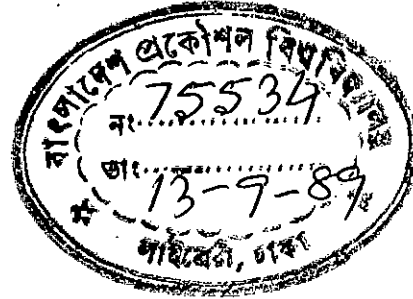


**EVALUATION OF PRODUCTION COST OF MULTI-
AREA INTERCONNECTED POWER SYSTEM
USING SEGMENTATION METHOD**

BY



A.S.M MANZURUL QUADER

A THESIS

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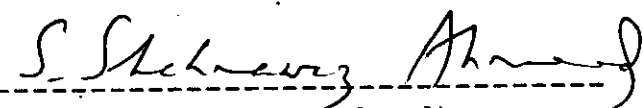
(Dr. Md. Quamrul Ahsan)
Professor, Department of Electrical
and Electronic Engineering, BUET, Dhaka. Chairman
(Supervisor)

2.  19.8.89.


(Dr. Md. Abdul Matin)
Professor and Head,
Department of Electrical and Electronic
Engineering, BUET, Dhaka. Member

3.  19.8.89

(Dr. Mohammad Ali Chowdhury)
Assistant Professor,
Department of Electrical and Electronic
Engineering, BUET, Dhaka. Member

4.  19/8/1989

(Dr. S. Shahnawaz Ahmed)
Assistant Professor,
Department of Electrical and Electronic
Engineering, BUET, Dhaka. Member

5. 

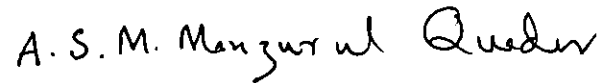
(Dr. A.M. Zahoorul Huq) 19.8.89
391, Baitul Aman Housing Estate
Road No. 6, Shymoli, Dhaka - 7. Member
(External)

CERTIFICATE

This is to certify that this work was done by me and it has not been submitted elsewhere for the award of any degree or diploma.

Countersigned

Signature of the Student



(Dr. Md. Quamrul Ahsan)

(A.S.M. Manzurul Quader)

Supervisor

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ABSTRACT

The evaluation of reliability and production cost of each alternative potential plan are two essential steps in generation expansion planning. Utilities interconnect with each other to decrease the production cost and increase the reliability of the global as well as individual system. The methodology to evaluate the reliability and the production cost is well established for single area system and also quite efficient methodology is available for two area interconnected systems. However, an efficient methodology of evaluating the production cost for multi-area interconnected systems (more than two system) has not yet been developed.

This thesis presents a methodology to evaluate the production cost of multi-area interconnected electrical power systems. The methodology is an extension of the segmentation method for evaluating the production cost of two area interconnected systems. The accuracy of the developed methodology is justified through a small example which can be solved by using a pocket calculator. The methodology is also applied to a realistic interconnected system.

ABBREVIATIONS

CLC	= Chronological Load Curve.
DNS	= Demand Not Served.
FD	= Frequency and Duration.
FOR	= Forced Outage Rate.
IEEE	= Institution of Electrical and Electronic Engineering.
IFC	= Incremental Fuel Cost.
LDC	= Load Duration Curve.
LOLP	= Loss of Load Probability.
PDF	= Probability Density Function.
RTC	= Residual Tie Line Capacity.
RTS	= Reliability Test System.
RV	= Random Variable.
TC	= Tie Line Capacity.

NOTATIONS

C_K = Capacity of the K-th generating unit.

CC_i = Capacity cost of unit i.

CF_i = Capacity factor of unit i.

EC_i = Production cost (Energy cost) of unit i.

e^I = Total export from system I to the importing systems.

$\text{exp}^{D^{I-K}}$ = Direct transfer of power from system I to the system K through the direct tie line connecting the I-th system and the K-th system.

$\text{exp}^{I^{I-K-M}}$ = Indirect transaction of power from system I to the M-th through the composite tie line (I-K) and (K-M).

exp^{I-K} = Total export from system I to system K.

FC_i = Fuel cost of unit i.

f_{LA} = Probability density function of available capacity.

f_{Lo} = Probability density function of outage capacity.

FCR_i = Fixed charge rate of unit i.

GEC = Global production cost.

GES = Expected energy generated by the global system.

GS = Global savings.

HR_i = Heat rate of unit i.

IC = Installed capacity.

IFC_i = Incremental fuel cost of unit i.

L = Random load.

Le = Equivalent load.

Lo_i = Random outage load corresponding to i-th unit.

m = Mean up time.

$m^I_{i,j,\dots,n}$ = First moment of load of system I corresponding to the segment (i,j, ..., n)

$m^I_{i,j,\dots,n}$ = Modified first moment of load of system I corresponding to the segment (i,j, ... n).

n_i = Total number of generating unit of system i.

OM_i = Operation and maintenance cost of unit i.

$P_{i,j,\dots,n}$ = Joint probability of the (i,j,...n) th segment.

PC_i = Plant cost of unit i.

q_k = FOR of the k-th unit.

r = Mean down time.

RTC^{I-K} = Residual tie line capacity from system I to system K.

S^I = Total number of committed units of system I.

SC = System cost.

T = Time period.

t_I = Number of segments in the direction of axis I which is attributed to any system I.

TC = Tie line capacity.

UE_k^- = Unserved energy before convolving the k-th unit.

UE_k = Unserved energy after convolving the k-th unit.

UC_i = Unit capacity cost of unit i.

ϵ (DNS) = Expected demand not served.

ϵ (ENS) = Expected energy not served.

ΔC = Segment size.

λ_k = Average incremental cost of the k-th unit.

ϵ (E_j) = Expected value of energy produced by unit j.

ϵ (E_g) = Expected value of energy consumed by the system s.

λ = Unit failure rate.

μ = Unit repair rate.

λ_k = Average incremental cost of k-th unit.

$\delta(\cdot)$ = Dirac-delta function.

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CHAPTER 1

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Generation expansion planning process begins with the estimate of the load growth and the total energy consumption [1]. The next important two aspects of planning are :

- i) The evaluation of reliability indices.
- ii) The production cost.

After identifying the need for generating capacity addition, the planner develops a number of feasible expansion alternatives on the basis of

1. Load growth.
2. Construction time.
3. Availability of sites.
4. Availability of fuel.

Each alternative plan is then evaluated on the basis of reliability [1]. The simple and most common of all reliability indices is the loss of load probability (LOLP) [2,3]. The plan that does not meet the reliability criteria is then eliminated or appropriately modified and plan which satisfies the required reliability level are then evaluated on the basis of economics.

The reliability and economic evaluation of multi-area interconnected systems is different from that of a single area system. If the available capacity in a geographical region can be transmitted wherever it is needed without tie line restrictions then this region may be treated as a 'single area' [4]. But if the available capacity in different geographical regions are transmitted wherever it is needed

through tie line restrictions then these regions are treated as 'area' interconnected systems.

The tumultuous events of the past decade have brought about financial pressures on electric utilities. Pressed for the fund and concerned about the return from their investment, electric utilities are increasingly placing renewed emphasis on the economics of their electric power system. Interconnection of electric utilities is one way by which economic benefits can be achieved. Such benefits would include lower production cost as well as deferral of plant construction due to reduced reserved capacity requirements.

1.2 BACKGROUND

* [At present for evaluating the reliability and production cost probabilistic simulation is widely used for generation expansion planning. The historical development of these methods is extremely interesting. In 1947, a large group of papers by Calabrese [7], Lyman[8], Seelye[9], Loane and Watchorn [10] proposed some of the basic concepts upon which some of the methods in use at the present time are based. In 1948, the first AIEE subcommittee on the Application of Probability Methods was organised. The subcommittee submitted several reports containing comprehensive definitions of equipment outage classifications in 1949 [11], 1954 [12] and 1957 [13]. The group of papers of 1947 proposed the methods which with some modification are now generally known as the 'Loss of Load Approach' and the 'Frequency and Duration Approach'. They are described in detail in a 1960 AIEE Committee Report [14]. * The effect of interconnections and the determination and allocation of capacity benefits resulting from interconnections were discussed by Watchorn [15] and Calabrese [16] in

1950 and 1953 respectively. In 1954, Watchorn [17] noted the benefits associated with using digital computer and in 1955, Kirchmayer et al. [18] illustrated it in the evaluation of economic unit additions in system expansion studies. In 1960, Brown et al. [19] published the results of a statistical study of five years of data on 387 hydroelectric generating units. Shortly after this in 1961, the AIRE subcommittee produced a manual [20] outlining reporting procedures and methods of analyzing forced outage data using digital equipment. Cook et al [3] proposed the basic method for evaluating LOLP of two interconnected systems. The initial approach to the calculation of outage frequency and duration indices in generating capacity reliability evaluation was modified by the introduction of a recursive approach. This technique is described in detail in a series of four publications [21,22,23,24].

The most important development in the evaluation of LOLP and production cost by probabilistic simulation was suggested by Baleriaux [25] and Booth [26]. In 1960, Rau et al. [27] proposed a computationally fast method, which approximates the discrete distribution of load (equivalent load) through Gram-Charlier series expansion as a continuous function. Rau et al. [28] proposed the utilization of the bivariate Gram-Charlier expansion to evaluate the LOLP of two interconnected systems. The bivariate Gram-Charlier expansion has also been utilized by Rau et al [27], by Noyes [30] and Ahsan et al [31] in the evaluation of production costs of two interconnected systems. Schenk et al. [32] proposed the segmentation method for the evaluation of expected energy generation and LOLP of a single area system. In this method the authors avoided the inherent inaccuracies of series expansion retaining the computational

efficiency. The segmentation method has been extended by Schenk, Ahsan and Vassos [33] to incorporate the reliability evaluation of two interconnected systems. Ahsan and Schenk [34] have utilized the segmentation technique to evaluate the production cost of two interconnected system. The segmentation method is accurate as well as computationally fast [32,33,34]. Recently, F.N. Lee [35] has proposed a methodology to evaluate the production cost of multi-area system based upon a reduced set of demand-supply feasibility conditions. However, it's computational accuracy and efficiency has not yet been tested.

1.3 THESIS ORGANIZATION:

This thesis presents a methodology to evaluate the production cost of multi-area interconnected systems. The methodology is an extension of the works of Ahsan and Schenk [34]. The proposed methodology utilizes the segmentation method for numerical evaluation. This thesis consists of seven chapters. In the first chapter the background and the motivation of the work is presented. A brief discussion of the generation expansion planning process is presented in chapter 2. In chapter 3, the generation and load models used in different probabilistic simulation techniques are derived. The generating system cost analysis is discussed in chapter 4. A brief discussion on corporate models is also included in this chapter. In chapter 5, the methodology of evaluation of multi-area production costing is presented. The numerical results and the generation and load model used in the numerical evaluation are presented in chapter 6. In chapter 7, salient observation and corresponding discussions and conclusions are presented on the basis of the results obtained in chapter 6. Some recommendations for further research are also presented in this chapter.

CHAPTER 2

CHAPTER 2

PLANNING PROCESS

2.1 INTRODUCTION:

The generation expansion planning is most important among the planning of all other sectors of power system. A suitable generation expansion plan must provide the electric utility with the capability of meeting customer demands during the plan period for a reasonable price and reliable quality. Of course, every investor owned utility must consider only those expansion plans that will enable it to maintain a sound financial posture.

In order to develop an economically optimal generation expansion plan it is necessary to introduce an economic indicator (cost functional) that measures the penalties associated with under and over expansion relative to the projected (expected) demand, the cost of capacity expansion and losses arising from residual capacities at the end of the plan period.

In this chapter, both the static and dynamic models of generation expansion planning are presented.

2.2 STATIC MODEL OF GENERATION EXPANSION PROCESS

Power system generation expansion planning process begins with a forecast of anticipated future load requirement. Estimate of both demand and energy requirement with time are crucial to effective system planning. Good forecast reflecting current and future trends, tempered with good judgement is the key to all planning, indeed to financial success. In addition to the uncertainty inherent in forecasting future load requirements, the planner must deal

with the uncertainties associated with

1. Unit reliability and maintenance schedules
2. Fuel costs.
3. Pollution abatement legislation and costs.
4. Construction cost.
5. Start up times.
6. Availability and cost of capital.

After identifying the need for generating capacity addition, the planner develops a number of feasible expansion alternatives on the basis of

1. load growth.
2. Construction time.
3. Availability of sites.
4. Availability of fuels.

In order to select the most suitable plan from the alternative plans, each alternate plan is evaluated first on the basis of reliability. There are different measures of reliability among them i) Loss of Load Probability (LOLP), ii) Unserved demand (DNS) iii) Frequency and Duration method (FD). The simple and most common of all reliability indices is the Loss of Load Probability [2,3].

Plans that do not meet the reliability criteria are eliminated or appropriately modified and the plan which satisfies the required reliability level are evaluated on the basis of economics.

The main factors which enter into the cost analysis of a given plan are : i) Capacity cost, ii) Production cost, iii) Timing of unit addition. The production cost includes the cost of fuel and the operation and maintenance cost. The evaluation of the energy production cost is by far the most complex part of cost analysis associated with a particular

expansion plan.

After cost analysis the environmental impact of the plan is considered. The waste of the nuclear, oil and coal units cause thermal and air pollutions. Thermally polluting air, rivers and lakes have serious effect on ecosystem which may cause environmental disaster. In Figure 2.1 the planning process is depicted in the form of block diagram.

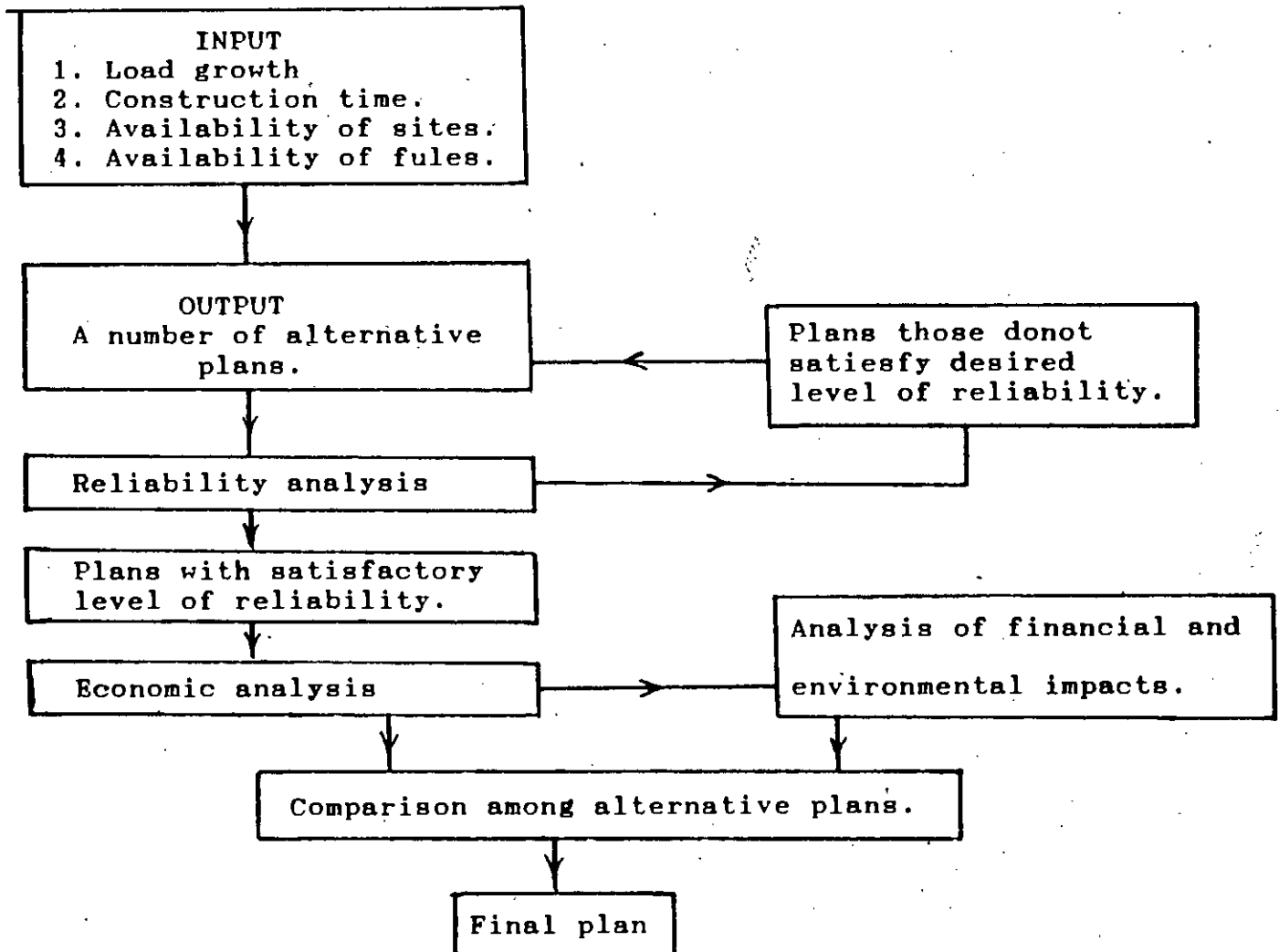


Figure 2.1 : Generation expansion planning process.

2.3 DYNAMIC MODEL OF GENERATION EXPANSION PROCESS

The dynamic expansion model includes the equations relating to the stock of generators at any given time with the retirement of old generators and new additions. It also includes the development of economic indicator model as well as the optimal policy of expansion. This follows the mathematical modeling of stock of generators considering the addition and the retirement of generators in discrete time.

2.3.1 GENERATION MODEL

The planning horizon is denoted by the time interval $(t, T]$, with both t and T assumed finite. The time interval $(t, T]$ is partitioned into M subintervals given by

$$t_0 \leq t_1 \leq t_2 \leq \dots \leq t_n \leq \dots \leq t_M = T, \text{ so that}$$

$$\bigcup_{k=1}^M (t_{k-1}, t_k) = (t, T]$$

It is assumed that there are m distinct classes of generators classified according to size, average lifetime and retirement characteristics as functions of age and existing number of generators of that age. The population (stock) of generators of class i , $i = 1, 2, 3, \dots, m$, at time t_n , $n = \{1, 2, 3, \dots, M\}$ in the system, denoted by X_t^i , is given by [38]

$$X_{t_n}^i = X_{t_n}^i(t_k) + N^i(X_{t_{n-1}}, (t_{n-1}, t_n]) \quad (2.1)$$

where $X_{t_n}^i(t_k)$, $0 \leq k \leq n-1$, denotes the number of generators of class i , present in the system at time t_n , which were commissioned at a previous time t_k . In other words, the age of the generators is given by $t_n - t_k$. The variable $N^i(X_{t_{n-1}}, (t_{n-1}, t_n])$ denotes the number of new generators of class i added to the system during the interval $(t_{n-1}, t_n]$,

given the previous stock level

$$X_{t_{n-1}} \equiv (X_{t_{n-1}}^i ; i=1,2,\dots,m)$$

For fixed but arbitrary $n \in \{1,2,\dots,M\}$ and $0 \leq k \leq n-1$, each of the variables $X_{t_n}^i(t_k)$, representing the no. of generators of age $t_n - t_k$ from class i surviving time t , is given by [28]

$$X_{t_n}^i(t_k) = X_{t_{n-1}}^i(t_k) - Q^i [t_n - t_k, X_{t_{n-1}}^i(t_k)] \quad (2.2)$$

where $X_{t_{n-1}}^i(t_k)$ is the no. of generators from class i which were added to the system at time t_k and survived time t_{n-1} ; $Q^i(a,y)$ is the no. of generators from class i that retire at age a given that the population of such machines is y .

It is noted that the function Q must satisfy

$$Q^i(a,y) \begin{cases} > 0 & \text{for } a > 0, y > 0 \\ = 0 & \text{otherwise} \end{cases}$$

Combining Equations (2.1) and (2.2)

$$\begin{aligned} X_{t_n}^i(t_k) &= X_{t_{n-1}}^i(t_k) - Q^i [t_n - t_k, X_{t_{n-1}}^i(t_k)] \\ X_{t_{n-1}}^i(t_{n-1}) &= N^i [X_{t_{n-1}}^i, (t_{n-1}, t_n)] \end{aligned} \quad (2.3)$$

$$0 \leq k \leq n-1, \quad 1 \leq n \leq M$$

The solutions of system(2.3) are the off diagonal elements of the lower triangular matrix:

$$\begin{bmatrix} X_{t_0}(t_0) & & & & & \\ X_{t_1}(t_0) & X_{t_1}(t_1) & & & & \\ X_{t_2}(t_0) & X_{t_2}(t_1) & X_{t_2}(t_2) & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ X_{t_M}(t_0) & X_{t_M}(t_1) & X_{t_M}(t_2) & \cdot & \cdot & \cdot & X_{t_M}(t_M) \end{bmatrix}$$

the element of this matrix, one for each class i , give the population distribution of generators by age. Summing the elements of a row, the total number of generators at a given time indicated by the subscript is obtained. On the other hand, each column represents the evolution of population as it diminishes with increasing time and eventually retires as $t_M \rightarrow \infty$.

2.4 ECONOMIC INDICATOR (COST FUNCTIONAL)

In order to determine the economic merit of any proposed generation expansion plan, economic indicator or cost functional philosophy should be used.

For the sake of simplicity and clarity, cost functional is the sum of three major items as follows:

- i) Cost of capacity additions, J_A .
- ii) Social cost arising from failure to meet the projected demand, J_B .
- iii) Penalty for residual generation capacity at the end of the plan period, J_C .

2.4.1 COST OF CAPACITY EXPANSION, J_A

The cost item i) denoted by J_A comprises generator costs, fuel (or production) costs and operation and maintenance cost, all combined for simplicity. Since the machines may be added to the system at any time and possibly several times during the period $(t_0, T]$ the total payment for the i -th class of machines is given by [38]

$$J_A = [T \wedge (\theta + \tau_i) - \theta] \times \alpha_i(\theta) \exp(-\delta_i \theta) N^i (X_\theta, d\theta) \quad (2.4)$$

where

$$T \wedge (\theta + \tau_i) = \min(T, \theta + \tau_i)$$

$\alpha_i(t)$ = average cost (per unit time) at time t for the generator of type i .

N^i = No. of machines of type i added.

τ_i = Average life time of machines of type i .

2.4.2 SOCIAL COSTS, J_B

It is difficult to assess the social cost caused by loss of load or failure to meet the expected demand. However, it is clear that loss of load has a serious economic impact on production and business, in addition to inconvenience suffered by domestic consumers. If C^i be the capacity of each generating units of class i and X denote the no. of such generator. Then the total generation capacity of the system at time t is given by [38]

$$C_t = \sum_{i=1}^m X_t^i C^i \quad (2.5)$$

If $D_t, t \in (t_0, T]$ denotes the expected demand, the deficit or surplus capacity is given by

$$R_t = D_t - C_t \quad (2.6)$$

If η_t denotes the social cost/unit loss of load/unit time, then the total social cost at time t is given by

$$J_B(t) = \eta_t R_t I(R_t) \quad (2.7)$$

where function I is called the indicator function and is given by

$$I(v) = \begin{cases} 1 & \text{for } v > 0 \\ 0 & \text{for } v \leq 0 \end{cases}$$

Hence the cumulative social cost denoted by J_B is given by

$$J_B = \int_{t_0}^T \eta_t R_t I(R_t) dt. \quad (2.8)$$

2.4.3 PENALTY FOR RESIDUAL GENERATION CAPACITY, J_C

The penalty for residual capacity is obtained by subtracting the salvage value from the cost of class i generators added to the system at time $t = \theta$ is [36]

$$P_i(\theta) = \{ \alpha_i(\theta) \exp(-\delta_i \theta) [\tau_i - (T - \theta)] \\ \times N^i(X_\theta, d\theta) - S_i(T - \theta) \\ \times N^i(X_\theta, d\theta) \} I[\tau_i - (T - \theta)] \quad (2.9)$$

$S_i(a)$ denote the salvage value of generators of class i and of age a . Integrating the above expression over the plan period and summing over all classes of generators we obtain the penalty for residual generation J_C given by

$$J_C = \sum_i \int_{t_0}^T P_i(\theta) d\theta \\ = \int_{t_0}^T \sum_i \{ \alpha_i(\theta) \exp(-\delta_i \theta) [\tau_i - (T - \theta)] \\ \times I[\tau_i - (T - \theta)] - S_i(T - \theta) \\ \times I[\tau_i - (T - \theta)] \} N^i(X_\theta, d\theta) \quad (2.10)$$

Summing (2.4), (2.8) and (2.10) we obtain the complete cost functional $J(N)$ given by

$$J(N) = J_A + J_B + J_C$$

$$= \int_{t_0}^T \sum_{i=1}^m ([T \wedge (\theta + \tau_i) - \theta] \alpha_i(\theta) \\ \times \exp(-\delta_i \theta) + \{ \alpha_i(\theta) \exp(-\delta_i \theta) \\ \times [\tau_i - (T - \theta)] - S_i(T - \theta) \} \\ \times I[\tau_i - (T - \theta)]) N^i(X_\theta, d\theta) \\ + \int_{t_0}^T \eta_\theta R_\theta I(R_\theta) d\theta \quad (2.11)$$

2.5 OPTIMAL POLICY

The cost function $J(N)$ given by (2.11) is a function of the control vector $N = (N^i, i = 1, 2, \dots, m)$. Minimizing this functional with respect to N , one obtains the optimal generation expansion policy. A policy N^* is optimal if $J(N^*) < J(N)$ for all admissible policies (N) . Principle of optimality is the minimum cost over the plan period. To obtain optimal expansion policy the cost functional equation (2.11) can be rewritten in the following form:

$$J(N) = \int_{t_0}^T \sum_{i=1}^m L_i(\theta) N^i(X_\theta, d\theta) + \int_{t_0}^T L(X_\theta, \theta) d\theta \quad (2.11a)$$

where the first term represents the cost of capacity expansions plus the penalty due to residual generation, while the second term represents the social cost. The function L_i and L can be easily be identified by comparing (2.11) with (2.11a).

In the numerical simulation the state equations (2.1) and (2.3) are used and the discrete version of the cost functional (2.11) becomes

$$J(N) = \sum_{k=1}^M \left(\sum_{i=1}^m L_i(t_{k-1}) \times N^i(X_t, (t_{k-1}, t_k]) + L(X_t, t_{k-1})(t_k - t_{k-1}) \right) \quad (2.12)$$

CHAPTER 3

CHAPTER 3

GENERATION AND LOAD MODELS

3.1 INTRODUCTION

The evaluation of LOLP and production costs for generation expansion planning by any method requires two basic models; the load model and the generation model. The various models for generation and those for system load, differ greatly in their degree of sophistication. The model suitable for incorporation of the probabilistic or stochastic nature of system behaviour are presented in this chapter. Such models are widely used in various probabilistic simulation techniques.

3.2 GENERATION CAPACITY MODEL

Different types of generating units are in use today and all types of units are randomly forced off-line because of technical problems during normal period of operation. To account for the random outage or availability of a unit, it is necessary to determine the probability density function (PDF) that describes the probability that a unit will be forced off-line or will be available during its normal period of operation. It may be assumed on the basis of historical data that the availability of the generating capacity of a given unit may be graphically represented as shown in Figure 3.1. This figure conveys the idea that random failure and repair of a unit can be defined as a two-state stochastic process. A stochastic process is defined as a process that develops in time in a manner controlled by probabilistic laws.

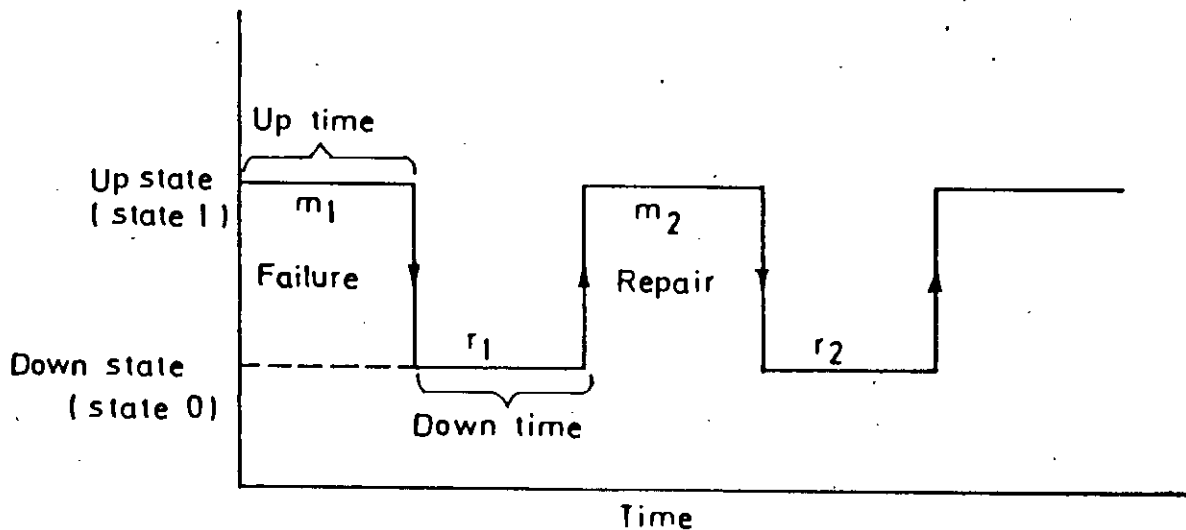


Figure 3.1 : Run-failure-repair-run cycle for a generating unit.

The system alternates between an operating state, or, up state, followed by a failed state, or down state, in which repair is effected.

For the i -th cycle, let

$$m_i = \text{UP time}$$

$$r_i = \text{DOWN time}$$

The random history of a generating unit may be represented in terms of an average (mean) UP time and an average DOWN time as follows:

$$m = \text{mean up time} = \frac{1}{N} \sum m_i$$

$$r = \text{mean down time} = \frac{1}{N} \sum r_i$$

where N is the total number of run-fail-repair-run cycles. Thus the unit failure rate λ and the repair rate μ may be expressed as

$$\lambda = \text{unit failure rate} = \frac{1}{m} \quad (3.1)$$

$$\mu = \text{unit repair rate} = \frac{1}{r} \quad (3.2)$$

With these two parameters the random failure and repair of a generating unit can be defined as a state-space diagram (two state) as shown in

Figure 3.2.

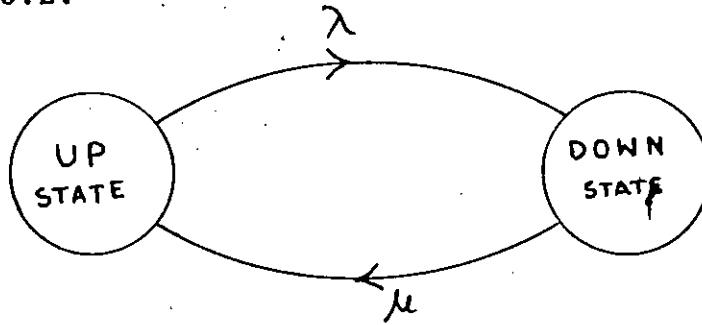


Figure 3.2 : Generating unit state -space diagram.

Two important parameters can be obtained from this model(36):

1. Unit availability - the long term probability that the unit will be in the , UP state.

2. Unit unavailability - the long term probability that the unit will be in the DOWN state.

To obtain the expressions for long-term availability and unavailability of a generating unit, it is first necessary to recognize that the stochastic process we are considering is a very special one, called a zero-order, discrete state, continuous transition Markov process. Such a stochastic process has the following properties [36]

1. Mutually exclusive and discrete states, that is, the generating unit can be in either the UP or the DOWN state, but not in both simultaneously.

2. Collectively exhaustive states, that is, since we assume that only possible states for a generating unit are the UP and the DOWN states, then these states define all the possible states we ever expect to find a unit in.

3. Changes of state are possible at any time.

4. The probability of departure from a state depends only on the current state and is independent of time.

5. The probability of more than one change of state during a small time interval Δt is negligible.

Let

$$P_1(t + \Delta t) = \text{Probability that the unit will be in the UP state at time } (t + \Delta t) \quad (3.3)$$

Thus

$$P(t + \Delta t) = \left[\begin{array}{l} \text{Probability of being} \\ \text{in state 1 at time } t \\ \text{and not leaving that} \\ \text{state during interval} \\ t. \end{array} \right] + \left[\begin{array}{l} \text{Probability of} \\ \text{being in state 2} \\ \text{at time } t \text{ and} \\ \text{moving to state 1} \\ \text{during interval } t \end{array} \right] \quad (3.4)$$

Consider that the distribution of a unit failure can be described by the exponential distribution.

$$F_1(t) = e^{-\lambda t} = \text{Probability of unit being available upto time } t \quad (3.5)$$

Expanding the right hand side of Equation (3.5) into infinite series and neglecting higher order terms, it is obtained as

$$F_1(t) = 1 - \lambda \Delta t + \frac{\lambda^2 (\Delta t)^2}{2!} + \dots$$

$$\approx 1 - \lambda \Delta t = \text{Probability of unit being available during time } \Delta t. \quad (3.6)$$

where

$$\lambda \Delta t = \text{Probability of transferring from state 1 to state 2 in time } \Delta t.$$

Again

$$F_2(t) = e^{-\mu t} = \text{Probability of unit being unavailable upto time } t \quad (3.7)$$

Expanding into an infinite series and neglecting higher order terms, it is obtained as

$$F_2(t) \triangleq 1 - \mu \Delta t = \text{Probability of unit being unavailable during time } \Delta t. \quad (3.8)$$

where

$\mu \Delta t$ = Probability of transferring from state 2 to state 1 in time t

Using the definitions of Equations(3.5) to (3.8), Equation(3.4) may be written as [36]

$$P_1(t + \Delta t) = P_1(t) [1 - \lambda \Delta t] + P_2(t) [\mu \Delta t] \quad (3.9)$$

Similarly,

$$P_2(t + \Delta t) = P_2(t) [1 - \mu \Delta t] + P_1(t) [\lambda \Delta t] \quad (3.10)$$

Rearranging these two Equations, we have

$$\frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = -\lambda P_1(t) + \mu P_2(t)$$

$$\frac{P_2(t + \Delta t) - P_2(t)}{\Delta t} = \lambda P_1(t) - \mu P_2(t)$$

Letting $\Delta t \rightarrow 0$, the following differential equations are obtained.

$$\frac{dP_1}{dt} = -\lambda P_1 + \mu P_2 \quad (3.11)$$

$$\frac{dP_2}{dt} = \lambda P_1 - \mu P_2 \quad (3.12)$$

with

$$P_1(t) + P_2(t) = 1$$

Equations (3.11) and (3.12) can be written in the matrix form as follows:

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix} = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \quad (3.13)$$

Solving (6)

$$P_1(t) = \frac{\mu}{\lambda + \mu} [P_1(0) + P_2(0)] + \frac{e^{-(\lambda + \mu)t}}{\lambda + \mu} [P_1(0) - P_2(0)] \quad (3.14)$$

$$P_2(t) = \frac{\lambda}{\lambda + \mu} [P_1(0) + P_2(0)] + \frac{e^{-(\lambda + \mu)t}}{\lambda + \mu} [P_2(0) - P_1(0)] \quad (3.15)$$

where $P_1(0)$ and $P_2(0)$ represent initial states(conditions) such that

$$P_1(0) + P_2(0) = 1$$

Consider that at $t = 0$ the generating unit is in the UP state, i.e, state 1.

$$P_1(0) = 1 \text{ and } P_2(0) = 0$$

$$P_1(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \quad (3.16)$$

$$P_2(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \quad (3.17)$$

In generation expansion planning long-term (steady state) probabilities are required. Hence, letting $t \rightarrow \infty$, Equations(3.16) and (3.17) are obtained as

$$P_1(\infty) = \frac{\mu}{\lambda + \mu}$$

$$P_2(\infty) = \frac{\lambda}{\lambda + \mu}$$

Thus the long-term probabilities of unit availability and unavailability are given by :

$$\text{Prob. (UP state)} = p = \frac{\mu}{\lambda + \mu} = \frac{r}{m + r} \quad (3.18)$$

$$\text{Prob. (DOWN state)} = q = \frac{\lambda}{\lambda + \mu} = \frac{m}{m + r} \quad (3.19)$$

so that

$$p + q = 1 \quad (3.20)$$

The traditional term for the unit unavailability is 'forced outage rate' (FOR), a misnomer in fact, since the concept is not a rate. An estimate for this important parameter may be given by

$$\text{FOR} = \frac{\text{forced outage hours}}{\text{forced outage hours} + \text{service hours}}$$

$$\text{or FOR} = \frac{\text{FOH}}{\text{FOH} + \text{SH}} \quad (3.21)$$

The usual method of accounting for partial outages is to increase the forced outage hours by an appropriate amount of time called 'equivalent force outage hours' (EFOH). This duration is obtained if the actual partial outage hours are multiplied by the corresponding fractional capacity reduction and these products are then totalled. Considering a single occurrence, for example, a unit operating at 60% capacity for 80 hours will have an equivalent forced outage duration of $80(0.4) = 32$ hours. Based on this approach, an estimate of 'equivalent forced outage rate' (EFOR) may be defined as

$$\text{EFOR} = \frac{\text{FOH} + \text{EFOH}}{\text{FOH} + \text{SH}} \quad (3.22)$$

where the service hours (SH) include the actual partial outage times as

well.

3.2.1 PROBABILITY DENSITY FUNCTION OF AVAILABLE AND OUTAGE CAPACITY:

For a generating unit of capacity C MW, FOR = q and availability p , the probability density functions (PDF) of available and forced outage capacity are given in Figure 3.3.

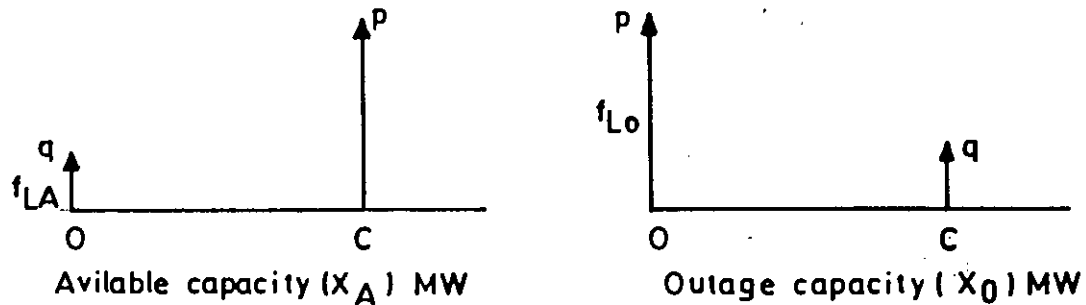


Figure 3.3 : PDFs of available and forced outage capacity.

The PDF of forced outage capacity may be conventionally expressed as [36]

$$f_{L_o}(X_o) = p \delta(X_o) + q \delta(X_o - C) \quad (3.23)$$

where

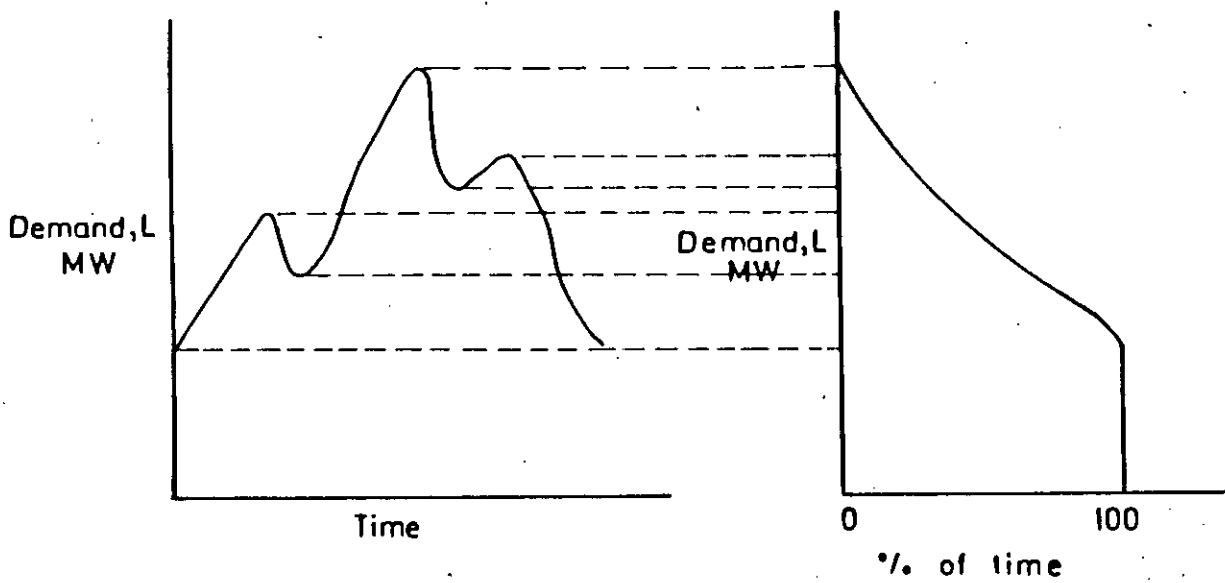
f_{L_o} = PDF of forced outage capacity.

$\delta(\cdot)$ = Dirace-delta function.

3.3 PROBABILISTIC LOAD MODELS:

Proper modelling of load is an important factor in the evaluation of LOLP and production cost. The probabilistic load model which is widely used describes the probability that load will exceed a certain value. The data required to develop such a model are readily available, since continuous readings of system demand and energy are usually obtained on a routine basis by electric utilities. If a recording of instantaneous demands were plotted for a particular period

of time, a curve such as depicted in Figure 3.4(a) might result. This is known as the 'Chronological Load Curve' (CLC). From this curve the 'Load Duration Curve' (LDC) in Figure 3.4(b) is easily constructed. The load duration curve is created by determining what percentage of time the demand exceeded a particular level.



(a) Instantaneous demand Vs. time

(b) Load duration curve

Figure 3.4 : Chronological load curve and load duration curve.

3.3.1 LOAD PROBABILITY DISTRIBUTION

For generation system studies it is necessary to interchange the axis parameters in Fig. 3.4(b) and normalize time, producing 'load probability distribution' in Figure 3.5. This curve is also called 'inverted load duration curve'. This load distribution will be denoted generally by $F_k(L)$, where k indicates the time period for which the distribution is applicable.

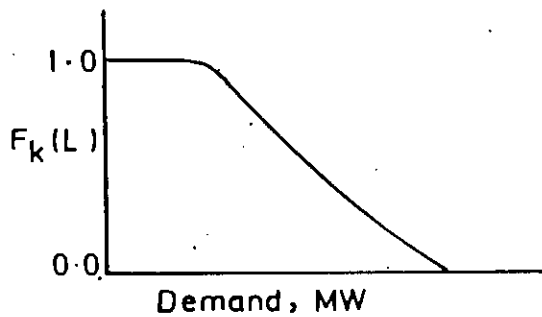


Figure 3.5 : Load probability distribution for week k.

3.3.2 HOURLY LOAD:

Another load model which is often used in various probability methods for evaluating LOLP and production cost is the hourly load. It is derived from the chronological load curve (CLC). Figure 3.6 shows a CLC, the time axis being divided into a number of small intervals between times $t_0, t_1, t_2 \dots t_{r-1}, t_r, \dots t_{n-1}, t_n$

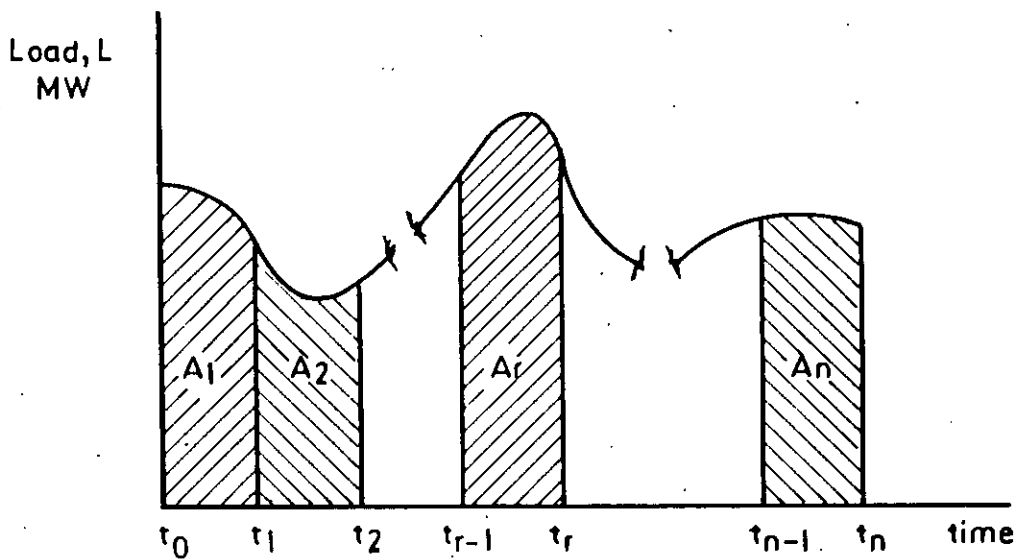


Figure 3.6 : CLC with time axis divided into n small intervals.

In Figure 3.6, the energy demand during the period between t_{r-1} and t_r is given by the area A_r under the CLC between t_{r-1} and t_r . Hence

$$A_r = \int_{t_{r-1}}^{t_r} L dt \quad (3.24)$$

Dividing this area by the period of time ($t_r - t_{r-1}$), the average load during that period is obtained. Thus

$$L_{avg}^r = \frac{A_r}{(t_r - t_{r-1})} \quad (3.25)$$

In this way the average load for all other time intervals are obtained. If the average load for each time interval is assumed to remain constant for the corresponding interval, then a distribution of load as shown in Figure 3.7 will result. Note that by such construction of load curve, the energy demand for each interval remains unchanged.

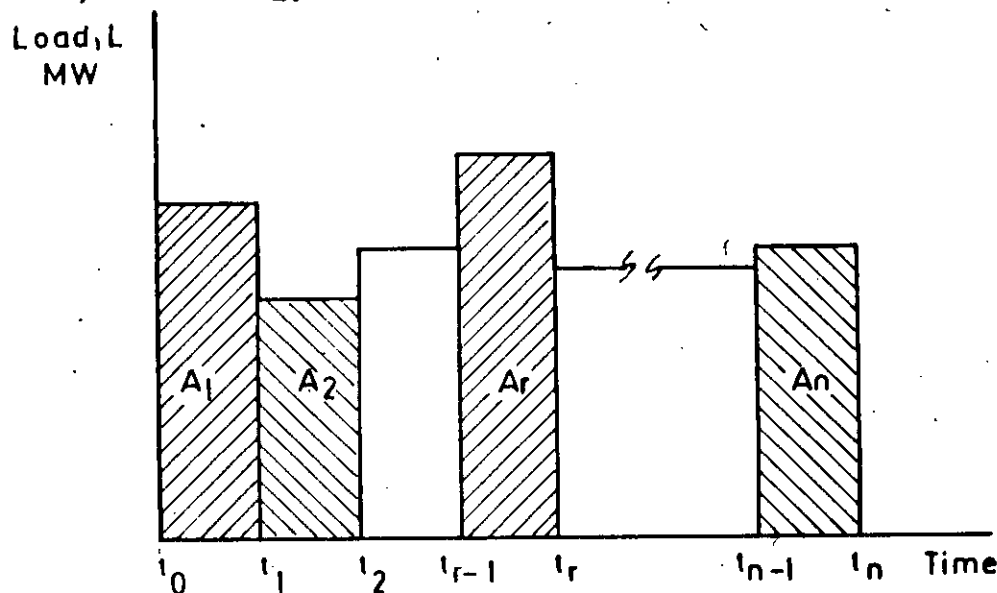


Figure 3.7 : Load distribution assuming constant load for each small interval

In each of the time intervals into which the time axis is divided equals to one hour then the resulting distribution is called 'hourly

load curve'.

3.3.3 EQUIVALENT LOAD:

The randomness in the availability of generation capacity is taken into consideration by defining a fictitious load, known as 'equivalent load' (L_e) [36]. Figure 3.8 depicts the relationship between the system load and generating units, where actual units have been replaced by fictitious perfectly reliable (100% reliable) units and fictitious random loads; whose probability density functions are the outage capacity density functions of the units.

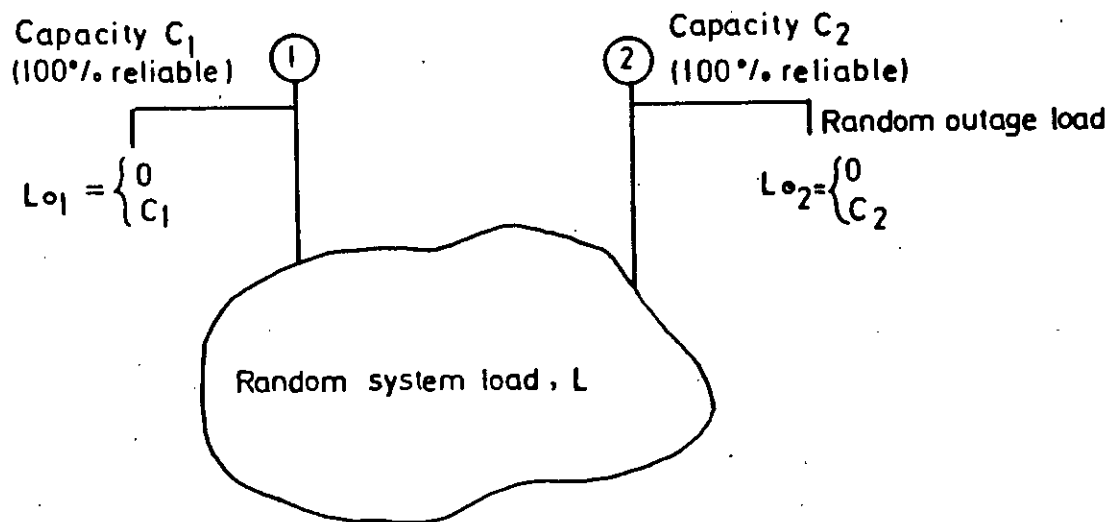


Figure 3.8 : Fictitious generating units and system load model.

If Lo_i represents the random outage load corresponding to the i -th unit, the equivalent load (L_e) may be expressed as

$$L_e = L + \sum_{i=1}^n Lo_i \quad (3.26)$$

where n is the total number generating units. When $Lo_i = C_i$, the net demand injected into the system is zero for the i -th unit, just as it would be if the actual unit of capacity C_i were forced off-line.

Note that the installed capacity of the system is given by

$$IC = \sum_{i=1}^n C_i \quad (3.27)$$

The outages of the generating units may be assumed independent of the system load. Then the distribution of the equivalent load will be the outcome of convolution of two distributions: f_{L_0} and f_L representing the PDFs of the outage capacity and the system load, respectively. For the discrete case the PDFs, f_L and f_{L_0} , may be written as

$$f_L(l) = \sum_i P_L (1-l_i) \quad (3.28)$$

$$f_{L_0}(l_0) = \sum_j P_{L_0j} (l_0 - l_{0j}) \quad (3.29)$$

Then the PDF of equivalent load f_{L_e} may be given as

$$\begin{aligned} f_{L_e}(l_e) &= f_L(l) * f_{L_0}(l_0) \\ &= \sum_{i,j} P_L P_{L_0j} (l_e - (l_i + l_{0j})) \quad (3.30) \end{aligned}$$

where * indicates the convolution and P_L and P_{L_0} are the probabilities of load and outages of machine, respectively. The small case letters within bracket of Equation (3.30) are the values of the corresponding random variables (RVs).

CHAPTER 4

CHAPTER 4

GENERATION SYSTEM COST ANALYSIS

4.1 INTRODUCTION

In chapter 2, the different steps in generation expansion process are presented. As it is discussed, the plans that satisfy the desired reliability level must be evaluated on the basis of economics in order to identify the one plan that impacts on the utility as a whole in the most favorable manner. In this chapter, the concepts used in analyzing the cost of a particular expansion plan, with most emphasis given to a conventional present-worth arithmetic method is presented, for completeness we include a brief presentation of corporate models as they are used in making a final selection. Since corporate models are computerized representations of the financial structure of a utility, a discussion in depth is not presented in this thesis.

4.2 COST ANALYSIS

In evaluating the cost associated with a particular expansion plan, it is essential that the following factors be considered:

1. Capacity cost.
2. Production cost.
3. Timing of unit additions.

These three quantities, more than any other, influence the overall expansion plan cost and hence must be taken into account.

4.2.1 CAPACITY COST

The capacity cost of unit i , denoted by CC_i , is usually

defined as follows:

$$CC_i = FCR_i UC_i C_i \text{ (US\$)} \quad (4.1)$$

where FCR_i = fixed charge rate

UC_i = unit capacity cost (US\$/MW)

C_i = capacity in MW

Generally the fixed charges consist of

1. Depreciation.
2. Rate of return.
3. Taxes.
4. Insurance.

1. Depreciation : In general man-made devices are impermanent or depreciate. This depreciation of equipment takes place as time proceeds. At any instant it is very difficult to determine the exact rate of depreciation or the total amount of depreciation since installation of the equipment. Many factors influence the depreciation, among them five main categories are: i) Life of enterprise ii) Life of the equipment, iii) inadequacy of equipment, iv) obsolescence of equipment and v) requirements of public authority.

2. Rate of return: If the difference between the utility service cost and the annual cost is the annuity A and the investment is P then rate of return i will be solved for by trial and error from

$$\frac{A}{P} = \frac{i}{1 - (1 + i)^{-n}} \quad (4.2)$$

3. Taxes : Taxes levied on an enterprise are many and quite diverse as to the basis used for computing them. The load upon which a plant stands is usually taxed at a rate depending upon its assessed valuation, which is determined from its location with regard to certain natural and

cultural advantages. The tax levied on the value of the capital equipment comprising the plant is usually known as property tax. There are numerous other forms of taxes such as social security, unemployment, income, excess profits and sales. Some taxes depend entirely upon the magnitude of the capital funds, others on the volume of business, and still others on a combination of the two factors.

4. Insurance : Every well-managed company carries some forms of insurance against accidents to equipments and personnel. Since the risks involved are diverse in nature, an insurance list is quite long. On it will be found fire, windstorms, hail, flood, earthquake, explosion, loss of use of public liability, workmen's compensation, automobile, marine, title, fidelity, forgery, credit and many others.

4.2.2 PRODUCTIONS COSTS

Production costs, the second important component in evaluating the cost of a particular expansion plan, can be accurately determined only if

1. A realistic load model is known for each future week or month in the planning period.

2. The units are committed to supply load in a manner that reflects actual operating procedures and conditions.

Production cost associated with unit i is given by

$$EC_i = FC_i + OM_i \quad (4.3)$$

where EC_i = Production or energy cost.

FC_i = Fuel cost in US \$

OM_i = Operations and maintenance cost in US \$.

4.2.3 PLANT COST

As it is sometimes convenient to calculate the cost associated with a particular plant, we can easily define an expression for plant cost (PC) in terms of Eqs.4.1 and 4.3:

$$\begin{aligned} PC_i &= CC_i + EC_i \\ &= CC_i + FC_i + OM_i. \end{aligned} \quad (4.4)$$

Equation 4.4 can be used to obtain accurate plant costs expressed either in dollars or in taka/kwh if the annual (E_i) produced by each unit is calculated using a very detailed production analysis in which all normal operating conditions are simulated.

4.2.4 TIMING OF UNIT ADDITIONS

The third and final major factor in cost analysis is the timing of unit additions. Since present-worth arithmetic is employed to determine the present-worth of a given expansion plan, it is obvious that the timing of the investment associated with the addition of a new unit is important. Further, the effect of new unit additions on total system production cost is a factor that should be reflected in the total present-worth of a particular expansion plan. Although this may not be obvious, we should realize that the addition of a new unit can be drastically change the operation of existing units and hence production cost. For instance, if a base load fossil unit were added prematurely to a system containing base load nuclears, the tendency to off-load the nuclear units could result in substantially higher production costs. When an approximate cost result is desired for a particular plan, it is not uncommon to assume that all the units are added simultaneously, in which case the system cost SC can be calculated as follows:

$$SC = \sum PC_j \quad (4.5)$$

where $\alpha_j \triangleq \epsilon(E_j) / \epsilon(E_s)$

$\epsilon(E_j)$ = expected value of energy produced by unit j

$\epsilon(E_s)$ = expected value of energy consumed by the system

Substituting the approximate expression for (E_j) and (E_s) in

Eq.4.5 we get

$$SC = \sum_j \frac{CF_j C_j}{CF_s C_s} PC_j \quad (4.6)$$

In choosing representative capacity factors for each unit it is necessary to satisfy the constraint

$$\sum_i \frac{CF_i C_i}{C_s} = CF_s \quad (4.7)$$

4.3 SYSTEM COST ANALYSIS

In this section an approach to system cost analysis, that relies on production cost data obtained from the simulation method that portrays the way units will probably be operated if the plan is implemented is presented. The method considers the time value of money in that the timing of unit addition is taken directly into account using conventional present-worth arithmetic. Also, the effect of new additions on total system production cost is appropriately factored into the cost estimates.

Specifically, the method involves calculating the annual capacity costs for each year and then multiplying this result by the appropriate present-worth factor to obtain the present-worth of the annual capacity cost for each year. Summing the present-worth of the annual capacity cost over all years in the planning horizon gives the present-worth of

the total annual capacity costs for the plan under study. To quantify this procedure, let $CC_i(k)$ be the annual capacity cost for unit i in year k , and let β_k be the present-worth factor for year k . The present-worth of the total annual capacity costs is given by [36].

$$CC = \sum_{k=1}^N \sum_{i=1}^G \beta_k CC_i(k) \quad (4.8)$$

If unit $i = m$ is not installed by year k , then

$$CC_m(k) = 0, \text{ otherwise } CC_m \text{ is given by Eq. 4.1.}$$

Similarly, let EC_k be the total system cost, as defined by Eq. 4.3, for year k . Note that this cost includes the production cost, not only for new units, but also for existing units. By incorporating the total system production cost, the effects of new unit additions on the operation and production cost of existing units are easily taken into account. By multiplying EC_k by β_k , the present-worth of the annual system production cost is obtained, and the sum of all present-worth annual production costs renders the present-worth of the total production costs incurred over the planning horizons

$$EC = \sum_{k=1}^N \beta_k EC_k \quad (4.9)$$

4.4 CORPORATE MODELS

A corporate model as the name implies is a model of the financial structure of a company. Through the use of such models a system planner can simulate the effect that a major expansion effort will have on the financial status of his company. Although such models are very detailed, we shall discuss in a simplified manner the basic structure and input-output requirements.

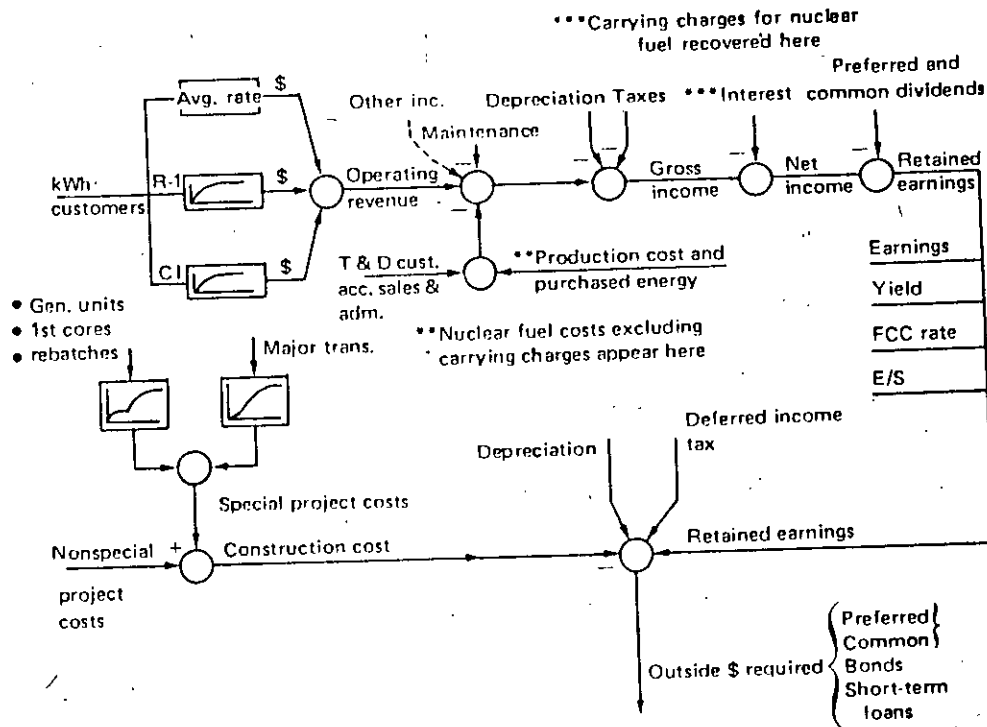


Fig. 4.1 Simplified corporate model. [36]

If we refer to standard statistical data published by most electric utilities, a very crude and oversimplified model of the cash flow in a utility can be obtained. For instance, if the "summary of earnings" and "statement of sources of funds used for construction" are translated into a standard block diagram, as used in classical control theory, a diagram like Fig. 4.1 will result.

4.4.1 INPUT-OUTPUT PARAMETERS

In effect we have in the corporate model an open-loop multiple input-multiple output system where the major inputs are:

1. Forecast energy and customers by rate class for the months being

simulated.

2. Construction expenditures for new plants, transmission facilities, etc., for each month.
3. Production costs for each month
4. Purchased energy costs.
5. Costs associated with transmission and distribution, customer, accounts, and sales and administration.

Similarly the major outputs are:

1. Gross income or operating income.
2. Retained earnings.
3. Earnings per share
4. Fixed charge coverage rate.
5. Yield
6. Outside capitalization requirements.

Clearly, such a planning tool is both convenient and necessary. To study the impact of a given expansion plan on any one of the six outputs, it is only necessary to input the initial cost of each plant and an expenditure-time-curve, which defines the rate at which money for the plant is to be spent, along with the other four inputs listed above.

Referring again to Fig.4.1 , we see that operating revenue is dependent on energy and on customers in addition to rate structure. Since energy consumption and customers are beyond the control of the utility, the only mechanism for adjusting operating revenue is rate relief. Altering operating revenue obviously results in a change in gross income, since gross income is simply operating revenue minus total operating expenses.

The major operating expenses, as shown in Fig. 4.1, are production and purchased energy costs, followed by transmission and distribution

and customer accounts expenses, with sales and general administrative costs following close behind. In addition, depreciation, taxes, and maintenance are also expense items covered by gross income. Depreciation is not a real expense, except for tax purposes, since it is subtracted to obtain (regulated) gross income, but added in again before outside capitalization requirements are determined. By handling depreciation in this manner, the utility is in effect able to establish rate schedules that will make it possible to replace a plant after it is retired. Note that construction costs are not considered operating expenses, with the exception of the construction of nuclear fuel cores.

CHAPTER 5

CHAPTER 5
THE METHODOLOGY OF MULTI-AREA
PRODUCTION COSTING

5.1 INTRODUCTION

This chapter presents the development of the methodology for evaluating the production cost of multi-area interconnected systems. The methodology is an extension of the method for evaluating the production cost of two-area interconnected system[34].The methodology also deals with the priority of the importing systems, one over the others, on the transactions of power from the exporting system. For clarification of the developed methodology, this chapter presents a simple example which can be solved by using a pocket calculator. To prove the validity of the method the same example is solved using heuristic approach in this chapter.

5.2 MATHEMATICAL MODEL

The starting point of the proposed method for evaluating the production cost of multi-area interconnected system is the sampling of chronological load curve (CLC) for each system for the time period under consideration. The loads are sampled every hour or any other appropriate time interval and each sample is assigned equal probability of occurrence . Thus the joint probabilities of the sample loads are obtained .

Then the load plane is subdivided into a grid structure. Each grid or segment has sides of equal size. Thus the load(equivalent) plane are covered with grids of N dimensions. Note that N is the number of interconnected systems; that is for three interconnected systems each

segment will be a cube and for two interconnected system each segment will be a square. The side of each grid will be equal to the maximum common factor of the generating unit capacities of all interconnecting systems as well as the tie line capacities.

As n dimensional array of segment is constructed each axis will be attributed to each system. The number of segments in any axis I which is attributed to system I is calculated using the following formula:

$$t_I = \frac{\sum_{K=1}^{n_I} C_K^I + TC_{\max}^I}{\Delta C} + 1 \quad (5.1)$$

where

C_K^I = The capacity of the K-th unit of system I.

n_I = Total number of units in system I.

TC_{\max}^I = Maximum capacity of the tie lines connecting system I.

ΔC = Segment size.

5.2.1 PARAMETERS OF A SEGMENT

Each segment of the n dimensional grid must have the following information:

i) The zeroeth moment , the joint probability of occurrence of the segment.

ii) The first moment of load of each system with in the range of the segment; that is, if N power systems are interconnected then each segment must have N first moments of load.

iii) The first moment of all residual tie line capacities.

The joint probability of occurrence of any segment is obtained by adding the probability of each load sample within the range of the

segment.

Similarly, the first moment of load corresponding to any system is the sum of the product of loads of that system and its probabilities. The load samples must be within the range of the segment.

The residual tie line capacity (RTC) for a system is that capacity of the tie line that remain at any stage of the loading process after having been utilized by the previously committed generating unit or units from the system. Initially, the first moment of RTC for each system is set equal to the product of the corresponding tie line capacity (TC) and the joint probability of the particular segment.

5.2.2 MERIT ORDER OF LOADING

The loading order of the units of systems is deduced from the knowledge of the average incremental cost of the units. In this order, the unit with the lowest average incremental cost comes first, then the unit with second lowest incremental cost and so on. Note that in the interconnected system, the choice is made from the units of the global system.

To incorporate the nonlinear characteristics of the incremental cost, the common strategy is to subdivide the capacity of a generating unit into capacity blocks, each block with a different average incremental cost. Clearly these blocks may occupy nonadjacent position in the merit order of loading. Multi block loading of generating units requires the application of the process of deconvolution. The segmentatin method accomplishes this effectively.

5.2.3 EVALUATION OF EXPECTED EXPORT/IMPORT

Before convolving any unit, in the loading order, the possible export or import must be evaluated. The fundamental strategy subsumed in the evaluation of the export is that each system must keep its own interest paramount. That is, a utility will only export power to another as long as it has excess capacity after having met its own demand.

The principal factors which affect the capacity transactions among the interconnected systems are:

1. The unserved load or demand of the exporting system.
2. The unserved load or demand of the importing system.
3. The capacity of the committed generating unit.
4. The residual tie line capacity from exporting system to importing system.

A system which export power to another is known as an exporting system and the system which receive power from the exporting system is known as importing system. A system becomes exporting one only when the unit is selected from this system for commitment (loading). However, the same system may be an importing system if the unit is selected from another system during the next loading stage.

Among the importing systems, some systems may have priority over others on the capacity transaction. It will be considered by considering percentage share of surplus generation capacity of exporting system.

Consider an interconnected electric power system as shown in Figure 5.1. Also consider that N number of utilities are connected in this interconnected system. Regarding the export /import in the multi-area interconnected system, it may be noted that the transaction of power from one system to another may be in two different ways :

i) Direct transaction : Directly from exporting system to importing system through a single tie line connecting the exporting and the importing system.

ii) Indirect transaction: Through composite tie lines, connecting the exporting system with the importing system . That is ,to transfer power more than one tie line is involved.

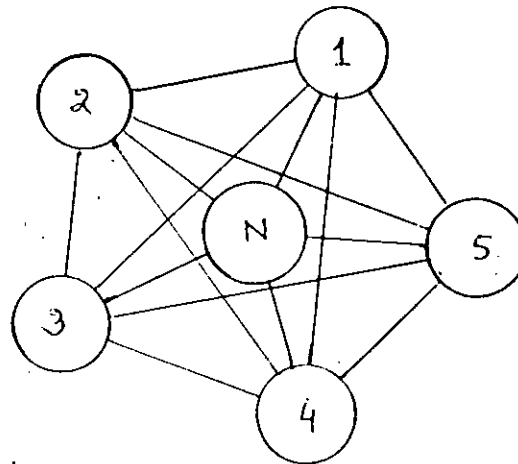


Figure 5.1 : Multi-area interconnected power systems.

To clarify the above transaction policy ,consider the example of transaction of power from system 1 to system 4 of Figure 5.1. When the power is transferred through the tie line between system 1 and 4 it will be referred to 'direct transaction'. If the power is transferred through the tie line between system 1 & 5 and the tie line between system 5 & 4, the transaction will be referred to 'indirect transaction'.

Direct transfer of power:

Export from exporting system to importing systems through a single tie line is calculated as follows:

Let system I is an exporting system then the surplus generation capacity of I-th system (SG^I) is

$$SG^I = \sum_{k=1}^{S^I+1} C_k^I \times P_{i,j,\dots,n} - m_{i,j,\dots,n}^I(BL) \quad (5.2)$$

where S^I is the total number of committed units of system I.

C_k^I is the capacity of the k-th unit of system I.

$P_{i,j,..n}$ is the joint probability of the (i,j..n) th segment.
 $m_{i,j,..n}^I$ is the first moment of load of system I corresponding to the segment (i,j..n).

The unserved demand of the system K (UD^K) may be expressed as

$$UD^K = m_{i,j,..n}^K (E1) - \sum_{k=1}^{S^K} C_k^K \times P_{i,j,..n} \quad (5.3)$$

Export $expD^{I-K}$ of any segment from exporting system I to system K is given by

$$expD^{I-K} = \text{Min} \{SG^I, UD^K, RTC^{I-K}\} \quad (5.4)$$

where K vary from 1 to N but $K \neq I$

RTC^{I-K} = Residual tie line capacity from system I to system K.

Next step is to modify the load of importing system as well as the corresponding tie line capacities.

The modified unserved demand of importing system K may be expressed as

$$UD^K = UD^K - exp^{IK} \quad (5.5)$$

and the expression of the modified first moment of RTC are

$$\begin{aligned} RTC^{I-K} &= RTC^{I-K} - exp^{IK}. \\ RTC^{K-I} &= RTC^{K-I} - exp^{IK}. \end{aligned} \quad (5.6)$$

Export/import with priority:

It is mentioned earlier that some importing systems may have priority over others. The priority of the importing systems one over the others on the transaction of power from exporting system to importing system should be taken into consideration by taking percentage share (PS) of the surplus generation capacity. Considering the priority of system K, the export from system I to system K may be given as

$$exp^{I-K} = \text{Min} \{ PS^K SG^I, UD^K, RTC^{I-K} \}. \quad (5.7)$$

where PS^K is the priority share of system K and

$$0 \leq PS \leq 1 \quad (5.8)$$

Export/import through composite tie lines:

The export from one system to other may be also through more than one tie lines. Consider indirect transaction from system I to system M through the tie lines I-K and K-M. That is, the surplus capacity after direct transaction may be exported from system I to system K via system M. Note that the surplus capacity of the exporting system I may be obtained by subtracting the export computed in Equation (5.4) or (5.7) from the surplus capacity, computed in Equation (5.2) after considering direct transaction.

Now the export through indirect transaction from system I to system M via system K may be given by

$$\text{expl}^{I-K-M} = \text{Min} (PS^M \text{SG}^I , UD^M , \text{RTC}^{I-K} , \text{RTC}^{K-M}) \quad (5.9)$$

where PS^M is the priority share of system M on the surplus generation of exporting system I.

To compute the total export from the exporting system all exports through direct transactions and indirect transactions should be added.

Total export from system I for the commitment of any unit j at any segment may be evaluated as

$$\begin{aligned}
e^I = & \sum_{\substack{K=1 \\ K \neq I}}^N e^{IK} + \sum_{\substack{K=1 \\ K \neq I}}^N \sum_{\substack{M=1 \\ M \neq K \neq I}}^N e^{IKM} \\
& + \sum_{\substack{K=1 \\ K \neq I}}^N \sum_{\substack{M=1 \\ M \neq K \neq I}}^N \sum_{\substack{L=1 \\ L \neq M \neq K \neq I}}^N e^{IKML} \\
& + \dots + \sum_{\substack{K=1 \\ K \neq I}}^N \dots \sum_{\substack{Z=1 \\ Z \neq \dots \neq M \neq K \neq I}}^N e^{IKML\dots Z}
\end{aligned} \tag{5.10}$$

In the above equation the first term indicate the directly export from exporting system I to importing systems K. The second term indicate the export from system I to system M through composite tie lines (I-K) and (K-M). The third term represent the export from system I to system L through three composite tie line (I-K), (K-M) and (M-L) and so on.

Similarly the total import of system K is the sum of direct and indirect import. That is,

$$\text{exp}^{I-K} = \text{exp}^D^{I-K} + \text{exp}^{I-K-M}$$

5.2.4 MODIFICATION OF FIRST MOMENT OF LOAD

When total export from the exporting system I is known then modified first moment of load of system I is given by.

$$m_{i,j,\dots,n}^I (El) = m_{i,j,\dots,n}^I (El) + e^I. \tag{5.11}$$

The first moment of load of importing system say K should also be modified through the equation

$$m_{i,j,\dots,n}^K (El) = m_{i,j,\dots,n}^K (El) - \text{exp}^{I-K} \tag{5.12}$$

5.2.5 EXPECTED ENERGY GENERATION

The expected energy generation by a given generating unit is obtained by evaluating the difference in unserved energies before and after the commitment of the generating unit. The expected unserved energy of the exporting system I before committing the k-th generating unit is given by

$$UE_k^I = T \sum_{i,j,\dots,n=1}^{t_i, t_j, \dots, t_n} (m_{i,j,\dots,n}^I (El) - C_t^I \times P_{i,j,\dots,n}) \quad (5.13)$$

where C_t^I is the total capacity of the already committed generating units of system I and is given by

$$C_t^I = \sum_{K=1}^{S^I} C_K^I \quad (5.14)$$

The limits l_1 in equation (5.13) is given by

$$l_1 = \text{Min} \{ w^I, w^J, w^K \dots w^N \} \text{ but } l_1 > 1.$$

where,

$$w^I = \sum_{K=1}^{S^I} C_K^I / C$$

Note that the unserved demand for any segment (i,j,k....n) must be positive; that is

$$(m_{i,j,\dots,n}^I (El) - C_t^I \times P_{i,j,\dots,n}) > 0 \quad (5.15)$$

5.2.6 PROCESS OF CONVOLUTION

The process of convolution is simply effected by shifting each segment appropriately as each generating unit, in the loading order, is committed to meet the equivalent load. For N interconnected systems an N dimensional approach is necessary, that is the direction of shift depends on the system that the generating unit belongs to. Assuming that the I-direction is attributed to system I then if a generating unit of system I is committed then the shift will be along the I-axis .

Considering the (i,j,...n)-th segment and assuming a generating unit of capacity C mw belongs to system I to be committed, the shifted first moment of load (equivalent load) of system I of the (i+w_I,j...n)-th segment may be expressed as

$$m_{i+w_I, j...n}^{new} = m_{i, j...n}^{old} + C^I \times P_{i, j...n} \quad (5.17)$$

where $w_I = C / \Delta C$

$\Delta C =$ Segment size.

Clearly, the first moments of load of other systems remain unchanged. Also the segment's probability stays unchanged by the shift since the zeroeth order moment are not affected by the shift. To obtain the final distribution of segments after convolving the k-th unit, the original distribution (before convolving k-th unit) is multiplied by the availability $(1 - q_k)$ and the shifted distribution is multiplied by $FOR(q_k)$ of the unit and these two results are added.

The unserved energy, after committment of the k-th generating unit, UE_k may be evaluated by using equation(5.13) with the capacity of the k-th unit added to C_t^I given by equation (5.14).

Hence the expected energy generation by the unit k is given by

$$E_k = UB_k^- - UB_k \quad (5.17)$$

Global expected energy generation by the systems are

$$GES = \sum_{k=1}^{n_1+n_2+\dots+n_i} E_k \quad (5.18)$$

Finally the production cost for the k-th generating unit is given by

$$EC_k = \lambda_k \times E_k \quad (5.19)$$

where λ_k = average incremental fuel cost of unit k.

The global production cost is then evaluated as follows :

$$GEC = \sum_{k=1}^{n_1+n_2+\dots+n_i} \lambda_k \times E_k \quad (5.20)$$

5.3 VERIFICATION OF THE METHODOLOGY

In order to verify the developed methodology, a simple but revealing system will be considered in what follows:

Consider three systems interconnected by tie lines each of 5 MW and having the chronological load curve for 2 hours as shown in Figure 5.2. The load is sampled every hour. The generation data are given in Table 5.1

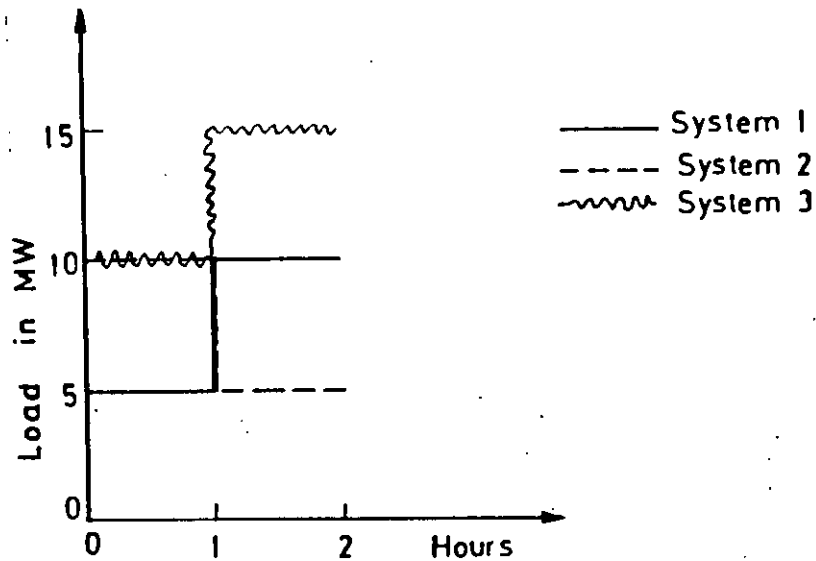


Figure 5.2: Chronological load curve for sample system.

Table 5.1 : Generation system description

System 1				System 2				System 3			
No.of Units	Cap. (MW)	FOR	Avg. IC	No.of Units	Cap. (MW)	FOR	Avg. IC	No.of Units	Cap. (MW)	FOR	Avg. IC
1	10	0.2	5	1	10	0.2	6	1	10	0.2	7

Assigning to each sampled hourly load equal probability (1/2 in this case) the joint occurrence of the 5 MW load in system 1, 10 MW load in system 2 and 10 MW load in system 3 has probability of 1/2. Note that there are three interconnected systems. Therefore, the grid structure will be three dimensional as shown in Figure 5.3; that is the structure will be cube shaped. Each side of the cube will be 5 MW, which is equal to the maximum common factor of the generating unit capacities. These segments are filled up as the loads are sampled. This is shown in Figure 5.4(a), 5.4(b) and 5.4(c). Since there is no impulse below base load, segments below base load are not required.

Each segment as shown in Fig. 5.4(a), 5.4(b) and 5.4(c) contain the following parameters, first row : the segment's probability or zeroeth moment, second row: the first moment of load of the system (system 1 shown first), third row : The first moment of residual tie line capacities.

From Table 5.1 the loading order can be easily deduced. The 10 MW unit of system 1 is loaded first. This is followed by the 10 MW unit of system 2 and finally 10 MW unit of system 3. Before the 10 MW unit of system 1 is committed, the possible export must be evaluated. The only segment for which export is possible is the one which corresponds to the 5 MW load of system 1, in figure 5.4(a) first segment in second row. For this segment export from system 1 to system 2 is 2.5 MW and to system 3 is 2.5 MW. Total export from system 1 is 5 MW. The modified first moment of load are $(5 + 5)/2 = 10/2$ for system 1, $(10-2.5)/2 = 7.5/2$ MW for system 2 and $(10 - 2.5)/2 = 7.5/2$ MW for system 3. Since system 1 is exporting 2.5 MW over the tie line 1-2, 2.5 MW over the tie line 1-3 the modified first moments of RTCs are

$$RTC^{1-2} = (5-2.5)/2 = 2.5/2$$

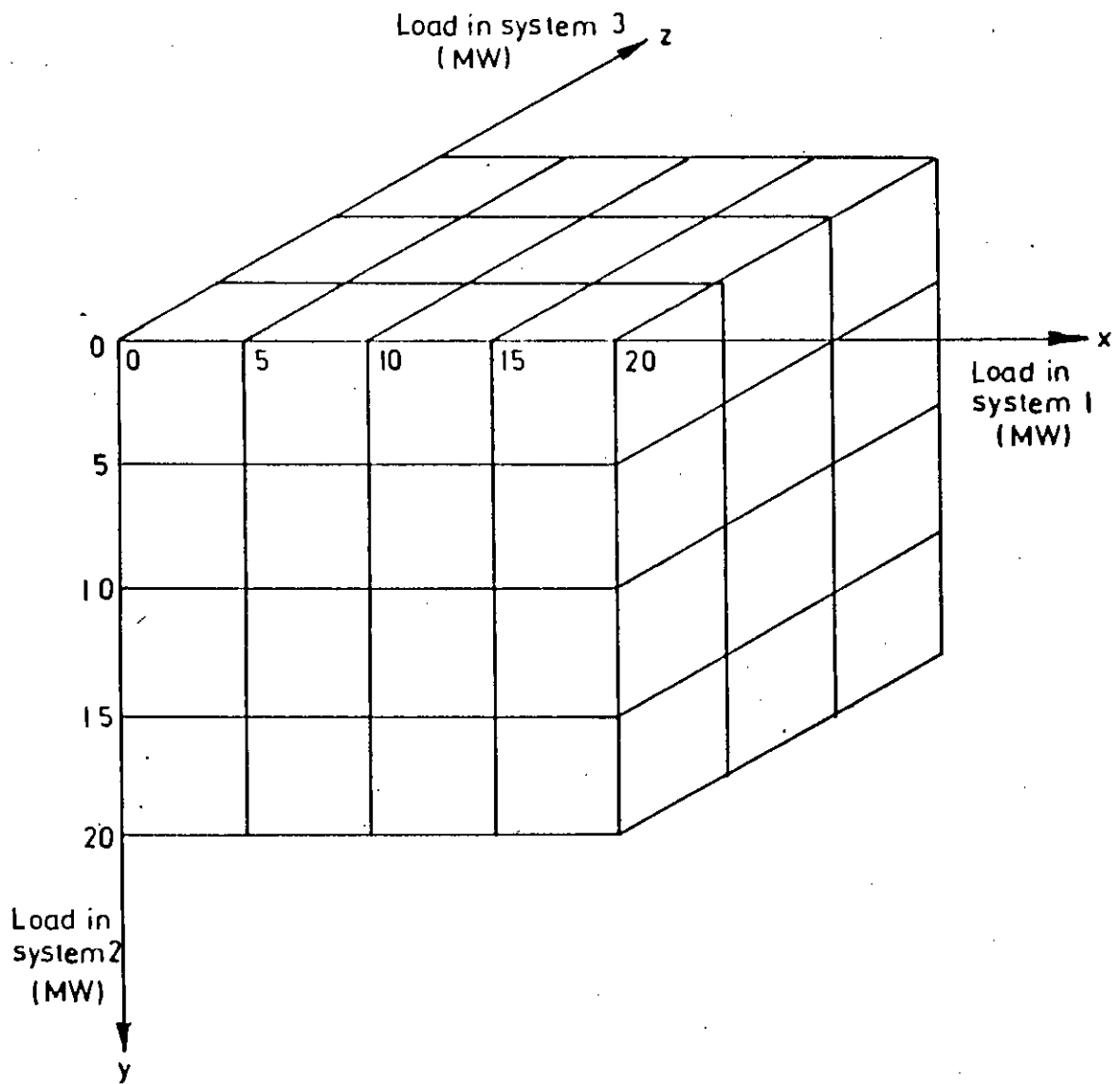


FIG. 5.3 SCHEMATIC THREE DIMENSIONAL REPRESENTATION OF JOINT PROBABILITIES AND CORRESPONDING FIRST MOMENTS.

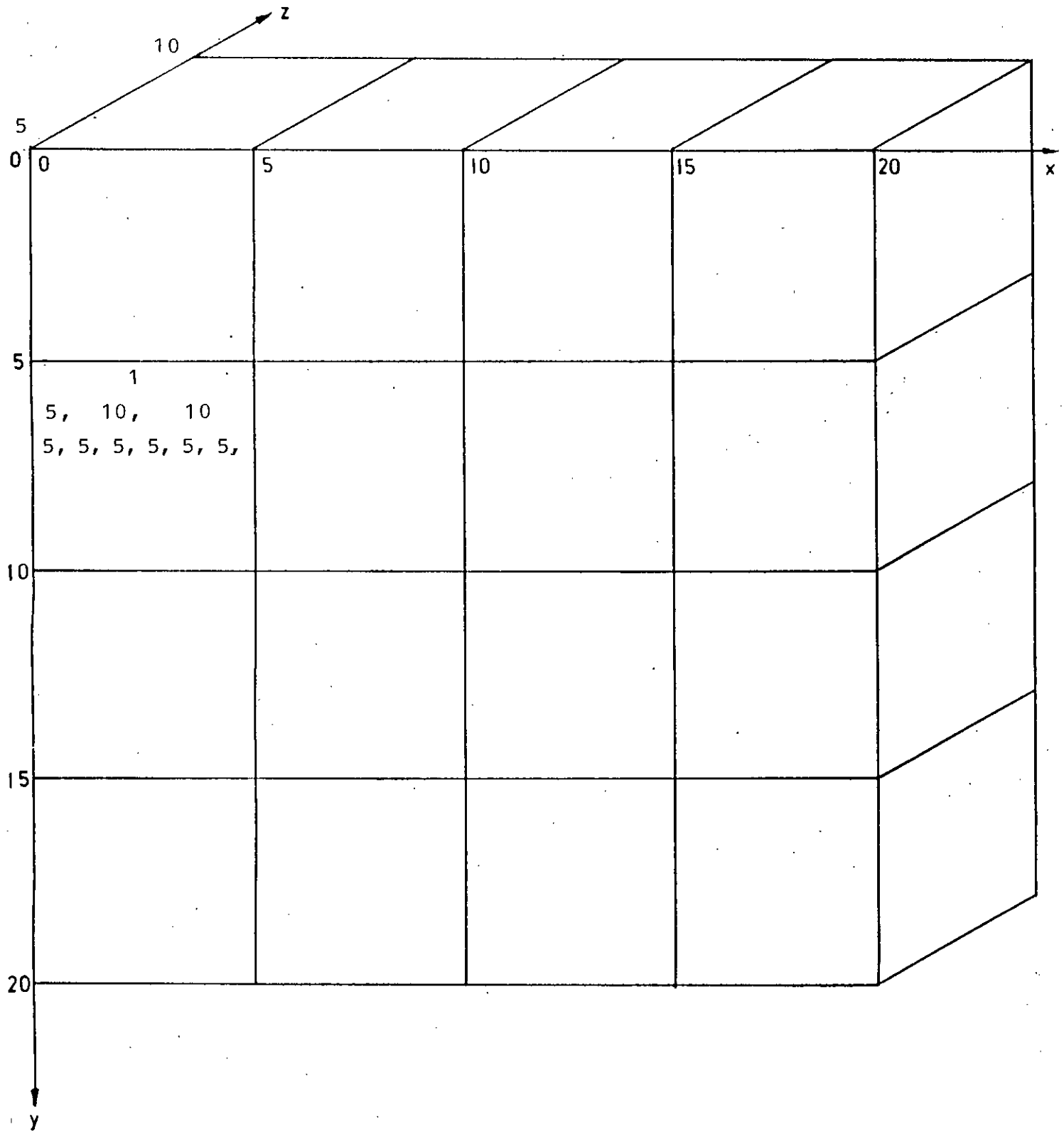


Fig. 5.4(a) Schematic representation of joint probabilities and corresponding first moments (all no. in the boxes to be divided by 2)

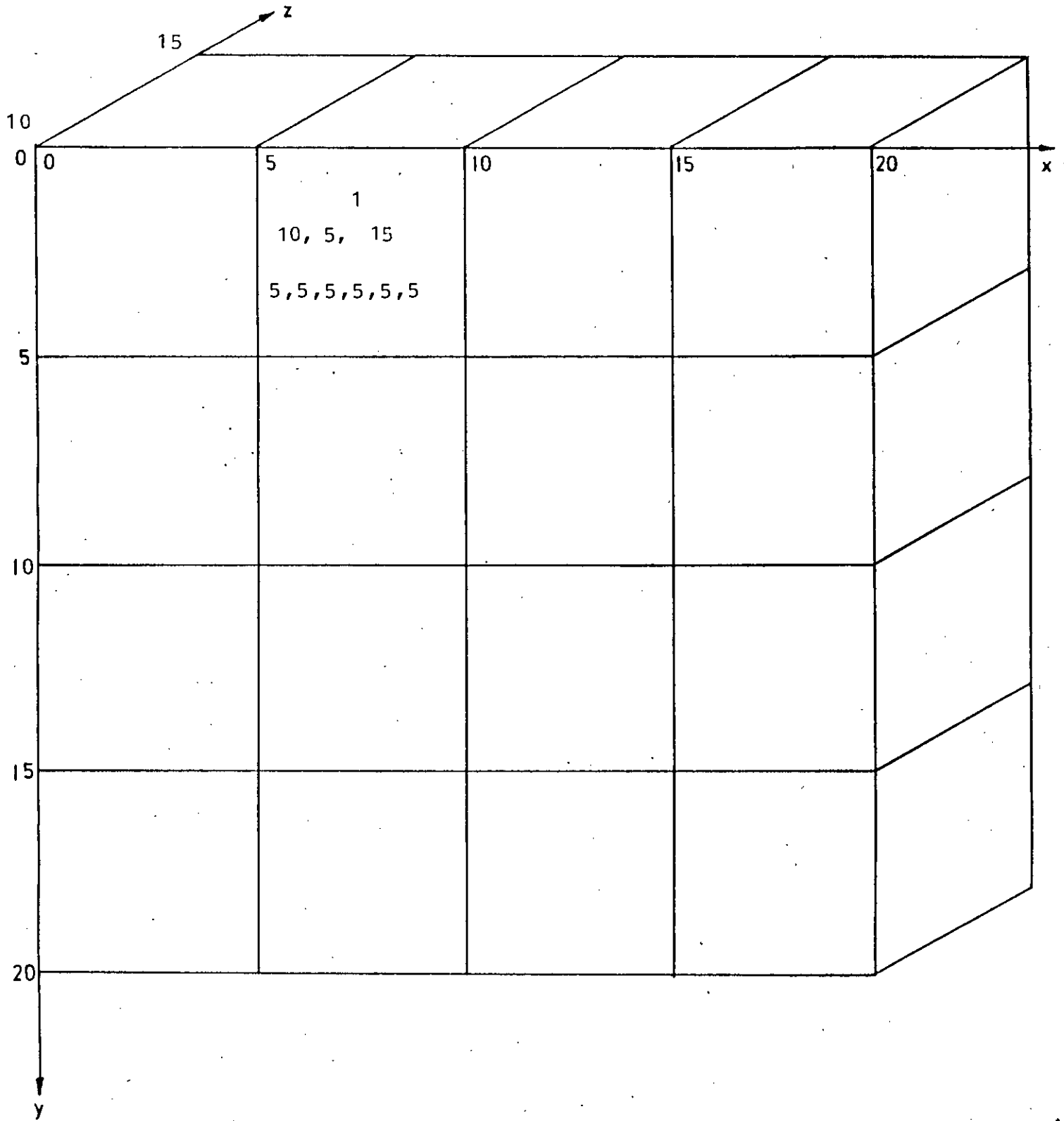


Fig. 5.4(b) Schematic representation of joint probabilities and corresponding first moments (all no. in the boxes to be divided by 2)

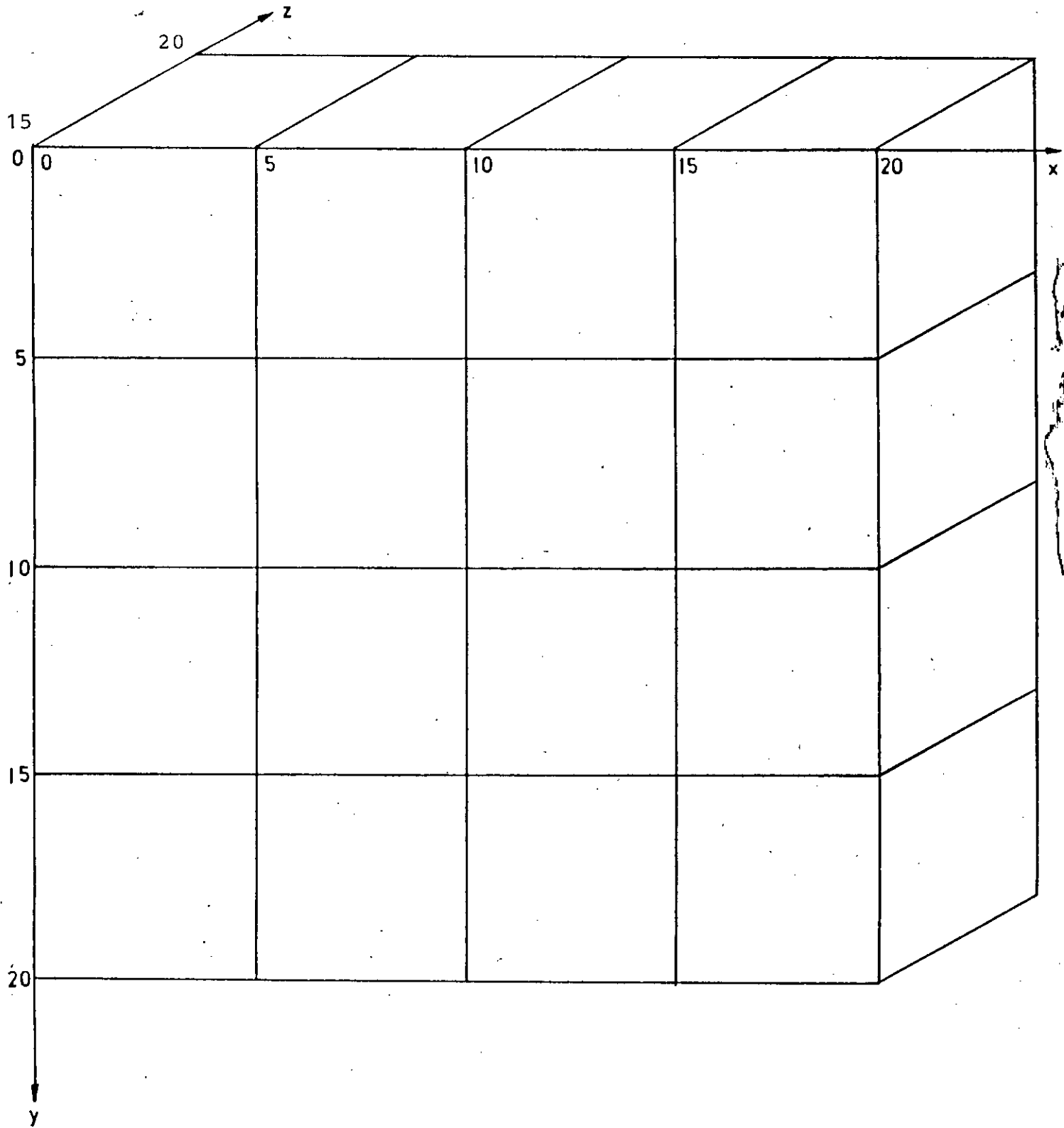


Fig. 5.4(c) Schematic representation of joint probabilities and corresponding first moments (all no. in the boxes to be divided by 2).

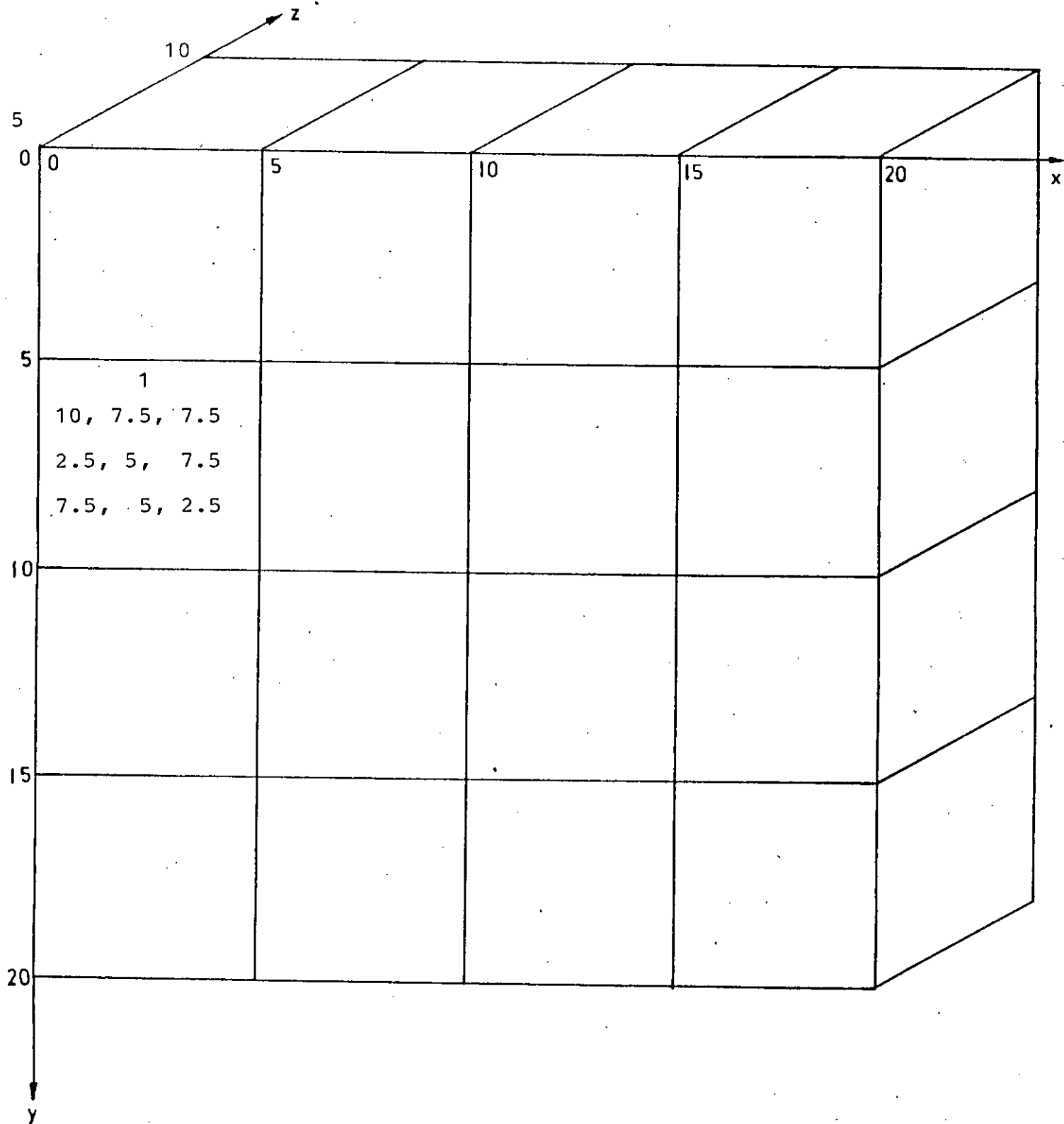


Fig. 5.5(a) Schematic representation of joint probabilities and corresponding modified first moments (all no. in the boxes to be divided by 2).

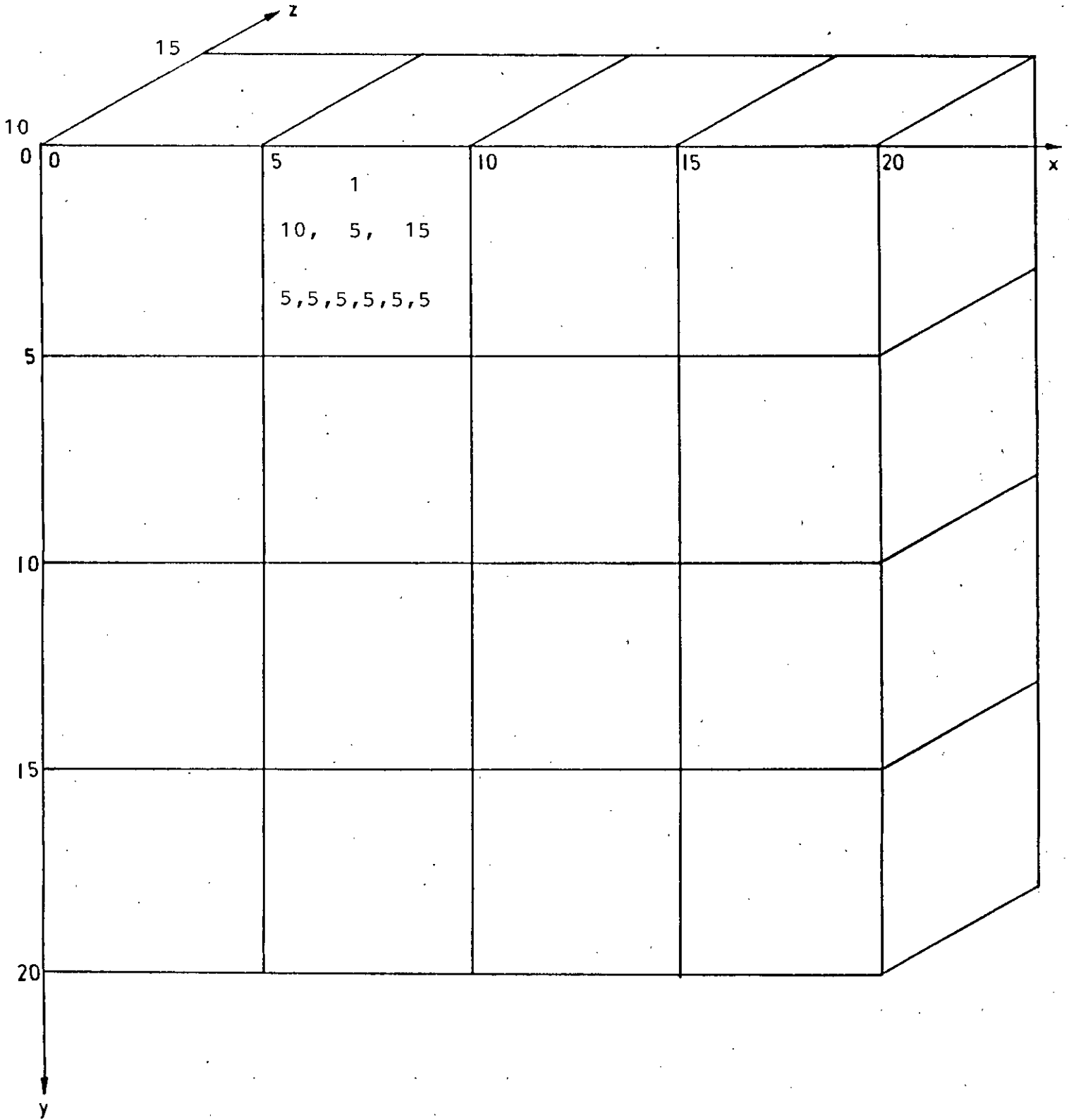


Fig. 5.5(b) Schematic representation of joint probabilities and corresponding modified first moments (all no. in the boxes to be divided 2).

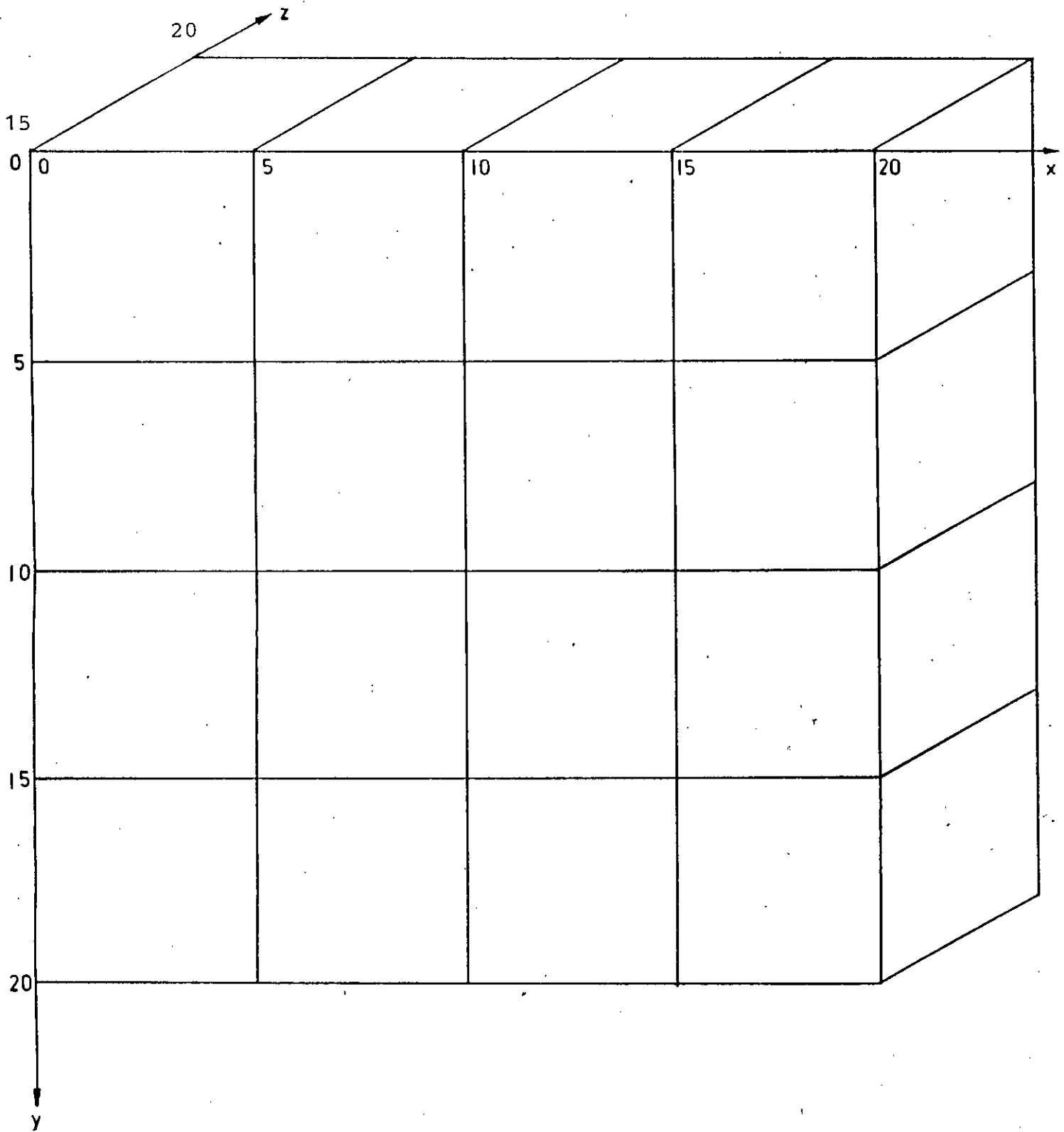


Fig. 5.5(c) Schematic representation of joint probabilities and corresponding modified first moments (all no. in the boxes to be divided by 2)

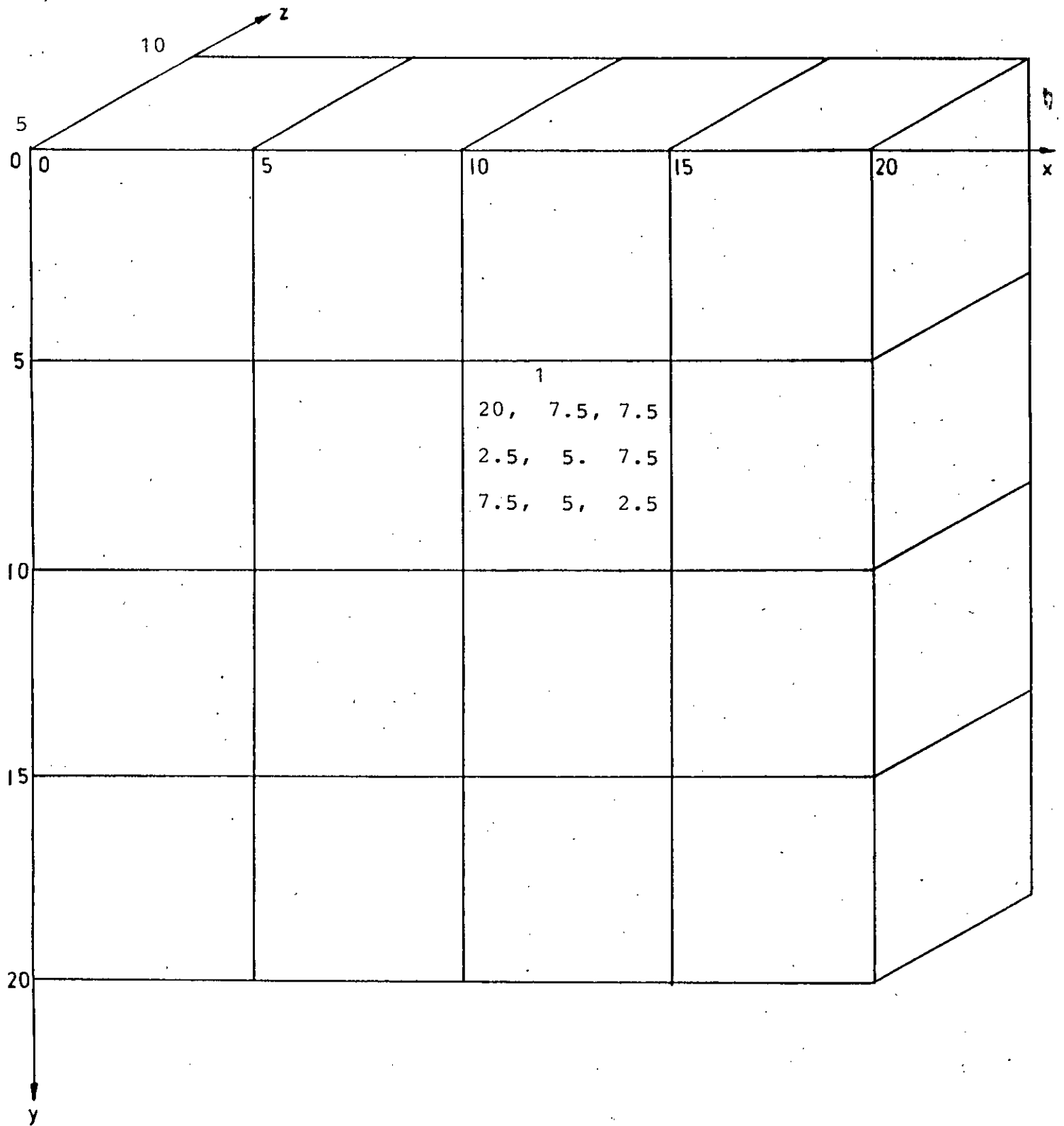


Fig. 5.6(a) Shift of joint probability and first moments during the convolution of the 10 mw unit of system 1 (all nos. in the boxes to be divided by 2).

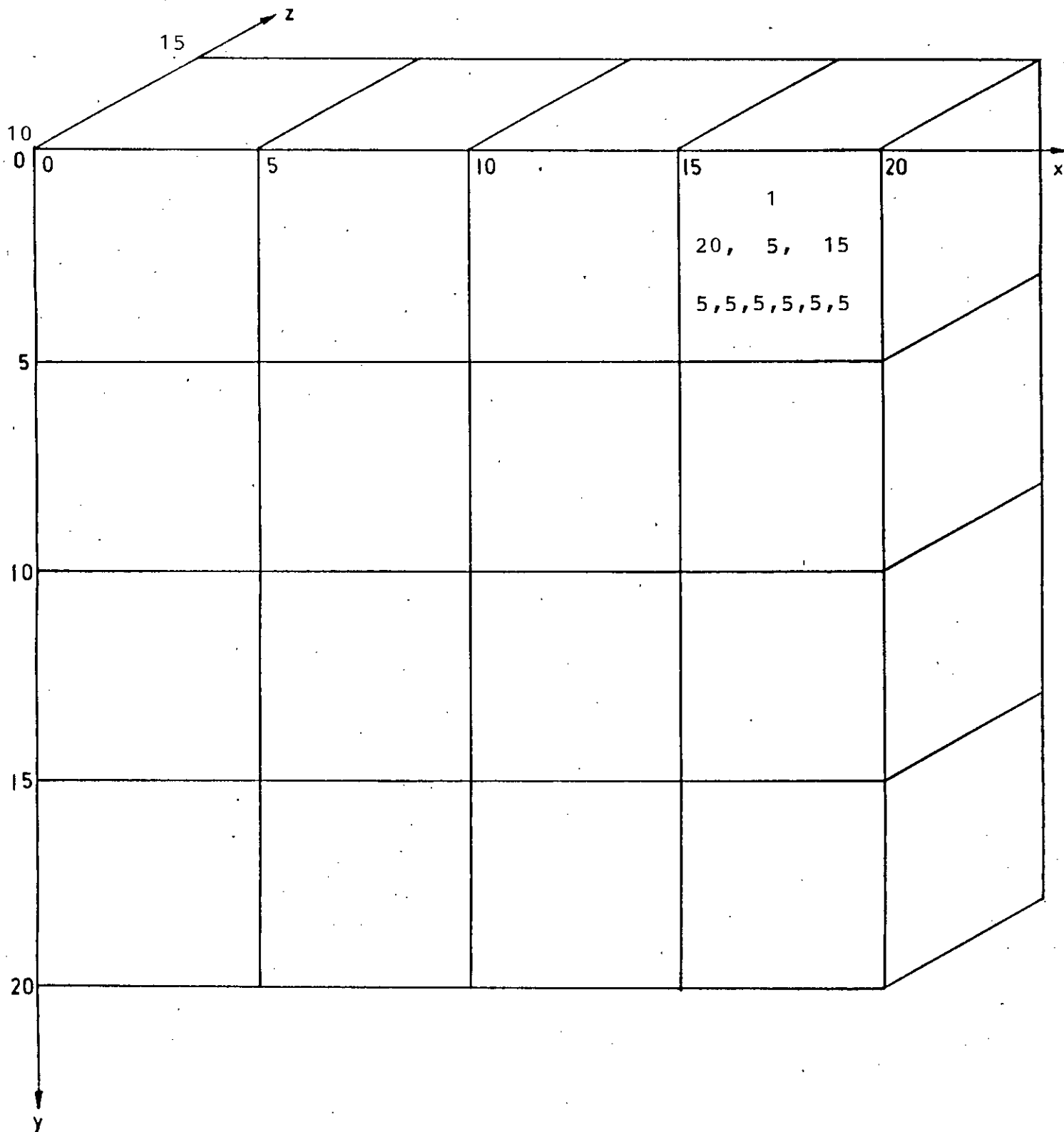


Fig. 5.6(b) Shift of joint probability and first moments during the convolution of the 10 mw unit of system 1 (all nos. in the boxes to be divided by 2).

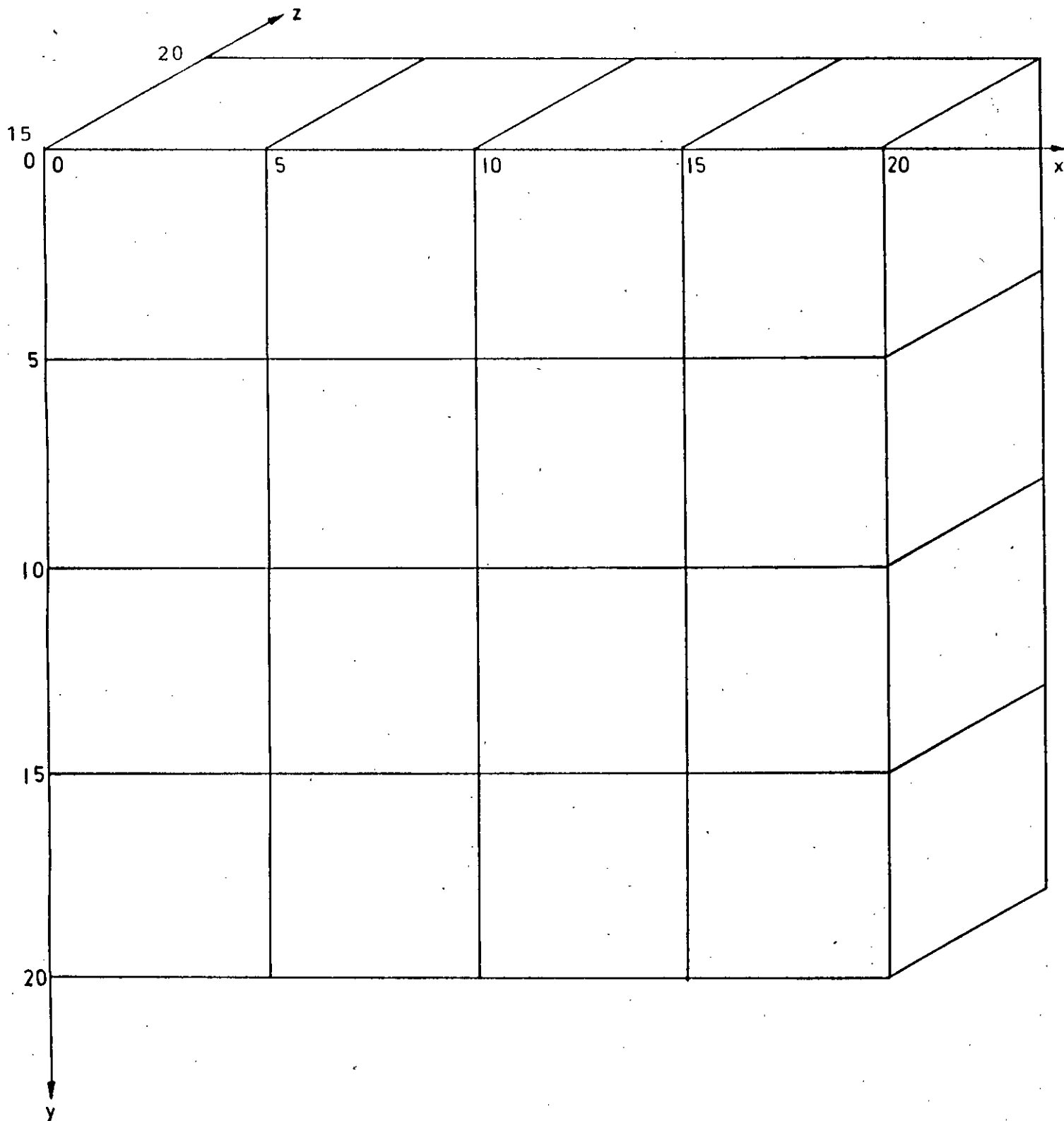


Fig. 5.6(c) Shift of joint probability and first moments during the convolution of the 10 mw unit of system 1 (all nos. in the boxes to be divided by 2).

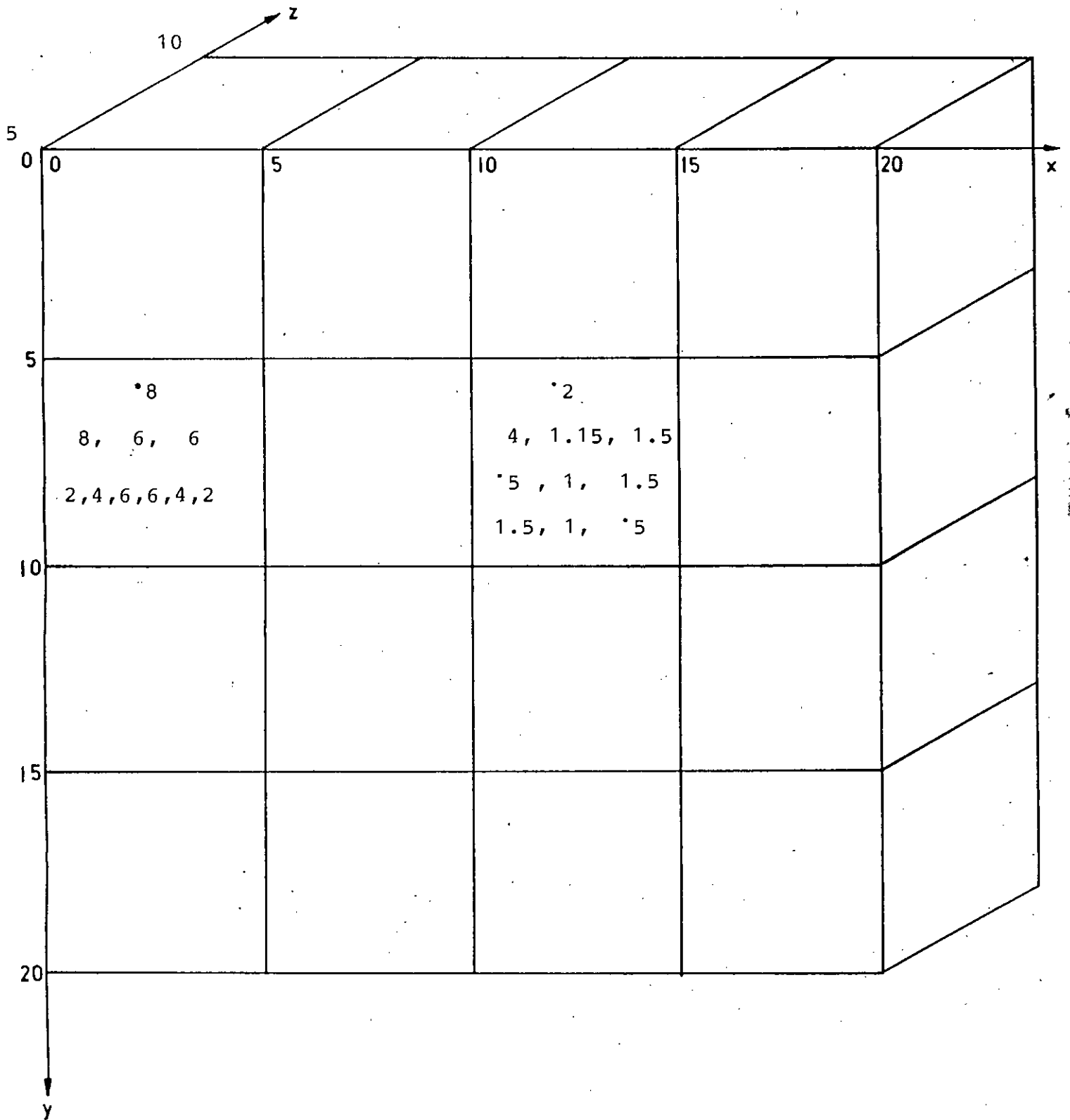


Fig: 5.7(a) Distribution of load and resulting first moments after convolving the 10 mw unit of system 1 (all no. in the boxes to be divided by 2).

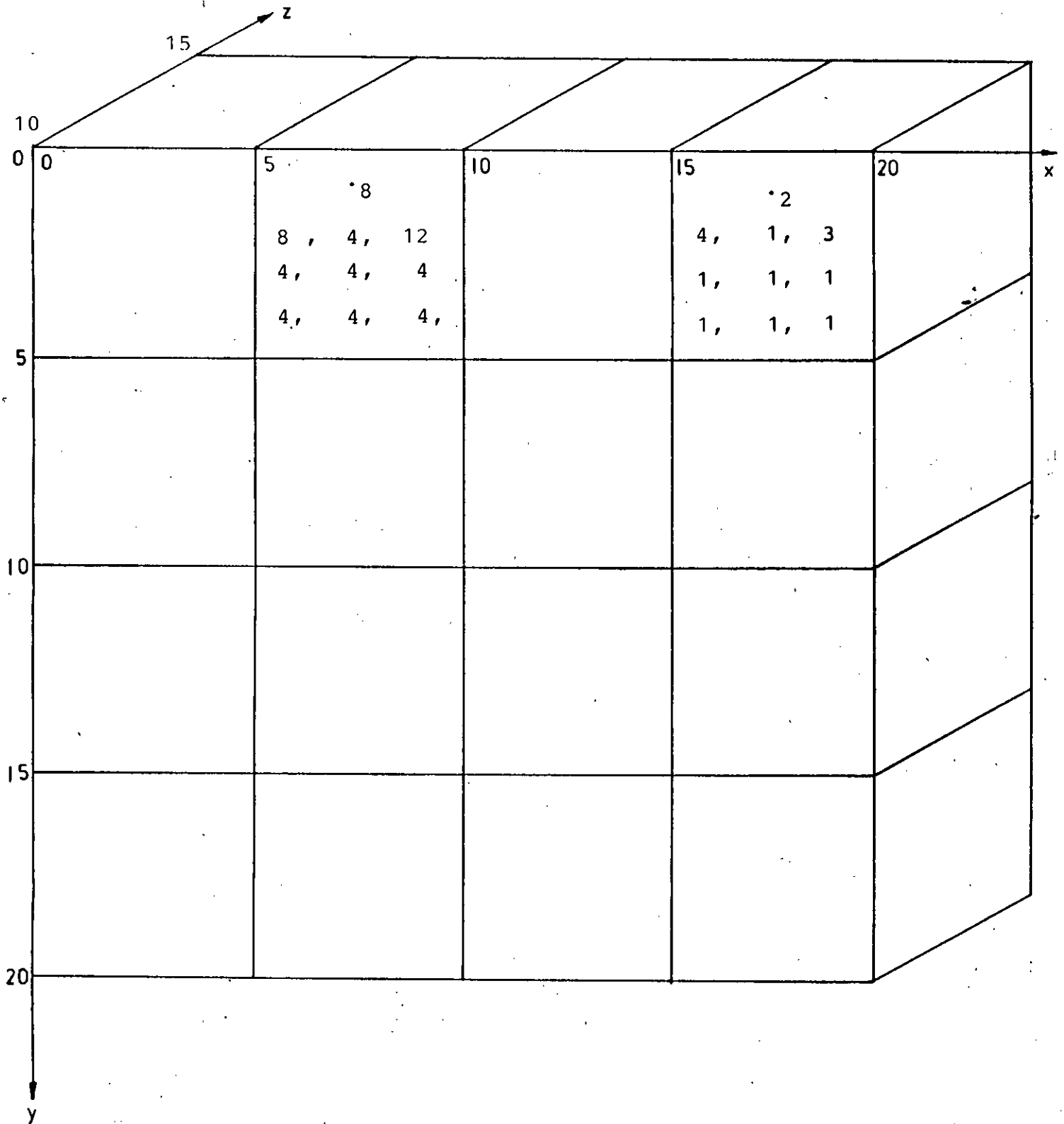


Fig. 5.7(b) Distribution of load and resulting first moments after convolving the 10 mw unit of system 1 (all no. in the boxes to be divided by 2).

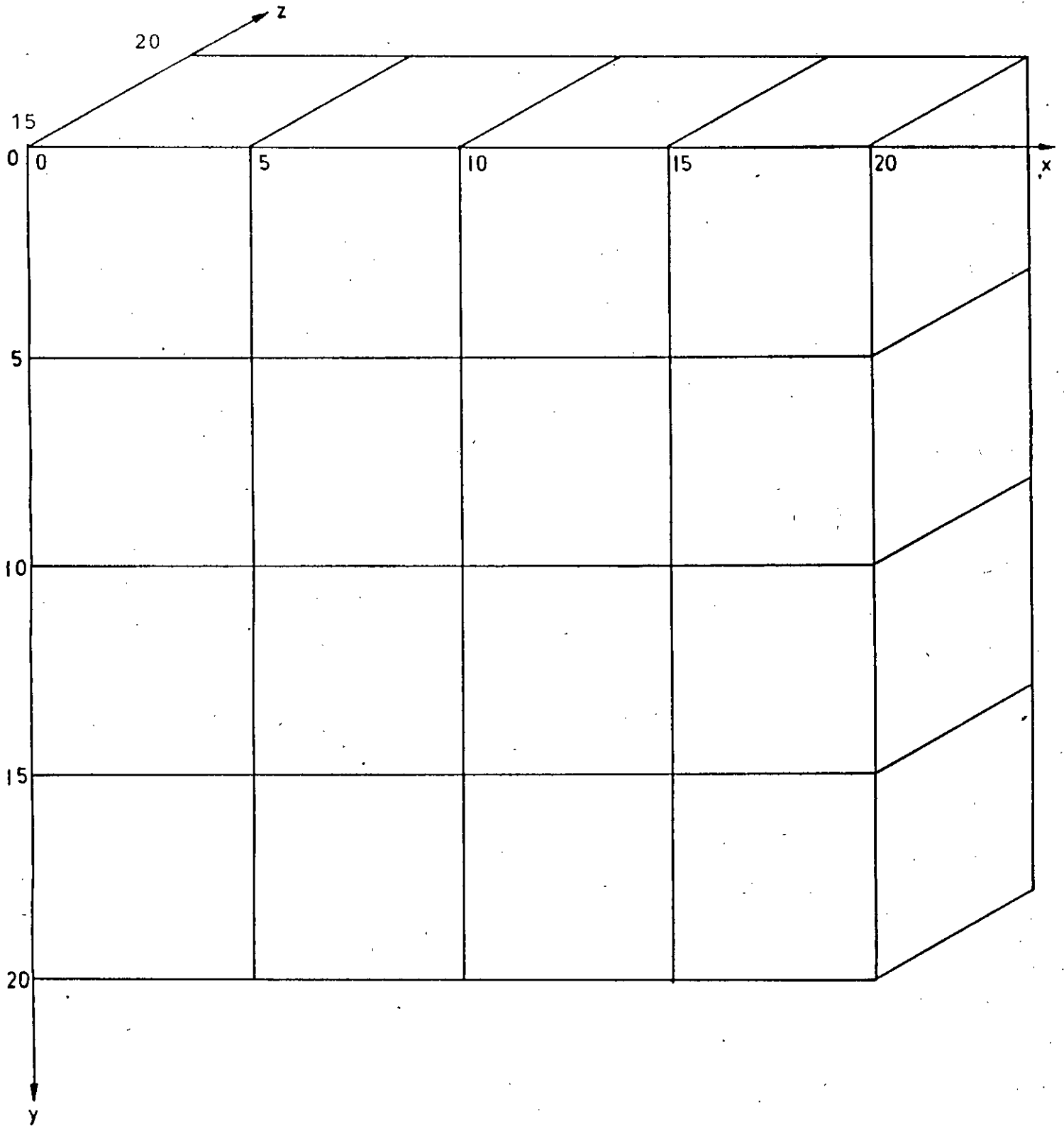


Fig. 5.7(c) Distribution of load and resulting first moments after convolving the 10 mw unit of system 1 (all no. in the boxes to be divided by 2).

$$RTC^{2-1} = (5+2.5)/2 = 7.5/2$$

$$RTC^{1-3} = (5-2.5)/2 = 2.5/2$$

$$RTC^{3-1} = (5+2.5)/2 = 7.5/2$$

The expected unserved energy of system 1 before committing the 10 MW unit is

$$UE_1^- = 1/2 (10 + 10)2 = 20 \text{ MWH.}$$

The convolution process is shown in Fig. 5.5(a), 5.5(b), 5.5(c). In figure 5.7(a), 5.7(b), 5.7(c) the distribution of load and corresponding first moment after the convolution of 10 MW unit of system 1 are shown. Unserved energy is recalculated as

$$UE_1 = 1/2 [(4+4) - 10(.2 + .2)]2 = 4 \text{ MWH.}$$

The expected energy generation of the 10 MW unit of system 1 is equal to

$$E_1 = UE_1^- - UE_1 = 20-4=16 \text{ MWH.}$$

The cost of energy generated by this unit is given by

$$EC_1 = \lambda_1 \times E_1 = 5 \times 16 = 80 \$$$

In a similar manner, the rest of the units are loaded. The expected energy generations and production costs are

For 10 MW unit of system 2

$$E_2 = 16 \text{ MWH.}$$

$$EC_2 = 96 \$$$

For 10 MW unit of system 3

$$E_3 = 13.44 \text{ MWH.}$$

$$EC_3 = 93.08 \$.$$

5.4 HEURISTIC APPROACH TO SOLVE THE SAMPLE EXAMPLE

Consider the three area interconnected system of section 5.3. The load model of Figure 5.2 shows that a 5 MW load in system 1, 10 MW load in system 2 and 10 MW load in system 3 occur 50% of the time, and 10 MW load in system 1, 5 MW load in system 2 and 15 MW load in system 3 occur remaining 50% of time.

From the generation data of Table 5.1, a capacity state table is developed and it is presented in Table 5.2. Based upon the joint probabilities of capacity states and load level we obtain the generating characteristics of different system which are shown in Figure 5.8(d), 5.8(h) and 5.8(l). The joint probability of capacity states and load level (5, 10, 10), whose duration is 50% of time for system 1, of which 38.4% of time generation is available from then upto 46.4% of time it remains unavailable from then upto 48% of time, it again becomes available and from then upto 50% of time it is unavailable, for the next load level it simply repeats. In the 50% of time generation capacity of system 1 is 10 MW but it's demand is 5 MW as shown in Figure 5.8(a). Hence it can export remaining 5 MW to system 2 and system 3, through the tie lines connecting them. Considering equal priority of the importing systems, system 2 receives 2.5 MW of load through the tie line connecting system 1 and system 2 (TLC) and system 3 receive 2.5 MW of load through the tie line connecting system 1 and system 3 (TLC). In a similar way, in the remaining 50% of time load transaction will occur.

The all possible capacity and duration of export and import over the tie lines from one system to the other systems is shown in Fig. 5.8.

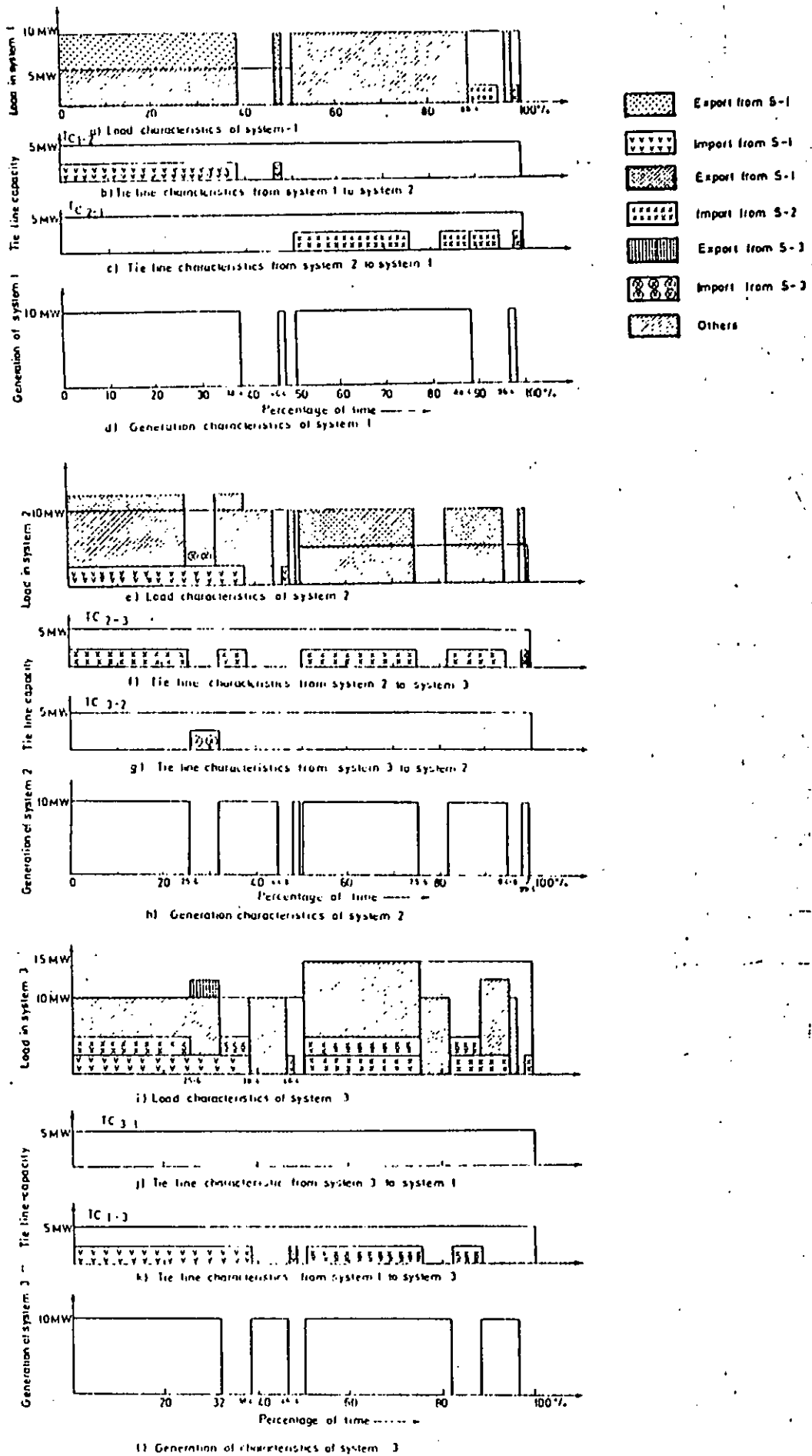


FIG. 5-6 LOAD LEVEL AND POSSIBLE TRANSACTIONS

Table 5.2 : Different states of the global system along with probabilities

Capacity out MW			Capacity available in the global system, MW	Probability	Joint probability of cap. states and load level	
Syst.1	Syst.2	Syst.3			(5,10,10)	(10,5,15)
0	0	0	30	0.512	0.256	0.256
0	10	0	20	0.128	0.064	0.064
0	0	10	20	0.128	0.064	0.064
10	0	0	20	0.128	0.064	0.064
10	10	0	10	0.032	0.016	0.016
0	10	10	10	0.032	0.016	0.016
10	0	10	10	0.032	0.016	0.016
10	10	10	0	0.008	0.004	0.004

75534

Expected energy supplied by the generator of system 1

$$E_1 = T (.384 + .016 + .384 + .016) \times 10 = 8T = 16 \text{ MWH}$$

where T is the time period of study = 2 hours.

Expected energy supplied by the generator of system 2

$$E_2 = (.256 + .064 + .064 + .016) \times 2 \times 10 T = 8 T = 16 \text{ MWH}$$

Expected energy supplied by the generator of system 3 is

$$E_3 = (.064 + .08 + .256 + .064 + .08) \times 10 \times T + .256 \times 5 \times T = 6.72 T = 13.44 \text{ MWH.}$$

5.5 ALLOCATION OF PRODUCTION COST AMONG THE MULTI-AREA INTERCONNECTED SYSTEMS:

The economic benefits of interconnected systems, in terms of global production cost savings for a particular tie line capacity is calculated by subtracting the global production cost at that tie line capacity from the global production cost at zero MW tie line capacity.

Thus

$$GS = GEC_0 - GEC$$

where GS = Global savings,

GEC_0 = Global production cost at zero MW tie line capacity.

GEC = Global production cost at any tie line capacity greater than zero MW.

There are a number of methods for allocation of the production cost among the interconnected systems. One of these is called 'split-the-savings'. This method is popular among the utilities of North America. In this method, the actual cost shared by each utility(system) is obtained on the basis of individual production costs, global production cost and the global savings. The production cost shared by the e-th system at a particular tie line capacity is obtained by subtracting 1/N -th of the global savings at that tie line capacity from its production cost at zero MW tie line capacity . That is

$$EC_e = EC_0 - \frac{GS}{N}$$

where EC_e = Production cost shared by the e-th system.

EC_0 = Production cost of the exporting system at zero MW tie line capacity

GS = Global savings.

CHAPTER 6

CHAPTER 6

NUMERICAL EVALUATION

6.1 INTRODUCTION

In chapter 5, a methodology has been developed to evaluate the production cost of multi-area interconnected power system and it is applied to a realistic interconnected system in this chapter.

This chapter includes a brief description of the realistic system's load and generation data used in evaluating the production cost. The results are presented in this chapter. The interconnected system considered for application is the three area interconnected systems, one of which is the IEEE Reliability test system (IEEE-RTS). The other two systems are hypothetical ones. The load models used for these systems are the hourly loads of IEEE. In what follows the IEEE-RTS is described.

6.2 IEEE RELIABILITY TEST SYSTEM (IEEE-RTS)[37]

The load model provides hourly loads on per unit basis expressed in chronological fashion. In the generation model some of the unit capacities are rounded off to reduce the computational requirements. The load model and generating system are briefly described in the following two sections.

6.3 LOAD DATA [37]

Each thirteen weeks hourly loads of winter, summer and spring are considered for the three interconnected systems. These three seasonal loads are assumed to occur simultaneously. The winter load is considered for system 1, summer load for system 2 and spring load for system 3.

The winter thirteen weeks are: 1-8 and 44 - 48, the summer weeks are: 18-30 and the spring weeks are: 9-17 and 31-34. The peak loads of system 1, 2, 3 are 2679, 2565, 2280 MW and the baseloads are 1102.6, 1000.56, 978.12 MW respectively. The energy requirements of systems 1, 2, 3 for the considered period are 4074.009, 3964.143 and 3481.064 GWH respectively. The three seasonal weekly peak loads in percentage of the annual peak load are given in Table 6.1. Table 6.2 gives a daily peak load in percentage of the weekly peak.

Table 6.1: Weekly peak load in percentage of annual peak load.

Winter		Summer		Spring	
Week	Peak load	Week	Peak load	Week	Peak load
1	0.862	18	0.837	9	0.74
2	0.900	19	0.870	10	0.737
3	0.878	20	0.880	11	0.715
4	0.834	21	0.856	12	0.727
5	0.880	22	0.811	13	0.704
6	0.841	23	0.900	14	0.750
7	0.832	24	0.887	15	0.721
8	0.806	25	0.896	16	0.800
44	0.881	26	0.861	17	0.754
45	0.885	27	0.755	31	0.722
46	0.909	28	0.816	32	0.776
47	0.940	29	0.801	33	0.800
48	0.890	30	0.880	34	0.729

Combining tables 6.1 and 6.2 together with annual peak load define a daily peak load model of $13 \times 7 = 91$ days.

Table 6.2: Daily peak load in percent of weekly peak

Day	Peak load
Monday	93.0
Tuesday	100.0
Wednesday	98.0
Thursday	96.0
Friday	94.0
Saturday	77.0
Sunday	75.0

Weekday and weekend hourly load models for the three seasons are given in Table 6.3. Combination of Tables 6.1, 6.2 and 6.3 with annual peak load of 2850 MW define an hourly load model of $91 \times 24 = 2184$ hours for each system.

Table 6.3: Hourly load in percent of daily peak.

Hour	Winter weeks (1-8 & 44-48)		Summer weeks (18-30)		Spring weeks (9-17 & 31-34)	
	Week day	Weekend	Week day	Weekend	Week day	Weekend
12-1 AM	67	78	64	74	63	75
1-2	63	72	60	70	62	73
2-3	60	68	58	66	60	69
3-4	59	66	56	65	58	66
4-5	59	64	56	64	59	65
5-6	60	65	58	62	65	65
6-7	74	66	64	62	72	68
7-8	86	70	76	66	85	74
8-9	95	80	84	81	95	83
9-10	96	88	95	86	99	89
10-11	96	90	99	91	100	92
11-12	95	91	100	93	99	94
12-1 PM	95	90	99	93	93	91
1-2	95	88	100	92	92	90
2-3	93	87	100	91	90	90
3-4	94	87	97	91	88	86
4-5	99	91	96	92	90	85
5-6	100	100	96	94	92	88
6-7	100	99	93	95	96	92
7-8	96	97	92	95	98	100
8-9	91	94	92	100	96	97
9-10	83	92	93	93	90	95
10-11	73	87	87	88	80	90
11-12	63	81	72	80	70	85

Hourly load for any hour of the week day may be expressed as:

$$HL = WKPK \times DPK \times HLWD \times APK$$

where, HL = Hourly load

WKPK = Weekly peak as a fraction of annual peak.

DPK = Daily peak as a fraction of weekly peak.

HLWD = Hourly load as a fraction of daily peak for week day.

APK = Annual peak load.

Similarly hourly load for any hour of the weekend day may be expressed as:

$$HL \times WKPK \times DPK \times HLWE \times APK$$

where, HLWE = Hourly load as a fraction of daily peak for weekend day.

6.4 GENERATION DATA [37]

The generation data of system 1 is the modified generation model of IEEE-RTS. The generation data of system 2 and system 3 are hypothetical ones. Generating system comprises nuclear, coal, oil and hydro generating unit capacities varying from 50 MW to 500 MW. System 1 has 19 generating units with an installed capacity of 2950 MW, system 2 has 15 generating units with an installed capacity of 3150 MW and system 3 has 11 generating units with an installed capacity of 2600 MW. The generation model of system 1, system 2, and system 3 are presented in Tables 6.4, 6.5 & 6.6 respectively.

Table 6.4: Generation data of system - 1.

Type of Unit	Unit Size(MW)	No.of Units	FOR	Avg. Incremental cost(\$/MWH.)
Hydro	50	6	0.01	0.0
Nuclear	400	2	0.12	5.592
Coal	150	4	0.04	11.16
Coal	350	1	0.08	11.40
Oil	200	3	0.05	19.87
Oil	100	3	0.04	20.08

Table 6.5: Generation data of System -2.

Type of Unit	Unit Size(MW)	No.of Units	FOR	Avg. Incremental cost (\$/MWH.)
Nuclear	500	1	0.13	4.5
Coal	400	2	0.13	14.3
Coal	350	1	0.13	15.1
Coal	250	1	0.08	18.6
Oil	350	1	0.14	30.4
Oil	200	2	0.1	35.0
Oil	100	3	0.01	38.4
Oil	50	4	0.11	43.2

Table 6.6: Generation data of system - 3.

Type of Unit	Unit Size(MW)	No.of Units	FOR	Avg.Incremental cost(\$/MWH.)
Nuclear	450	1	0.15	4.65
Coal	400	2	0.15	13.65
Coal	350	1	0.11	16.27
Coal	250	1	0.01	18.27
Oil	250	1	0.15	18.27
Oil	200	1	0.2	28.27
Oil	100	2	0.2	36.87
Oil	50	2	0.12	39.87

6.5 COMPUTER PROGRAM

For numerical evaluation a computer program is developed in FORTRAN, based on the methodology developed in chapter 5. The program is applicable to three-area interconnected systems with all different connections .

6.6 NUMERICAL RESULTS

The above three-area interconnected systems are simulated to evaluate the production cost of individual system as well as the global system. The production cost is evaluated for two different conditions; (i) Without considering priority; that is in case of transaction of

export to importing systems both the importing systems are considered to get the same privilege ,(ii) Considering priority, that is, in the transaction of export, one of the importing system is considered to get preference over the other.

Tables 6.7 to 6.10 present the results for equal priority while Tables 6.11 to 6.13 present the result for the condition where the preference is given to one importing system. In this simulation, the privileged system is system 1. The tie line capacity is varied from 0 MW to 350 MW in steps of 50 MW.

In Table 6.7, the expected energy generation of individual generating unit for different tie line capacity is presented. The first column of this table represents the system, second and third column present the capacity and the FOR of the unit respectively. Columns 4 to 6 give the expected energy generations of the individual units for different tie line capacities. The tie line capacity is indicated at the second row from the top of the table.

In Tables 6.8 and 6.11, the total expected energy generations and the corresponding production cost of individual systems are shown. In Figures 6.1 the expected energy generations of individual systems for different tie line capacities are depicted.

The global expected energy generation, production cost and savings at different tie line capacities are given in Tables 6.9 and 6.12.

Global production cost vs. tie line capacity, global savings vs. tie line capacities are presented in Figure 6.2 and 6.4 respectively. In Figure 6.4 the upper dotted line represents maximum savings or upper limit of savings. The allocation of production cost among the three systems using 'split-the-savings' principle is also shown in Tables 6.10 and 6.13.

The variation of global unserved energies with tie line capacity is shown in Figure 6.3. The lower dotted line indicates the lower limit of unserved energy.

It is noted that graphical representation is only for the condition of equal priority. The numerical results in considering priority for system 1 is not presented graphically as the results which are obtained at this consideration has negligible difference with those obtained in first consideration .

Table 6.7: Expected energy generation of individual generators for different tie line capacities considering equal priority.

System	Capacity MW	FOR	Expected energy gen.(GWhr)/ Tie line capacity			
			0 MW	100 MW	200 MW	300 MW
	50	0.01	108.1091	108.1091	108.1091	108.1091
	50	0.01	108.1091	108.1091	108.1096	108.1096
	50	0.01	108.1091	108.1091	108.1090	108.1090
	50	0.01	108.1091	108.1091	108.1091	108.1091
	50	0.01	108.1091	108.1091	108.1089	108.1089
	50	0.01	108.1091	108.1091	108.1088	108.1088
	400	0.12	768.7668	768.7668	768.7668	768.7668
	400	0.12	768.7668	768.7666	768.7666	768.7666
	150	0.04	311.9500	314.4954	314.4954	314.4954
1	150	0.04	296.8181	313.7811	314.4952	314.4952
	150	0.04	267.1819	303.8802	314.4032	314.4952
	150	0.04	233.6969	279.8496	309.0259	314.4950
	350	0.08	399.5722	497.6590	601.0083	675.2216
	200	0.05	174.6169	181.0031	185.6008	188.7872
	200	0.05	106.7278	138.5095	162.0160	170.6078
	200	0.05	56.2488	86.0509	126.8847	147.0916
	100	0.04	16.4631	27.1949	46.7553	62.7529
	100	0.04	10.3364	19.1562	35.0723	52.7872
	100	0.04	6.5881	12.9367	25.4437	41.7068

System	Capacity MW	FOR	Expected energy gen.(GWHR)/ Tie line capacity			
			0 MW	100 MW	200 MW	300 MW
	500	0.13	950.0394	950.0394	950.0394	950.0394
	400	0.13	760.0292	759.6639	749.5674	721.7287
	400	0.13	743.4065	713.1611	675.3526	650.9405
	350	0.13	545.3305	532.1409	513.9049	504.7859
	250	0.08	331.3886	294.9959	268.0520	254.6319
	350	0.14	322.0494	245.6786	176.1840	130.6172
	200	0.1	124.8180	91.6061	63.5860	47.1086
2	200	0.1	83.0970	57.4653	39.3181	29.8496
	100	0.01	31.9321	19.5448	12.1867	8.4617
	100	0.01	23.1426	13.9658	8.5505	6.1729
	100	0.01	16.4621	9.6799	5.9899	4.8370
	50	0.11	5.6719	3.1295	2.1794	1.3341
	50	0.11	4.8246	2.8478	2.1141	0.8411
	50	0.11	4.0609	2.8241	1.8040	0.7115
	50	0.11	3.8319	2.8238	0.6996	0.5974

System	Capacity MW	FOR	Expected energy gen.(GWHR)/ Tie line capacity			
			0 MW	100 MW	200 MW	300 MW
	450	0.15	835.3791	835.3791	835.3791	835.3791
	400	0.15	742.5575	742.5575	742.5574	742.5574
	400	0.15	717.9152	719.1729	721.8205	725.8874
	350	0.11	522.5287	491.8364	466.0274	457.7196
	250	0.01	316.0960	318.2809	318.4762	321.4559
3	250	0.15	161.5939	214.9776	238.3707	246.1158
	200	0.2	78.4515	74.6465	72.7681	71.4504
	100	0.2	27.9615	17.9290	11.8015	9.5695
	50	0.2	8.8375	5.6212	4.0101	4.8073
	50	0.12	8.4261	6.2799	3.0959	4.6265

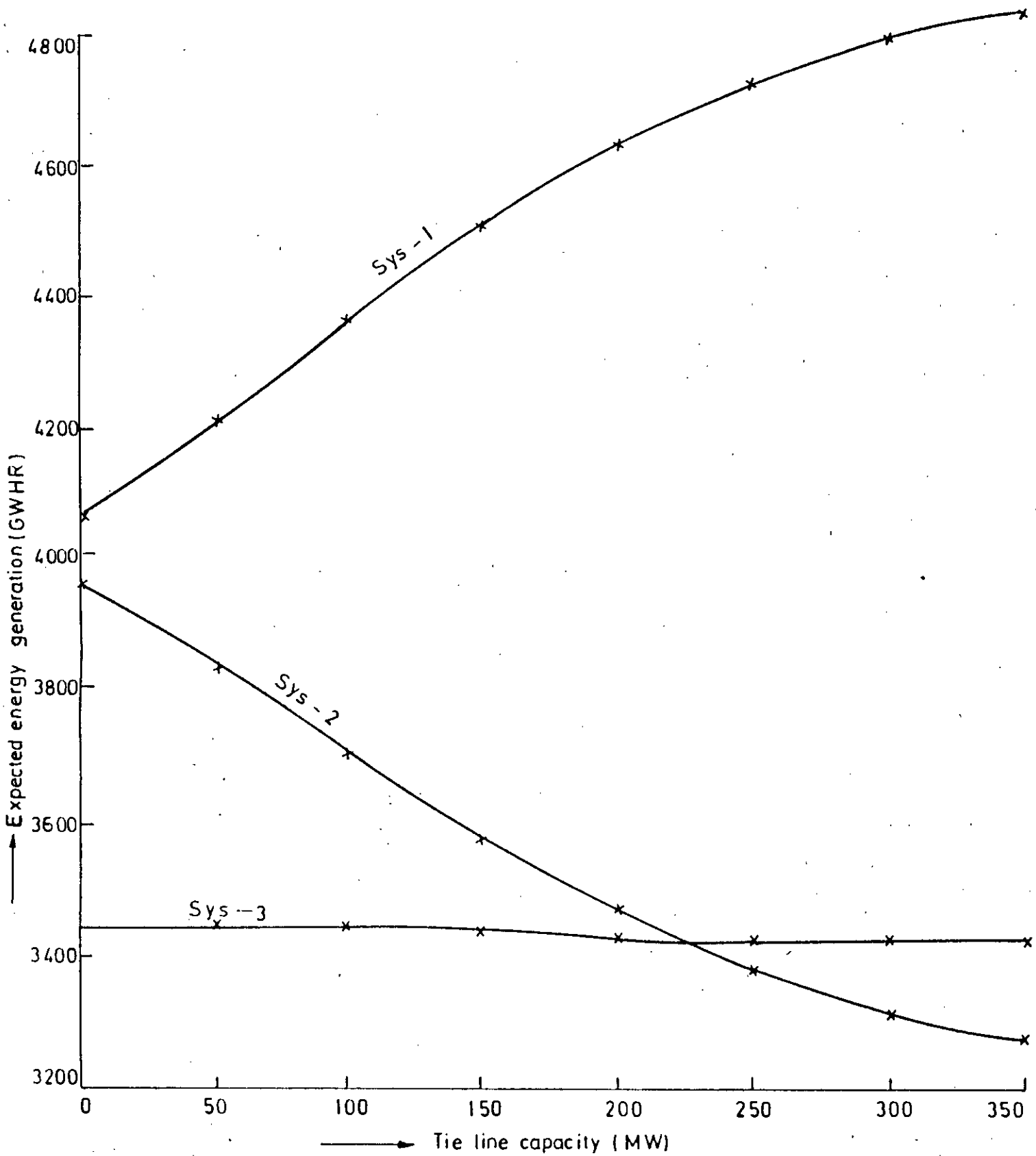


FIG. 6.1 EXPECTED ENERGY GENERATION OF INDIVIDUAL SYSTEM VS TIE LINE CAPACITY

Table 6.8: Expected energy generation and production cost of individual system.

Tie line Cap. (MW)	Individual expected energy gen. (GWhr)			Individual production cost (10000 \$)		
	Syst.1	Syst.2	Syst. 3	Syst.1	Syst.2	Syst. 3
0	4066.3882	3950.0848	3441.1512	3291.5073	6078.1385	4577.5142
50	4214.1388	3824.6990	3443.7406	3494.1471	5717.5343	4556.5006
100	4360.7047	3699.5670	3440.2873	3704.6235	5370.5682	4532.9408
150	4500.0663	3579.0606	3432.2857	3913.3058	5049.3177	4505.3522
200	4621.3885	3469.5287	3424.5846	4100.8731	4767.4863	4478.3664
250	4716.5064	3379.5493	3419.7607	4250.1943	4544.5265	4455.9954
300	4783.1240	3312.6575	3420.4388	4355.5886	4382.5288	4450.5150
350	4822.8095	3270.6936	3422.7396	4419.5345	4279.9185	4451.9556

Table 6.9: Global expected energy generation, production cost and savings.

Tie line Capacity (MW)	Global expected energy gen. (GWhr)	Global Production cost (10000 \$)	Global Savings (10000\$)	Global Unserved energy (GWhr)
0	11457.6241	13947.1600	0	61.5915
50	11482.5784	13768.1821	178.9779	36.6372
100	11500.5590	13608.1326	339.0274	18.6566
150	11511.4126	13467.9758	479.1842	7.8030
200	11515.5018	13346.7258	600.4342	3.7138
250	11515.8164	13250.7162	696.4438	3.3992
300	11516.2203	13188.6324	758.5276	2.9953
350	11516.2426	13151.4086	795.7514	2.973

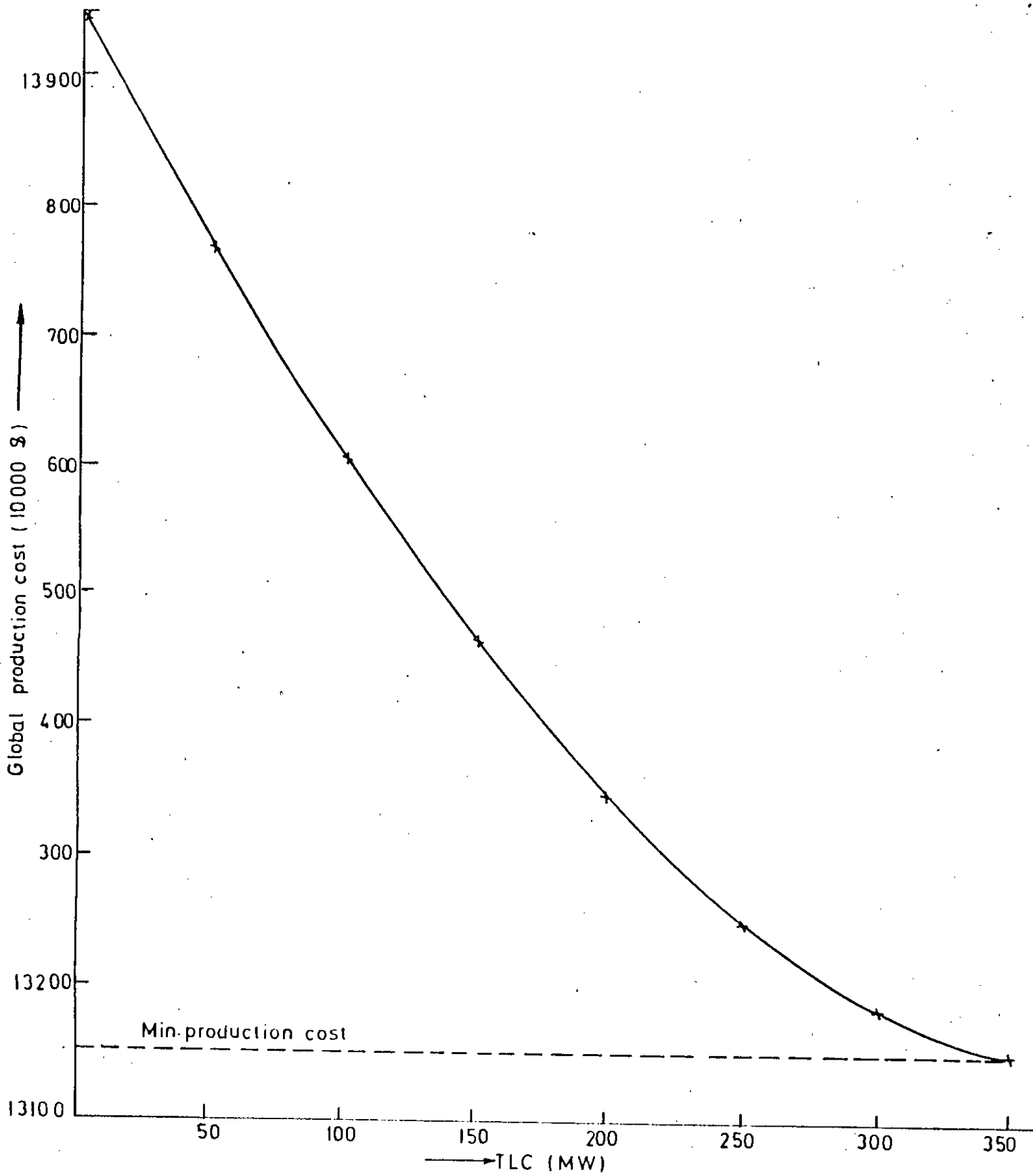


FIG 6 2 GLOBAL PRODUCTION COST VS. TIE LINE CAPACITY

Table 6.10: Expected global savings and production cost shared by each system for different tie line capacities.

Tie line cap. (MW)	Global savings (10000\$)	Individual Cost of gen. (10000 \$)			Individual cost actually incurred(10000 \$)		
		Syst.1	Syst.2	Syst.3	Syst.1	Syst.2	Syst.3
0	0	3291.5073	6078.1385	4577.5142	3291.5073	6078.1385	4577.5142
50	178.9779	3494.1471	5717.5343	4556.5006	3231.8480	6018.4792	4517.8549
100	339.0274	3704.6235	5370.5682	4532.9408	3178.4981	5965.1293	4464.5050
150	479.1842	3913.3058	5049.3177	4505.3522	3131.7792	5918.4104	4417.7861
200	600.4342	4100.8731	4767.4863	4478.3664	3091.3625	5877.9937	4377.3694
250	696.4438	4250.1943	4544.5265	4455.9954	3059.3593	5845.9905	4345.3662
300	758.5276	4355.5886	4382.5288	4450.5150	3038.6647	5825.2959	4324.6716
350	795.7514	4419.5345	4279.9185	4451.9556	3026.2568	5812.8880	4312.2637

Table 6.11: Expected energy generation and production cost of individual system considering priority for system 1.

Tie line Cap. (MW)	Individual expected energy gen. (GWHR)			Individual production cost (10000 \$)		
	Syst.1	Syst.2	Syst. 3	Syst.1	Syst.2	Syst. 3
0	4066.3882	3950.0848	3441.1512	3291.5073	6078.1385	4577.5142
50	4214.1389	3824.6991	3443.7406	3494.1473	5717.5348	4556.5008
100	4360.7049	3699.5670	3440.2874	3704.6239	5370.5685	4532.9411
150	4500.0664	3579.0608	3432.2857	3913.3060	5049.3182	4505.3524
200	4621.3885	3469.5288	3424.5846	4100.8730	4767.4867	4478.3665
250	4716.5063	3379.5496	3419.7605	4250.1939	4544.5271	4455.9952
300	4783.1238	3312.6577	3420.4387	4355.5881	4382.5294	4450.5148
350	4822.8094	3270.6935	3422.7395	4419.5344	4279.9184	4451.9555

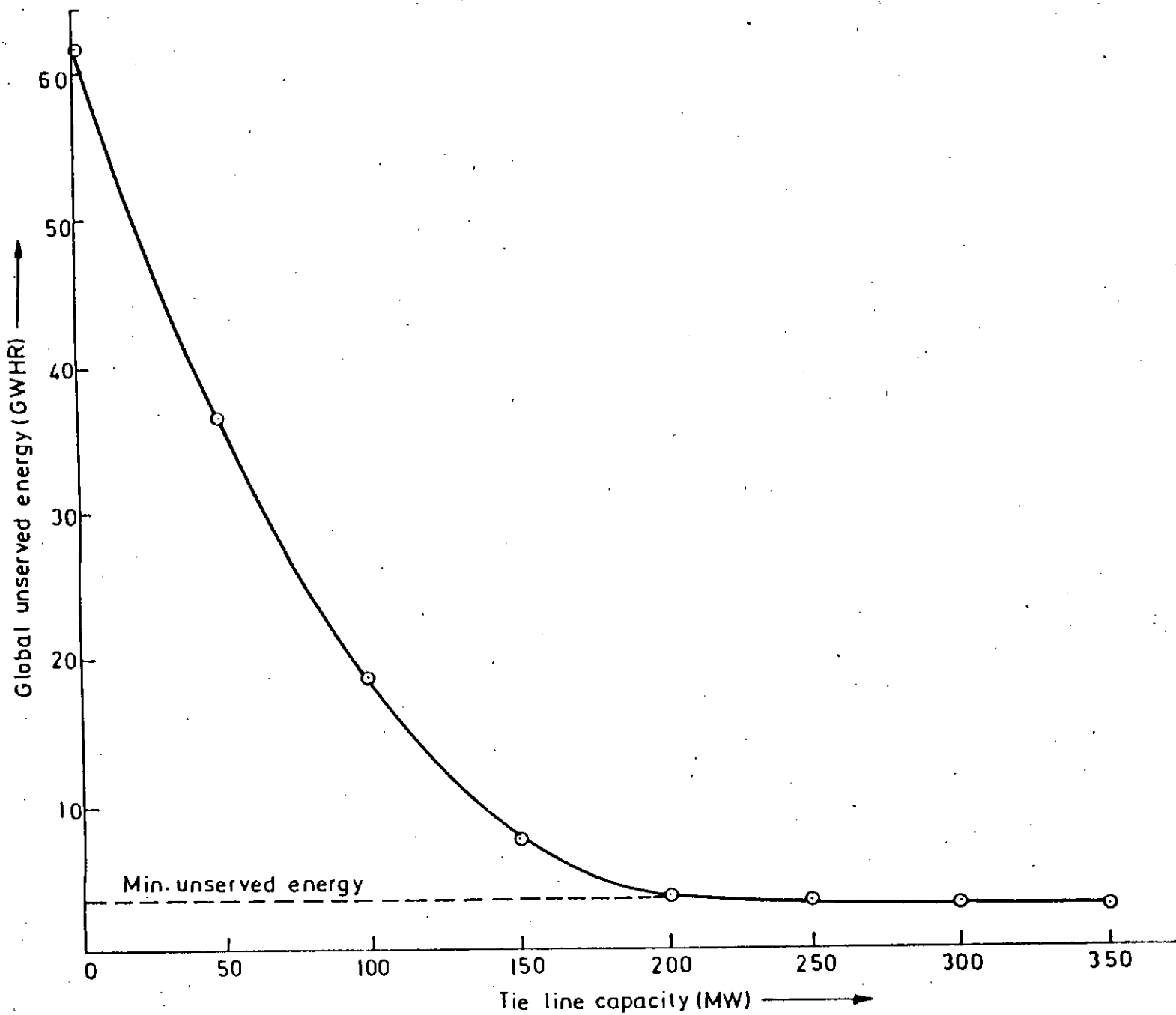


FIG. 6.3 GLOBAL UNSERVED DEMAND VS TIE LINE CAPACITY

Table 6.12: Global expected energy generation, production cost and savings considering priority for system 1.

Tie line Capacity (MW)	Global expected energy gen. (GWHR)	Global Production cost (10000 \$)	Global Savings (10000\$)	Global Unserved energy (GWHR)
0	11457.6241	13947.1600	0	61.5915
50	11482.5787	13768.1828	178.9772	36.6370
100	11500.5593	13608.1334	339.0266	18.6563
150	11511.4129	13467.9766	479.1834	7.8027
200	11515.5020	13346.7263	600.4337	3.7136
250	11515.8164	13250.7162	696.4438	3.3993
300	11516.2203	13188.6323	758.5277	2.9954
350	11516.2426	13151.4086	795.7514	2.9730

Table 6.13: Expected global savings and production cost shared by each system for different tie line capacities considering priority for system 1.

Tie line cap. (MW)	Global savings (10000\$)	Individual Cost of gen. (10000 \$)			Individual cost actually incurred(10000 \$)		
		Syst.1	Syst.2	Syst.3	Syst.1	Syst.2	Syst.3
0	0	3291.5073	6078.1385	4577.5142	3291.5073	6078.1385	4577.5142
50	178.9772	3494.1473	5717.5348	4556.5008	3231.8482	6018.4794	4517.8551
100	339.0266	3704.6239	5370.5685	4532.9411	3178.4984	5965.1296	4464.5053
150	479.1834	3913.3060	5049.3182	4505.3524	3131.7795	5918.4107	4417.7864
200	600.4337	4100.8730	4767.4867	4478.3665	3091.3627	5877.9939	4377.3696
250	696.4438	4250.1939	4544.5271	4455.9952	3039.3593	5845.9905	4345.3662
300	758.5277	4355.5881	4382.5294	4450.5148	3038.6647	5825.2959	4324.6716
350	795.7514	4419.5345	4279.9185	4451.9556	3026.2568	5812.8880	4312.2637

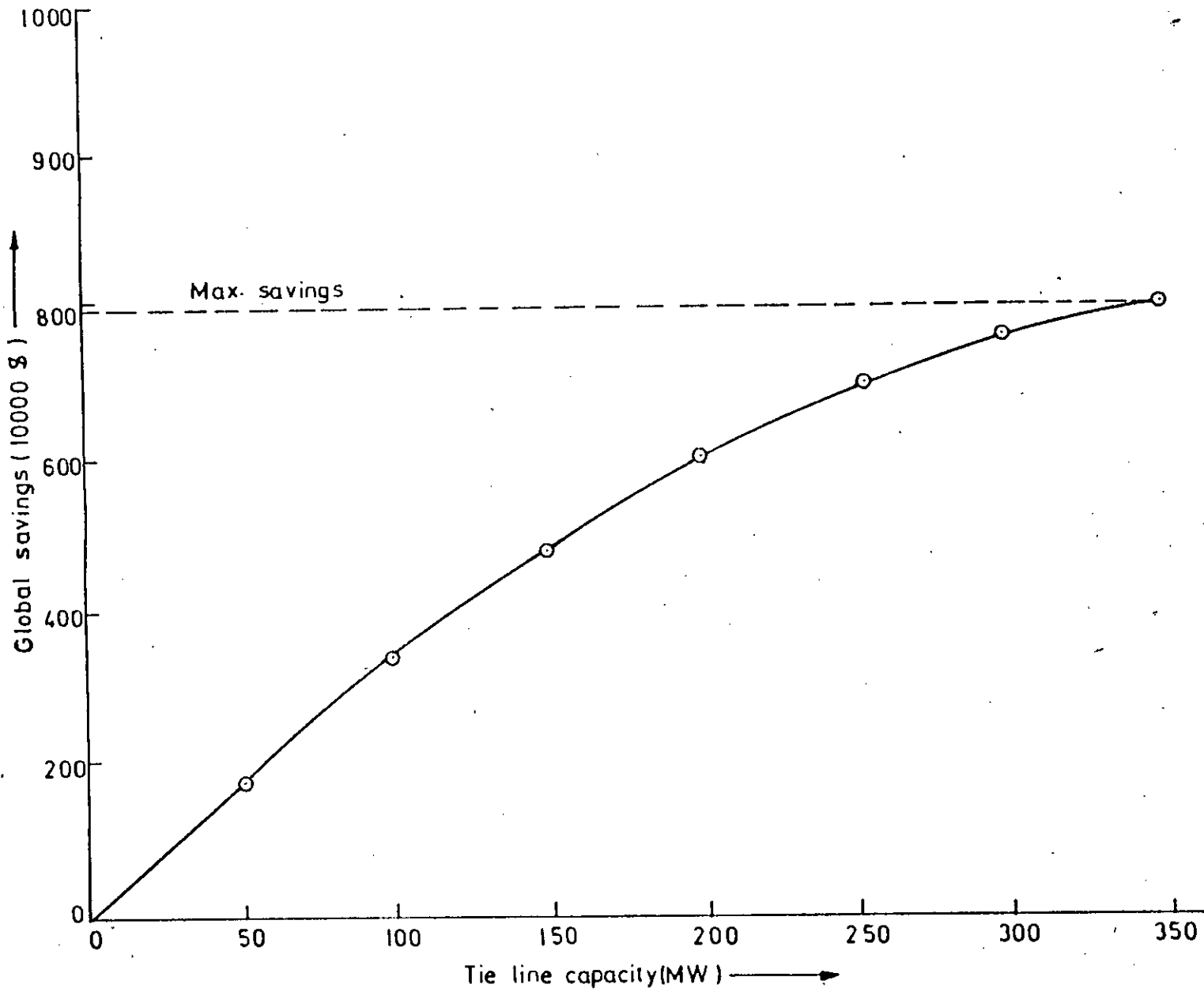


FIG. 6.4 GLOBAL SAVINGS VS. TIE LINE CAPACITY

CHAPTER 7

CHAPTER 7

OBSERVATION AND CONCLUSION

7.1 INTRODUCTION

This chapter presents the observations and discussions on the simulation results presented so far. This chapter also presents the conclusions of the thesis. The chapter is concluded with few comments and recommendations for further research.

7.2 OBSERVATIONS AND CONCLUSIONS

It is clearly observed from Tables 6.4 to 6.6 that the generating units of system 1 are of much lower incremental cost than those of System 2 and System 3. Therefore, it is expected that System 1 would export major part of the time. This is confirmed by Tables 6.7 and 6.8. It is observed from Table 6.7 that the expected energy generation of the generating units of System 1 increases while those of the units of System 2 decreases and those of System 3 sometimes increases and sometimes decreases with the increase of tie line capacity. The reason is for loading order which is based upon incremental cost, as the generating unit of one system are not loaded first, as a result the total expected energy generation of System 1 increases while those of System 2 decreases and System 3 remains almost constant. It is observed from Table 6.8 and from Fig. 6.1. The saturation effect is pronounced at 350 MW of tie line capacities, that is the expected energy generation by System 1, System 2 and System 3 become almost constant for tie line capacity above 350 MW. From Table 6.8 it is also clearly observed that production cost of System 1 increases and System 2 decreases while that of System 3 remain almost constant with the increases of tie line capacity. However

from Table 6.9 it is observed that global production cost decreases with the increase of tie line capacity and it is also observed from Fig. 6.2, above 350 MW of TLC global production cost becomes almost constant i.e saturation effect occur.

Table 6.9 also indicate that global expected savings increases with the increase of TLC and reaches the upper limit i.e maximum savings occur at or above 350 MW of TLC ,which is also observed from Fig. 6.4. Global unserved energy decreases with the increase of TLC and above 350 MW of TLC saturation occurs (Table 6.9 and Fig 6.3).

As mentioned earlier while the global production cost decreases with the increase of TLC the production cost of System 1 increases and that of System 2 decreases and that of System 3 remains almost constant. If the three systems are independent and there is no interconnection among them then the production cost shared by each system is equal to the individual production cost at zero tie line capacity .However, for interconnection the production cost shared by each system decreases with the increase of TLC (Table 6.10).At one time it will happen that system 1 have -ve values of production cost sharing which indicate that system 1 does not have to incur any expense for meeting its demand ; moreover, it earns some money by exporting energy to System 2 and System 3 Also for these cases, the production cost shared by System 2 and System 3 is less than the amount at zero MW tie line capacity.

The same observations are made by taking into consideration System 1 priority over System 2 and System 3. These are justified through Table 6.11 to Table 6.13 .

The accuracy of the developed methodology is shown in this thesis by solving a simple numerical example using this method and the heuristic approach. The methodology is also applied to a three-area

interconnected system. The results show the common trend.

7.3 RECOMENDATION FOR FUTHER RESEARCH

The following recomendation are made in continuation of this research.

- (i) The method may be applied to more than three interconnected systems.
- (ii) The load correlation may be varied in evaluating production cost.
- (iii) The transaction policies of different nature among the systems may be used .

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