a stair of typical dimension is designed in accordance with different codes of practice.

2.2.1 American Design Practice

ACI code provides virtually no guideline for designing stair slabs supported on landings [4]. The followers of ACI building code, however, design stair slabs as a simple one way slab. They, through their continual use, have pushed this convention so far that such a practice has nearly assumed to be institutionalized as a quasi-ACI method of designing stair slab .

To design a stair slab supported on beams at top and bottom kink the span will be taken equal to the center to center horizontal distance between the supports. This method of design requires steel to be placed only in the direction of the slab. Transverse steel, usually one bar to each tread, is used only to assist in distribution of the load and to provide temperature reinforcement[4].

To design a flight starting and ending at landing the simple span would be the horizontal distance between supports. In this method of design the inclination of the stair slab is disregarded.

2.2.2 British Code of Practice

The British code[5] provisions for the design of stair slabs are more specific about the effective span and the way load is to be distributed. The code requirements regarding various design aspects of stair are :

a. Effective span of staircases:

"When a staircase without stringer beams is built monolithically at its end into structural members spanning at right angles to the span of the staircase, the effective span should be taken as the sum of the clear horizontal distance between the supporting members and half the breadths of the supporting members subject to a maximum addition of 900 mm at both ends.

When a stair case without stringer beams is simply supported the effective span should be taken as the horizontal distance between the centre lines of the supports.

For the purpose of this sub-clause a staircase may be taken to include a section of landing, spanning in the same direction and continuous with the stair flight".

b. Distribution of loading

"In general the ultimate load should be assumed to be uniformly distributed over of a stair case. When however, staircases surrounding open wells include two spans which interest at right angles, the load on the area common to both spans may be assumed to be divided equally between the two spans". (Fig.2.1(a)).

"When staircases or landings, which span in the direction of the flight, are built at least 100 mm into walls along part or all of their length, a 150 mm strip adjacent to the wall may be deducted from the loaded area". (Fig.2.1(b)).

c. Effective breadth of staircase:

"The effective breadth of a staircase without stringer beams should normally be taken as the actual breadth of the staircase. When a staircase is built into a wall along part or all of its span, two-thirds of the embedded breadth upto a maximum of 80 mm should be included in the effective breadth". (Fig.2.1(b)).

2.2.3 Indian Design Practice

For the purpose of designing stair slabs the followers[9,11,13,14] of Indian code divide stair slabs into two categories:

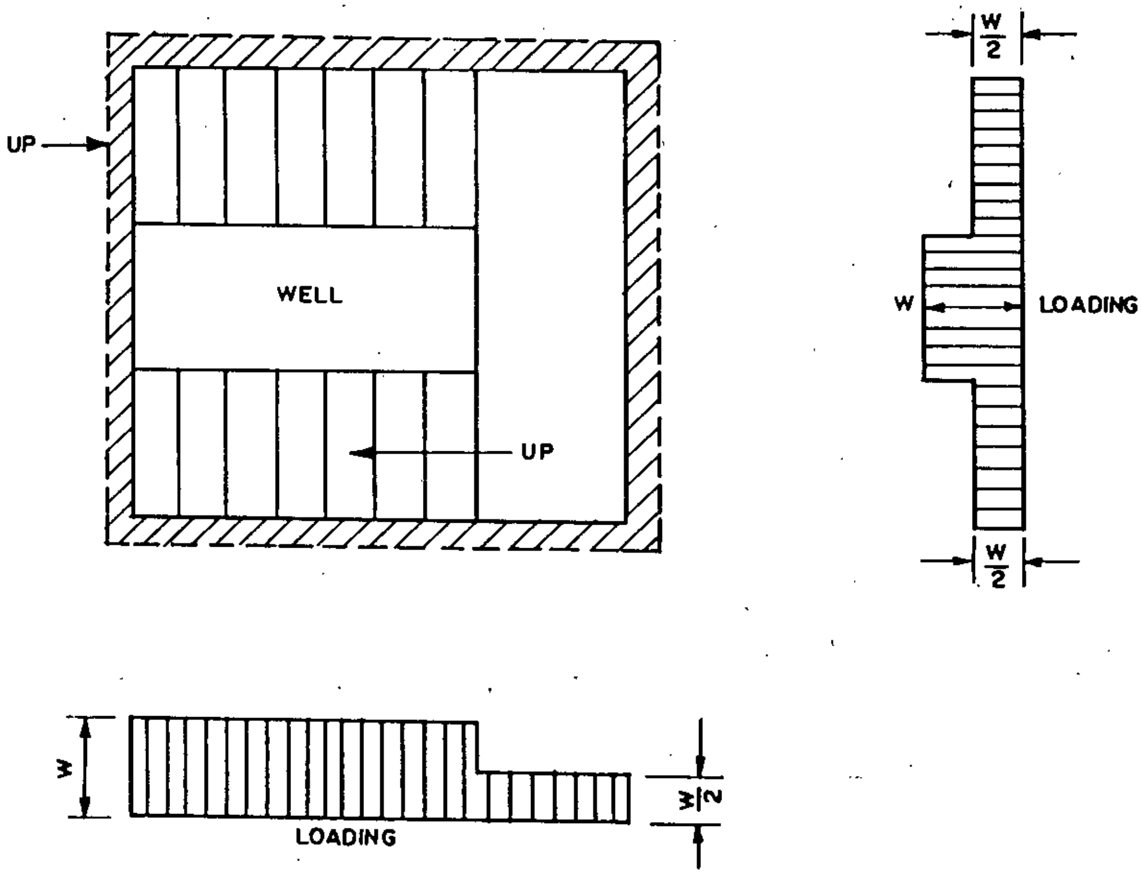


Fig. 2.1(a) Loading on an open-well stair

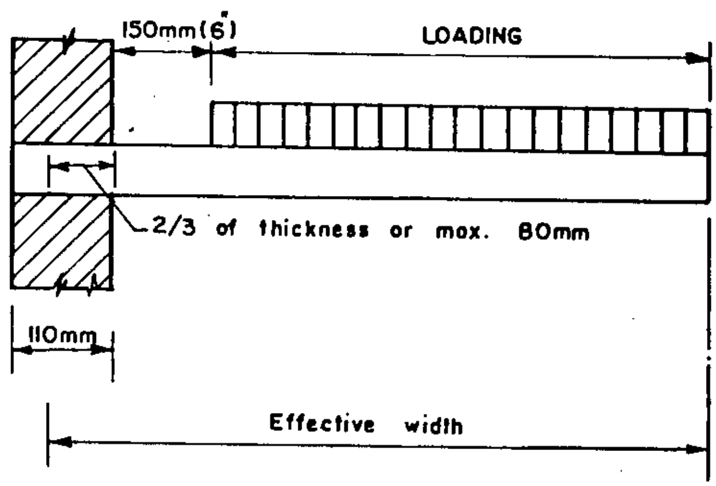


Fig. 2.1(b) Loading on stairs built into wall.

i) Stair slab spanning horizontally

ii) Stair slab spanning longitudinally.

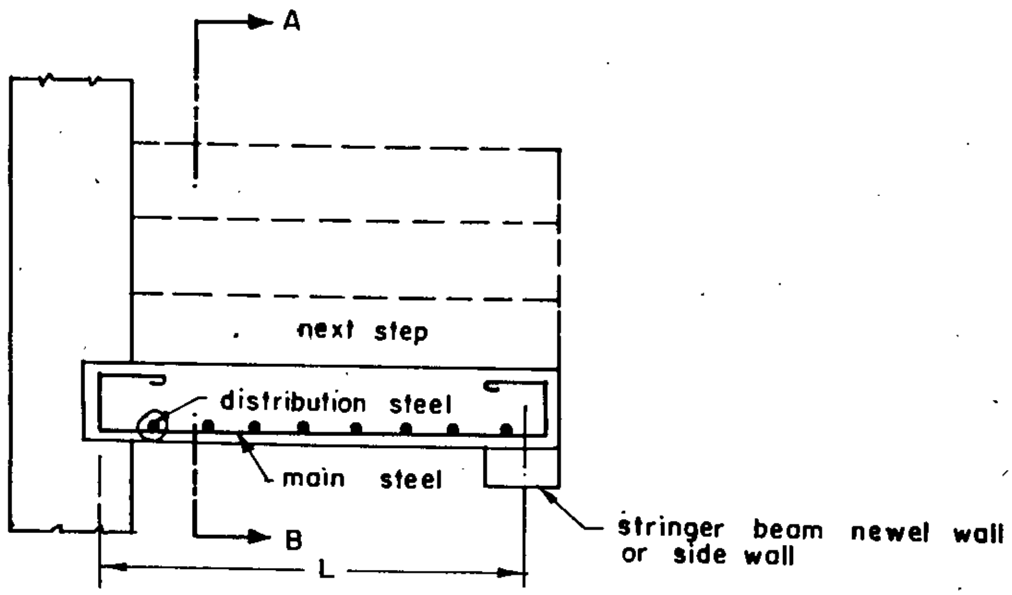
These cases are discussed here:

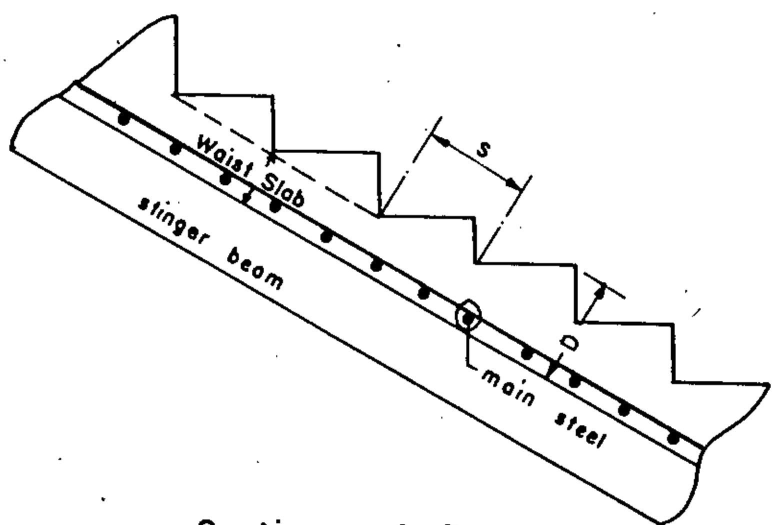
i) Stair slab spanning horizontally

In this category, the slab is supported on each side by side wall or stringer beam on one side and beam on the other side. Sometimes, as in the case of straight stair, the slab may also be supported on both the sides by the two side walls. The slab may also be supported horizontally by side wall on one side of each flight and the common newel on the other side between the backward and forward flights. In such a case the effective span (L) is the horizontal distance between center to center of supports. Each step is designed as spanning horizontally with a bending moment equal to $qL^2/8$,where "q" is the total load per unit area. Each step is considered equivalent to a rectangular beam of width, s (measured parallel to the slope of the stair) and an effective depth equal to D/2, as shown in Fig.2.2

ii) Stairs spanning longitudinally

In this type, the waist slab is supported at bottom and top of the flight and unsupported at the sides. The flight of the stair is continuous having beams both at top and bottom. The waist slab may be designed with a horizontal span equal to the center to center distance of the supporting beams. The waist slab is designed for a





Section A-B

Fig. 2.2 Stair slab spanning horizontally bending moment of $qL^2/10$; where q is the total load per unit area. Where the flights or landing are built into wall, the landing slab is assumed to span in the same direction as the stairs. They should be considered as acting together to form a single slab. The effective span for each flight shall be taken as clear distance covered by the flight including the whole of landing plus 8 cm at either end for end bearing. The formula for bending moment is given by $qL^2/8$.

Sometimes the landing slab may span at right angles to the direction of stairs. In such a case the landing slab may be supposed to be acting as a beam supporting the flights. The effective span in such a case should be taken as a distance equal to the going of the stairs plus at each end either half the width of the landing or one meter whichever is smaller (similar to the British code of practice). Regarding the distribution of loading on stairs with open wells Indian code of practice follows the British code.

2.3 Loading on Stairs

In this article the specification of loading for a stair slab as recommended by various codes of practice are discussed .

2.3.1 Live load

i) The American Concrete Institution specify a live load of 100 psf[4] for stairs.

ii) According to British Code [5,18] the characteristic load imposed on stairs should be 5 kN/sq.m (104.4 psf).

iii) The Indian Standard Institution (I.S.I)[10] specify live load of stairs as shown in Table 2.1

TABLE 2.1 Live Load on Stairs

Types of stairs and landings	Minimum live load	Alternative minimum live load
i) Stairs in dwelling houses, tenements, hospital wards, bed rooms and private sitting rooms in hostels and dormitories, but not liable to overcrowding.	300 kg/sq.m (61.5 psf)	Subject to minimum of 130 kg concentrated load at the unsupported end of each step for stair constructed out of structurally independent cantilever step.
ii) Stairs and landings mentioned in (i) but liable to over crowding and for all other classes.	500 kg/sq.m (102 psf)	——

2.3.2 Dead Load

The dead weight of stair consists of:

i) dead weight of the waist slab and
ii) dead weight of steps.

For the purpose of calculating design loads, the dead weight of inclined waist slab is magnified over a horizontal projection. The dead weight of the steps is calculated by taking the steps to be an equivalent horizontal slab of thickness equal to half the rise.

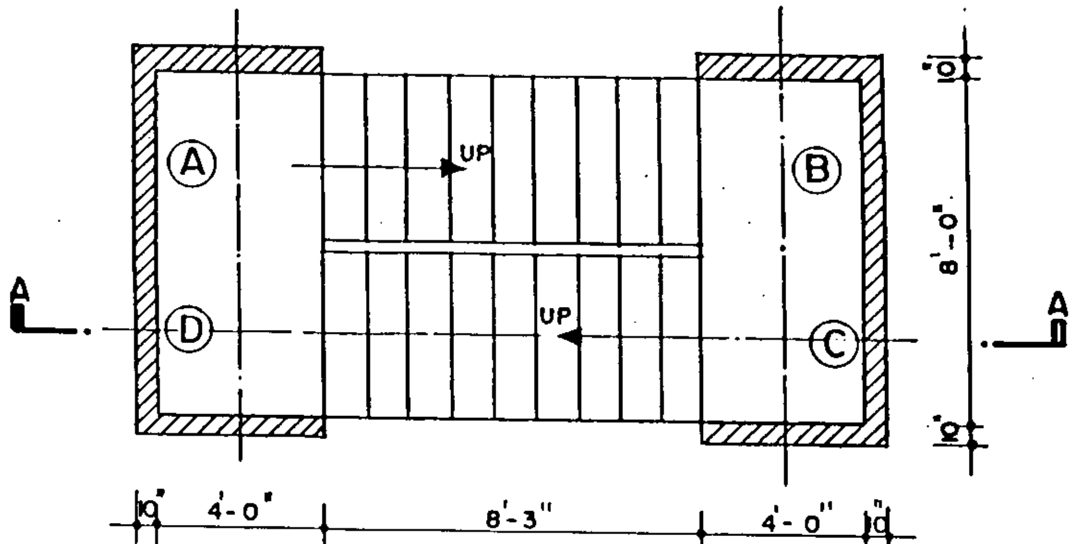
2.4 Case Studies: Examples of Conventional Design Approaches

In this article design examples based on the leading codes of practice are furnished. A Dog-legged and an Open-well type of stair of dimension shown in Fig. 2.3 and Fig. 2.4, are designed. Later a comparison of the designs by conventional and the proposed method will be presented (Art. 5.5).

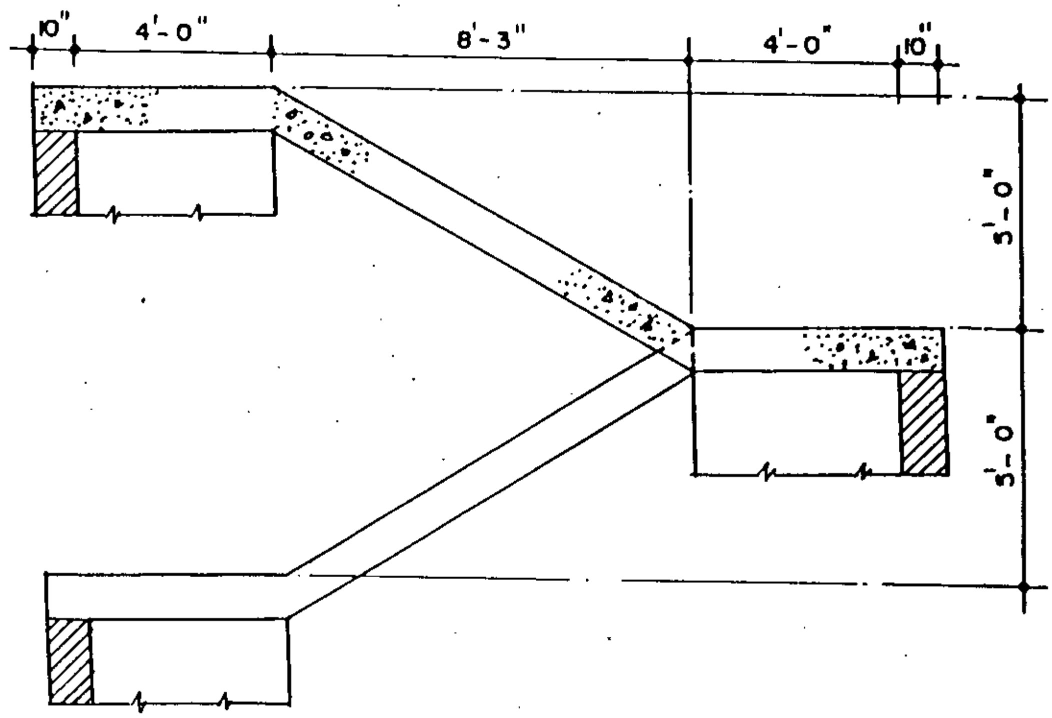
2.4.1 Design of Dog-legged Stair

a) Design by Conventional American Approach [4]

Span = c/c of support
= 17.08 ft.



PLAN



SECTION A - A

Fig. 2.3 Typical dimension of Dog - legged stair under study.

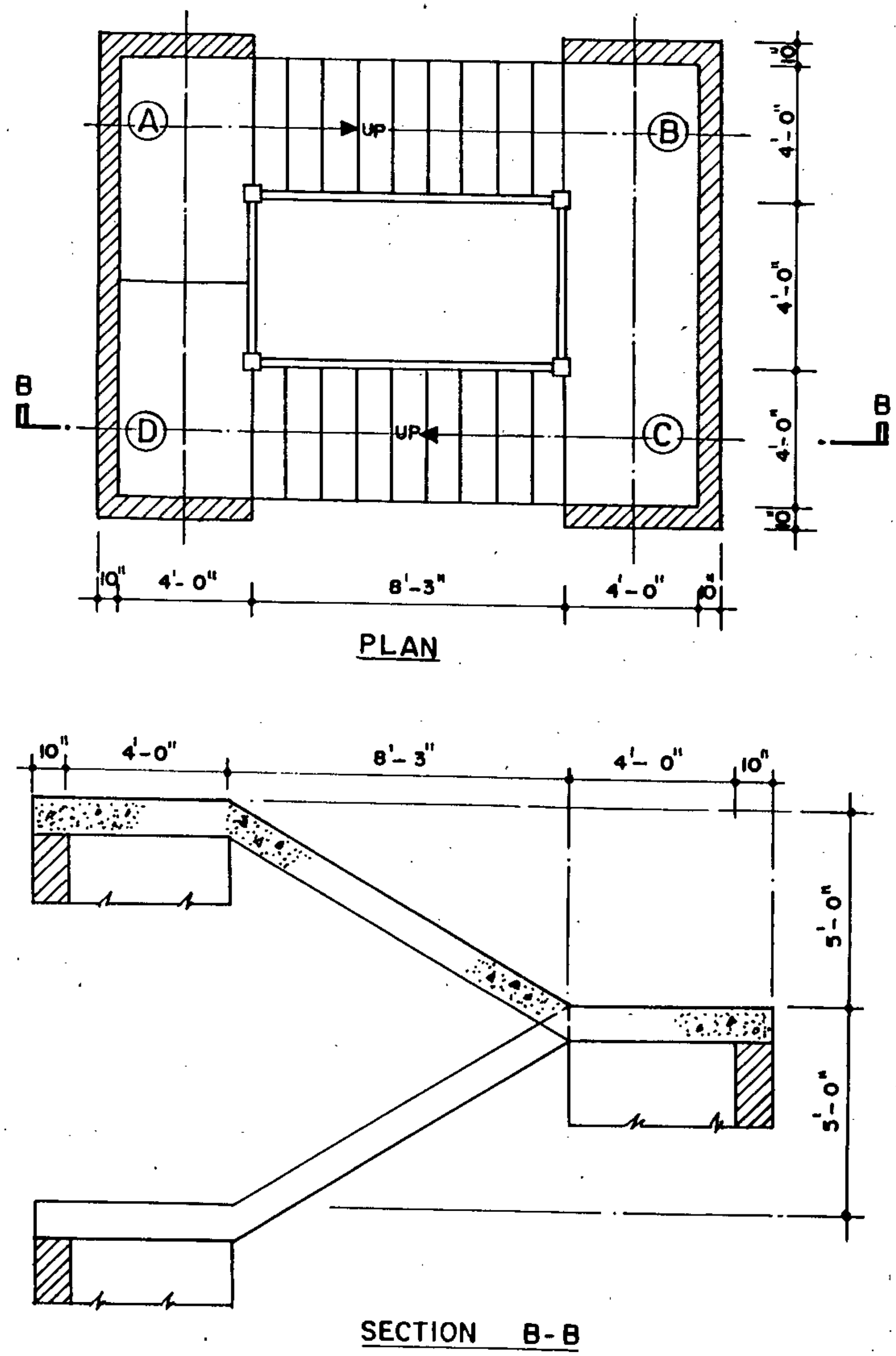


Fig. 2.4 Typical dimension of open-well type of stair under study.

For a slab thickness of 7.5" and with an assumed floor finish of 20 psf and a live load of 100 psf the load becomes:

On landing = 214 psf

On waist portion = 267 psf (duly magnified on a horizontal projection)

The maximum moment = 9213 ft.-lb./ft.

For a concrete with crushing strength of 3000 psi and a grade 40 steel, an overall thickness of 7.5" with a steel of 0.973 sq.in/ft. are required.

b) Design by British Approach[5]

Span = going + (half of landing) x 2

= 12.25 ft.

For a slab thickness of 6" with the other loads as assumed in (a) above the total load becomes

On landing = 175 psf

On waist portion = 245 psf

Therefore maximum moment at mid span = 4308 ft.-lb./ft.

Which calls for an overall thickness of 5.5 inch and steel area of 0.66 sq.in. /ft.

2.4.2 Design of Open-Well Stair

a) Design by Conventional American Approach:

Design of span AB: (Fig. 2.4)

Span = 17.08 ft.

Design of this span is the same as that of a Dog-legged stair. Hence a slab thickness of 7.5" with steel area of 0.97 sq.in. /ft. are required (Art. 2.4.1(a)).

Design of span BC: (Fig. 2.4)

Span = 12.833 ft.

With a total load of 214 psf on landing the design moment in this case is 4398 ft.-lb./ft. which calls for a total thickness of 5.5 inch. So design of span AB governs here and a thickness of 7.5" is chosen, with steel of 0.46 sq.in./ft. in span BC

b) Design by British Approach[5]:

Design of span AB:

Span = 12.25 ft.

As presented in Art. 2.4.1(b), this span requires a slab thickness of 5.5 inch with a steel area of 0.66 sq.in /ft.

Design of span BC:

Maximum moment = 3063 ft. .lb./ft.

So design of AB governs here and a thickness of 5.5 inch with a steel of 0.47 sq.in /ft. is required.

2.5 Remarks on the Codes and Practices

From the review of the major codes of practice regarding design of stair slabs, followed by case studies presented in the preceding article, following observations can be made:

a) In the absence of specific guidelines for designing stairs, the practitioners of ACI code, design stairs which are overly safe and wasteful.

b) Although the British code for design of stair slabs shed light on the problem from a more rational stand point, the requirements of CP 110 are still conservative.

c) Indian code of practices are very similar to the British one. However, it takes into account the effect of continuity of the stair slab supported by beams at top and bottom kink, by specifying a positive mid-span moment of $ql^2/10$. But it specifies nothing regarding the bending moment at the beam supports.

d) In general, none of the codes give

appropriate bending moment co-efficients for stair.

e) Although it is common practice to provide negative steel at the top and bottom kink, no direct/straight forward specification is available to quantify this steel.

f) The general tendency of designing stair slab as simple horizontal one way slab disregards the interaction of adjoining flights. The presence of such adjoining flights obviously restricts the deflections that would be the case in an otherwise simple slab.

g) The assumption that the direction of the reactions at the upper and lower supports is vertical will result in a vertical shearing forces at the kinks. The component of this vertical shearing forces along the inclined slab will result in axial tension or compression at the kink. No code provides due account in this regard.

CHAPTER 3
FINITE ELEMENT ANALYSIS OF STAIR SLAB

3.1 General

Among the available numerical methods, finite element method is one of the most versatile and powerful tools for analyzing structures of complicated shape with arbitrary boundary conditions. In the present study, this method was used to investigate the behavior of the stair slabs. In this chapter the analysis scheme and the results of the present analysis are presented.

3.2 Finite Element Computer Programs

The present study comprises of investigating the actual behavior of stair slab with the help of finite element technique. The generalized thick shell finite element program developed by Ahmad[6,7] and ANSYS[8] (stiff63 element quadrilateral thin shell) were used for the purpose of analysis of stair slabs, in the present study. The salient features of these shell elements together with brief description of the programs are presented in APPENDIX-II and III.

3.3.1 Shell as a Structural Element

The analysis of shell structures often embraces two distinct, commonly applied theories. The first of them, the membrane theory identifies the shell action in which the external loads are carried entirely by inplane forces like the skin forces in a balloon under air pressure. A membrane, either flat or curved is identified as a body incapable of transferring moments and shear forces. These skin forces expressed in terms of force per unit length, are termed as "membrane stress resultants". The second, the bending theory include the effects of bending. Thus a shell structure can develop in-plane forces in addition to those forces and moments existing in a plate or beam. In-plane and bending effects may then be analyzed separately and superposed. Hence a flat shell element may be developed as a combination of a membrane element and a plate element of the same shape.

3.3.2 Choice of Element for the Problem:

The thick shell elements often called generalized shell element include bending as well as shear deformations. In this element nodal lines are straight and normal to the undeformed middle surface. They are inextensible and remain straight after deformation . However, they are in general not normal to the deformed middle surface, allowing the calculation of transverse shear stresses and strains. As a result ,this element is well suited for modeling plates and shells where transverse shear is important[19]. Transverse shear is important for shells made of composite materials which may have a low shear modulus compared to their elastic modulus, and is of increasing importance for all materials as the shell thickness increases.

In the quadrilateral thin shell, the nodal lines which are straight and normal to the undeformed middle surface are inextensible and remain straight and normal to the deformed middle surface after deformation. That means that the transverse shear stresses are neglected in this thin shell element. As such, this element is said to implement a "classical"(19) shell theory, and well suited for modeling thin plates and shells.

3.4 Assumptions and Limitations

The study was carried out with the following assumptions:

a) Structural Idealization of the Problem:

i) A typical double flight dog-legged stair having an intermediate landing and an open well type stair were analyzed. For this purpose boundary conditions were applied taking the symmetry condition

UY=O, ROTX=0 (Fig.3.1 & Fig.3.2).

ii) Waist slab and landing slab were assumed to have same thickness.

iii) The additional stiffening effect provided by the treads to the waist slab was disregarded, though clearly present.

iv) Slab thickness was assumed to withstand the stresses only and no account is given to slenderness (vibration etc.)

b) <u>Properties of Material:</u>

i) The material is linearly elastic, homogeneous and isotropic.

ii) Modulus of elasticity, $E = 3x10^6$ and poisson's ratio $= 0.18$

c) <u>Loading:</u>

i) Both Live load and dead weights are applied as gravity loads.

With a live load of 100 psf (as recommended by most of the leading codes of practice [4,5]), a floor finish 20 psf and 50 psf self weight of the slab (for a 4" thickness) the total load on landing was taken to be equal to 170 psf. And in the inclined waist portion, with waist slab of same thickness with steps of 6" rise, a total load of 207.5 psf was considered. These figures were converted into equivalent material density of 0.30 $lb./in^3$ and 0.36 $lb./in^3$ for landing and waist slab respectively.

3.5 Idealization of the Structure

For the purpose of analysis, a Dog-legged and an Open-well stair of typical dimension were considered. For the Dog-legged type, first analysis was carried out for a two-flight stair (Fig.3.1) and later on a three-flight one (Fig.3.2). Boundary conditions are shown along with the line of symmetry in Fig.3.1 & Fig.3.2 . Nodal lines on the plane of symmetry were kept restrained against displacement in the perpendicular direction of the plane of symmetry and against rotation out of the plane of symmetry .

3.6 Element Meshes Used for Analysis

Zahedi(3) tried a number of different element meshes to analyze the single flight stair imposing boundary conditions at the line of symmetry. By analyzing the single flight stair, with simplifying boundary conditions imposed at the line of symmetry, he came out with a suitable mesh (Fig. 3.3) arrangement that could tackle the complicated stress conditions at kink zone, with thick shell element(6,7). Details of the treatment of kink is shown in Fig.3.3. In the present analysis the same mesh configuration of elements was extended for a two-flight and later for a three flight analysis, with thick shell program.

Element sub-division for a open-well type of stair was made in the same way as shown in Fig.3.6 for analysis by thick shell program.

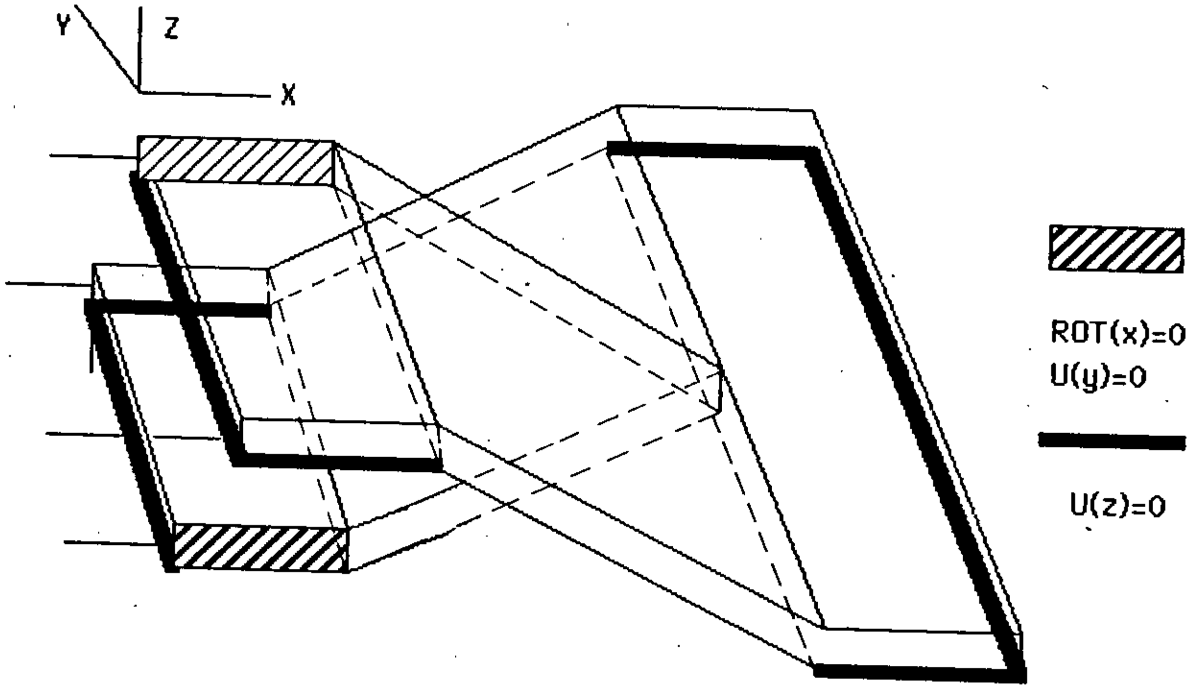


Fig. 3.1 Boundary conditions for a 2-flight analysis

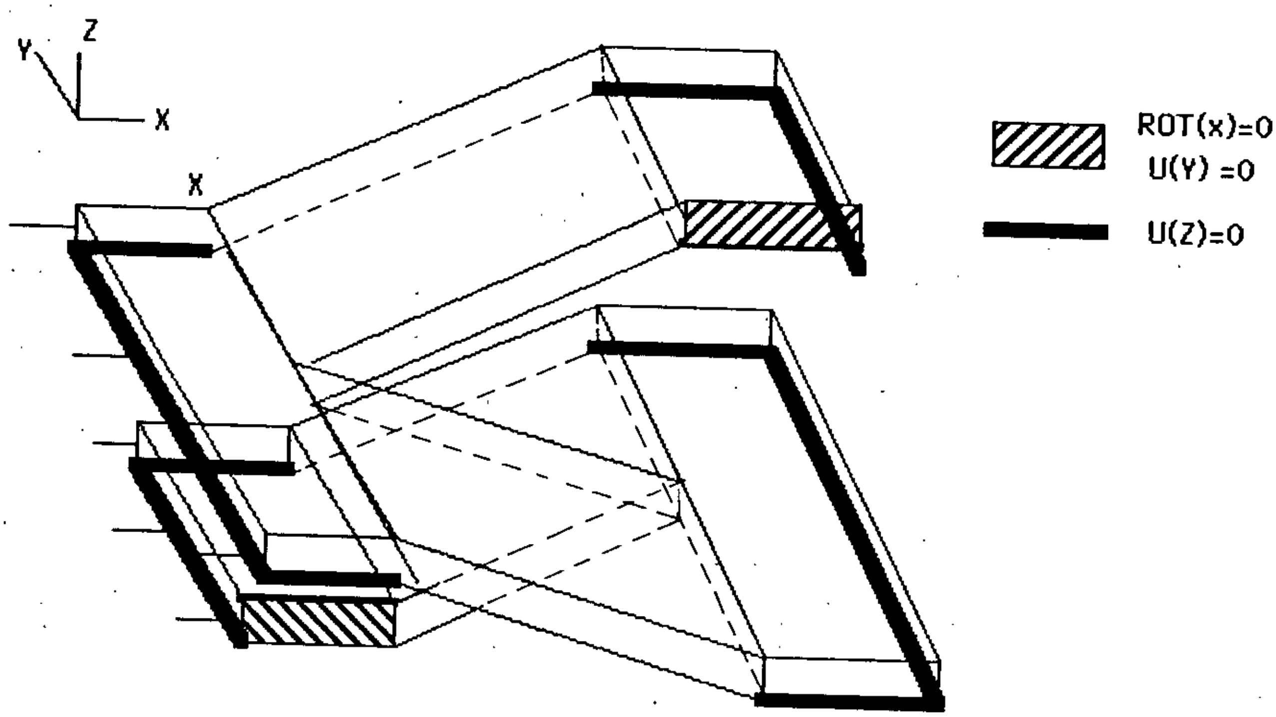


Fig 3.2 Boundary conditions for a 3-flight analysis

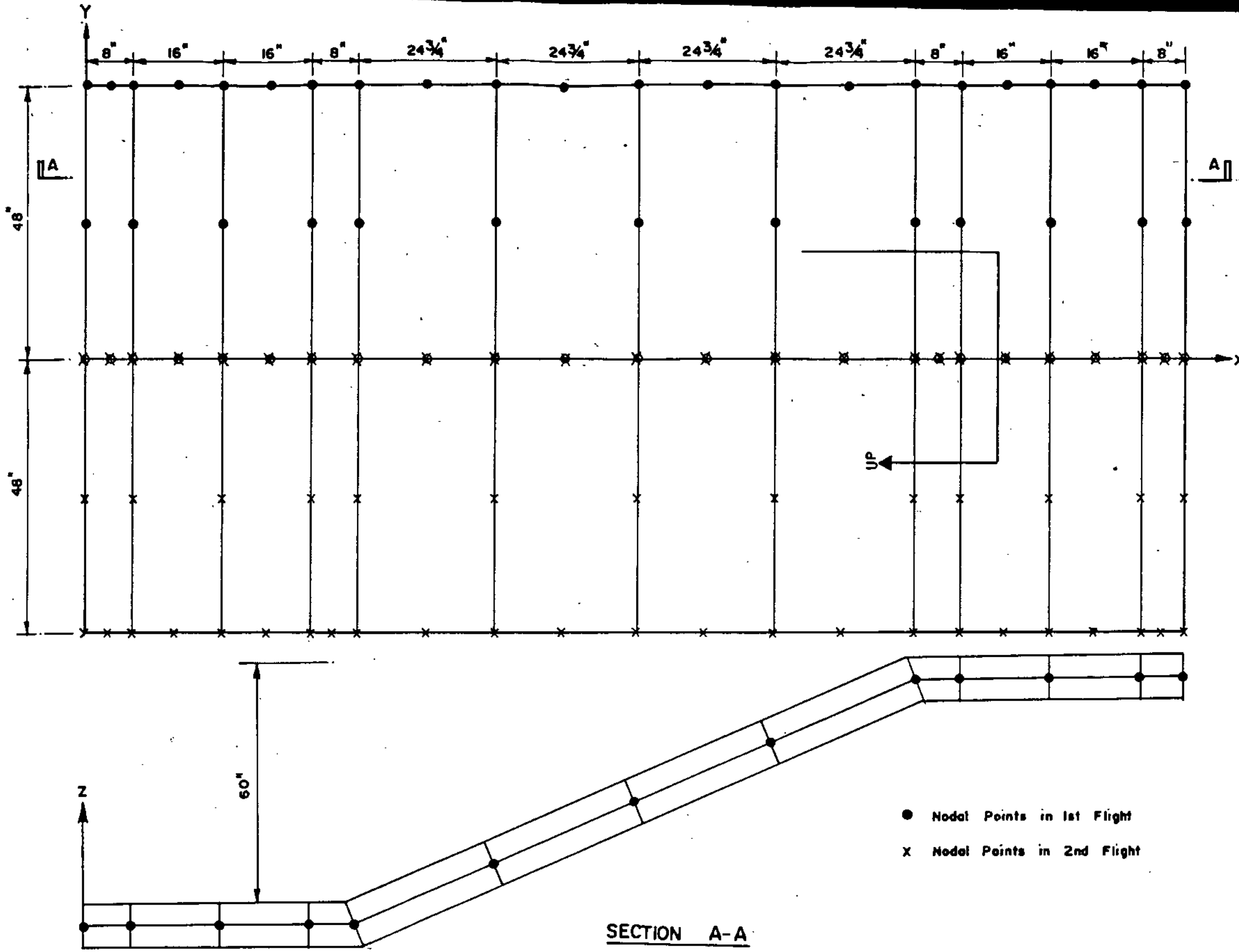


Fig. 3.3 Element mesh for thick shell analysis of Dog-logged stair.

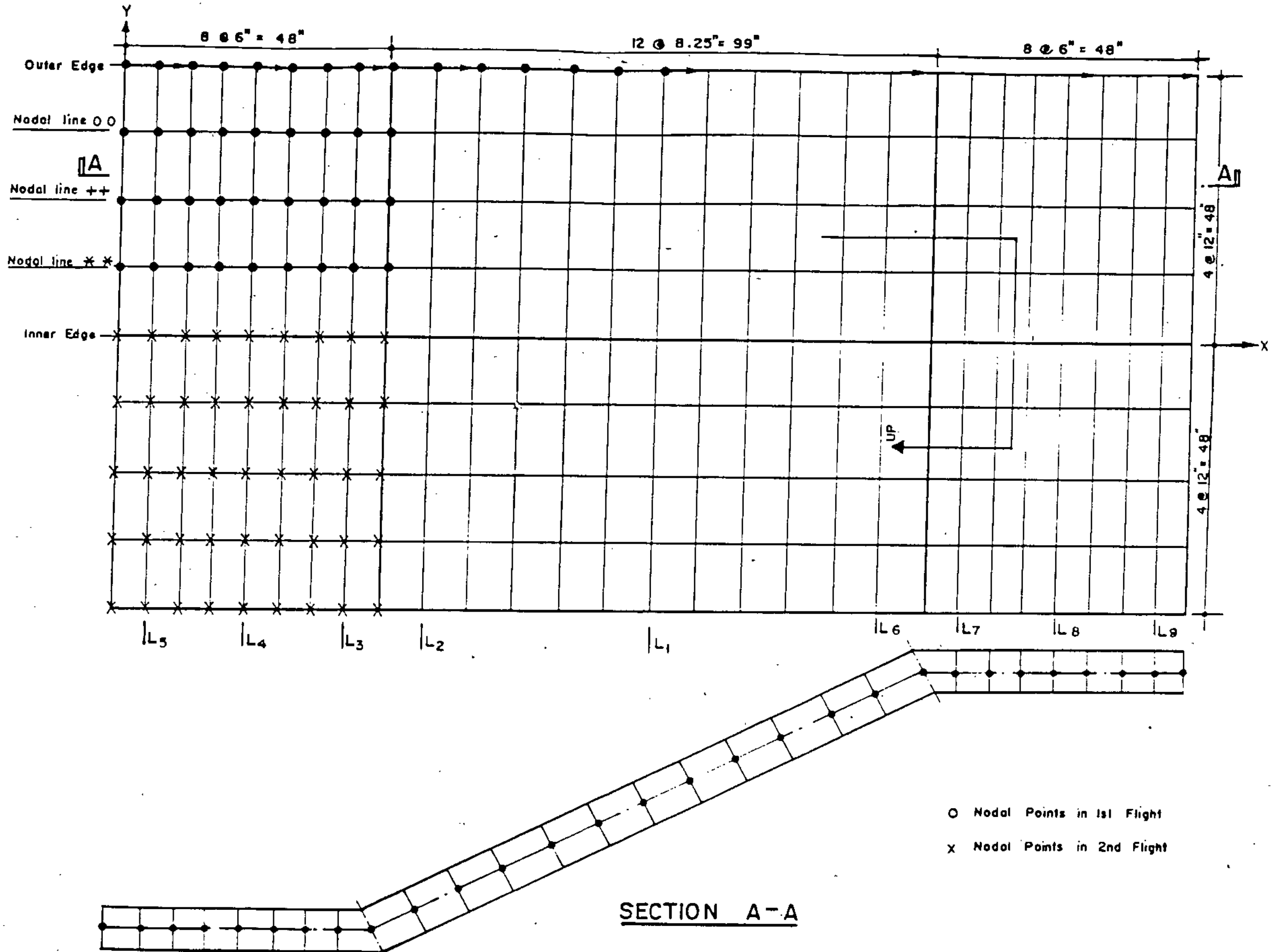


Fig. 3.4 Element mesh for thin shell analysis of Dog-legged stair.

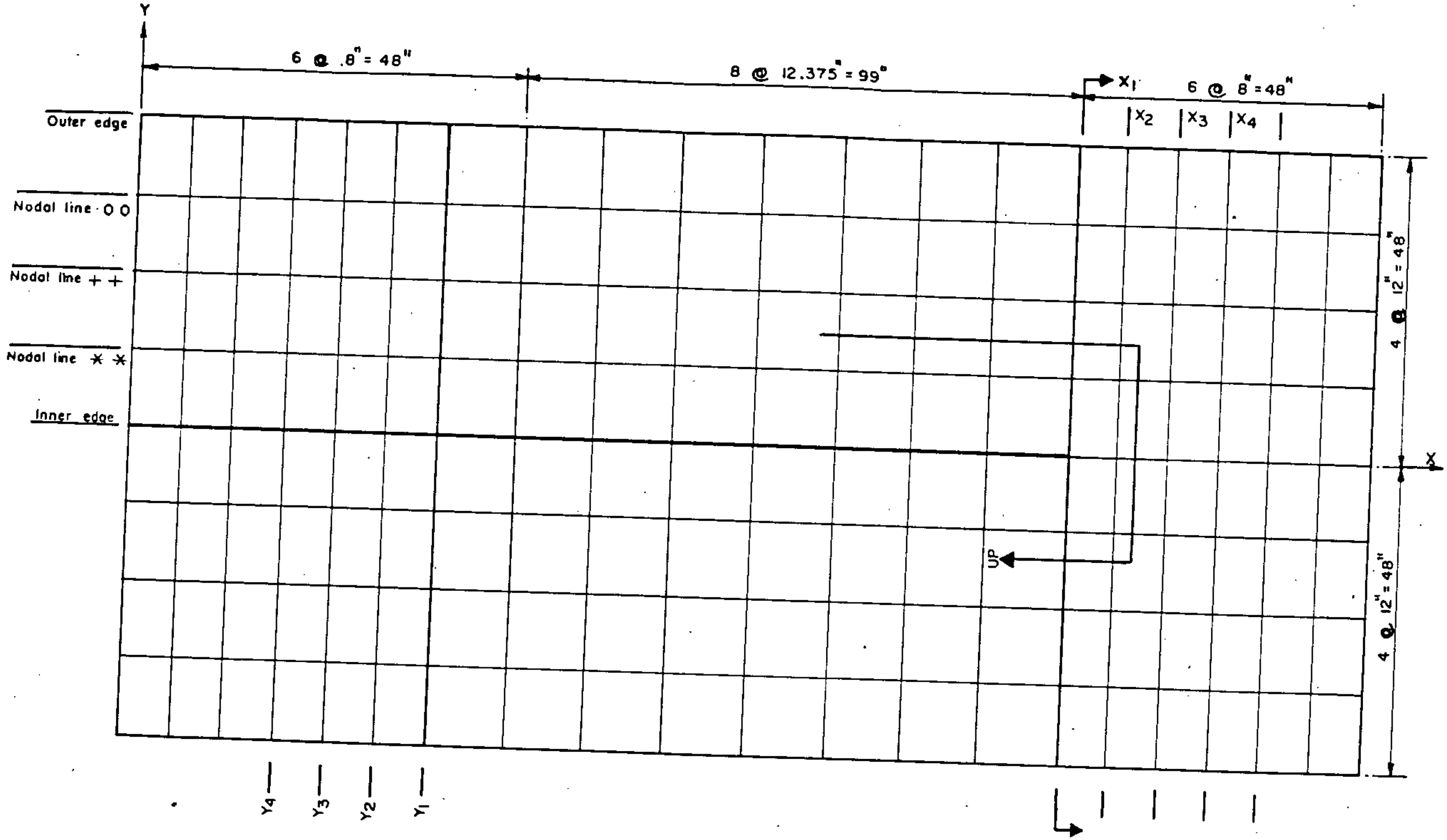


Fig. 3.5 Element mesh for thin shell analysis of Dog-legged stair (3 flight analysis)

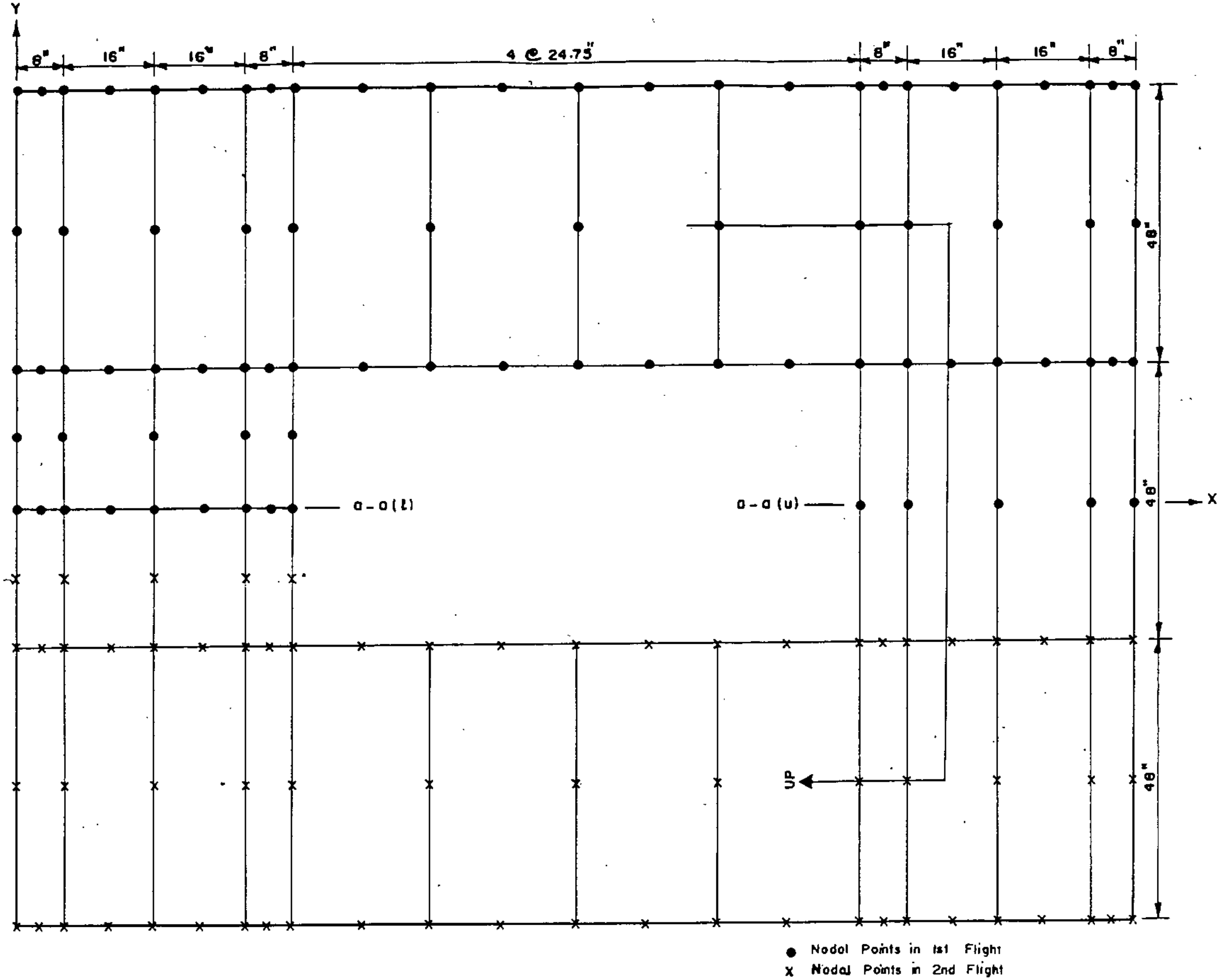


Fig. 3.6 Element mesh for analysis of Open-well stair by thick shell program.

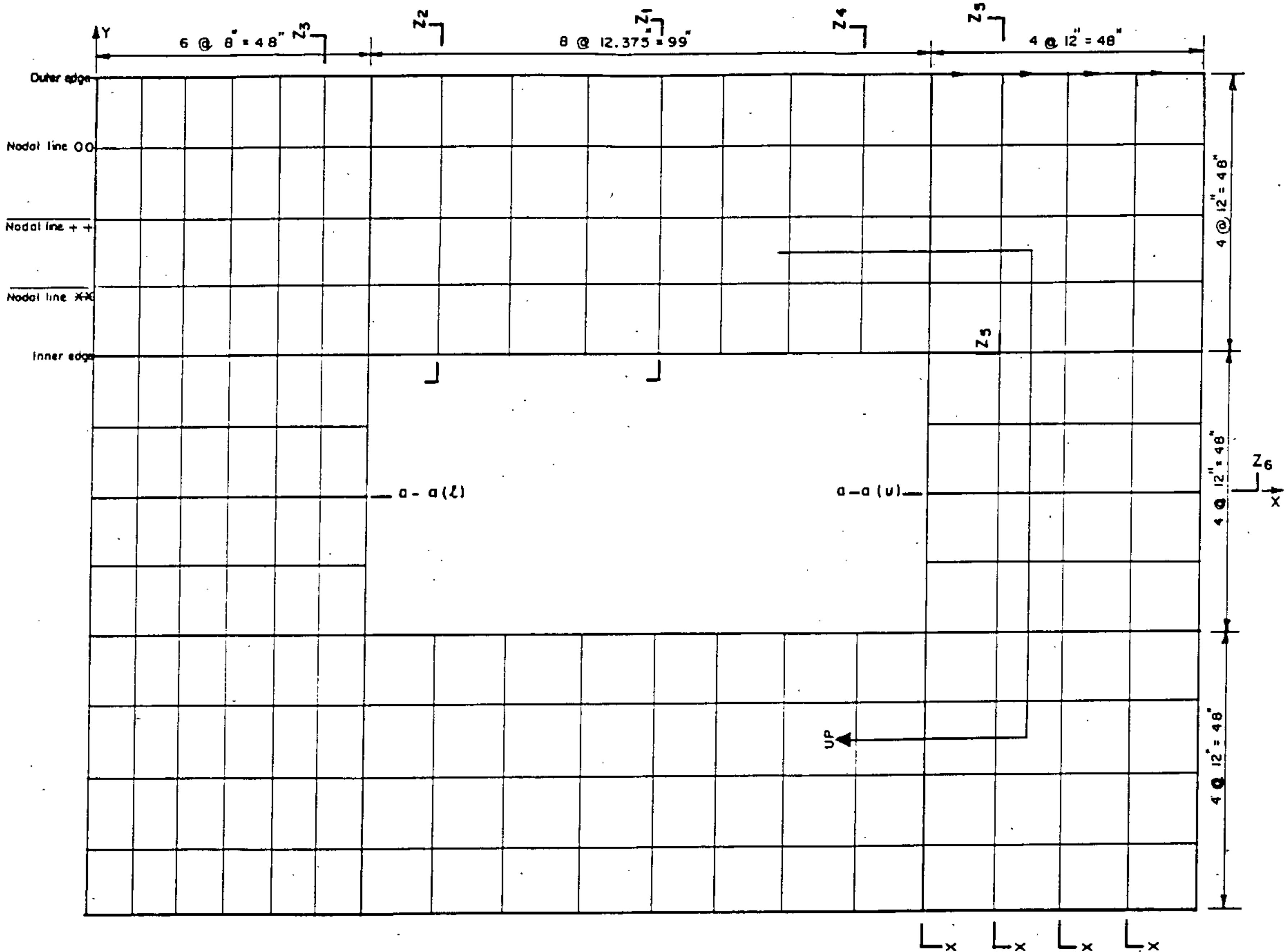


Fig. 3.7 Element mesh for thin shell analysis of Open-well stair

The element meshes used for the thin shell element (Stiff63, ANSYS[8]) is shown in Fig.3.4 and 3.6. These meshes have been found to be suitable after a number of trials. Mesh configuration of Fig.3.4 has been used in the analysis of two flight while that of Fig.3.5 has been used for the three flight analysis.

The thin shell analysis of the Open-well type stair has been carried out with the mesh shown in Fig.3.7.

The finite element method is an approximate method based on an assumed displacement. The displacement formulated finite element gives a lower bound solution of the problem. With the increasing number of nodes, the continuum being more flexible, approaches the "exact" solution.

The mesh of Fig. 3.4 is the result of a number of trial meshes, for stiff63 element. The results of those trial meshes is presented in Fig. 3.8. The mesh of Fig. 3.4 with 281 nodes for two-flight gives sufficiently accurate result, as appears from Fig. 3.8 . It is clear that for an increase in the number of nodes from 150 to 281 the improvement of the result is only insignificant, to justify latter's use economically. So for a three flight extension a mesh (Fig.3.5) with 204 nodes (per two flight) has been selected consistently and the results presented on the basis of this are sufficiently accurate.

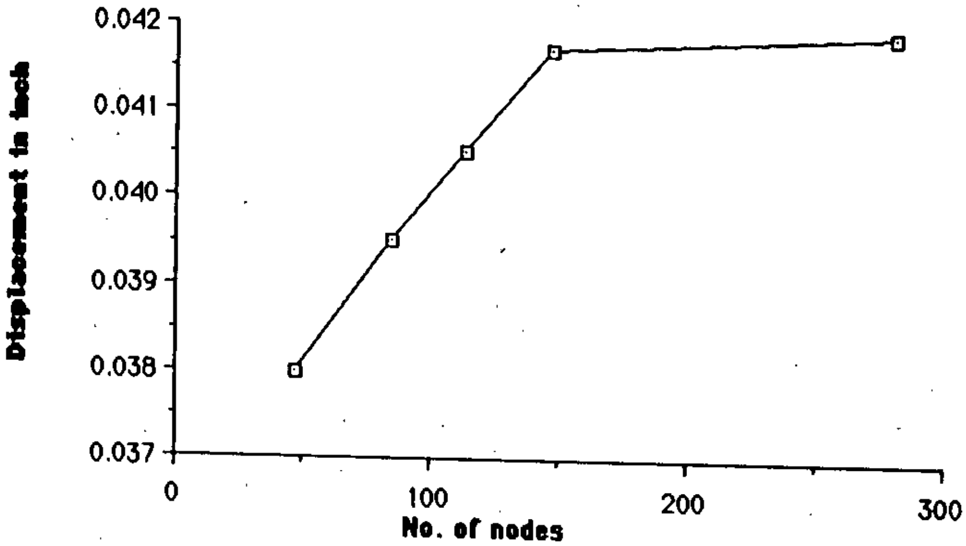


FIG. 3.8 Displacement against number of nodes (ANSYS Stiff 63 element)

3.7 Presentation of the Results

The results of the finite element analysis are summarized in the following articles. The results will be arranged in such a way that the bending moment and the vertical displacement at any location are placed one upon another. The axial forces along the length of the flight, as obtained from the corresponding analysis, are then plotted.

The X-direction is selected in the direction of the length of the flight . The moments and axial forces computed from the stresses in X-direction are termed as MX and FX respectively. Similar quantities in Y-direction (direction perpendicular to the length of the flight) are termed as MY and FY respectively.

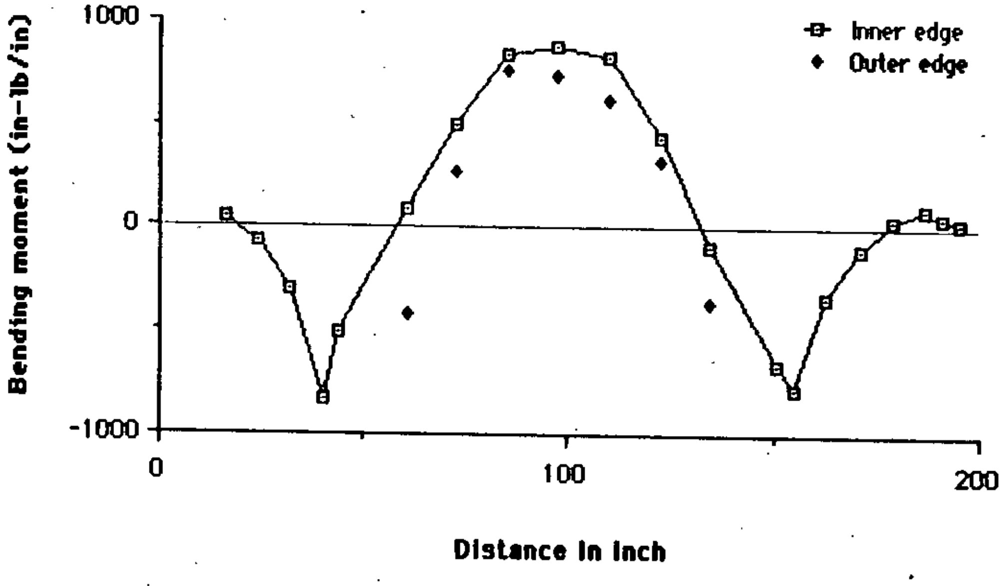
The bending moments(MX) at inner and outer edge along the length of the flight are presented in the figures. The bending moment (MX) values in between these two extreme boundaries are smaller in magnitude (compared to those values at inner and outer edges) and are not shown in the figures. A moment causing compression at top is positive. Axial forces are presented considering the tensile axial force as positive and compressive one as negative.

3.7.1 Analysis of Dog-legged Stair

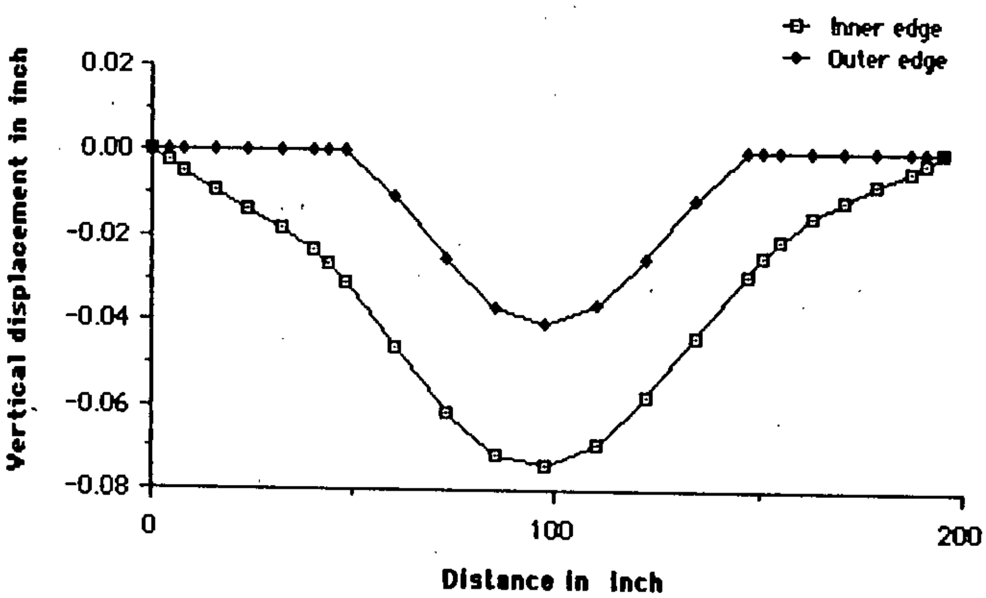
a) Analysis for Two Flights:

A double flight dog-legged stair having typical dimensions shown in Fig.2.7 have been analyzed using thick-shell finite element program. The element mesh is shown in Fig.3.3. The boundary conditions for the problem is defined in article 3.6. The landing slabs , for the present study, being supported on walls on three sides, act as support for the waist slab. The bending moment (MX) diagram and vertical displacements for first flight are shown in Fig.3.9. The axial forces (FX) for first and second flight are shown in Fig.3.10. Since the the bending moment diagram for the first and second flight are found to be same, the second flight bending moment diagram is not presented here.

The same problem has also been analyzed using thin shell element of ANSYS[8]. The mesh configuration is shown in Fig.3.4. The results are shown in Fig.3.11 and Fig.3.12. The bending moments (MX) for both the flights are same.

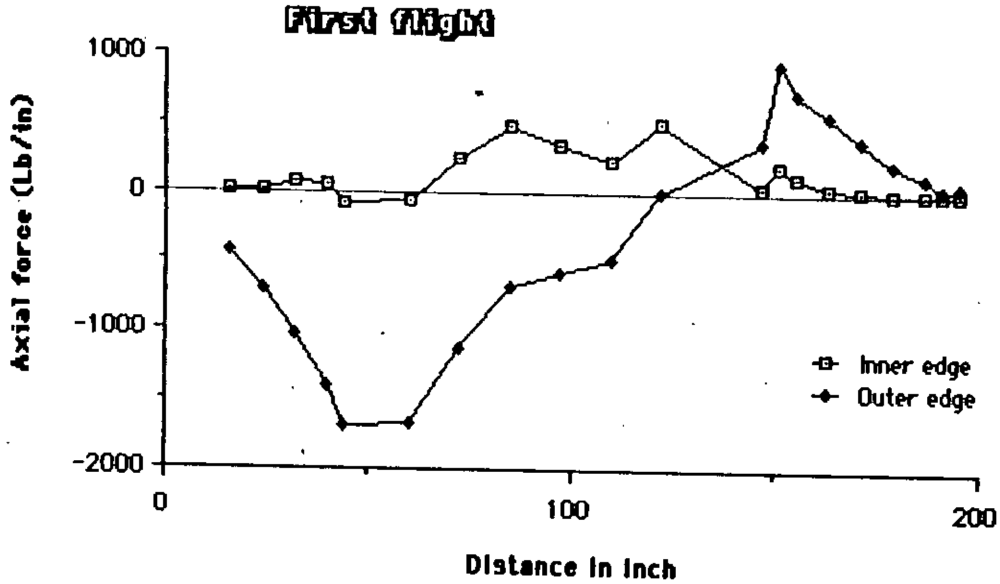


a) Bending moment (MX)

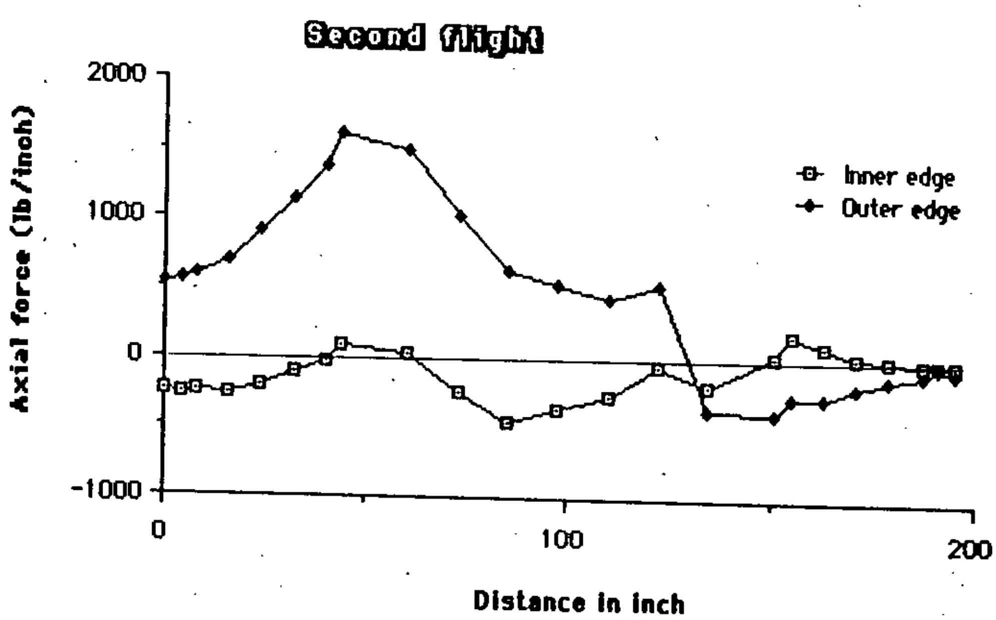


b) Deflection

Fig. 3.9 Results of 2-flight analysis using thick shell elements.

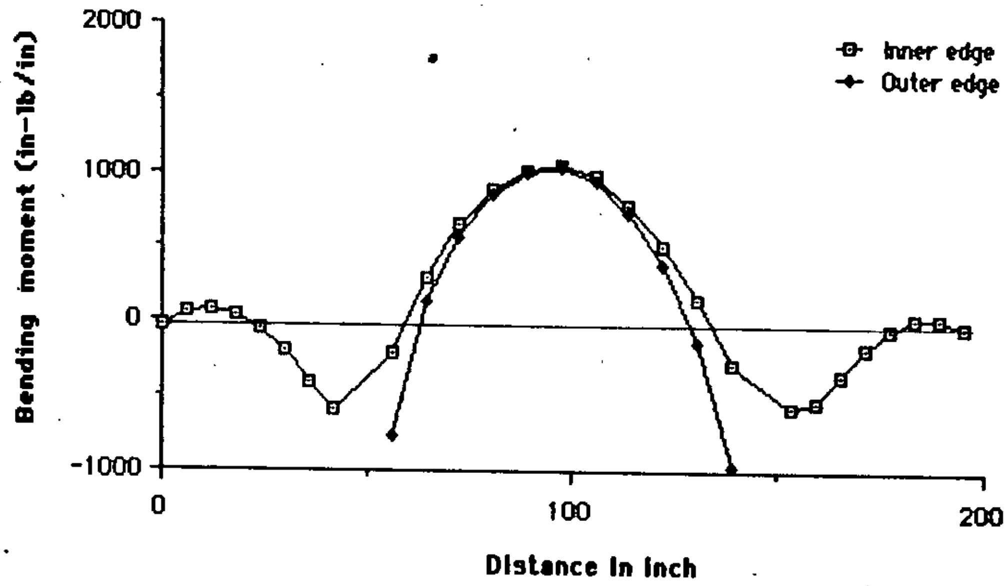


a) Axial force (FX) in first flight.

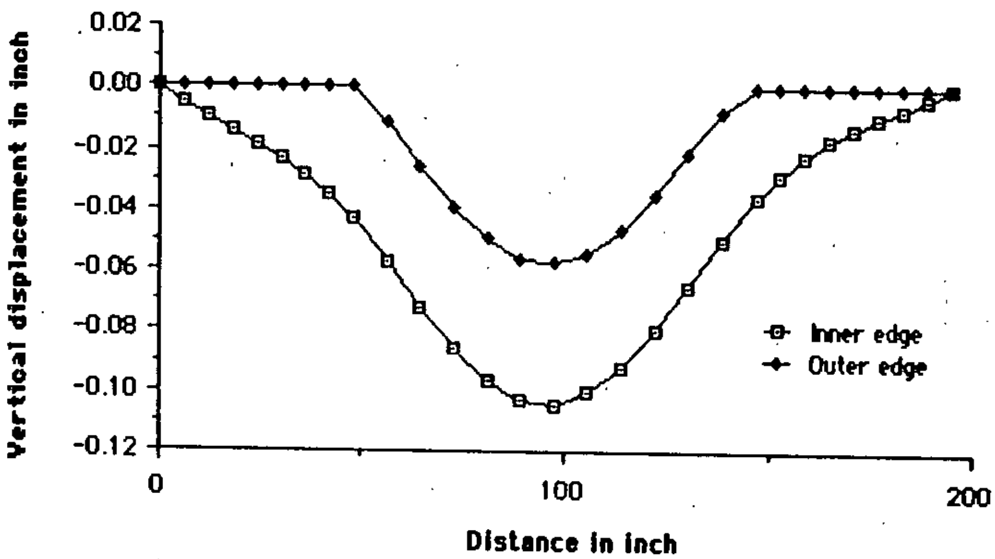


b) Axial force (FX) in second flight.

Fig. 3.10 Results of 2-flight analysis using thick shell elements

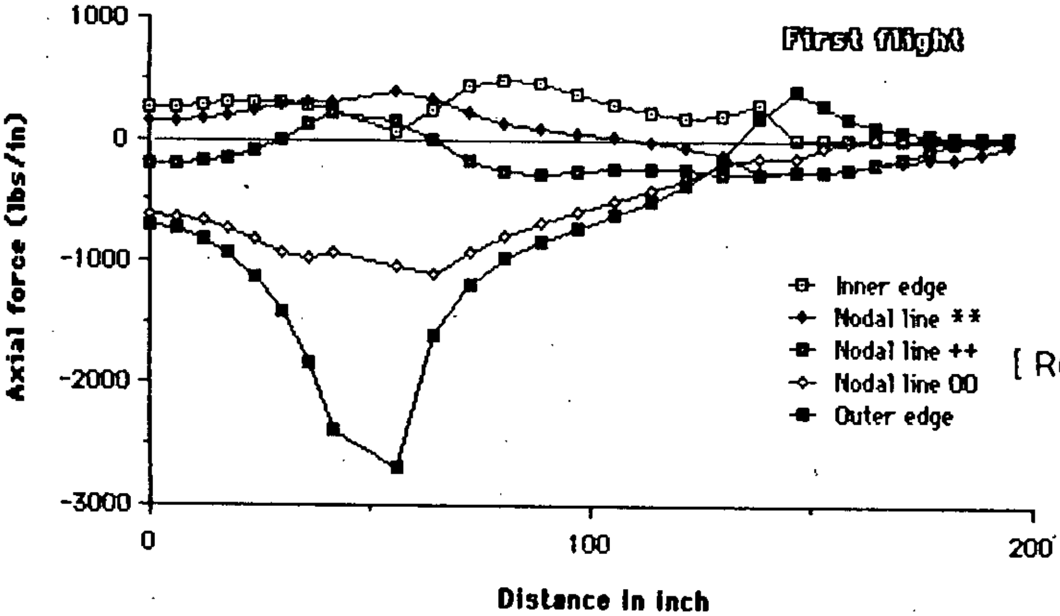


a) Bending moment (MX)



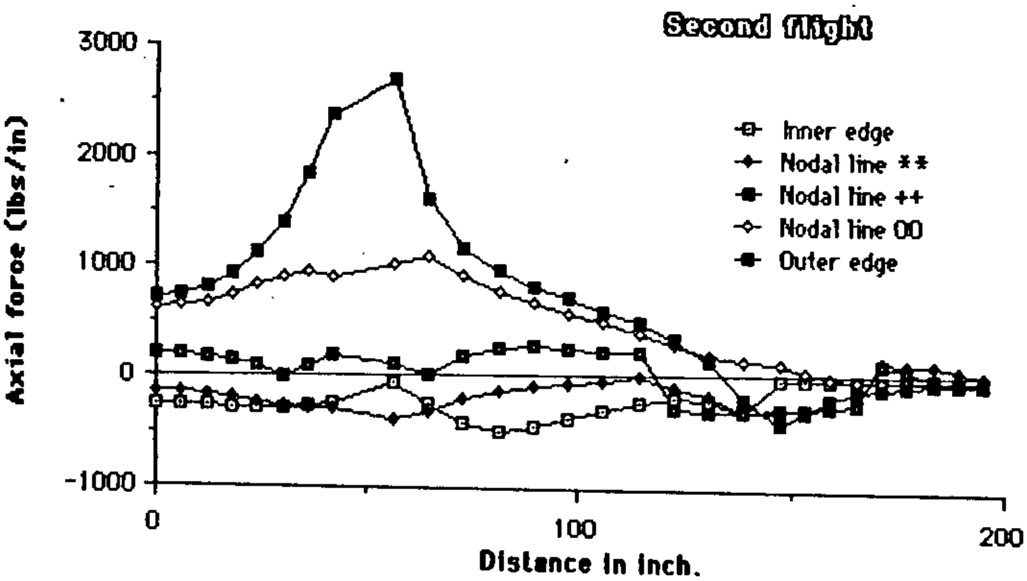
b) Deflection

Fig. 3.11 Results of 2-flight analysis using thin shell elements



[Ref. Fig 3.4]

a) Axial force (FX) in first flight.



b) Axial force (FX) in second flight.

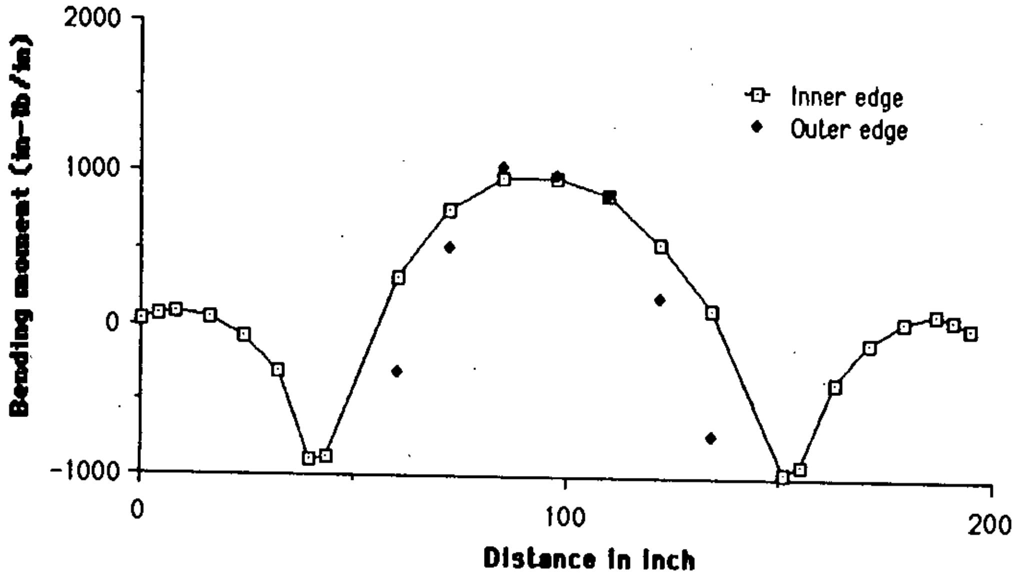
Fig. 3.12 Results of 2-flight analysis using thin shell elements

b) Analysis for Three Flights:

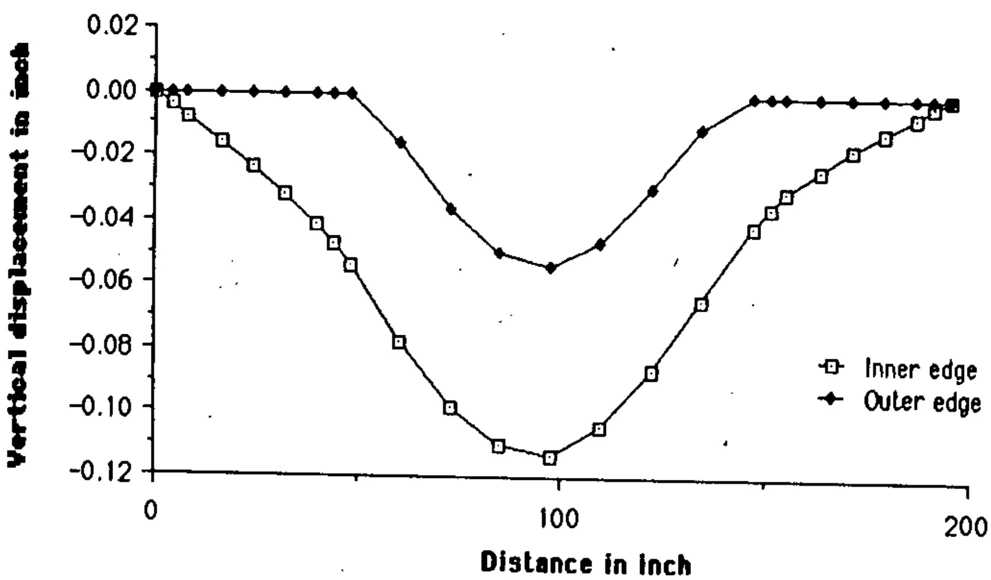
In order to make the analysis further free from the effect of imposed boundary conditions attempts were made to analyze the same dog-legged stair by extending the geometry into another flight (i.e. three flight, where two consecutive flight serves between floors). The imposed boundary conditions are shown in Fig.3.2. The typical element sub-division for a thick shell analysis is shown in Fig.3.3. The bending moments (MX), and displacements for the middle flight (intermediate flight) are shown in Fig. 3.13. Axial forces (FX) are shown in Fig.3.14.

The thin shell linear element of ANSYS has also been implemented for this problem. Element sub-division for this case is shown in Fig.3.5. Bending moment (MX), vertical displacement along length the stair for second (i.e. intermediate) flight are presented in Fig.3.15. Axial forces (FX) for this flight is in Fig.3.16. Figure 3.17 and 3.18 show the bending moments (MX) and axial forces (FX) for first and third flight.

The moments and forces in lateral (Y) direction of the stair slab are small except at landing. However, at landing the magnitude of these moments (MY) are maximum near kink and gradually fades away towards the end of the landing slab. Variations of the moments (MY) and displacements(UZ) at four different sections X1,X2,X3, and X4 (Fig.3.5) at intermediate landing and at another four sections Y1,Y2,Y3,Y4 (Fig. 3.5) at floor level landing are shown in Fig.3.19 and Fig.3.20.



a) Bending moment (MX)



b) Deflection

Fig. 3.13 Results of 3-flight analysis using thick shell elements (2nd flight values are plotted)

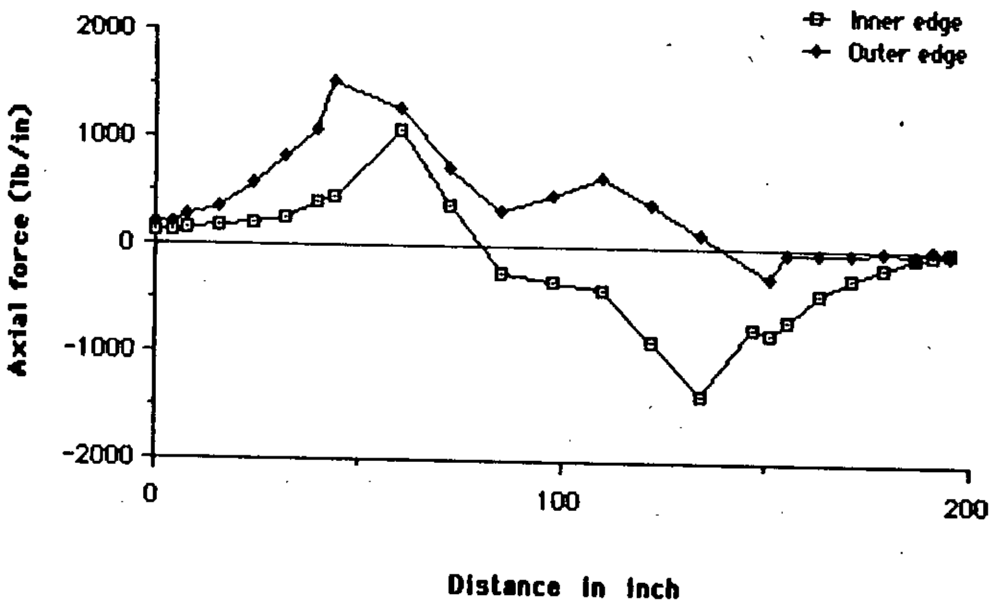
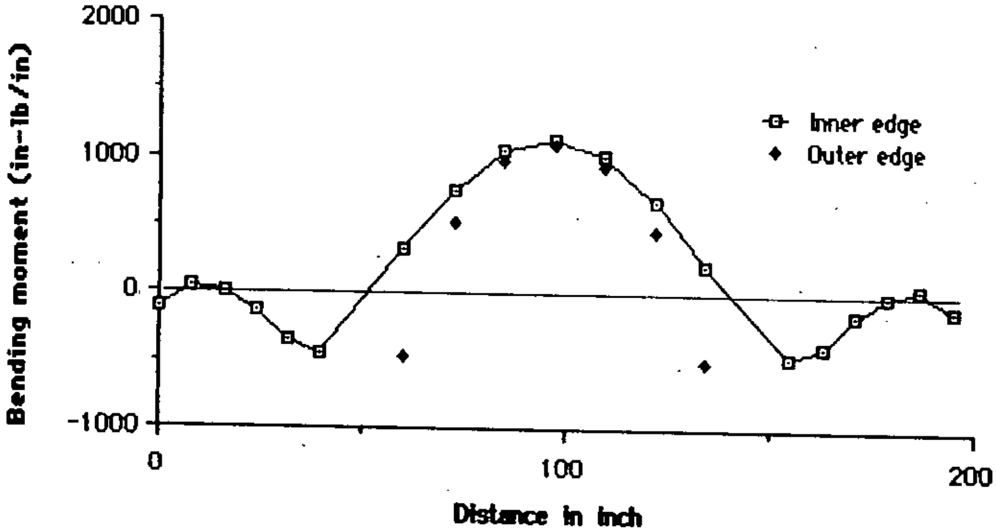
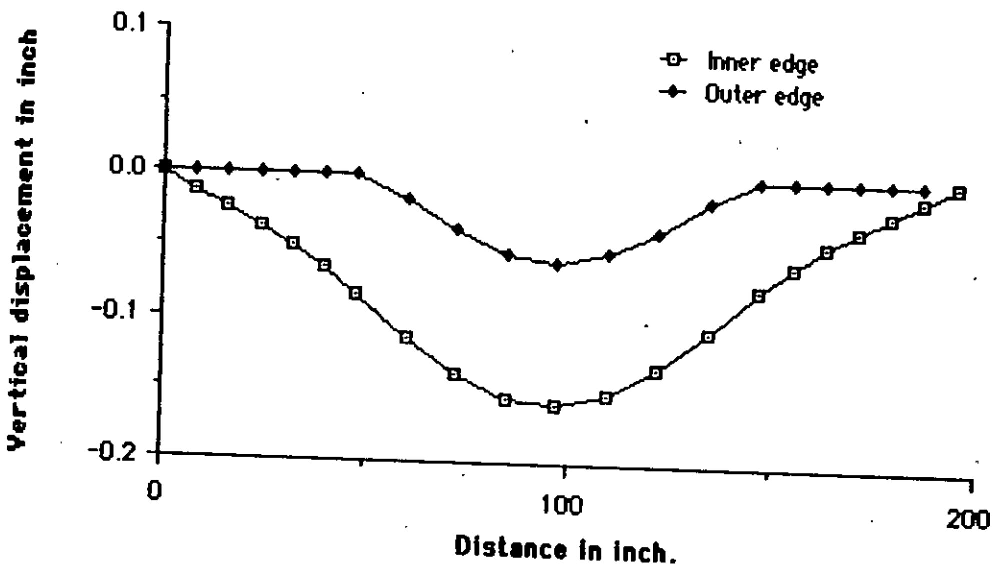


Fig. 3.14 Results of 3-flight analysis using thick shell elements (2nd flight values are plotted)

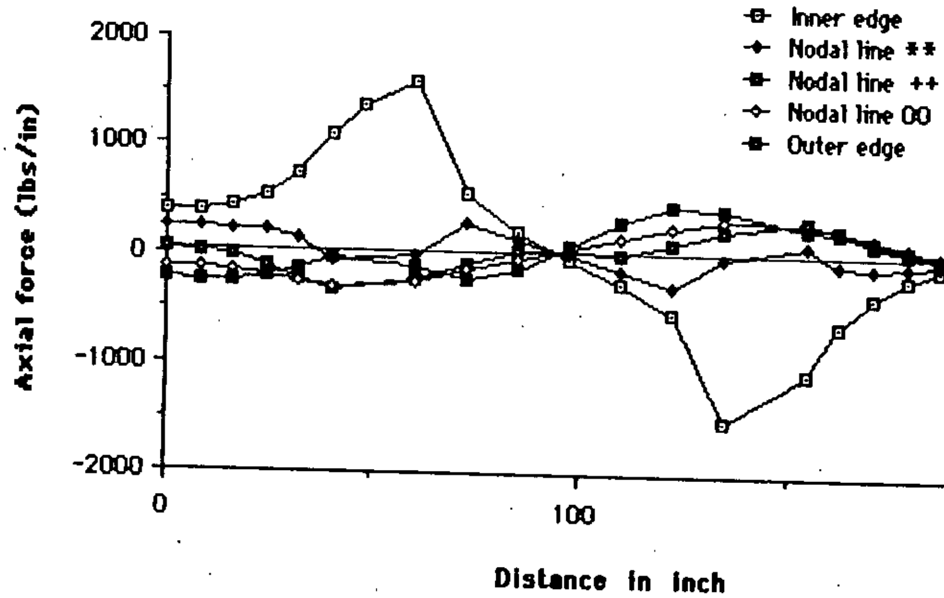


a) Bending moment (MX)



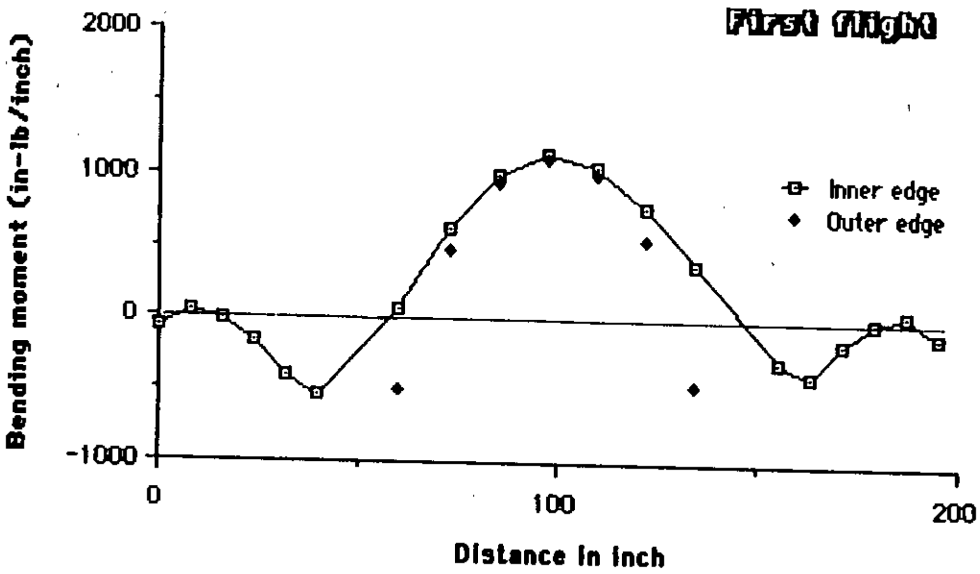
b) Deflection

Fig. 3.15 Results of 3-flight analysis using thin shell elements
(2nd flight values are plotted)

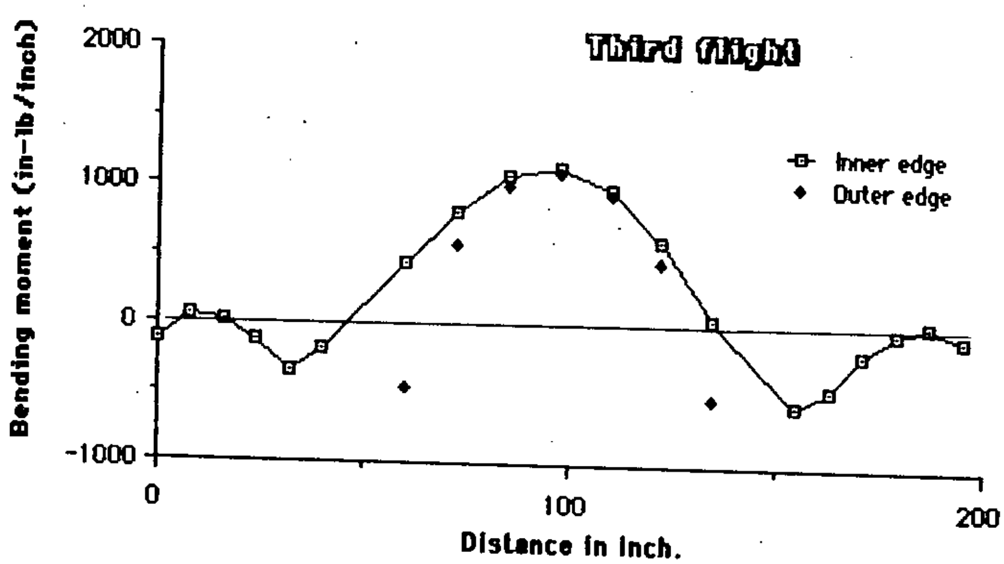


[Ref. Fig 3.5]

Fig. 3.16 Results of 3-flight analysis using thin shell elements (2nd flight values are plotted)

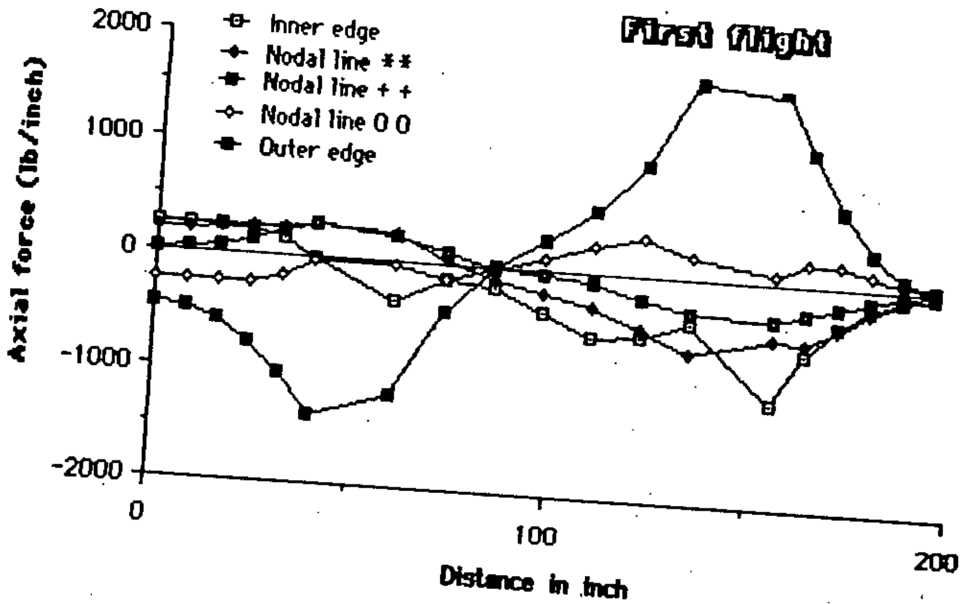


a) Bending moment (MX) in first flight.

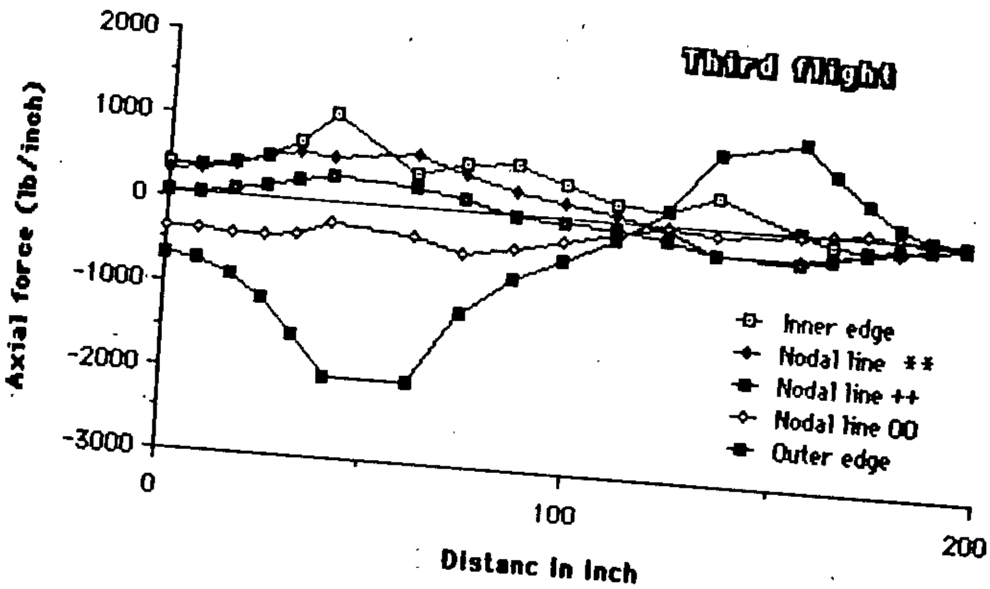


b) Bending moment (MX) in third flight.

Fig. 3.17 Results of 3-flight analysis using thin shell elements
(Ist and 3rd flight values are plotted)

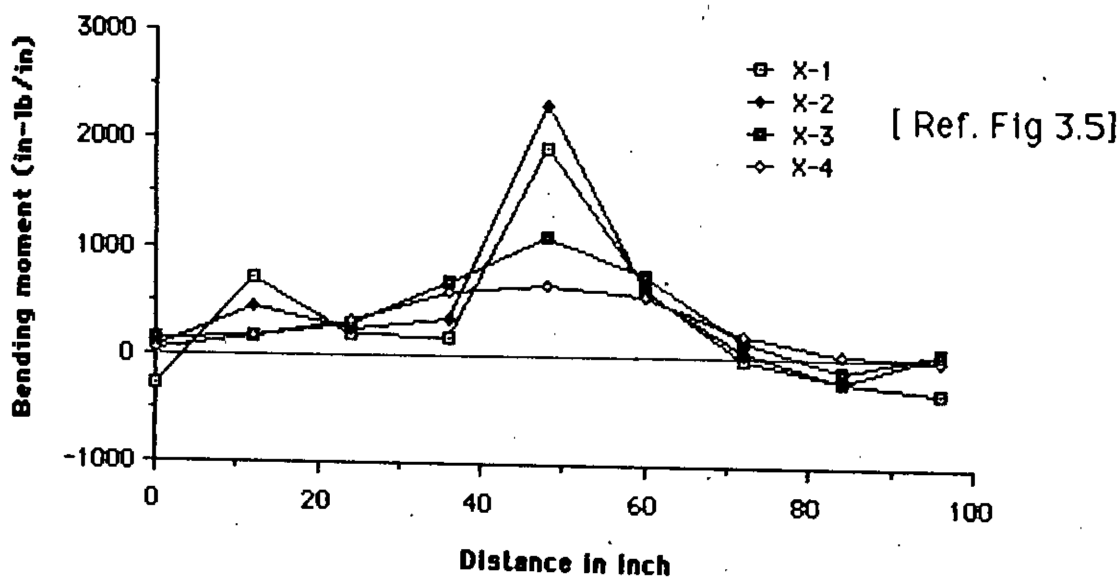


a) Axial force (FX) in first flight.

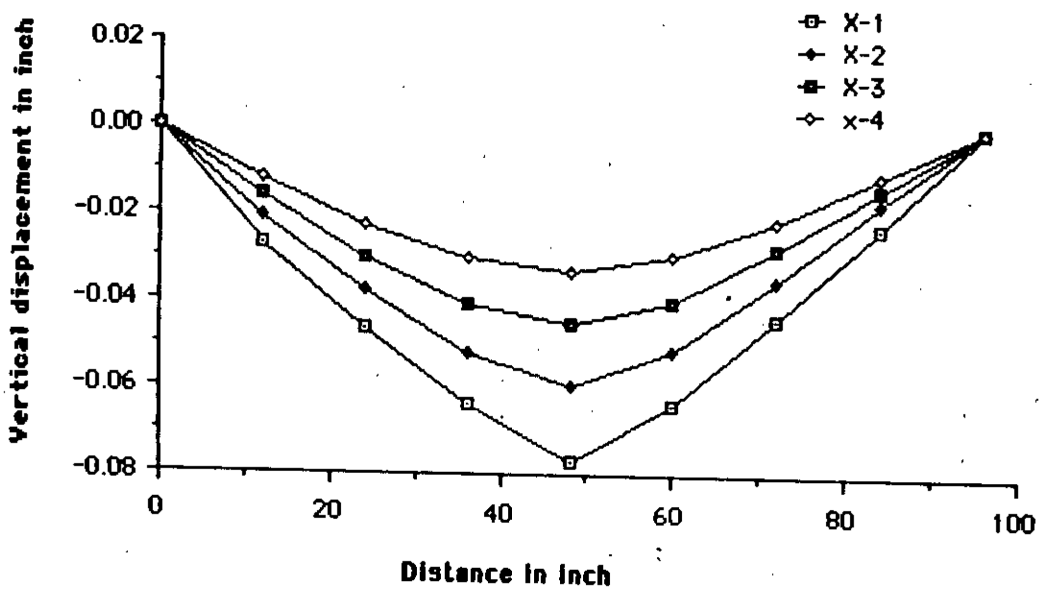


b) Axial force (FX) in third flight.

Fig. 3.18 Results of 3-flight analysis using thin shell elements (Ist and 3rd flight values are plotted)

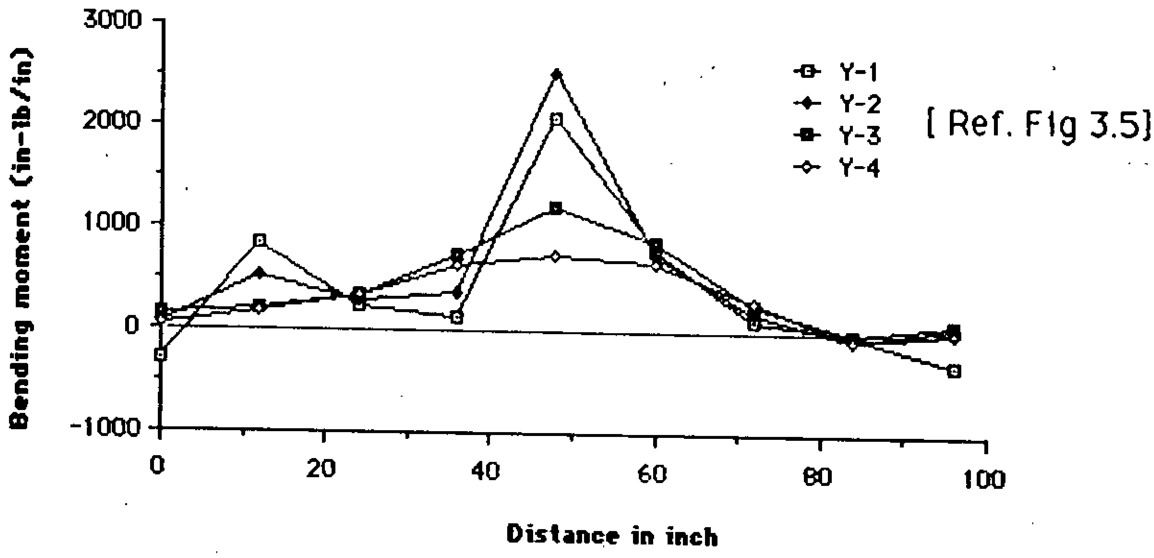


a) Bending moment (MY)

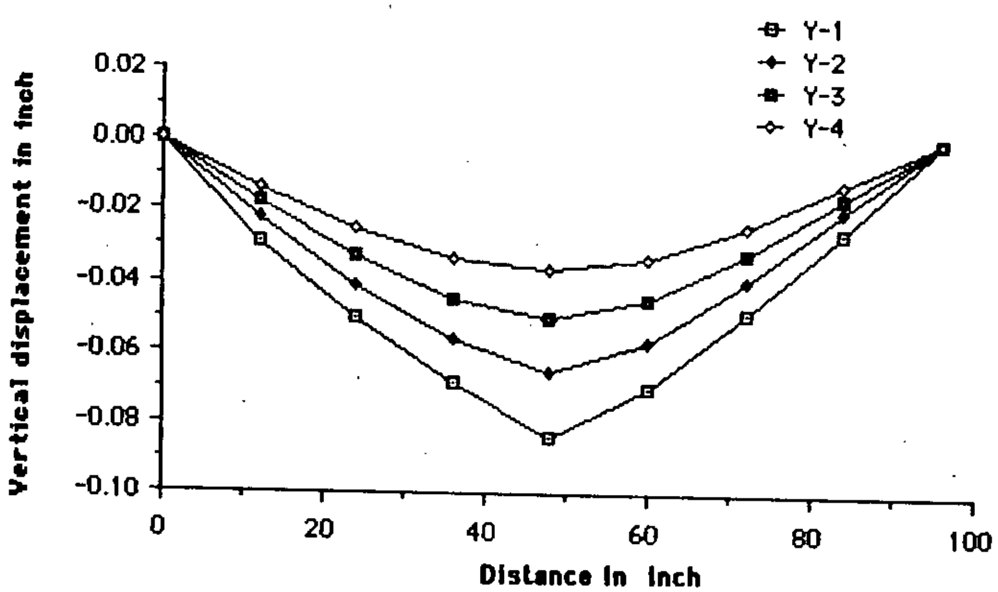


b) Deflection

Fig. 3.19 Results of 3-flight analysis using thin shell elements



a) Bending moment (MY)



b) Deflection

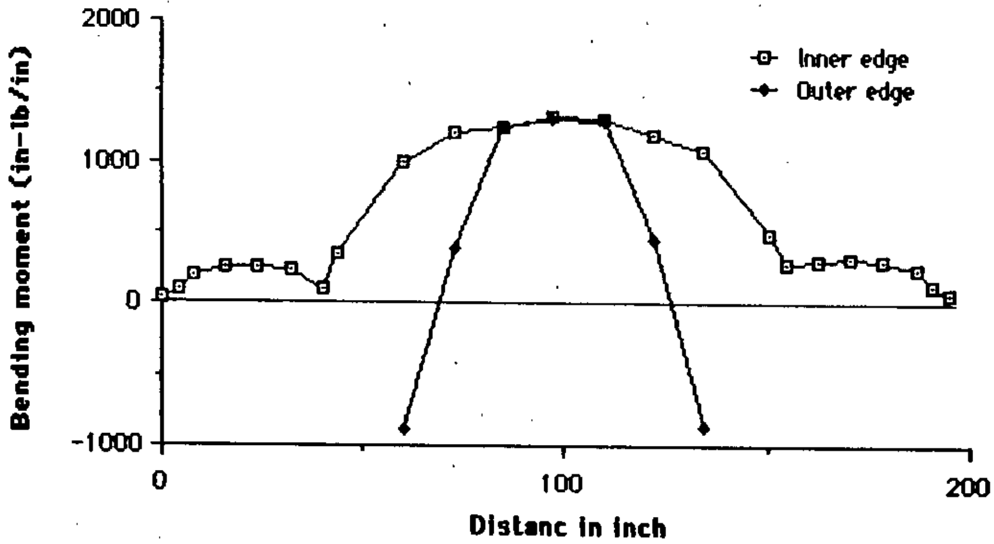
Fig. 3.20 Results of 3-flight analysis using thin shell elements

3.7.2 Analysis of Open Well Stair

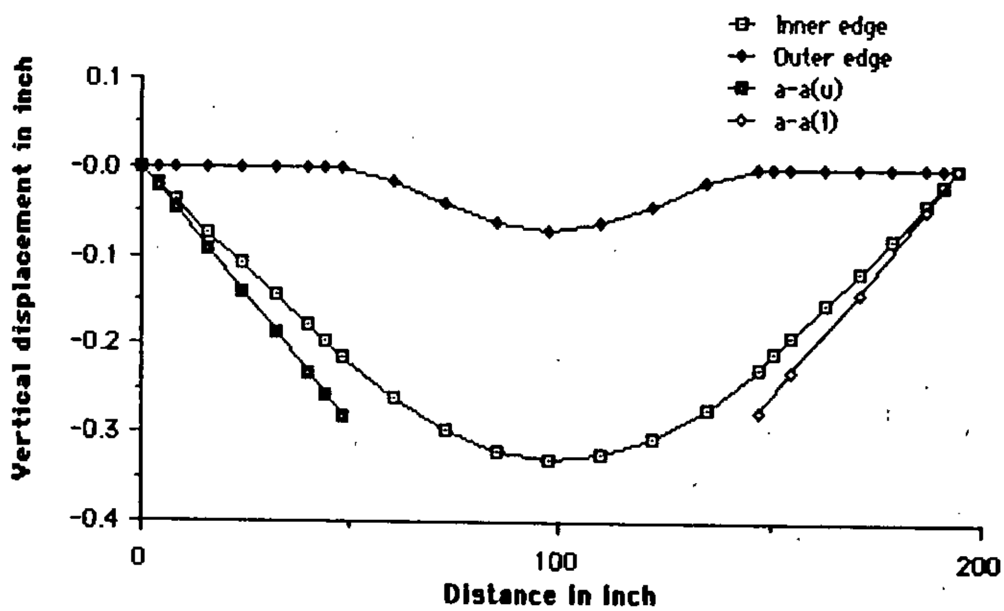
A double flight open-well stair shown in Fig.2.8, was analyzed using thick shell element with the finite element mesh configuration of Fig.3.6. From this result bending moments (MX) and vertical displacements (UZ) along the length of the stair are plotted and shown in Fig.3.21.

The results of ANSYS-thin shell analysis for the same problem are plotted in Fig.3.22 to Fig.3.24. The element mesh used in this case is shown in Fig.3.7. Figure.3.22 shows the bending moment(MX) and vertical displacement along the length of the stair. Besides plotting the inner edge and outer edge values of bending moment(MX) and displacements, the corresponding values at the center of landing (i.e. along section a-a in Fig.3.6 & 3.7) are also shown. An a-a(l) symbol in these figures indicates that the landing is at lower(l) level and a-a (u) indicates that at upper (u) level section for the first flight.

Axial forces in both longitudinal(X) and lateral(Y) direction are small in the case of a Open-well stair. Both thin shell and thick shell analyses results are in good agreement in this respect. Axial forces (FX) in the longitudinal direction is plotted in Fig.3.23 for a well opening of 48". The value of Lateral moments(MY) at landing are maximum near kink and gradually fades away towards the end of the landing. Variations of this moments and the vertical displacements at four different sections x1,x2,x3 and x4 (Fig.3.7) at intermediate landing are plotted in

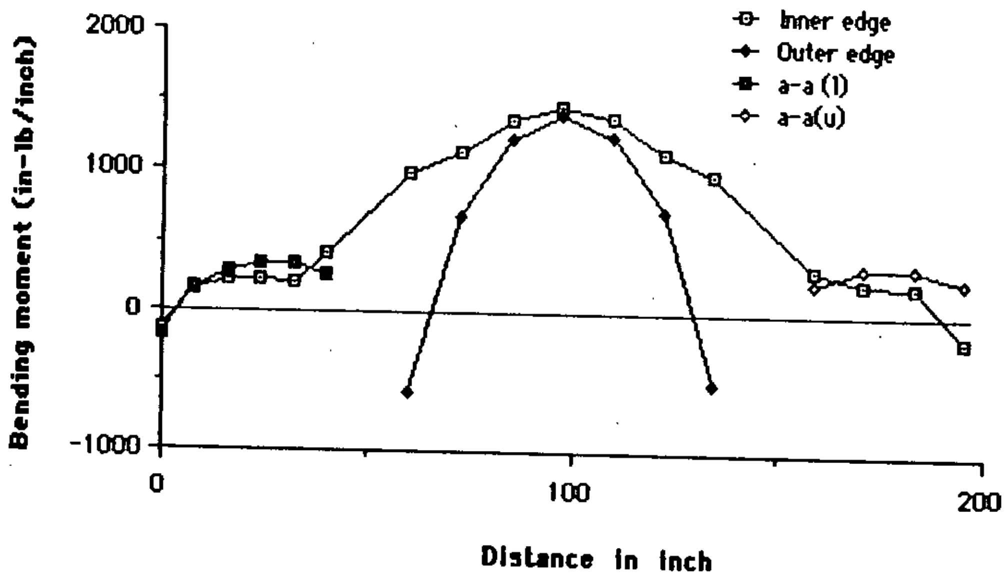


a) Bending moment (MX)

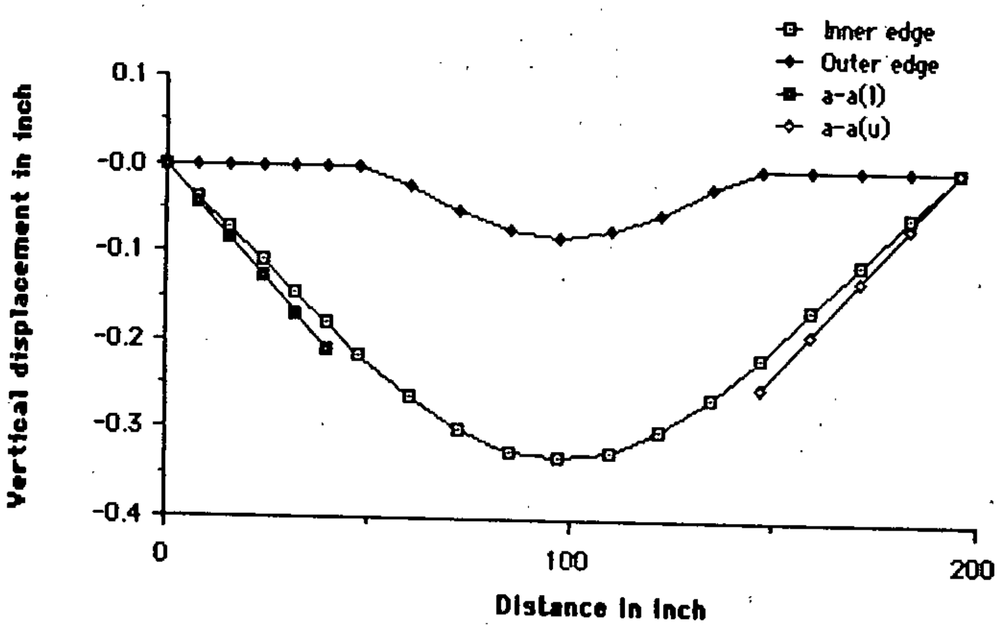


b) Deflection

Fig. 3.21 Results of Analysis of Open-well stair (c=48") using thick shell elements.



a) Bending moment (MX)



b) Deflection

Fig. 3.22 Results of Analysis of Open-well stair (c=48") using thin shell elements.

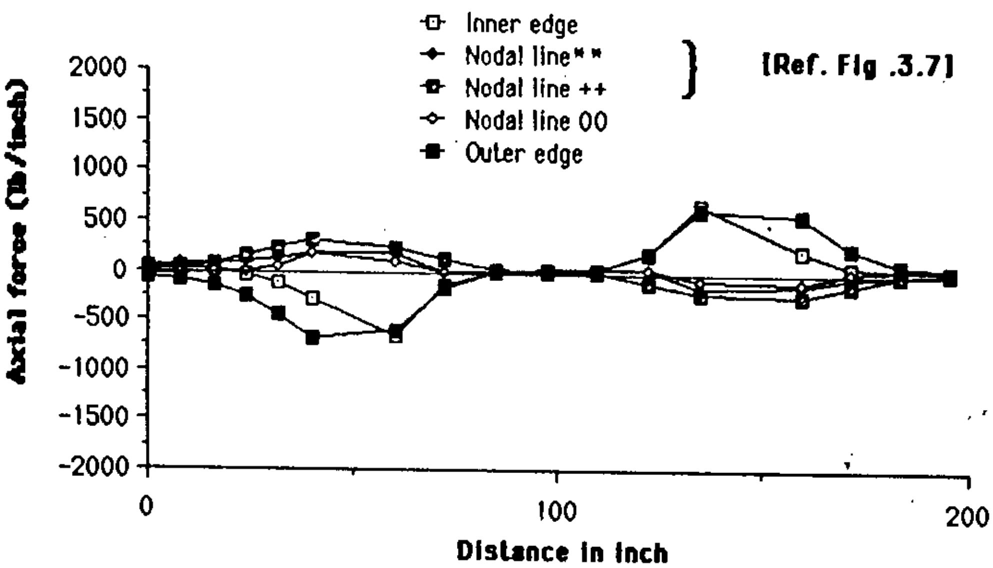
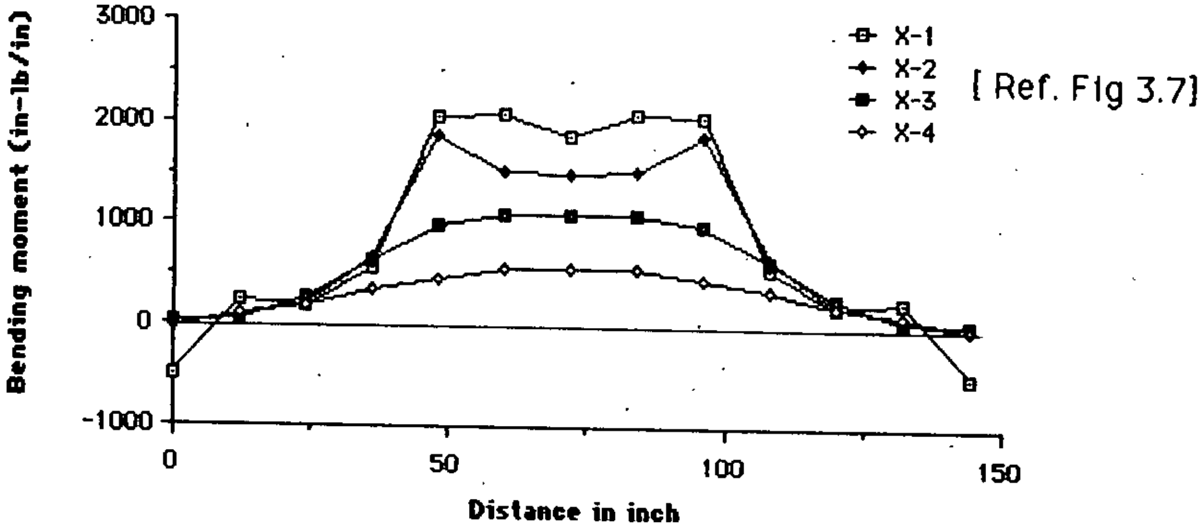
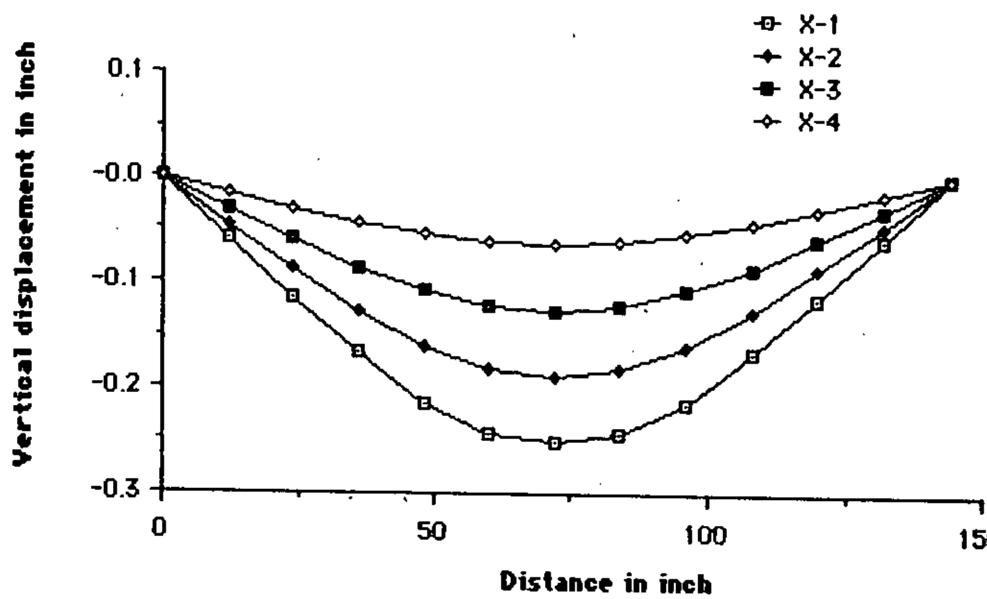


Fig. 3.23 Results of Analyis of Open-well stair (c=48") using thin shell elements.



a) Bending moment (MY)



b) Deflection

Fig. 3.24 Results of Analysis of Open-well stair (c=48") using thin shell elements.

Fig.3.24.

3.7.3 Displacement in Horizontal Plane

During the analysis of the stair slab it has been observed that a flight, at its upper kink have a tendency of moving sideways(Y) as a rigid body towards the next flight. This phenomenon has been observed in all the analyses, so far described. Fig.3.25(i) and Fig. 3.25(ii) show such displacements (in a horizontal plane) of inner edges for a 3-flight analysis of Dog-legged stair and a 2-flight analysis of open-well stair respectively. Both are the results of ANSYS stiff63 elements. Similar behavior is also observed with thick shell analysis.

3.8 Parametric Study

In an attempt to investigate the relative importance of the geometric parameters of the stair, a parametric study has been carried out. The parameters considered in this study are:

- i. Length of the landing slab, "a"
- ii. Horizontal projection of the waist slab (Going) ,"b"
- iii. Height of the flight, "h"
- iv. Width of the flight , "w"
- v. Opening ("c") of an Open-well stair

The relevant stair parameters are shown in Fig.3.26 .

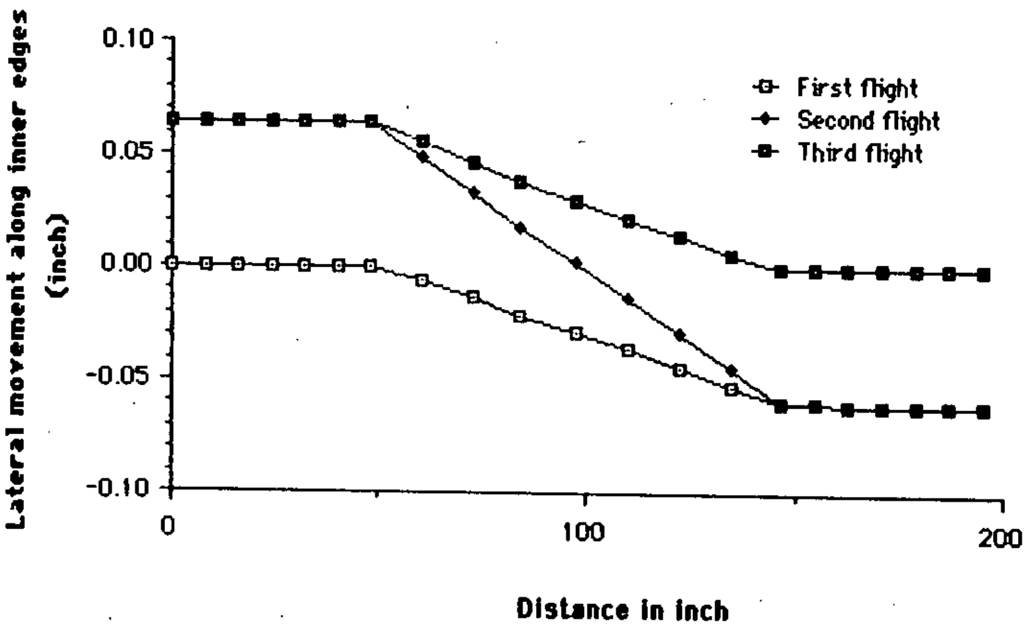


Fig. 3.25 (i) Movement of different flights of a Dog-legged Stair in a Horizontal Plane.

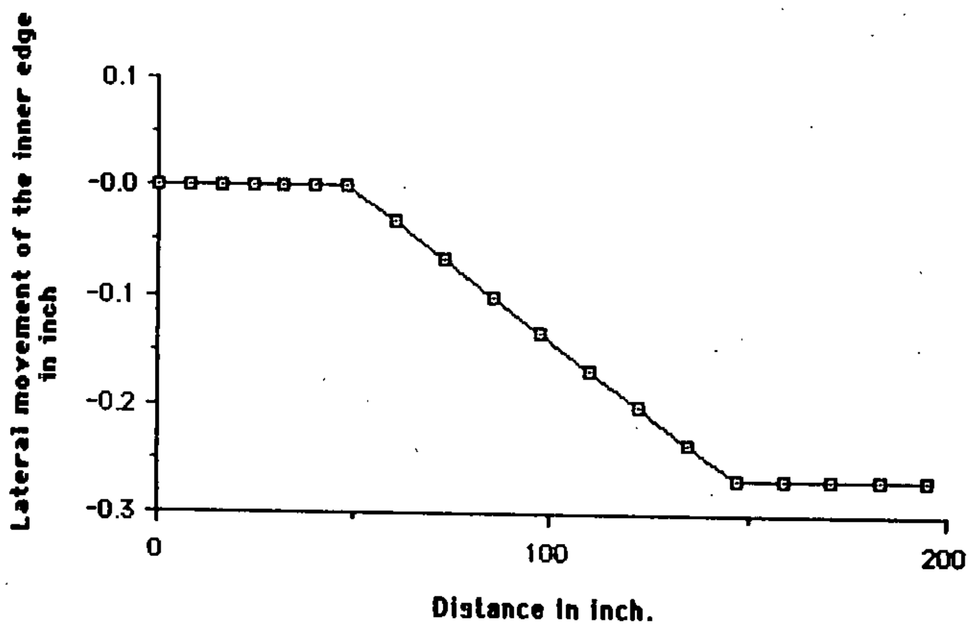


Fig. 3.25(ii) Movement of a flight of Open-well Stair in a Horizontal Plane.

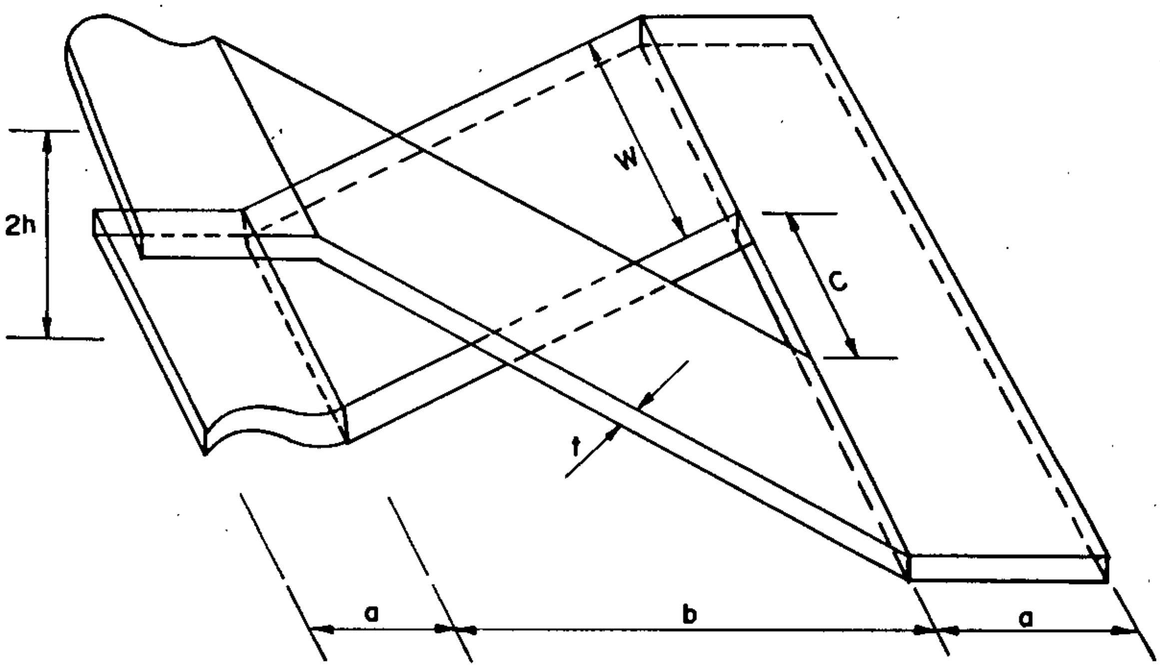


Fig. 3.26 Stair Parameters.

For a dog-legged stair parameter c is taken as zero. Each of the parameters were varied independently keeping the remaining parameters constant. Results of these variation are shown in Table 3.1 to 3.5. Effective span shown in these tables were computed on the basis that $qb_e^2/8$ should be equal to the maximum positive moment (mid span moment); Where b_e is the effective span (distance between the points of contraflexures), and q is the total load per unit area on the waist slab duly magnified on a horizontal projection.

In addition to studying the effect of geometric parameters of stair , effects of varying distribution of live load and varying support arrangements have also been studied. Results are summarized in the following articles.

3.8.1 Varying the Geometric Parameters

i) Varying the length of landing slab "a":

The parameter "a" shown in Fig 3.26 was varied from 48" to 60" at an interval of 6" .The variation of maximum positive moment and maximum negative moment with the variation of dimensionless parameter a/b are shown in fig 3.27(i). Corresponding variation of maximum vertical displacement is in fig.3.27 (ii).

ii) Varying the Height of the Flight "h":

The parameter ,h varies as the direct consequence of variation of height between floors , being served by the

TABLE 3.1 Variation of parameter 'a'

t in	a in	b in	l=(2a+b) in	h in	w in	a/b	Max. +ve moment. in-lb/in	Effective span(b_e) inch	$\frac{b_e}{b}$ x100 %
4	48	99	195	60	48	.485	975	71.78	72.5
	54	99	207	60	48	.545	923	69.83	70.5
	60	99	219	60	48	.606	913	69.46	70.16

TABLE 3.2 Variation of parameter 'h'

t in	a in	b in	l=(2a+b) in	h in	w in	h/b	Max. +ve moment. in-lb/in	Effective span(b_e) inch	$\frac{b_e}{b}$ x100 %
4	48	99	195	57	48	.575	941	70.52	71.22
	48	99	195	60	48	.606	975	71.78	72.50
	48	99	195	63	48	.636	998	72.62	73.35

TABLE 3.3 Variation of parameter 'b'

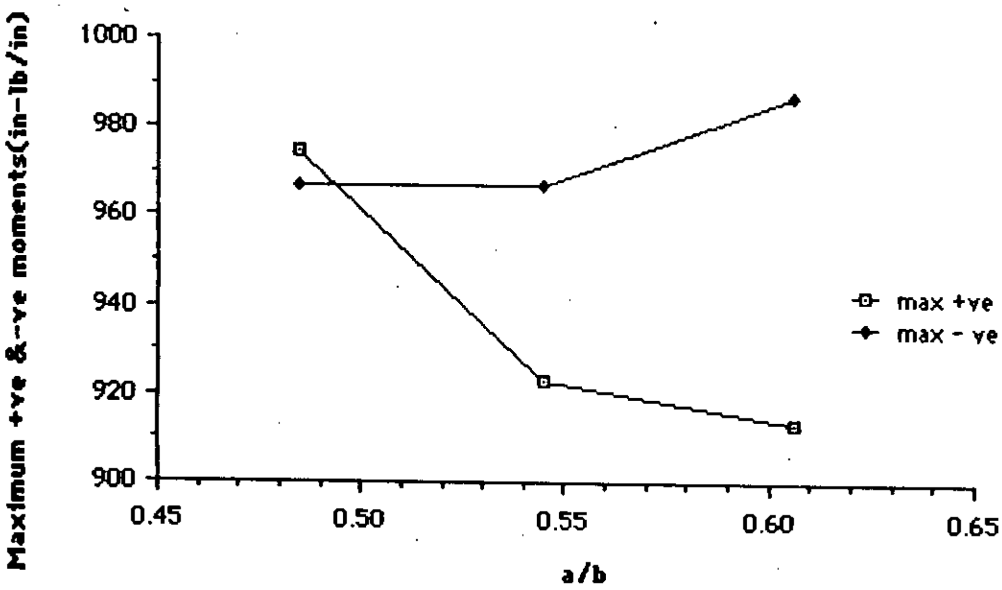
t in	a in	b in	l=(2a+b) in	h in	w in	b/l	Max. +ve moment. in-lb/in	Effective span(b_e) inch	$\frac{b_e}{b}$ x100 %
4	48	99	195	60	48	0.508	975	71.78	72.5
	48	105	201	60	48	0.522	1010	73.05	69.57
	48	111	207	60	48	0.536	1092	75.96	68.43

TABLE 3.4 Variation of flight width 'w' in a Dog-legged stair

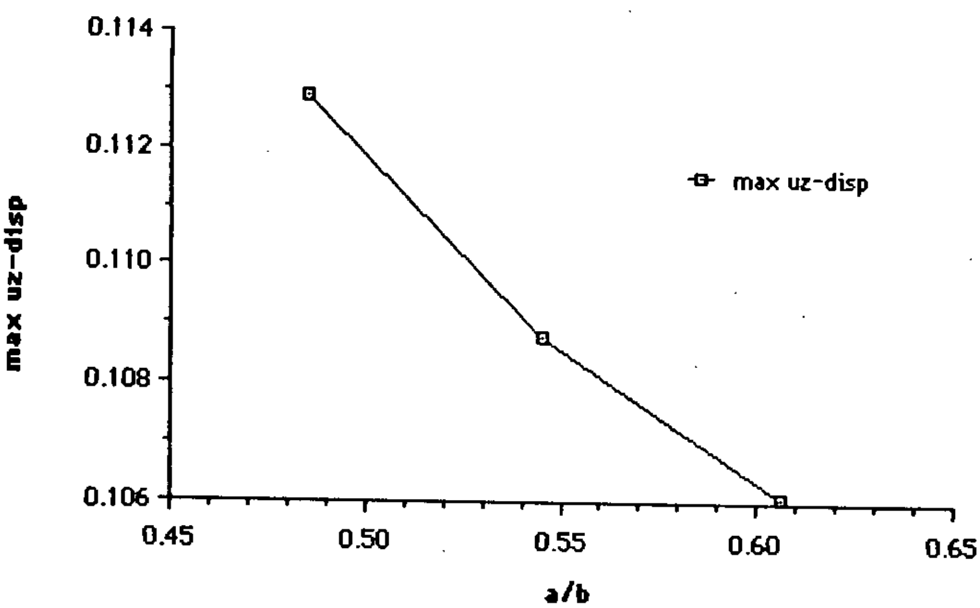
t in	a in	b in	l in	h in	w in	w/b	Max. +ve moment in-lb/in	Effective span b_e in	$\frac{b_e}{b} \times 100$
4	48	99	195	60	36	0.363	1013	73.16	73.9
	48	99	195	60	48	0.484	1138	77.54	78.33
	48	99	195	60	54	0.545	1210	79.96	79.96
	48	99	195	60	60	0.606	1289	82.53	83.36

TABLE 3.5 Variation of flight width 'w' in an Open-well stair

t in	a in	b in	l in	h in	c in	w in	Max. +ve moment in-lb/in
4	48	99	195	60	48	36	1188
	48	99	195	60	48	48	1474
	48	99	195	60	48	60	1833



(i) Effect of variation of parameter 'a'. on moment (MX)



(ii) Effect of variation of parameter 'a'. on deflection

Fig 3.27 Results of variation of the length of landing 'a' stair. Parameter "h" was varied from 57" to 66" at an interval of 3".The variation of maximum positive moment and maximum negative moment with the variation of dimensionless parameter h/b are shown in fig 3.28(i). Corresponding variation of maximum vertical displacement is shown in Fig.3.28(ii).

iii) Varying the Going of the Stair "b":

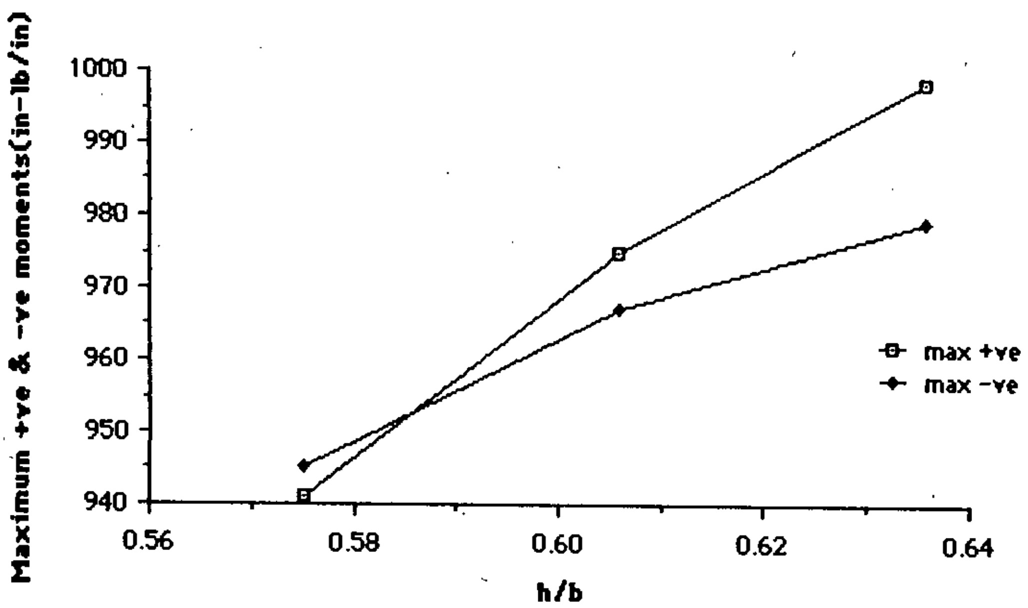
The parameter "b" was varied from 99" to 111" at an interval of 6". The variation of maximum positive moment and maximum negative moment with the variation of dimensionless parameter b/l are shown in Fig.3.29(i). Corresponding variation of maximum vertical displacement is in Fig.3.29(ii).

iv) Varying the Width of the Flight "w"

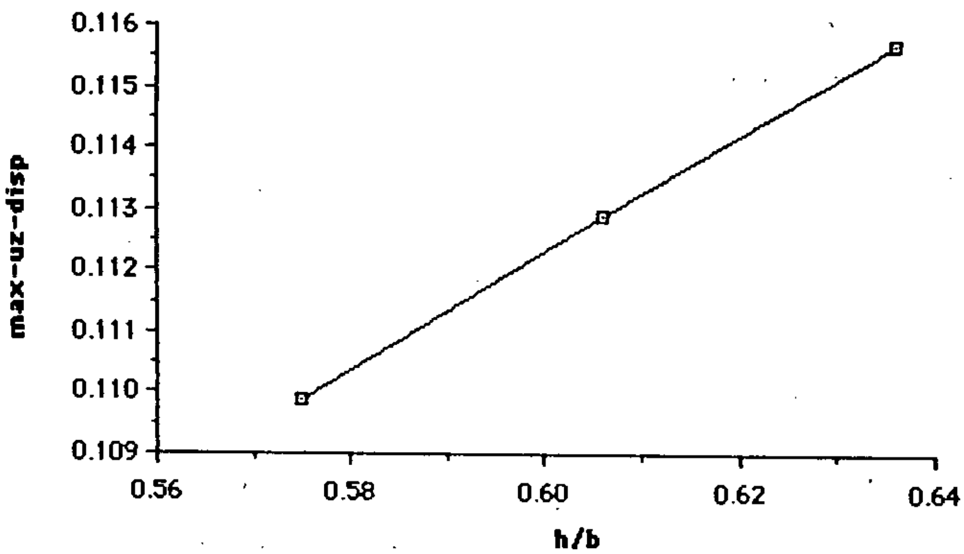
Parameter,w , the width of the flight has been varied from 36" to 60". Results of this variation are presented as function of dimensionless quantity w/b in Fig. 3.30

v) Varying the well opening ("c") in an open well stair:

In an open-well stair the effect of varying the well opening from 12" to 48" was studied. Unlike Dog-legged stair, the open well stair analysis by both thin and thick shell elements did not produce negative bending moment(MX) near kink in the inner edge.(Fig.3.21 and 3.22) . But the behavior at outer edge is similar to the Dog-legged stair;

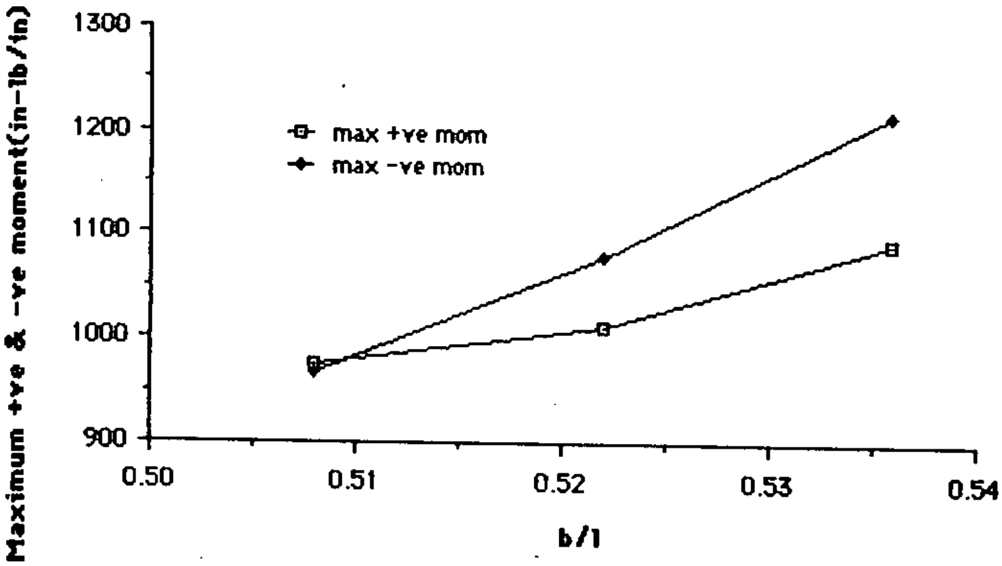


(i) Effect of variation of parameter 'h'. on moment (MX)

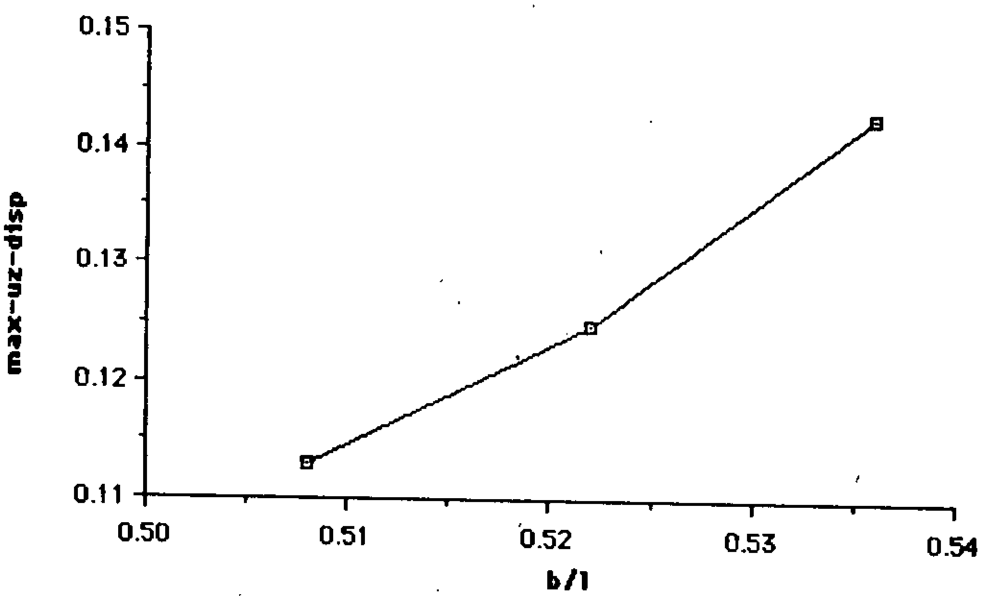


(ii) Effect of variation of parameter 'h'. on deflection

Fig 3.28 Results of variation of the height 'h' of the flight



(i) Effect of variation of parameter 'b' on moment (MX).



(ii) Effect of variation of parameter 'b'.on deflection

Fig 3.29 Results of variation of the going 'b' of the stair

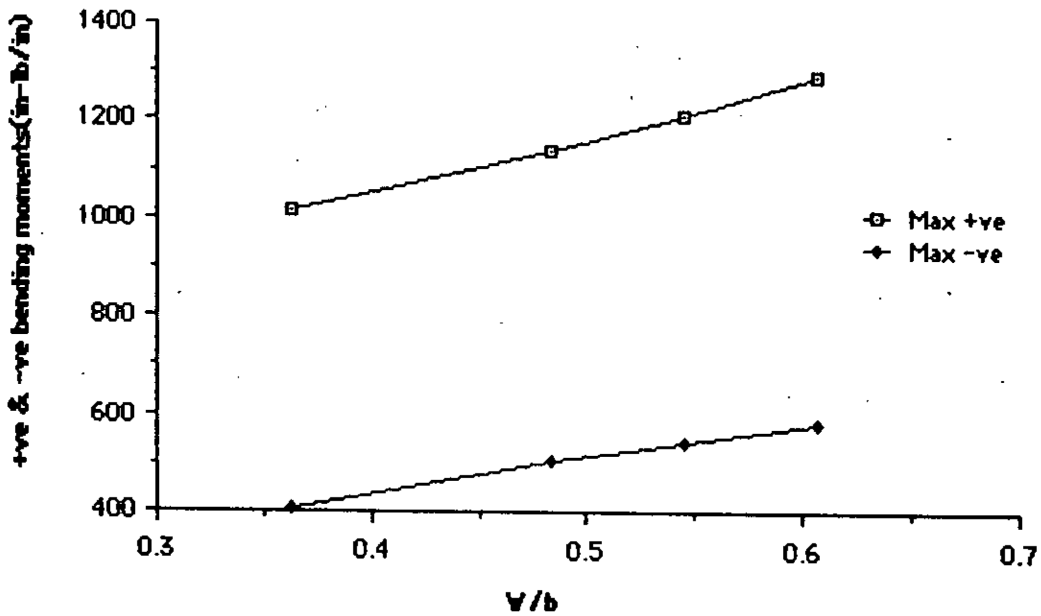


Fig 3.30 Results of variation of the width 'w' of the flight

i.e. sufficient restraining effect is present to produce negative bending, near kink zone. The restraining effect provided by the supporting outside wall could only travel a small distance towards the inner edge. This is shown in Fig. 3.31(i) to Fig. 3.31(v) by drawing the variation of bending moments(MX) at line Z1, Z2, Z3, Z4 and Z5 (shown in Fig. 3.7) for well opening 12" to 48".

The bending moment in lateral direction (Y) are significant at landing level. For obvious reason these moments(MY) are maximum near kink (x1 line on Fig. 3.7) and gradually they fade away towards the end of landing. This was reported earlier in Fig. 3.23 for well opening of 48". The variation of this lateral moment(MY) along Z6 line (Fig. 3.7) for different well opening (c=12" to 48") is shown in Fig. 3.32.

vi) Varying the width ("w") of the flight for an Open-well stair:

The effect of varying the flight width, w on the behavior of a Open-well stair (with C=48") are studied and results are shown in Fig3.33. and Fig.3.34. In this case parameter w was varied from 36" to 60" at an interval of 12". Figure 3.33 shows the way the maximum positive and negative moment(MX) varies with the dimensionless quantity w/b. Figure 3.34 shows the variation of lateral moment(MY) along Z6 line for different values of w/b.

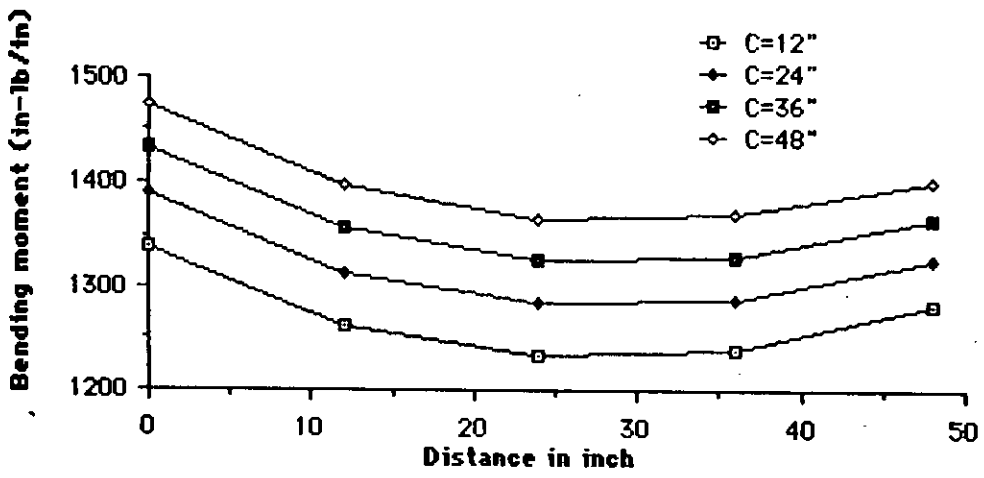


Fig. 3.31(i) Variation of moment (MX) at Z1 line

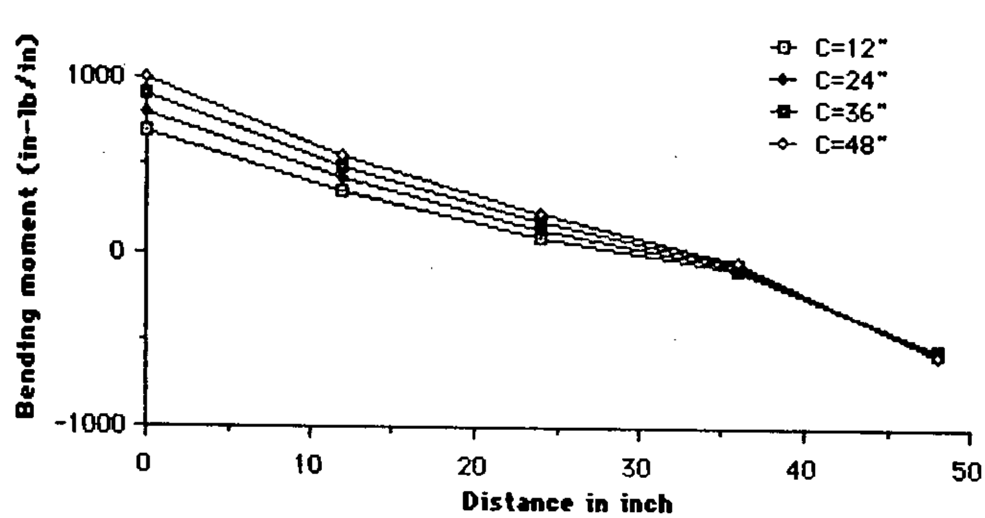


Fig. 3.31(ii) Variation of moment(MX) at Z2 line

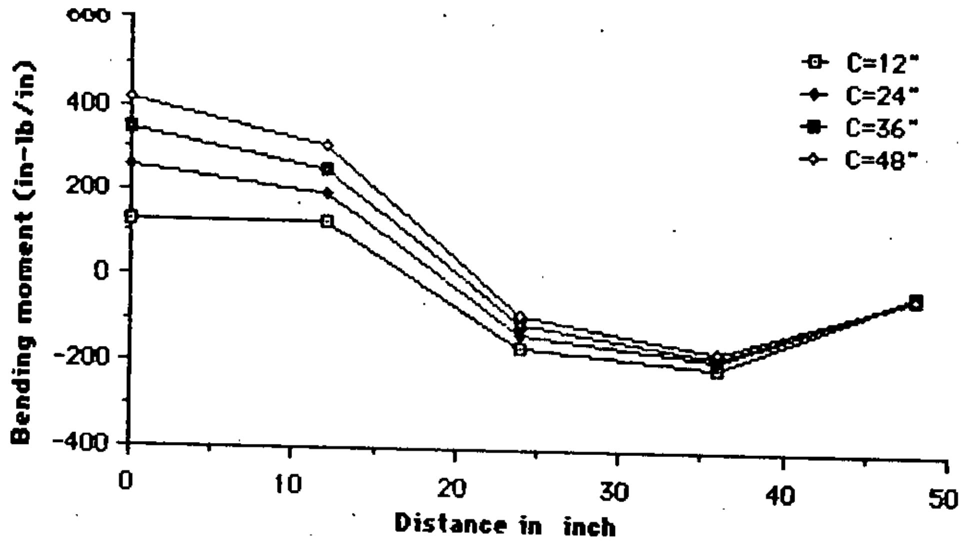


Fig. 3.31(iii) Variation of moment along Z3 line

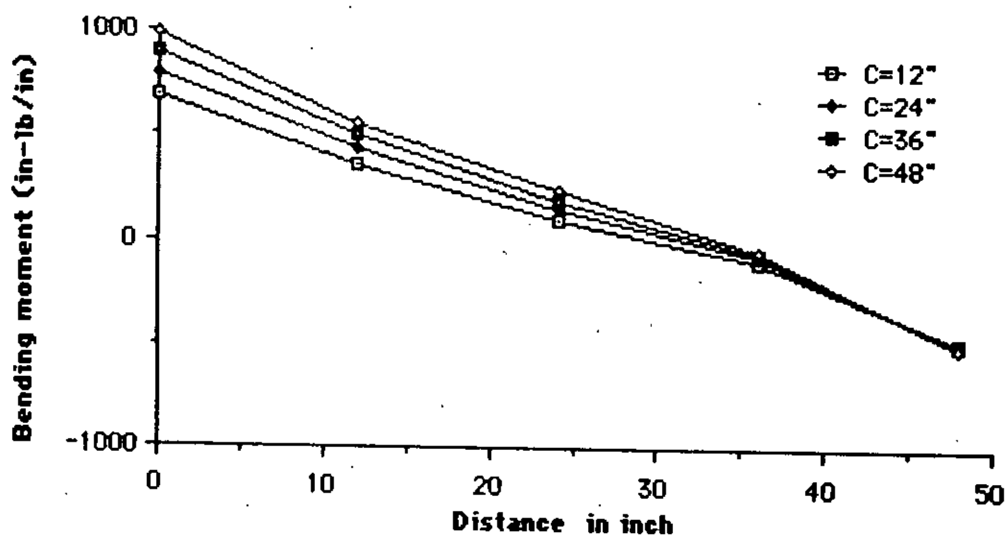


Fig. 3.31(iv) Variation of moment along Z4 line

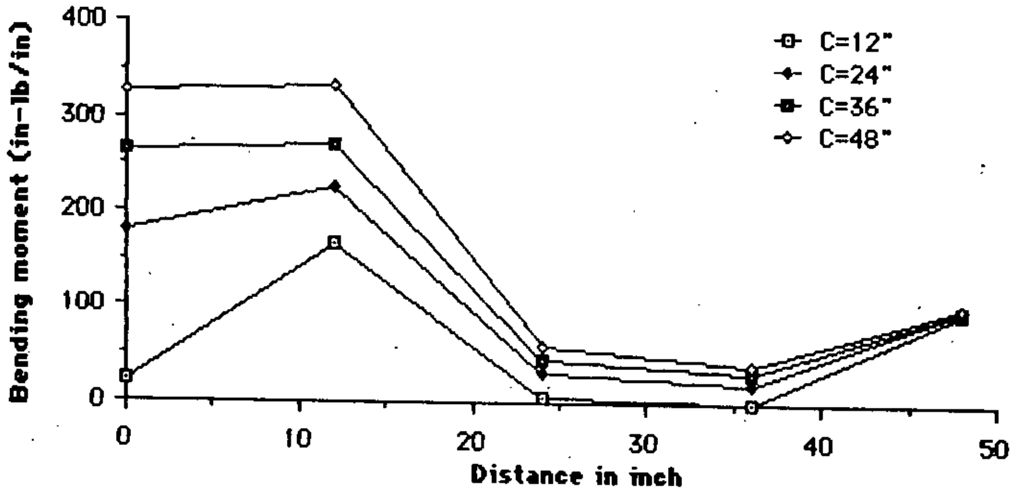


Fig. 3.31(v) Variation of moment along Z5 line

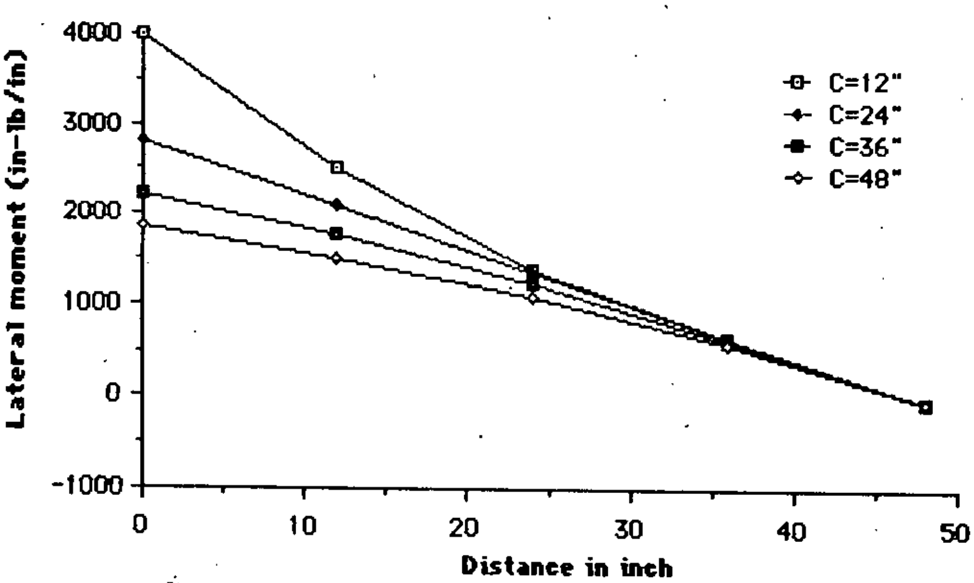


Fig. 3.32 Variation of moment (MY) along Z6 line

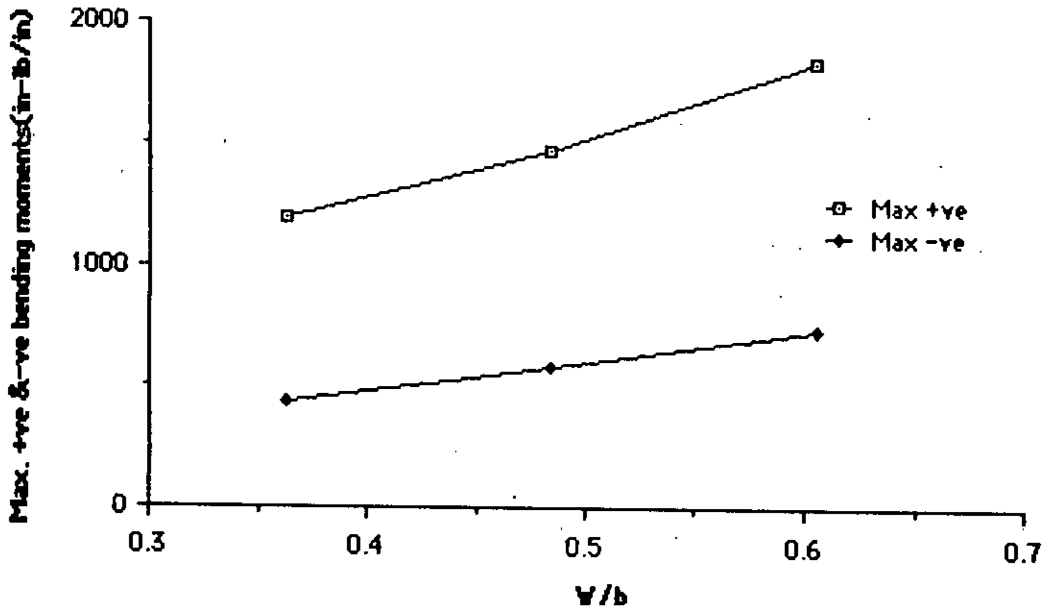


Fig. 3.33 Effects of variation of parameter 'w' in an Open-well stair. (c=48")

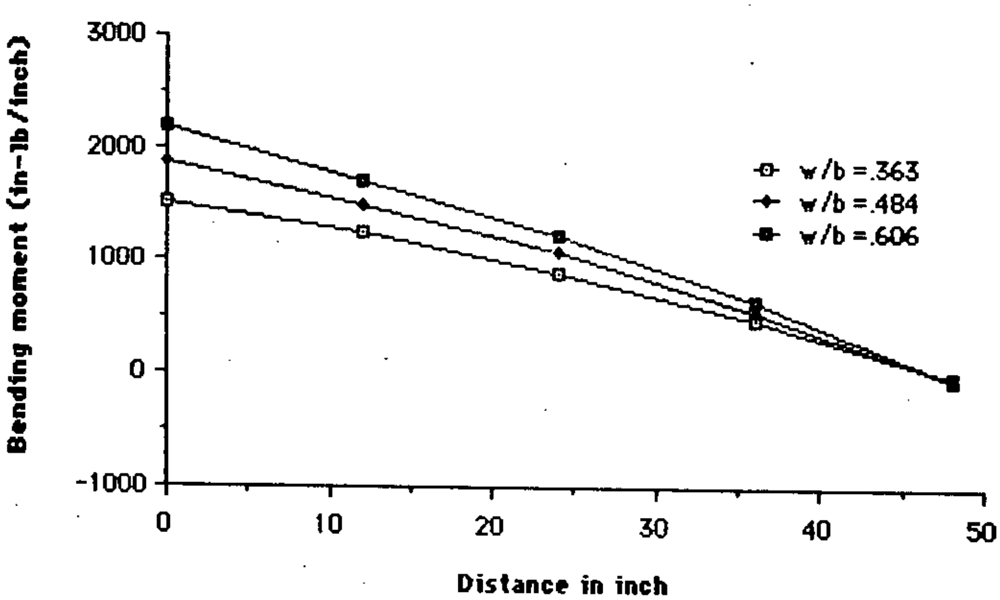


Fig. 3.34 Variation of moment [MY] at Z6 line for varying parameter 'w'. (c=48").

3.8.2 Varying Load Cases

The stiff 63 (ANSYS) element with element mesh of Fig. 3.5 and 3-flight configuration was implemented by varying live load. First, full live load was applied on the waist slab and the lower landing (of 2nd flight) while the upper landing was loaded with dead load only; the other two flights were loaded with full live load. The results of this loading variation is compared in Fig. 3.35 with the results of loads on all panel.

The next variation of load was done by imposing full live load on the waist slab portion of the 2nd flight while the two landing slabs of the 2nd flight were loaded with dead load only. The first and 3rd flight was completely loaded with full live load in both cases. Loading arrangement and resulting bending moment diagram for 2nd flight is plotted in Fig. 3.36. The bending moment diagram of the same structure with live load on all waist and landing slabs is also shown in the same diagram.

3.8.3 Varying Support Arrangements

So far , only one type of supporting arrangements for the stair slab has been considered (Fig.2.3, 2.4, 3.1, 3.2) for the present study. In order to make the present study more comprehensive other supporting arrangements have also been considered. They are described here:

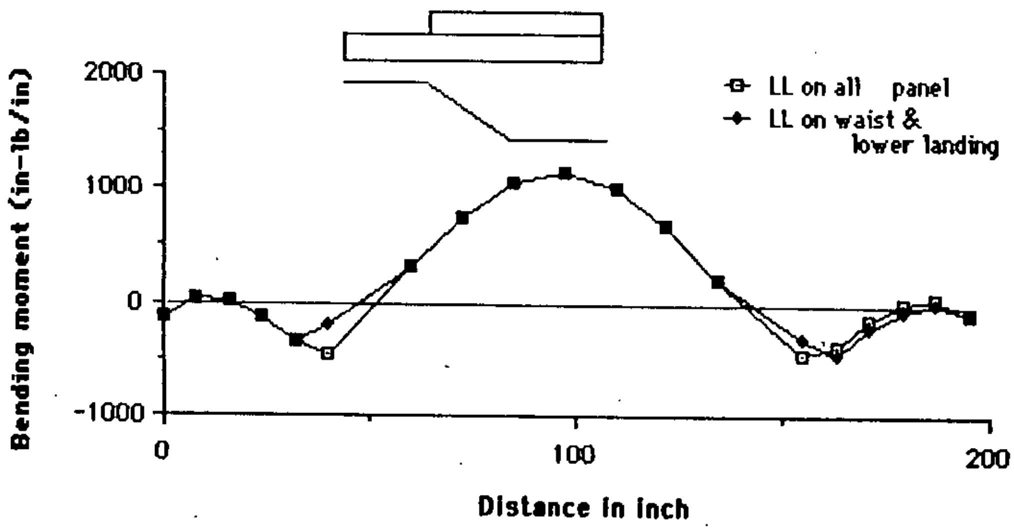


Fig 3.35 Inner edge bending moment for different load cases

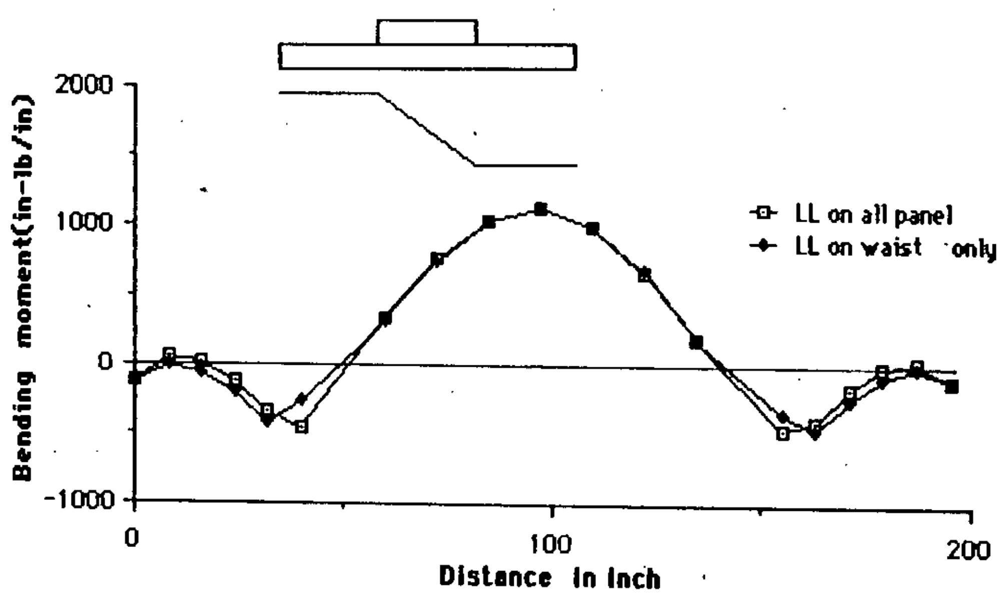
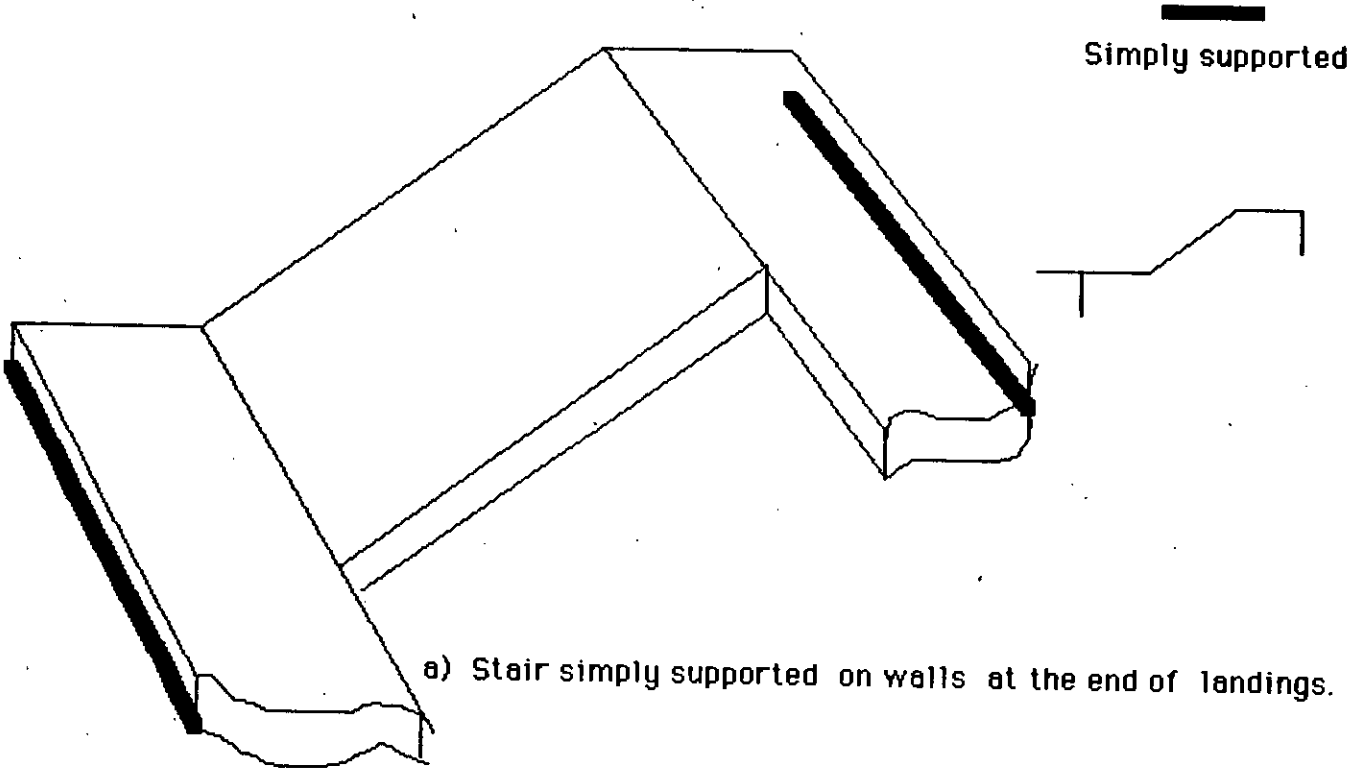


Fig 3.36 Inner edge bending moment for different load cases.

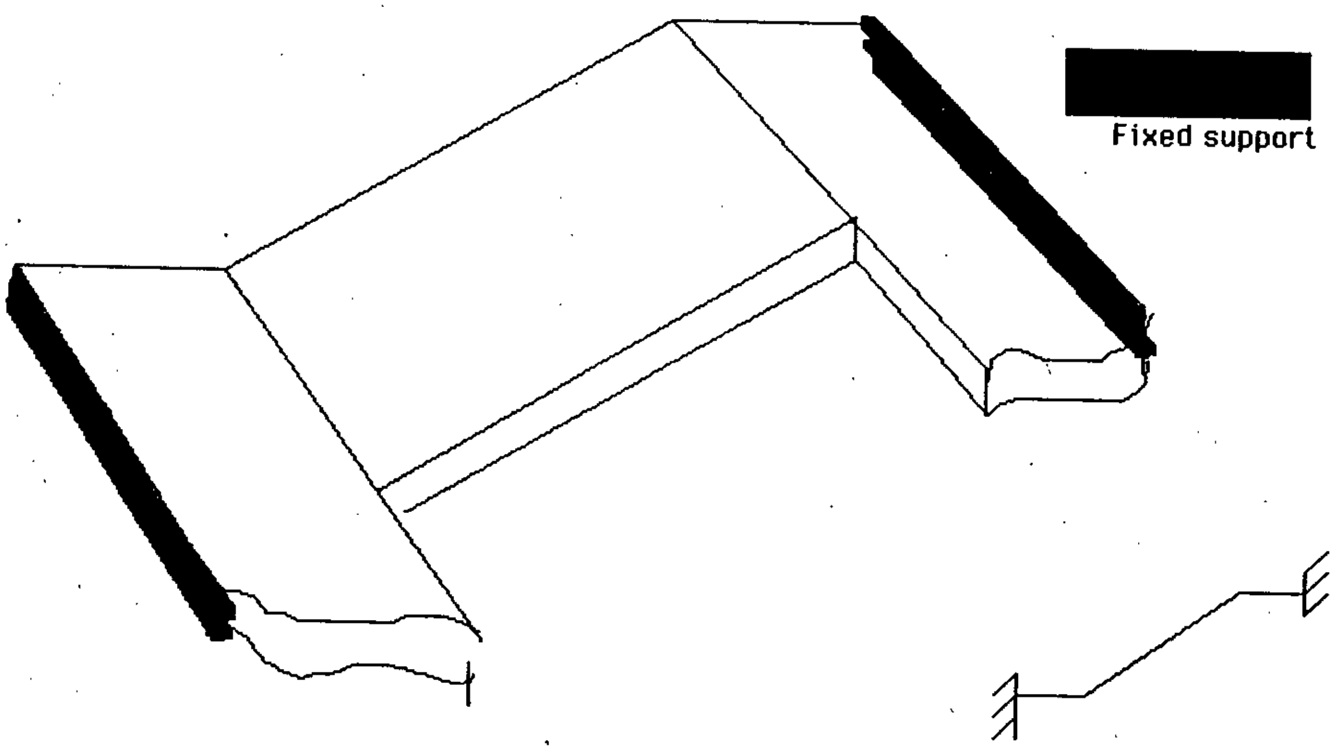
i. Stair slab simply supported on walls at the end of both landings.(Fig.3.37(a))

ii. Stair slab completely fixed at wall at the end of both landings.(Fig3.37(b))

Analyses of these cases have been made using 3-flight analysis by ANSYS with full live load. Both inner and outer edge values of bending moment along the length of flight are reported along with the corresponding vertical displacements in Fig.3.38 and Fig.3.39 respectively for the above two cases.

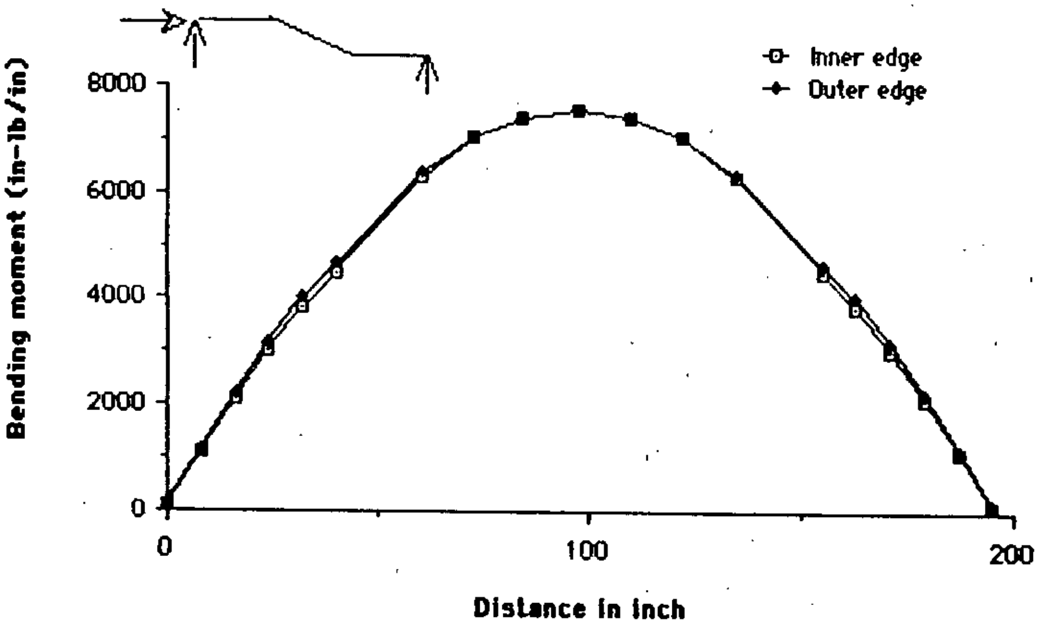


a) Stair simply supported on walls at the end of landings.

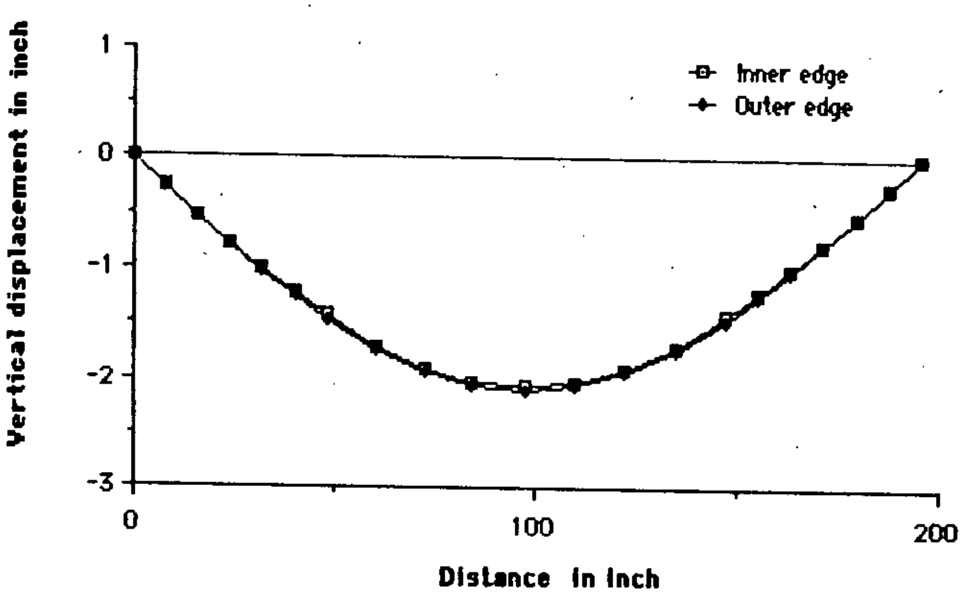


b) Stair, fixed on walls at the end of landings.

Fig . 3.37 Showing support arrangements considered for analysis scheme of Art.3.9.3

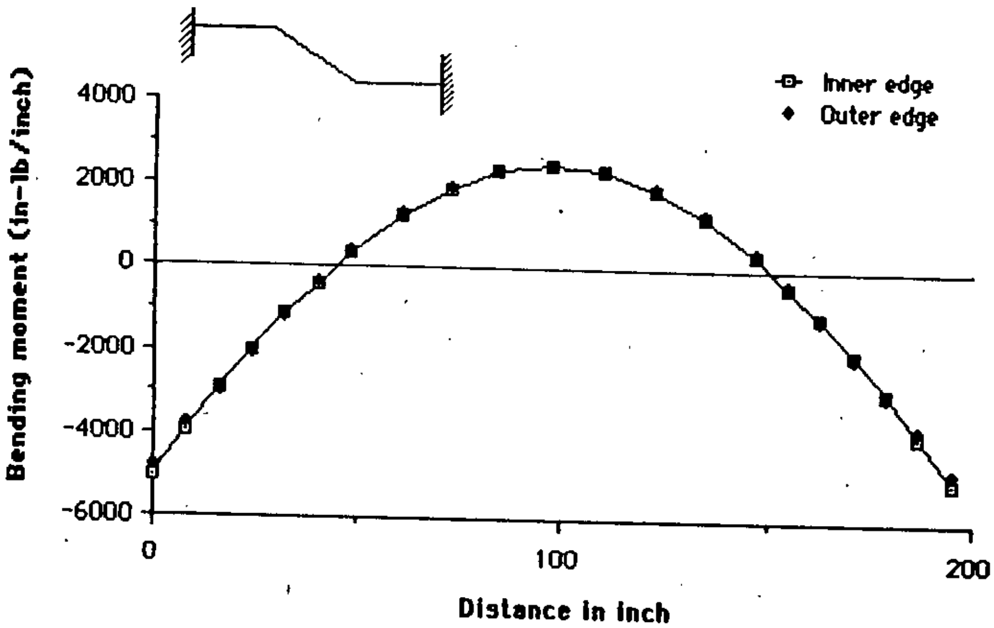


a) Bending Moment (MX)

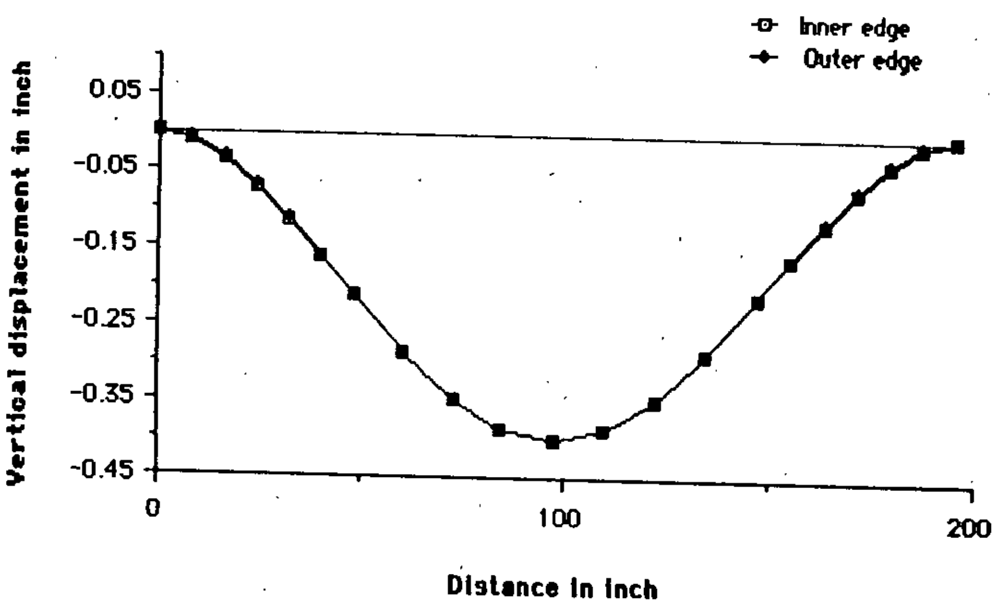


b) Deflection (UZ)

Fig 3.38 Results of analyis of stair simply supported at end



a) Bending Moment (MX)



b) Deflection (UZ)

Fig 3.39 Results of analyis of stair completely fixed at end.

CHAPTER 4
INTERPRETATION OF RESULTS

4.1 General

The aim of the present study was to investigate the behavior of stair slabs and make necessary recommendations to improve design guidelines for stair slabs. In order to establish the behavior of stair under the action of load , the results of chapter 3 is discussed in the following articles.

The present study restricts its discussions to the general arrangement of stair described in Fig. 3.1 and Fig.3.2, in which the landing slab supports the waist slab, running perpendicular to the landing slab. The landing slab is supported by bearing walls or beams along its three edges, while the fourth edge, which supports the waist slab is free. This free edge provides an effective support for the waist slab. The restraint provided by this support is clearly visible from the bending moment diagrams of the stair slabs.

With the general boundary condition described above, different features of the findings relating to the Dog-legged and Open-well stair are discussed in the following articles.

4.2 Analysis of Dog-legged Stair

For Dog-legged stair, results of analysis for two-flight and three-flight have already been presented in Chapter 3.

Those results will be discussed here to establish the general behavior of stair.

4.2. 1 Analysis for 2-flight

a) Flexural Behavior:

The bending moments are computed from the nodal stresses, which are average values at the node for the adjacent elements. The nodal stress values are in element co-ordinate system (local) and it will not be appropriate to average the nodal stress for the nodes along the kink line. Since adjacent element stresses at a kink node differ in direction. For this reason the kink point nodal stress has been neglected for computation of bending moments. However, when a finer mesh is used the average trend for the distribution of moment may be clearly visualized.

In Fig. 3.9 the bending moments for 1st flight is shown as obtained from thick shell analysis for 2-flights. The element near to the left wall support gave stresses that are unduly high. This edge was simply vertically and horizontally restrained (Fig. 3.1). And at this edge existence of such high stresses are very unlikely to occur. This could be because of some spurious behavior of the edge element, coupled with the undue effect of using a rather coarse mesh. However in other locations the stresses are quite reasonable.

The landing slab, supported by edge wall at outer edges provides sufficient restraint to reduce the mid span positive moments and produce negative moments of appreciable magnitude near kink line. Despite the fact that only outer edges (of a flight) are supported by edge wall through the whole length of the landing, the travel of this restraining effect towards the inner edge is also significant.

From Fig. 3.9, it can be seen that the flexural behavior at inner and outer edge are not that much different as to demand separate consideration , so far as flexural design is concerned. Although in the positive moment region the outer edge bending moments are smaller than the corresponding inner edge value, the negative values near kink zone are higher at outer edge. In spite of the fact that the location for the maximum negative moment is at kink, we do not have the moment values there, for the reason stated earlier. However a visual extrapolation of these outer edge bending moment values gives maximum negative moment (at kink) of comparable magnitude with the positive moment at midspan. As an alternative attempt this negative moment will be calculated from the stresses of the individual elements at kink location. This attempt is deferred at this stage and will be presented (Art: 4.4) in a separate article as soon as the preliminary discussions on the overall flexural behavior is completed.

The results obtained by using the thin shell element (stiff63, ANSYS) with a finer mesh, reflects the same flexural behavior(Fig.3.11). Due to this refinement reasonably smooth curves are obtained simply by joining the points by line graph. The critical values for negative moments (at kink) are again of magnitude almost equal to the magnitude of maximum positive (mid span) moment. The distance between the point of contraflexures is about 80% of the going. This will in fact, be of smaller value because the bending moment diagrams, plotted here, (Fig.3.9 and 3.11), have, in general, a tendency of getting flat near kink in the absence of the kink point bending moment values. Had the kink point values been included, the moment diagram would have indicated a smaller distance between the point of contraflexures, i.e. a smaller effective span would have resulted. This effective span, then must give the maximum mid span positive moment by the relation $qb_e^2/8$, for which the effective span b_e is about 75 %. (q, be the total load per unit area on the waist slab, duly magnified over a horizontal projection.)

The displacement diagram of Fig. 3.9 and Fig. 3.11 also connote the flexural response reflected in the bending moment diagrams. The outer edge, being supported over the entire lengths of the landings at both ends and hence deflects less than the inner edge. The inner edge deflection curve matches very closely with the outer edge one. The hogging shape near kink zone reflects the same tone as do the bending moment diagrams.

b) Inplane Forces

In addition to the flexural behavior of stair slab as a one way slab along the length of the flight inplane forces of appreciable magnitude has also been reported in Chapter 3. These axial forces along the length of the slab are presented in Fig. 3.10. Fig. 3.12 for 2-flight analysis and in Fig. 3.14, 3.16 and 3.18 for a 3-flight analysis.

For the 2-flight analysis by thick shell elements axial force of appreciable magnitude are present at outer edges and near kink zone. From Fig. 3.10 it can be seen that at the second flight, the forces are simply opposite in sense to the corresponding forces at the first flight. Also it is worth noticing that such high stresses are only at floor level kink zone, while the mid-landing kink zone is not subjected to such high axial stresses. The opposing nature of the stresses in first and second flight indicates the anti-symmetric behavior.

The ANSYS stiff63 elements provide the same pattern of axial forces as do the thick shell elements(Fig 3.12). The outer edges are highly stressed at floor level kink. The magnitude of the peak stress, in this case, is much higher than the previous analysis by thick shell elements. Because of finer mesh used in implementing ANSYS, the structure is more flexible in this case. The variation of axial stresses between the inner edge and outer edge is also shown in Fig. 3.12 (Nodal lines as shown in Fig. 3.12 are defined in Fig. 3.4).

The presence of high axial stress near floor level kink point seems unrealistic. Because if it is assumed that the waist portion of the slab is simply hanged at top and bottom kink with full of its weight, the resulting axial stress will not be greater than 20 psi. The high axial stress obtained, could be either a spurious behavior of the edge element or the effect of the imposed boundary conditions. In an attempt to investigate whether such high axial stresses actually exist or not, the mesh of Fig.3.4 (mesh-1) was changed to such a configuration so that the the outer strip of element is only 6" in width(let us call it mesh-2). The idea was that, had there been any disturbance in the edge element it would be limited within the small strip of outer edge elements. The variation of axial stresses due to this change in mesh configuration is presented in Fig. 4.1 (i) to Fig.4.1(ix). These figures shows the distribution of axial forces(FX) along the width of the stair slab at different location (Fig. 3.4). The axial forces for both the flights of a two-flight analysis are presented(Fig 3.12). These figures also reveal the anti-symmetric response of the two-flight dog-legged stair. That means at any particular location if there acts an axial compression in the first flight, axial tension of same magnitude acts in the second flight. It can be seen explicitly from these figures that change in element mesh configuration from mesh-1 to mesh-2 do not change the stresses. This implies that the stresses obtained here are the correct stresses (if not true stresses) for the model considered for analysis.

4.2.2 Analysis for Three flight

The flexural behavior for a two-flight Dog-legged stair were quite reasonable. But the presence of high axial stress, although very localized, can hardly be justified. Figure 3.1 shows the boundary conditions for 2-flight analysis, where

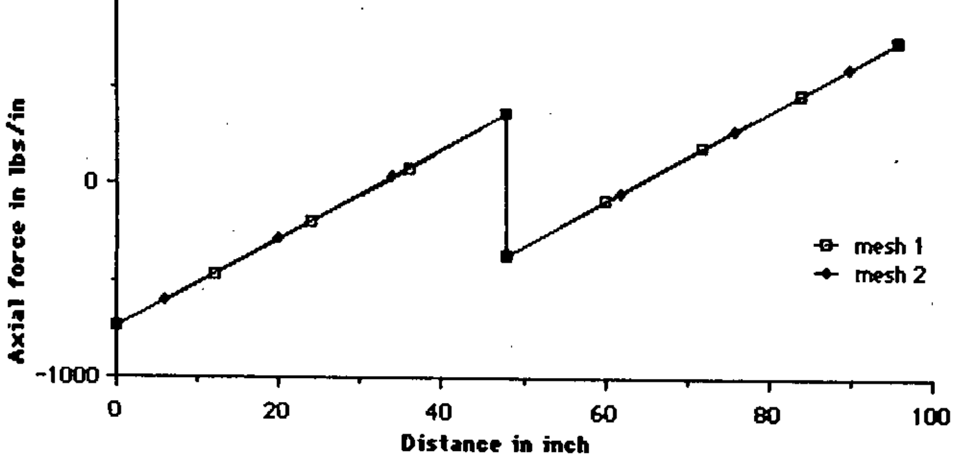


Fig 4.1 (i) Distribution of axial forces along line L.1

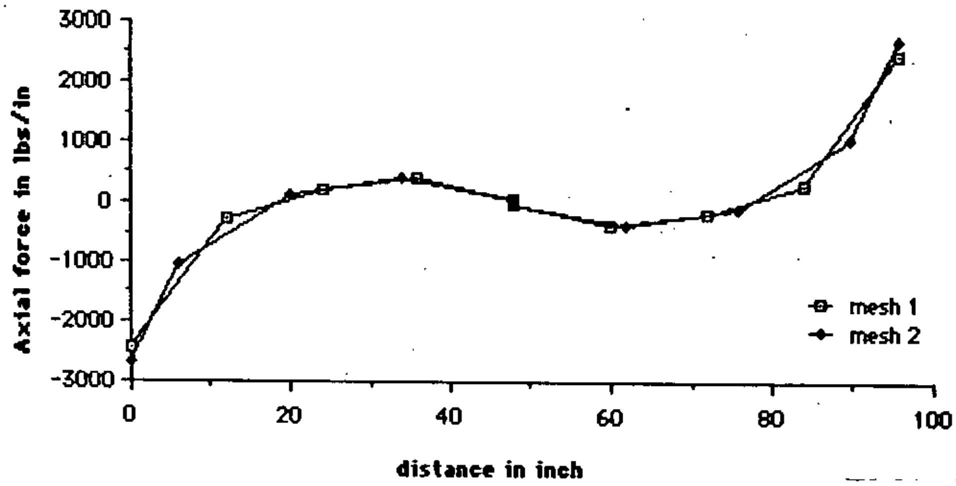


Fig 4.1 (ii) Distribution of axial forces along line L2

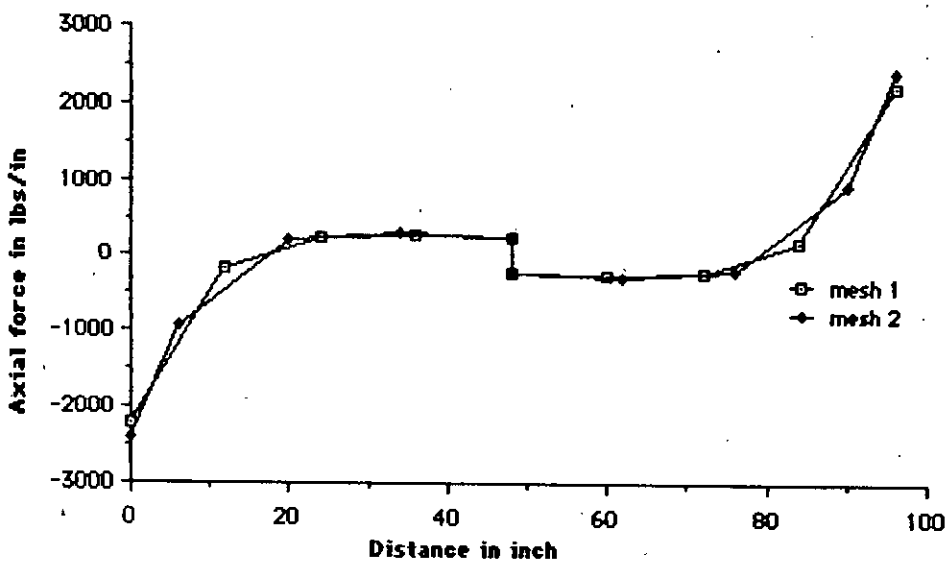


Fig 4.1 (iii) Distribution of axial forces along line L3

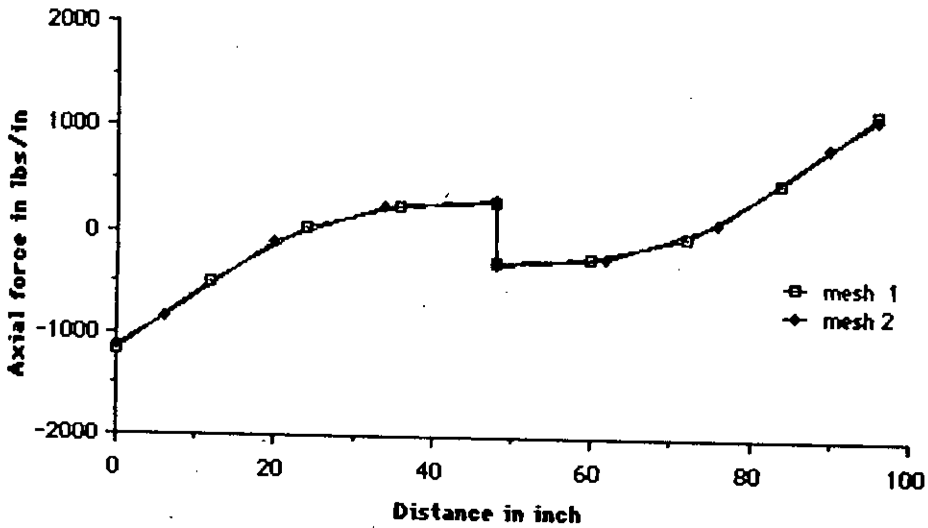


Fig 4.1 (iv) Distribution of axial forces along line L4

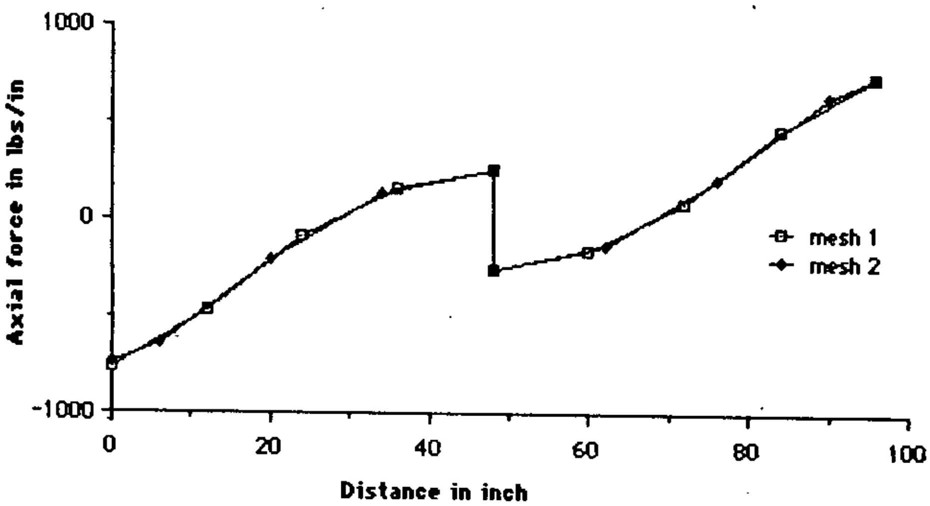


Fig 4.1 (v) Distribution of axial forces along line L5

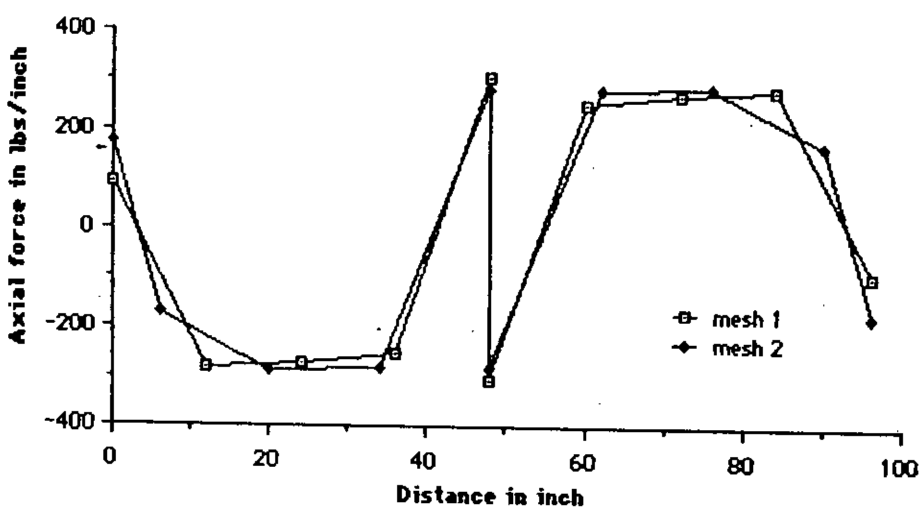


Fig 4.1 (vi) Distribution of axial forces along line L6

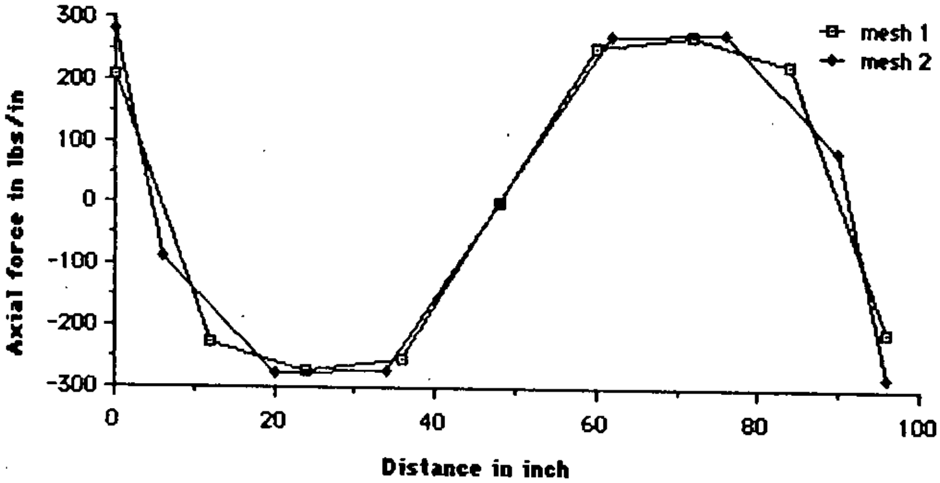


Fig 4.1 (vii) Distribution of axial forces along line L7

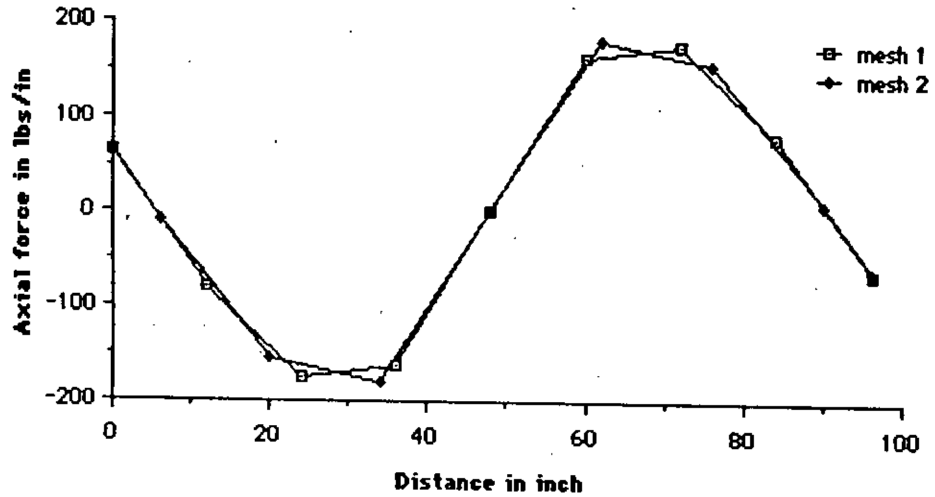


Fig 4.1 (viii) Distribution of axial forces along line L8

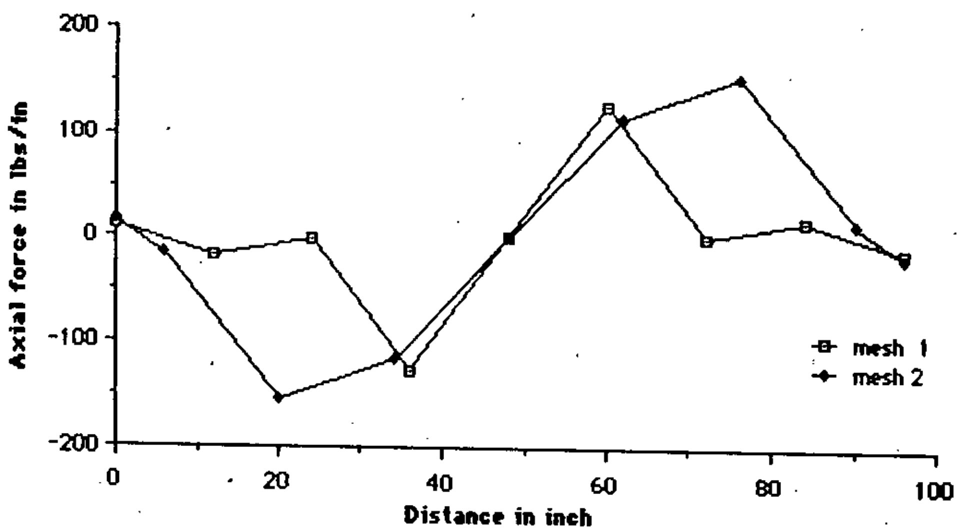


Fig 4.1 (ix) Distribution of axial forces along line L9

the possibility of local disturbance due to the applied boundary condition can not be completely ruled out. In a bid to disentangle the analysis from such imposed conditions, analysis of a 3-flight extension for the Dog-legged stair of same dimension was performed. The boundary condition for this case is given in Fig. 3.2, where from it is clear that the 2nd-flight will be free from local disturbances due to the applied boundary conditions.

In Fig. 4.2 the vertical displacements of the inner edges of the first and second flight of a three flight analysis are compared. From this comparison, the local disturbances in first flight at the locations of applied boundary conditions are quite clear. The symmetry conditions UY=0 and ROTX=0 were applied(article 3.6), allowing free vertical movement at the inner edge of the landing slab. But in actual case where the flight is continued to the next, there will be some restraint in the vertical direction, as well. In the second flight, the displacement is (from Fig. 4.2) more rational and complete symmetry about the mid-span is visible.

It is also worth mentioning, that overall vertical displacement is higher for the 2nd flight than that of the 1st flight, and the maximum values of these displacements vary by more than 20%. Inspite of this variation, the curvature of these two cases are comparable at sufficient distances from the location of imposed boundary conditions. The hogging curvature near kink zone is an indication of the much restraint provided by landing slab.

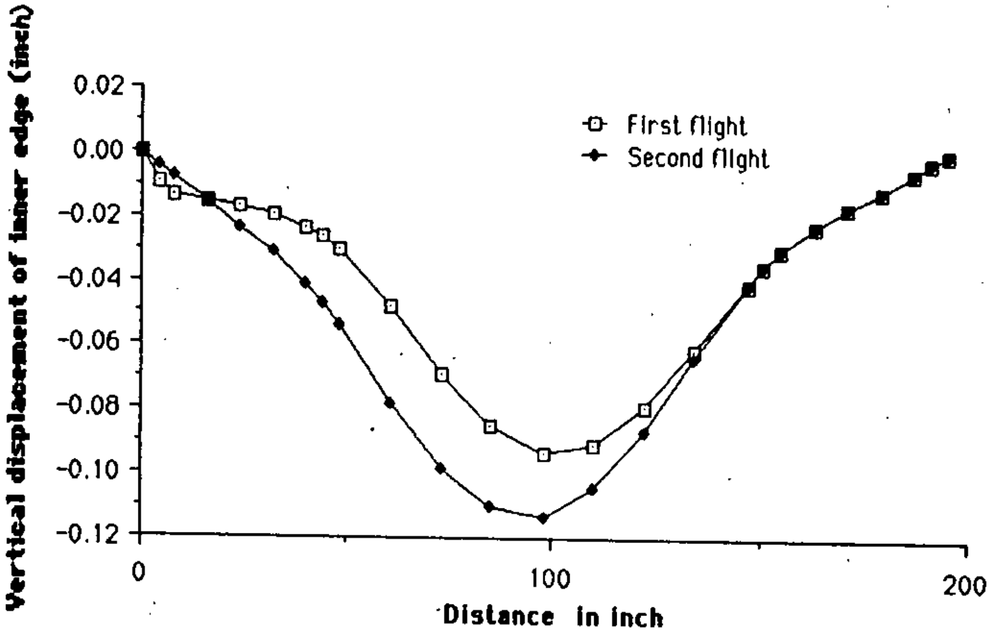


Fig. 4.2 Vertical displacements along inner edges of 1st and 2nd flight of a 3- flight analysis.

a) Flexural Behavior

The flexural response of the 2nd flight of a 3-flight analysis can be visualized from Fig. 3.13, and 3.15. Figure 3.13 is the result obtained using thick element and Fig 3.15 is due to the ANSYS stiff63 element. The overall nature of the bending moment diagram is very similar to that obtained for a 2-flight analysis. Interpretation of this diagram, therefore will be a repetitive one. It is worth noticing that maximum values of moment (both positive and negative one) are slightly higher (of the order of 3-4%) in this case than the analysis for 2-flights. The reason is quite perceivable. In the later case the flight under consideration is subjected to the effect of two other adjoining flights while in the previous case (Analysis for two flight) effect of one adjoining flight was there at one landing while the other landing was free of such effect. The vertical displacement for the adjoining flights are also indicative of this fact. The thick shell result on a 3-flight analysis produced maximum vertical displacement for second flight about 20% higher than that in 1st or 3rd flight.

In addition to the bending moment diagram for 2nd flight, the corresponding diagram for the 1st and third flight as obtained from ANSYS thin shell analysis are also presented. These can be seen in Fig. 3.17. The behavior of 1st and 3rd flight are similar in this respect.

b) Inplane Forces

The inplane axial forces are presented in Fig. 3.14 and Fig. 3.16 for the 2nd flight of a 3-flight analysis by thick and thin shell elements respectively. Because of taking single element across the width, thick shell results of axial force are shown in inner and outer edges only. A rather finer mesh implemention for ANSYS thin shell allows the presentation of distribution of the longitudinal axial forces along the width of the stair.

The general pattern of inner edge axial forces shown in Fig. 3.14 and Fig. 3.16 are similar; although the later records a higher value. On the other hand, considerable difference is visible in the outer edge axial force values(Fig. 3.14 and Fig. 3.16).

Although the behavior of the 2nd flight of a 3-flight analysis is of our primary interest, the longitudinal axial forces of the 1st and 3rd flight may also be of some interest in interpreting the overall behavior. These are presented in Fig. 3.18. From Fig. 3.18 it is evident that behavior of 1st and 3rd flight is anti-symmetric. It is to be noted that 1st and 3rd flight have same geometrical layout i.e. for both of them the lower kink is at the left end and the upper kink is at right end. It appears that only the outer edge values near kink zone are significant and the lower end (kink) value of the 1st flight corresponds with the upper end value of the 3rd flight. The vice-versa is also true.

The first and third flight are both 'end' flights and correspondence in axial forces reveals the fact that the important consideration is that whether the flight is continuous with a next one or not. This behavior is same , irrespective of the location of the kink (whether upper or lower).

c) Moment in Lateral Direction (MY)

The assumption that flights of a stair case behave as a one way slab in the direction of the flight is true in a general sense. It has been observed from the result of the present analysis that the stresses in lateral direction (Y) are insignificant except at the landings. Since the landings, in the present analysis, are considered to be supported by side walls, a portion of the load is carried in the direction perpendicular to the length of the flight, at landing level.

Fig. 3.19 and Fig. 3.20 shows the bending moments (MY) and corresponding vertical displacements at mid-floor landing and floor level landing respectively for a 3-flight analysis by ANSYS staff63 element. Lines X1,X2, X3, X4 (Fig.3.5) are along the nodal lines starting from kink towards the end of the mid-floor landing. And lines Y1, Y2, Y3, Y4 (Fig.3.5) are at floor level landing. These figures reveal that the strip closest to the kink is subjected to considerably high moment (MY) in the lateral direction. Average value of which is 2000 ft.-lb./ft. for the first 1 ft. strip near kink. Such high magnitude is prevalent at the location where a forward and backward flight meet at a landing, over a very localized area. Beyond a 1 ft. strip away from the kink this moment is of trivial magnitude. The overall characteristics of this moment is similar at mid-floor landing and floor-level landing, though the later records a slightly higher value.

The concentration of rather high lateral moment(MY) can also be interpreted from the corresponding displacement pattern. At kink line, abrupt change in slope occur where a forward and a backward flight meet together (Fig.3.19 & 3.20). This abrupt change in slope gradually reduces , away from the kink.

4.3 Analysis of Open-well type Stair

A stair, with a gap between the forward and backward flight is frequently chosen for residential as well as office buildings. Many of such stairs are supported by side walls or beams at landing levels in addition to the supports at the end of landings. Such a stair case is analyzed using both thick shell and ANSYS stiff63 elements. Results of these analyses have already been presented in Chapter 3 (Art:3.7.2).

For the purpose of analysis a open-well stair with well opening (parameter C, in Fig.3.26 , which is the clear distance between a forward and backward flight) of 48" is considered. In a later stage the effect of varying this well opening on the behavior of stair slab will be discussed.

a) Flexural Behavior:

Figure 3.21 shows the bending moment diagram and vertical displacements along the length of 1st flight as obtained by thick shell analysis. Behavior for 2nd flight is similar. This analysis is for 2-flight with the symmetry condition applied along line a-a (Fig.3.6 and 3.7) at either floor level.

The same problem was analyzed using ANSYS stiff 63 element. Results are presented in Fig. 3.22. Identical behavior has been found in both the above mentioned analyses. The restraining effect near kink zone is clearly visible which restricts the high moment that would have otherwise been resulted. Although maximum positive moment at outer and inner edge is same, there is significant difference in negative moments near kink. Displacements along a-a line (Fig.3.21 & 3.22) closely matches with the inner edge displacement and so does the moment. Moment along a-a line was omitted in Fig. 3.21 but included in Fig. 3.22. Because in the previous case those moment values were unreasonably high. Since the corresponding values of Fig. 3.22 are reasonable and concordant with displacements of both Fig. 3.21 and Fig.3.22, this omission of moments along line a-a(in Fig 3.21) is not of much concern.

In these figures the moment values at kink location was disregarded for the reason stated earlier (Art.4.2.1). An attempt to recover this value from the individual element stresses is made in Art. 4.4.

The important point to note here is that the maximum negative moment (at kink) along outer edge is not greater (with the exception of kink point moment) than the maximum positive moment at mid span.

Unlike the behavior of Dog-legged stair, the open-well stair suffers a negative moment at outer edge kinks but a positive one at inner edge kink. For developing design guidelines it is necessary to identify the location, in between the inner and outer edge, where this change occur. This effort is deferred here and will be discussed in Art. 4.6.2.

b) Inplane Forces:

The longitudinal axial forces, which were present in all the analyses of the present study of Dog-legged stair, were found to be negligible (Fig.3.23) in the case of open-well stair(opening of 48") average value of axial stress were of the order of 20 psi, a value that may be disregarded for the purpose of design. In order to check the condition of axial stresses for a smaller well opening (c= 4 inch) axial force(FX) along length have been plotted in Fig. 4.3. It is seen that for even such small opening, axial forces are not important.

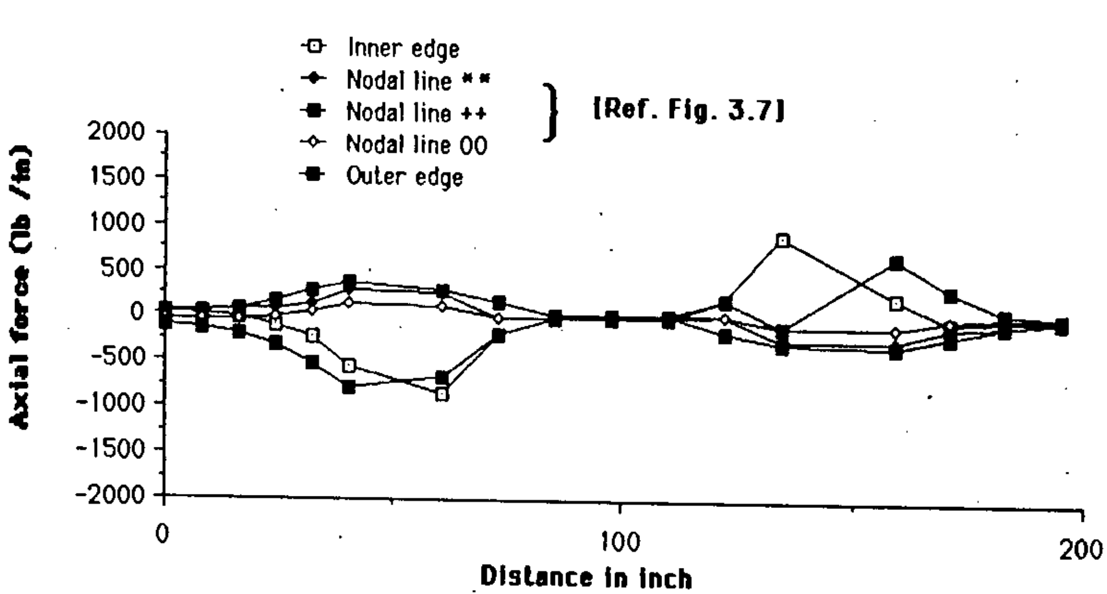


Fig. 4.3 Axial force (FX) distribution for a stair with C =4°

c) Moment in Lateral Direction(MY)

It has been discussed in the preceding article that stresses and resulting moments are small in lateral(Y) direction (direction perpendicular to the length of flight) except at landing level. It has also been discussed in a previous article (Art.4.2) that moments in lateral direction are present in landing level of Dog-legged stairs. Such moments are also present in case of Open-well type of stair. This is shown in Fig. 3.24. As the landing is supported along 3-sides, this lateral moment have higher values along the free edge (X1 line; Fig.3.7 is referred for location of X1, X2, X3, and X4 lines). These higher values of moments are dominant only within a 1-ft. strip adjacent to the kink (between X1 and X2 line). Average value of this moment is 1750 ft.-lb./ft. It is important to note that this moment is present only in well portion (segment between the forward and backward flight) of the strip between X1 and X2 line. At other locations, this moment is negligible.

4.4 Moment at Kink Point

In all the preceding discussions regarding the flexural behavior of stairs the kink point bending moment values are intentionally kept out of consideration . On the basis of foregoing discussions, it can be concluded that Fig.3.13 and 3.15 characterizes the flexural behavior of stair slab of the type and supporting arrangement considered .Attempts can now be made to retrieve the kink point bending moment values from the element stresses of the corresponding analysis. For the Dog-legged type the second flight element stresses of a 3-flight analysis and for a Open-well type of stair the first flight values of element stresses are considered for obtaining kink point bending moments. Both of these analyses is by ANSYS stiff63 element.

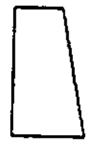
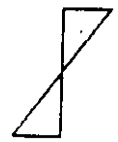
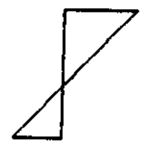
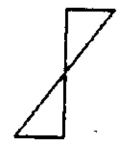
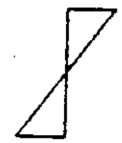
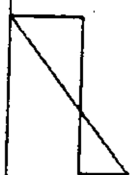
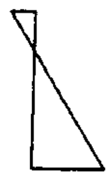
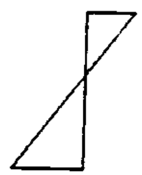
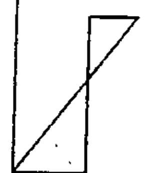
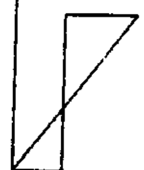
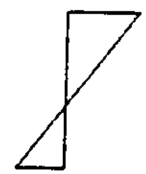
Both outer and inner edge stresses of the horizontal and inclined element meeting at upper and lower kink locations are separately shown in TABLE 4.1

Table 4.1 shows an average value of moment of -2000 in-lb./in exist at the outer edge kink of a Dog-legged stair. While at the inner edge the corresponding value of moment of +700 in-lb. /in (if only horizontal elements are considered). These values seem to be unreasonable, if compared with the usual trend of the plotted values of bending moment in Fig 3.13.

Similar disagreement will also be evidenced if the corresponding values of Table 4.1 for an Open well stair is compared with the plotted values of Fig.3.22. In these figures(Fig. 3.13 ,3.22) the next to kink point values differ considerably with the kink point values of Table 4.1. (Although physical location of these points are only of 8 to 12 inch apart). The only conclusions that can be drawn from this , is that the kink point stresses tabulated in Table 4.1 is unreliable.

A simple extrapolation of the outer edge bending moment values of Fig 3.13 and Fig.3.22 will show that at the outer edge kink point the maximum negative moment values can conservatively be estimated to be equal to the maximum positive mid-span moment.

TABLE 4.1 KINK POINT STRESSES

Analysis scheme	Inner edge x-stresses(psi) & moments(in-lb/in)				Outer edge x-stresses(psi) & moments(in-lb/in)			
	Lower kink		Upper kink		Lower kink		Upper kink	
	Horizontal element	Inclined element	Horizontal element	Inclined element	Horizontal element	Inclined element	Horizontal element	Inclined element
3-flight analysis of Dog-legged stair	-558  -95 M=617	-559  -595 M=-48	28  635 M=807	553  675 M=163	788  -649 M=-1982	872  -683 M=-2073	758  -739 M=-1996	803  -782 M=-2113
Analysis of Open-well stair	-444  258 M=935	-801  276 M=1435	-258  419 M=901	-261  790 M=1400	823  -1238 M=-2741	829  -1239 M=-2755	1113  -735 M=-2464	1209  -755 M=-2618

This criterion , for ascertaining outer edge kink point bending moment will cover both Dog-legged and Open-well type of stair. The restraining effect of the outer-edge support travels inward significantly . As a result , at the inner edge , although the support is at the end of landing slab , sufficient restraining effect is present near the kink location . This fact is true for both Dog-legged and Open-well type of stair. For a Dog-legged stair , the same magnitude of moment can be assumed at both inner and outer kinks. But for an Open-well stair, it will be sufficiently conservative , if we assign , at the inner edge, a moment value of 50% of the magnitude of the corresponding maximum negative moment.

4.5 Displacement in Horizontal Plane

In article 3.7.3 it has been reported that the flights of a stair slab has a tendency to move sideways (Y-direction), towards the forward flight (going upward) at landing level. This feature is discussed here.

In Fig. 3.25(i), due to the imposed boundary condition, preventing the lateral (UY=0) movement of landing as applied at left landing of 1st flight and at right landing of 3rd flight, these landings do not move from their position.

It is quite interesting to note that the other two landings, which are free to move laterally, move by the same amount through the entire length of the landing. And the inclined flights suffers a linear variation in-between the landings. The corresponding outer edge locations also move by the same amount, meaning a rigid body movement.

This sort of behavior is quite difficult to perceive, at the first instance . Since one would expect that due to anti-symmetrical arrangements at landing, the landing will not move along the line of anti-symmetry (on the basis of which , the symmetry condition of this study has been set, as in Fig.3.2). However, it seems that where a forward and a backward flight meet together at a landing, the landing slab derive much resistance from the background (going downward) flight than the forward one.

This observation might lead to the question about the validity of our assumption of anti-symmetry (UY=0) at the location where a flight is of discontinued at landing. This objection can be defended by two arguments:

i) We have finally based our conclusions on the observation of the behavior of a 2nd flight of a 3-flight analysis. (The second flight being free of such imposed effects.)

ii) Figure 3.15 through Fig. 3.18 presents the flexural behavior of 1st, 2nd and 3rd flight without any abrupt differences.

4.6 Effects of Various Parameters

The stair parameters, defined in Fig. 3.26, were studied to establish their influence on the overall behavior of the stair slab. The scheme of this parameter study is described in Art:3.8. The findings are presented in Fig. 3.27 through Fig.3.39. In the following articles the main features of these findings will be presented. The general arrangements and boundary condition defined in Fig. 3.2 is maintained throughout the study of the parameters unless otherwise specified. The parametric study , in general , was carried out by the thick shell program, except for studying the effect of parameter 'w' and 'c' , which were done by thin shell element.

4.6.1 Parameter Relating to Dog-legged Stair

i)Effect of the length of landing slab 'a'

The effect of the variation in landing length 'a'(in the direction of the length of a flight) on moment and displacement are shown in Fig. 3.27(i) and Fig. 3.27(ii). This is shown against non-dimensional quantity a/b. It is evident that the maximum positive moment decreases with the increasing ratio of a/b, while maximum negative moment shows an increase. This implies that the fixity at the landing increases as a direct consequence of increasing the landing length. Hence it has been observed that an increase in landing length decreases the positive design moments. This fact is also endorsed in Fig. 3.27(i), from which it is seen that maximum vertical displacement (at mid span) reduces with increasing a/b ratio. Hence it can be concluded that the landing slab having supports on three sides provide restraint to the inclined waist slab to a significant extent.

ii) Effect of Variation in Flight Height 'h'

The scheme is described in Art. 3.8.1 and results are presented in Fig. 3.28(i) and Fig. 3.28(ii) as function of dimension less parameter h/b. It is to be noted that parameter h/b indicates the inclination of the waist slab. For the range of h/b considered in this analysis the inclination of the stair slab varies from 29.93° to 32.47° . And for this variation the positive moment varies by 5% and the negative one by 3%, both variations show an increase. Figure 3.28(ii) records the way maximum vertical displacement varies with h/b. It appears that , 'h' is not a very sensitive parameter.

iii) Effects of Length of Going 'b'

The behavior of a stair slab supported by landings reveals that the landing slab provides an effective restraint on the inclined waist slab [(Fig 3.29 (i) and (ii)]. And as a consequence of this, the point of contra-flexure lies within the inclined waist portion of the slab. So parameter 'b' (the 'going' of the stair) controls the effective span of the stair.

It has been found that [from Fig.3.29(i) and (ii)] for changing "b" from 99 inch to 111 inch, effective span is about 70% of the going (TABLE 3.3).

iv) Effects of varying flight width 'w'

The assumption that the stair slab behaves as a one way slab in the direction of the flight is in general, not represented in Fig. 3.30 when parameter "w" was varied from 36" to 60", other parameters remaining constant for a dog-legged stair. There is more or less a linear increase in both positive and negative moment values with increasing flight width, W. The positive moment values plotted here are the maximum inner-edge values (mid span) and the negative values are the outer edge value at node next to the kink (In absence of exact kink point moment a value next to kink point are considered. However, so long we are interested about the general trend of the variation this discrepancy is of little consequence). It appears from Fig. 3.30 that the rate increase of both positive and negative moments are almost same for an increasing ratio of w/b.

The deviation of a stair slab, in this case, from behaving like a one-way slab is the result of interaction of the adjoining flights connected by the landings at either end. To test this the behavior a single flight with symmetry conditions applied at either landing level has been investigated (with and without outer wall support).The results are presented in Table 4.2. The effect of adjoining flights and that of the outer edge support is clearly evident in Table 4.2.

TABLE 4.2 Effect of varying 'w' on the flexural behaviour for different conditions.

Analysis scheme	w=48"		w=60"		% change of moment for changing "w"	
	Positive moment in-lb/in	Negetive moment in-lb/in	Positive moment in-lb/in	Negetive moment in-lb/in	+ ve	- ve
2nd flight of a 3-flight analysis	1138	503	1289	580	13.27	15.3
Single flight with landings supported along outer edge wall also.	970	438	1039	467	7.1	6.6
Single flight with no outer edge support at landing	7516	——	7567	——	0.67	——

4.6.2 Parameter Relating to Open-well Stair

i) Effects of Well Opening("c") in Open-well Stair

a) Effects on longitudinal flexural behavior

The flexural behavior of open-well type of stair is given in Fig. 3.21 and Fig. 3.22. It can be observed that behavior along outer edge is somewhat similar to that of the Dog-legged stair. But along inner edge, positive moment of small magnitude occurs. So it is important to investigate the behavior of the stair for a range of well opening. The scheme and the results are in Art: 3.8.1.

Fig. 3.31(i) shows the variation of bending moment(MX) along Z1 line (Referred to Fig.3.7 for location of Z1, Z2, Z3, Z4, Z5 and Z6) for well opening (c) ranging from 12 inch to 48.inch. As Z1 line is at mid-span these moments are the maximum positive design moments. In-between the inner edge (distance =0) and outer edge (distance =48) the values of bending moments are smaller. It is seen that maximum positive design moment increases as the gap between the forward and the backward flight (well opening, c) widens, although the general characteristics of distribution of these moments are maintained.

Fig. 3.31(ii) records the moment values along Z2 line. The location of Z2 line is important, because it is near the kink line where the inner edge (distance,0") moment is positive and the outer edge value is negative. The variation between outer edge and inner edge is more or less a linear one with practically zero moment at center (distance 24"). The inner edge value of bending moment at this location (Z2) varies considerably with well opening. For the variation of well opening (c=) 12" to 48" the inner edge moment varies from +600 ft.-lb./ft. to +1000 ft.-lb./ft. On the other hand, the outer edge value of this moment along line Z2 remains unchanged whatever be the well opening. This has a very significant implication that the fixity is derived from the outer edge wall support which is transmitted inward. And the efficiency of this transfer is dependent on the well opening.

Fig. 3.31(iii) shows the variation of longitudinal moment along Z3. Although moment values at this location are small, the inner edge records a positive moment and the outer edge shows a negative moment. And the center of the flight , in-between the outer and the inner edge, can again be taken as the line of demarcation between the positive and negative moment regions, for simplicity. However, the error or discrepancy arising out of this simplification is trivial.

Fig. 3.31(iv) has the same characteristics as Fig.3.31(ii). This is due to the fact that location Z2 and Z4 are same, the only difference is that the former corresponds to a lower kink and the later to an upper kink. This similarity again confirms the symmetric behavior of the stair about the midspan (Z1 line), inspite of the fact that one kink is at lower level and the other one is at upper level.

Fig. 3.31(v) has comparable features as that of Fig.

3.31(iii). Of course, there are variations in the corresponding magnitudes of moment along Z3 and Z5 line. It must be noted that Z5 line is 12" right to upper (right) Kink while Z3 line is 8" left of lower (left) kink. This also leads to a useful conclusion that beyond 12" right of right kink and 12" left of left kink(Fig. 3.7) outer edge negative values of moment do not exist.

b) Effects on Lateral Moment(MY)

As it has already been reported (Fig. 3.24) that moments in lateral direction (Y) are significant in the landing slab of a Open-well stair, (see Art: 4.3) it is now become necessary to investigate the response of this lateral moment for various well-opening. The magnitude of this moment is high along Z6 line with maximum values at the kink line; progressively decreasing towards the end of landing. Fig. 3.32 records the variation of this lateral moment along Z6 line for well opening of 12" to 48". It is interesting to note that this lateral moment increases with a decrease in well opening. This, at a first instance, might seem to be spurious. But this can be well explained if one compares the vertical displacements along various transverse line (X1 through X4) of landing in Fig.3.20 and Fig. 3.24. As the distance between forward and backward flight decreases this moment is significantly affected. Of course, the values of this moment at kink location (X1 line) are more sensitive to a corresponding variation of well-opening C. But away from the kink line this sensitivity diminishes. Beyond half the length of the landing (Distance 24"), measured from the kink line the values of this moment remains practically same, regardless of well opening.

ii) Effects of Flight width "w"

<u>a) Effects on Longitudinal Flexural Behavior</u>

The influence of the flight width 'w' with respect to the flexural behavior of Dog-legged stair has already been described (Art 4.7.1). In the present article the effect of varying w on the flexural behavior of an open-well stair is being considered. For this purpose flight width was varied from 36" to 60" for an open-well stair of well opening of 48".

The flexural response similar to that of the Dog-legged stair has been evidenced in this case also (Fig 3.33). But in the case of open-well stair, the rate of increase of bending moment (with a increase in w/b ratio) is 2.33 times than the corresponding rate of a increase in a Dog-legged stair. Both positive and negative moments are equally sensitive with w/b ratio (Fig.3.33).

<u>b) Effects on Moment in lateral Direction (MY)</u>

As is already discussed (Art.4.3) that in an open-well stair bending moments(MY) of appreciable magnitude are present in the direction perpendicular to the length of the flight at landing level. The effect of varying flight width on this lateral moment along Z6 line(Fig. 3.7) has been presented in Fig. 3.34. From this figure the sensitivity of this moment on w/b ratio is apparent. For changing 'W' from 36 inch to 60 inch the maximum value of this lateral moment has changed by more than 46%.

4.6.3 Load Cases

The effect of changing live load positions are compared with the result of full live load on all panels, in Fig. 3.35 and Fig. 3.36. The influence of the position of live load considered is not very significant for the positive moments. For all the three cases under consideration live load has been present in the middle portion ('waist slab'). This portion , particularly governs the magnitude of the maximum positive moment. The Live load on the landing has only trivial effect on the maximum positive moment. Landing slab , with or without live load has some influence on the moments near kink , although the effect is not that important.

4.6.4 Support Arrangements

All the analyses in the preceding articles have been carried out with the support arrangement described in Fig. 3.1 and Fig. 3.2 (in general, a similar supporting condition for the open-well stair has also been considered). It is now considered that stair slab is supported at the far end of the landing (Fig.3.37). The moment(MX) distribution along the length of the flight are shown in Fig. 3.38 and 3.39 for simply supported and completely fixed conditions respectively. These results are obtained from the same mesh (Fig.3.5) as that of a 3-flight analysis by ANSYS. Inner edge and outer edge values match closely, despite the presence of adjoining flights. The displacement diagram of Fig. 3.38 and Fig. 3.39 are very similar to the displacements of a simple and a fixed ended beam respectively (under UD load). In the simply supported case the midspan moment is equal to the moment of a simply supported beam with span equal to the center to center of the support. No special behavior at or near is kink visible. And axial forces are not at all significant.

In the case of the stair of Fig. 3.37(b) the bending moment diagram of Fig. 3.39 represents the behavior of a horizontal fixed ended beam. No special behavior due to inclination of the waist slab is observed , except some inplane transfer of load (although small in magnitude).

Due to complete fixity at end, the effect of adjoining flights has been eliminated considerably, in the case of fixed ended stair. This is evident from the same behavior at outer and inner edge in the fixed ended stair. But in the case of simply supported stair [Fig 3.37 (b)] some effect of the adjoining flights are visible.

CHAPTER 5

A NEW DESIGN RATIONALE

5.1 General

With the results presented in chapter-3 followed by threadbare discussions on each of the items in chapter-4, attempts will be made to summarize the different important features of the behavior of both Dog-legged and Open-well stair, in the following articles . Proposal for a new design guide for both of the above mentioned types of stair will also be made .

5.2 Behavior of Stair Slab

The structural behavior of Dog-legged stair slab shows that the load is transferred pre-dominantly by flexural action .The presence of localized axial stress does not seem to require attention from the designers view point. Moreover when analyzing an Open-well stair it has been observed that axial forces are of negligible magnitude(Fig.3.23). This is true even if there is an existence of nominal well opening(Fig.4.3). In reality it is common practice to provide some small gap between forward and backward flight of a Dog-legged stair.

When stairs with support at far end (Art: 3.8.3) have been considered, flexural behavior has been predominant with only insignificant values of axial stresses. This behavior is more or less similar to that of a simple one way slab having similar support arrangements.

The introduction of outer edge support for the landing, provides significant rigidity to the structure , as a whole . The restraining effect of the the stair slab, derived from outer edge travel inward . And significant restraint is also present even along the inner edge(Fig3.15 and 3.22). It is observed that the bending moment diagram of the Dog-legged type and Open-well type stair at outer edge is similar to a fixed ended beam. From the sensitivity analysis of the various parameters it is seen that , in general , parameter "w" (width of the flight) and parameter "c" (gap between the forward and backward flight) are the most sensitive parameters , for the type of stairs considered in this study. The behavior at or near outer edge remains almost unaffected by these parameters and the travel of the restraining effect towards the inner edge essentially depends on these parameters. With the increasing value of both "w" and/or "c" the restraining effect in the inner edge becomes progressively smaller. However, corresponding mid-span moment remains unaffected(Fig 3.22).

5.3 Proposal for a Design Guide

In view of the above characterization of the behavior of stair slab , proposal for a design guide is formulated here. The response of the stair slab is predominantly flexural , for both Open-well and Dog-legged type of stair.

For the purpose of design , the critical locations of moments are :

i)Mid span of the flight for a positive moment in the longitudinal direction

ii) Kink location, for a negative moment in the longitudinal direction. And

iii) Landing , near the kink for a moment in lateral direction (perpendicular to the direction of the flight)

In the following articles ,guidelines for ascertaining design moments are proposed. Dog-legged and Open-well type of stair are considered separately for this purpose.

5.3.1 Dog-legged Stair

ii) Mid-Span Positive Moment:

Because of the sensitivity of this moment values with the various geometrical parameters , it is not easy to specify a general rule to quantify the maximum positive moment . However, it will be sufficiently conservative to assume an effective span of 90 % (Art:3.8.1) of the 'going' of the stair slab. In other words, a moment formula of $qb^2/10$ can be assigned; where q is the total load per unit area on the waist slab, duly magnified on a horizontal plane and b is the going of the stair.

The above recommendation will be adequate for determining mid span positive moment for a stair with the landing slab supported on outer edge walls.

ii) Negative Moment at Kink

The negative moment at kink will be taken to be equal to the positive maximum moment specified earlier; ie. a moment value of $qb^2/10$ is proposed for kink location.

iii) Moment in Lateral Direction at Landing Level:

At the landing level of a Dog-legged stair positive moment of considerable magnitude occur. This moment is dominant only over a small region of the landing and this moment is estimated to be equal to $qb^2/10$. Since, the existence of this moment is concentrated over a small region , considerable redistribution of stresses will result , before a failure takes place. Hence , the above simplification in ascertaining this moment will , adequately serve the purpose of design.

5.3.2 Open-well Stair

i) Mid-Span Positive Moment:

The mid span positive moment , for an Open-well stair is susceptible to the parameters 'c' (the opening between the consecutive flights) and 'w' (the width of the flight).

However, an effective span equal to the 'going' of the stair will give positive moment , satisfactorily for the common range of these parameters. Therefore, a value of $qb^2/8$ is proposed as a means of quantification of the maximum positive moment , where 'q' is the total load on the waist slab duly magnified over a horizontal projection and 'b' is the 'going' of the stair.

ii) Negative Moment at Kink:

For the Open-well stair the critical location for negetive moment is along the outer edge kink. The magnitude of inner edge bending moment is influenced by the parameter,'c'. However, the maximum value of this negative moment can conservatively be estimated to be equal to the maximum positive moment proposed earlier for a common range of c. And in the inner strip of half the width of the flight, 50 % of the specified maximum negative moment may be assumed.

iii) Moment in Lateral Direction at Landing level:

This moment , although exists over a small region at landing , significantly depends on the well opening , 'c' . Because of this dependence , it is difficult to propose a unified value that will ,in general , cover all possible geometrical dimensions. Considering this fact, a rather conservative value of this moment can be obtained by the formula, $q_l l_l^2/11$ is recommended for a w/c ratio equal to unity. Where, q_l=total load per unit area on the landing slab; l_l=2w+c , is the span of the landing slab in the direction perpendicular to the direction of the flight.

w= width of the flight

c= the opening between the flights.

And in general , for cases where w/c is not unity , the above quantity should be multiplied by the factor , $k=4/c \geq 1$, where c is in feet.

5.4 Design Examples

For the purpose of delineating the proposed method , a Dog-legged and and Open-well stair of dimension shown in Fig. 2.3 and 2.4 respectively , are designed here. This will also enable us to compare the cost-effectiveness of the proposed method.

5.4.1 Dog-legged Stair

Design of span AB:

span = 'going'

= 99 in.

=8.25 ft.

For a slab thickness of 4 inch and other dead and live load as assumed in Art: 2.4 , the total load on waist slab ,

q =217.15 psf (duly magnified on a horizontal projection)

Design moment = $qb^2/10$

= 1478 ft.lb./ft.

For which a slab thickness of 4" with a steel area of 0.34 sq.in /ft. is required.

The negative section is also controlled by the same requirement.

Design of span BC:

This span is also controlled by the moment 1478 ft.-lb./ft.

The arrangement of steel for this case is shown in Fig.5.1.

5-4-2 Open-well Stair

Let a slab thickness of 4.5 "

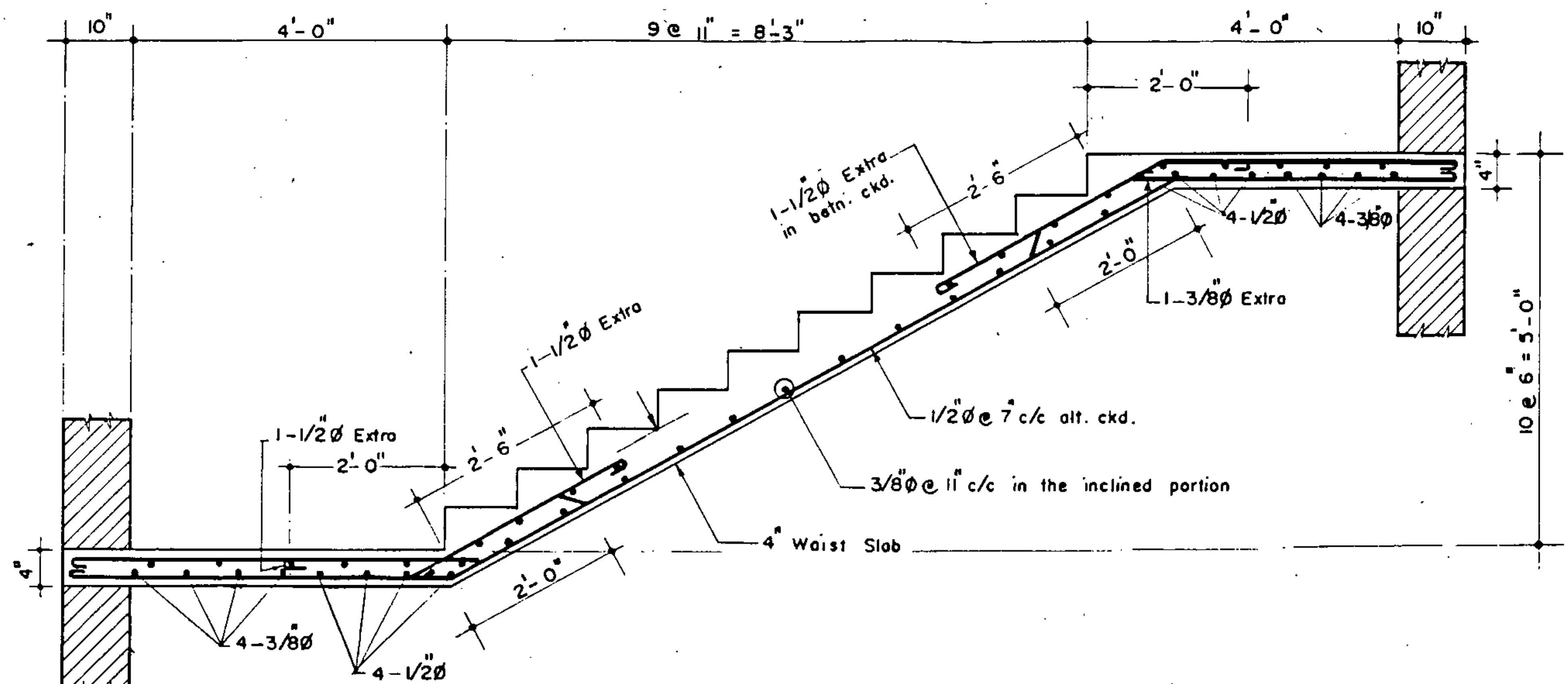
Design of span AB:

The total load on waist slab, q=224.2 psf

Design moment =$qb^2/8$

=1908 ft.lb./ft., which can be tackled by an effective depth of 3.5 in. with a steel area of 0.374 sq.in. /ft.

The negetive section is also controlled by the same requirement. However, at the inner strip (of half the width of the flight) 50 % of the steel required by negetive moment may be provided.



NOTES: At landing level, transverse steel is provided as required by transverse (y) bending moment. This steel is required in a strip of half the length (a) of landing near kink
In the remaining half of landing, steel may be provided as required by temperature/shrinkage requirement.

Fig. 5.1 Reinforcement detail of the stair slab, designed by proposed method.

Design of span BC:

$$\text{span}, l_1 = 2w+c$$
$$= (2\times48 + 48)$$
$$= 12 \text{ ft.}$$

The total load on landing , q_1=176.25 psf

Design moment $= q_1 l_1^2/11$

$= 2307$ ft.-lb. /ft.

Which calls for a 4.5 in. thickness and a steel requirement of 0.452 in ^{2}/ ft.

Placement of the reinforcement for this case should be made in the same way as shown in Fig.5.1 . However, as mentioned earlier, some reduction in the negative steel can be made in the inner strip of half the width of the flight.

5.5 Comparison of the Proposed Method with the Conventional Methods

The results of the design example presented in the preceding article are compared with the results of the conventional practices (presented in Art: 2.4) in Table 5.1 . From the table it appears that the proposed design practice is very cost effective.

TABLE 5.1 COMPARISON OF DIFFERENT DESIGN APPROACHES

Method of analysis	Dog-legged stair					Open-well stair				
	over-all thickness in.	Span AB		Span BC		over-all thickness in.	Span AB		Span BC	
		Positive steel sq.in/ft.	Negative steel sq.in/ft.	Positive steel sq.in/ft.	Negative steel sq.in/ft.		Positive steel sq.in/ft.	Negative steel sq.in/ft.	Positive steel sq.in/ft.	Negative steel sq.in/ft.
ACI Code	7.5	0.97	***	***	—	7.5	0.97	***	0.46	—
British Code	5.5	0.66	***	***	—	5.5	0.66	***	0.47	—
Proposed method	4	0.34	0.34	0.34	—	4.5	0.374	0.374	0.452	—

Note : *** marked quantities can not be determined by the corresponding method

CHAPTER 6
CONCLUSIONS

6.1 General

In this chapter conclusions derived from the present study are summarized. Unless otherwise specified, the conclusions listed here are limited for the types of stair with the particular supporting arrangements considered under the purview of the present study. A guideline for future study in the area is also indicated.

6.2 Behavior of Stair Slabs

The behavior of stair slabs can, in general, be summarized as follows:

i) The stair slab does not behave like a simple one way slab.

ii) The stair slab carries load by flexural action. The inplane stresses are insignificant.

iii) At kink, negative moment of appreciable magnitude occurs in the longitudinal direction of the flight. However, in the case of Open-well stair a small positive moment occurs at the inner kink .

iv) Moments in the direction, perpendicular to the direction of the flight are small in the inclined portion of the stair slab, but these are of appreciable magnitude at landing near the kink zone.

v) When the stair slab is supported only at the far ends of the landing slabs , its (stair slab) behavior is similar to that of a simple slab.

6.3 Proposed Design Guidelines

The salient features of the proposed guidelines for the design of stairs are summarized here. It should be noted that these conclusions are subject to the assumptions and limitations described in Art:3.4.

a) Dog-legged Stair:

(i) The maximum positive design moment in the longitudinal direction can be obtained by the formula:

$$qb^2/10$$

where q = total dead and live load on the waist slab per unit area, duly magnified over a horizontal projection.

b = the going of the stair

(ii) The magnitude of the negative moment in the longitudinal direction can be safely taken as equal to the positive design moment.

(iii) The transverse moment at landing level is also assumed to be equal to the design moment as given under (i) and positive in sense.

b) Open-well Stair:

(i) The longitudinal positive moment can conservatively be assumed to be equal to $qb^2/8$

where q and b , are as defined in (a) above.

(ii) A negative moment of same value ($qb^2/8$) is also considered appropriate near kink. This moment is to be considered only in an outer strip of half the width ('w') of the flight in the case of $c \simeq w$. In the inner strip this moment is not significant.

(iii) The transverse moment at landing level can be ascertained by the formula $kq_1 l_1^2/11$

where q_1 = total load per unit area on the landing slab; l=(2w+c), is the span of the landing slab(measured in the direction perpendicular to the direction of flight); w= width of the flight; and c= opening between a forward and backward flight.

and $k = 4/c \geq 1$ where c is in feet.

The proposed design guide enables straight forward computation of design forces and at the same time it is a cost-effective one compared to the conventional design methods.

6.4 Scope for Future Studies

Consistent with the objectives of the present study, the over-all behaviors of the stair slab have been established and specific design guidelines have been formulated. The results presented here compare well with the findings of the experimental studies described in Ref. 2 and 3. However, in order to have confidence in the design guides presented in this thesis , it may be advisable to carry out model tests to the point of destruction. A limitation of the present study is that due to significant susceptibility to some of the geometrical parameters, conclusions have been based from a conservative footing, covering possible geometrical variations. Further economy can be achieved if design curves can be formulated for a wide combination of geometrical parameters.

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APPENDIX I

A.1.1 Technical Terms Used in Stairways

The commonly used technical terms[9,10] of stair case construction and design shown in Fig.A.1.1 are briefly defined below:

i) Steps: A portion of stairway comprising of tread and riser which permits ascent or descent from one floor to another.

ii) Tread: The horizontal upper part of a step on which foot is placed in ascending or descending a stairway.

iii) Riser: The vertical portion of a step providing a support to the tread.

iv) Flight: A series of steps without any platform, break or landing in their direction.

v) Landing: A platform or resting place provided between two flights. A landing extending right across stair-case is termed as half space landing and the one extending only half across a stair-case is called a quarter-space landing.

vi) Pitch: The angle of inclination of the stair with the floor is known as pitch.

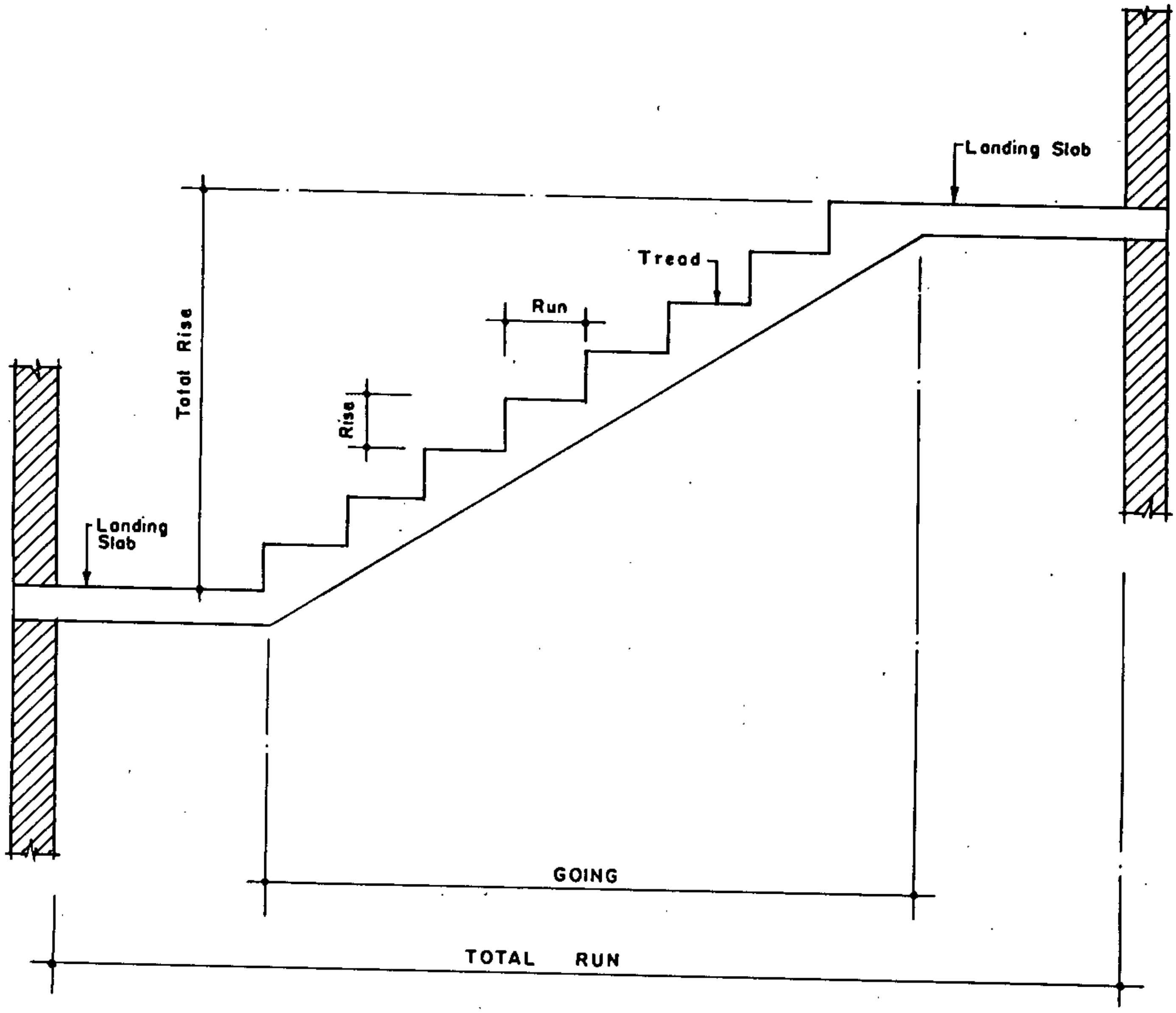


Fig. A.I.I Stair Terminology

vii) Rise: This is the vertical distance between two successive treads.

viii) Run: The width of a tread in the direction of the flight is called run. The total length of a stair in a horizontal plane is known as total run and it includes the length of the landings also.

ix) Waist: The thickness of structural slab in case of a R.C.C stair is known as waist.

The inclined portion of a flight is known as waist slab.

x) Stringer beam: Beam running in the direction of flight to support the steps in a stair.

A.1.2 Types of Stairs

a) Classification based on geometrical lay-out:

Generally, stairs are classified[9-15] depending on the geometrical arrangement of the adjacent flights and landings. On the basis of this, stairs may be of following types:

i) Straight run stair:

In this simple form of stair there is no change in direction on any flight between consecutive floors, (Fig.A.1.2(a)).

ii) L shaped stair with Landing:

This is a stair with adjacent flights at right angle to each other having an intermediate landing (Fig.A.1.2(b)).

iii) Dog-legged stair:

This is the most common geometrical shape used in residential buildings (Fig.A.1.2(c)). A stair of this type has virtually no gap or opening in-between the forward and the backward flight, meeting at a landing.

iv) Open-well stairs:

These consists of two or more straight flights arranged in such a manner that a clear space, called a 'well' occurs between the backward and forward flights. (Fig.A.1.2(d))

v) Bifurcated stairs:

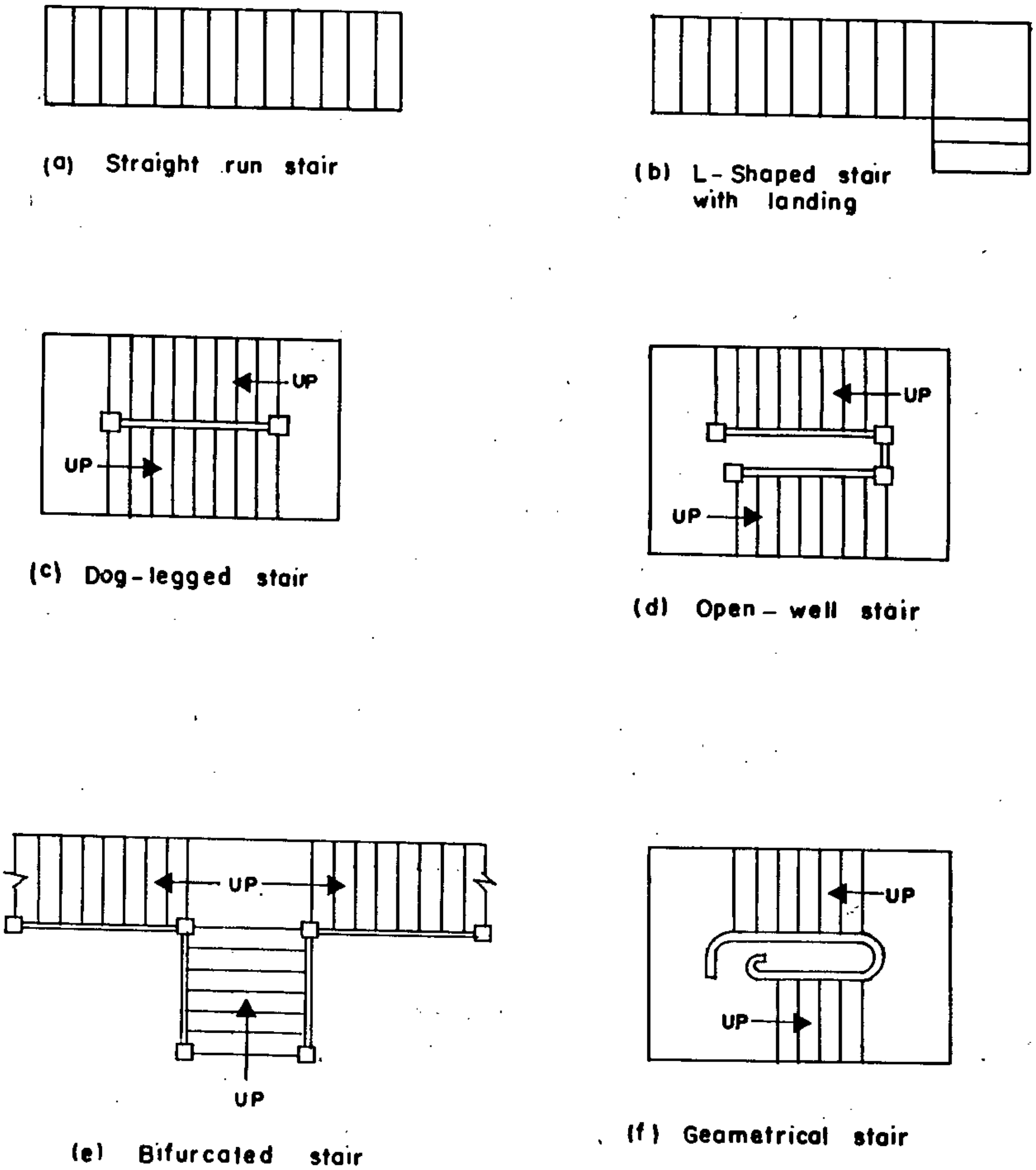
These stairs are so arranged that there is a wide flight at the start which is subdivided into two narrow flights at the mid-landing. The two narrow flights start from either side of the mid-landing. (Fig.A.1.2(e)).

vi) Geometrical stairs:

These are similar to the open newel stair with the difference that the open well between the forward and backward flight are curved (Fig.A.1.2(f)).

vii) Circular stairs:

The plan of this type of stair appears to follow a circle with a single center and having large radius of curvature (Fig.A.1.2(g)).



(a) Straight run stair

(b) L-Shaped stair with landing

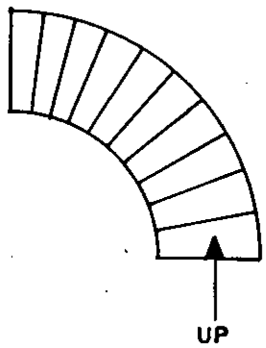
(c) Dog-legged stair

(d) Open-well stair

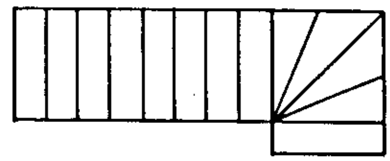
(e) Bifurcated stair

(f) Geometrical stair

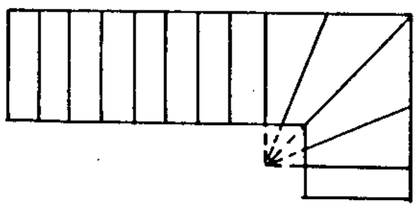
Fig. A.1.2 Types of Stairs.



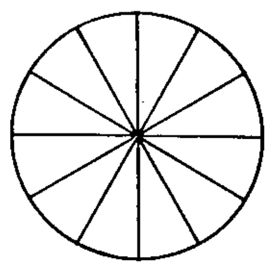
(9) Circular stair



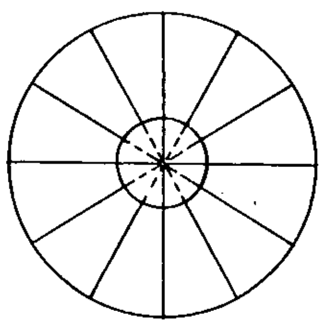
(h) L – Shaped stairs with winders



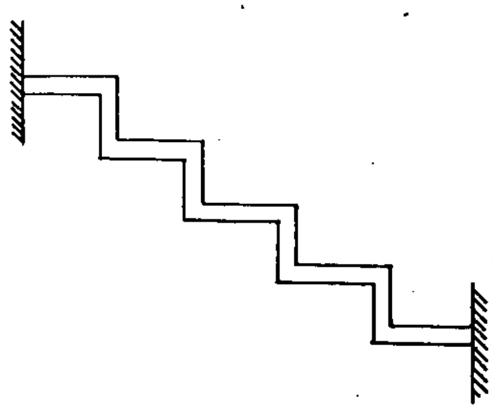
(i) L – Shaped stairs with winders



(j) Spiral stairs



(k) Spiral stairs



(l) Slabless stair

Fig. A.I.2 Types of Stairs (Continued)

viii) L-shaped stairs with winders:

This type of stair has triangular treads called winders in the landing portion of the otherwise L-shaped stair. Winders are helpful in compressing a stair into a much smaller space but are perilously steep where they converge, and their treads become much too shallow for comfort and safety; and many codes do not permit winders. The two possible arrangements of L-shaped stair with winders are shown in Fig.A.1.2(h) and Fig.A.1.2(i).

ix) Spiral stairs:

It is a misnomer, because in reality the structure is a helix, not a spiral. It is also known as helicoidal stair. A right helicoid has a wrapped surface generated by moving a straight line touching a helix so that the moving line is always perpendicular to the axis of the helix. Plan of two different helicoidal stair are shown in Fig.A.1.2(j) and Fig.A.1.2(k) .

x) Slab-less stairs:

In recent years the saw-tooth like structure as shown in Fig.A.1.2(l) is sometimes used as stair.

b) Classification Based on Material and Support Conditions:

Besides the above mentioned classifications stairs may also be classified[16] according to the material of construction; namely concrete stair, wooden stair, steel stair etc. Of them concrete stair is most commonly used.

Concrete stairs:

Reinforced concrete stairs are extensively used because of their high resistance to fire and suitability in construction of complicated shapes . The flight of a reinforced concrete stair may be supported either on the beams at landing level or on the landing slab itself. The various possible ways of supporting a reinforced concrete stair flights are shown in Fig.A.1.3.

i) Flight supported on beams

Concrete stairs may be designed to be supported on beams at both top and bottom landing levels(Fig.A.1.3(a)). This arrangement reduces the bending moment in stair by reducing the span. Such stairs are designed as simple one way slab. Transverse steel are provided only for temperature and load distribution purposes. However, the beams hanging out of the slab is objectionable from aesthetic sense and is seldom used now a days.

ii) Flight ending at landing

The lower end of such flight is supported on a beam, running perpendicular to the direction of flight and at the upper kink the flight is supported on the landing slab, which inturn is supported on wall or beam at the other end or along three edges. The arrangement is shown in Fig.A.1.3(b).

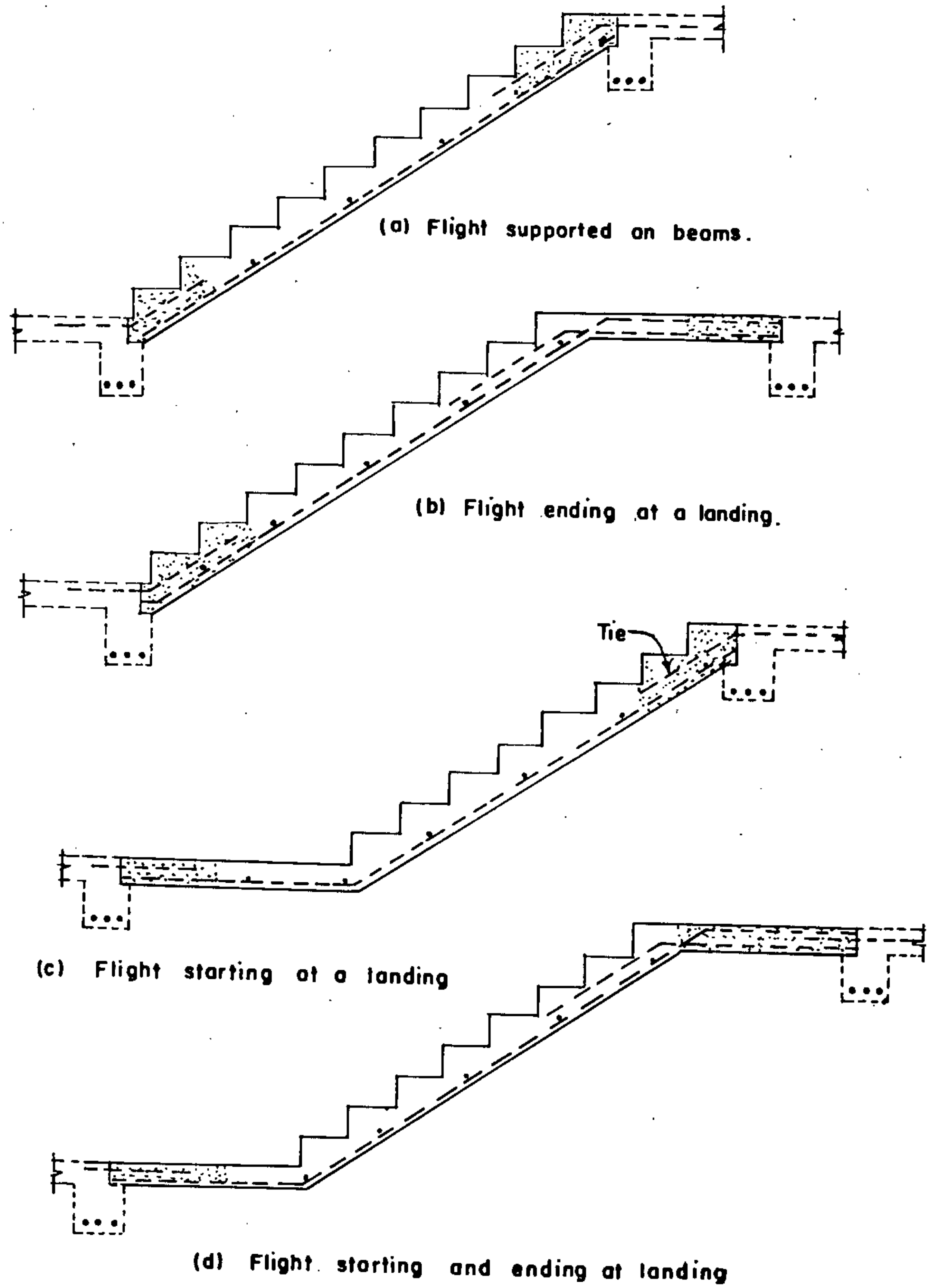


Fig. A.1.3 Types of concrete stairs and support arrangement. (Ref. 16)

iii) Flight starting at a landing

The structural arrangement in this case is similar to the previous case. Here the supporting beam is at the upper kink. (Fig.A.1.3(c)).

iv) Flight starting and ending at landing

This type of structural arrangement is extensively used, because of their aesthetic appearance. The inclined waist slab is supported on landing slabs at either end (Fig.A.1.3(d)). The supporting landing slab may run in the same direction as the flight or at right angle to it. However, the arrangement of the reinforcement near kink should be carefully detailed depending on these two cases. The arrangement shown in Fig. A.1.3 (d) is for the case where the landing runs in the same direction as the flight. But where the landing runs in the perpendicular direction of the flight, steel at the kink location should be placed as shown in Fig. 5.1.

APPENDIX II

A.2.1 Brief Description of Thick Shell Element

Thick shell program is a Fortran code to implement the general thick shell element developed by Ahmad[6,7]. A brief description of the program is given in Art: A.2.2

Typical thick shell elements are shown in Fig. A.2.1. In thick shells bending effect can be expected to be significant. The transverse shear deformation is also significant. From a three dimensional point of view the elements have two degeneracies. Firstly, the original normals to the middle surface are assumed to remain straight. Secondly, the distance of a point along the normal from the middle surface remains unaffected.

a) Geometric Definition of the Element:

The external faces of the element are curved, while the sections across the thickness are generated by straight lines, pairs of points i_{top} and i_{bottom}, each with given cartesian co-ordinates, describe the shape of the element.

If ξ, η be the two curvilinear coordinates in the middle plane of the shell (Fig.A.2.2) and ζ a linear co-ordinate in the thickness direction. If further it is assumed that ξ, η, ζ vary between 1 and -1 on the respective faces of the element then it can be written a relationship between the cartesian co-ordinates of any point

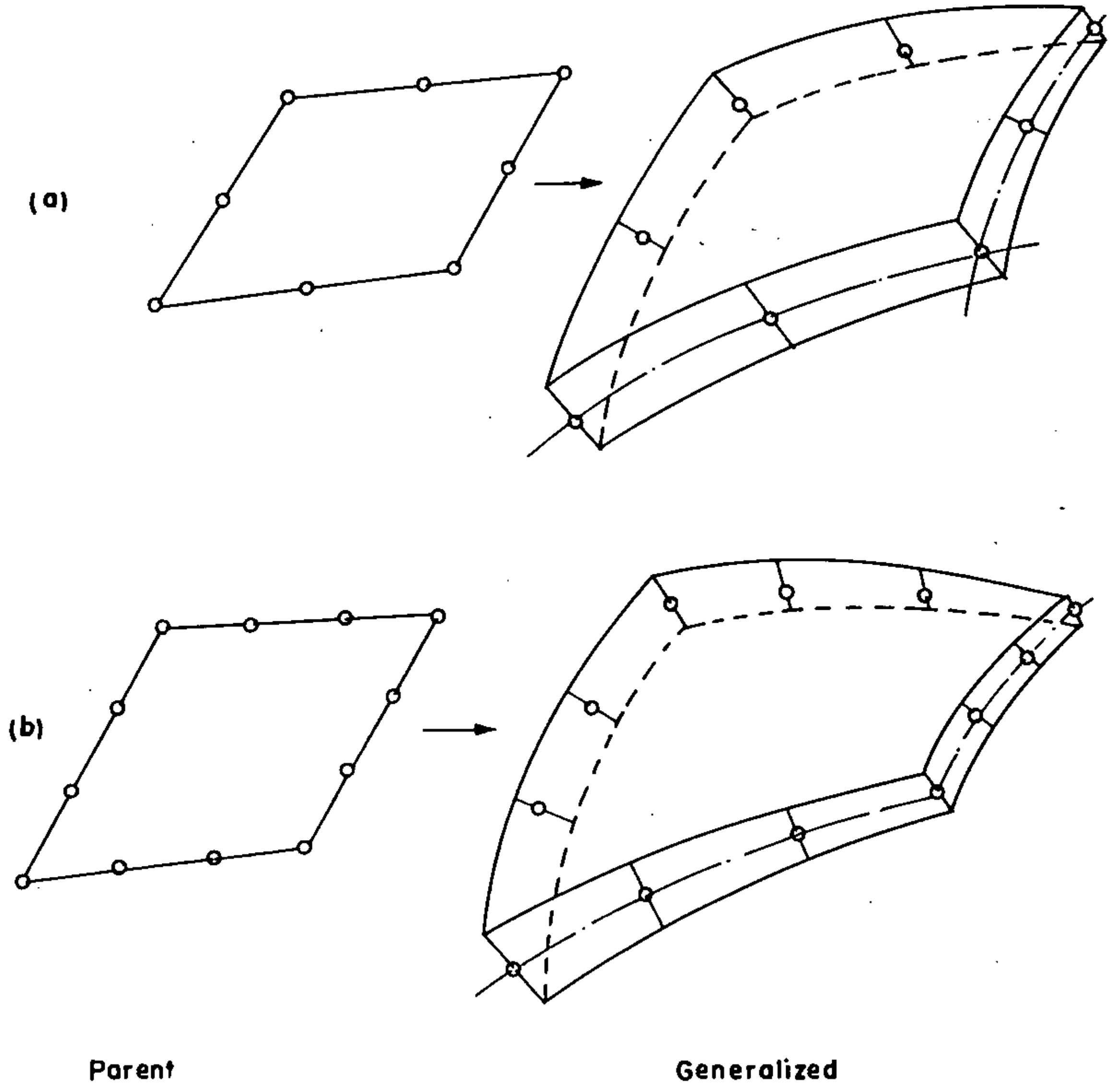


Fig. A.2.1 Thick Shell Elements (a) Parabolic (b) Cubic of the shell and the curvilinear co-ordinates in the form

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma N_i\,(\xi,\eta)\ \frac{(1+\zeta)}{2} \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{Top} + \Sigma N_i\,(\xi,\eta)\ \frac{(1+\zeta)}{2} \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{Bottom}$$

Here Ni(ξ , η) is a shape function taking a value of unity at the nodes i and zero of all other nodes. If the basic functions Ni are derived as 'shape functions', of a 'parent', two dimensional element, square or even triangular in plan and are so designed that compatibility is achieved at interfaces, then the curved shape elements will fit into each other. Arbitrary curved shapes of the element can be achieved by using shape functions of different orders. Only parabolic and cubic types are shown in Fig.A.2.1. For the purpose of present analysis a parabolic element has been used. By placing a larger number of nodes on the surfaces of the element more elaborate shapes can be achieved if so designed. It should be noted that the co-ordinate direction is only approximately normal to the middle surface. The relationship between the cartesian co-ordinates and curvilinear co-ordinates can be written conveniently in a form specified by the 'vector' connecting the upper and lower points (i.e. a vector of length equal to the shell thickness, t) and mid-surface co-ordinates (shown in Fig. A.2.2)

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma N_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{mid} + \Sigma N_i \frac{\zeta}{2} \bar{v}_{3i}$$

with

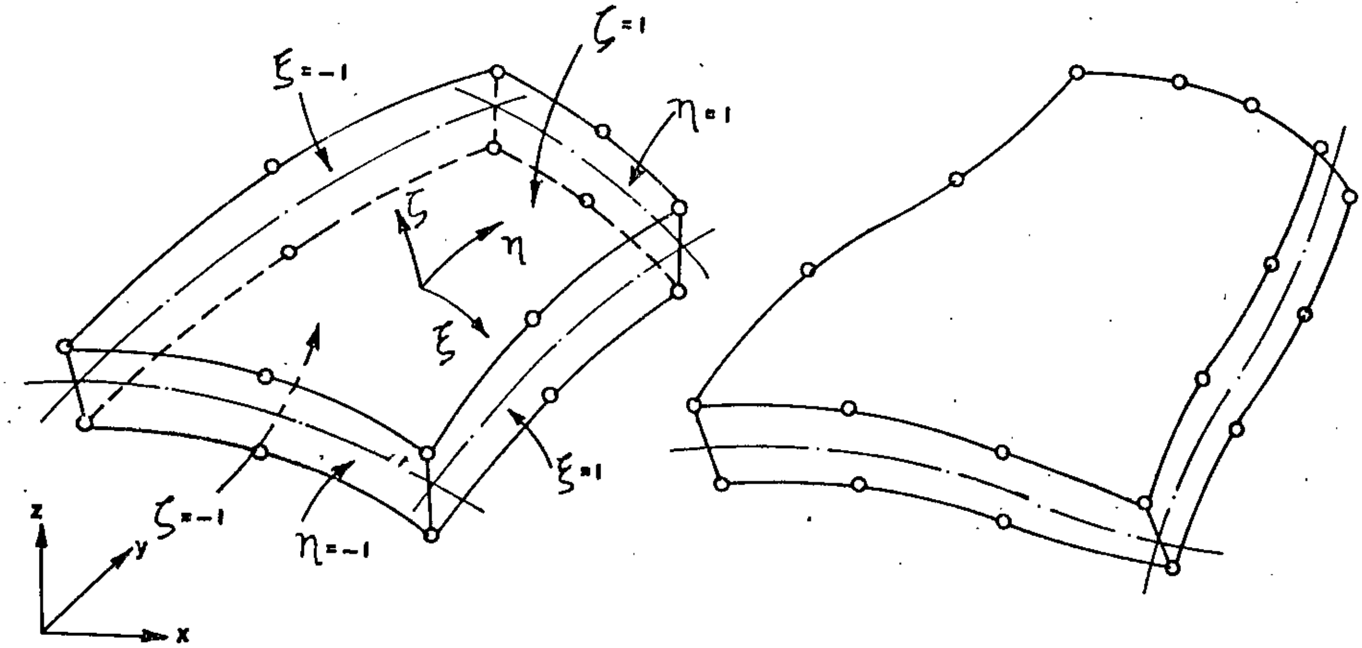
$$\bar{v}_{3i} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{top} - \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom}$$

defining a vector whose length is the shell thickness.

Displacement Field

Since the strains in the direction normal to the mid-surface is assumed to be negligible, the displacement throughout the element will be taken to be uniquely defined by the three cartesian components of the mid-surface node displacement and two rotations of the nodal vector $\bar{V}_{3i}$ about orthogonal directions normal to it. If two such orthogonal directions are given by vector $\bar{V}_{2i}$ and $\bar{V}_{1i}$ of unit magnitude, with corresponding (scalar) rotation. α_i and β_i, it be written, similar to the previous equation, but now dropping the suffix mid for simplicity.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \Sigma N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \Sigma N_i \zeta \frac{t_i}{2} \begin{bmatrix} \bar{v}_{1i} & - \bar{v}_{2i} \end{bmatrix} \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix}$$



(a) Parabolic

(b) Cubic

Fig. A.2.2 Geometry of the element.

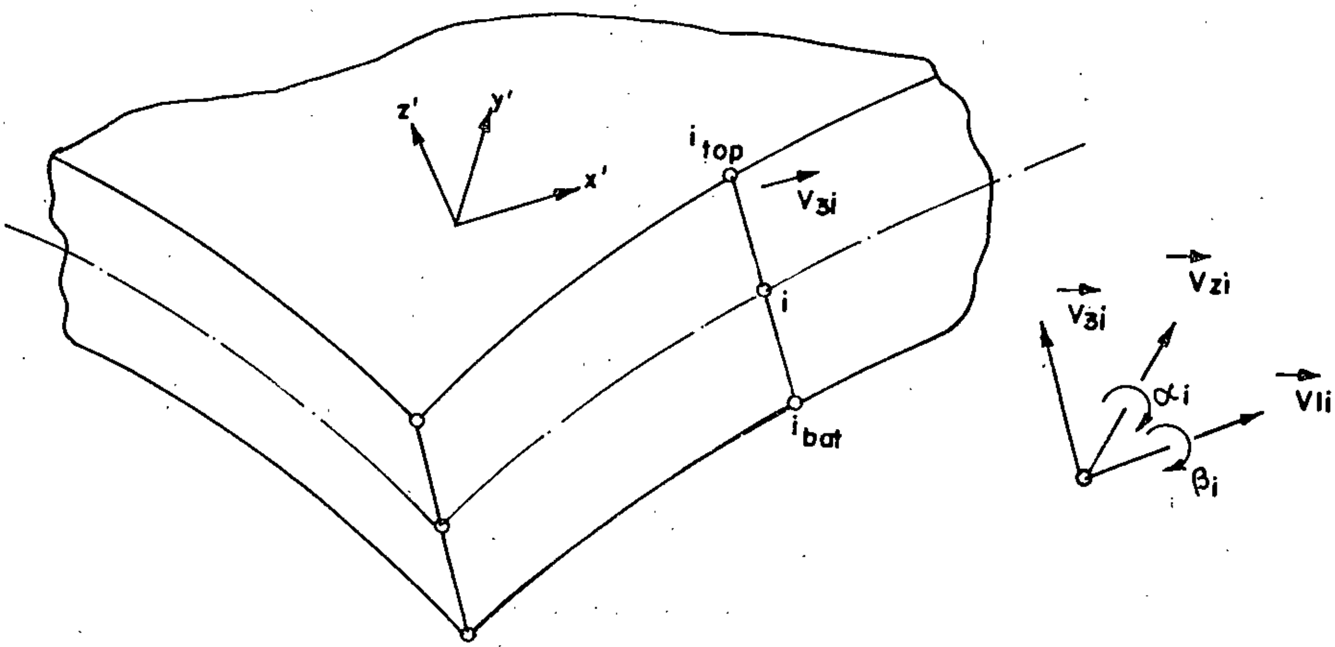


Fig. A.2.3 Local Coordinates and Nodal Degrees of Freedom.

A.2.2 General Features of the Program

The general thick shell finite element program is a generalized program. The geometry of a structure is defined in a global system i.e. rectangular cartesian. The loading and boundary conditions must be given in the same unit as the nodal displacements of an element. The stresses are usually calculated at the nodal points in the global system.

The top and bottom co-ordinates of each node with respect to cartesian co-ordinate are fed into the program. Co-ordinates for non-corner nodes lying on straight edges are not required to be given. If these co-ordinates of the nodes are fed into the program, then the shape of the element is automatically defined in the program. Therefore the thickness of the element can vary from node to node and the edges may be curved parabolically and cubically depending upon the type of element used. The program as at present can handle isotropic elastic material. The material properties are defined for every element, thus allowing the program to deal with materials varying from element to element. The temperature and pressure can be varied from node to node.

Output from the Program

The displacements are calculated and printed against each node in the ascending order for every loading case. Stresses are first calculated in the local orthogonal system and then transformed to the global cartesian system. For every node the top surface stresses are followed by the bottom surface stresses.

The global stresses are also stored separately for top and bottom surfaces against nodal numbers and at the end a simple averaging is performed on them. The average stresses are then printed out in the ascending order of the nodal numbers. The top surface stresses for all the loading cases are followed by the bottom surface stresses.

Division of Structure into Elements

The structure is first of all divided into suitable elements and the order numbered in any suitable way as shown for example in Fig. A.2.4. The elements are also suitably numbered in some sequence on which they are fed in the computer. Two probable sequence are shown in Figs.A.2.4(a) and A.2.4(b). Each element is topologically defined by its nodal numbers in a consistent right hand screw system shown in Fig.A.2.5(a) and A.2.5(b).

Front Width and Selection of Order of Elimination

To carry out the analysis of a structure using the minimum possible computer storage, the elements are selected in such a sequence that the maximum number of variables to be handled at any particular time (the front width) is minimum. For example, the prescribed order of elements in Fig. A-2.4(a) will give the smallest front

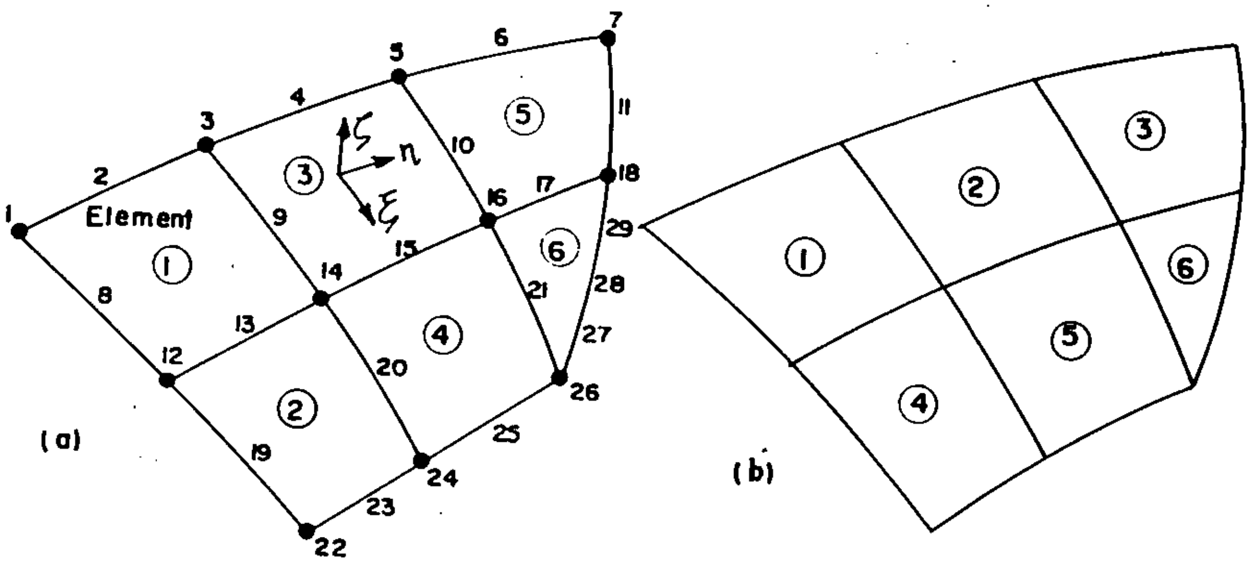


Fig. A.2.4 Division of Structure with Parabolic Elements.

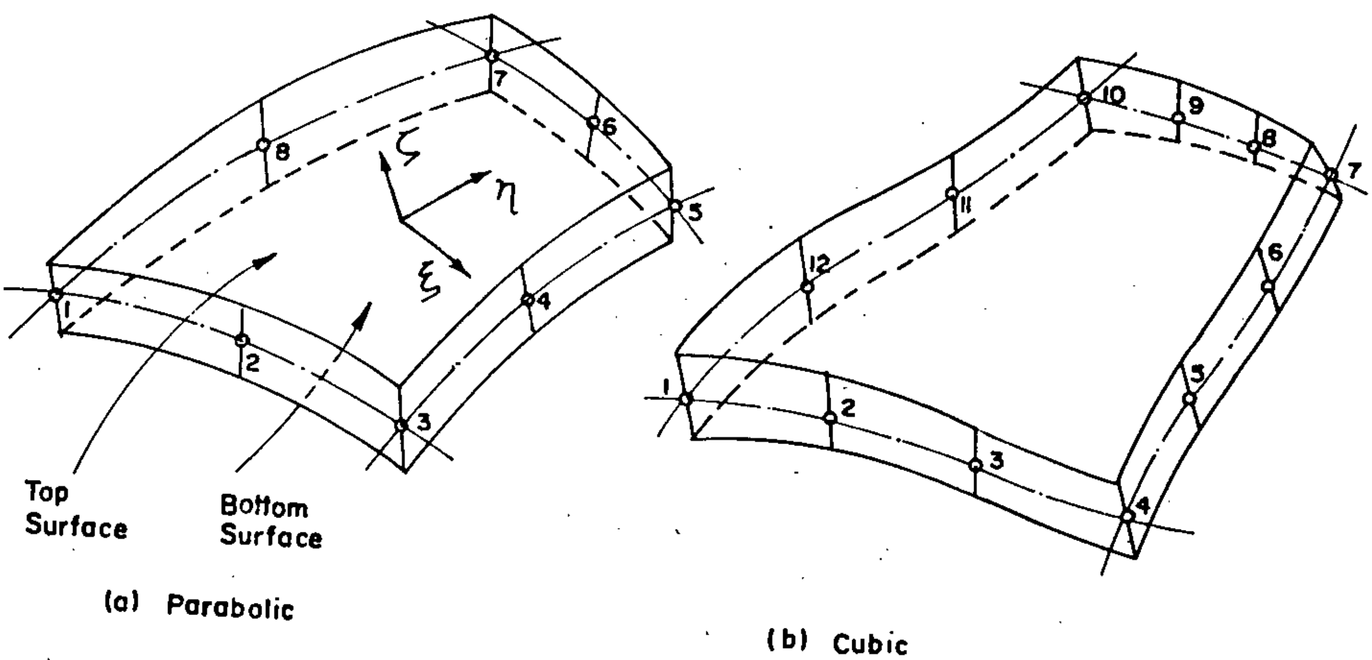


Fig. A.2.5 Difinition of Element Topology.

width. This is evident even from inspection in a simple structure.

The thick shell program uses the Frontal Solution technique. Here the assembly of an element stiffness and the corresponding right hand sides is immediately followed by the process of elimination of the variables corresponding to nodes which occur for the last time. This is indicated to the program by inserting a -ve sign before these nodes. This can easily be put in most shell structures once the element sequence has been selected.

APPENDIX III

A.3.1 Brief Description of Thin Shell Element

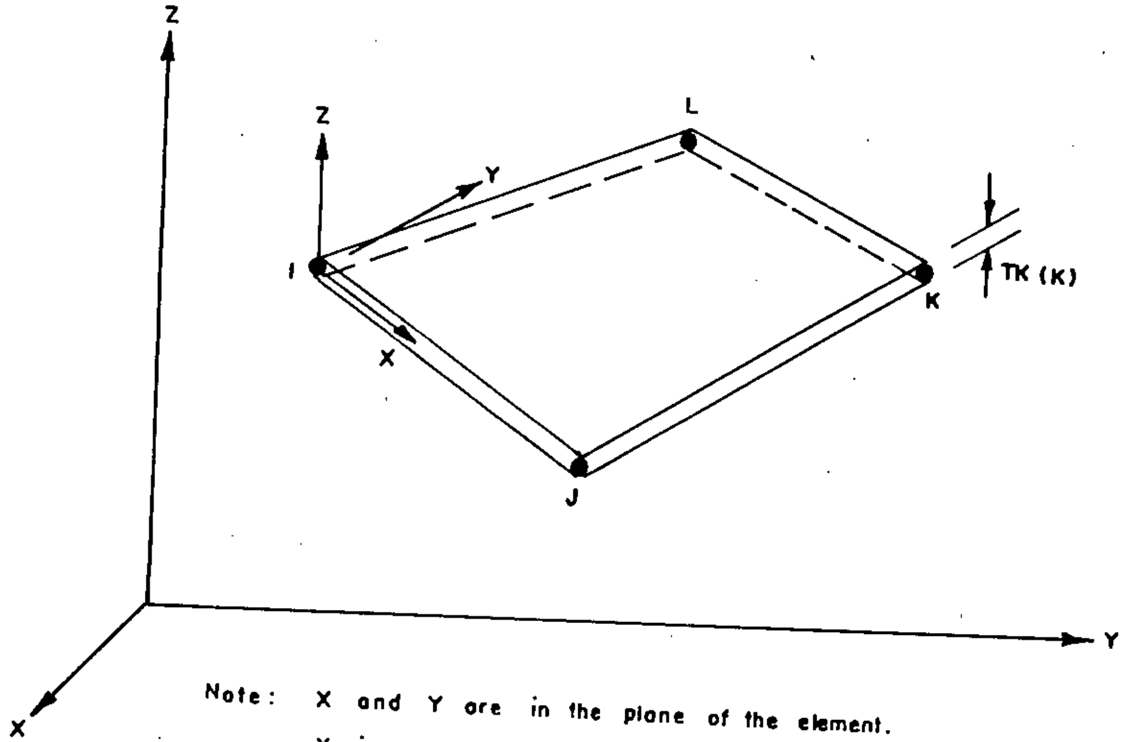
This quadrilateral thin shell element (Stiff63, ANSYS) has both bending and membrane capabilities. Both in-plane and normal loads can be applied. The element has six degrees of freedom at each node: translations in nodal X,Y,Z directions and rotations about nodal X,Y and Z axes. Shear deformation is neglected in this thin shell element.

The gometry , nodal point locations , loading , and the co-ordinate system for this element is shown in Fig. A.3.1. A brief description of ANSYS features, together with an example input data are presented in the following article.

A.3.2 General Features of ANSYS

The ANSYS computer program is a large-scale, general purpose computer program for the solution of several classes of engineering problems. Analysis capabilities include static and dynamic; elastic, plastic, creep and swelling; buckling; small and large deflections; steady state and transient heat transfer, electrostatics, magnetostatics, and fluid flow.

The matrix displacement method of analysis based upon finite element idealization is employed throughout the program. The library of finite elements available number more than forty for static and dynamic analyses, and twenty for



Note: X and Y are in the plane of the element.
X is parallel to IJ

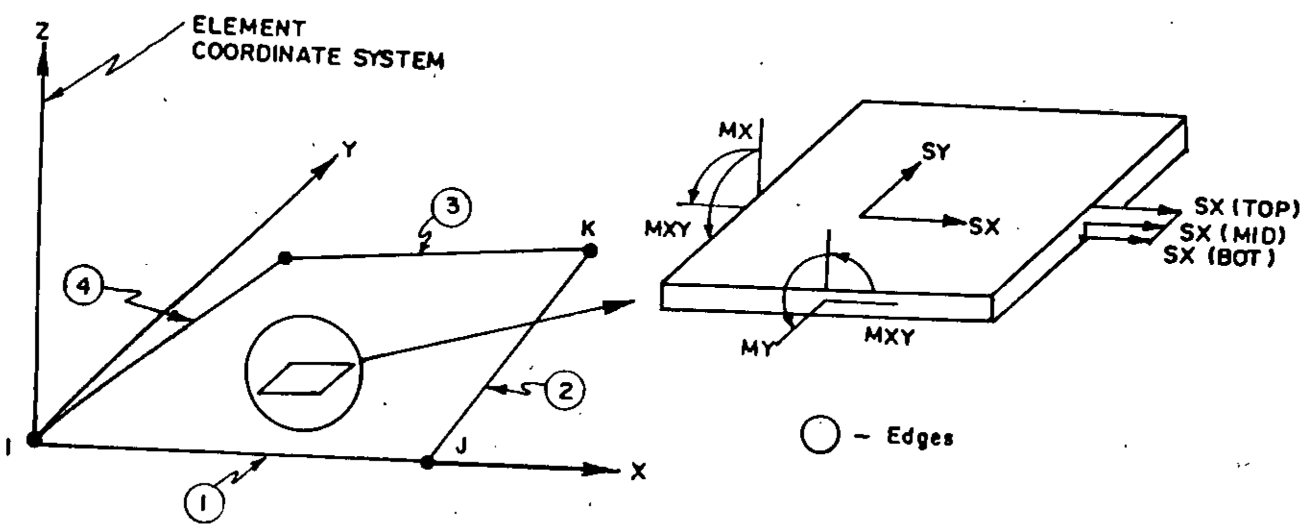


FIG. A.3.1 STIFF 63 ELEMENT (ANSYS)

heat transfer analyses. This variety of elements gives the ANSYS program the capability of analyzing two-and three-dimensional frame structures, piping systems. two-dimensional plane and axisymmetric solids, three-dimensional solids, flat plates, axisymetric and three-dimensional shells and nonlinear problems including interfaces and cables.

Loading on the structure may be forces, displacements, pressures, temperatures of response spectra. Loadings may be arbitrary functions of time of linear and nonlinear dynamic analyses. Loadings for heat transfer analyses include internal heat generation, convection and radiation boundaries, and specified temperatures or heat flows.

The ANSYS program used the wave-front (or "frontal") direct solution method for the system of simultaneous linear equations developed by the matrix displacement method, and gives results of high accuracy in a minimum of computer time. The program has the capability of solving large structures. There is no limit on the number of elements used in an analysis. There is no "band width" limitation in the analysis definition; however, there is a "wave-front" restriction depends on the amount of core storage available for a given problem. Up to 3000 degrees of freedom on the wave-front can be handled in a large core. For extremely large problems an out-of-core wave-front procedure (which effectively removes the "wave-front" limit with an increased run time penalty) is available.

The input data for the ANSYS program has been designed to make it as easy as possible to define the problem to the computer. A preprocessor (PREP7) contains (real constants, material properties, constraints, loads etc) as well. Geometry plotting is available for all elements in the ANSYS library, including isometric, perspective, section, edge, and hidden-line plots of three-dimensional structures.

ANSYS is capable of generating substructures (or superelements). These substructures may be stored in a library file for use in other analyses. Substructuring portions of a model can result in considerable computer-time savings for nonlinear analyses.

Postprocessing routines are available for algebratic modification, differentiation, and integration of calculated results. Root-sum-square operations may be perfomed on seismic model results. Response spectra may be generated from dynamic aanalysis results. Results from various loading modes may be combined for harmonically loaded axisymmetric structures. Post routines also plot distorted geometries, stress contours, safety factor contours, temperature contours, mode shapes, time history graphs, and stress-strain curves.

A.3.3 Element Input

To run ANSYS following Element parameters are be given in put file:

Element type

Nodal co-ordinates

Element properties (dimensions)

Material properties

Loading etc.

An example in-put data for a simple eight element beam structure (Fig A.3.2) is given here:

Example: Beam Deflection

```
RB  /PREP7                  *BEGIN PREP7 PREPROCESSING
   /TITLE,PR-16,BEAM *DEFLECTION EXAMPLE
    ET,1,3                 *DEFINE MODULUS FO ELASTICITY MATERIAL PROPERTY
    EX,1,30E6               *DEFINE MODULUS OF ELASTICYTY MATERIAL PROPERTY
    DENS,1,.00073           *DEFINE MASS DENSITY PROPERTY
    R,1, 13,.0014,.36*DEFINE REAL CONSTANTS (AREA, MOM. OF INERTIA,
                              THICKNESS)
    N,1                     *DEFINE NODE 1
    N,9,4                   *DEFINE NODE 9
    FILL                    *FILL BETWEEN PREVIOUS TWO NODES
    E,1.2                   *DEFINE ELEMENT 1
    EGEN,8,1,1              *GENERATE 8 ELEMENTS (TOTAL) FROM ELEMENT 1
    ITER, 1,1               *DEFINE ITERATIONS, PRINT AND POST CONTROLS
    ACEL,,,386.4            *DEFINE ACELY (ACELERATION IN THE Z DIRECTION)
    D. 1,all                *CONSTRAIN ALL DISPLACEMENTS AT NODE 1
    ,9,UY                   *CONSTRAIN OF DISPLACEMENT AT NODE 9
    F,5,FY,100              *DEFINE FROCE OF 100 IN FY DIRECTION AT NODE 5
    P,5,6,8.5,,8           *DEFINE PRESSURE OF 8.5 ON ELEMENTS FROM NODES 5
                              TO 9
    TDBC.1  SRDBC, 1 *INCLUDE DISPLACEMENT BOUNDARY CONDITIONS ON PLOT
    FBC.1                  *INCLUDE FORCE BOUNDARY CONDITIONS ON PLOT
    PRBC.1                 *INCLUDE PRESSURE BOUNDARY CONDITIONS ON PLOT
    NPLOT, 1               *PROCEDURE NODE PLOT INCLUDING NODE NUMBERS
```

```
    ENUM.1              *INCLUDE ELEMENT NUMBERS OF PLOT
    EPLOT               *PRODUCE ELEMENT PLOT
    AFMRIT              *WRITE ANALYSIS FILE
    FINISH              *TERMINATE PREP7 ROUTINE
---------------
RB  /INPUT,27           *SUBMIT ANALYSIS FILE TO SOLUTION PHASE
     FINISH             *TERMINATE SOLUTION PHASE
     SET. 1, 1          *DEFINE DATA SET (LOAD STEP 1. LITERATION 1)
     PLDISP.1           *PLOT DISPLACED AND UNDISPLACED SHARES
     FINISH             *TERMINATE POST1 ROUTINE
```

A.3.3 Solution Printout

The solution print out from a fall execution run consists of the nodal solution and element solution.

The nodal solution from a structural analysis consists of displacements at the degrees of freedom: Nodal displacements are output in the nodal coordinate system. However, other options are also available (See Ref.8).

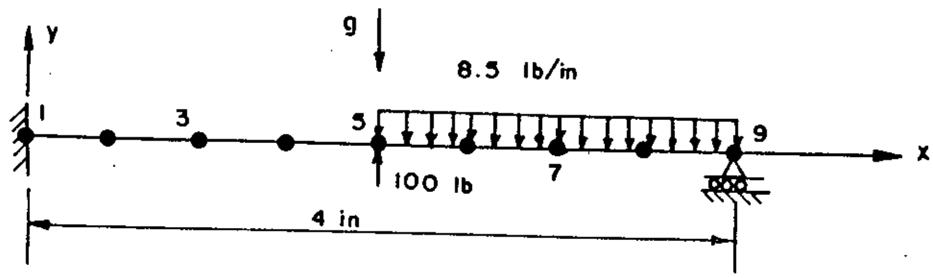


Fig. A.3.2 Beam Model for Example I.